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## TAM 212 Worksheet 1

### *Solutions*

1. What are the positions of Urbana and Delhi spherical coordinates  $r, \theta, \phi$  (use  $R_E = 6371$  km)?

$$\begin{array}{lll} \text{Urbana} & \phi = 40^\circ 6' 35'' \text{ N} & \lambda = 88^\circ 12' 15'' \text{ W} \\ \text{Delhi} & \phi = 28^\circ 36' 36'' \text{ N} & \lambda = 77^\circ 13' 48'' \text{ E} \end{array}$$

$$\begin{array}{llll} \text{Urbana} & r = 6371 \text{ km} & \theta = -1.539 \text{ rad} & \phi = 0.700 \text{ rad} \\ \text{Delhi} & r = 6371 \text{ km} & \theta = 1.348 \text{ rad} & \phi = 0.499 \text{ rad} \end{array}$$

2. What are the positions of Urbana and Delhi in  $x, y, z$  coordinates?

*Convert spherical to rectangular coordinates:*

$$\begin{array}{llll} \text{Urbana} & x = 153 \text{ km} & y = -4870 \text{ km} & z = 4104 \text{ km} \\ \text{Delhi} & x = 1236 \text{ km} & y = 5454 \text{ km} & z = 3051 \text{ km} \end{array}$$

3. How far is it from Urbana to Delhi in a straight line through the Earth?

$$\|\vec{r}_U - \vec{r}_D\| = 10\,435 \text{ km}.$$

4. How far is from Urbana to Delhi along a great circle route?

*Angle between  $\vec{r}_U$  and  $\vec{r}_D$  gives 12 227 km.*

5. What is the maximum latitude reached as we travel along the great circle between Urbana and Delhi?

*Take normal vector  $\vec{n} = \vec{r}_U \times \vec{r}_D$  and find angle from  $\hat{k}$ , giving  $79.7^\circ$ .*

6. What is the unit vector from Urbana directly towards Delhi in the  $\hat{i}, \hat{j}, \hat{k}$  basis?

*Normalize  $\vec{r}_D - \vec{r}_U$  to give  $0.1038 \hat{i} + 0.9895 \hat{j} - 0.1010 \hat{k}$ .*

7. What is an initial tangent vector to the great circle route from Urbana to Delhi in the  $\hat{i}, \hat{j}, \hat{k}$  basis?

*Take the cross product  $\vec{n} \times \vec{r}_U$  or the orthogonal complement of the last answer with respect to  $\vec{r}_U$  to obtain  $0.1235 \hat{i} + 0.3634 \hat{j} + 0.4266 \hat{k}$  (this is not normalized).*

8. What are the spherical basis vectors  $\hat{e}_r, \hat{e}_\theta, \hat{e}_\phi$  at Urbana, written in the  $\hat{i}, \hat{j}, \hat{k}$  basis?

*Evaluate the explicit formulas:*

$$\hat{e}_r = 0.0240 \hat{i} - 0.7644 \hat{j} + 0.6443 \hat{k}$$

$$\hat{e}_\theta = 0.9995 \hat{i} + 0.0313 \hat{j}$$

$$\hat{e}_\phi = -0.0202 \hat{i} + 0.6439 \hat{j} + 0.7648 \hat{k}$$

9. When we take off from Urbana to fly to Delhi, which compass direction should we initially head in? Is this compass direction maintained as we fly along the great circle?

*The tangent vector is in the  $\hat{e}_\theta, \hat{e}_\phi$  plane. The basis vector  $\hat{e}_\phi$  points North, so find the angle between the tangent vector and  $\hat{e}_\phi$ . This gives a compass bearing of  $13.59^\circ$  (clockwise from North). This is not maintained as we fly along a great circle.*

10. (Bonus question) How many direct flights are there per day from Urbana to Delhi? What factors determine this (e.g., economic, technical, political)?

*There is no direct technical obstacle. For example, Singapore Airlines flies Newark to Singapore direct in an Airbus A340-500, which is 15 345 km, see [http://en.wikipedia.org/wiki/Non-stop\\_flight](http://en.wikipedia.org/wiki/Non-stop_flight). Probably Champaign Airport would require upgrading to land such a plane. There seems little economically reason to fly Urbana-Delhi direct, however.*

11. (Bonus question) What would the path from Urbana to Delhi look like if we did wanted to follow a constant compass direction? What compass direction would this be? This can be best visualized on the Mercator projection.

*Straight line on the Mercator projection gives a bearing of  $94.83^\circ$  (clockwise from North).*