TAM 212. Midterm 1. Feb 21, 2013.

- There are 20 questions, each worth 5 points.
- You must not communicate with other students during this test.
- No electronic devices allowed.
- This is a 2 hour exam.
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- There are several different versions of this exam.

1. Fill in your information:

Full Name: _____

UIN (Student Number): _____

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2. Circle your discussion section:

	Monday	**Tuesday**	**Wednesday**	**Thursday**
8–9		ADI (260) Karthik		
9–10		ADC (260) Venanzio		ADK (260) Aaron
10–11		ADD (256) Aaron ADQ (344) Jan	ADS (252) Ray	ADT (243) Aaron ADU (344) Jan
11–12		ADE (252) Jan		ADL (256) Kumar
12–1	ADA (243) Ray ADP (135) Seung	ADF (335) Seung ADG (336) Kumar	ADJ (256) Ray ADR (252) Lin	ADN (260) Kumar
1–2				
2–3				
3–4				
4–5	ADV (252) Karthik		ADO (260) Mazhar ADW (252) Lin	
5–6	ADB (260) Mazhar	ADH (260) Karthik	ADM (243) Mazhar	

3. Fill in the following answers on the Scantron form:

94. A

95. D

96. C

1. (5 points) Points P and Q are moving in circular paths around the origin O with angular velocities ω_P and ω_Q and speeds v_P and v_Q, respectively.

The two particles are moving with the same angular velocity, so $\omega_P = \omega_Q$. Which statement is true?

(A) $2v_Q < v_P$

(B) $\frac{1}{2}v_Q < v_P \leq v_Q$

(C) $v_P \leq \frac{1}{2}v_Q$

(D) $v_Q < v_P \leq 2v_Q$

2. (5 points) A particle P has position, velocity, and acceleration vectors given by:

$$\vec{r} = -2\hat{\imath} + 3\hat{\jmath}\ \text{m}$$
$$\vec{v} = -4\hat{\imath} - \hat{\jmath}\ \text{m/s}$$
$$\vec{a} = 2\hat{\imath} + 6\hat{\jmath}\ \text{m/s}^2.$$

Consider the following statements:

(i) The particle is moving closer to the origin.

(ii) The particle is moving further from the origin.

(iii) The particle is speeding up.

(iv) The particle is slowing down.

Which statements are true?

(A) none of the other options

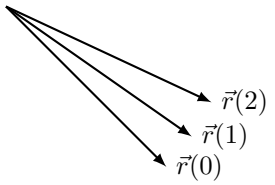
(B) (ii) and (iv)

(C) (ii) and (iii)

(D) (i) and (iii)

(E) (i) and (iv)

3. (5 points) The position vector $\vec{r}(t)$ of a point is shown below at $t = 0$ s, $t = 1$ s, and $t = 2$ s.



Which direction is the closest to the direction of the acceleration $\vec{a}(0)$ at time $t = 0$ s?

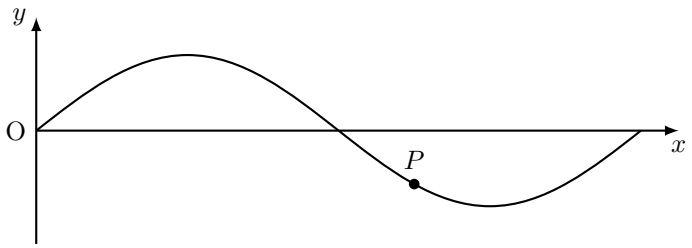
(A) ↖

(B) ↙

(C) ↗

(D) ↘

4. (5 points) A particle is moving to the left along a variable-height ground with ground height given by $y(x) = \sin(x/20)$ m. The particle's horizontal velocity component is a constant $v_x = -4$ m/s. What is the vertical component of velocity v_y when $x = 25\pi$ m?



(A) $v_y < -1$ m/s

(B) 0 m/s $\leq v_y < 1$ m/s

(C) $v_y = 0$ m/s

(D) -1 m/s $\leq v_y < 0$ m/s

(E) 1 m/s $\leq v_y$

5. (5 points) A car driving down the road travels a distance $s = \frac{3}{2}t^2$ m from its starting point. At $t = 1$ s the car is driving around a curve and the magnitude of its acceleration is $a = 5$ m/s^2. What is the radius of curvature ρ of the curve?

(A) $1 \text{ m} \leq \rho < 2 \text{ m}$

(B) $3 \text{ m} \leq \rho < 4 \text{ m}$

(C) $2 \text{ m} \leq \rho < 3 \text{ m}$

(D) $4 \text{ m} \leq \rho$

(E) $0 \text{ m} \leq \rho < 1 \text{ m}$

6. (5 points) The particle P has polar coordinates $r = 4$ m, $\theta = -135^\circ$ and velocity $\vec{v} = \hat{\jmath}$. Which statement is true?

