# TAM 212. Midterm 2 Practice. Mar 25, 2013. (V4)

- There are 50 questions, each worth 1 point.

- You must not communicate with other students during this test.

- No electronic devices allowed.

- This is a 2 hour exam.

- Do not turn this page until instructed to do so.

- There are several different versions of this exam.

- The notation $\vec{r}_{PQ}$ denotes the position vector from $P$ to $Q$.

## 1. Fill in your information:

**Full Name:** _____

**UIN (Student Number):** _____

**NetID:** _____

## 2. Circle your discussion section:

| | Monday | Tuesday | Wednesday | Thursday |
|---|---|---|---|---|
| **8–9** | | ADI (260) Karthik | | |
| **9–10** | | ADC (260) Venanzio | | ADK (260) Aaron |
| **10–11** | | ADD (256) Aaron ADQ (344) Jan | ADS (252) Ray | ADT (243) Aaron ADU (344) Jan |
| **11–12** | | ADE (252) Jan | | ADL (256) Kumar |
| **12–1** | ADA (243) Ray ADP (135) Seung | ADF (335) Seung ADG (336) Kumar | ADJ (256) Ray ADR (252) Lin | ADN (260) Kumar |
| **1–2** | | | | |
| **2–3** | | | | |
| **3–4** | | | | |
| **4–5** | ADV (252) Karthik | | ADO (260) Mazhar ADW (252) Lin | |
| **5–6** | ADB (260) Mazhar | ADH (260) Karthik | ADM (243) Mazhar | |

## 3. Fill in the following answers on the Scantron form:

91. A

92. A

93. A

94. A

95. D

96. C

1. (1 point) A circular rigid body with radius $r = 2$ m is rolling clockwise without slipping on a flat surface in 2D as shown, with angular velocity $\vec{\omega} = -\omega\hat{k}$. The speed of the center is $v_C = 6$ m/s.
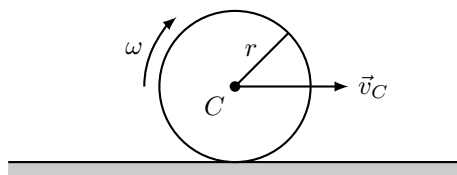


What is the angular velocity $\omega$?

(A) $0 \text{ rad/s} \leq \omega < 1 \text{ rad/s}$

(B) $1 \text{ rad/s} \leq \omega < 2 \text{ rad/s}$

(C) $2 \text{ rad/s} \leq \omega < 3 \text{ rad/s}$

(D) $4 \text{ rad/s} \leq \omega$

(E) $3 \text{ rad/s} \leq \omega < 4 \text{ rad/s}$

2. (1 point) A rigid body is moving in 2D as shown below with angular velocity $\vec{\omega} = \omega\hat{k}$. A pin at point $Q$ constrains that point to move in a horizontal slot.
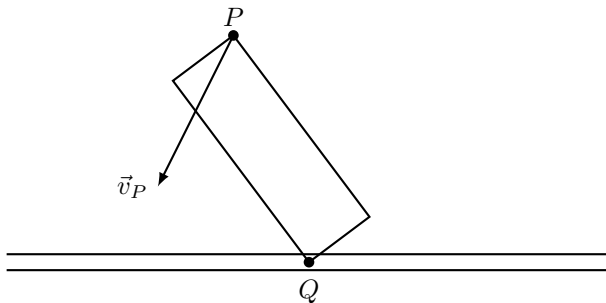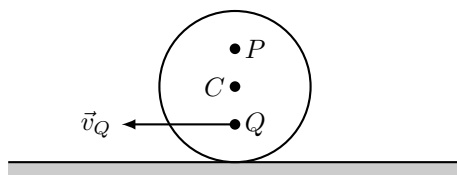


Point $P$ on the body has:

$$\vec{r}_{PQ} = \hat{i} - 3\hat{j} \text{ m}$$
$$\vec{v}_P = -\hat{i} - 2\hat{j} \text{ m/s.}$$

What is the speed $v_Q$ of point $Q$?

(A) $4 \text{ m/s} \leq v_Q$

(B) $1 \text{ m/s} \leq v_Q < 2 \text{ m/s}$

(C) $2 \text{ m/s} \leq v_Q < 3 \text{ m/s}$

(D) $3 \text{ m/s} \leq v_Q < 4 \text{ m/s}$
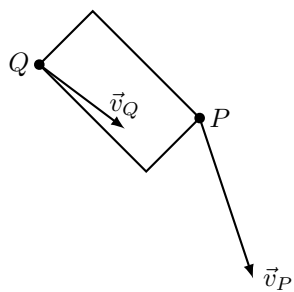
(E) $0 \text{ m/s} \leq v_Q < 1 \text{ m/s}$

3. (1 point) A circular rigid body is rolling without slipping on a flat surface in 2D as shown. The velocity of point $Q$ is $\vec{v}_Q = -\hat{\imath}$ m/s.



What is the $\hat{\imath}$ component $v_{Px}$ of point $P$?

(A) $2$ m/s $\leq v_{Px}$

(B) $-2$ m/s $\leq v_{Px} < 0$ m/s

(C) $v_{Px} = 0$ m/s
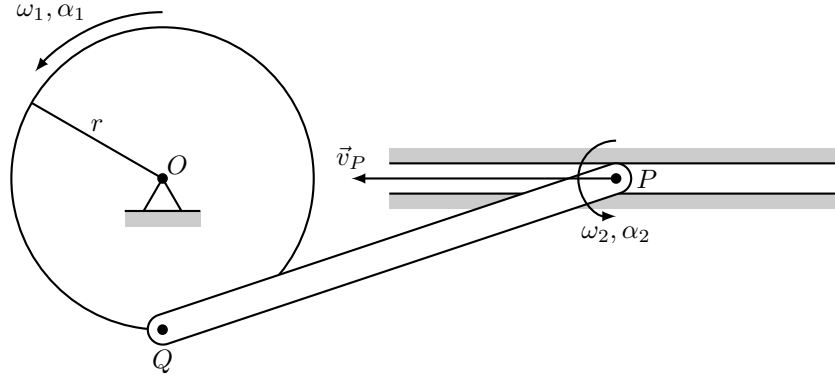
(D) $v_{Px} < -2$ m/s

(E) $0$ m/s $< v_{Px} < 2$ m/s

4

4. (1 point) A rigid body is moving in 2D as shown below.



What is the direction of the angular velocity of the body?

(A) ↻ (clockwise)

(B) ↺ (counterclockwise)

5. (1 point) A circular rigid body with radius $r = 2$ m rotates about the fixed center $O$ as shown. A rigid rod connects pins $P$ and $Q$, and point $P$ is constrained to only move horizontally. Point $P$ has velocity $\vec{v}_P = -4\hat{\imath}$ m/s and acceleration $\vec{a}_P = 0$. The angular velocity and angular acceleration of the circular body are $\vec{\omega}_1 = \omega_1 \hat{k}$ and $\vec{\alpha}_1 = \alpha_1 \hat{k}$, while those of the rod are $\vec{\omega}_2 = \omega_2 \hat{k}$ and $\vec{\alpha}_2 = \alpha_2 \hat{k}$.
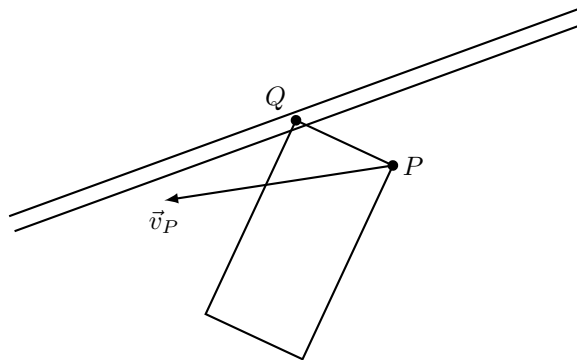


The position vectors are:

$$\vec{r}_{OQ} = -2\hat{\jmath} \text{ m}$$
$$\vec{r}_{PQ} = -6\hat{\imath} - 2\hat{\jmath} \text{ m}.$$

What is $\alpha_1$?

