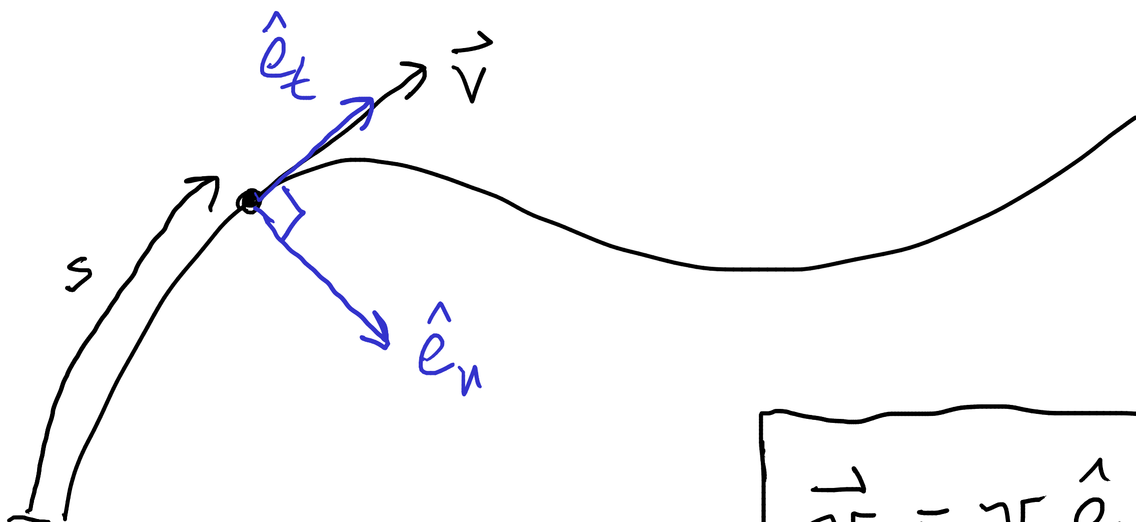
Tangential / Normal basis



$\hat{e}_t$ tangential

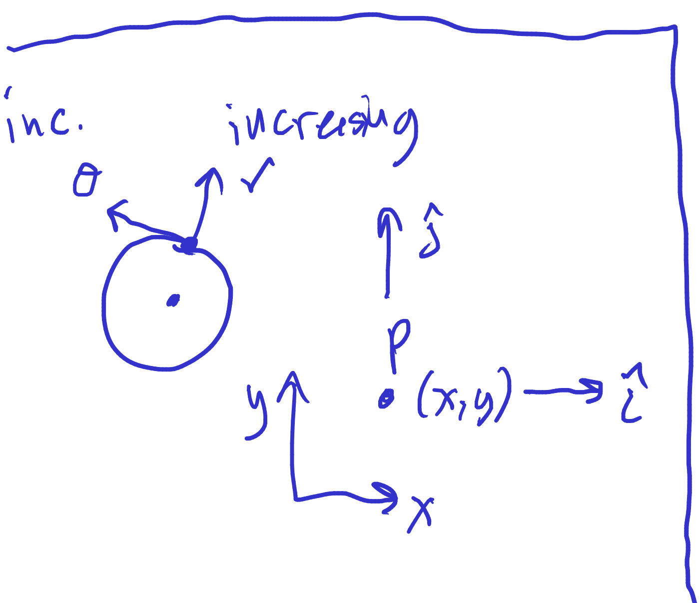
$\hat{e}_n$ normal (inwards)

$$\boxed{\vec{v} = v\,\hat{e}_t} + 0\,\hat{e}_n \qquad \boxed{\hat{e}_t = \hat{v}}$$

?

$$\vec{v} = v\,\hat{v}$$

$\hat{e}_t$



s = path length = arc length

= distance traveled.

$$\boxed{v = \dot{s}} \qquad s = \int_0^t v(\tau)\,d\tau$$

$$\vec{v} = \dot{s}\,\hat{e}_t$$

$$\vec{a} = \dot{\vec{v}} = \frac{d}{dt}\left(\dot{s}\,\hat{e}_t\right) = \ddot{s}\,\hat{e}_t + \dot{s}\,\dot{\hat{e}}_t$$

