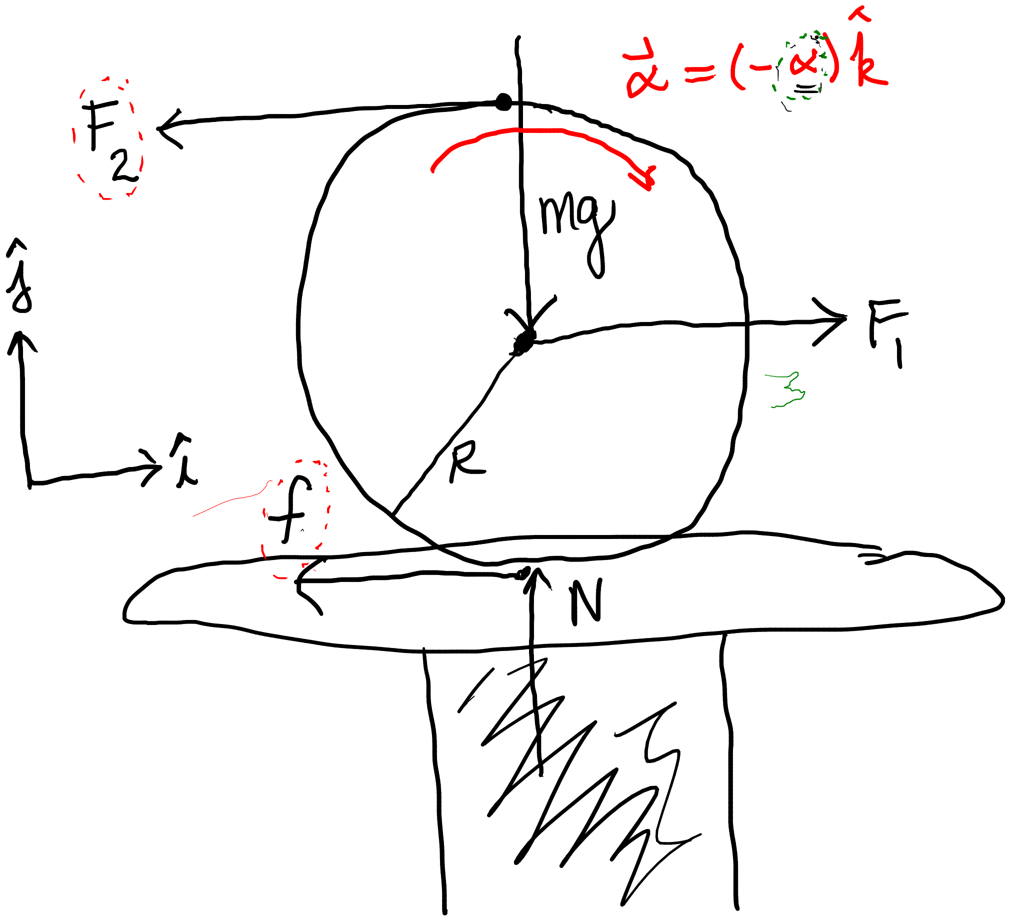
TAM 212 Stone of Orthanc (HW 10C)



Find $\vec{\alpha}$, f

$\vec{F}_1 = 65\hat{i}$ N

$\vec{F}_2 = -40\hat{i}$ N $= -F_2\hat{i}$ $F_2 = 40$

$R = 0.43$ m

$R_{table} = 5$m

$\mu = 0.225$

$m = 13$ kg

$k_{zc} = 0.54$ m Radius of Gyration

case 1: assume rolling without slip
"method of assumed motion"
solve for f, then check that $|f| \leq \mu N$

kinematics: rolling without slip $\boxed{\vec{a}_c = R\alpha\hat{i}}$

① $\sum F_x = m a_{cx}$

$$\boxed{F_1 - F_2 - f = mR\alpha}$$

② $\sum F_y = m a_{cy}$

$$\boxed{N - mg = 0}$$

③ $\sum \vec{M}_c = I_c \vec{\alpha}$

$$\left.\begin{aligned} \sum \vec{M}_c &= (F_2 R - fR)\,\hat{k} \\ I_c &= m k_{zc}^2 \\ \vec{\alpha} &= (-\alpha)\,\hat{k} \end{aligned}\right\} \quad \boxed{(F_2 R - fR) = -m k_{zc}^2 \alpha}$$

algebra: from ① $f = F_1 - F_2 - mR\alpha$

into ③ $F_2 R - (F_1 - F_2 - mR\alpha) R = -m k_{zc}^2 \alpha$

$$\alpha = \frac{(F_1 - 2F_2)R}{m(R^2 + k_{zc}^2)} = -1.04125$$

$$\vec{\alpha} = -\alpha\hat{k} = 1.04125\,\hat{k} \text{ rad/s}^2 \qquad \text{CCW}$$

then $f = F_1 - F_2 - mR\alpha = 30.82 \text{ N}$

But: checking slip condition - is $|f| \leq \mu N$?

$\mu N = \mu mg = 28.69 \text{ N}$ $\Rightarrow$ violated assumption

$\Rightarrow$ stone is slipping

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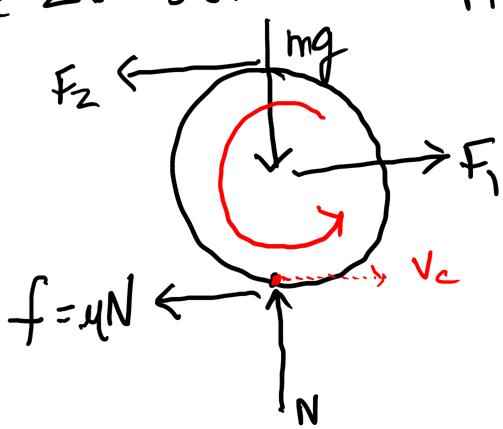
case 2: stone is slipping



Method of assumed forces: $f = \mu N$

VERY IMPORTANT !

(1) $\sum F_x = m a_{cx} \neq m R \alpha$

$$\boxed{F_1 - F_2 - \mu N = m a_{cx}}$$

(2) $\sum F_y = m a_{cy}$

$$\boxed{N - mg = 0}$$

(3) $\sum \vec{M}_c = I_c \vec{\alpha}$

$$\left.\begin{aligned} \sum \vec{M}_c &= F_2 R - \mu N R \\ I_c &= m k_{zc}^2 \\ \vec{\alpha} &= (-\alpha)\hat{k} \end{aligned}\right\} \quad \boxed{F_2 R - \mu N R = -m k_{zc}^2 \alpha}$$

solving: (1) $a_{cx} = \dfrac{F_1 - F_2 - \mu N}{m} = -0.284 \text{ m/s}^2$

(3) $\alpha = \dfrac{\mu N R - F_2 R}{m k_{zc}^2} = -1.282 \text{ rad/s}^2$

$\vec{\alpha} = -\alpha \hat{k}$

$= 1.282 \hat{k} \text{ rad/s}^2$

CCW

check: direction of friction force opposes velocity of contact point?

Yes ✓