Exam revision

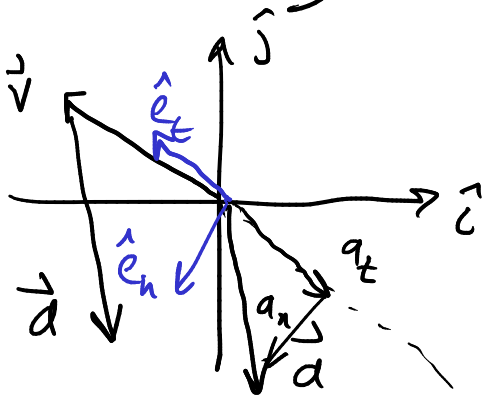
diagrams

$\vec{v} = -3\hat{\imath} + 2\hat{\jmath}$

$\vec{a} = \hat{\imath} - 6\hat{\jmath}$

A: speeding up

B: slowing down



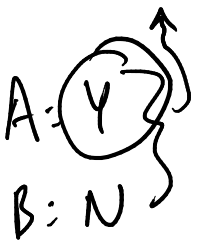
A: CW

B: CCW

$a_t < 0 \Rightarrow$ slowing down

$\dot{v} = \ddot{s} = a_t = \vec{a} \cdot \hat{e}_t = \vec{a} \cdot \hat{v}$

$\vec{a} \cdot \vec{v} < 0$



Magnitudes and derivatives

$\vec{r}$

$\vec{v} = \dot{\vec{r}}$

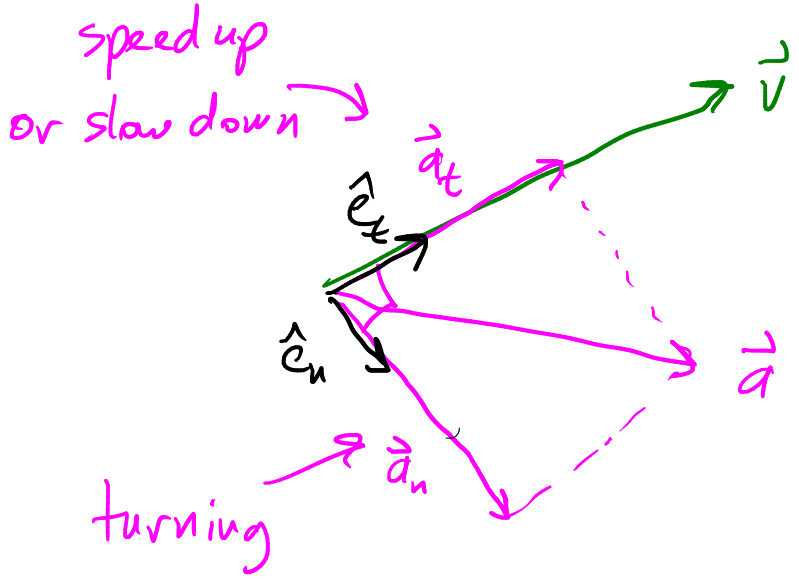
$\vec{a} = \dot{\vec{v}} = \ddot{\vec{r}}$

$\boxed{\dot{s} = v} = \|\vec{v}\| = \|\dot{\vec{r}}\| = \left\|\frac{d}{dt}\vec{r}\right\|$

$\ddot{s} = \dot{v} \neq \left\|\frac{d}{dt}\vec{v}\right\| = \|\vec{a}\| = a$

↑ equal A

not equal B



$$a_t^2 + a_n^2 = a^2$$

$\vec{a}_t$ $\vec{a}_n$

$$\vec{a} = a_t \hat{e}_t + a_n \hat{e}_n$$

$$= \ddot{s}\,\hat{e}_t + \frac{\dot{s}^2}{\rho}\,\hat{e}_n$$

$$a_t = \ddot{s} = \dot{v}$$

$$a_n = \frac{\dot{s}^2}{\rho} = \frac{v^2}{\rho}$$

$$\boxed{\left\| \frac{d}{dt}\vec{v} \right\| \neq \frac{d}{dt}\|\vec{v}\|}$$

not equal

$$\left\| \frac{d}{dt}\vec{v} \right\| = \left\| \dot{v}_x \hat{i} + \dot{v}_y \hat{j} \right\| = \sqrt{\dot{v}_x^2 + \dot{v}_y^2}$$

$$\frac{d}{dt}\|\vec{v}\| = \frac{d}{dt}\sqrt{v_x^2 + v_y^2} = \frac{d}{dt}\left(v_x^2 + v_y^2\right)^{1/2}$$

$$= \frac{1}{2}\left(v_x^2 + v_y^2\right)^{-1/2}\left(2v_x\dot{v}_x + 2v_y\dot{v}_y\right)$$

$$\vec{v} = v\,\hat{v}$$

$$\dot{\vec{v}} = \dot{v}\,\hat{v} + v\,\dot{\hat{v}}$$

$$= \ddot{s}\,\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$