(A) $-1 \text{ rad/s}^2 \le \alpha_1 < 0 \text{ rad/s}^2$

(B) $0 \text{ rad/s}^2 < \alpha_1 < 1 \text{ rad/s}^2$

(C) $\alpha_1 = 0 \text{ rad/s}^2$

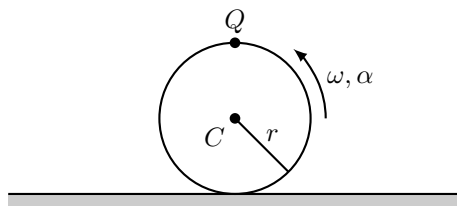(D) $\alpha_1 < -1 \text{ rad/s}^2$

(E) $1 \text{ rad/s}^2 \le \alpha_1$

6. (1 point) A rigid body is moving in 2D as shown below, with point $Q$ constrained to move in the angled slot.

$Q$

$P$

$\vec{v}_P$

What is the direction of the angular velocity of the body?

(A) ↻ (clockwise)

(B) ↺ (counterclockwise)

7. (1 point) A circular rigid body with radius $r = 1$ m is rolling without slipping with angular velocity $\vec{\omega} = 2\hat{k}$ on a flat surface in 2D as shown. The body is speeding up and has angular acceleration $\vec{\alpha} = \alpha\hat{k}$. Point $Q$ is at the top of the body and has acceleration magnitude $a_Q = 5$ m/s$^2$.
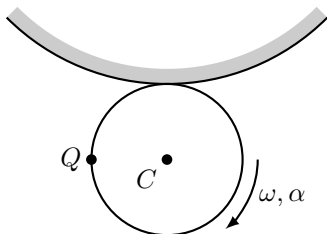


What is $\alpha$?

(A) $1$ rad/s$^2 \leq \alpha < 1.5$ rad/s$^2$

(B) $1.5$ rad/s$^2 \leq \alpha < 2$ rad/s$^2$

(C) $0.5$ rad/s$^2 \leq \alpha < 1$ rad/s$^2$

(D) $0$ rad/s$^2 \leq \alpha < 0.5$ rad/s$^2$

(E) $2$ rad/s$^2 \leq \alpha$

8. (1 point) A circular rigid body is rolling without slipping on a curved surface in 2D as shown. At the current instant the body is rotating clockwise and rate of rotation is increasing ($\omega$ and $\alpha$ are both positive in the direction shown), such that $r\alpha = r\omega^2$.



What is the direction of the acceleration $\vec{a}_Q$ of point $Q$?

(A) $\leftarrow$

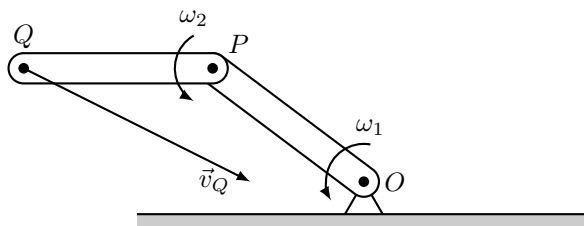(B) $\rightarrow$

(C) $\uparrow$

(D) $\nearrow$

(E) $\nwarrow$

9. (1 point) A rigid body is moving in 2D as shown, with a clockwise rotation ($\omega$ is positive in the direction indicated). The angular velocity $\omega$, distance $r_{PQ}$, and speed $v_P$ satisfy $\omega r_{PQ} = 2v_P$.
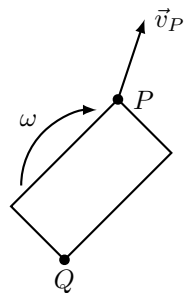


What is the direction of $\vec{v}_Q$?

(A) ↗

(B) ↘

(C) ↖

(D) ↙

10. (1 point) Two rods are connected with pin joints at $O$, $P$, and $Q$ as shown. Rod $OP$ has angular velocity $\vec{\omega}_1 = \omega_1 \hat{k}$ and rod $PQ$ has angular velocity $\vec{\omega}_2 = \omega_2 \hat{k}$.



The positions and velocities at the current instant are:
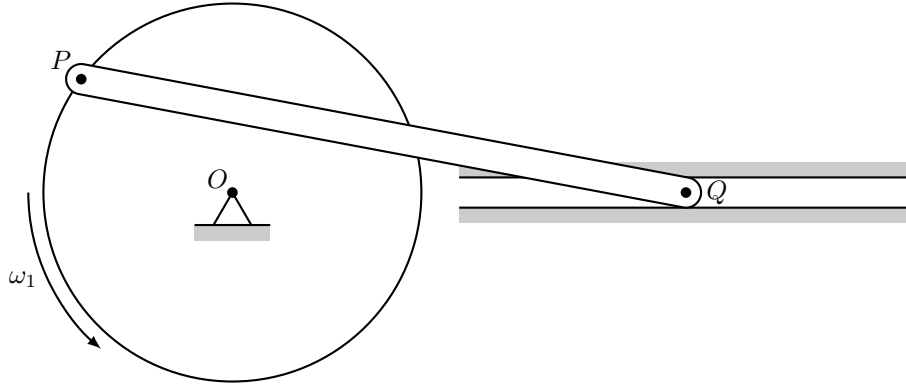
$$\vec{r}_{OP} = -4\hat{i} + 3\hat{j} \text{ m}$$
$$\vec{r}_{PQ} = -5\hat{i} \text{ m}$$
$$\vec{v}_Q = 6\hat{i} - 3\hat{j} \text{ m/s}.$$

What is $\omega_1$?

(A) $\omega_1 = 0$ rad/s

(B) $1 \text{ rad/s} \leq \omega_1$

(C) $0 \text{ rad/s} < \omega_1 < 1 \text{ rad/s}$

(D) $-1 \text{ rad/s} \leq \omega_1 < 0 \text{ rad/s}$

(E) $\omega_1 < -1$ rad/s

11. (1 point) A circular rigid body rotates about the fixed center $O$ with angular velocity $\vec{\omega}_1 = 2\hat{k}$ rad/s as shown. A rigid rod connects pins $P$ and $Q$, and point $Q$ is constrained to only move horizontally.
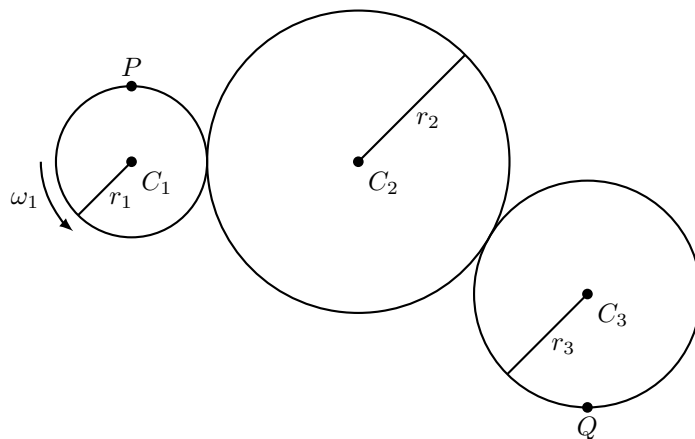


At the current instant the positions are:

$$\vec{r}_{OP} = -4\hat{i} + 3\hat{j} \text{ m}$$
$$\vec{r}_{PQ} = 16\hat{i} - 3\hat{j} \text{ m.}$$

What is the speed $v_Q$ of point $Q$?

