Name: ________________ Group members: ________________ ________________ ________________

TAM 212 Worksheet 2

Solutions

1. The earth revolves around the sun in the counterclockwise direction, completing one full revolution every 365 days. In reality the orbit is elliptical (with an eccentricity of 0.0167), but we'll pretend that the orbit is a perfect circle. How many revolutions per day does the earth make around the sun? What is the angular velocity $\omega_{\rm SE}$ (radians/day) of the earth around the sun?

$1/365$ *revolutions per day;* $\omega_{\rm SE} = 2\pi/365$ *rad/day.*

2. With the sun at the origin, write the position vector for the earth as a function of time in both the cartesian and polar basis. You can leave your answer in terms of the earth's angular position $\theta_{\rm E}$ and the distance between the sun and the earth $R_{\rm SE} = 1$ AU (1 AU = 149,597,871 km, but we recommend working in AU).

cartesian: $\vec{r} = R_{\rm SE}\cos(\theta_{\rm E})\ \hat{\imath} + R_{\rm SE}\sin(\theta_{\rm E})\ \hat{\jmath}$
polar: $\vec{r} = R_{\rm SE}\ \hat{e}_r$.

3. What is the expression for the velocity and acceleration of the earth in the cartesian basis? In the polar basis? Leave your answer in terms of θ_E and $\dot{\theta}_E$, recalling that $\dot{\theta}_E = \omega_{\rm SE}$.

cartesian:
$\vec{v} = -R_{\rm SE}\dot{\theta}_{\rm E}\sin(\theta_{\rm E})\ \hat{\imath} + R_{\rm SE}\dot{\theta}_{\rm E}\cos(\theta_{\rm E})\ \hat{\jmath}$
$\vec{a} = -R_{\rm SE}\dot{\theta}_{\rm E}^2\cos(\theta_{\rm E})\ \hat{\imath} - R_{\rm SE}\dot{\theta}_{\rm E}^2\sin(\theta_{\rm E})\ \hat{\jmath}$

polar:
$\vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta = R_{\rm SE}\dot{\theta}_{\rm E}\,\hat{e}_\theta$
$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{e}_\theta = -R_{\rm SE}\dot{\theta}_{\rm E}^2\,\hat{e}_r$

4. On the diagrams provided, plot the radial and tangential velocity and acceleration of the earth.

5. We all know that the moon revolves around the earth. However, when observed from the vantage point of a fixed sun, the path of the moon can be surprising. The moon orbits the earth in the counterclockwise direction, completing one full revolution every 28 (solar) days at a distance of $R_{\mathrm{EM}} = 0.0026$ AU. Write down in the cartesian basis the position of the moon relative to the earth in terms of R_{EM} and the moon's angular position θ_{M}. Then write down in the cartesian basis the position of the moon relative to the sun.

$$\vec{r}_{\mathrm{M/E}} = R_{\mathrm{EM}} \cos(\theta_{\mathrm{M}})\ \hat{\imath} + R_{\mathrm{EM}} \sin(\theta_{\mathrm{M}})\ \hat{\jmath}$$
$$\vec{r}_{\mathrm{M/S}} = (R_{\mathrm{SE}} \cos(\theta_{\mathrm{E}}) + R_{\mathrm{EM}} \cos(\theta_{\mathrm{M}}))\ \hat{\imath} + (R_{\mathrm{SE}} \sin(\theta_{\mathrm{E}}) + R_{\mathrm{EM}} \sin(\theta_{\mathrm{M}}))\ \hat{\jmath}$$

6. Sketch the trajectory of the moon, as the earth travels around the sun, on the diagram provided. In the reference frame of the sun, what object in the sky does the moon appear to be orbiting? The earths are spaced apart in $\sim$28 day segments on the pictured orbit.

appears to orbit the sun.

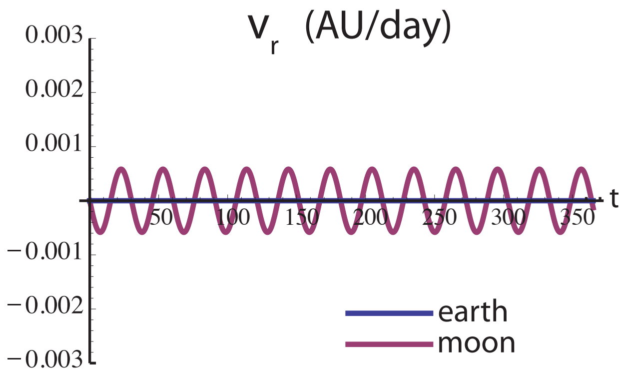
7. What are the expressions for the velocity and the acceleration of the moon, relative to the sun, in the cartesian basis? Leave your answer in terms of θ_E, $\dot{\theta}_E$, θ_M, and $\dot{\theta}_M$ where $\dot{\theta}_M$ is the angular velocity of the moon around the earth.

