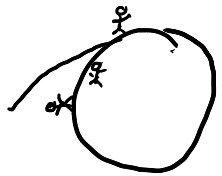
TAM 212 Review



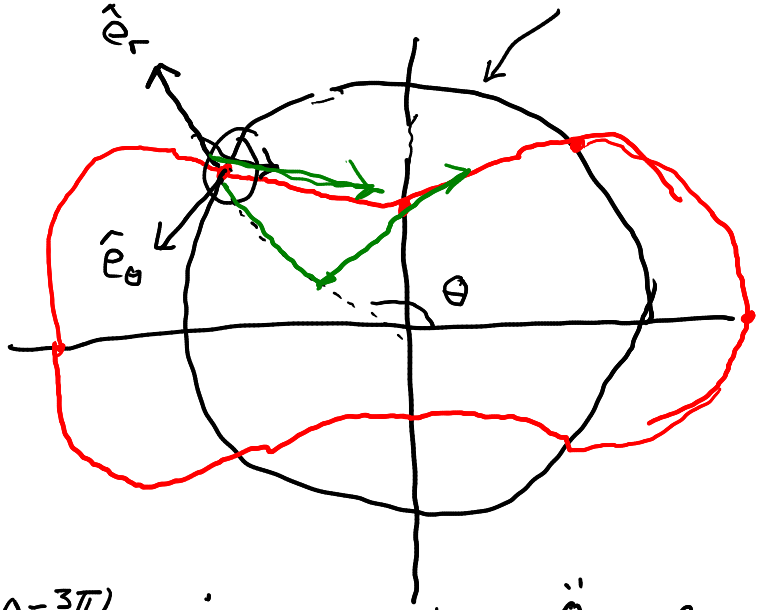
Example: polar & tangential/normal coordinate system

A car is driving on a track defined by $r = 2 + \cos 2\theta$ m.
At an instant, we measure: $\theta = \frac{3\pi}{4}$ radians, $\dot{\theta} = -2$ rad/s, $\ddot{\theta} = -2$ rad/s^2.

(a) sketch the track
(b) what are $\vec{v}, \vec{a}$, in the polar basis?
(c) is the car speeding up or slowing down?
(d) what is the instantaneous radius of curvature?

} ⇒ tangential/normal

(a) $r = 2 + \cos 2\theta$



θ	$\cos 2\theta$	$r = 2 + \cos 2\theta$
0°	1	3
45°	0	2
90°	-1	1
135°	0	2
180°	1	3

$\theta = 3\pi/4$ $\quad \dot{\theta} = -2$ rad/s $\quad \ddot{\theta} = -2$ rad/s^2 $\quad$ at instant shown

$r = 2 + \cos 2\theta \longrightarrow 2 + \cos\frac{3\pi}{2} = 2$

$\dot{r} = (-\sin 2\theta)(2\dot{\theta}) \longrightarrow -2\dot{\theta}\sin 2\theta = -4$

$\ddot{r} = \frac{d}{dt}(-2\dot{\theta}\sin 2\theta)$

$= (-2\ddot{\theta}\underbrace{\sin 2\theta}_{-1} - 4\dot{\theta}^2\underbrace{\cos 2\theta}_{0}) \longrightarrow -4$

(b) $\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

$\vec{v} = -4\hat{e}_r + 2(-2)\hat{e}_\theta$

$\boxed{\vec{v} = -4\hat{e}_r - 4\hat{e}_\theta}$

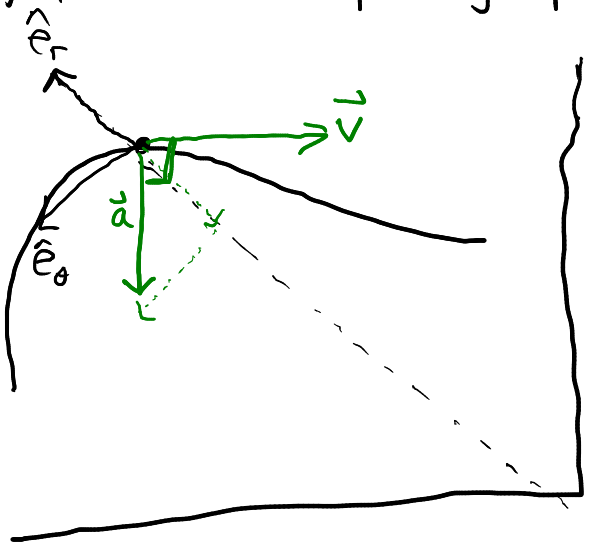
$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$

$= (-4 - 2(-2)^2)\hat{e}_r + (2(-2) + 2(-4)(-2))\hat{e}_\theta$

$\boxed{\vec{a} = -12\hat{e}_r + 12\hat{e}_\theta}$

(c) Is the car speeding up or slowing down?

$$\vec{a} = -12\hat{e}_r + 12\hat{e}_\theta$$



Need to know a_t

$$\vec{a} = a_t \hat{e}_t + a_n \hat{e}_n$$

$$\vec{v} = v\hat{e}_t \Rightarrow \hat{e}_t = \frac{\vec{v}}{v} = \frac{1}{\sqrt{2}}\left(-\hat{e}_r - \hat{e}_\theta\right)$$

$$a_t = \vec{a} \cdot \hat{e}_t$$

$$= \left(-12\hat{e}_r + 12\hat{e}_\theta\right) \cdot \frac{1}{\sqrt{2}}\left(-\hat{e}_r - \hat{e}_\theta\right)$$

$$=$$

Is the car:

A) speeding up

B) slowing down

C) neither