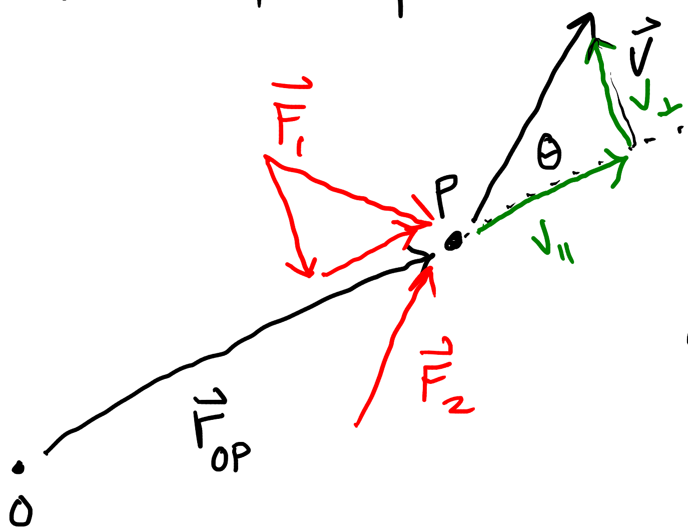
TAM 212

Recall: for point masses (mass m)



linear momentum : $\vec{p} = m\vec{v}$

angular momentum : $\vec{H}_O = \vec{r}_{OP} \times m\vec{v}$

$$= r_{OP}\, m\, v_\perp$$

$$= r_{OP}\, m\, \underline{v \sin\theta}$$

(1) Externally applied forces $\vec{F}_1, \vec{F}_2$ change the linear momentum of particle

$$\sum \vec{F} = \vec{F}_1 + \vec{F}_2 = \dot{\vec{p}} = m\dot{\vec{v}} = m\vec{a} \quad \text{(constant mass)}$$

(2) Externally applied moments $\vec{M}$ change the angular momentum

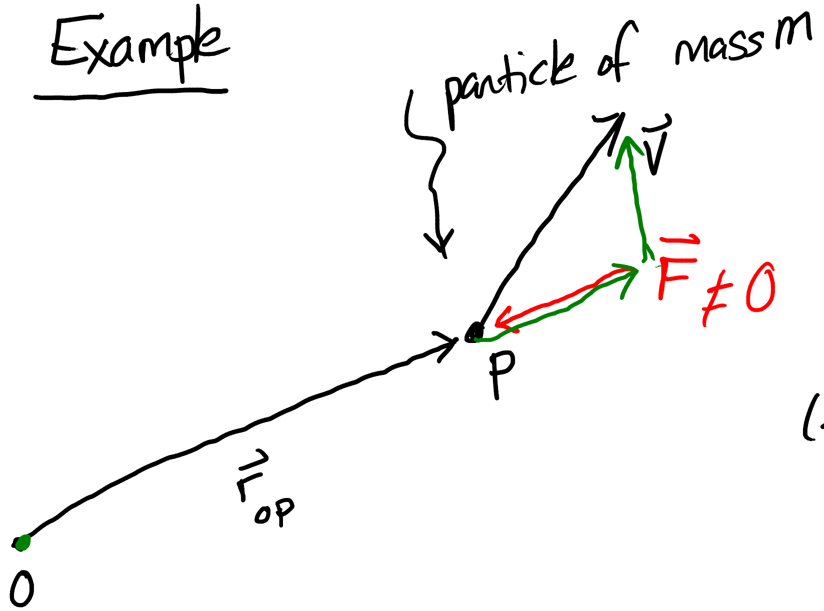
$\vec{M} = \vec{r}_{OP} \times \vec{F}$

properties of system:

linear momentum $\vec{p}$ $\longrightarrow$ externally applied forces $\vec{F}$

angular momentum $\vec{H}_o$ $\longrightarrow$ externally applied moments $\vec{M}$

Example



At this instant:

(1) Is the linear momentum $\vec{p}$ changing?

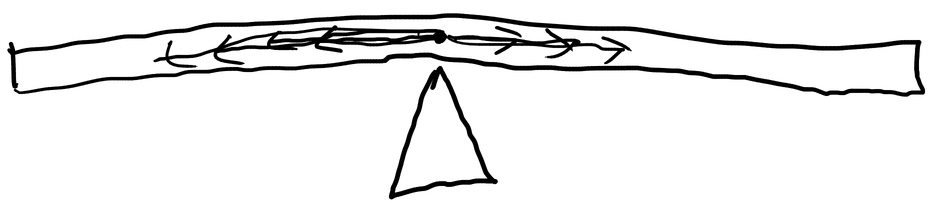
A) YES B) NO

(2) Is the angular momentum about O changing?

A) YES B) NO

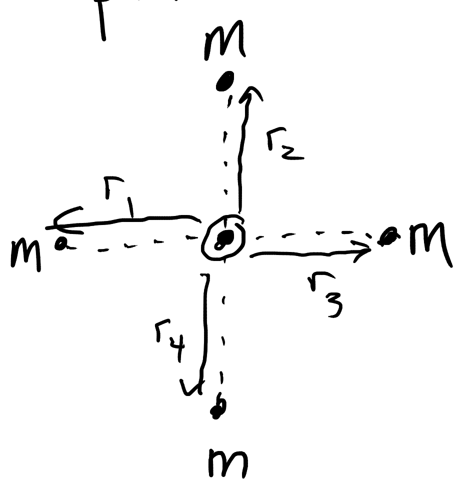
Formulation of Laws for Rigid Bodies

CENTER OF MASS of a rigid body - unique point of a rigid body where the weighted relative position of the distributed mass sums to zero.