(A) $0 \text{ m/s} \le v_Q < 3 \text{ m/s}$

(B) $9 \text{ m/s} \le v_Q < 12 \text{ m/s}$

(C) $6 \text{ m/s} \le v_Q < 9 \text{ m/s}$

(D) $3 \text{ m/s} \le v_Q < 6 \text{ m/s}$
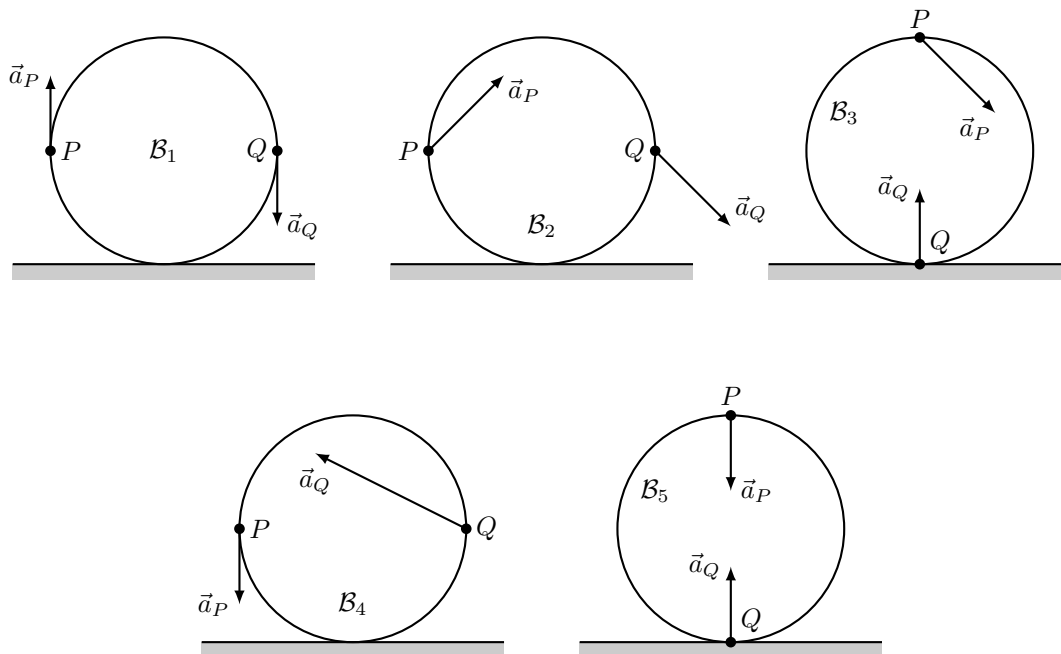
(E) $12 \text{ m/s} \le v_Q$

12

12. (1 point) Three meshed gears rotate about fixed centers as shown. The radii are $r_1 = 2$ m, $r_2 = 4$ m, and $r_3 = 3$ m and the gear at $C_1$ is rotating counterclockwise as shown.



What is the relationship between the speeds $v_P$ and $v_Q$ of points $P$ and $Q$?

(A) $v_p > v_Q$

(B) $v_p < v_Q$

(C) $v_p = v_Q$

13. (1 point) Five circular rigid bodies are rolling without slipping as shown, with the accelerations of points $P$ and $Q$ on the bodies as drawn.

$\vec{a}_P$ $P$ $\mathcal{B}_1$ $Q$ $\vec{a}_Q$

$\vec{a}_P$ $P$ $Q$ $\vec{a}_Q$ $\mathcal{B}_2$

$P$ $\mathcal{B}_3$ $\vec{a}_P$ $\vec{a}_Q$ $Q$

$\vec{a}_Q$ $P$ $Q$ $\vec{a}_P$ $\mathcal{B}_4$

$P$ $\mathcal{B}_5$ $\vec{a}_P$ $\vec{a}_Q$ $Q$

Which body does *not* have physically possible accelerations for points $P$ and $Q$?

(A) $\mathcal{B}_1$

(B) $\mathcal{B}_4$

(C) $\mathcal{B}_2$

(D) $\mathcal{B}_5$
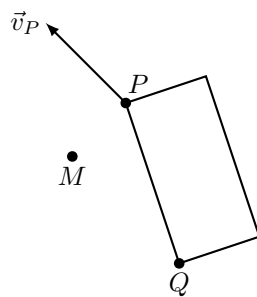
(E) $\mathcal{B}_3$

14

14. (1 point) A rigid body is moving in 2D as shown below. The accelerations of points $P$ and $Q$ on the body are as shown.
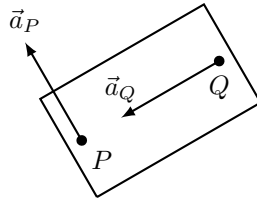


What is the direction of the angular acceleration $\alpha$ of the body?

(A) ↻ (clockwise)

(B) ↺ (counterclockwise)

15. (1 point) A rigid body is moving in 2D as shown below with points $P$ and $Q$ attached to the body. The instantaneous center of the body is at point $M$.



What is the direction of the velocity $\vec{v}_Q$ of point $Q$?

(A) ↗

(B) ↙

(C) ↘

(D) ↖

16. (1 point) A circular rigid body with radius $r = 2$ m is rolling without slipping on a curved surface with radius of curvature $\rho$ in 2D as shown. The angular velocity of the body is a constant $\vec{\omega} = 2\hat{k}$ rad/s. Point $P$ is fixed to the edge of the body and, at the instant shown, is the contact point. The magnitude of acceleration of $P$ is $a_P = 10$ m/s$^2$.
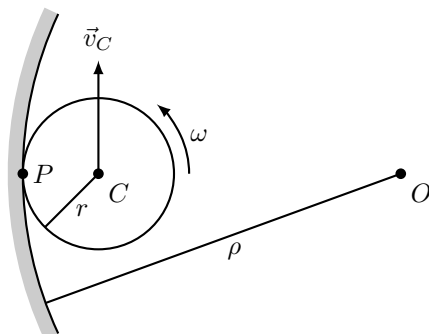


What is the radius of curvature $\rho$ of the surface?

(A) $9 \text{ m} \le \rho < 12 \text{ m}$

(B) $12 \text{ m} \le \rho$

(C) $3 \text{ m} \le \rho < 6 \text{ m}$

(D) $0 \text{ m} \le \rho < 3 \text{ m}$

(E) $6 \text{ m} \le \rho < 9 \text{ m}$

17. (1 point) A rigid body is moving in 2D as shown below with zero angular acceleration and some angular velocity. The acceleration of point $P$ is shown, as is the acceleration $\vec{a}_Q$ of one of the $Q_i$ points.
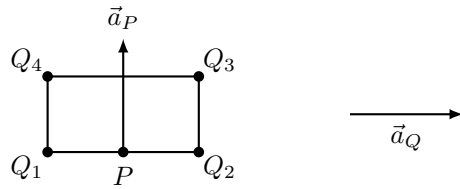


Which point $Q_i$ has the given acceleration $\vec{a}_Q$?

(A) $Q_4$
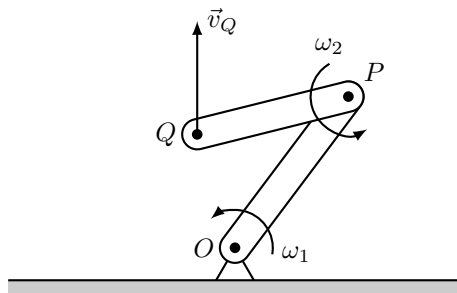
(B) $Q_1$

(C) $Q_2$

(D) $Q_3$

18. (1 point) A circular rigid body is rolling without slipping on a flat surface in 2D in a counterclockwise direction as shown.
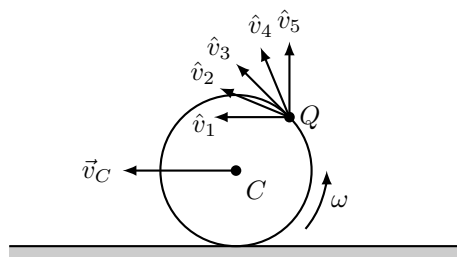


What is the direction of the velocity $\vec{v}_Q$ of point $Q$?

