TAM 212. Midterm 2. Apr 4, 2013.

- There are 20 questions, each worth 5 points.
- You must not communicate with other students during this test.
- No electronic devices allowed.
- This is a 2 hour exam.
- Do not turn this page until instructed to do so.
- There are several different versions of this exam.
- The notation $\vec{r}_{PQ}$ denotes the position vector from P to Q.

1. Fill in your information:

Full Name: _____

UIN (Student Number): _____

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2. Circle your discussion section:

	Monday	**Tuesday**	**Wednesday**	**Thursday**
8–9		ADI (260) Karthik		
9–10		ADC (260) Venanzio		ADK (260) Aaron
10–11		ADD (256) Aaron ADQ (344) Jan	ADS (252) Ray	ADT (243) Aaron ADU (344) Jan
11–12		ADE (252) Jan		ADL (256) Kumar
12–1	ADA (243) Ray ADP (135) Seung	ADF (335) Seung ADG (336) Kumar	ADJ (256) Ray ADR (252) Lin	ADN (260) Kumar
1–2				
2–3				
3–4				
4–5	ADV (252) Karthik		ADO (260) Mazhar ADW (252) Lin	
5–6	ADB (260) Mazhar	ADH (260) Karthik	ADM (243) Mazhar	

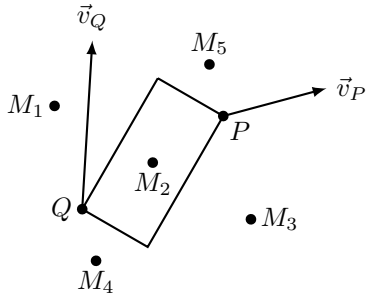
3. Fill in the following answers on the Scantron form:

94. A

95. D

96. C

1. (5 points) A rigid body is moving in 2D as shown below.



Which point M_i is the instantaneous center?

(A) M_5

(B) M_2

(C) M_3

(D) M_1

(E) M_4

2. (5 points) A rigid body is moving in 2D with angular velocity $\vec{\omega} = -2\hat{k}$ rad/s. Two points P and Q are fixed to the body and the offset between them is in the direction $\hat{r}_{PQ} = \frac{1}{5}(3\hat{\imath} - 4\hat{\jmath})$ (note that this is the unit vector in the direction of the offset vector $\vec{r}_{PQ}$, not the actual offset vector $\vec{r}_{PQ}$). The velocities are:

$$\vec{v}_P = 3\hat{\imath} + 4\hat{\jmath} \text{ m/s}$$
$$\vec{v}_Q = -5\hat{\imath} - 2\hat{\jmath} \text{ m/s}.$$

What is the distance r_{PQ} between P and Q?

(A) $8 \text{ m} \leq r_{PQ}$

(B) $4 \text{ m} \leq r_{PQ} < 6 \text{ m}$

(C) $2 \text{ m} \leq r_{PQ} < 4 \text{ m}$

(D) $6 \text{ m} \leq r_{PQ} < 8 \text{ m}$

(E) $0 \text{ m} \leq r_{PQ} < 2 \text{ m}$

3. (5 points) A rigid body is moving in 2D with angular velocity $\vec{\omega} = -2\hat{k}$ rad/s and angular acceleration $\vec{\alpha} = -\hat{k}$ rad/s^2. Points P and Q on the body have:

$$\vec{r}_{PQ} = 2\hat{\imath} - 2\hat{\jmath} \text{ m}$$
$$\vec{a}_P = 3\hat{\imath} - 3\hat{\jmath} \text{ m/s}^2.$$

What is the $\hat{\jmath}$ component a_{Qy} of the acceleration $\vec{a}_Q$ of point Q?

