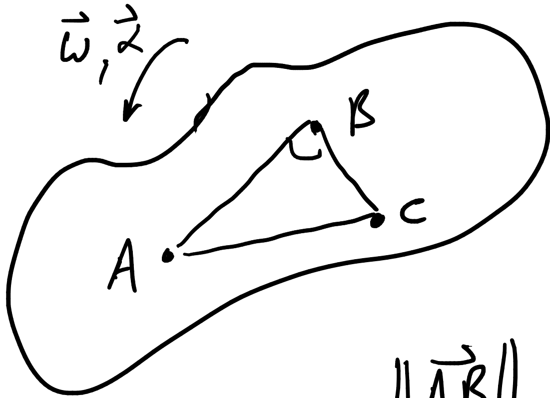
# Rigid Bodies




rigid body: distances and angles between all points on the body are fixed.

$$\|\vec{AB}\| = \text{constant}$$

$$\angle ABC = \text{constant}$$

center of mass:

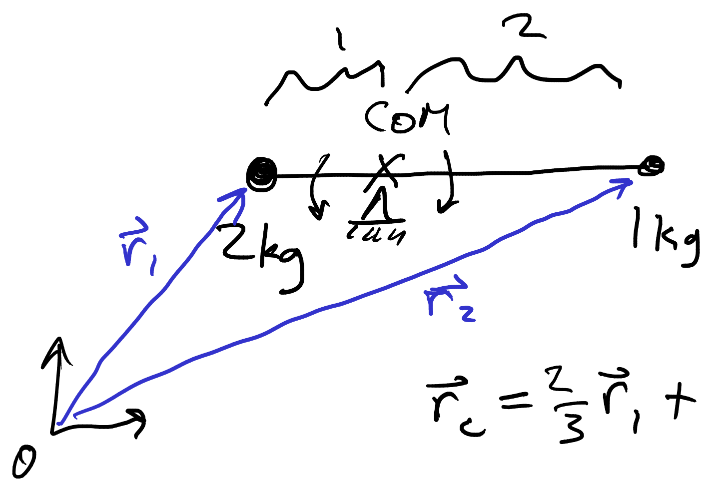
$$\vec{r}_c = \frac{1}{m}\int_V \rho \vec{r}\, dV \qquad m = \int_V \rho\, dV$$

density

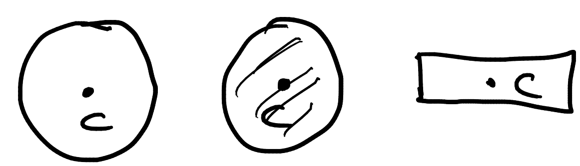
point masses $m_i$ at $\vec{r}_i$:

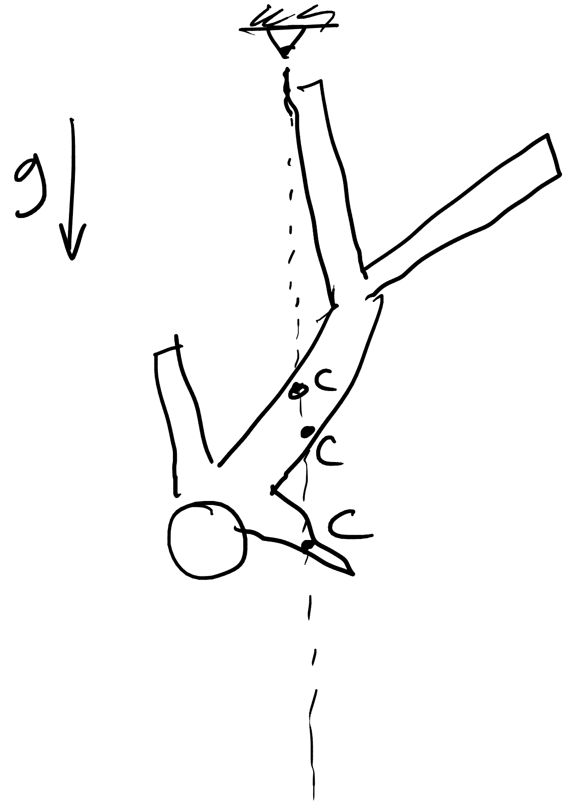
$$\vec{r}_c = \frac{1}{m}\sum_i m_i \vec{r}_i \qquad m = \sum_i m_i$$

$$= \sum_i \frac{m_i}{m}\vec{r}_i$$

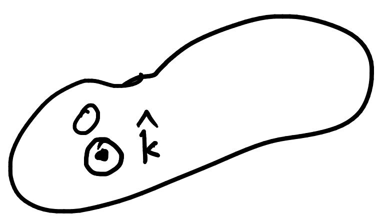



$$\vec{r}_c = \frac{2}{3}\vec{r}_1 + \frac{1}{3}\vec{r}_2$$






**Moment of inertia:** rotation about a fixed axis.



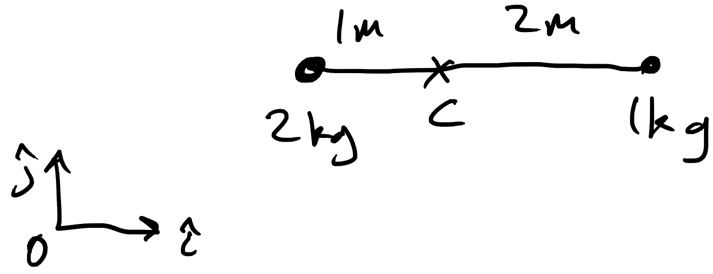

$$I_{O,\hat{k}} = \int_V \rho r^2 \, dV$$

$r$ = distance from axis $\hat{k}$

point masses: $I_{O,\hat{k}} = \sum_i m_i r_i^2$

$r_i$ = distance from $\hat{k}$ to $m_i$

ex




$$I_{C,\hat{k}} = 2\cdot 1^2 + 1\cdot 2^2 = 6 \ kgm^2$$

point

axis of rotation.

## Equations of motion:


$$\vec{M}_{C,1} = \vec{r}_1 \times \vec{F}_1$$

$$\vec{M}_{C,2} = \vec{r}_2 \times \vec{F}_2$$

$$m\vec{a}_C = \sum_i F_i$$

$$I_{C,\hat{k}}\,\vec{\alpha} = \sum_i M_{C,i}$$