1.2.22

AI24BTECH11024-Pappuri Prahladha

Question:
Rain is falling vertically at a speed of $35ms^{-1}$.Winds start blowing after sometime with a speed of $12ms^{-1}$ in east to west direction.In which direction should a boy waiting at a bus stop hold his umbrella ?
Solution:

Term	Description
$\hat{i}$ and $-\hat{i}$	Unit vectors along East and West directions
$\hat{j}$ and -$\hat{j}$	Unit vectors along North and south directions
$\mathbf{V_1}$ or Vector 1 and $\mathbf{V_2}$ or Vector 2	velocity vectors of Rain and Wind
$\mathbf{V_3}$ or Vector 3	Resultant velocity vector of Rain and Wind
θ	Required angle with the horizontal

TABLE 1: Terms used

The representation of a vector in matrices is as follows:

$$\mathbf{V_1} = x\hat{i} + y\hat{j} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{0.1}$$

Velocity vector of rain:

$$\mathbf{V_1} = -35\hat{j}$$

$$\mathbf{V_1} = \begin{pmatrix} 0 \\ -35 \end{pmatrix} \tag{0.2}$$

Velocity vector of Wind:

$$\mathbf{V_2} = -12\hat{i}$$

$$\mathbf{V_2} = \begin{pmatrix} -12 \\ 0 \end{pmatrix} \tag{0.3}$$

The trajectory of Rain Drops is along the resultant velocity vectors of Rain and Wind.

The addition of two vectors is as follows:

$$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a+c \\ b+d \end{pmatrix} \tag{0.4}$$

The resultant velocity vector:

$$\mathbf{V_1} + \mathbf{V_2} = \begin{pmatrix} 0 \\ -35 \end{pmatrix} + \begin{pmatrix} -12 \\ 0 \end{pmatrix} \tag{0.5}$$

$$\mathbf{V_1} + \mathbf{V_2} = \mathbf{V_3} = \begin{pmatrix} -12 \\ -35 \end{pmatrix} \tag{0.6}$$

Let the origin be O:

$$O = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.7}$$

The direction vector of AB is defined as:

$$\mathbf{M} = \mathbf{B} - \mathbf{A} = k \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{0.8}$$

Where m is the slope of AB.we can also say that

$$\mathbf{m} \equiv k \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{0.9}$$

So the direction vector of O and v_3 is:

$$\mathbf{D} = \mathbf{V_3} - \mathbf{O} = \begin{pmatrix} -12 \\ -35 \end{pmatrix} = -12 \begin{pmatrix} 1 \\ \frac{35}{12} \end{pmatrix} \tag{0.10}$$

From equation 0.9 slope of direction vector OV_3 is

$$Slope = \frac{35}{12} \tag{0.11}$$

The required angle(θ) made by umbrella with the horizontal is;

$$\theta = \tan^{-1}\left(\frac{35}{12}\right) = 71.565^\circ \tag{0.12}$$

$\therefore$ The boy hold the umbrella in the direction of 71.565° South of West

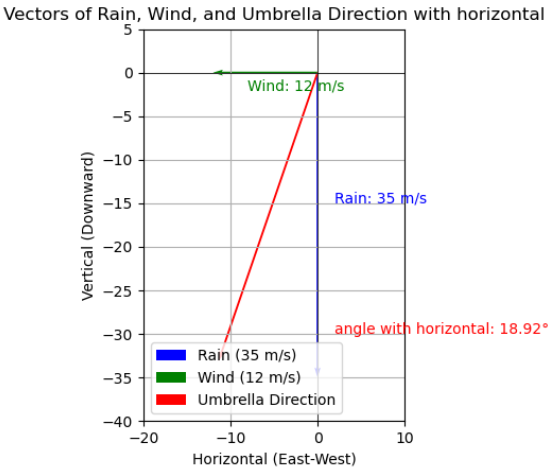

Fig. 0.1: Plot showing the velocity vectors