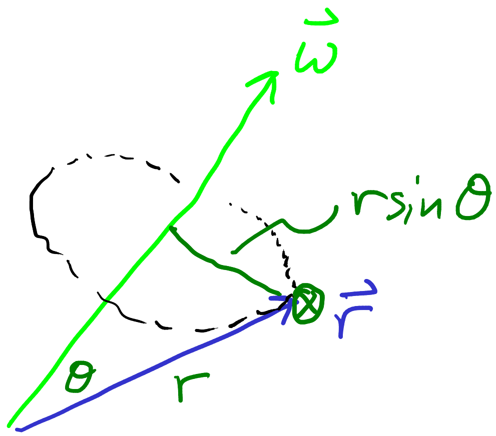
Rotations and angular velocity

$\vec{\omega}$ = angular velocity vector

$\hat{\omega}$ = axis / direction of rotation

$\omega = \|\vec{\omega}\|$ = speed of rotation

$$\left.\begin{aligned} A&: \vec{\omega}\times\vec{r} \\ B&: \vec{r}\times\vec{\omega} \end{aligned}\right\} \vec{v}$$



$\vec{r}$ rotates with ang. vel. $\vec{\omega}$

A: into board

B: out of board

C: NOTA.

What is $\vec{v} = \dot{\vec{r}}$?

$\hat{v}$ = into board

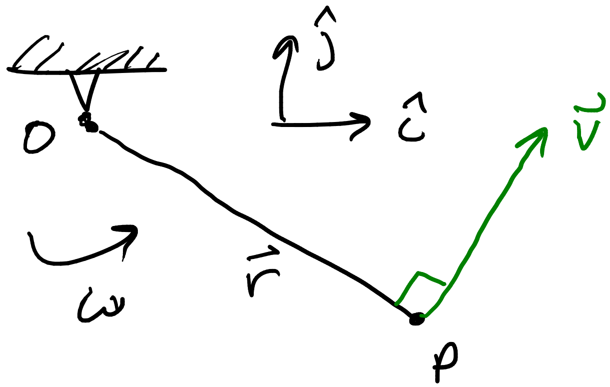
$$\left\{\begin{aligned} &r = \|\vec{r}\| = \text{constant} \\ &\Rightarrow \quad \vec{v} \perp \vec{r} \qquad \vec{v}\cdot\vec{r} = 0 \\ &\text{rotate about } \vec{\omega} \\ &\Rightarrow \quad \vec{v} \perp \vec{\omega} \qquad \vec{v}\cdot\vec{\omega} = 0 \end{aligned}\right.$$

$\operatorname{dir}(\vec{v}) = \operatorname{dir}(\vec{\omega}\times\vec{r})$

$\left\{ v = \omega(r\sin\theta) = \omega r\sin\theta \right.$

$$\rightarrow \boxed{\vec{v} = \dot{\vec{r}} = \vec{\omega}\times\vec{r}}$$

ex



$\vec{r}_{op} = 4\hat{i} - 3\hat{j}$ m

$\omega = 2$ rad/s

$\vec{\omega} = 2$ out of board

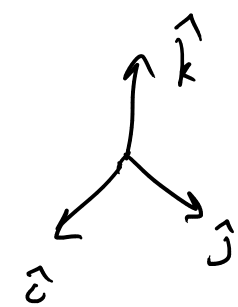
$= 2\hat{k}$ rad/s

$$\vec{V}_p = \vec{\omega} \times \vec{r}_{op}$$

$$= 2(\overbrace{3\hat{i} + 4\hat{j}}^{\text{length } r = 5\,m}) = 6\hat{i} + 8\hat{j} \text{ m/s.}$$

$\cancel{-3\hat{i} - 4\hat{j}}$

$$= 2\hat{k} \times (4\hat{i} - 3\hat{j})$$ ← or use cross-product formulas.



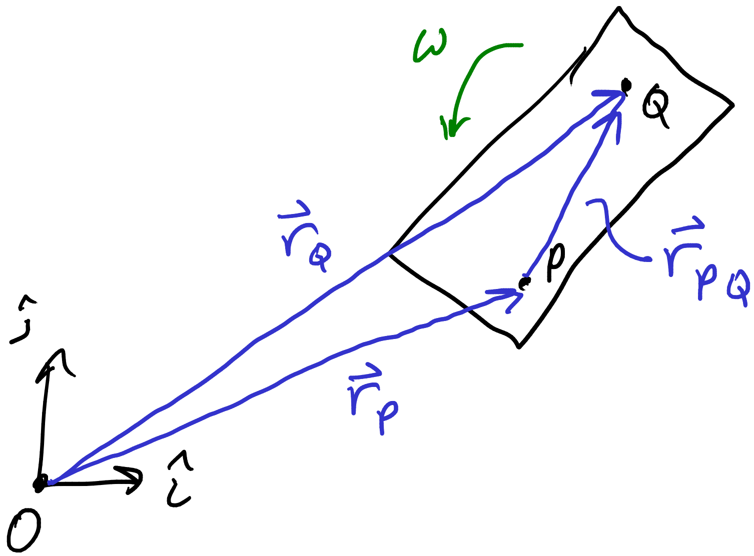
$$= 2\hat{k} \times 4\hat{i} - 2\hat{k} \times 3\hat{j}$$

$$= 8(\hat{k} \times \hat{i}) - 6(\hat{k} \times \hat{j})$$

$$= 8\hat{j} - 6(-\hat{i})$$

$$= 6\hat{i} + 8\hat{j}$$

Rotating Rigid Bodies



<u>positions</u>

$$\vec{r}_P = \vec{r}_{OP}$$
$$\vec{r}_Q = \vec{r}_{OQ}$$

$$\vec{r}_{OQ} = \vec{r}_{OP} + \vec{r}_{PQ}$$

$$\boxed{\vec{r}_Q = \vec{r}_P + \vec{r}_{PQ}}$$

rigid bodies can:

① translate ← specify any velocity

② rotate. ← $\vec{\omega}$ ang. vel. of body

<u>velocity</u>

$$\vec{r}_Q = \vec{r}_P + \vec{r}_{PQ}$$
$$\dot{\vec{r}}_Q = \dot{\vec{r}}_P + \dot{\vec{r}}_{PQ}$$
$$\boxed{\vec{v}_Q = \vec{v}_P + \vec{\omega} \times \vec{r}_{PQ}}$$

$\vec{r}_{PQ}$ rotating $\vec{\omega}$

$\Rightarrow \dot{\vec{r}}_{PQ} = \vec{\omega} \times \vec{r}_{PQ}$