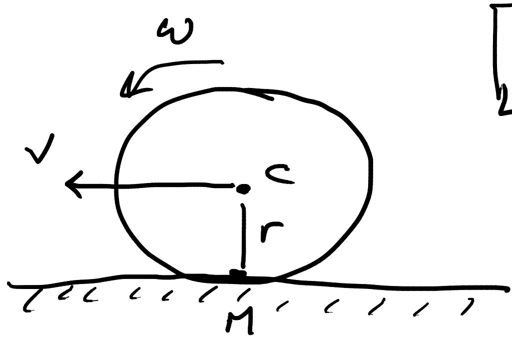
Midterm 2 material

- velocity and acceleration of rigid bodies
- constrained motion of RBs.
- instantaneous center
- gears and chains
- multiple coupled rigid bodies $(\vec{v}, \vec{a}, \vec{\omega}, \vec{\alpha})$
- rolling on flat surfaces
- rolling on curved surfaces.

Sign conventions



$\boxed{V = \omega r}$

only if consistent sign choices

	$\vec{v} = v\hat{i}$	$\vec{v} = -v\hat{i}$
$\vec{\omega} = \omega\hat{k}$	A	B
$\vec{\omega} = -\omega\hat{k}$	C	D

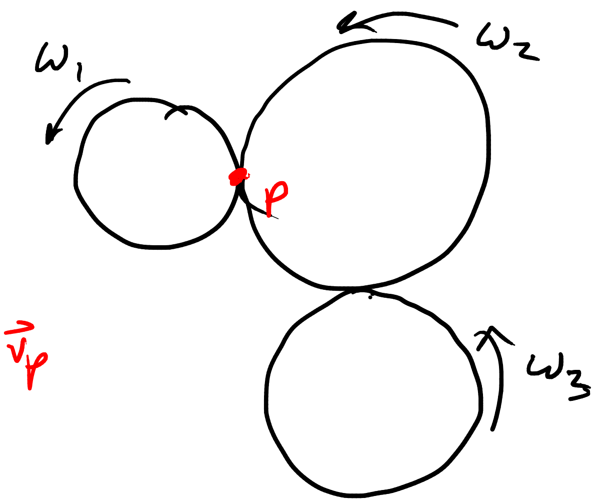
$\vec{v}_c = \vec{\omega} \times \vec{r}_{MC} \Rightarrow A, B, C, D\,?$

always true.

distinguish between: – direction defined as positive

– actual direction of movement

ex



draw arrows in +ve direction for scalar variables.

P direction $\vec{v}_P$

A: ↑

B: ↓

C: ?

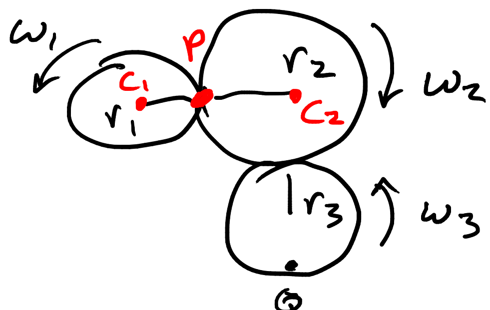
actual direction of $\vec{v}_P$ is unknown until we know whether ω_1 is +ve or −ve.

	A	B
$\vec{\omega}_1 = +\omega_1 \hat{k}$	+	−
$\vec{\omega}_2 = +\omega_2 \hat{k}$		
$\vec{\omega}_3 = +\omega_3 \hat{k}$		

↑ valid choice

$$\vec{v}_P = \vec{\omega}_1 \times \vec{r}_{C_1P} = \vec{\omega}_2 \times \vec{r}_{C_2P}$$