(A) $\hat{v}_1$

(B) $\hat{v}_4$

(C) $\hat{v}_2$

(D) $\hat{v}_5$

(E) $\hat{v}_3$

19. (1 point) Two rods are connected with pin joints at $O$, $P$, and $Q$ as shown. Rod $OP$ has angular velocity $\vec{\omega}_1 = -\hat{k}$ rad/s and rod $PQ$ has angular velocity $\vec{\omega}_2 = \omega_2 \hat{k}$.



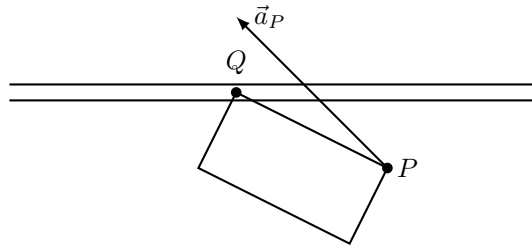The velocity $\vec{v}_Q$ of point $Q$ is directly upwards and the positions of the rods are:

$$\vec{r}_{OP} = 3\hat{i} + 4\hat{j} \text{ m}$$
$$\vec{r}_{PQ} = -4\hat{i} - \hat{j} \text{ m}$$

What is the speed $v_Q$ of point $Q$?

(A) $9 \text{ m/s} \leq v_Q < 12 \text{ m/s}$

(B) $3 \text{ m/s} \leq v_Q < 6 \text{ m/s}$

(C) $6 \text{ m/s} \leq v_Q < 9 \text{ m/s}$

(D) $12 \text{ m/s} \leq v_Q$

(E) $0 \text{ m/s} \leq v_Q < 3 \text{ m/s}$

20. (1 point) A rigid body is moving in 2D as shown below with angular velocity $\vec{\omega} = \omega\hat{k}$ and zero angular acceleration. A pin at point $Q$ constrains that point to move in a horizontal slot.



Point $P$ on the body has:

$$\vec{r}_{PQ} = -2\hat{i} + \hat{j} \text{ m}$$
$$\vec{a}_P = -2\hat{i} + 2\hat{j} \text{ m/s}^2.$$

What is the magnitude $a_Q$ of the acceleration $\vec{a}_Q$ of point $Q$?

(A) $1 \text{ m/s}^2 \leq a_Q < 2 \text{ m/s}^2$

(B) $3 \text{ m/s}^2 \leq a_Q < 4 \text{ m/s}^2$

(C) $0 \text{ m/s}^2 \leq a_Q < 1 \text{ m/s}^2$

(D) $4 \text{ m/s}^2 \leq a_Q$

(E) $2 \text{ m/s}^2 \leq a_Q < 3 \text{ m/s}^2$

21. (1 point) Four gears are positioned with fixed centers and two chains connect pairs of gears as shown. The gears have radii $r_1 = 3$ m, $r_2 = 2$ m, $r_3 = 4$ m, and $r_4 = 3$ m, and the two gears centered at $C_2$ are locked together so they have the same angular velocity. The gear centered at $C_1$ has angular velocity $\vec{\omega}_1 = 3\hat{k}$ rad/s, while the gear at $C_4$ has angular velocity $\vec{\omega}_4 = \omega_4\hat{k}$.
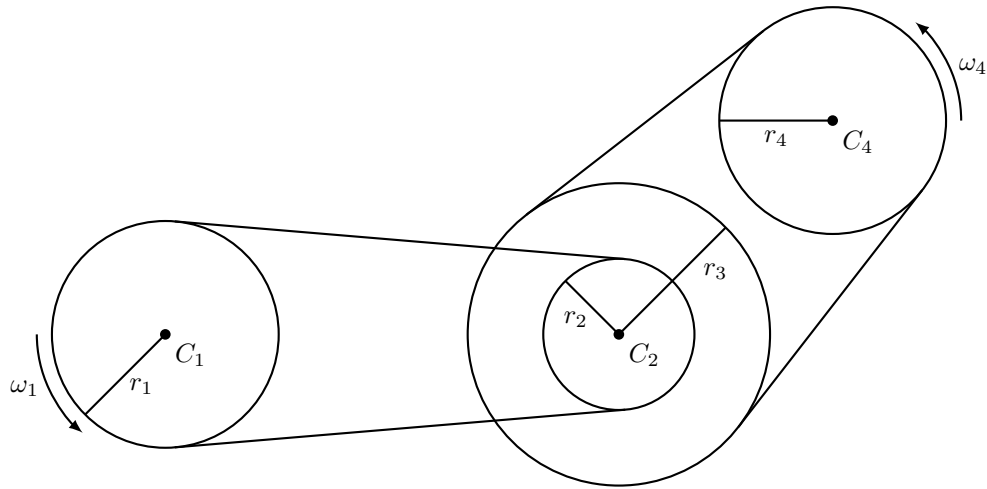


What is $\omega_4$?

(A) $4$ rad/s $\leq \omega_4$

(B) $3$ rad/s $\leq \omega_4 < 4$ rad/s

(C) $0$ rad/s $\leq \omega_4 < 1$ rad/s

(D) $2$ rad/s $\leq \omega_4 < 3$ rad/s

(E) $1$ rad/s $\leq \omega_4 < 2$ rad/s

22. (1 point) Two rods are connected with pin joints at $O$, $P$, and $Q$ as shown. The rods have angular velocities and angular accelerations as indicated.
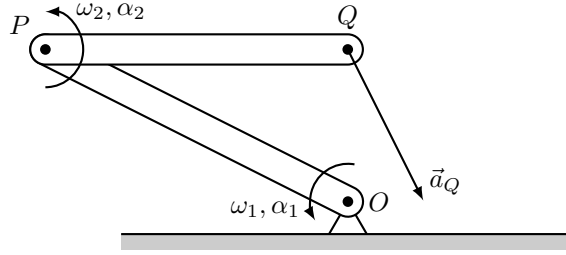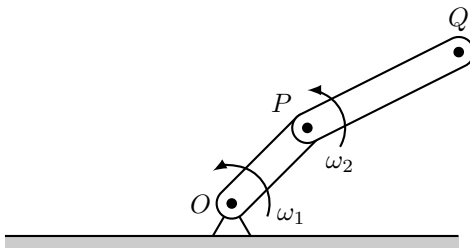


At a particular instant we observe:

$$\vec{r}_{OP} = -2\hat{\imath} + \hat{\jmath} \text{ m} \qquad\qquad \vec{r}_{PQ} = 2\hat{\imath} \text{ m}$$
$$\vec{\omega}_1 = \hat{k} \text{ rad/s} \qquad\qquad\qquad \vec{\omega}_2 = -\hat{k} \text{ rad/s}$$
$$\vec{\alpha}_1 = \alpha_1 \hat{k} \qquad\qquad\qquad\quad \vec{\alpha}_2 = \alpha_2 \hat{k}$$
$$\qquad\qquad\qquad\qquad\qquad \vec{a}_Q = \hat{\imath} - 2\hat{\jmath} \text{ m/s}^2.$$

What is $\alpha_2$?

(A) $-1 \text{ rad/s}^2 \le \alpha_2 < 0 \text{ rad/s}^2$

(B) $1 \text{ rad/s}^2 \le \alpha_2$

(C) $0 \text{ rad/s}^2 < \alpha_2 < 1 \text{ rad/s}^2$

(D) $\alpha_2 = 0 \text{ rad/s}^2$

(E) $\alpha_2 < -1 \text{ rad/s}^2$

23. (1 point) Two rods are connected with pin joints at $O$, $P$, and $Q$ as shown. Rod $OP$ has angular velocity $\vec{\omega}_1 = \omega_1 \hat{k}$ and rod $PQ$ has angular velocity $\vec{\omega}_2 = \omega_2 \hat{k}$.



