Energy

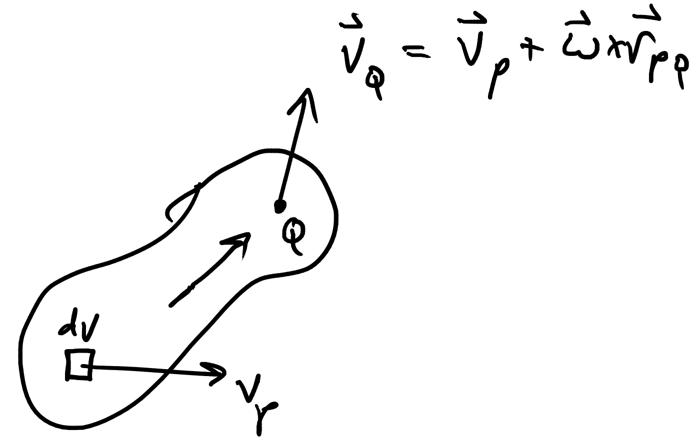
$$E = T + V$$

total energy = kinetic + potential

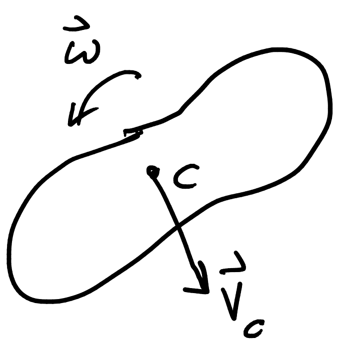
point mass: $T = \frac{1}{2} m v^2$

rigid body: $T = \int_V \frac{1}{2} \rho v_p^2 \, dV$

density

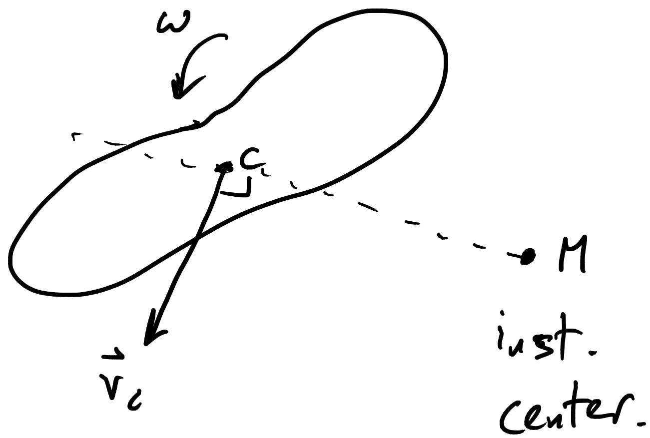


energy is extensive quantity ⇒ energy adds



$$\boxed{T = \underbrace{\frac{1}{2} m v_c^2}_{\text{translational K.E.}} + \underbrace{\frac{1}{2} I_c \omega^2}_{\text{rotational K.E.}}}$$

(König's Thm)



$\vec{v}_M = 0$

$\vec{v}_c = \vec{\omega} \times \vec{r}_{MC}$

$2D \Rightarrow \quad v_c = \omega r_{MC}$

$$T = \frac{1}{2} m (\omega r_{MC})^2 + \frac{1}{2} I_C \omega^2$$

$$= \frac{1}{2} \big(\underbrace{m r_{MC}^2 + I_C}_{I_M} \big) \omega^2$$

$$\boxed{T = \frac{1}{2} I_M \omega^2}$$

pure rotation about M

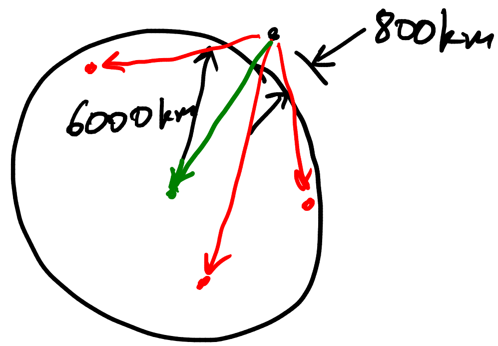
two forms of T: $\quad T = \frac{1}{2} m v_c^2 + \frac{1}{2} I_C \omega^2 \quad$ C center of mass.

