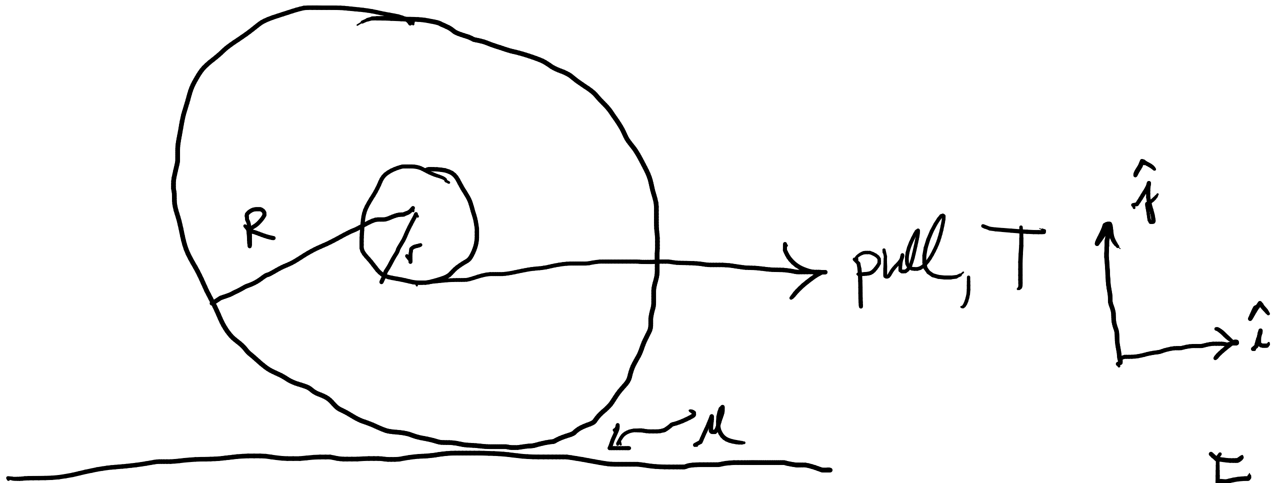
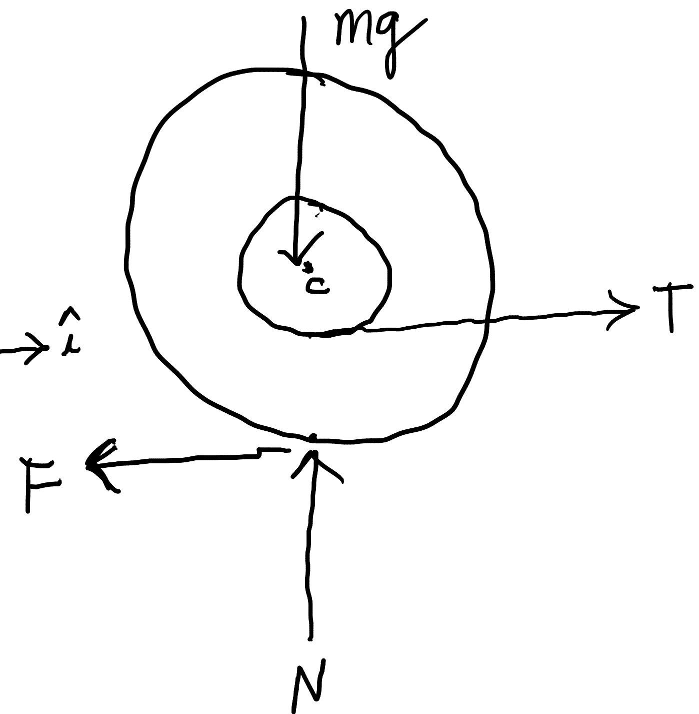
TAM 212

Find $\vec{\alpha}$, friction force arising from applied tension T

$\vec{\alpha} = (-\alpha)\hat{k}$





kinematics $\vec{a}_c = R\alpha\hat{\imath}$

(1) $\sum F_x = m a_{cx} = mR\alpha$

(2) $\sum F_y = m a_{cy} = 0$

(3) $\sum \vec{M}_c = I_{c,\hat{k}}\vec{\alpha}$

(1) $T - F = mR\alpha$

$$\boxed{\alpha = \frac{T-F}{mR}}$$

(2) $N - mg = 0$

$$\boxed{N = mg}$$

(3) $(Tr - FR)\hat{k} = I_{c,\hat{k}}\vec{\alpha}$

$$I_{c,\hat{k}} = mR^2$$

$$\vec{\alpha} = (-\alpha)\hat{k}$$

$$Tr - FR = -mR^2\alpha$$

$$F = \frac{Tr + mR^2\alpha}{R}$$

$$\boxed{F = T\left(\frac{r}{R}\right) + mR\alpha}$$

Solving the system of equations:

$$F = T\left(\frac{r}{R}\right) + mR\alpha$$

$$F = T\left(\frac{r}{R}\right) + \cancel{m}R\left(\frac{T-F}{\cancel{mR}}\right)$$

$$\boxed{F = \frac{1}{2}T\left(1 + \frac{r}{R}\right)}$$

$$\alpha = \frac{T-F}{mR}$$

$$= \frac{T}{mR} - \frac{1}{2mR}\,T\left(1 + \frac{r}{R}\right)$$

$$\boxed{\alpha = \frac{T}{2mR}\left(1 - \frac{r}{R}\right)}$$

$\alpha > 0$, CW rolling

Where is the onset of slipping?

