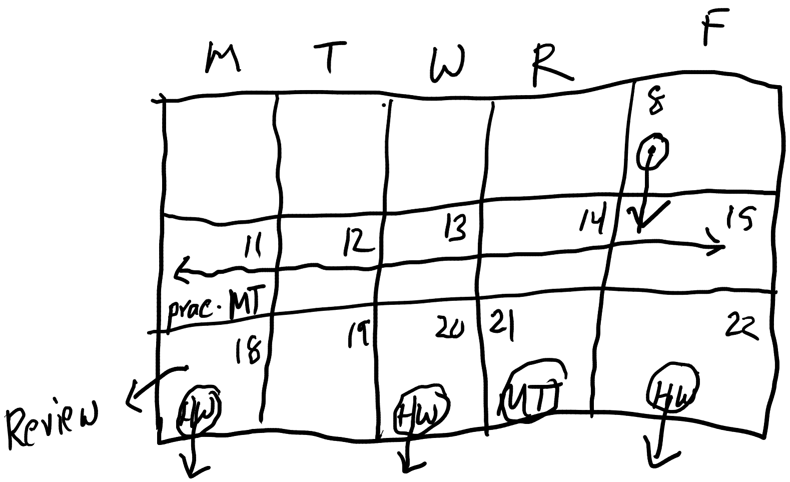
Normal, Tangential Basis

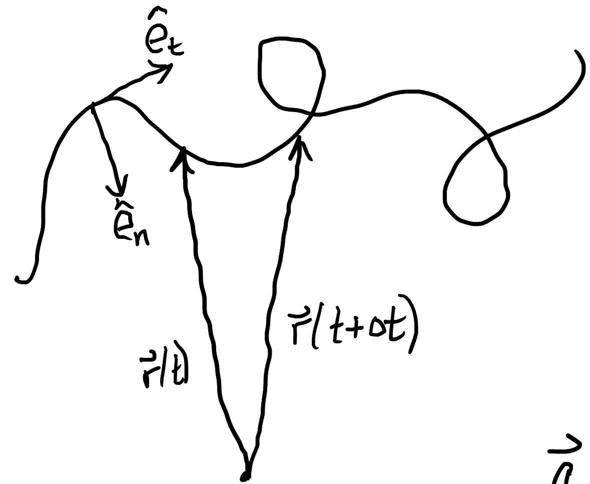
__topics__
- cartesians
- polar
- tangential / normal
- angular velocity



no new HW
discussion: roller coaster

discussion section:
Open office hours

Recall:



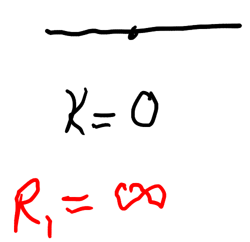
path $s(t)$ = distance covered

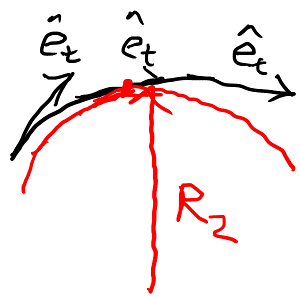
$$\vec{v} = \frac{d\vec{r}}{dt} = \underset{\text{speed}}{\dot{s}}\ \underset{\text{direction}}{\hat{e}_t} \qquad \hat{e}_t = \frac{\vec{v}}{|v|}$$

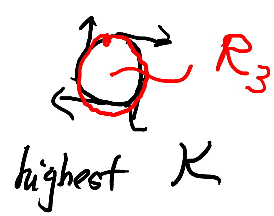
$$\vec{a} = \frac{d\vec{v}}{dt} = \ddot{s}\,\hat{e}_t + \underbrace{(\dot{s})^2 \kappa}_{\text{rate of change of direction w.r.t } t} \hat{e}_n$$

rate of change of speed w.r.t. time

$\kappa \equiv$ curvature, describes how $\hat{e}_t$ changes direction with respect to distance covered