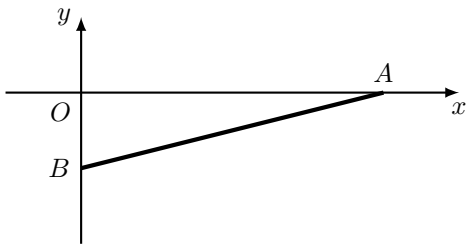
(A) $\dot{r} < 0$ and $\dot{\theta} \geq 0$

(B) $\dot{r} \geq 0$ and $\dot{\theta} \geq 0$

(C) $\dot{r} < 0$ and $\dot{\theta} < 0$

(D) $\dot{r} \geq 0$ and $\dot{\theta} < 0$

7. (5 points) A rod with fixed length is positioned as shown, with x the horizontal coordinate of end A and y the vertical coordinate of end B. End A can only move horizontally, while end B can only move vertically.



At an instant of time, we have $x = 4$ m and $y = -1$ m, and the rod is moving so that $\dot{y} = -2$ m/s. What is $\dot{x}$?

(A) $\dot{x} < -1$ m/s

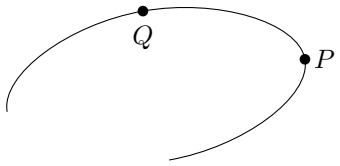
(B) -1 m/s $\leq \dot{x} < 0$ m/s

(C) 0 m/s $< \dot{x} < 1$ m/s

(D) 1 m/s $\leq \dot{x}$

(E) $\dot{x} = 0$ m/s

8. (5 points) A point is moving around the curve shown below with varying speed.



The radius of curvature and normal acceleration at P and Q are given by:

$$\rho_P = 2 \text{ m} \qquad \rho_Q = 4 \text{ m}$$
$$a_{P,n} = 4 \text{ m/s}^2 \qquad a_{Q,n} = 3 \text{ m/s}^2.$$

Which of the following is true about the velocities v_P at P and v_Q at Q?

(A) $v_P < v_Q$

(B) $v_P = v_Q$

(C) $v_P > v_Q$

9. (5 points) A point is currently at position $x = -2$ m, $y = 1$ m, $z = 4$ m and is rotating in the x–y plane about the origin with angular velocity $\vec{\omega} = -3\hat{k}$ rad/s. The velocity $\vec{v}$ of the point is:

(A) $\vec{v} = -3\hat{\imath} - 6\hat{\jmath}$ m/s

(B) $\vec{v} = 3\hat{\imath} - 6\hat{\jmath}$ m/s

(C) $\vec{v} = -3\hat{\imath} + 6\hat{\jmath}$ m/s

(D) $\vec{v} = 3\hat{\imath} + 6\hat{\jmath}$ m/s

10. (5 points) A particle moves so that its position in polar coordinates is given by

$$r = (3 + \sin t) \text{ m}$$
$$\theta = t \text{ rad.}$$

What is the $\hat{\jmath}$ component of velocity v_y at $t = \pi$ s?

(A) $0 \text{ m/s} < v_y < 2 \text{ m/s}$

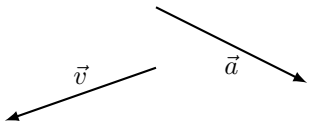
(B) $-2 \text{ m/s} \leq v_y < 0 \text{ m/s}$

(C) $v_y < -2 \text{ m/s}$

(D) $2 \text{ m/s} \leq v_y$

(E) $v_y = 0 \text{ m/s}$

11. (5 points) The velocity $\vec{v}$ and acceleration $\vec{a}$ for a single particle P are shown below at a particular instant.



Which statement is true at this instant?

(A) the particle is slowing down

(B) the particle's speed is not changing

(C) the particle is speeding up

12. (5 points) A particle starts at the origin at time $t = 0$ s and its velocity is given by

$$\vec{v} = 4t^3\,\hat{\imath} - t^2\,\hat{\jmath}\ \text{m/s}.$$

At time $t = 1$ s, what is the particle's distance r from the origin?

(A) $2\ \text{m} \leq r < 4\ \text{m}$

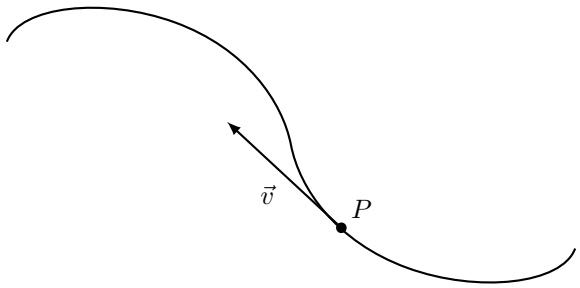
(B) $6\ \text{m} \leq r < 8\ \text{m}$

(C) $4\ \text{m} \leq r < 6\ \text{m}$

(D) $0\ \text{m} \leq r < 2\ \text{m}$

(E) $8\ \text{m} \leq r$

13. (5 points) A car is driving on a curved track with the top view shown below. At a given instant the car is at point P with velocity $\vec{v}$ and its speed is decreasing, such that the tangential and normal components of its acceleration are equal in magnitude.



Which direction is the closest to the direction of the acceleration $\vec{a}$ at the instant shown?

(A) $\rightarrow$

(B) $\downarrow$

(C) $\leftarrow$

(D) $\uparrow$

14. (5 points) A car is observed moving in the plane with velocity $\vec{v} = -2\hat{\imath} + 4\hat{\jmath}$ m/s and acceleration $\vec{a} = -\hat{\imath} - 3\hat{\jmath}$ m/s^2. At this instant, is it:

(A) stationary

(B) driving around a curve clockwise

(C) driving in a straight line

(D) driving around a curve counterclockwise

15. (5 points) If a particle has position vector $\vec{r}(t) = 2\cos(2t)\,\hat{\imath} + \sin(3t)\,\hat{\jmath} + (t^3 - 2t)\,\hat{k}$ m, what is its speed $v(0)$ at time $t = 0$ s?