- to maintain rolling $\quad 0 \leq F \leq \mu N$
- slipping occurs exactly when F reaches critical value μN

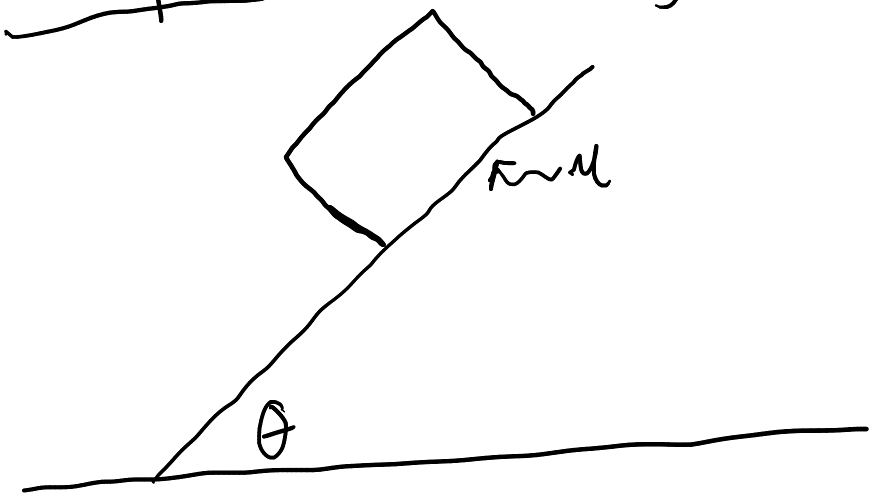
$$F_{cr} = \mu N = \frac{1}{2} T_{cr} \left(1 + \frac{r}{R}\right)$$

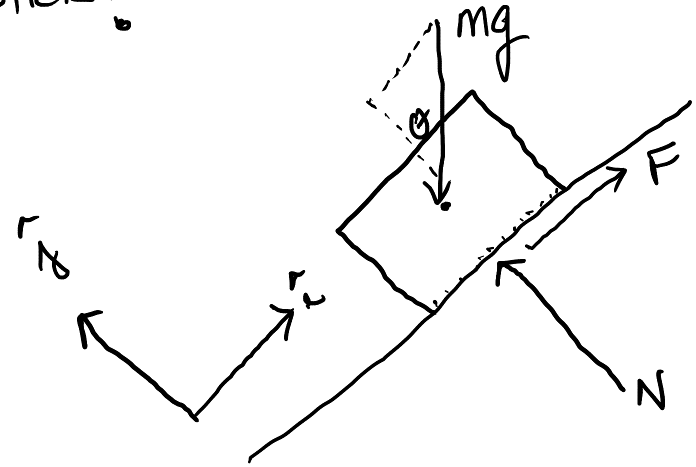
$$T_{cr} = \frac{2\mu N}{1 + \frac{r}{R}} = \frac{2\mu N R}{R + r}$$

to induce slipping: ① pull so that $T > T_{cr}$

② low friction surface (μ small)

Example) Block sitting on inclined surface. Does it slide or stick?





We need to consider both possibilities:

① block sticks

$\boxed{|F| < \mu N \iff V_c = \text{speed of any contact point} = 0}$

V_c is known but $|F|$ is not known

② slipping occurs

$\boxed{|F| = \mu N \iff V_c \neq 0}$

$|F|$ is known

direction of is known (opposes direction of slip) } but V_c unknown complimentarity condition $\underbrace{(|F| - \mu N)\, v_c = 0}$

case of sticking: $v_c = 0$

case of sliding $|F| = \mu N$

case 1: assume block sticks

$\sum F_x = m a_{cx} = 0 \implies F - mg \sin\theta = 0$ $\quad \boxed{F = mg \sin\theta}$

$\sum F_y = m a_{cy} = 0 \implies N - mg \cos\theta = 0$ $\quad \boxed{N = mg \cos\theta}$

Now: check assumption. Is $|F| \leq \mu N$?

$mg \sin\theta \leq \mu\, mg \cos\theta$?

"Method of assumed motion"

Yes. we're done, block sticks.

No. The block is slipping. $|F| = \mu N$. Need to solve for $\vec{a}_c$.

case 2) Block is slipping

"Method of assumed forces"

$$\sum F_x = m a_{cx}$$

$$\sum F_y = m a_{cy} = 0$$

$$\boxed{\mu N - mg \sin\theta = m a_{cx}}$$

$$\boxed{N - mg\cos\theta = 0}$$

$$a_{cx} = \frac{\mu N - mg\sin\theta}{m} = \frac{\mu mg\cos\theta - mg\sin\theta}{m}$$

$$a_{cx} = \mu g\cos\theta - g\sin\theta$$