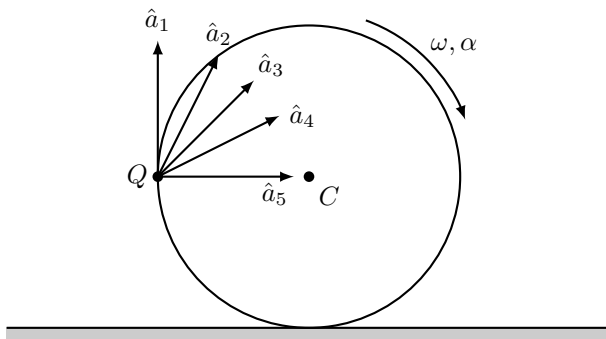The positions and angular velocities of the rods at the current instant are:

$$\vec{r}_{OP} = \hat{i} + \hat{j} \text{ m} \qquad\qquad \vec{r}_{PQ} = 2\hat{i} + \hat{j} \text{ m}$$
$$\vec{\omega}_1 = 2\hat{k} \text{ rad/s} \qquad\qquad \vec{\omega}_2 = -\hat{k} \text{ rad/s}.$$

What is the $\hat{j}$ component $v_{Qy}$ of the velocity $\vec{v}_Q$ of point $Q$?

(A) $-2$ m/s $\leq v_{Qy} < 0$ m/s

(B) $v_{Qy} = 0$ m/s

(C) $0$ m/s $< v_{Qy} < 2$ m/s

(D) $v_{Qy} < -2$ m/s

(E) $2$ m/s $\leq v_{Qy}$

24. (1 point) A circular rigid body is rolling without slipping on a flat surface in 2D as shown. The angular velocity $\vec{\omega} = -\omega\hat{k}$ and angular acceleration $\vec{\alpha} = -\alpha\hat{k}$ satisfy $\alpha = \omega^2$.



What is the direction of the acceleration $\vec{a}_Q$ of point $Q$?

(A) $\hat{a}_5$

(B) $\hat{a}_3$

(C) $\hat{a}_4$

(D) $\hat{a}_2$

(E) $\hat{a}_1$

25. (1 point) Two equal-length rods are connected with pin joints at $O$, $P$, and $Q$ as shown, so that the line $OP$ is at 45° from horizontal. Both rods are rotating counterclockwise with the same angular velocity.
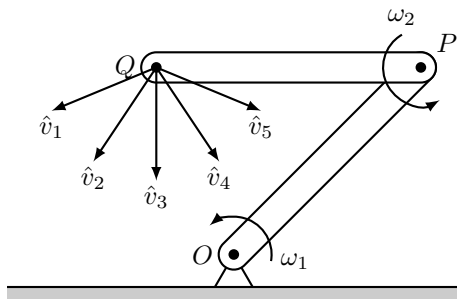


What is the direction of the velocity $\vec{v}_Q$ of point $Q$?

(A) $\hat{v}_4$

(B) $\hat{v}_3$

(C) $\hat{v}_2$

(D) $\hat{v}_1$

(E) $\hat{v}_5$

26. (1 point) A rigid body is moving in 2D as shown below with angular velocity $\vec{\omega} = \omega\hat{k}$. A pin at point $Q$ constrains that point to move in a vertical slot.
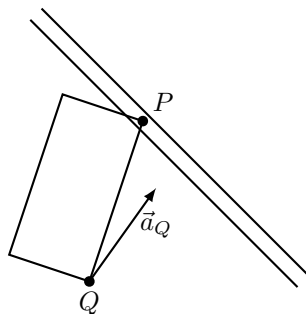


Point $P$ on the body has:

$$\vec{r}_{PQ} = -\hat{\imath} - 2\hat{\jmath} \text{ m}$$
$$\vec{v}_P = \hat{\imath} + \hat{\jmath} \text{ m/s}.$$
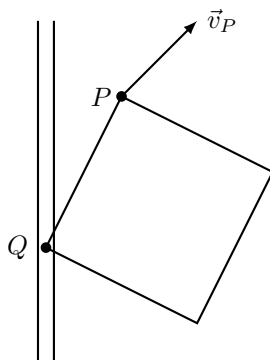
What is $\omega$?

(A) $1 \text{ rad/s} \leq \omega$

(B) $-1 \text{ rad/s} \leq \omega < 0 \text{ rad/s}$

(C) $0 \text{ rad/s} < \omega < 1 \text{ rad/s}$

(D) $\omega < -1 \text{ rad/s}$

(E) $\omega = 0 \text{ rad/s}$

27. (1 point) A rigid body is moving in 2D as shown below with zero angular acceleration. A pin at point $P$ constrains that point to move in an angled slot.



In which direction is the acceleration $\vec{a}_P$ of point $P$?

(A) ↙

(B) ↖

(C) ↗

(D) ↘

28. (1 point) A circular rigid body with radius $r = 1$ m is rolling without slipping on a curved surface with radius of curvature $\rho = 3$ m in 2D as shown. The speed $v_C$ of the center is a constant $v_C = 2$ m/s. Point $Q$ is fixed to the edge of the body and, at the instant shown, the points $O$-$Q$-$C$ form a horizontal line.
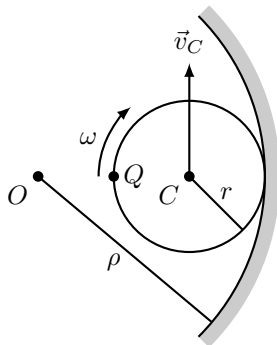


What is the $\hat{\imath}$ component $a_{Qx}$ of the acceleration $\vec{a}_Q$ of point $Q$?

(A) $-2 \text{ m/s}^2 \leq a_{Qx} < 0 \text{ m/s}^2$

(B) $a_{Qx} < -2 \text{ m/s}^2$

(C) $0 \text{ m/s}^2 < a_{Qx} < 2 \text{ m/s}^2$

(D) $a_{Qx} = 0 \text{ m/s}^2$

(E) $2 \text{ m/s}^2 \leq a_{Qx}$

29. (1 point) A rigid body is moving in 2D with angular velocity $\vec{\omega} = \omega\hat{k}$. Two points $P$ and $Q$ are fixed to the body and have:

$$\vec{r}_{PQ} = 2\hat{i} - 2\hat{j} \text{ m}$$
$$\vec{v}_P = 3\hat{j} \text{ m/s}$$
$$\vec{v}_Q = -2\hat{i} + \hat{j} \text{ m/s.}$$

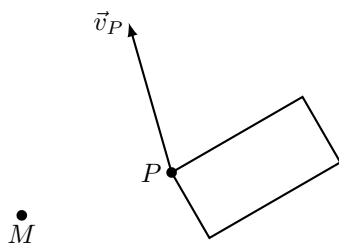What is $\omega$?

(A) $2 \text{ rad/s} \leq \omega$

(B) $\omega = 0 \text{ rad/s}$

(C) $\omega < -2 \text{ rad/s}$

(D) $0 \text{ rad/s} < \omega < 2 \text{ rad/s}$

(E) $-2 \text{ rad/s} \leq \omega < 0 \text{ rad/s}$

30. (1 point) A rigid body is moving in 2D as shown below. The instantaneous center of the body is at point $M$.



What is the direction of the angular velocity of the body?

