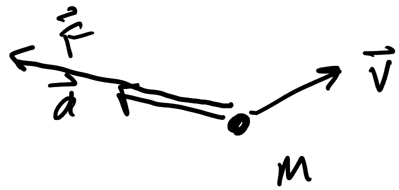
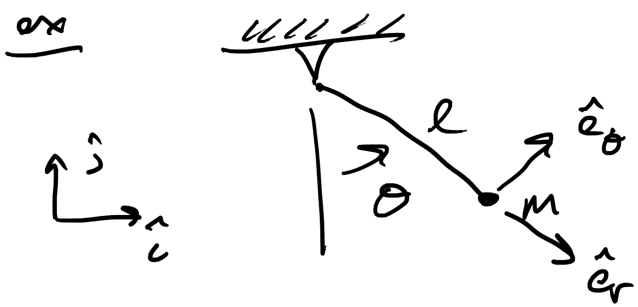
Linear momentum for point masses



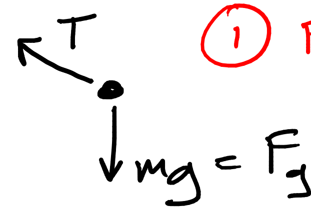
$\vec{F} = m\vec{a}$ lin. mom. $\vec{p} = m\vec{v}$

$$\underbrace{\dot{\vec{p}} = \frac{d}{dt}(m\vec{v}) = m\vec{a}}_{\text{assuming } \dot{m}=0.} = \vec{F} \qquad \boxed{\vec{F} = \dot{\vec{p}}}$$

ex



FBD



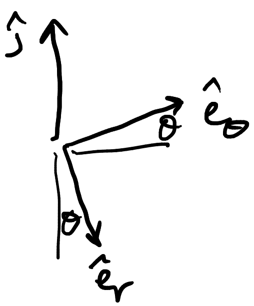
1 FBD

$\vec{T} = -T\hat{e}_r$

$\vec{F}_g = -mg\hat{j}$

2 kinematics

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$



$\hat{j} = -\cos\theta\,\hat{e}_r + \sin\theta\,\hat{e}_\theta$

$$= -\ell\dot{\theta}^2\hat{e}_r + \ell\ddot{\theta}\hat{e}_\theta$$

$$\underline{-T\hat{e}_r} - mg\hat{j} = \vec{T} + \vec{F}_g = \vec{F} = \underbrace{m\vec{a}}_{\dot{\vec{p}}} = m(\underline{-\ell\dot{\theta}^2\hat{e}_r} + \ell\ddot{\theta}\hat{e}_\theta)$$

3 Newton

$\cancel{-T = -m\ell}\,\dot{\theta}^2$

4 algebra.

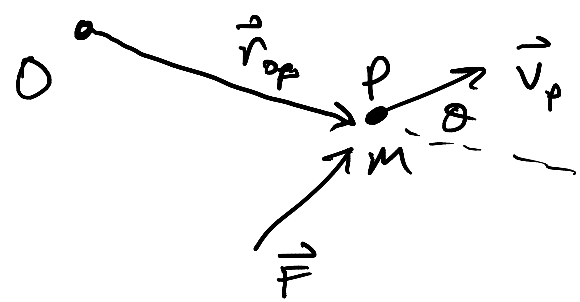
$$(-T + mg\cos\theta)\hat{e}_r - mg\sin\theta\,\hat{e}_\theta = -m\ell\dot{\theta}^2\hat{e}_r + m\ell\ddot{\theta}\hat{e}_\theta$$

* Can compare components if only one orthogonal basis is used. $\hat{e}_r, \hat{e}_\theta$

$$m l \ddot{\theta} = -mg\sin\theta \Rightarrow \ddot{\theta} = -\frac{g}{l}\sin\theta$$

$$-T + mg\cos\theta = -ml\dot{\theta}^2 \Rightarrow T = mg\cos\theta + ml\dot{\theta}^2$$

Angular momentum for point masses



ang. mom. of mass about O

$$\vec{H}_O = \vec{r}_{op} \times m\vec{v}_p$$

$$H_o = mrv\sin\theta$$

Moment of $\vec{F}$ applied at P about O

$$\vec{M}_o = \vec{r}_{op} \times \vec{F}$$

<u>moment equ</u>

$$\dot{\vec{H}}_o = \frac{d}{dt}\left(\vec{r}_{op} \times m\vec{v}_p\right)$$ O fixed.

$$= \dot{\vec{r}}_{op} \times m\vec{v}_p + \vec{r}_{op} \times m\dot{\vec{v}}_p$$

$$= \cancel{\vec{v}_p \times m\vec{v}_p}^{\,0} + \vec{r}_{op} \times m\vec{a}_p$$

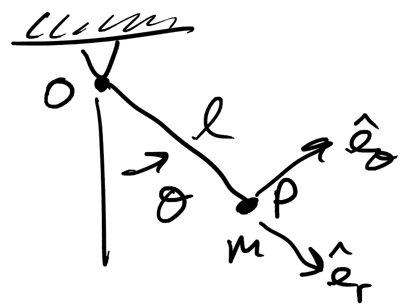
$$= \vec{r}_{op} \times \vec{F}$$

$$= \vec{M}_o$$

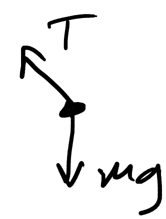
$$\boxed{\vec{M}_o = \dot{\vec{H}}_o}$$

Careful: $\vec{M}_o \neq \dot{\vec{H}}_c$

<u>ex</u>



FBD



$\vec{F}_g = -mg\hat{j}$

$$\vec{H}_o = \vec{r}_{op} \times m\vec{v}_p$$

$$\vec{v}_p = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$= l\hat{e}_r \times ml\dot{\theta}\hat{e}_\theta$$

$$= ml^2\dot{\theta}\hat{k}$$

$$\vec{M}_o = \vec{r}_{op} \times \vec{F}$$

$$= l\hat{e}_r \times \left(-mg\hat{j} - T\hat{e}_r\right)$$

$$= -mgl\sin\theta\,\hat{k}$$