(A) $v(0) = 0$ m/s

(B) $1 \text{ m/s} \leq v(0) < 3 \text{ m/s}$

(C) $0 \text{ m/s} < v(0) < 1 \text{ m/s}$

(D) $3 \text{ m/s} \leq v(0) < 5 \text{ m/s}$

(E) $5 \text{ m/s} \leq v(0)$

16. (5 points) A point is moving with position vector

$$\vec{r} = (t^2 - 2t)\,\hat{\imath} + (e^{2t} - 2t)\,\hat{\jmath}\ \text{m}.$$

What is the radius of curvature ρ at $t = 0$ s?

(A) $\frac{1}{2}\ \text{m} \leq \rho < 1\ \text{m}$

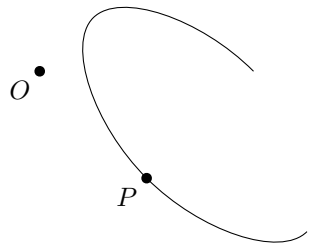
(B) $\frac{3}{2}\ \text{m} \leq \rho < 2\ \text{m}$

(C) $2\ \text{m} \leq \rho$

(D) $1\ \text{m} \leq \rho < \frac{3}{2}\ \text{m}$

(E) $0\ \text{m} \leq \rho < \frac{1}{2}\ \text{m}$

17. (5 points) A point is moving around the curve shown and is currently at position P. Consider a polar basis $\hat{e}_r, \hat{e}_\theta$ at P from the origin O and a tangential/normal basis $\hat{e}_t, \hat{e}_n$ at P.



Which of the following is true?

(A) $\hat{e}_n = -\hat{e}_r$

(B) $\hat{e}_n = -\hat{e}_\theta$

(C) $\hat{e}_n = \hat{e}_r$

(D) $\hat{e}_n = \hat{e}_\theta$

18. (5 points) A position P has an associated polar basis with

$$\hat{e}_\theta = -\frac{1}{2}\hat{\imath} - \frac{\sqrt{3}}{2}\hat{\jmath}.$$

What is θ?

(A) $0 \leq \theta < \frac{1}{2}\pi$

(B) $\frac{3}{2}\pi \leq \theta < 2\pi$

(C) $\pi \leq \theta < \frac{3}{2}\pi$

(D) $\frac{1}{2}\pi \leq \theta < \pi$

19. (5 points) A particle is moving in the plane so that at a particular instant its polar coordinates have $\dot{r} = 3$ m/s and $\dot{\theta} = -4$ rad/s. The speed is $v = 5$ m/s. What is r?

(A) $0 \text{ m} \leq r < 1 \text{ m}$

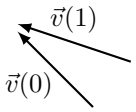
(B) $3 \text{ m} \leq r < 4 \text{ m}$

(C) $1 \text{ m} \leq r < 2 \text{ m}$

(D) $4 \text{ m} \leq r$

(E) $2 \text{ m} \leq r < 3 \text{ m}$

20. (5 points) The velocity $\vec{v}(t)$ of a point is shown below at $t = 0$ s and $t = 1$ s.



Which direction is the closest to the direction of the acceleration $\vec{a}(0)$ at time $t = 0$ s?

(A) ↖

(B) ↙

(C) ↘

(D) ↗

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5–6	ADB (260) Mazhar	ADH (260) Karthik	ADM (243) Mazhar	

3. Fill in the following answers on the Scantron form:

94. B

95. E

96. D

1. (5 points) A particle is moving in the plane so that at a particular instant its polar coordinates have $\dot{r} = 3$ m/s and $\dot{\theta} = -1$ rad/s. The speed is $v = 5$ m/s. What is r?

(A) $3 \text{ m} \leq r < 4 \text{ m}$

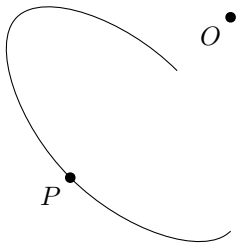
(B) $1 \text{ m} \leq r < 2 \text{ m}$

(C) $2 \text{ m} \leq r < 3 \text{ m}$

(D) $4 \text{ m} \leq r$

(E) $0 \text{ m} \leq r < 1 \text{ m}$

2. (5 points) A point is moving around the curve shown and is currently at position P. Consider a polar basis $\hat{e}_r, \hat{e}_\theta$ at P from the origin O and a tangential/normal basis $\hat{e}_t, \hat{e}_n$ at P.



Which of the following is true?

(A) $\hat{e}_n = \hat{e}_\theta$

(B) $\hat{e}_n = \hat{e}_r$

(C) $\hat{e}_n = -\hat{e}_\theta$

(D) $\hat{e}_n = -\hat{e}_r$

3. (5 points) A particle P has position, velocity, and acceleration vectors given by:

$$\vec{r} = \hat{\imath} - 3\hat{\jmath} \text{ m}$$
$$\vec{v} = -2\hat{\imath} - 5\hat{\jmath} \text{ m/s}$$
$$\vec{a} = 2\hat{\imath} - 2\hat{\jmath} \text{ m/s}^2.$$

Consider the following statements:

(i) The particle is moving closer to the origin.

