

9.5.3

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Question:

Find the area of the region bounded by the curves $(x - 1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$.

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Solution:

Term	Description
$\mathbf{V}_1, \mathbf{u}_1$ and f_1	conic parameters of circle $(x - 1)^2 + y^2 = 1$
$\mathbf{V}_2, \mathbf{u}_2$ and f_2	conic parameters of circle $(x)^2 + y^2 = 1$

TABLE 1: Terms used

The conic parameters for the two circles can be expressed as

$$\begin{aligned}\mathbf{V}_1 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u}_1 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}, f_1 = 0, \\ \mathbf{V}_2 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u}_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f_2 = -1.\end{aligned}\tag{0.1}$$

On substituting from (0.1) in (??), we obtain

$$\begin{vmatrix} 1 + \mu & 0 & -1 \\ 0 & 1 + \mu & 0 \\ -1 & 0 & -\mu \end{vmatrix} = 0\tag{0.2}$$

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$$\mu = -1.\tag{0.3}$$

Substituting (0.1) in (??), we obtain

$$\mathbf{x}^\top \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} + 1 = 0\tag{0.4}$$

$$\implies \begin{pmatrix} -2 & 0 \end{pmatrix} \mathbf{x} = -1\tag{0.5}$$

Therefore the intersection of the two circles is a line with parameters

$$\mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \mathbf{h} = \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}.\tag{0.6}$$

The intersection parameters of the chord in (0.5) with the first circle in (0.1) is obtained from (??) as

$$\mu_i = \pm \frac{\sqrt{3}}{2} \quad (0.7)$$

Hence the point of intersection are obtained from (??) as

$$\mathbf{a}_0 = \left(\frac{1}{2}, \frac{\sqrt{3}}{2} \right), \mathbf{a}_2 = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2} \right). \quad (0.8)$$

The desired area of region is given as

$$\begin{aligned} & 2 \left(\int_0^{\frac{1}{2}} \sqrt{1 - (x-1)^2} dx + \int_{\frac{1}{2}}^1 \sqrt{1 - x^2} dx \right) \\ &= 2 \left[\frac{1}{2} (x-1) \sqrt{1 - (x-1)^2} + \frac{1}{2} \sin^{-1} (x-1) \right]_0^{\frac{1}{2}} \\ &+ 2 \left[\frac{x}{2} \sqrt{1 - x^2} + \frac{1}{2} \sin^{-1} x \right]_{\frac{1}{2}}^1 = \frac{2\pi}{3} - \frac{\sqrt{3}}{2} = 1.22837 \quad (0.9) \end{aligned}$$

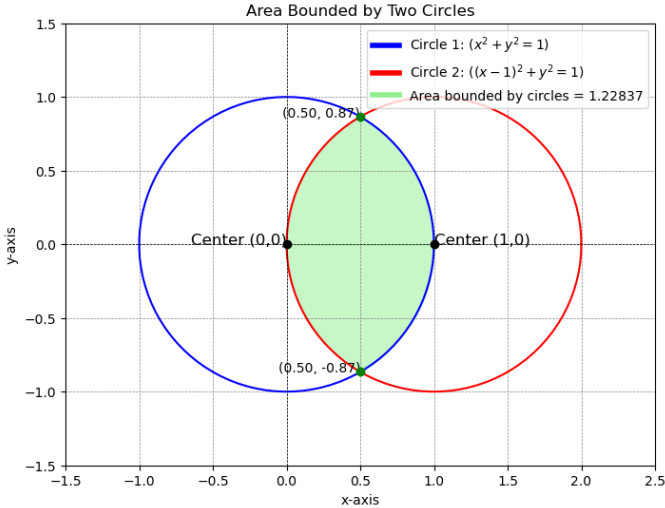


Fig. 0.1: Plot showing area bounded by the circles