TAM 212

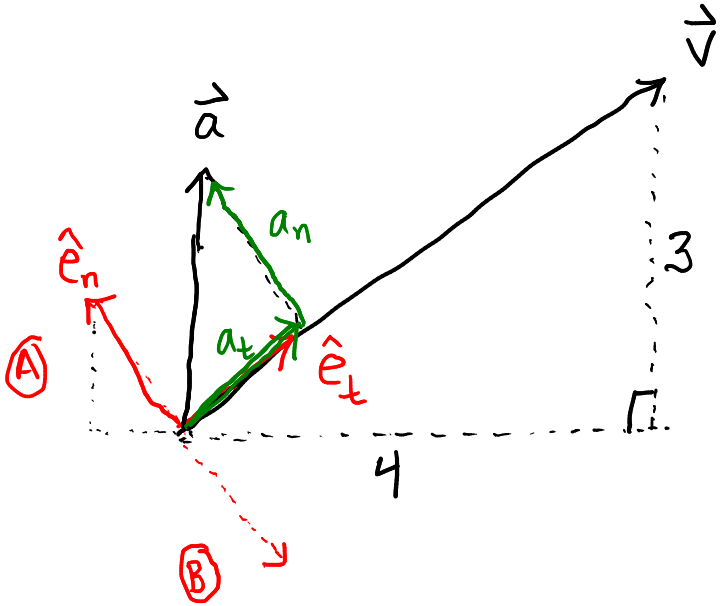
Question 1: A satellite tracks a car moving with velocity $\vec{v} = 4\hat{i} + 3\hat{j}$ m/s, and acceleration $\vec{a} = 2\hat{j}$ m/s^2.

Find $\hat{e}_t$, $\hat{e}_n$, radius of curvature ρ.

Sketch the trajectory of the car at the instant shown.

Is the car speeding up or slowing down?

SOLUTION



$$\vec{v} = v\hat{e}_t$$

$$\hat{e}_t = \frac{\vec{v}}{v} = \frac{4\hat{i} + 3\hat{j}}{5}$$

$\hat{e}_n$ = (A) or (B) — Know this trick!

(A) $\dfrac{-3\hat{i} + 4\hat{j}}{5}$ or $\dfrac{+3\hat{i} - 4\hat{j}}{5}$

Radius of Curvature: $\vec{a} = a_t\hat{e}_t + a_n\hat{e}_n$ $\qquad a_n = \vec{a}\cdot\hat{e}_n$

$$a_t = \frac{d}{dt}|\vec{v}| > 0$$

$$a_n = |\vec{v}|^2/\rho$$

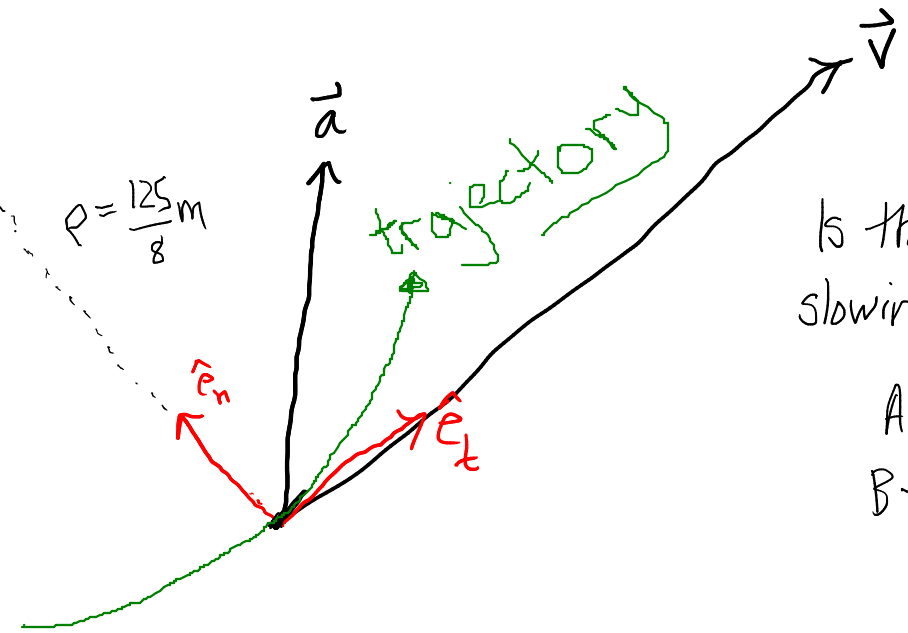
$$a_n = |v|^2/\rho = \vec{a} \cdot \hat{e}_n$$

$$= (2\hat{j}) \cdot \frac{1}{5}(-3\hat{i} + 4\hat{j}) = 8/5$$

$$|v|^2/\rho = 8/5$$

$$\rho = |v|^2 \cdot \frac{5}{8} = \frac{5^3}{8} = \boxed{125/8 \ m}$$

$\frac{d}{dt}\vec{v}$ ($\vec{a}$) vs. $\underbrace{\frac{d}{dt}|\vec{v}|}_{a_t}$



Is the car speeding up or slowing down?

A - speeding up

B - slowing down

Question 2 A car is driving on track defined by $r = 2 + \cos 2\theta$ m. At an instant, we measure $\theta = 3\pi/4$ rad, $\dot{\theta} = -2$ rad/s, $\ddot{\theta} = -2$ rad/s^2.

(a) Sketch the track

(b) What are $\vec{v}$, $\vec{a}$ in the polar basis?

(c) Is the car speeding up or slowing down?

(d) What is the radius of curvature?