(A) $a_{Qy} < -3$ m/s^2

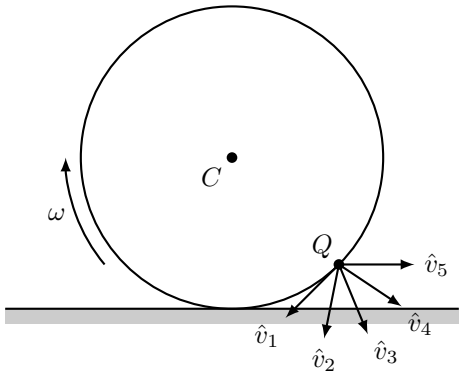
(B) $a_{Qy} = 0$ m/s^2

(C) -3 m/s$^2 \leq a_{Qy} < 0$ m/s^2

(D) 3 m/s$^2 \leq a_{Qy}$

(E) 0 m/s$^2 < a_{Qy} < 3$ m/s^2

4. (5 points) A circular rigid body is rolling without slipping on a flat surface in 2D in a clockwise direction as shown.



What is the direction of the velocity $\vec{v}_Q$ of point Q?

(A) $\hat{v}_2$

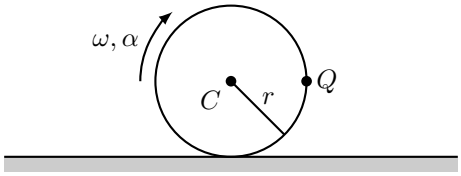
(B) $\hat{v}_3$

(C) $\hat{v}_1$

(D) $\hat{v}_4$

(E) $\hat{v}_5$

5. (5 points) A circular rigid body with radius $r = 2$ m is rolling without slipping with angular velocity $\vec{\omega} = -\hat{k}$ rad/s on a flat surface in 2D as shown. The body is speeding up and has angular acceleration $\vec{\alpha} = -\alpha\hat{k}$. Point Q is at the right edge of the body and has acceleration $\vec{a}_Q = 3\hat{\imath} - 5\hat{\jmath}$ m/s^2.



What is α?

(A) 1 rad/s^2 $\leq \alpha <$ 1.5 rad/s^2

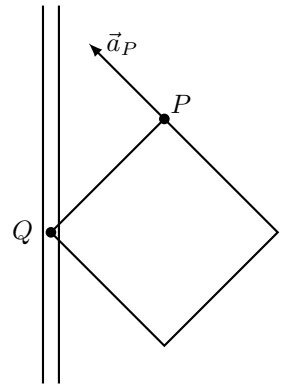
(B) 0 rad/s^2 $\leq \alpha <$ 0.5 rad/s^2

(C) 1.5 rad/s^2 $\leq \alpha <$ 2 rad/s^2

(D) 0.5 rad/s^2 $\leq \alpha <$ 1 rad/s^2

(E) 2 rad/s^2 $\leq \alpha$

6. (5 points) A rigid body is moving in 2D as shown below with angular velocity $\vec{\omega} = \omega\hat{k}$ and zero angular acceleration. A pin at point Q constrains that point to move in a vertical slot.



Point P on the body has:

$$\vec{r}_{PQ} = -\hat{\imath} - \hat{\jmath}\ \text{m}$$
$$\vec{a}_P = -\hat{\imath} + \hat{\jmath}\ \text{m/s}^2.$$

What is the magnitude a_Q of the acceleration $\vec{a}_Q$ of point Q?

(A) $0\ \text{m/s}^2 \leq a_Q < 1\ \text{m/s}^2$

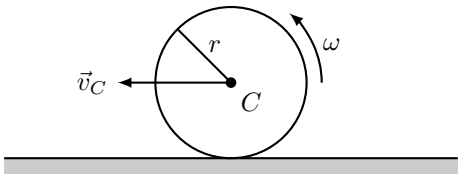
(B) $2\ \text{m/s}^2 \leq a_Q < 3\ \text{m/s}^2$

(C) $3\ \text{m/s}^2 \leq a_Q < 4\ \text{m/s}^2$

(D) $4\ \text{m/s}^2 \leq a_Q$

(E) $1\ \text{m/s}^2 \leq a_Q < 2\ \text{m/s}^2$

7. (5 points) A circular rigid body with radius $r = 2$ m is rolling without slipping on a flat surface in 2D as shown. The speed of the center is $v_C = 9$ m/s.



