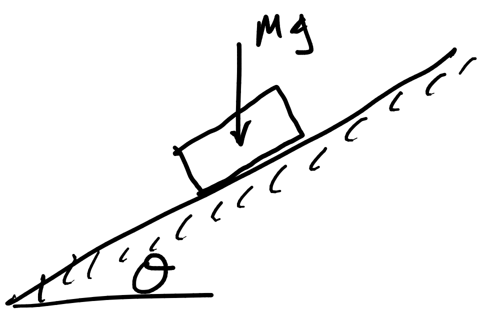
Dry Friction – Coulomb Friction



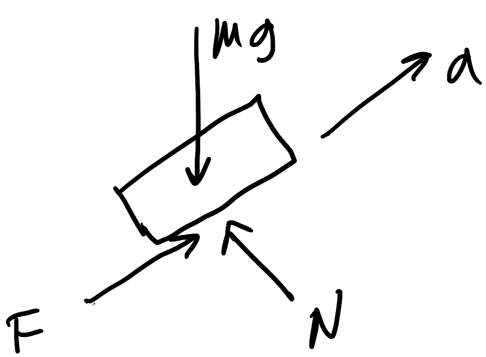
stick: $v_{contact} = 0$, $|F| \le \mu |N|$

slip: $v_{contact} > 0$, $|F| = \mu |N|$

transition: $v = 0$, $|F| = \mu |N|$

direction of F opposes the motion.

point mass model



ex block on slope $\theta = 45°$, $\mu = 0.7$

Q: what happens if we start from rest?

① method of assumed motion: assume sticking $\Rightarrow v = 0,\ a = 0.$

solve for F

$$\sum \vec{F} = m\vec{a} \quad \begin{cases} N - mg\cos\cancel{\theta}^{45°} = 0 \\ F - mg\sin\cancel{\theta}^{45°} = ma = 0 \end{cases}$$

$$N = \frac{mg}{\sqrt{2}}$$

$$F = \frac{mg}{\sqrt{2}}$$

check $|F| \le \mu |N|$: $\cancel{\frac{mg}{\sqrt{2}} \le 0.7 \frac{mg}{\sqrt{2}}}$ false

$\Rightarrow$ physically impossible

$\Rightarrow$ our original assumption (sticking) is wrong.

② method of assumed force: assume slipping

$\Rightarrow |F| = \mu |N|$ opposing v/a

choose signs: N, F both positive as shown.

$F = \mu N$, solve for a:

$$\sum \vec{F} = m\vec{a} : \begin{cases} N - mg\cos\theta = 0 \Rightarrow N = mg\cos\theta \\ \cancel{F} - mg\sin\theta = ma \end{cases}$$

μN

$$a = g(\mu\cos\theta - \sin\theta)$$

$$= \frac{g}{\sqrt{2}}(0.7 - 1)$$

$$= -\frac{0.3}{\sqrt{2}} g$$

ex $\mu = 2$, $\theta = 45°$

① assume stick: $v = 0$, $a = 0$

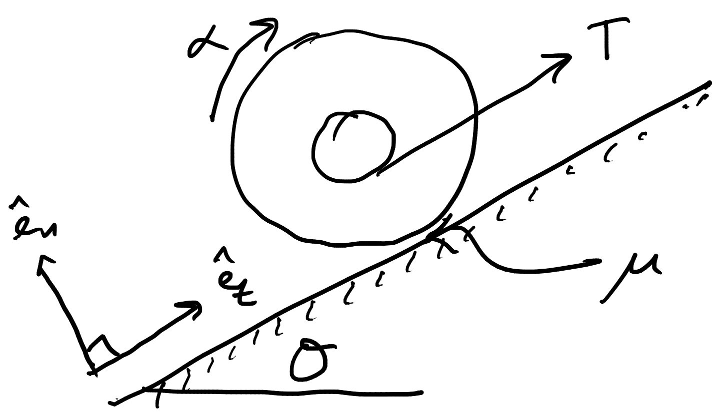
$$\Rightarrow F = \frac{mg}{\sqrt{2}}, \quad N = \frac{mg}{\sqrt{2}}$$

check: $F \le \mu N$

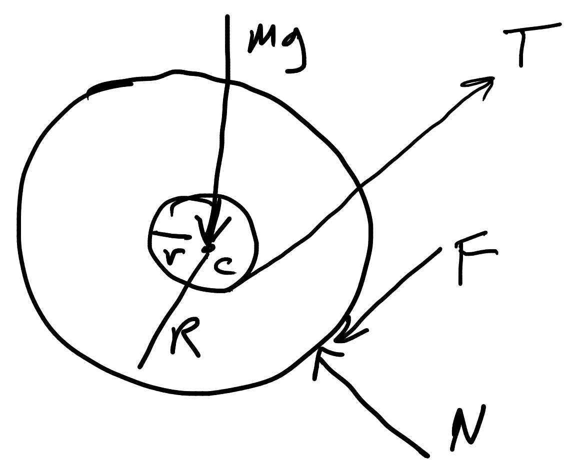
$$\frac{mg}{\sqrt{2}} \le 2 \cdot \frac{mg}{\sqrt{2}} \quad \text{ok.}$$

physically possible. $\Rightarrow a = 0$, $F = N = \frac{mg}{\sqrt{2}}$.

ex



$$\vec{a} = a\hat{e}_t$$
$$\vec{\alpha} = -\alpha\hat{k}$$



$\hat{e}_t$: $T - F - mg\sin\theta = ma$

$\hat{e}_n$: $N - mg\cos\theta = 0$

$$\sum M_c: \quad Tr\,\hat{k} - FR\,\hat{k} = I_c \vec{\alpha} = -I_c \alpha \hat{k}$$

$$Tr - FR = -I_c \alpha$$

also: $a = R\alpha \leftarrow$ assuming stick.

solve $\Longrightarrow$

$$\left.\begin{aligned} \alpha &= \frac{1}{I_c + mR^2}\left(T(R-r) - mgR\sin\theta \right) \\ F &= T\frac{r}{R} + \frac{I_c}{R}\alpha \\ N &= mg\cos\theta \end{aligned}\right\} \text{sticking case.}$$