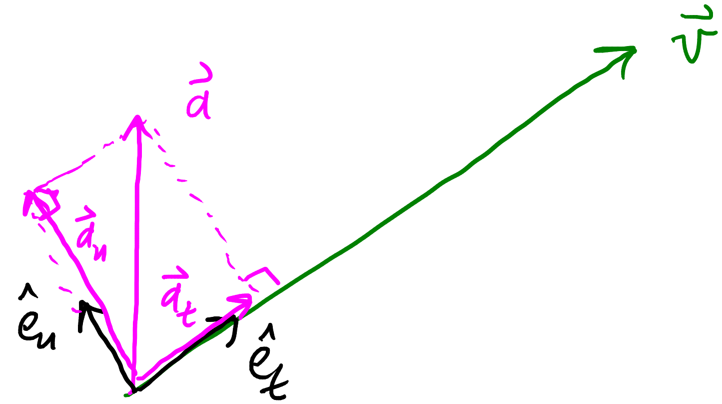
Question: A satellite tracks a car moving with $\vec{v} = 4\hat{i} + 3\hat{j}$ m/s and $\vec{a} = 2\hat{j}$ m/s$^2$.

What are $\hat{e}_t$, $\hat{e}_n$ and $\rho$? Sketch the car's likely trajectory.

$$\hat{e}_t = \hat{v} = 0.8\hat{i} + 0.6\hat{j}$$




$$a_t = \vec{a} \cdot \hat{e}_t = 1.2 \text{ m/s}^2$$

$$\vec{a}_t = a_t \hat{e}_t = 1.2 (0.8\hat{i} + 0.6\hat{j})$$

$$= 0.96\hat{i} + 0.72\hat{j} \text{ m/s}^2$$

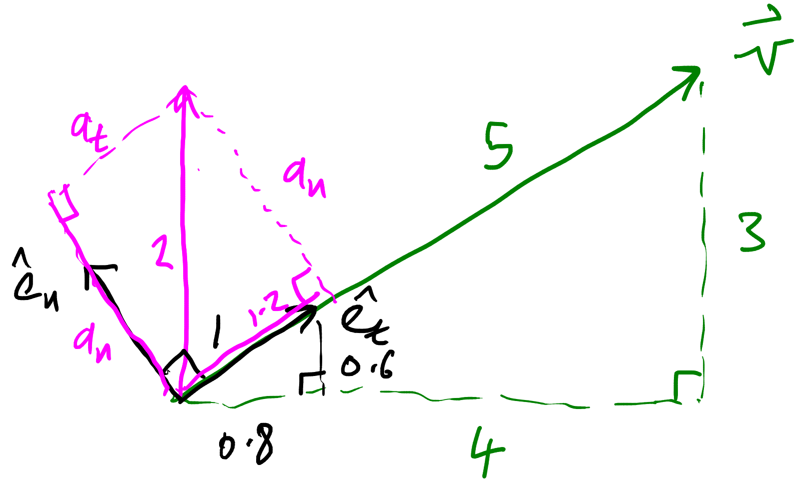
$$\vec{a}_n = \vec{a} - \vec{a}_t = -0.96\hat{i} + 1.28\hat{j} \text{ m/s}^2 \qquad a_n = 1.6 \text{ m/s}^2$$

$$\hat{e}_n = \frac{1}{a_n}\vec{a}_n = \frac{1}{1.6}(-0.96\hat{i} + 1.28\hat{j}) = -0.6\hat{i} + 0.8\hat{j}$$

$$\left.\begin{aligned} \vec{a} &= a_t \hat{e}_t + a_n \hat{e}_n = 1.2\,\hat{e}_t + 1.6\,\hat{e}_n \\ &= \ddot{s}\,\hat{e}_t + \frac{\dot{s}^2}{\rho}\hat{e}_n \end{aligned}\right\}$$

$$\frac{\dot{s}^2}{\rho} = 1.6 \qquad \dot{s} = v = 5$$

$$\rho = \frac{25}{1.6} = 15.625 \text{ m}$$




$a = 2$

$a_t = 1.2$

$a^2 = a_t^2 + a_n^2$

$4 = 1.44$

$a_n^2 = 2.56$

$a_n = 1.6$

Want $\hat{e}_n \cdot \hat{e}_t = 0$ $\quad \|\hat{e}_n\| = 1$ ← two possible answers in 2D

$$(X\hat{i} + Y\hat{j}) \cdot (0.8\hat{i} + 0.6\hat{j}) = 0$$

$$X \times 0.8 + Y \times 0.6 = 0$$

$$-0.6 \times 0.8 + 0.8 \times 0.6 = 0 \qquad \hat{e}_n = -0.6\hat{i} + 0.8\hat{j}$$

$$\cancel{0.6 \times 0.8 - 0.8 \times 0.6 = 0} \qquad \cancel{\hat{e}_n = 0.6\hat{i} - 0.8\hat{j}}$$ ← wrong based on $\vec{a}$ direction.

to get perpendicular vector in 2D: swap components, change one sign.

Question: A car is driving on a track defined by $r = 2 + \cos 2\theta$ m. At an instant we measure: $\theta = \frac{3\pi}{4}$ rad, $\dot{\theta} = -2$ rad/s, $\ddot{\theta} = -2$ rad/s$^2$.

(a) Sketch the track.

(b) What are $\vec{v}$ and $\vec{a}$ in the polar basis?

(c) Is the car speeding up or slowing down?

(d) What is the instantaneous radius of curvature?

(a)