What is the angular velocity ω?

(A) $0 \text{ rad/s} \leq \omega < 1 \text{ rad/s}$

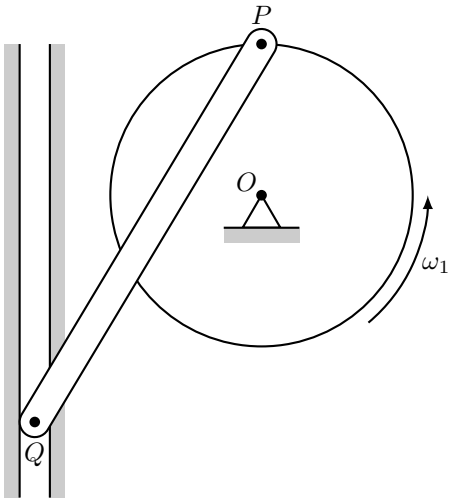
(B) $1 \text{ rad/s} \leq \omega < 2 \text{ rad/s}$

(C) $2 \text{ rad/s} \leq \omega < 3 \text{ rad/s}$

(D) $4 \text{ rad/s} \leq \omega$

(E) $3 \text{ rad/s} \leq \omega < 4 \text{ rad/s}$

8. (5 points) A circular rigid body rotates about the fixed center O with angular velocity $\vec{\omega}_1 = 5\hat{k}$ rad/s as shown. A rigid rod connects pins P and Q, and point Q is constrained to only move vertically.



At the current instant the positions are:

$$\vec{r}_{OP} = 2\hat{\jmath} \text{ m}$$
$$\vec{r}_{PQ} = -3\hat{\imath} - 5\hat{\jmath} \text{ m}.$$

What is the speed v_Q of point Q?

(A) $8 \text{ m/s} \leq v_Q$

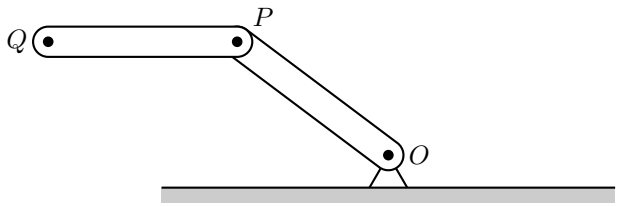
(B) $2 \text{ m/s} \leq v_Q < 4 \text{ m/s}$

(C) $4 \text{ m/s} \leq v_Q < 6 \text{ m/s}$

(D) $6 \text{ m/s} \leq v_Q < 8 \text{ m/s}$

(E) $0 \text{ m/s} \leq v_Q < 2 \text{ m/s}$

9. (5 points) Two rods are connected with pin joints at O, P, and Q as shown. The angular velocity and acceleration for rod OP are $\vec{\omega}_1$ and $\vec{\alpha}_1$, while the angular velocity and acceleration for rod PQ are $\vec{\omega}_2$ and $\vec{\alpha}_2$.



The positions and angular velocities of the rods at the current instant are:

$$\vec{r}_{OP} = -4\hat{\imath} + 3\hat{\jmath} \text{ m} \qquad \vec{r}_{PQ} = -5\hat{\imath} \text{ m}$$
$$\vec{\omega}_1 = 0 \qquad \vec{\omega}_2 = 2\hat{k} \text{ rad/s}$$
$$\vec{\alpha}_1 = 1\hat{k} \text{ rad/s}^2 \qquad \vec{\alpha}_2 = -\hat{k} \text{ rad/s}^2.$$

What is the $\hat{\jmath}$ component a_{Qy} of the acceleration $\vec{a}_Q$ of point Q?