OR



$\vec{\omega}_1 = +\omega_1 \hat{k}$

$\vec{\omega}_2 = -\omega_2 \hat{k}$

$\vec{\omega}_3 = +\omega_3 \hat{k}$

$r_1\omega_1 = r_2\omega_2 = r_3\omega_3$

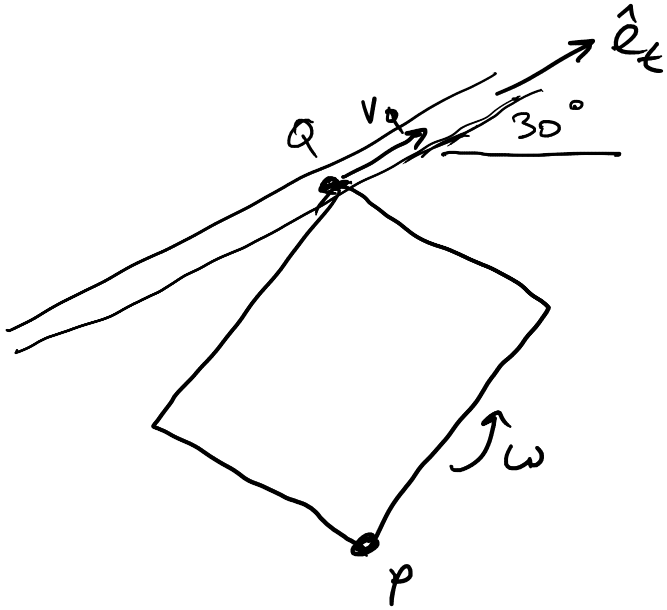
A: true

B: false

C: ?

Q: given $\vec{v}_Q = -2\hat{i}$, what is ω_1?

ex



$\vec{V}_P = 4\hat{\imath} + v_{Py}\hat{\jmath}$

$\vec{\omega} = 2\hat{k}$

$\vec{r}_{PQ} = 2\hat{\jmath}$

Q: what is $\vec{v}_Q$?

$\hat{e}_t = \cos 30° \, \hat{\imath} + \sin 30° \, \hat{\jmath}$

$= \frac{\sqrt{3}}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}$

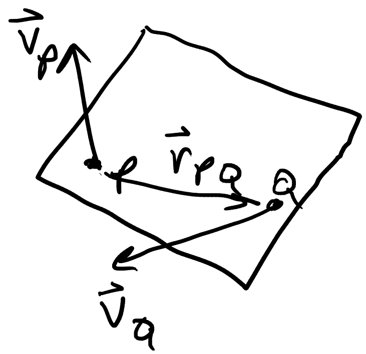
$\vec{V}_Q = v_Q \hat{e}_t = \vec{V}_P + \vec{\omega} \times \vec{r}_{PQ}$

$\Longrightarrow$ solve for v_{Py}, v_Q.

v_Q might be +ve or −ve.

instantaneous centers

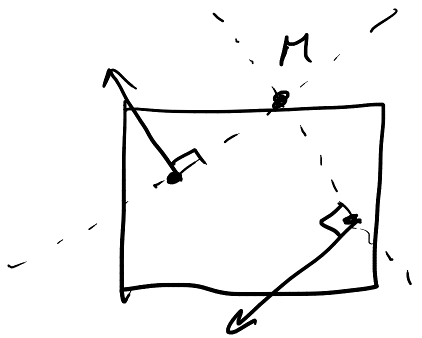
- algebraic: $\vec{v}_M = 0 = \vec{v}_P + \vec{\omega} \times \vec{r}_{PM} \Rightarrow \vec{r}_{PM} = \frac{1}{\omega^2} \vec{\omega} \times \vec{v}_P$

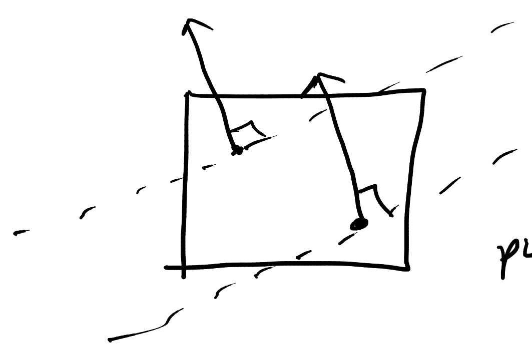


$\vec{v}_Q = \vec{v}_P + \vec{\omega} \times \vec{r}_{PQ} \Rightarrow$ find $\vec{\omega}$