(A) ↻ (clockwise)

(B) ↺ (counterclockwise)

31. (1 point) A rigid body is moving in 2D with points $P$ and $Q$ attached to it. We have:

$$\vec{r}_P = -\hat{\imath} - 2\hat{\jmath} \text{ m} \qquad\qquad \vec{r}_Q = \hat{\imath} - \hat{\jmath} \text{ m}$$
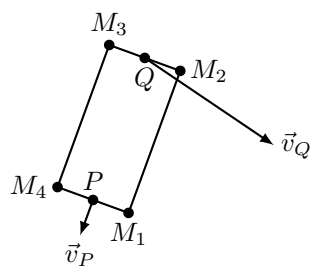$$\vec{v}_P = 2\hat{\imath} + \hat{\jmath} \text{ m/s} \qquad\qquad \vec{v}_Q = 3\hat{\imath} - \hat{\jmath} \text{ m/s.}$$

What is the $x$ coordinate $M_x$ of the instantaneous center $M$ of the body?

(A) $M_x = 0$ m

(B) $-2 \text{ m} \le M_x < 0$ m

(C) $0 \text{ m} < M_x < 2$ m
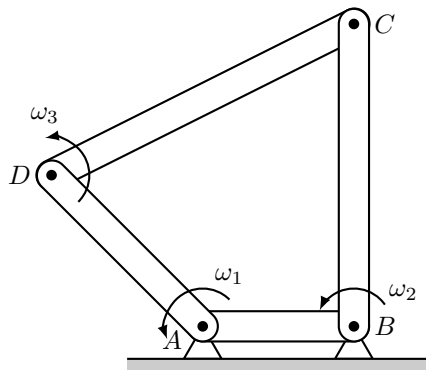
(D) $M_x < -2$ m

(E) $2 \text{ m} \le M_x$

32. (1 point) A rigid body is moving in 2D as shown below.



Which point $M_i$ is the instantaneous center?

(A) $M_1$

(B) $M_3$

(C) $M_2$

(D) $M_4$

33. (1 point) A four-bar linkage has rigid rods connecting pins at $A$, $B$, $C$, and $D$, as shown. The angular velocities are $\vec{\omega}_1 = 2\hat{k}$ for rod $AD$, $\vec{\omega}_2 = \omega_2\hat{k}$ for rod $BC$, and $\vec{\omega}_3 = \omega_3\hat{k}$ for rod $DC$.



At the current instant the positions are:

$$\vec{r}_{AB} = \hat{i} \text{ m}$$
$$\vec{r}_{BC} = 2\hat{j} \text{ m}$$
$$\vec{r}_{AD} = -\hat{i} + \hat{j} \text{ m}$$
$$\vec{r}_{DC} = 2\hat{i} + \hat{j} \text{ m.}$$

What is $\omega_2$?
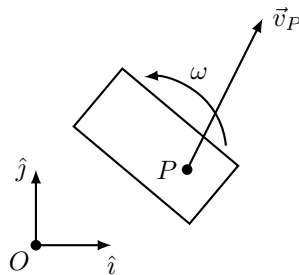
(A) $0 \text{ rad/s} \le \omega_2 < 0.5 \text{ rad/s}$

(B) $2 \text{ rad/s} \le \omega_2$

(C) $1.5 \text{ rad/s} \le \omega_2 < 2 \text{ rad/s}$

(D) $1 \text{ rad/s} \le \omega_2 < 1.5 \text{ rad/s}$

(E) $0.5 \text{ rad/s} \le \omega_2 < 1 \text{ rad/s}$

34. (1 point) A rigid body is moving in 2D as shown below with angular velocity $\vec{\omega} = 2\hat{k}$ rad/s.



Relative to the origin $O$, the point $P$ has:

$$\vec{r}_P = 2\hat{\imath} + \hat{\jmath} \text{ m}$$
$$\vec{v}_P = \hat{\imath} + 2\hat{\jmath} \text{ m/s}.$$

What is the $y$ coordinate $M_y$ of the instantaneous center $M$ of the body?

(A) $4 \text{ m} \leq M_y < 5 \text{ m}$

(B) $3 \text{ m} \leq M_y < 4 \text{ m}$

(C) $M_y < 2 \text{ m}$

(D) $5 \text{ m} \leq M_y$

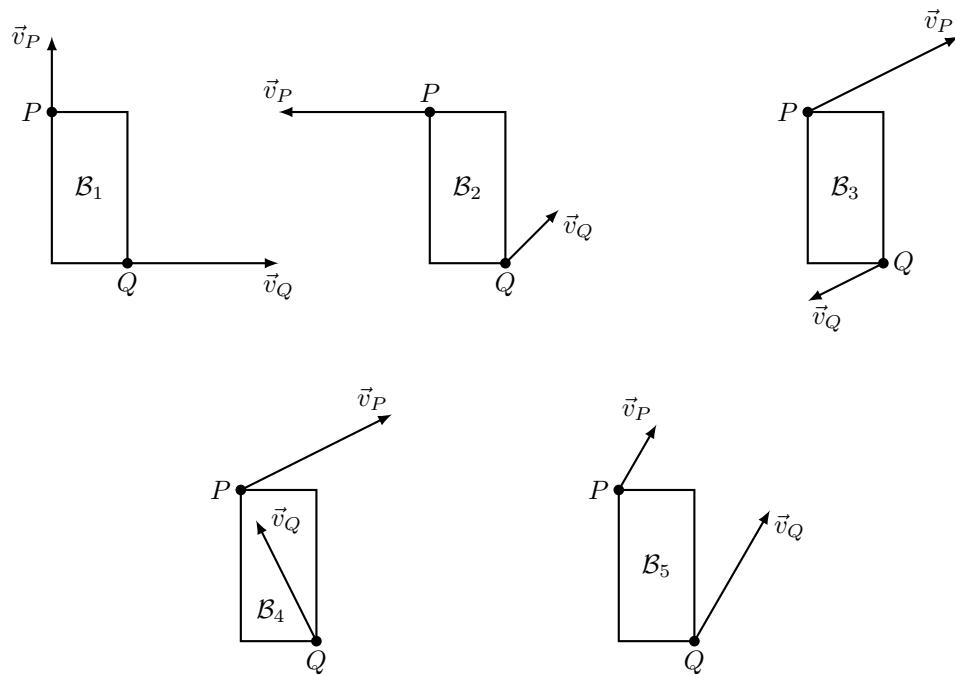(E) $2 \text{ m} \leq M_y < 3 \text{ m}$

35. (1 point) A rigid body is moving in 2D as shown below with counterclockwise angular velocity. The velocity of point $P$ is shown, as is the velocity $\vec{v}_Q$ of one of the $Q_i$ points.
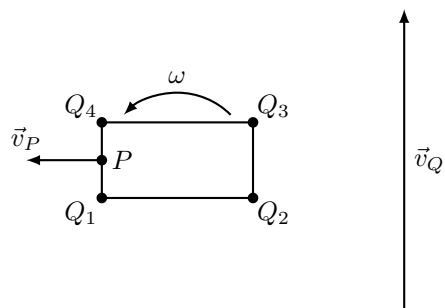


Which point $Q_i$ has the given velocity $\vec{v}_Q$?

(A) $Q_4$

(B) $Q_1$

(C) $Q_3$

(D) $Q_2$

36. (1 point) Five bodies moving in 2D are shown below with the velocities of points $P$ and $Q$ on the bodies as drawn.



Which body has physically possible velocities for point $P$ and $Q$?

(A) $\mathcal{B}_1$

(B) $\mathcal{B}_2$

(C) $\mathcal{B}_3$

(D) $\mathcal{B}_5$

(E) $\mathcal{B}_4$

37. (1 point) Two equal-length rods are connected with pin joints at $O$, $P$, and $Q$ as shown, so that the line $OP$ is at $45°$ from horizontal. Both rods are rotating counterclockwise with the same constant angular velocity.
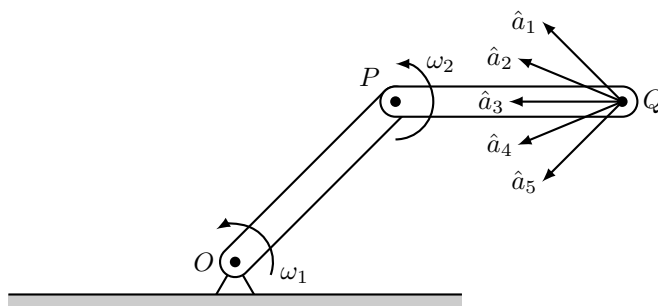


What is the direction of the acceleration $\vec{a}_Q$ of point $Q$?