(A) $2 \text{ m/s}^2 \leq a_{Qy}$

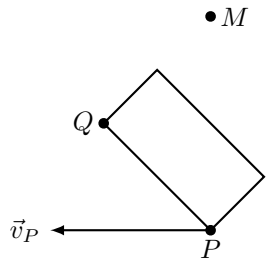
(B) $-2 \text{ m/s}^2 \leq a_{Qy} < 0 \text{ m/s}^2$

(C) $a_{Qy} = 0 \text{ m/s}^2$

(D) $a_{Qy} < -2 \text{ m/s}^2$

(E) $0 \text{ m/s}^2 < a_{Qy} < 2 \text{ m/s}^2$

10. (5 points) A rigid body is moving in 2D as shown below with points P and Q attached to the body. The instantaneous center of the body is at point M.



What is the direction of the velocity $\vec{v}_Q$ of point Q?

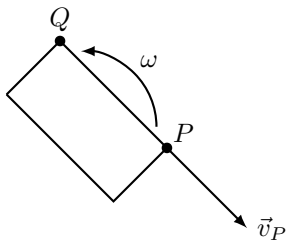
(A) ↘

(B) ↙

(C) ↗

(D) ↖

11. (5 points) A rigid body is moving in 2D as shown, with a counterclockwise rotation (ω is positive in the direction indicated). The angular velocity ω, distance r_{PQ}, and speed v_P satisfy $\omega r_{PQ} = v_P$.



What is the direction of $\vec{v}_Q$?

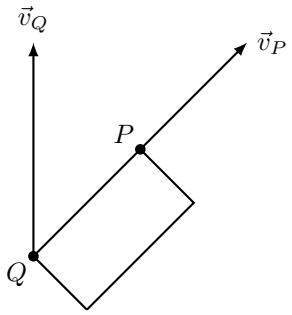
(A) $\leftarrow$

(B) $\uparrow$

(C) $\downarrow$

(D) $\rightarrow$

12. (5 points) A rigid body is moving in 2D as shown below.

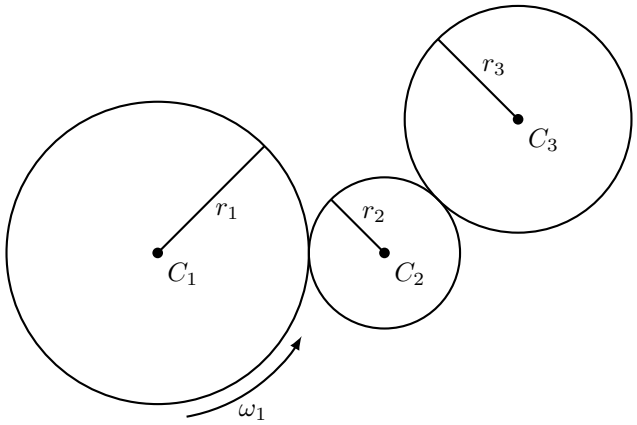


What is the direction of the angular velocity of the body?

(A) ↺ (counterclockwise)

(B) ↻ (clockwise)

13. (5 points) Three meshed gears rotate about fixed centers as shown. The radii are $r_1 = 4$ m, $r_2 = 2$ m, and $r_3 = 3$ m and the corresponding angular velocities are $\vec{\omega}_1 = 3\hat{k}$ rad/s, $\vec{\omega}_2 = \omega_2\hat{k}$, and $\vec{\omega}_3 = \omega_3\hat{k}$.



What is ω_3?

(A) 3 rad/s $\leq \omega_3 <$ 4 rad/s

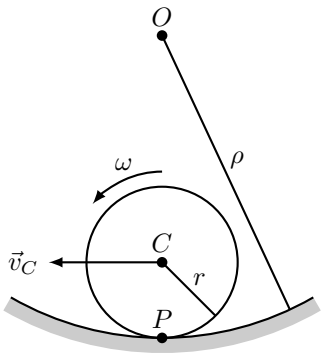
(B) 1 rad/s $\leq \omega_3 <$ 2 rad/s

(C) 4 rad/s $\leq \omega_3$

(D) 2 rad/s $\leq \omega_3 <$ 3 rad/s

(E) 0 rad/s $\leq \omega_3 <$ 1 rad/s

14. (5 points) A circular rigid body with radius $r = 3$ m is rolling without slipping on a curved surface with radius of curvature ρ in 2D as shown. The angular velocity of the body is a constant $\vec{\omega} = \hat{k}$ rad/s. Point P is fixed to the edge of the body and, at the instant shown, is the contact point. The magnitude of acceleration of P is $a_P = 6$ m/s^2.