(ii) The particle is moving further from the origin.

(iii) The particle is speeding up.

(iv) The particle is slowing down.

Which statements are true?

(A) (ii) and (iii)

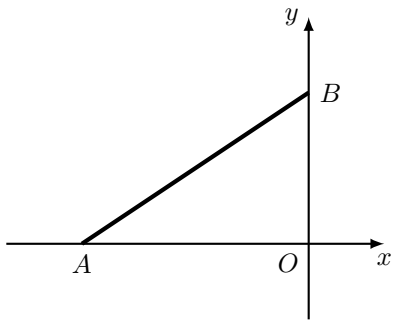
(B) (i) and (iii)

(C) (ii) and (iv)

(D) none of the other options

(E) (i) and (iv)

4. (5 points) A rod with fixed length is positioned as shown, with x the horizontal coordinate of end A and y the vertical coordinate of end B. End A can only move horizontally, while end B can only move vertically.



At an instant of time, we have $x = -3$ m and $y = 2$ m, and the rod is moving so that $\dot{y} = 2$ m/s. What is $\dot{x}$?

(A) $1 \text{ m/s} \leq \dot{x}$

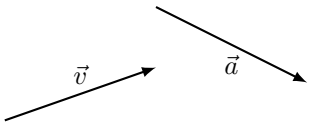
(B) $\dot{x} = 0$ m/s

(C) $-1 \text{ m/s} \leq \dot{x} < 0$ m/s

(D) $\dot{x} < -1$ m/s

(E) $0 \text{ m/s} < \dot{x} < 1$ m/s

5. (5 points) The velocity $\vec{v}$ and acceleration $\vec{a}$ for a single particle P are shown below at a particular instant.



Which statement is true at this instant?

(A) the particle's speed is not changing

(B) the particle is slowing down

(C) the particle is speeding up

6. (5 points) A particle starts at the origin at time $t = 0$ s and its velocity is given by

$$\vec{v} = -t\,\hat{\imath} + 3t^2\,\hat{\jmath}\ \text{m/s}.$$

At time $t = 2$ s, what is the particle's distance r from the origin?

(A) $6\ \text{m} \leq r < 8\ \text{m}$

(B) $8\ \text{m} \leq r$

(C) $4\ \text{m} \leq r < 6\ \text{m}$

(D) $2\ \text{m} \leq r < 4\ \text{m}$

(E) $0\ \text{m} \leq r < 2\ \text{m}$

7. (5 points) A particle moves so that its position in polar coordinates is given by

$$r = (3 + \cos t)\ \text{m}$$
$$\theta = t\ \text{rad}.$$

What is the $\hat{\jmath}$ component of velocity v_y at $t = \pi$ s?

(A) $v_y < -2\ \text{m/s}$

(B) $v_y = 0\ \text{m/s}$

(C) $2\ \text{m/s} \leq v_y$

(D) $0\ \text{m/s} < v_y < 2\ \text{m/s}$

(E) $-2\ \text{m/s} \leq v_y < 0\ \text{m/s}$

8. (5 points) A point is currently at position $x = -2$ m, $y = -1$ m, $z = 4$ m and is rotating in the x–y plane about the origin with angular velocity $\vec{\omega} = -3\hat{k}$ rad/s. The velocity $\vec{v}$ of the point is:

(A) $\vec{v} = 3\hat{\imath} + 6\hat{\jmath}$ m/s

(B) $\vec{v} = -3\hat{\imath} + 6\hat{\jmath}$ m/s

(C) $\vec{v} = -3\hat{\imath} - 6\hat{\jmath}$ m/s

(D) $\vec{v} = 3\hat{\imath} - 6\hat{\jmath}$ m/s

9. (5 points) A point is moving with position vector

$$\vec{r} = (t^2 - 6t)\,\hat{\imath} + (e^{3t} - 3t)\,\hat{\jmath} \text{ m}.$$

What is the radius of curvature ρ at $t = 0$ s?

(A) $1 \text{ m} \leq \rho < \frac{3}{2}$ m

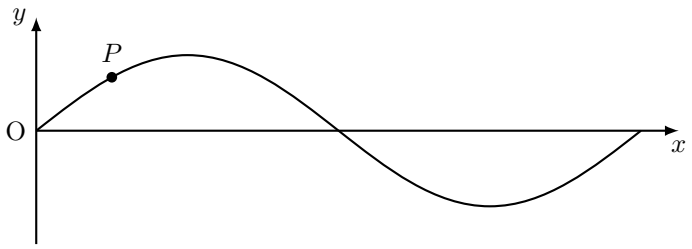
(B) $\frac{3}{2} \text{ m} \leq \rho < 2$ m

(C) $2 \text{ m} \leq \rho$

(D) $0 \text{ m} \leq \rho < \frac{1}{2}$ m

(E) $\frac{1}{2} \text{ m} \leq \rho < 1$ m

10. (5 points) A particle is moving to the left along a variable-height ground with ground height given by $y(x) = \sin(x/20)$ m. The particle's horizontal velocity component is a constant $v_x = -4$ m/s. What is the vertical component of velocity v_y when $x = 5\pi$ m?