(A) $\hat{a}_2$

(B) $\hat{a}_4$

(C) $\hat{a}_5$

(D) $\hat{a}_3$

(E) $\hat{a}_1$

38. (1 point) A circular rigid body is rolling without slipping with angular velocity $\vec{\omega} = -\hat{k}$ and angular acceleration $\vec{\alpha} = 2\hat{k}$ on a flat surface in 2D as shown. Point $Q$ is offset from the center $C$ by $\vec{r}_{CQ} = -3\hat{i}+4\hat{j}$ m.
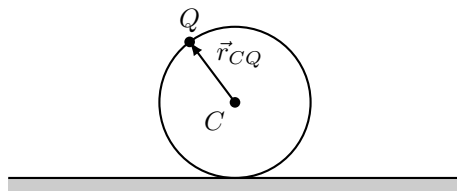


What is the $\hat{j}$ component $a_{Qy}$ of the acceleration $\vec{a}_Q$ of point $Q$?

(A) $a_{Qy} < -8$ m/s$^2$

(B) $0$ m/s$^2 < a_{Qy} < 8$ m/s$^2$

(C) $8$ m/s$^2 \leq a_{Qy}$

(D) $-8$ m/s$^2 \leq a_{Qy} < 0$ m/s$^2$

(E) $a_{Qy} = 0$ m/s$^2$

39. (1 point) A rigid rod is moving in 2D as shown below.
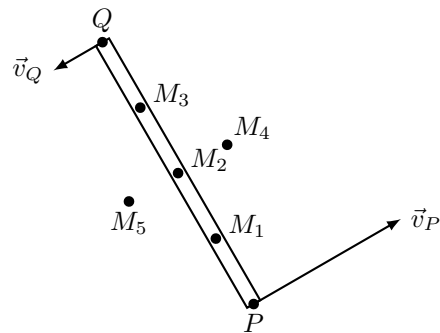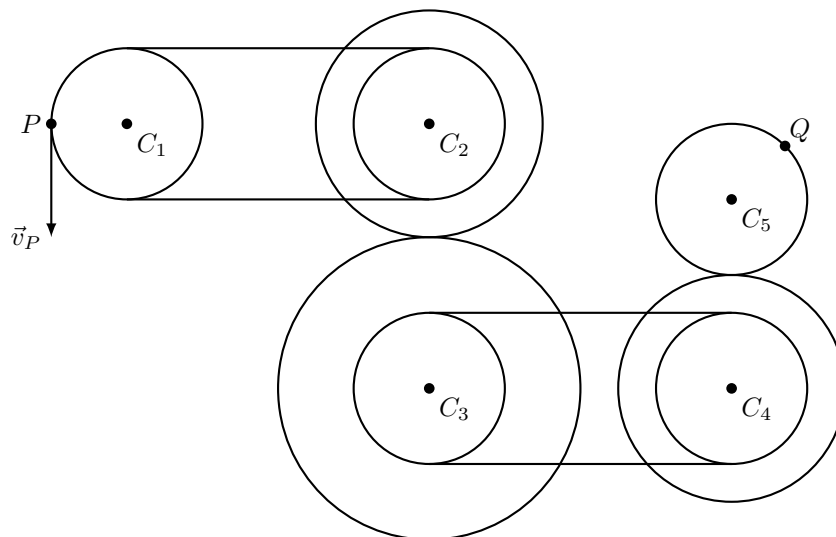


Which point $M_i$ is the instantaneous center?

(A) $M_1$

(B) $M_3$

(C) $M_5$

(D) $M_4$

(E) $M_2$

40. (1 point) A set of gears and chains are positioned as shown with fixed centers $C_1$ through $C_5$. All gear pairs that have the same center are fixed together to rotate at the same angular velocity. All gears and chains are meshed so there is no slip. Point $P$ has velocity $\vec{v}_P$ in the direction indicated.



What is the direction of the velocity $\vec{v}_Q$ of point $Q$?

(A) ↘

(B) ↗

(C) ↙

(D) ↖

41. (1 point) A rigid body is moving in 2D with angular velocity $\vec{\omega} = -\hat{k}$ rad/s and angular acceleration $\vec{\alpha} = 2\hat{k}$ rad/s$^2$. Points $P$ and $Q$ on the body have:

$$\vec{r}_{PQ} = -2\hat{i} + \hat{j} \text{ m}$$
$$\vec{a}_P = \hat{i} - \hat{j} \text{ m/s}^2.$$

What is the $\hat{i}$ component $a_{Qx}$ of the acceleration $\vec{a}_Q$ of point $Q$?

(A) $a_{Qx} < -3$ m/s$^2$

(B) $-3$ m/s$^2 \leq a_{Qx} < 0$ m/s$^2$

(C) $a_{Qx} = 0$ m/s$^2$

(D) $3$ m/s$^2 \leq a_{Qx}$

(E) $0$ m/s$^2 < a_{Qx} < 3$ m/s$^2$

42. (1 point) A rigid body is moving in 2D with angular velocity $\vec{\omega} = \omega\hat{k}$, for a non-negative value of $\omega$, and zero angular acceleration. Points $P$ and $Q$ on the body are a distance $r_{PQ} = 0.2$ m apart and it is observed that:

$$\vec{a}_P = 2\hat{\imath} - \hat{\jmath} \text{ m/s}^2$$
$$\vec{a}_Q = -\hat{\imath} + 3\hat{\jmath} \text{ m/s}^2.$$

What is $\omega$?

(A) $5 \text{ rad/s} \leq \omega < 10 \text{ rad/s}$

(B) $10 \text{ rad/s} \leq \omega < 15 \text{ rad/s}$

(C) $0 \text{ rad/s} \leq \omega < 5 \text{ rad/s}$

(D) $20 \text{ rad/s} \leq \omega$

(E) $15 \text{ rad/s} \leq \omega < 20 \text{ rad/s}$

43. (1 point) Three meshed gears rotate about fixed centers as shown. The radii are $r_1 = 3$ m, $r_2 = 2$ m, and $r_3 = 4$ m and the corresponding angular velocities are $\vec{\omega}_1 = 2\hat{k}$ rad/s, $\vec{\omega}_2 = \omega_2\hat{k}$, and $\vec{\omega}_3 = \omega_3\hat{k}$.
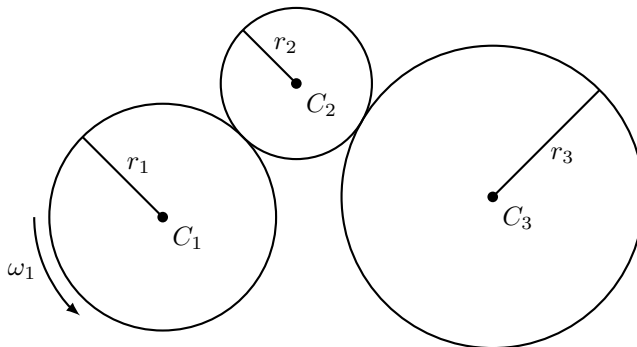


What is $\omega_3$?