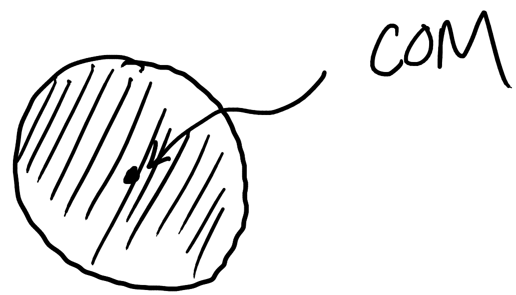


Ex)

point masses:



rigid body



total mass $M = \sum m_i$

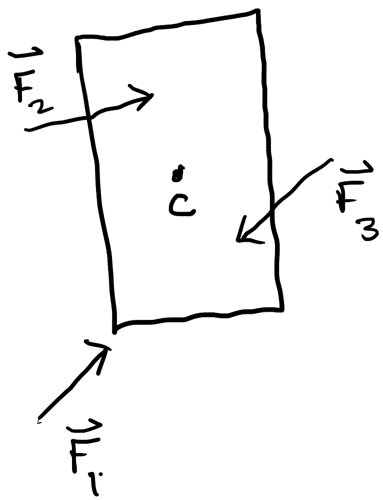
center of mass: $\vec{r}_c = \frac{1}{M} \sum m_i \vec{r}_i$

$$M = \int_V \rho(\vec{r})\, dV$$

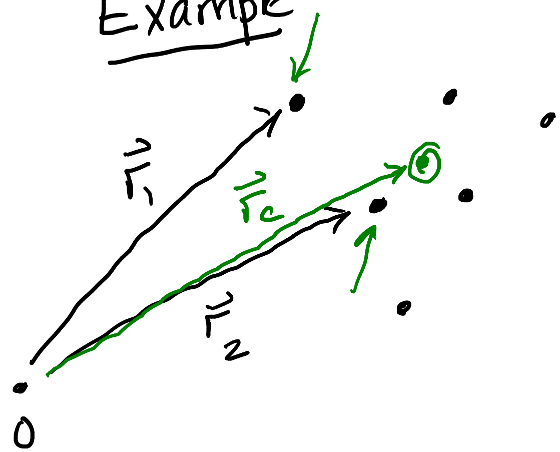
$$\vec{r}_c = \frac{1}{M} \int \rho(\vec{r})\, \vec{r}\, dV$$

Linear Momentum Law for Rigid Bodies

$$\boxed{\sum \vec{F}_i = M\vec{a}_c} = M\dot{\vec{v}}_c$$



Example



$$\vec{r}_i = (\vec{r}_i - \vec{r}_c) + \vec{r}_c$$

$$\vec{v}_i = \frac{d\vec{r}_i}{dt}$$

$$= \frac{d}{dt}(\vec{r}_i - \vec{r}_c) + \frac{d\vec{r}_c}{dt}$$

$$= \frac{d}{dt}(\vec{r}_i - \vec{r}_c) + \vec{v}_c$$

linear momentum:

$$\vec{P} = \sum m_i \vec{v}_i$$

$$= \sum m_i \left[\frac{d}{dt}(\vec{r}_i - \vec{r}_c) + \vec{v}_c\right]$$

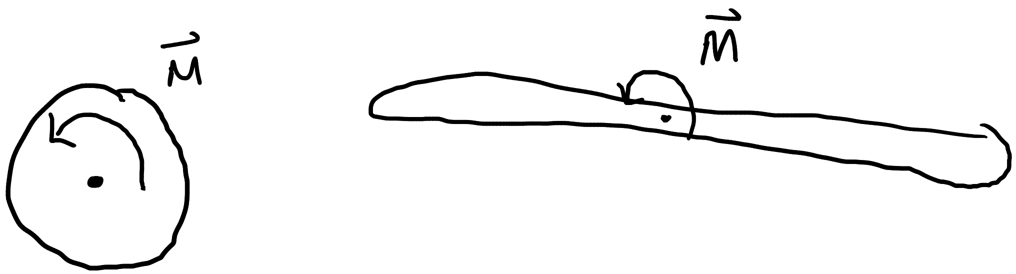
$$= \frac{d}{dt}\Big(\underbrace{\sum m_i(\vec{r}_i - \vec{r}_c)}_{0}\Big) + \left(\sum m_i\right)\vec{v}_c = M\vec{v}_c$$

Angular Momentum Law for Rigid Bodies

MOMENT OF INERTIA I_c

$\vec{F} = m\vec{a}$ $\quad$ $\vec{a} = \vec{F}/m$ $\quad$ m = resistance to linear acceleration

by analogy $\quad$ I_c = resistance to angular acceleration



Formally, the moment of inertia is