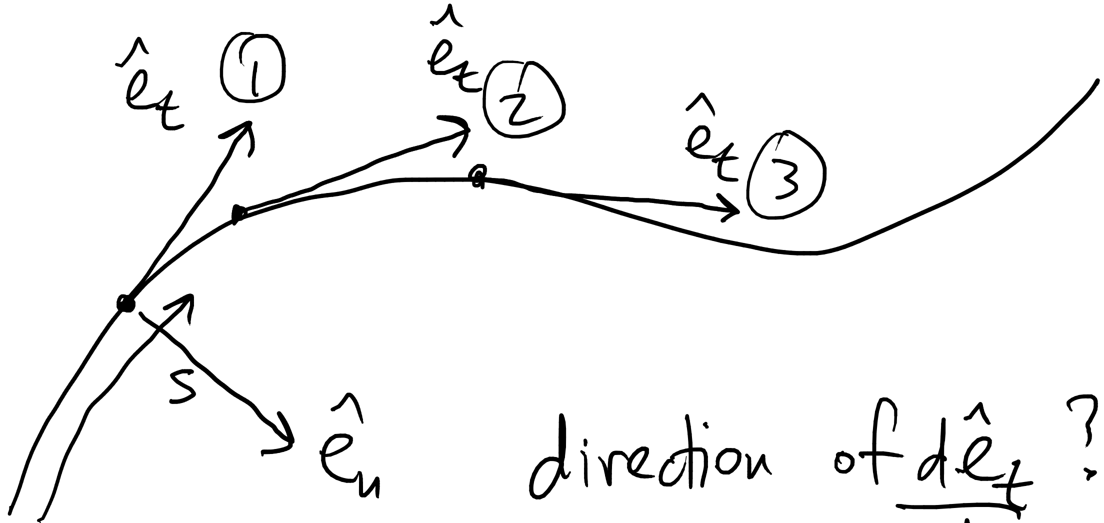
car: accelerator/brake.
tangential acceleration

Convert t-derivative to s-derivative

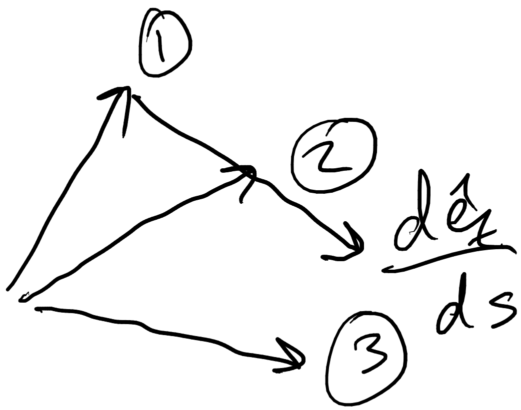
$$\frac{d}{dt}\hat{e}_t = \frac{ds}{dt}\frac{d}{ds}\hat{e}_t = \dot{s}\,\frac{d\hat{e}_t}{ds}$$

$$\vec{a} = \ddot{s}\,\hat{e}_t + \dot{s}^2\,\frac{d\hat{e}_t}{ds}$$

$$\frac{d\hat{e}_t}{ds}$$



direction of $\frac{d\hat{e}_t}{ds}$?



A : parallel to $\hat{e}_t$

B : perpendicular to $\hat{e}_t$

C : neither

$$\frac{d\hat{e}_r}{dt} = \dot{\theta}\,\hat{e}_\theta$$

is $\frac{d\hat{e}_t}{ds} = \hat{e}_n$?

A : Yes

B : No

C : Neither

kappa

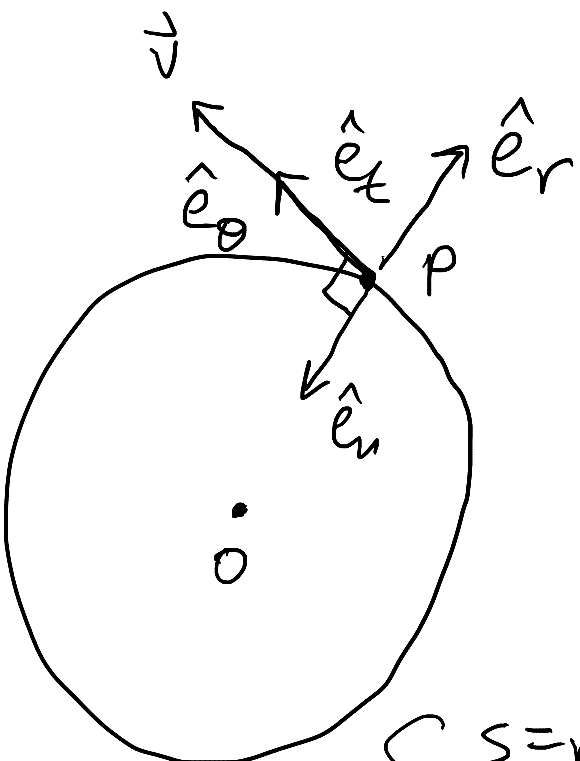
$$\frac{d\hat{e}_t}{ds} = \kappa \hat{e}_n$$

curvature

$$\boxed{\kappa = \frac{1}{r}}$$

radius of curvature

$$\boxed{\vec{a} = \ddot{s}\,\hat{e}_t + \dot{s}^2 \kappa\, \hat{e}_n}$$



$\hat{e}_r = -\hat{e}_n$

always

true for circles. r constant

A : Y

B : N

$$\vec{a} = \underbrace{r\ddot{\theta}}_{\ddot{s}}\,\hat{e}_\theta - \underbrace{r\dot{\theta}^2}_{\frac{v^2}{r} = \frac{\dot{s}^2}{r}}\,\hat{e}_r$$

$-\hat{e}_n$

circle
$$\begin{cases} s = r\theta \\ \dot{s} = r\dot{\theta} \\ \ddot{s} = r\ddot{\theta} \end{cases}$$

$$\boxed{\vec{a} = \ddot{s}\,\hat{e}_t + \dot{s}^2/r\;\hat{e}_n}$$