(A) $\omega_3 < -2$ rad/s

(B) $2$ rad/s $\leq \omega_3$

(C) $0$ rad/s $< \omega_3 < 2$ rad/s

(D) $-2$ rad/s $\leq \omega_3 < 0$ rad/s

(E) $\omega_3 = 0$ rad/s

44. (1 point) A rigid body is moving in 2D with angular velocity $\vec{\omega} = 3\hat{k}$ rad/s. Points $P$ and $Q$ on the body have:

$$\vec{r}_{PQ} = \hat{\imath} + 4\hat{\jmath} \text{ m}$$
$$\vec{v}_P = -3\hat{\imath} - 2\hat{\jmath} \text{ m/s.}$$

What is the $\hat{\jmath}$ component $v_{Qy}$ of the velocity $\vec{v}_Q$ of point $Q$?

(A) $-2$ m/s $\leq v_{Qy} < 0$ m/s

(B) $v_{Qy} = 0$ m/s

(C) $0$ m/s $< v_{Qy} < 2$ m/s

(D) $2$ m/s $\leq v_{Qy}$

(E) $v_{Qy} < -2$ m/s

45. (1 point) A rigid body is moving in 2D with angular velocity $\vec{\omega} = 2\hat{k}$ rad/s. Two points $P$ and $Q$ are fixed to the body and the offset between them is in the direction $\hat{r}_{PQ} = \frac{1}{5}(-3\hat{i} + 4\hat{j})$ (note that this is the unit vector in the direction of the offset vector $\vec{r}_{PQ}$, not the actual offset vector $\vec{r}_{PQ}$). The velocities are:
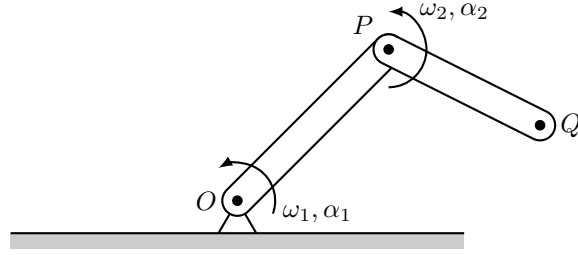
$$\vec{v}_P = 2\hat{i} + 3\hat{j} \text{ m/s}$$
$$\vec{v}_Q = -2\hat{i} \text{ m/s.}$$

What is the distance $r_{PQ}$ between $P$ and $Q$?

(A) $2 \text{ m} \leq r_{PQ} < 3 \text{ m}$

(B) $3 \text{ m} \leq r_{PQ} < 4 \text{ m}$

(C) $1 \text{ m} \leq r_{PQ} < 2 \text{ m}$

(D) $0 \text{ m} \leq r_{PQ} < 1 \text{ m}$

(E) $4 \text{ m} \leq r_{PQ}$

46. (1 point) Two rods are connected with pin joints at $O$, $P$, and $Q$ as shown. The rods have angular velocities and angular accelerations as indicated.



The positions and angular velocities of the rods at the current instant are:

$$\vec{r}_{OP} = 2\hat{i} + 2\hat{j} \text{ m} \qquad\qquad \vec{r}_{PQ} = 2\hat{i} - \hat{j} \text{ m}$$

$$\vec{\omega}_1 = \hat{k} \text{ rad/s} \qquad\qquad \vec{\omega}_2 = -\hat{k} \text{ rad/s}$$

$$\vec{\alpha}_1 = 0 \qquad\qquad \vec{\alpha}_2 = \hat{k} \text{ rad/s}^2$$

What is the $\hat{i}$ component $a_{Qx}$ of the acceleration $\vec{a}_Q$ of point $Q$?

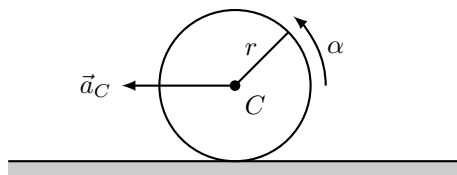(A) $0 \text{ m/s}^2 < a_{Qx} < 2 \text{ m/s}^2$

(B) $a_{Qx} < -2 \text{ m/s}^2$

(C) $2 \text{ m/s}^2 \leq a_{Qx}$

(D) $a_{Qx} = 0 \text{ m/s}^2$

(E) $-2 \text{ m/s}^2 \leq a_{Qx} < 0 \text{ m/s}^2$

47. (1 point) A circular rigid body is rolling without slipping with angular acceleration $\vec{\alpha} = 3\hat{k}$ on a flat surface in 2D as shown. The acceleration of the center is $\vec{a}_C = -2$ m/s².
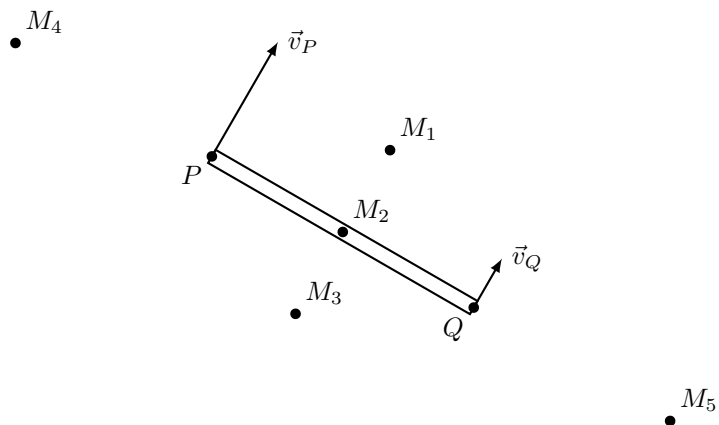


What is the radius $r$ of the rigid body?

(A) $3 \text{ m} \leq r < 4 \text{ m}$

(B) $2 \text{ m} \leq r < 3 \text{ m}$

(C) $4 \text{ m} \leq r$

(D) $0 \text{ m} \leq r < 1 \text{ m}$

(E) $1 \text{ m} \leq r < 2 \text{ m}$
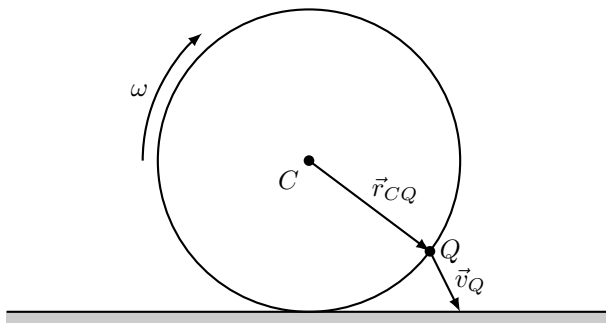
48. (1 point) A rigid rod is moving in 2D as shown below.



Which point $M_i$ is the instantaneous center?

(A) $M_4$

(B) $M_3$

(C) $M_2$

(D) $M_5$

(E) $M_1$

49. (1 point) A circular rigid body with is rolling without slipping on a flat surface in 2D as shown, with angular velocity $\vec{\omega} = -\omega\hat{k}$.
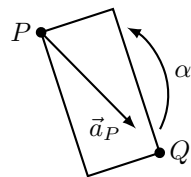


Point $Q$ has:

$$\vec{r}_{CQ} = 4\hat{\imath} - 3\hat{\jmath} \text{ m}$$
$$\vec{v}_Q = \hat{\imath} - 2\hat{\jmath} \text{ m/s}.$$

What is the angular velocity $\omega$?

(A) 2 rad/s $\leq \omega <$ 3 rad/s

(B) 0 rad/s $\leq \omega <$ 1 rad/s

(C) 4 rad/s $\leq \omega$

(D) 3 rad/s $\leq \omega <$ 4 rad/s

(E) 1 rad/s $\leq \omega <$ 2 rad/s

50. (1 point) A rigid body is moving in 2D as shown below, with a counterclockwise angular acceleration and points $P$ and $Q$ on the body. We know that $a_P = 2\omega^2 r_{PQ}$ and $\alpha = \omega^2$.



What is the direction of the acceleration $\vec{a}_Q$?

(A) $\rightarrow$

(B) $\uparrow$

(C) $\leftarrow$

(D) $\downarrow$