TAM 212

Euler's Laws: (rigid bodies)

$$\sum \vec{F} = m\vec{a}_c$$

$$\sum \vec{M}_c = I_{c,\hat{k}}\,\vec{\alpha}$$

"moment of inertia"

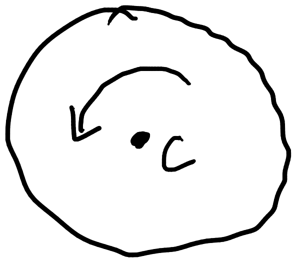
$$I_{c,\hat{k}} = \int \rho r^2 \, dV$$

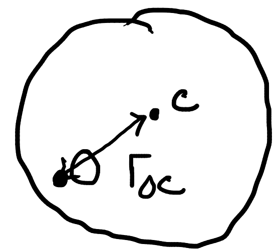
ρ = mass density

r = distance from <u>axis</u> <u>of</u> <u>rotation</u>

sometimes: $I_{c,\hat{k}} = \int r^2 \, dV$

Parallel Axis Theorem: to obtain the moment of inertia $I_{O,\hat{k}}$ about some point O, not necessarily the center of mass (CoM)



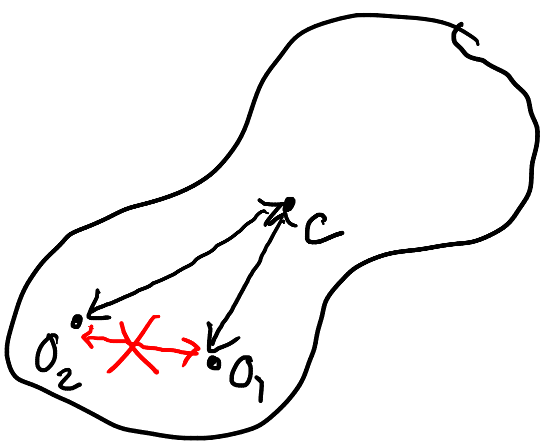


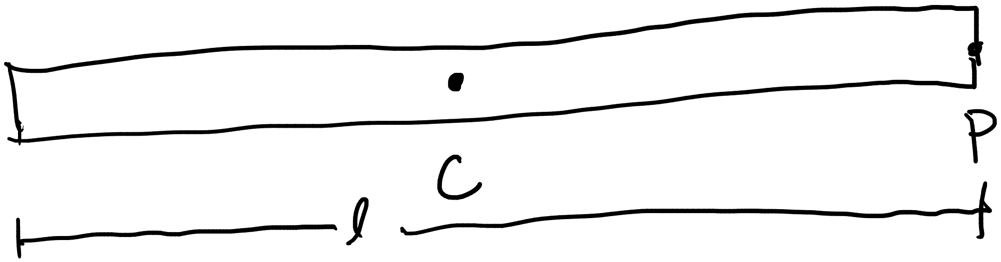
$$I_{c,\hat{k}} < I_{O,\hat{k}}$$

total mass of body

parallel axis thm: $I_{O,\hat{k}} = I_{c,\hat{k}} + m r_{oc}^2$

must be C.O.M.





Ⓐ $I_c > I_p$

Ⓑ $I_c = I_p$

Ⓒ $I_c < I_p$

* moment of inertia is smallest about C

* as we move away from C, moment

$$I_{c,\hat{k}} = \frac{1}{12} ml^2$$

$$I_{p,\hat{k}} = I_{c,\hat{k}} + m\left(\frac{l}{2}\right)^2$$

$$= \frac{1}{12} ml^2 + \frac{1}{4} ml^2$$

$$= \frac{1}{3} ml^2$$

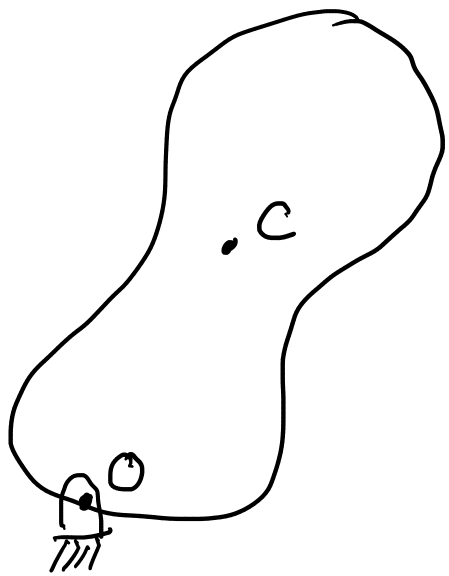
Euler's 2nd Law: $\sum \vec{M}_c = I_{c,\hat{k}} \vec{\alpha}$ ← always true

