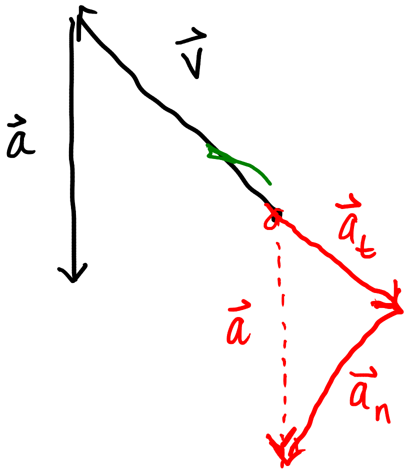
TAM 212



slowing down or speeding up?

$$\vec a = \vec a_t + \vec a_n$$

$$= \underbrace{a_t \hat e_t} + a_n \hat e_n \leftarrow \text{turning of particle}$$

slowing down: $a_t < 0$

speeding up: $a_t > 0$

$$\vec a \cdot \frac{\vec v}{v} = a_t = \vec a \cdot \hat e_t$$

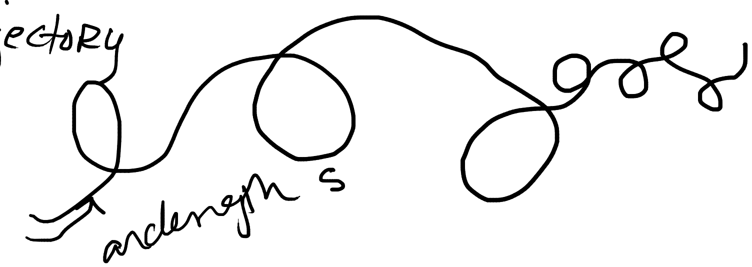
$$\underbrace{\begin{aligned}&\vec r\\ &\vec v = \dot{\vec r}\\ &\vec a = \dot{\vec v} = \ddot{\vec r}\end{aligned}}_{\text{vectors}}$$

s = arclength along a trajectory

= scalar

~~vector~~

~~component~~



$$\vec a = a_t \hat e_t + a_n \hat e_n$$

$$= \ddot s\, \hat e_t + \frac{(\dot s)^2}{\rho} \hat e_n$$

$$= \frac{d}{dt} v\, \hat e_t + \frac{v^2}{\rho} \hat e_n$$

$$\dot s = \text{speed} = \|\vec v\| = \|\dot{\vec r}\| = \left\|\frac{d}{dt}\vec r\right\|$$

BUT:

$$\underbrace{\ddot s = \dot v = \frac{d}{dt}\|\vec v\|}_{a_t} \neq \underbrace{\left\|\frac{d}{dt}\vec v\right\| = \|\vec a\|}_{\text{magnitude of total acceleration}} = a$$

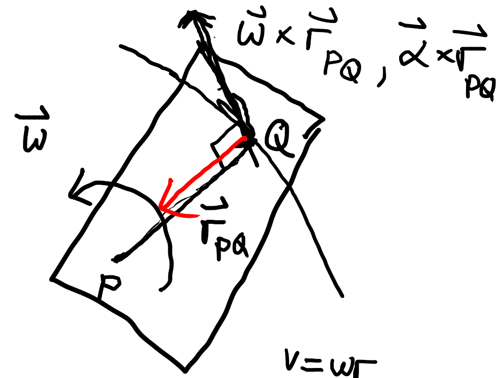
Rigid Bodies

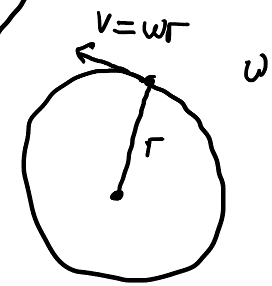
$\vec{\omega}_P \quad \vec{\omega}_Q \quad$ vs. $\vec{\omega}$

$$\vec{r}_Q = \vec{r}_P + \vec{r}_{PQ}$$

$$\vec{V}_Q = \vec{V}_P + \underline{\underline{\vec{\omega}\times\vec{r}}}_{PQ} = \vec{V}_P + \vec{V}_{Q/P}$$

$$\vec{a}_Q = \vec{a}_P + \vec{\alpha}\times\vec{r}_{PQ} + \vec{\omega}\times(\vec{\omega}\times\vec{r}_{PQ})$$





① velocity. say $\vec{\omega} = \omega\hat{k}$

What is the direction of $\vec{\omega}\times\vec{r}_{PQ}$?

Is $\perp$ to $\vec{r}_{PQ}$

② accelerations.

$$\vec{a}_Q = \vec{a}_P + \underbrace{\vec{\alpha}\times\vec{r}_{PQ}}_{\text{direction?}} + \underbrace{\vec{\omega}\times(\vec{\omega}\times\vec{r}_{PQ})}_{\text{direction?}}$$

$\vec{\alpha}$ = angular acceleration

$\vec{\alpha} = \alpha\hat{k}$

tangential acceleration, change of speed of pt Q due to change of rate of spinning

inwards, normal acceleration, changing direction

$-\omega^2 r$