(A) $v_y = 0$ m/s

(B) -1 m/s $\leq v_y < 0$ m/s

(C) 0 m/s $\leq v_y < 1$ m/s

(D) $v_y < -1$ m/s

(E) 1 m/s $\leq v_y$

11. (5 points) A position P has an associated polar basis with

$$\hat{e}_\theta = \frac{1}{2}\hat{\imath} + \frac{\sqrt{3}}{2}\hat{\jmath}.$$

What is θ?

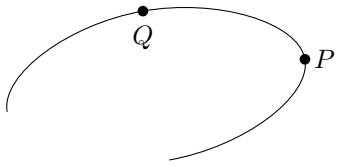
(A) $\pi \leq \theta < \frac{3}{2}\pi$

(B) $0 \leq \theta < \frac{1}{2}\pi$

(C) $\frac{1}{2}\pi \leq \theta < \pi$

(D) $\frac{3}{2}\pi \leq \theta < 2\pi$

12. (5 points) A point is moving around the curve shown below with varying speed.



The radius of curvature and normal acceleration at P and Q are given by:

$$\rho_P = 2 \text{ m} \qquad \rho_Q = 4 \text{ m}$$
$$a_{P,n} = 4 \text{ m/s}^2 \qquad a_{Q,n} = 2 \text{ m/s}^2.$$

Which of the following is true about the velocities v_P at P and v_Q at Q?

(A) $v_P = v_Q$

(B) $v_P < v_Q$

(C) $v_P > v_Q$

13. (5 points) If a particle has position vector $\vec{r}(t) = (2t - t^2)\,\hat{\imath} - 3\cos(t)\,\hat{\jmath} - \sin(4t)\,\hat{k}$ m, what is its speed $v(0)$ at time $t = 0$ s?

(A) $v(0) = 0$ m/s

(B) $6 \text{ m/s} \leq v(0)$

(C) $4 \text{ m/s} \leq v(0) < 6 \text{ m/s}$

(D) $0 \text{ m/s} < v(0) < 2 \text{ m/s}$

(E) $2 \text{ m/s} \leq v(0) < 4 \text{ m/s}$

14. (5 points) A car is observed moving in the plane with velocity $\vec{v} = 2\hat{\imath} - 4\hat{\jmath}$ m/s and acceleration $\vec{a} = -\hat{\imath} - 3\hat{\jmath}$ m/s^2. At this instant, is it:

(A) stationary

(B) driving around a curve clockwise

(C) driving around a curve counterclockwise

(D) driving in a straight line

15. (5 points) The particle P has polar coordinates $r = 4$ m, $\theta = -45°$ and velocity $\vec{v} = -\hat{\jmath}$. Which statement is true?

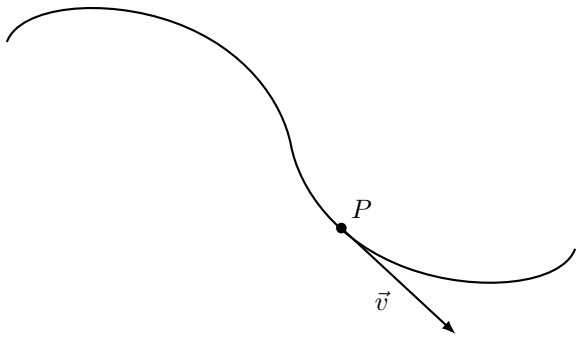
(A) $\dot{r} \geq 0$ and $\dot{\theta} \geq 0$

(B) $\dot{r} < 0$ and $\dot{\theta} < 0$

(C) $\dot{r} < 0$ and $\dot{\theta} \geq 0$

(D) $\dot{r} \geq 0$ and $\dot{\theta} < 0$

16. (5 points) A car is driving on a curved track with the top view shown below. At a given instant the car is at point P with velocity $\vec{v}$ and its speed is decreasing, such that the tangential and normal components of its acceleration are equal in magnitude.



Which direction is the closest to the direction of the acceleration $\vec{a}$ at the instant shown?

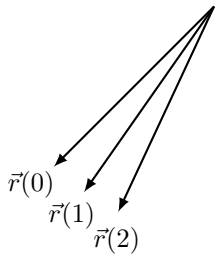
(A) $\rightarrow$

(B) $\uparrow$

(C) $\downarrow$

(D) $\leftarrow$

17. (5 points) The position vector $\vec{r}(t)$ of a point is shown below at $t = 0$ s, $t = 1$ s, and $t = 2$ s.



Which direction is the closest to the direction of the acceleration $\vec{a}(0)$ at time $t = 0$ s?

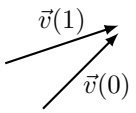
(A) ↗

(B) ↖

(C) ↘

(D) ↙

18. (5 points) The velocity $\vec{v}(t)$ of a point is shown below at $t = 0$ s and $t = 1$ s.



Which direction is the closest to the direction of the acceleration $\vec{a}(0)$ at time $t = 0$ s?

(A) ↘

(B) ↙

(C) ↖

(D) ↗

19. (5 points) A car driving down the road travels a distance $s = (\frac{1}{2}t^3 + \frac{1}{2}t)$ m from its starting point. At $t = 1$ s the car is driving around a curve and the magnitude of its acceleration is $a = 5$ m/s^2. What is the radius of curvature ρ of the curve?

