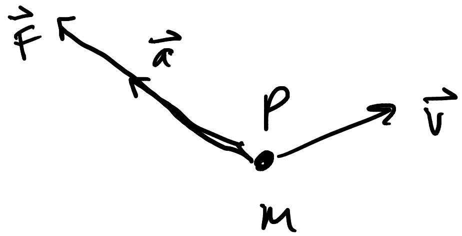
Linear momentum for point masses

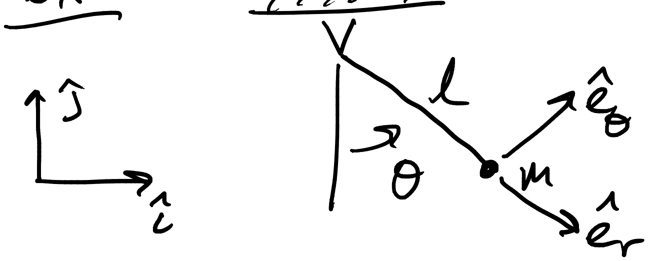


$\vec F = m\vec a$ lin. mom. $\vec p = m\vec v$

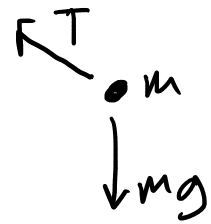
$$\dot{\vec p} = \frac{d}{dt}(m\vec v) = m\vec a = \vec F \quad \text{(constant mass)}$$

$$\boxed{\vec F = \dot{\vec p}}$$

ex



1 FBD



$\vec F_g = -mg\hat j$

$\vec T = -T\hat e_r$

2 Kinematics $\longrightarrow \vec a$?

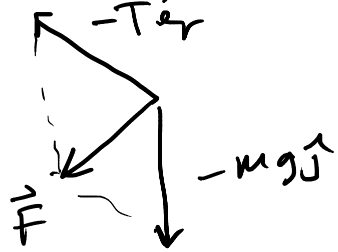
$$\vec a = (\ddot r - r\dot\theta^2)\hat e_r + (r\ddot\theta + 2\dot r\dot\theta)\hat e_\theta$$

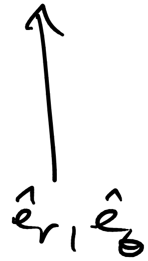
$$= -l\dot\theta^2\hat e_r + l\ddot\theta\hat e_\theta$$

3 Newton $\vec F = m\vec a$

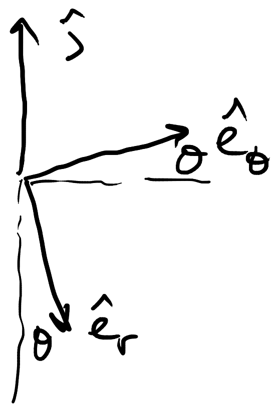
$$\vec F_g + \vec T = m(-l\dot\theta^2\hat e_r + l\ddot\theta\hat e_\theta)$$

4 algebra $-mg\hat{j} - T\hat{e}_r = -ml\dot{\theta}^2\hat{e}_r + ml\ddot{\theta}\hat{e}_\theta$ ← correct but hard to work with





$$\cancel{T = ml\dot{\theta}^2}$$

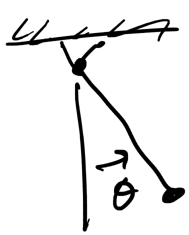


$$\hat{j} = -\cos\theta\,\hat{e}_r + \sin\theta\,\hat{e}_\theta$$

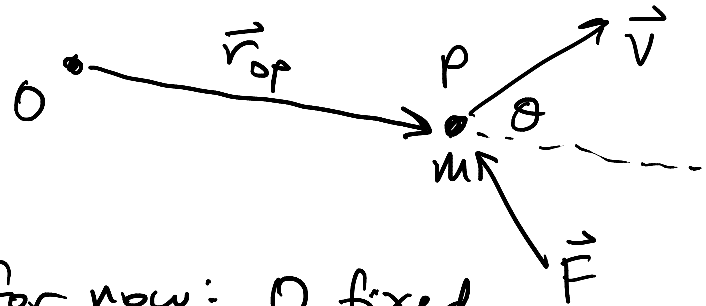
$$(-T + mg\cos\theta)\,\hat{e}_r - mg\sin\theta\,\hat{e}_\theta = -ml\dot{\theta}^2\hat{e}_r + ml\ddot{\theta}\hat{e}_\theta$$

one basis $\Rightarrow$ compare components.

$$\left.\begin{aligned} ml\ddot{\theta} &= -mg\sin\theta \\ -T + mg\cos\theta &= -ml\dot{\theta}^2 \end{aligned}\right. \Longrightarrow \begin{cases} \ddot{\theta} = -\dfrac{g}{l}\sin\theta \\ T = mg\cos\theta + ml\dot{\theta}^2 \end{cases}$$



Angular momentum for point masses

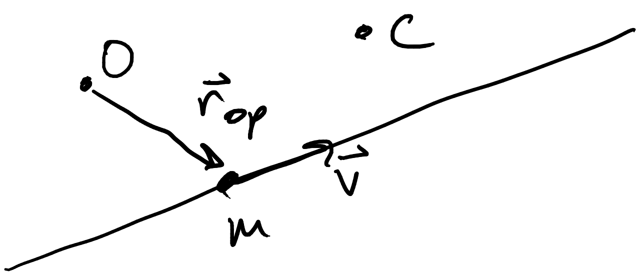


for now: O fixed

ang. mom. of P about O

$$\boxed{\vec{H}_O = \vec{r}_{op} \times m\vec{v}_p}$$

$$H_O = r_{op}\, m\, v_p \sin\theta$$



moment of $\vec{F}$ applied at P about O

$$\boxed{\vec{M}_O = \vec{r}_{op} \times \vec{F}}$$

moment eqn.

$$\dot{\vec{H}}_O = \frac{d}{dt}\left(\vec{r}_{op} \times m\vec{v}_p\right)$$

$$= \dot{\vec{r}}_{op} \times m\vec{v}_p + \vec{r}_{op} \times m\dot{\vec{v}}_p$$

$$= \cancel{\vec{v}_p \times m\vec{v}_p}^{\,0} + \vec{r}_{op} \times m\vec{a}_p$$

$$= \vec{r}_{op} \times \vec{F}$$

$$= \vec{M}_O$$

$$\boxed{\vec{M}_O = \dot{\vec{H}}_O}$$

careful $\cancel{\vec{M}_O \neq \dot{\vec{H}}_C}$