Point masses

$$\boxed{\vec{F} = m\vec{a}}$$

$$\boxed{\dot{\vec{p}} = \vec{F}}$$

defns
$$\left\{\begin{array}{l} \vec{p} = m\vec{v} \\ \vec{H}_o = \vec{r}_{op} \times m\vec{v}_p \\ \vec{M}_o = \vec{r}_{op} \times \vec{F} \end{array}\right.$$

$\vec{F}$ is applied at P.

$$\boxed{\vec{M}_o = \dot{\vec{H}}_o}$$

Momentum formulas: $\vec{F} = 0$ (no net external force)

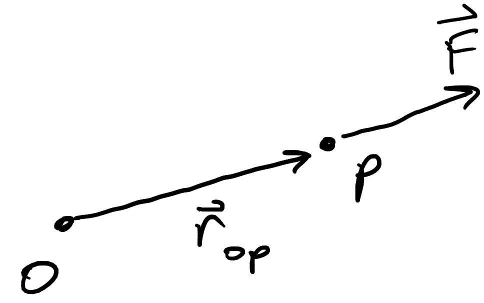
$\Rightarrow \dot{\vec{p}} = 0 \Rightarrow \vec{p}$ constant.

also $\vec{M}_o = 0$ (no net external moment)

$\Rightarrow \dot{\vec{H}}_o = 0 \Rightarrow \vec{H}_o =$ constant.

when is $\vec{M}_o = 0$?

<u>radial force</u>



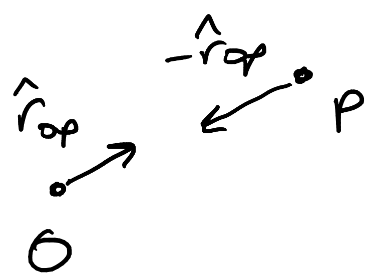
$$\vec{M}_o = \vec{r}_{op} \times \vec{F}$$

zero moment.

<u>ex</u> $\vec{F} = \dfrac{-GMm}{r_{op}^2}\,\hat{r}_{op}$

sun M at O

Earth m at P



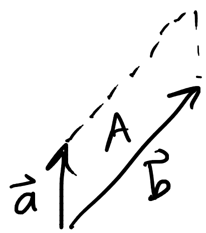
moment due to gravity

$$\vec{M}_o = \vec{r}_{op} \times \vec{F}$$

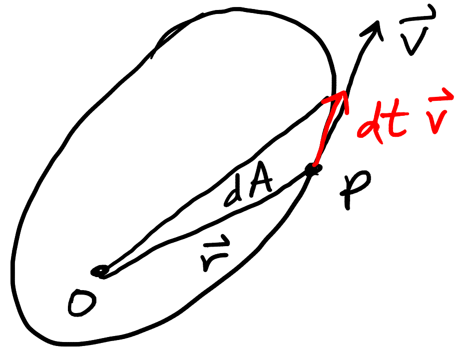
$$= \vec{r}_{op} \times \left(-\frac{GMm}{r_{op}^2}\,\hat{r}_{op}\right)$$

$$= 0$$

for gravity, $\vec{H}_o = \text{constant}$.



$A = \|\vec{a} \times \vec{b}\|$



dA is the area covered in time dt

$$dA = \_\_\ \vec{r}\ \_\_\ dt\,\vec{v}$$

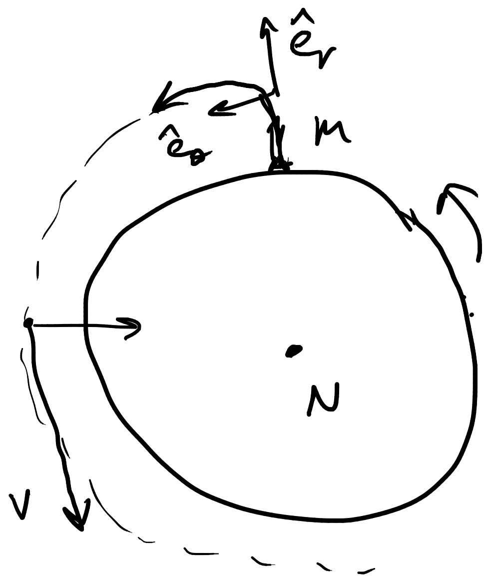
$$= \frac{1}{2}\|\vec{r} \times dt\,\vec{v}\|$$

$$= \frac{dt}{2m}\|\vec{r} \times m\vec{v}\|$$

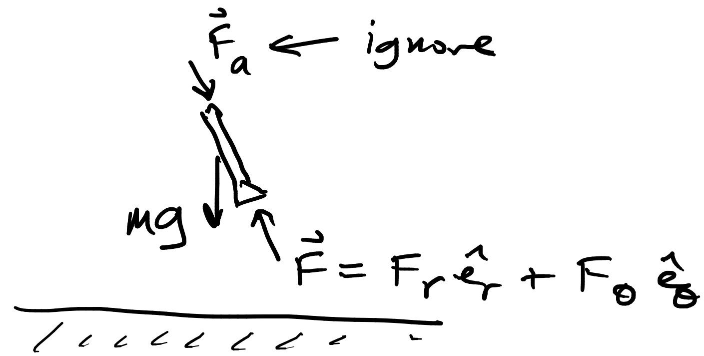
$$\dot{A} = \frac{dA}{dt} = \frac{1}{2m}\|\vec{H}_o\| = \text{constant}.$$

the area sweep rate of a planet is constant. — Kepler's 2nd law.

ex lauch to orbit.



Sun rises in the E.



$$m\vec{a} = \vec{F}$$

$$m(\ddot{r} - r\dot{\theta}^2)\hat{e}_r + m(r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta = -mg\,\hat{e}_r + F_r\hat{e}_r + F_\theta\hat{e}_\theta$$

$$m\ddot{r} - mr\dot{\theta}^2 = -mg + F_r$$

$$mr\ddot{\theta} + 2m\dot{r}\dot{\theta} = F_\theta$$