What is the radius of curvature ρ of the surface?

(A) $0 \text{ m} \leq \rho < 3 \text{ m}$

(B) $6 \text{ m} \leq \rho < 9 \text{ m}$

(C) $3 \text{ m} \leq \rho < 6 \text{ m}$

(D) $9 \text{ m} \leq \rho < 12 \text{ m}$

(E) $12 \text{ m} \leq \rho$

15. (5 points) A rigid body is moving in 2D with angular velocity $\vec{\omega} = -2\hat{k}$ rad/s. Points P and Q on the body have:

$$\vec{r}_{PQ} = 2\hat{\imath} - \hat{\jmath} \text{ m}$$
$$\vec{v}_P = -\hat{\imath} + \hat{\jmath} \text{ m/s}.$$

What is the $\hat{\imath}$ component v_{Qx} of the velocity $\vec{v}_Q$ of point Q?

(A) $-2 \text{ m/s} \leq v_{Qx} < 0 \text{ m/s}$

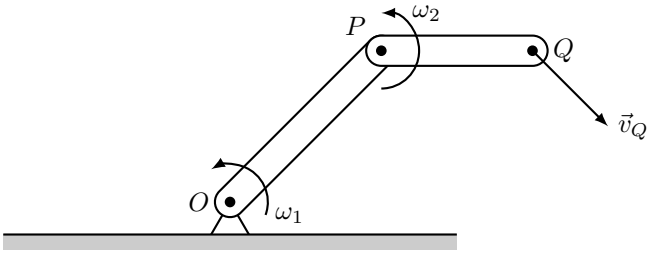
(B) $0 \text{ m/s} < v_{Qx} < 2 \text{ m/s}$

(C) $2 \text{ m/s} \leq v_{Qx}$

(D) $v_{Qx} = 0 \text{ m/s}$

(E) $v_{Qx} < -2 \text{ m/s}$

16. (5 points) Two rods are connected with pin joints at O, P, and Q as shown. Rod OP has angular velocity $\vec{\omega}_1 = \omega_1 \hat{k}$ and rod PQ has angular velocity $\vec{\omega}_2 = \omega_2 \hat{k}$.



The positions and velocities at the current instant are:

$$\vec{r}_{OP} = 2\hat{\imath} + 2\hat{\jmath} \text{ m}$$
$$\vec{r}_{PQ} = 2\hat{\imath} \text{ m}$$
$$\vec{v}_Q = 2\hat{\imath} - 2\hat{\jmath} \text{ m/s}.$$

What is ω_2?

(A) $\omega_2 = 0$ rad/s

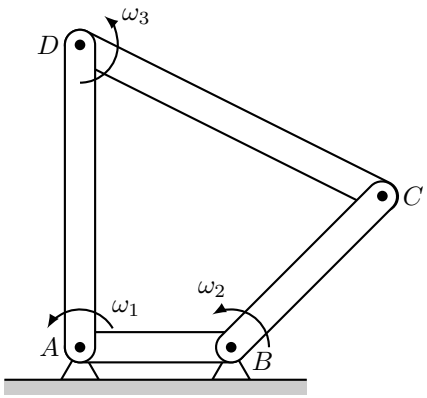
(B) -1 rad/s $\leq \omega_2 < 0$ rad/s

(C) 1 rad/s $\leq \omega_2$

(D) $\omega_2 < -1$ rad/s

(E) 0 rad/s $< \omega_2 < 1$ rad/s

17. (5 points) A four-bar linkage has rigid rods connecting pins at A, B, C, and D, as shown. The angular velocities are $\vec{\omega}_1 = \hat{k}$ for rod AD, $\vec{\omega}_2 = \omega_2\hat{k}$ for rod BC, and $\vec{\omega}_3 = \omega_3\hat{k}$ for rod DC.



