Point masses

$$\vec{p} = m\vec{v}$$

$$\vec{H}_o = \vec{r}_{op} \times m\vec{v}_p$$

$$\vec{M}_o = \vec{r}_{op} \times \vec{F}$$

$$\vec{F} = m\vec{a}$$

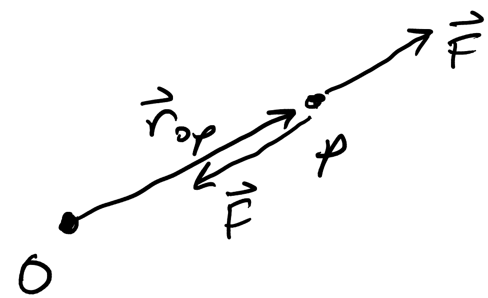
$$\vec{F} = \dot{\vec{p}}$$

$$\vec{M}_o = \dot{\vec{H}}_o$$

momentum: $\vec{F} = 0 \Rightarrow \dot{\vec{p}} = 0 \Rightarrow \vec{p} = \text{constant}$

$\vec{M}_o = 0 \Rightarrow \dot{\vec{H}}_o = 0 \Rightarrow \vec{H}_o = \text{constant}$

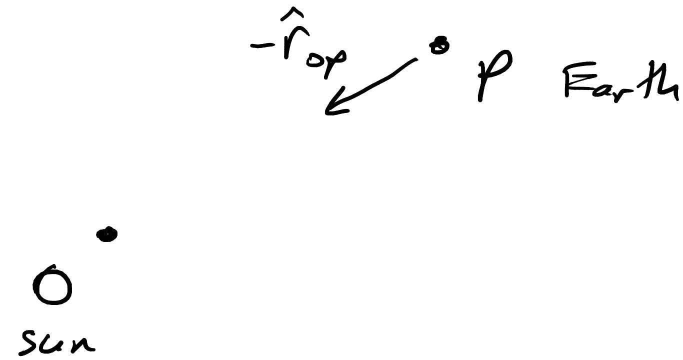
when is $\vec{M}_o = 0$?



radial forces

ex

$$\vec{F} = -\frac{GMm}{r_{op}^2}\hat{r}_{op}$$



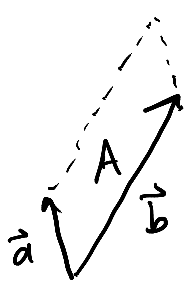
$$\vec{M}_o = \vec{r}_{op} \times \left(-\frac{GMm}{r_{op}^2}\hat{r}_{op}\right) = 0$$

$$\vec{H}_o = \vec{r}_{op} \times m\vec{v}_p$$

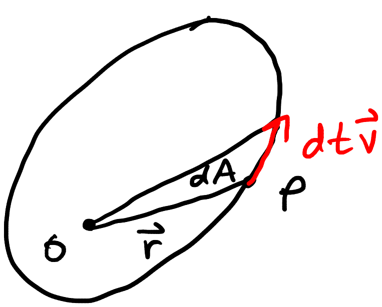
$$\Rightarrow \vec{H}_o = \text{constant}$$

Kepler's 2nd law: "equal area in equal time"

$$\dot{A} = \text{Constant}$$



$A = \|\vec{a} \times \vec{b}\|$



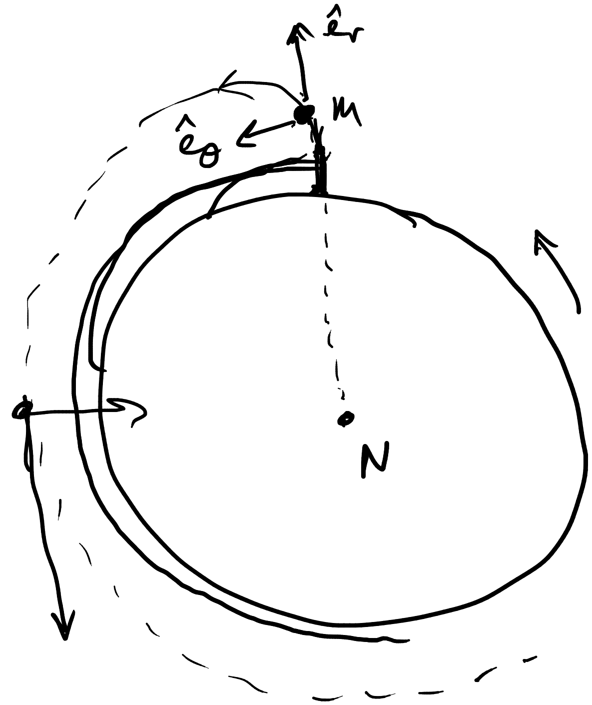
sweep area dA in time dt

$$dA = \frac{1}{2}\|\vec{r} \times dt\vec{v}\|$$

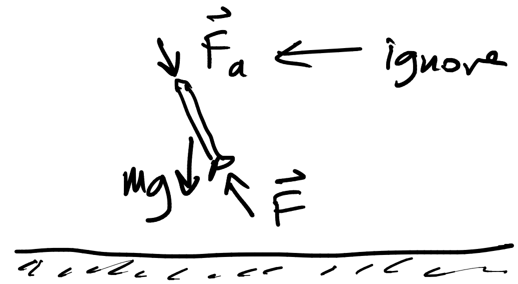
$$= \frac{dt}{2m}\|\vec{r} \times m\vec{v}\| = \frac{dt}{2m}\|\vec{H}_o\|$$

$$\dot{A} = \frac{dA}{dt} = \frac{1}{2m}\|\vec{H}_o\| = \text{Constant}.$$

ex launch satellites to orbit.



Sun rises in the E



$$m\vec a = \vec F$$

$$m(\ddot r - r\dot\theta^2)\hat e_r + m(r\ddot\theta + 2\dot r\dot\theta)\hat e_\theta = -mg\hat e_r + F_r\hat e_r + F_\theta\hat e_\theta$$

$$m\ddot r - mr\dot\theta^2 = -mg + F_r$$

$$mr\ddot\theta + 2m\dot r\dot\theta = F_\theta$$