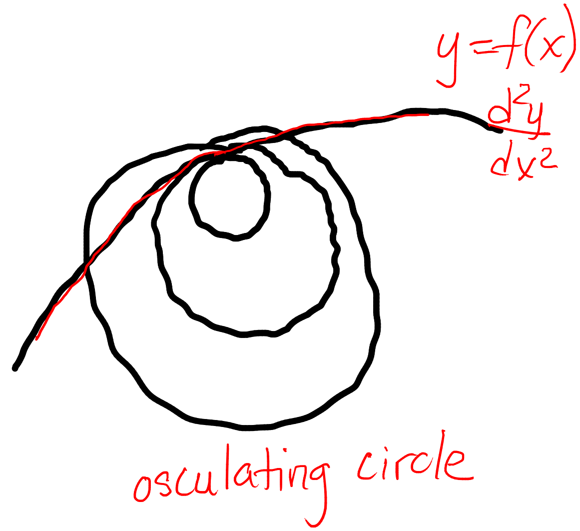


$R = 1/\kappa$

$$\vec{a} = \ddot{s}\,\hat{e}_t + (\dot{s})^2 K \,\hat{e}_n$$

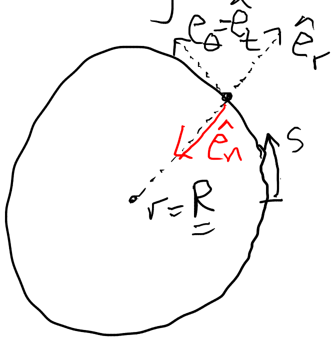
$$\dot{s} = v = |\vec{v}|$$

$$= \left(\frac{d}{dt}|v|\right)\hat{e}_t + v^2 K\,\hat{e}_n$$



Example: particle moving on a circular path, with a constant angular velocity $\underline{\underline{\omega}}$ (rad/sec)

$x = R\cos\omega t$
$y = R\sin\omega t$



polar basis: $\vec{v} = \omega R\, \hat{e}_\theta$

$$\boxed{\vec{a} = -\omega^2 R\, \hat{e}_r}$$

tangential/normal basis:

arclength $s(t) = \omega R t$

$\dot{s} = \omega R$

$\ddot{s} = 0$

$$\vec{v} = \dot{s}\,\hat{e}_t = \omega R\,\hat{e}_t$$

$$\vec{a} = \cancel{\ddot{s}\,\hat{e}_t} + \frac{(\dot{s})^2}{R}\,\hat{e}_n$$

$$= \frac{(\omega R)^2}{R}\,\hat{e}_n$$

$$\boxed{\vec{a} = \omega^2 R\,\hat{e}_n}$$

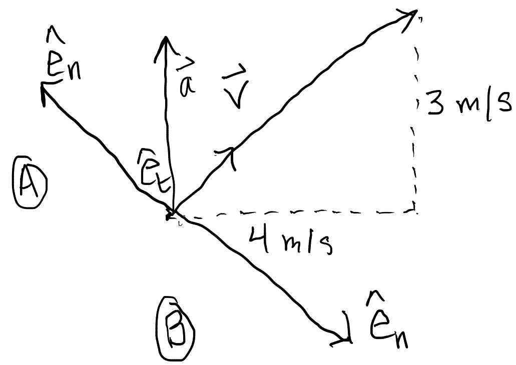
A satellite tracks a car and finds:

$$\vec{v} = 4\hat{i} + 3\hat{j} \ \text{m/s}$$

$$\underline{\vec{a} = 2\hat{j}} \ \text{m/s}^2$$

What are $\hat{e}_t$, $\hat{e}_n$, and radius of curvature ρ?

Sketch trajectory of the car.



$$\hat{e}_t = \frac{\vec{v}}{|\vec{v}|} = \frac{4\hat{i} + 3\hat{j}}{5}$$

$\hat{e}_n$