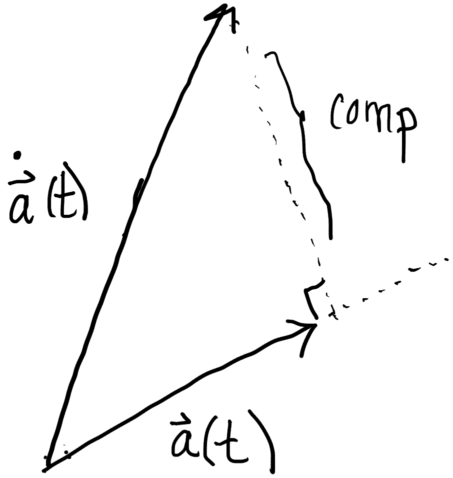


$$\vec{a}(t+\Delta t) \approx \vec{a}(t) + (\Delta t)\dot{\vec{a}}(t)$$

if $\dot{\vec{a}}(t)$ has a non-zero component $\parallel$ to $\vec{a}$

$\Rightarrow$ magnitude of $\vec{a}$ changes

if $\dot{\vec{a}}(t)$ has a non-zero component $\perp$ to $\vec{a}$

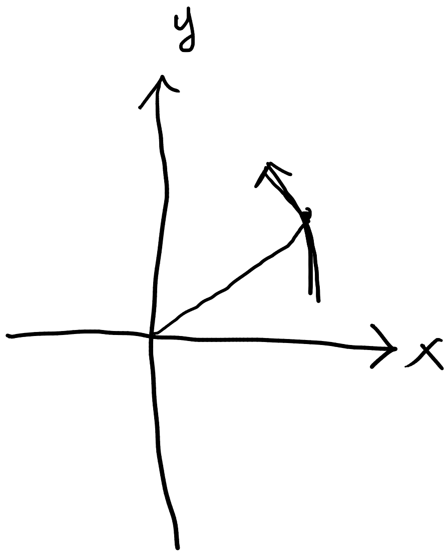
$\Rightarrow$ direction of $\vec{a}$ changes last time: $\vec{a}(t)$

$$\frac{d}{dt} a = \frac{d}{dt} (\vec{a} \cdot \vec{a})^{1/2} = \cdots\cdots = \underbrace{\hat{a} \cdot \dot{\vec{a}}}_{}$$

What about $\frac{d}{dt}(\hat{a})$?

$$\frac{d}{dt}(\hat{a}) = \frac{d}{dt}\left(\frac{\vec{a}}{a}\right) \qquad \hat{a} = \frac{\vec{a}}{a}$$

$$= \frac{a\dot{\vec{a}} - \vec{a}\,\dot{a}}{a^2} = \frac{1}{a}\left[\dot{\vec{a}} - (\dot{\vec{a}} \cdot \hat{a})\hat{a}\right] = \frac{1}{a} \operatorname{comp}(\dot{\vec{a}}, \vec{a})$$



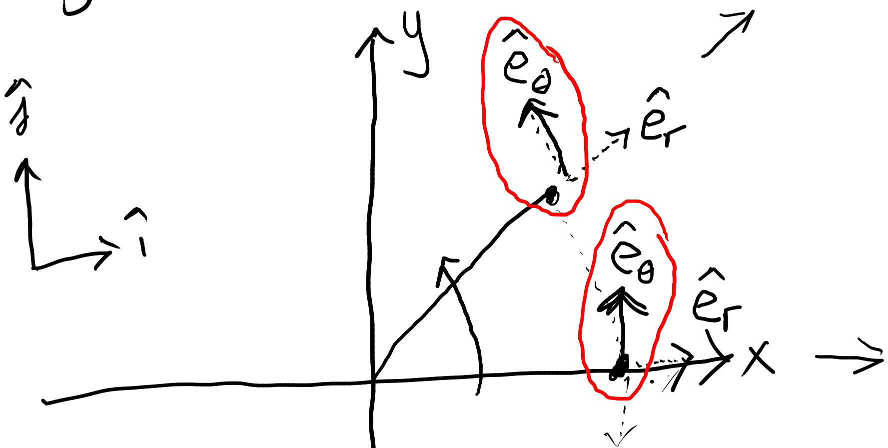
particle moving on a circular trajectory.

The velocity of the particle lies in the ________ direction

A) radial

B) tangential

Cyllindrical Coordinates



particle moving at a constant angular velocity $\dot{\theta}$ (rad/sec)

Question: how do $\hat{e}_r$, $\hat{e}_\theta$ vary in time?

$$\dot{\hat{e}}_r = \dot{\theta}\,\hat{e}_\theta$$

$$\dot{\hat{e}}_\theta = -\dot{\theta}\,\hat{e}_r$$

$$\hat{e}_r = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$$\hat{e}_\theta = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

$$\frac{d}{dt}(\hat{e}_r) \qquad \frac{d}{dt}(\hat{e}_\theta)$$

$\frac{d}{dt}(\hat{e}_r)$ → does $\hat{e}_r$ change in magnitude? No

→ does $\hat{e}_r$ change in direction? Yes

What direction does $\frac{d}{dt}(\hat{e}_r) = \dot{\hat{e}}_r$ lie? $\hat{e}_\theta$

$$\hat{e}_r = \cos\theta \, \hat{\imath} + \sin\theta \, \hat{\jmath}$$

$$\dot{\hat{e}}_r = (-\sin\theta)\dot{\theta}\hat{\imath} + (\cos\theta)\dot{\theta}\hat{\jmath} = \dot{\theta}\,[\underbrace{-\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}}] = \dot{\theta}\,\hat{e}_\theta$$

$\frac{d}{dt}(\hat{e}_\theta)$ → Does the magnitude of $\hat{e}_\theta$ change? No

→ Does the direction of $\hat{e}_\theta$ change? Yes

$\Rightarrow$ The direction of $\dot{\hat{e}}_\theta = \frac{d}{dt}(\hat{e}_\theta)$ is $\underline{\hat{e}_r}$

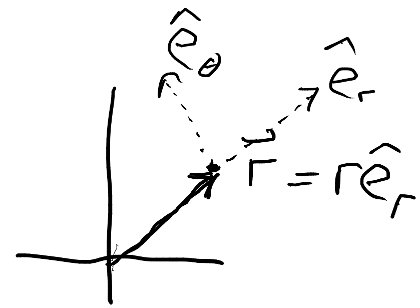
$$\hat{e}_\theta = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

$$\dot{\hat{e}}_\theta = (-\cos\theta)\dot{\theta}\,\hat{i} + (-\sin\theta)\dot{\theta}\,\hat{j} = -\dot{\theta}\left[\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right]$$

$$= -\dot{\theta}\,\hat{e}_r$$

$\vec r = (r, \theta)$ $\qquad$ $\vec r = r\hat e_r$

position in cyllindrical coordinates?



$$\vec v = \frac{d\vec r}{dt} = \frac{d}{dt}(r\hat e_r) = \dot r \hat e_r + r\dot{\hat e}_r \qquad \dot{\hat e}_r = \dot\theta \hat e_\theta$$

$$\vec v = \dot r \hat e_r + r\dot\theta \hat e_\theta$$

for a circular trajectory $r(t) = R$ $\qquad$ $\theta(t) = \dot\theta t$

$$\vec v = 0\hat e_r + r\dot\theta \hat e_\theta$$