$T = \frac{1}{2} I_M \omega^2 \quad$ M inst. center

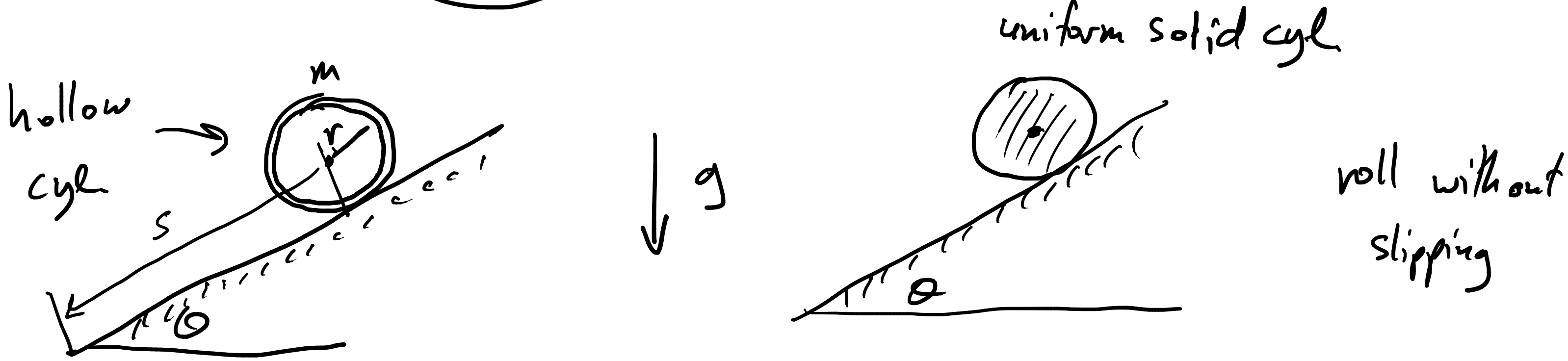
$T = \frac{1}{2} I_O \omega^2 \quad$ O fixed point

potential energy: gravity applies at C. $\quad V = m g h_c$

general form of gravity $\quad V = \dfrac{G m_1 m_2}{r_{12}}$



ex

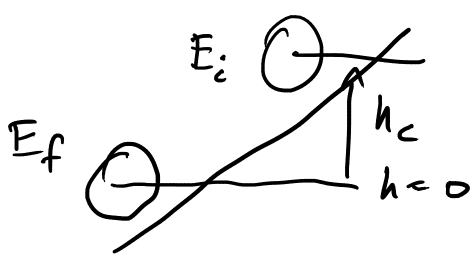


both cyl released from rest at same height

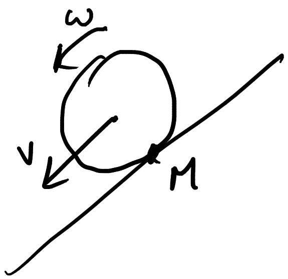
which reaches the bottom first?

after travelling distance s, what is v_c?

no friction or damping $\Rightarrow$ energy constant.
loss



$$E_f = E_i$$



$$E_f = T_f + \cancel{V_f}^{\,0}$$

$$E_i = \cancel{T_i}^{\,0} + V_i$$

$$= mgh_c$$

$$= \tfrac{1}{2}mv_c^2 + \tfrac{1}{2}I_c\omega^2$$

$$= \tfrac{1}{2}m(\omega r)^2 + \tfrac{1}{2}I_c\omega^2$$

$$= \tfrac{1}{2}\underbrace{(mr^2 + I_c)}_{I_M}\omega^2$$

mr^2

hollow cyl $\Rightarrow$

$$= mr^2\omega^2$$

$$= mv_c^2$$

$$E_f = E_i$$

$$mv_c^2 = mgh_c$$

$$v_c = \sqrt{gh_c}$$

solid cyl $\Rightarrow$ $$T_f = \tfrac{1}{2}(mr^2 + \tfrac{1}{2}mr^2)\omega^2$$

$$= \tfrac{3}{4}mr^2\omega^2$$

$$= \tfrac{3}{4}mv_c^2 \quad \Rightarrow \quad v_c = \sqrt{\tfrac{4}{3}gh_c}$$

solid cyl is faster.