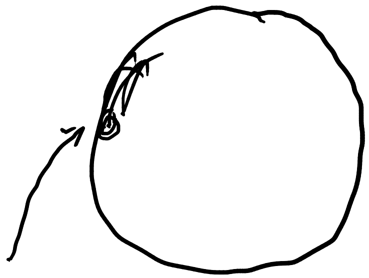
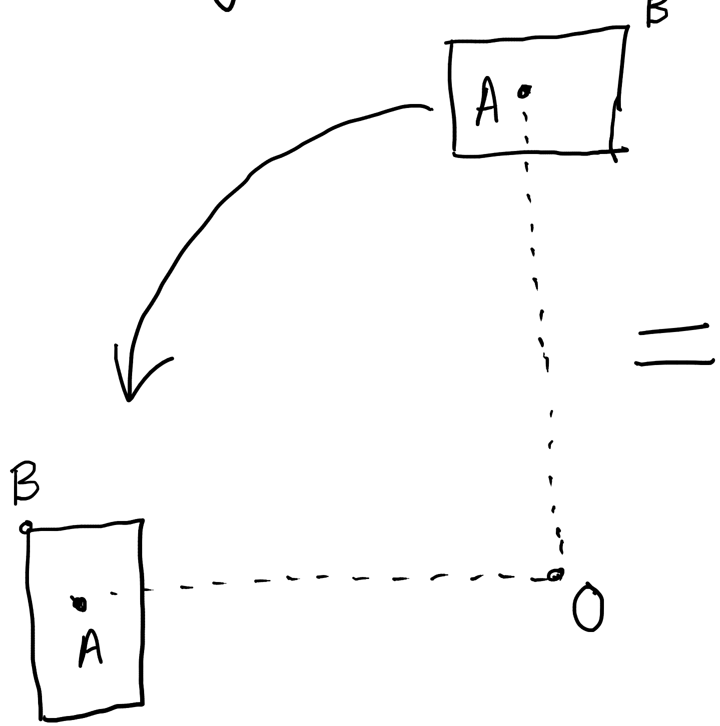
TAM 212

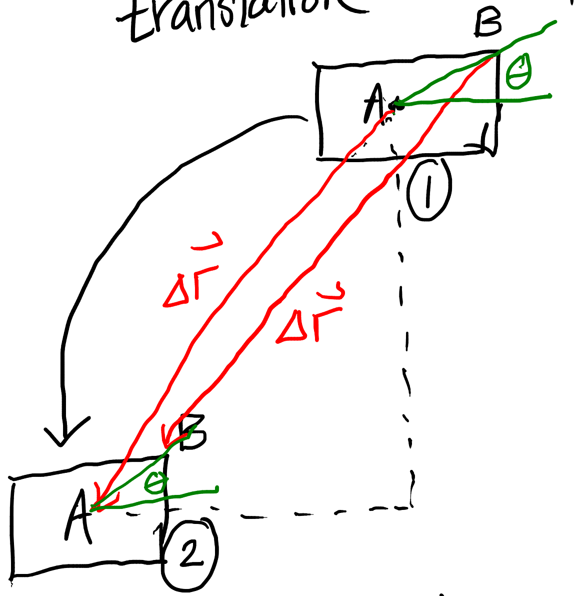


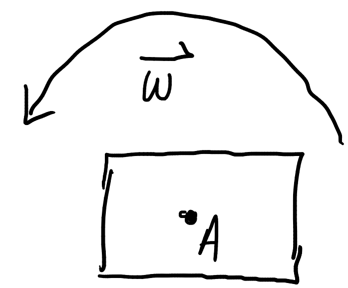
Rigid Body – non-deformable



=

translation + rotation





change in the orientation (θ) of pts A, B

$$\vec{r}_{A,2} = \vec{r}_{A,1} + \Delta\vec{r}$$

$$\vec{r}_{B,2} = \vec{r}_{B,1} + \Delta\vec{r}$$

$$\vec{r}_{OB} = \vec{r}_{OA} + \vec{r}_{AB}$$

relationship between positions

derivative w.r.t. time:

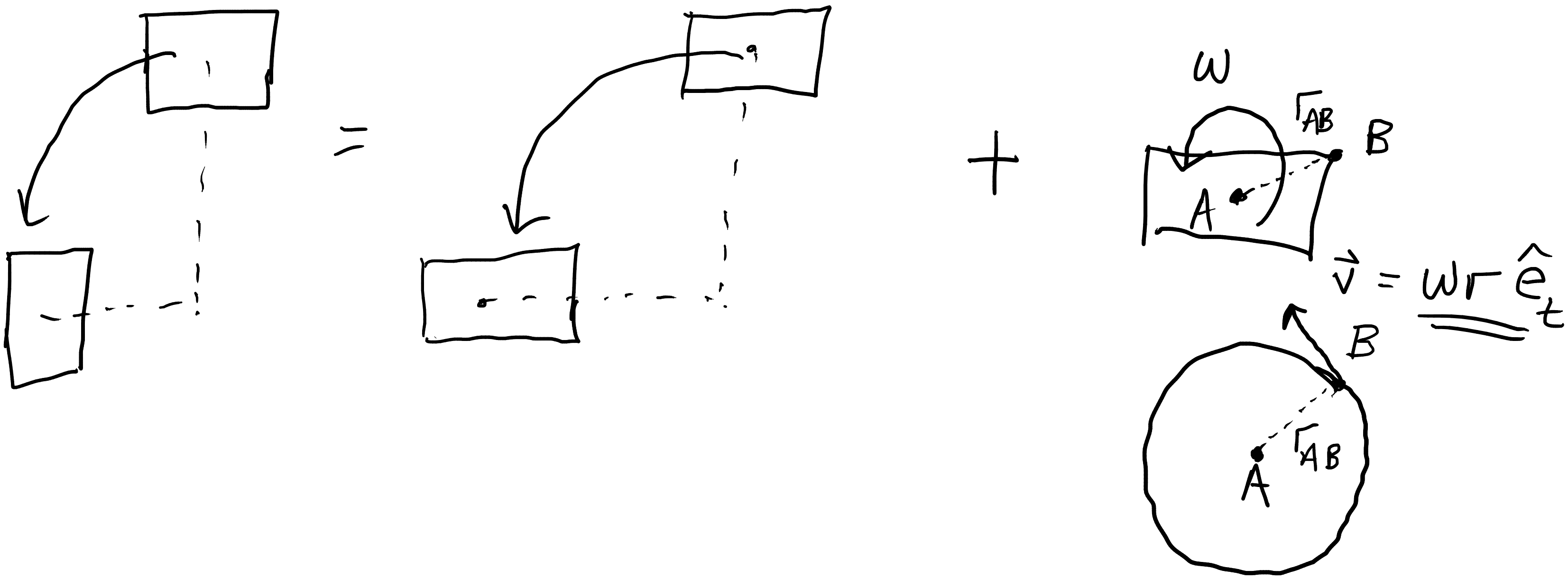
$$\underbrace{\frac{d\vec{r}_{OB}}{dt}}_{\vec{v}_B} = \underbrace{\frac{d\vec{r}_{OA}}{dt}}_{\vec{v}_A} + \frac{d\vec{r}_{AB}}{dt}$$

relative velocity of B wrt A

$= \vec{\omega} \times \vec{r}_{AB}$

motion = translation + rotation

$$\vec{V}_B = \vec{V}_A + \vec{w} \times \vec{r}_{AB}$$



- points on a rigid body have position, velocity, and acceleration vectors
- rigid bodies are associated with orientations, angular velocities, angular accelerations

$$\vec{r}_B = \vec{r}_A + \vec{r}_{AB}$$

$$\vec{V}_B = \vec{V}_A + \underline{\vec{\omega} \times \vec{r}_{AB}}$$

What about accelerations?

$$\frac{d\vec{V}_B}{dt} = \frac{d\vec{V}_A}{dt} + \frac{d}{dt}\left(\vec{\omega} \times \vec{r}_{AB}\right)$$

$$\vec{a}_B = \vec{a}_A + \dot{\vec{\omega}} \times \vec{r}_{AB} + \underbrace{\vec{\omega} \times \dot{\vec{r}}_{AB}}$$

$$\dot{\vec{\omega}} \equiv \vec{\alpha}$$

angular acceleration

WARNING!

$$\vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{AB}\right) \stackrel{?}{=} \left(\vec{\omega} \times \vec{\omega}\right) \times \vec{r}_{AB}$$

A $=$

B $\neq$

$$\vec{a}_B = \vec{a}_A + \underbrace{\vec{\alpha} \times \vec{r}_{AB}}_{\text{rate of change of rotational speed}} + \underbrace{\vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{AB}\right)}_{\text{"centrepatal" acceleration } -\omega^2 r}$$