(A) $0 \text{ m} \leq \rho < 1 \text{ m}$

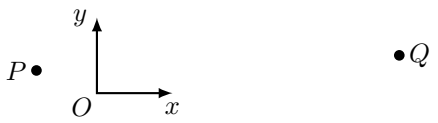
(B) $1 \text{ m} \leq \rho < 2 \text{ m}$

(C) $3 \text{ m} \leq \rho < 4 \text{ m}$

(D) $2 \text{ m} \leq \rho < 3 \text{ m}$

(E) $4 \text{ m} \leq \rho$

20. (5 points) Points P and Q are moving in circular paths around the origin O with angular velocities ω_P and ω_Q and speeds v_P and v_Q, respectively.



The two particles are moving with the same angular velocity, so $\omega_P = \omega_Q$. Which statement is true?

(A) $2v_Q < v_P$

(B) $v_P \leq \frac{1}{2}v_Q$

(C) $\frac{1}{2}v_Q < v_P \leq v_Q$

(D) $v_Q < v_P \leq 2v_Q$

TAM 212. Midterm 1. Feb 21, 2013.

- There are 20 questions, each worth 5 points.
- You must not communicate with other students during this test.
- No electronic devices allowed.
- This is a 2 hour exam.
- Do not turn this page until instructed to do so.
- There are several different versions of this exam.

1. Fill in your information:

Full Name: _____

UIN (Student Number): _____

NetID: _____

2. Circle your discussion section:

	Monday	**Tuesday**	**Wednesday**	**Thursday**
8–9		ADI (260) Karthik		
9–10		ADC (260) Venanzio		ADK (260) Aaron
10–11		ADD (256) Aaron ADQ (344) Jan	ADS (252) Ray	ADT (243) Aaron ADU (344) Jan
11–12		ADE (252) Jan		ADL (256) Kumar
12–1	ADA (243) Ray ADP (135) Seung	ADF (335) Seung ADG (336) Kumar	ADJ (256) Ray ADR (252) Lin	ADN (260) Kumar
1–2				
2–3				
3–4				
4–5	ADV (252) Karthik		ADO (260) Mazhar ADW (252) Lin	
5–6	ADB (260) Mazhar	ADH (260) Karthik	ADM (243) Mazhar	

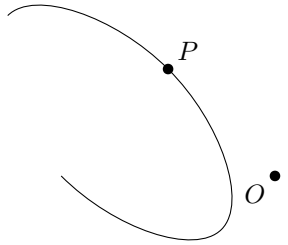
3. Fill in the following answers on the Scantron form:

94. C

95. A

96. E

1. (5 points) A point is moving around the curve shown and is currently at position P. Consider a polar basis $\hat{e}_r, \hat{e}_\theta$ at P from the origin O and a tangential/normal basis $\hat{e}_t, \hat{e}_n$ at P.



Which of the following is true?

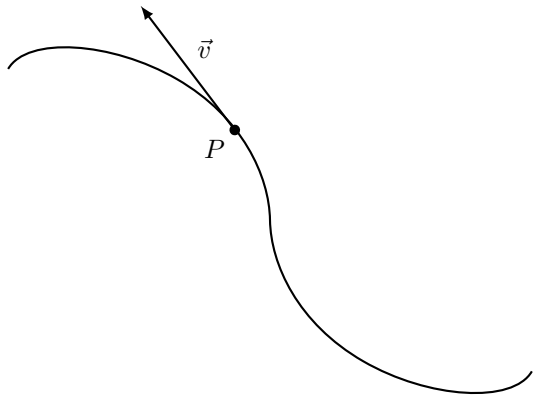
(A) $\hat{e}_n = \hat{e}_\theta$

(B) $\hat{e}_n = -\hat{e}_r$

(C) $\hat{e}_n = \hat{e}_r$

(D) $\hat{e}_n = -\hat{e}_\theta$

2. (5 points) A car is driving on a curved track with the top view shown below. At a given instant the car is at point P with velocity $\vec{v}$ and its speed is decreasing, such that the tangential and normal components of its acceleration are equal in magnitude.



Which direction is the closest to the direction of the acceleration $\vec{a}$ at the instant shown?

(A) $\uparrow$

(B) $\leftarrow$

(C) $\downarrow$

(D) $\rightarrow$

3. (5 points) A car driving down the road travels a distance $s = 2t^2$ m from its starting point. At $t = 1$ s the car is driving around a curve and the magnitude of its acceleration is $a = 5$ m/s^2. What is the radius of curvature ρ of the curve?

(A) $3 \text{ m} \leq \rho < 4 \text{ m}$

(B) $0 \text{ m} \leq \rho < 1 \text{ m}$

(C) $4 \text{ m} \leq \rho$

(D) $1 \text{ m} \leq \rho < 2 \text{ m}$

(E) $2 \text{ m} \leq \rho < 3 \text{ m}$

4. (5 points) If a particle has position vector $\vec{r}(t) = -\sin(2t)\,\hat{\imath} + (t + t^3)\,\hat{\jmath} + 3\cos(2t)\,\hat{k}$ m, what is its speed $v(0)$ at time $t = 0$ s?

