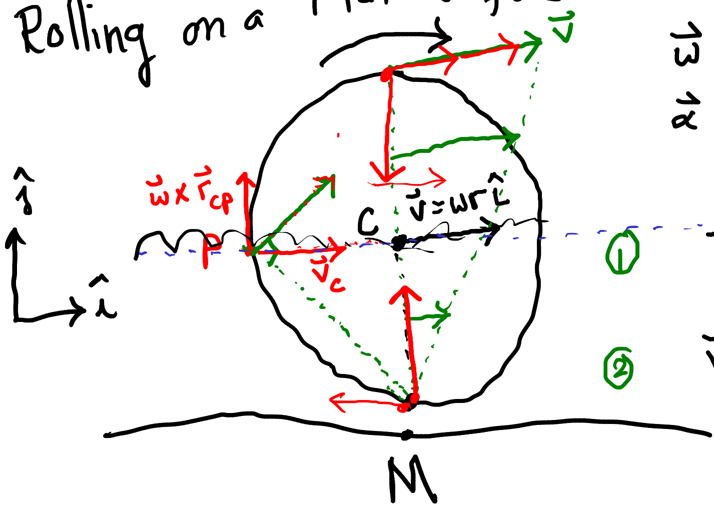
TAM 212

Rolling on a Flat Surface



$\vec{\omega} = -\omega \hat{k}$

$\vec{\alpha} = -\alpha \hat{k}$

① $\vec{v}_P = \vec{v}_C + \vec{\omega} \times \vec{r}_{CP}$

② $\vec{v}_P = \cancel{\vec{v}_M}^{\,0} + \vec{\omega} \times \vec{r}_{MP} = \vec{\omega} \times \vec{r}_{MP}$

$|\vec{v}_P| = \omega r_{MP}$

accelerations:

$\omega = \dot{\theta}$

$\alpha = \ddot{\theta}$

$$\vec{a}_C = \frac{d}{dt}(\vec{v}_C) = \frac{d}{dt}(\omega r \hat{i})$$

$$= \frac{d}{dt}(\omega r)\,\hat{i} + \cancel{\left[\frac{d}{dt}(\hat{i})\right]}^{\,0} \omega r$$

$$= \boxed{r \alpha \hat{i}}$$

$\uparrow \frac{d\omega}{dt}$

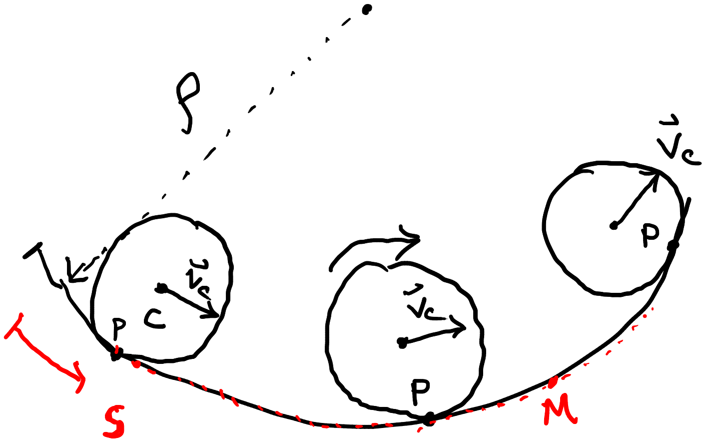
straightforward tangential inwards circular

$$\boxed{\vec{a}_M = \vec{a}_C + \vec{\alpha} \times \vec{r}_{CM} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{CM})}$$

$\vec{r}_{CM} = -r\hat{j}$

$$= (r\alpha\hat{i}) + (-\alpha\hat{k}) \times (-r\hat{j}) - \omega^2 \vec{r}_{CM}$$

$$= (\cancel{r\alpha\hat{i}}) + (\cancel{-r\alpha\hat{i}}) + \omega^2 r \hat{j} = \omega^2 r \hat{j}$$



$$\vec{\omega} = -\omega \hat{k}$$

$$\vec{\alpha} = -\alpha \hat{k}$$

P = instantaneous center of velocity

$$\vec{V}_p = 0$$

$\vec{r}_M$ = position of instantaneous center

$$= \vec{r}_M(t)$$

$$\vec{v} = \dot{s}\hat{e}_t$$

$$\vec{a} = \ddot{s}\hat{e}_t + \frac{\dot{s}^2}{\rho}\hat{e}_n$$

$$\vec{V}_M = \frac{d\vec{r}_M}{dt} \neq 0 \qquad \vec{V}_M = \dot{s}\hat{e}_t$$

$$\vec{a}_M = \frac{d\vec{V}_M}{dt} = \frac{d}{dt}(\dot{s}\hat{e}_t)$$

$$= \ddot{s}\hat{e}_t + \dot{s}\,\dot{\hat{e}}_t$$

$$= \ddot{s}\hat{e}_t + \dot{s}\left(\frac{\dot{s}}{\rho}\right)\hat{e}_n$$