

TAM 212

$$\textcircled{F} = ma$$

$$\vec{F} = m\vec{a}$$

Newton's Law's of Motion - basis for classical mechanics

- (1) If there is no net force on an object, then it's velocity is a constant.

Either (1) object is at rest $\vec{v} = 0$

(2) it's moving with constant speed & direction

- (2) The acceleration of a body is parallel and proportional to the net force \vec{F}_{net} acting on it, and inversely proportional to the body's mass m .

$$\vec{F}_{\text{net}} = m\vec{a} = m \frac{d\vec{v}}{dt}$$

alternative expression (equivalent)

$$\vec{p} = \text{momentum} = m\vec{v}$$

$$\vec{F}_{\text{net}} = \dot{\vec{p}}$$

When no net forces act on an object, momentum is conserved.

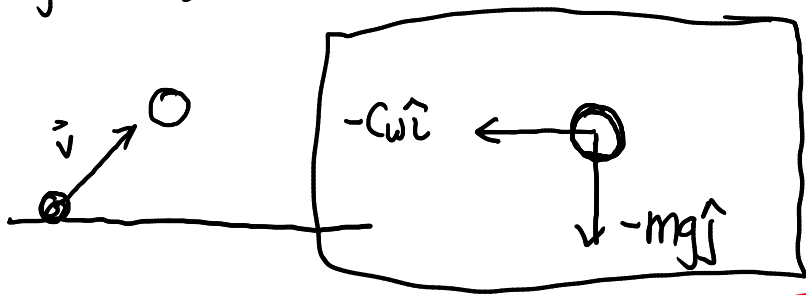
TRUE (A)

FALSE (B)

(3) When a first body exerts a force \vec{F}_1 on a second body, the second body simultaneously exerts a force $\vec{F}_2 = -\vec{F}_1$ on the first body.

Example 1: Know \vec{F} , compute \vec{a}
"Method of Assumed Forces"

A cannonball of mass m is fired from the origin, with initial velocity $\vec{v}_0 = v_{0x} \hat{i} + v_{0y} \hat{j}$. It experiences a force $\vec{F}_g = -mg \hat{j}$ and a wind force $\vec{F}_w = -C_w \hat{i}$. Find $\vec{a}(t)$, $\vec{v}(t)$, $\vec{r}(t)$



$$\textcircled{1} \quad m\vec{a} = \vec{F}_{\text{net}} = \vec{F}_w + \vec{F}_g$$

$$\vec{a}(t) = \frac{1}{m} (-C_w \hat{i} - mg \hat{j})$$

$$a_x(t) = -\frac{C_w}{m} \quad a_y = -g$$

$$\textcircled{2} \quad \vec{v}(t) = \vec{v}_0 + \int_0^t \vec{a}(\tau) d\tau$$

$$= \vec{v}_0 + \int_0^t \left(-\frac{C_w}{m} \hat{i} - g \hat{j} \right) d\tau$$

$$\vec{v}(t) = \vec{v}_0 - \frac{C_w}{m} t \hat{i} - gt \hat{j}$$

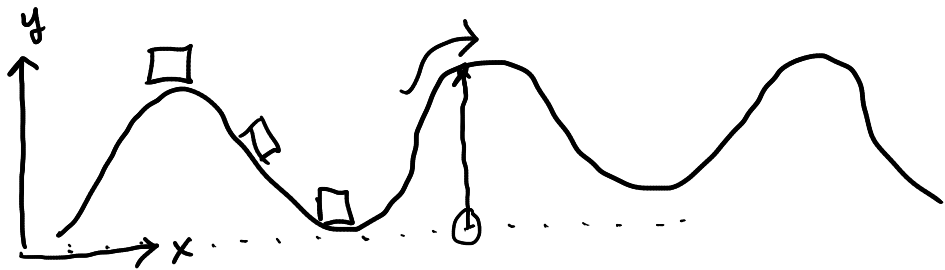
$$\textcircled{3} \quad \vec{r}(t) = \cancel{\vec{r}_0} + \int_0^t \vec{v}(\tau) d\tau$$

$$= \int_0^t \left[\vec{v}_0 - \frac{C_\omega}{m} \tau \hat{i} - g\tau \hat{j} \right] d\tau$$

$$\boxed{\vec{r}(t) = \vec{v}_0 t - \frac{1}{2} \frac{C_\omega}{m} t^2 \hat{i} - \frac{1}{2} g t^2 \hat{j}}$$

Example 2 } Know \vec{a} , compute \vec{F}
"Method of Assumed Motion"

A car of mass m drives on a sinusoidal road at a constant horizontal speed v_0 . The road surface is given by $y = A \sin(kx)$. What is the force of the road on the car?



$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$

$$\begin{aligned}x(t) &= v_0 t \\ y(t) &= A \sin(kx(t)) \\ &= A \sin(kv_0 t)\end{aligned}$$

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = v_0 \hat{i} + A k v_0 \cos(kv_0 t) \hat{j}$$

$$\vec{a}(t) = -A(kv_0)^2 \sin(kx(t)) \hat{j} \quad \vec{F} = m\vec{a}$$