At the current instant the positions are:

$$\vec{r}_{AB} = \hat{\imath} \text{ m}$$
$$\vec{r}_{BC} = \hat{\imath} + \hat{\jmath} \text{ m}$$
$$\vec{r}_{AD} = 2\hat{\jmath} \text{ m}$$
$$\vec{r}_{DC} = 2\hat{\imath} - \hat{\jmath} \text{ m}.$$

What is ω_2?

(A) $1 \text{ rad/s} \leq \omega_2 < 1.5 \text{ rad/s}$

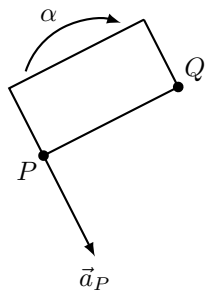
(B) $0 \text{ rad/s} \leq \omega_2 < 0.5 \text{ rad/s}$

(C) $0.5 \text{ rad/s} \leq \omega_2 < 1 \text{ rad/s}$

(D) $1.5 \text{ rad/s} \leq \omega_2 < 2 \text{ rad/s}$

(E) $2 \text{ rad/s} \leq \omega_2$

18. (5 points) A rigid body is moving in 2D as shown below, with a counterclockwise angular acceleration and points P and Q on the body (α is positive in the direction shown). We know that $a_P = \omega^2 r_{PQ} = \alpha r_{PQ}$.



What is the direction of the acceleration $\vec{a}_Q$?

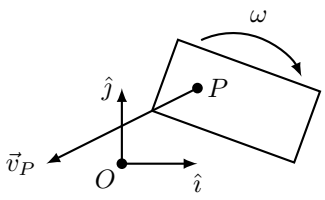
(A) $\uparrow$

(B) $\rightarrow$

(C) $\downarrow$

(D) $\leftarrow$

19. (5 points) A rigid body is moving in 2D as shown below with angular velocity $\vec{\omega} = -\hat{k}$ rad/s.



Relative to the origin O, the point P has:

$$\vec{r}_P = \hat{\imath} + \hat{\jmath} \text{ m}$$
$$\vec{v}_P = -2\hat{\imath} - \hat{\jmath} \text{ m/s}.$$

What is the x coordinate M_x of the instantaneous center M of the body?

(A) $M_x < -1$ m

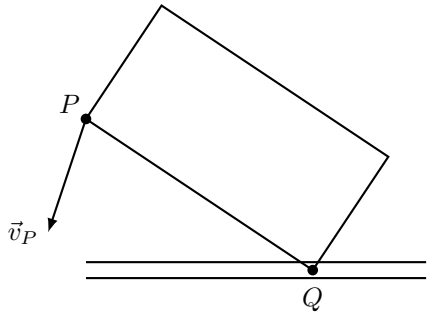
(B) 0 m $< M_x < 1$ m

(C) -1 m $\leq M_x < 0$ m

(D) $M_x = 0$ m

(E) 1 m $\leq M_x$

20. (5 points) A rigid body is moving in 2D as shown below with angular velocity $\vec{\omega} = \omega\hat{k}$. A pin at point Q constrains that point to move in a horizontal slot.



Point P on the body has:

$$\vec{r}_{PQ} = 3\hat{\imath} - 2\hat{\jmath} \text{ m}$$
$$\vec{v}_P = -\hat{\imath} - 3\hat{\jmath} \text{ m/s}.$$

What is ω?