(A) $1 \text{ m/s} \leq v(0) < 2 \text{ m/s}$

(B) $0 \text{ m/s} < v(0) < 1 \text{ m/s}$

(C) $2 \text{ m/s} \leq v(0) < 3 \text{ m/s}$

(D) $3 \text{ m/s} \leq v(0)$

(E) $v(0) = 0 \text{ m/s}$

5. (5 points) A particle is moving in the plane so that at a particular instant its polar coordinates have $\dot{r} = 3$ m/s and $\dot{\theta} = -2$ rad/s. The speed is $v = 5$ m/s. What is r?

(A) $1 \text{ m} \leq r < 2 \text{ m}$

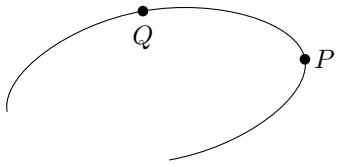
(B) $3 \text{ m} \leq r < 4 \text{ m}$

(C) $4 \text{ m} \leq r$

(D) $2 \text{ m} \leq r < 3 \text{ m}$

(E) $0 \text{ m} \leq r < 1 \text{ m}$

6. (5 points) A point is moving around the curve shown below with varying speed.



The radius of curvature and normal acceleration at P and Q are given by:

$$\rho_P = 2 \text{ m} \qquad \rho_Q = 4 \text{ m}$$
$$a_{P,n} = 6 \text{ m/s}^2 \qquad a_{Q,n} = 2 \text{ m/s}^2.$$

Which of the following is true about the velocities v_P at P and v_Q at Q?

(A) $v_P < v_Q$

(B) $v_P > v_Q$

(C) $v_P = v_Q$

7. (5 points) A particle P has vector, velocity, and acceleration vectors given by:

$$\vec{r} = -3\hat{\imath} - 2\hat{\jmath}\ \text{m}$$
$$\vec{v} = \hat{\imath} + 4\hat{\jmath}\ \text{m/s}$$
$$\vec{a} = -9\hat{\imath} + 2\hat{\jmath}\ \text{m/s}^2.$$

Consider the following statements:

(i) The particle is moving closer to the origin.

(ii) The particle is moving further from the origin.

(iii) The particle is speeding up.

(iv) The particle is slowing down.

Which statements are true?

(A) (ii) and (iii)

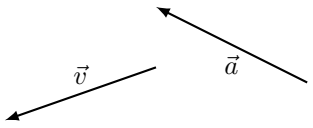
(B) (ii) and (iv)

(C) none of the other options

(D) (i) and (iv)

(E) (i) and (iii)

8. (5 points) The velocity $\vec{v}$ and acceleration $\vec{a}$ for a single particle P are shown below at a particular instant.



Which statement is true at this instant?

(A) the particle's speed is not changing

(B) the particle is speeding up

(C) the particle is slowing down

9. (5 points) The particle P has polar coordinates $r = 4$ m, $\theta = -135^\circ$ and velocity $\vec{v} = -\hat{\jmath}$. Which statement is true?

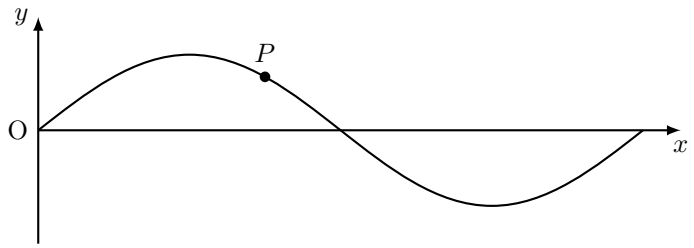
(A) $\dot{r} \geq 0$ and $\dot{\theta} \geq 0$

(B) $\dot{r} < 0$ and $\dot{\theta} \geq 0$

(C) $\dot{r} < 0$ and $\dot{\theta} < 0$

(D) $\dot{r} \geq 0$ and $\dot{\theta} < 0$

10. (5 points) A particle is moving to the left along a variable-height ground with ground height given by $y(x) = \sin(x/20)$ m. The particle's horizontal velocity component is a constant $v_x = -4$ m/s. What is the vertical component of velocity v_y when $x = 15\pi$ m?



(A) $0 \text{ m/s} \leq v_y < 1 \text{ m/s}$

(B) $v_y < -1 \text{ m/s}$

(C) $-1 \text{ m/s} \leq v_y < 0 \text{ m/s}$

(D) $v_y = 0 \text{ m/s}$

(E) $1 \text{ m/s} \leq v_y$

11. (5 points) A particle moves so that its position in polar coordinates is given by

$$r = (3 + \cos t) \text{ m}$$
$$\theta = -t \text{ rad.}$$

What is the $\hat{\jmath}$ component of velocity v_y at $t = \pi$ s?

