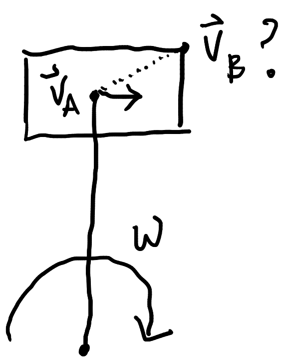
Rigid Bodies, Angular Velocity

rigid body → non-deformable

distance between any two points on the body remains the same throughout the motion

Who cares? (Significance)

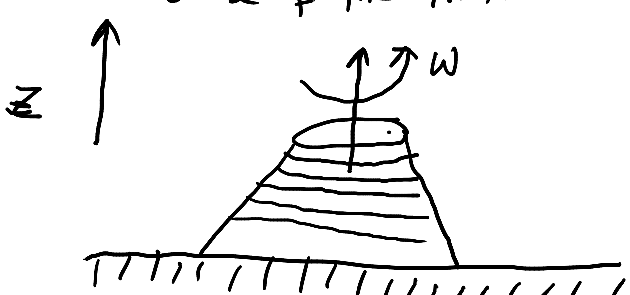
→ velocities of different points on a rigid body are related to each other via the angular velocity of the body



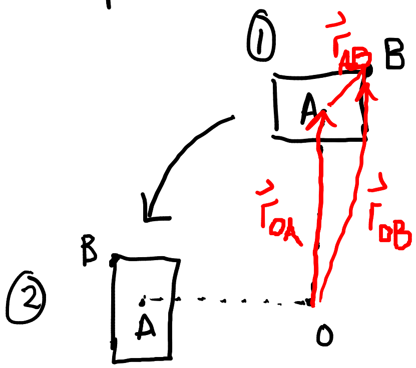
We can find $\vec{v}_B$ given only $\vec{v}_A$, $\vec{\omega}$

simplest case of rigid body motion: plane motion of a rigid body

- all points of the R.B. stay in the same plane over the course of the motion



planar motion :



if we know : (1) $\vec{r}_{OA}$

(2) orientation of pt B with respect to A

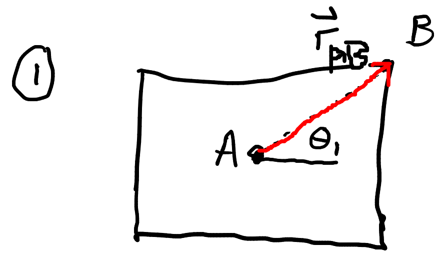
then we know the position of pt B

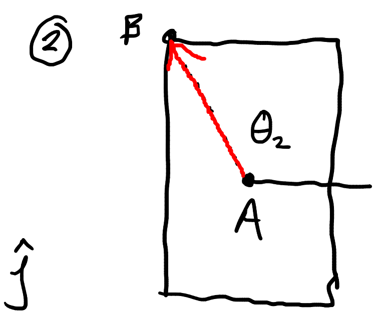
$$\boxed{\vec{r}_{OB} = \vec{r}_{OA} + \vec{r}_{AB}}$$ rigid body

Relationship Between Velocities

$$\frac{d}{dt}\left(\vec{r}_{OB} = \vec{r}_{OA} + \vec{r}_{AB}\right)$$

$$\boxed{\vec{V}_{OB} = \vec{V}_{OA} + \dot{\vec{r}}_{AB}}$$





$\vec{r}_{AB} = r_{AB}\,\hat{u}$ $\qquad \hat{u} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

$= r_{AB}\left(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right)$

$$\frac{d}{dt}\vec{r}_{AB} = r_{AB}\left(-(\sin\theta)\dot{\theta}\,\hat{i} + (\cos\theta)\dot{\theta}\,\hat{j}\right) = r_{AB}\,\dot{\theta}\left(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\right)$$

$$\dot{\vec{r}}_{AB} = r_{AB}\,\dot{\theta}\left(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\right)$$

we can write this more compactly:

$$= \left(\dot{\theta}\,\hat{k}\right) \times \underbrace{\left(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right) r_{AB}}$$

that is:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \dot{\theta} \\ r\cos\theta & r\sin\theta & 0 \end{vmatrix} = r\,\dot{\theta}\left(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\right)$$

$\dot{\theta}\hat{k}$ = angular velocity of rigid body $= \vec{\omega}$

$r_{AB}\left(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right) = r_{AB}\,\hat{u} = \vec{r}_{AB}$

$$\boxed{\dot{\vec{r}}_{AB} = \vec{\omega} \times \vec{r}_{AB}}$$

$$\vec{V}_{OB} = \vec{V}_{OA} + \vec{\omega} \times \vec{r}_{AB}$$