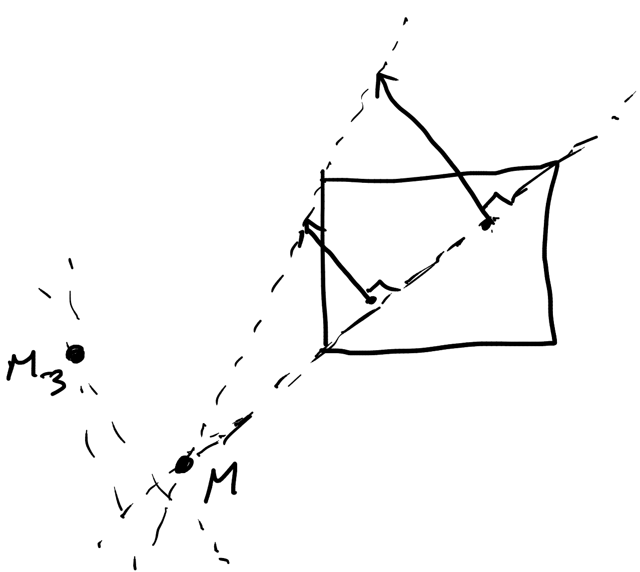
$\Rightarrow$ find $\vec{r}_{PM}$

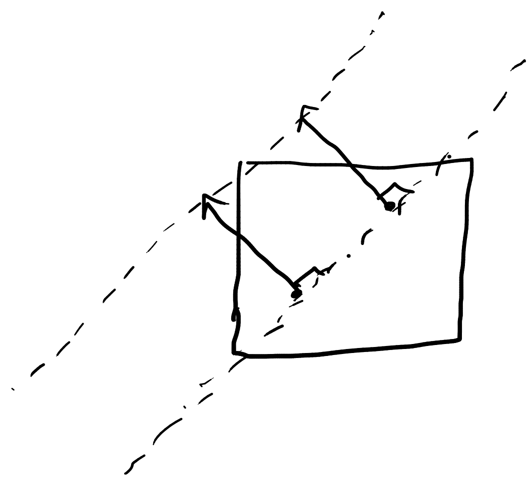
- geometric:





no M
pure trans.





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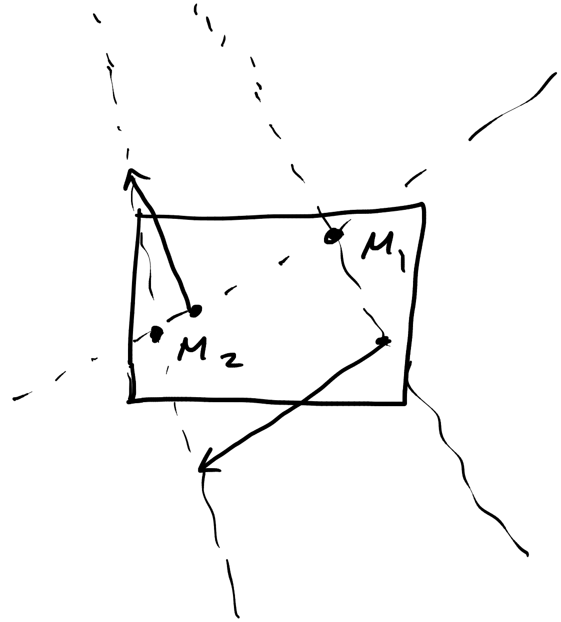

real M?

A: M_1

B: M_2

C: M_3

D: impossible