(A) $2 \text{ m/s} \leq v_y$

(B) $v_y < -2 \text{ m/s}$

(C) $v_y = 0 \text{ m/s}$

(D) $-2 \text{ m/s} \leq v_y < 0 \text{ m/s}$

(E) $0 \text{ m/s} < v_y < 2 \text{ m/s}$

12. (5 points) A point is currently at position $x = 2$ m, $y = -1$ m, $z = 4$ m and is rotating in the x–y plane about the origin with angular velocity $\vec{\omega} = -3\hat{k}$ rad/s. The velocity $\vec{v}$ of the point is:

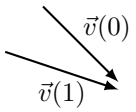
(A) $\vec{v} = 3\hat{\imath} - 6\hat{\jmath}$ m/s

(B) $\vec{v} = 3\hat{\imath} + 6\hat{\jmath}$ m/s

(C) $\vec{v} = -3\hat{\imath} - 6\hat{\jmath}$ m/s

(D) $\vec{v} = -3\hat{\imath} + 6\hat{\jmath}$ m/s

13. (5 points) The velocity $\vec{v}(t)$ of a point is shown below at $t = 0$ s and $t = 1$ s.



Which direction is the closest to the direction of the acceleration $\vec{a}(0)$ at time $t = 0$ s?

(A) $\nwarrow$

(B) $\searrow$

(C) $\nearrow$

(D) $\swarrow$

14. (5 points) A point is moving with position vector

$$\vec{r} = (t^2 - 2t)\,\hat{\imath} + (8t - 8e^t)\,\hat{\jmath}\ \text{m}.$$

What is the radius of curvature ρ at $t = 0$ s?

(A) $\frac{1}{2}\ \text{m} \leq \rho < 1\ \text{m}$

(B) $1\ \text{m} \leq \rho < \frac{3}{2}\ \text{m}$

(C) $0\ \text{m} \leq \rho < \frac{1}{2}\ \text{m}$

(D) $\frac{3}{2}\ \text{m} \leq \rho < 2\ \text{m}$

(E) $2\ \text{m} \leq \rho$

15. (5 points) A car is observed moving in the plane with velocity $\vec{v} = -2\hat{\imath} + 4\hat{\jmath}$ m/s and acceleration $\vec{a} = \hat{\imath} + 3\hat{\jmath}$ m/s^2. At this instant, is it:

(A) driving in a straight line

(B) driving around a curve counterclockwise

(C) driving around a curve clockwise

(D) stationary

16. (5 points) A position P has an associated polar basis with

$$\hat{e}_\theta = -\frac{1}{2}\hat{\imath} + \frac{\sqrt{3}}{2}\hat{\jmath}.$$

What is θ?

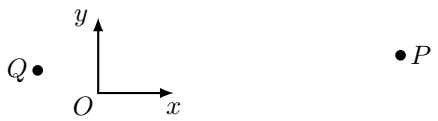
(A) $0 \leq \theta < \frac{1}{2}\pi$

(B) $\frac{1}{2}\pi \leq \theta < \pi$

(C) $\frac{3}{2}\pi \leq \theta < 2\pi$

(D) $\pi \leq \theta < \frac{3}{2}\pi$

17. (5 points) Points P and Q are moving in circular paths around the origin O with angular velocities ω_P and ω_Q and speeds v_P and v_Q, respectively.



The two particles are moving with the same angular velocity, so $\omega_P = \omega_Q$. Which statement is true?

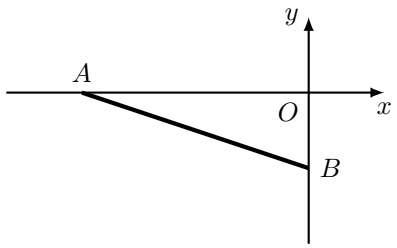
(A) $\frac{1}{2}v_Q < v_P \leq v_Q$

(B) $2v_Q < v_P$

(C) $v_P \leq \frac{1}{2}v_Q$

(D) $v_Q < v_P \leq 2v_Q$

18. (5 points) A rod with fixed length is positioned as shown, with x the horizontal coordinate of end A and y the vertical coordinate of end B. End A can only move horizontally, while end B can only move vertically.



At an instant of time, we have $x = -3$ m and $y = -1$ m, and the rod is moving so that $\dot{y} = -2$ m/s. What is $\dot{x}$?

(A) $\dot{x} = 0$ m/s

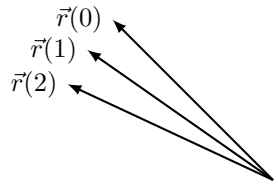
(B) -1 m/s $\leq \dot{x} < 0$ m/s

(C) 0 m/s $< \dot{x} < 1$ m/s

(D) 1 m/s $\leq \dot{x}$

(E) $\dot{x} < -1$ m/s

19. (5 points) The position vector $\vec{r}(t)$ of a point is shown below at $t = 0$ s, $t = 1$ s, and $t = 2$ s.



Which direction is the closest to the direction of the acceleration $\vec{a}(0)$ at time $t = 0$ s?

(A) ↙

(B) ↘

(C) ↗

(D) ↖

20. (5 points) A particle starts at the origin at time $t = 0$ s and its velocity is given by

$$\vec{v} = -t^2\,\hat{\imath} + 4t\,\hat{\jmath}\ \text{m/s}.$$

At time $t = 1$ s, what is the particle's distance r from the origin?

(A) $0\ \text{m} \leq r < 2\ \text{m}$

(B) $2\ \text{m} \leq r < 4\ \text{m}$

(C) $4\ \text{m} \leq r < 6\ \text{m}$

(D) $8\ \text{m} \leq r$

(E) $6\ \text{m} \leq r < 8\ \text{m}$