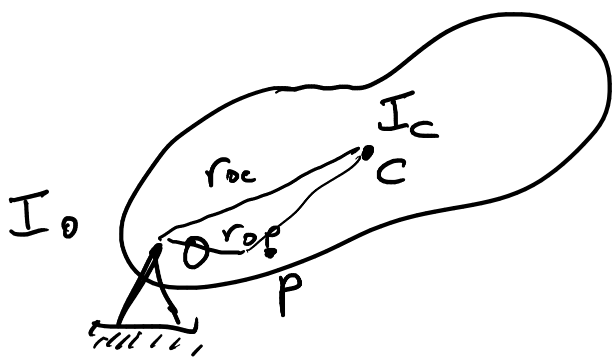
Rigid Bodies

$$\sum \vec{F} = m\vec{a}_c$$

$$\sum \vec{M}_c = I_{c,\hat{k}} \, \vec{\alpha}$$

$$I_{c,\hat{k}} = \int_V \rho r^2 \, dV$$

r = distance from axis of rotation



parallel axis thm:

$$I_o = I_c + m r_{oc}^2$$

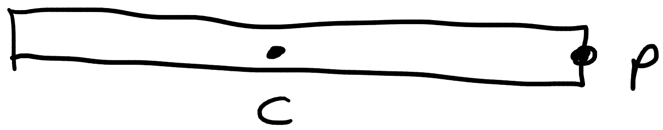
must be C

total mass of body

$$I_p = I_o + m r_{op}^2$$

$$I_c = I_p + m r_{pc}^2 = I_o + m r_{op}^2 + m r_{pc}^2$$

$$= I_c + m r_{oc}^2 + m r_{op}^2 + m r_{pc}^2$$



A $I_c > I_p$

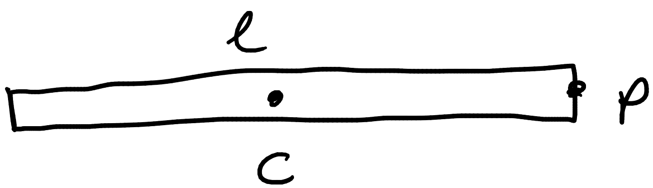
B $I_c = I_p$

C $I_c < I_p$ *

$I = \int \rho r^2 dV$

- move away from C always increases I
- moment of inertia is smallest at C.

<u>ex</u>



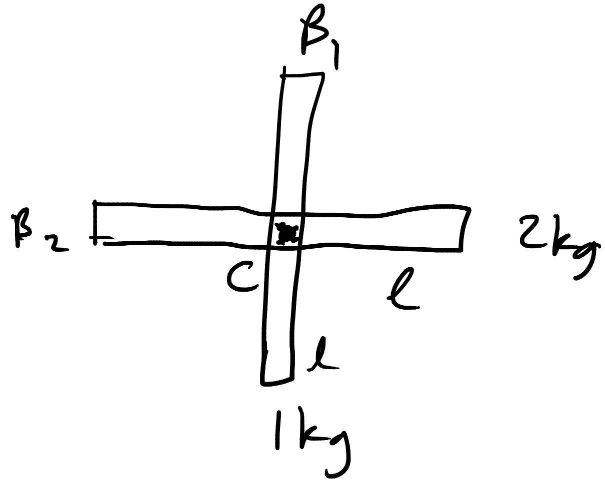
$$I_c = \frac{1}{12} m l^2$$

$$I_p = I_c + m\left(\frac{l}{2}\right)^2$$

$$= \frac{1}{12} m l^2 + \frac{1}{4} m l^2$$

$$= \frac{1}{3} m l^2$$

moments of inertia add.



$$I_C = I_{C,B1} + I_{C,B2}$$

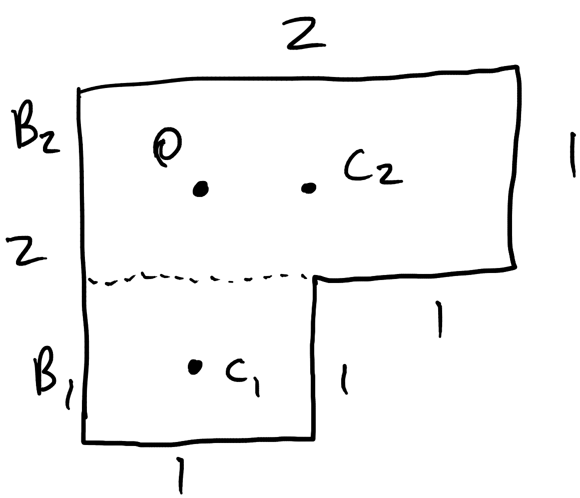
$$= \frac{1}{12} \cdot 1 \cdot l^2 + \frac{1}{12} \cdot 2 \cdot l^3$$