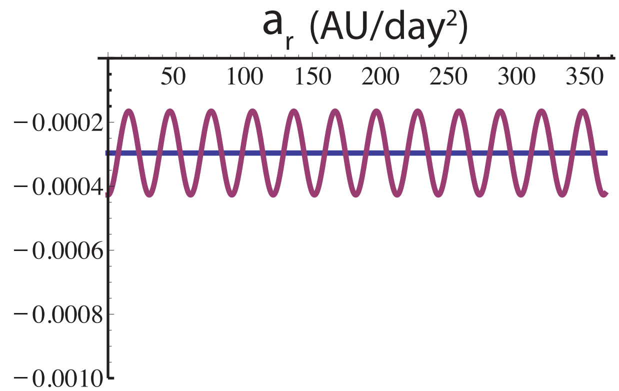
$$\vec{v}_{\mathrm{M/E}} = -\left(\dot{\theta}_{\mathrm{E}} R_{\mathrm{SE}} \sin(\theta_{\mathrm{E}}) + \dot{\theta}_{\mathrm{M}} R_{\mathrm{EM}} \sin(\theta_{\mathrm{M}})\right)\ \hat{\imath} + \left(\dot{\theta}_{\mathrm{E}} R_{\mathrm{SE}} \cos(\theta_{\mathrm{E}}) + \dot{\theta}_{\mathrm{M}} R_{\mathrm{EM}} \cos(\theta_{\mathrm{M}})\right)\ \hat{\jmath}$$
$$\vec{a}_{\mathrm{M/S}} = -\left(\dot{\theta}_{\mathrm{E}}^2 R_{\mathrm{SE}} \cos(\theta_{\mathrm{E}}) + \dot{\theta}_{\mathrm{M}}^2 R_{\mathrm{EM}} \cos(\theta_{\mathrm{M}})\right)\ \hat{\imath} - \left(\dot{\theta}_{\mathrm{E}}^2 R_{\mathrm{SE}} \sin(\theta_{\mathrm{E}}) + \dot{\theta}_{\mathrm{M}}^2 R_{\mathrm{EM}} \sin(\theta_{\mathrm{M}})\right)\ \hat{\jmath}$$

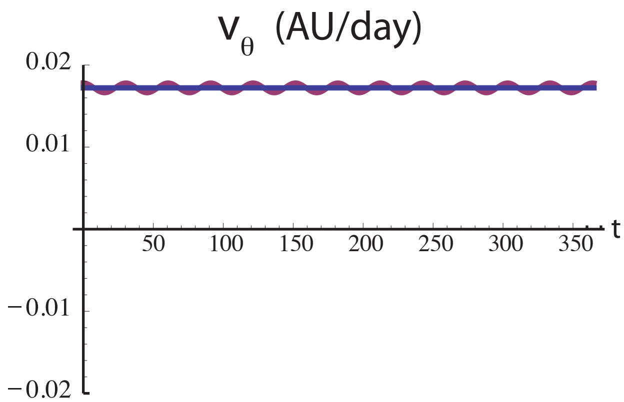
8. We'll now see how the radial and tangential components of the velocity and acceleration are different for the moon's trajectory in comparison to the earth's trajectory. Making use of the expressions in the question above, fill out the chart below for the radial and tangential components of the moon's velocity and acceleration for the given orientations:

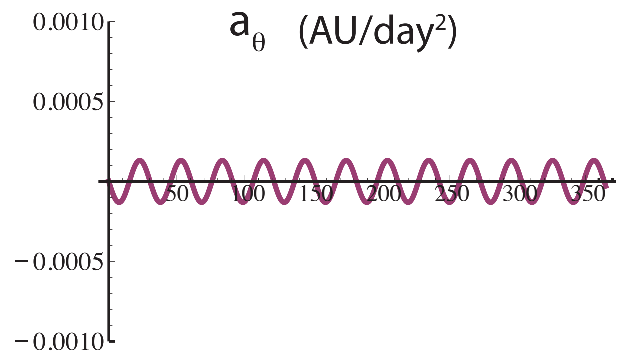
θ_{E}	0	0	0	0
θ_{M}	0	$\pi/2$ rad	π rad	$-\pi/2$ rad
v_{r}	0	$-\dot{\theta}_{\mathrm{M}} R_{\mathrm{EM}}$	0	$\dot{\theta}_{\mathrm{M}} R_{\mathrm{EM}}$
v_θ	$\dot{\theta}_{\mathrm{E}} R_{\mathrm{SE}} + \dot{\theta}_{\mathrm{M}} R_{\mathrm{EM}}$	$\dot{\theta}_{\mathrm{E}} R_{\mathrm{SE}}$	$\dot{\theta}_{\mathrm{E}} R_{\mathrm{SE}} - \dot{\theta}_{\mathrm{M}} R_{\mathrm{EM}}$	$\dot{\theta}_{\mathrm{E}} R_{\mathrm{SE}}$
a_{r}	$-\dot{\theta}_{\mathrm{E}}^2 R_{\mathrm{SE}} - \dot{\theta}_{\mathrm{M}}^2 R_{\mathrm{EM}}$	$-\dot{\theta}_{\mathrm{E}}^2 R_{\mathrm{SE}}$	$-\dot{\theta}_{\mathrm{E}}^2 R_{\mathrm{SE}} + \dot{\theta}_{\mathrm{M}}^2 R_{\mathrm{EM}}$	$-\dot{\theta}_{\mathrm{E}}^2 R_{\mathrm{SE}}$
a_θ	0	$-\dot{\theta}_{\mathrm{M}}^2 R_{\mathrm{EM}}$	0	$\dot{\theta}_{\mathrm{M}}^2 R_{\mathrm{EM}}$

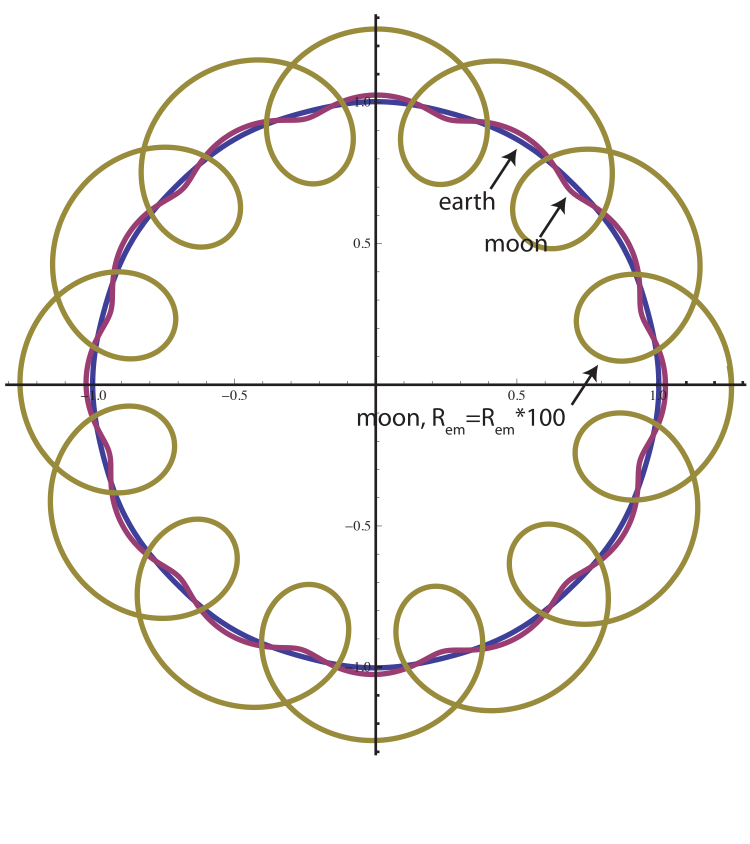
9. Using the table above as a guide, sketch the tangential and radial components for the velocity and acceleration of the moon on the same plots that you used for the earth's velocity and acceleration.


v_r (AU/day)
0.003
0.002
0.001
−0.001
−0.002
−0.003
50
100
150
200
250
300
350
t
earth
moon


a_r (AU/day²)
t
50
100
150
200
250
300
350
−0.0002
−0.0004
−0.0006
−0.0008
−0.0010


v_θ (AU/day)
0.02
0.01
−0.01
−0.02
50
100
150
200
250
300
350
t


a_θ (AU/day²)
0.0010
0.0005
−0.0005
−0.0010
50
100
150
200
250
300
350
t


1.0
0.5
−0.5
−1.0
−1.0
−0.5
0.5
1.0
earth
moon
moon, R_em=R_em*100