Alternate form of Euler's 2nd Law:

$$\sum \vec{M}_O = I_{O,\hat{k}} \vec{\alpha}$$

← true if O is a fixed point

"pivot equation"

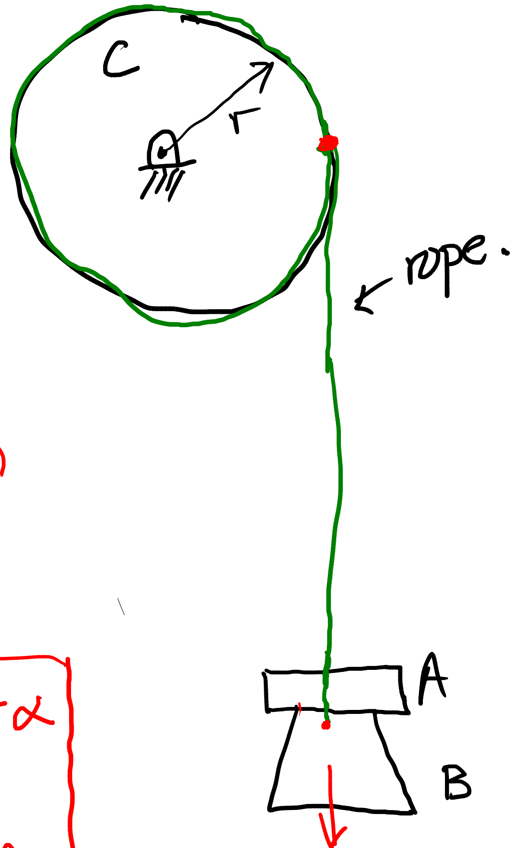


Example:

Rope wrapped around a 10 lbf cylinder, passing through the annular disk A (5 lbf) that is tied to 15 lbf block.

Let the system go from rest.

What is the reaction force exerted by rigid body B on rigid body A?



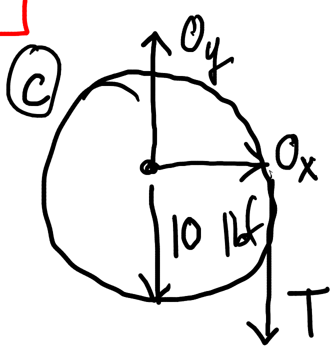
T, R, α,

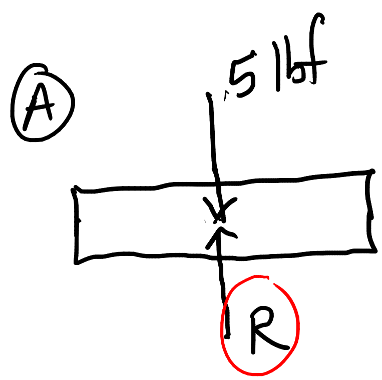
$\ddot{y}_A$, $\ddot{y}_B$

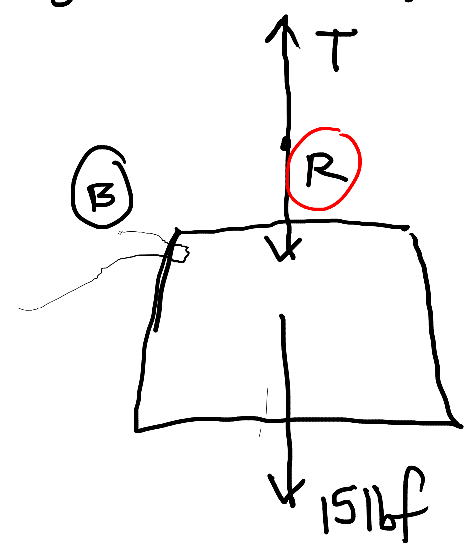
① $\ddot{y}_B = r\alpha$

② $\ddot{y}_B = \ddot{y}_A$

FBD's:







① $\sum \vec{M}_o = I_o \vec{\alpha}$

$$Tr = \left(\tfrac{1}{2} m r^2\right) \alpha$$

$$\boxed{T = \tfrac{1}{2} m_c r \alpha}$$

② $\sum F_y = m_B \ddot{y}_B$

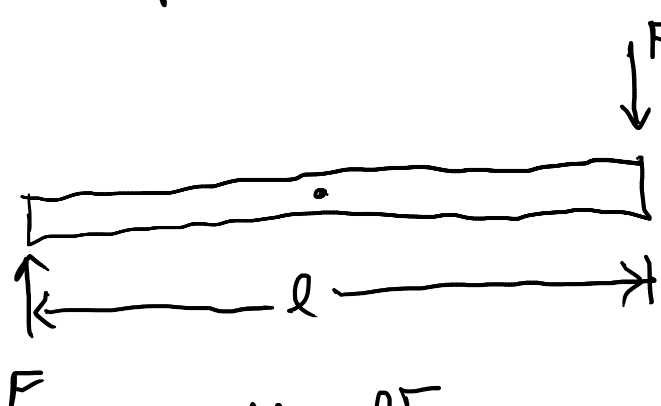
$$\boxed{15 - T + R = m_B \ddot{y}_B}$$

③ $\sum F_y = m_A \ddot{y}_A$

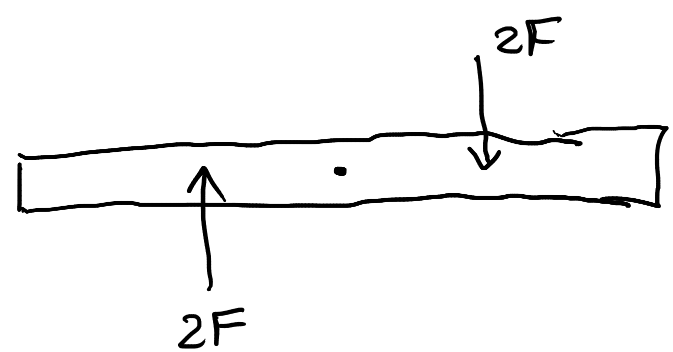
$$\boxed{5 - R = m_A \ddot{y}_A}$$

PURE MOMENTS vs. Regular Moments

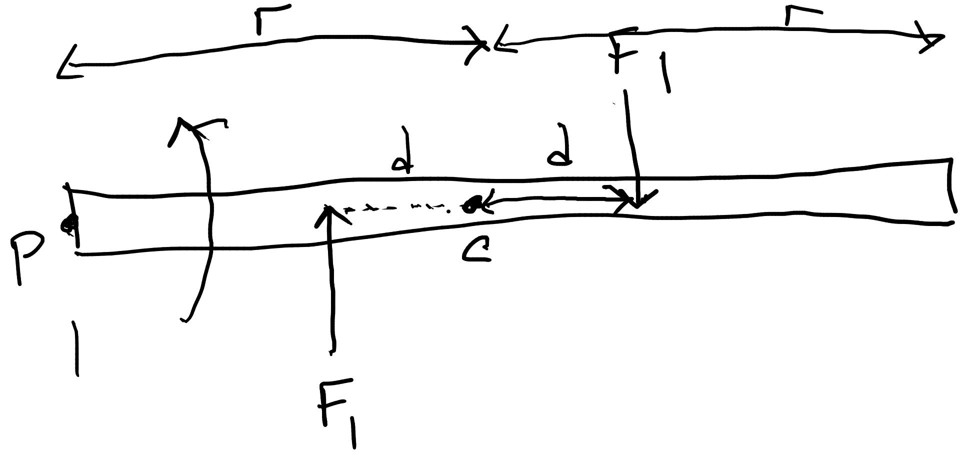
- "pure moments" or "couples" are a system of forces w/ a resultant moment but no resultant force
- creates a pure rotation (but no translation)
- no acceleration of the C.O.M.
- "pure moments" are free vectors, their effect on a body is independent of where we apply the moment



$M = lF$



$M = lF$



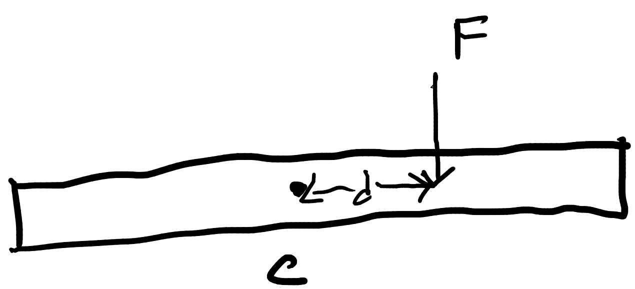
$$\vec{M}_c = -2Fd$$

$$\vec{M}_p = F(r-d) - F(r+d) = 2Fd$$

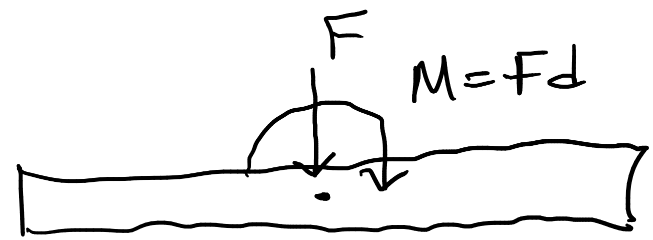
forces : applied to rigid bodies at a distance d from the COM have the same effect as

① the same force applied to the COM **and**

② a pure moment applied to the system Fd



$\Leftrightarrow$



F applied to COM $\Rightarrow$ pure transl.

M $\Rightarrow$ pure rotation