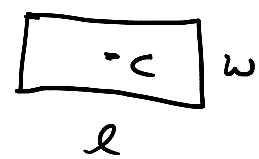
$$= \frac{3}{12} l^2$$

$$= \frac{1}{4} l^2$$

ex

$\rho = 1 kg/m^2$

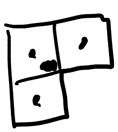




$$I_c = \frac{1}{12} m (l^2 + w^2)$$

$$I_{C_1}^{B_1} = \frac{1}{12} m_1 \cdot 2 = \frac{1}{6} m_1 = \frac{1}{6}$$

$$I_{C_2}^{B_2} = \frac{1}{12} \underset{2}{\cancel{m_2}} (2^2 + 1^2) = \frac{5}{6}$$

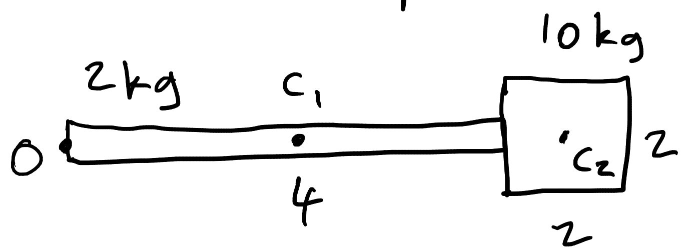


$$I_O^{B_1} = I_{C_1}^{B_1} + m_1 r_{OC_1}^2 = \frac{1}{6} + 1.1 = \frac{7}{6}$$

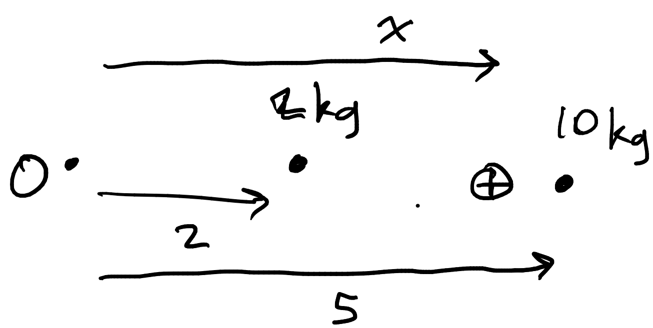
$$I_O^{B_2} = I_{C_2}^{B_2} + m_2 r_{OC_2}^2 = \frac{5}{6} + 2.\frac{1}{4} = \frac{8}{6}$$

$$I_O = I_O^{B_1} + I_O^{B_2} = \frac{7}{6} + \frac{8}{6} = \frac{15}{6}$$

center of mass of coupled bodies:



when combing COM, regard each body as concentrated at its own COM.



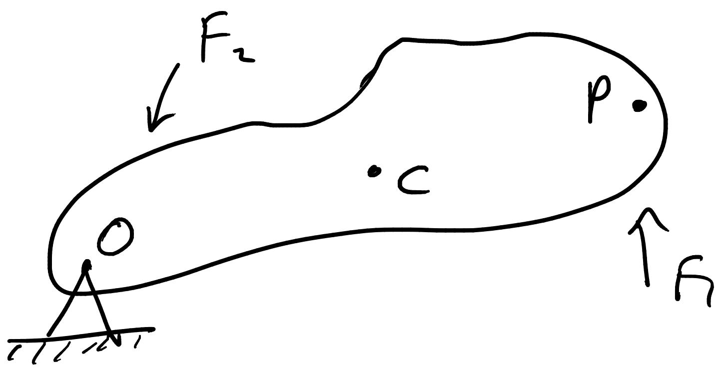
$$x = \frac{1}{2+10}(2.2 + 10.5)$$

$$= \frac{54}{12}\ m.$$

Alternative form of rotation equ.

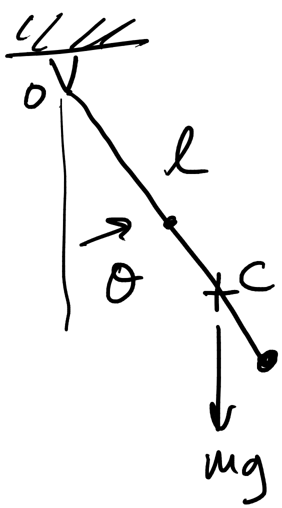
$$\sum \vec{M}_c = I_c \vec{\alpha} \quad \text{always true.}$$

$$\sum \vec{M}_O = I_O \vec{\alpha} \quad \text{if } O \text{ is a fixed point.}$$



$$\cancel{\sum \vec{M}_P = I_P \vec{\alpha}}$$

ex



rod mass 1kg

point mass 1kg

$$r_c = \frac{3}{4} l$$

what is $\ddot{\theta}$?