(A) $\omega < -2$ rad/s

(B) $\omega = 0$ rad/s

(C) $2 \text{ rad/s} \leq \omega$

(D) $0 \text{ rad/s} < \omega < 2 \text{ rad/s}$

(E) $-2 \text{ rad/s} \leq \omega < 0 \text{ rad/s}$

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- No electronic devices allowed.
- This is a 2 hour exam.
- Do not turn this page until instructed to do so.
- There are several different versions of this exam.
- The notation $\vec{r}_{PQ}$ denotes the position vector from P to Q.

1. Fill in your information:

Full Name: _____

UIN (Student Number): _____

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2. Circle your discussion section:

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9–10		ADC (260) Venanzio		ADK (260) Aaron
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1–2				
2–3				
3–4				
4–5	ADV (252) Karthik		ADO (260) Mazhar ADW (252) Lin	
5–6	ADB (260) Mazhar	ADH (260) Karthik	ADM (243) Mazhar	

3. Fill in the following answers on the Scantron form:

94. B

95. E

96. D

1. (5 points) A rigid body is moving in 2D with angular velocity $\vec{\omega} = -\hat{k}$ rad/s. Two points P and Q are fixed to the body and the offset between them is in the direction $\hat{r}_{PQ} = \frac{1}{5}(3\hat{\imath} + 4\hat{\jmath})$ (note that this is the unit vector in the direction of the offset vector $\vec{r}_{PQ}$, not the actual offset vector $\vec{r}_{PQ}$). The velocities are:

$$\vec{v}_P = 2\hat{\jmath} \text{ m/s}$$
$$\vec{v}_Q = 4\hat{\imath} - \hat{\jmath} \text{ m/s}.$$

What is the distance r_{PQ} between P and Q?

(A) $6 \text{ m} \leq r_{PQ} < 8 \text{ m}$

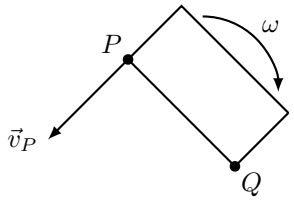
(B) $4 \text{ m} \leq r_{PQ} < 6 \text{ m}$

(C) $0 \text{ m} \leq r_{PQ} < 2 \text{ m}$

(D) $8 \text{ m} \leq r_{PQ}$

(E) $2 \text{ m} \leq r_{PQ} < 4 \text{ m}$

2. (5 points) A rigid body is moving in 2D as shown, with a clockwise rotation (ω is positive in the direction indicated). The angular velocity ω, distance r_{PQ}, and speed v_P satisfy $\omega r_{PQ} = v_P$.



What is the direction of $\vec{v}_Q$?

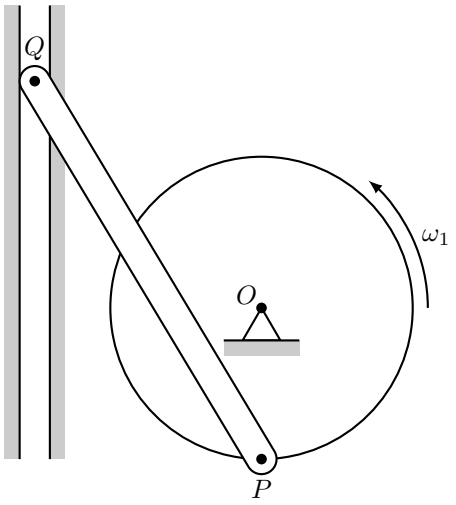
(A) ↖

(B) ↗

(C) ↘

(D) ↙

3. (5 points) A circular rigid body rotates about the fixed center O with angular velocity $\vec{\omega}_1 = 5\hat{k}$ rad/s as shown. A rigid rod connects pins P and Q, and point Q is constrained to only move vertically.



At the current instant the positions are:

$$\vec{r}_{OP} = -2\hat{\jmath} \text{ m}$$
$$\vec{r}_{PQ} = -3\hat{\imath} + 5\hat{\jmath} \text{ m}.$$

What is the speed v_Q of point Q?

(A) $2 \text{ m/s} \leq v_Q < 4 \text{ m/s}$

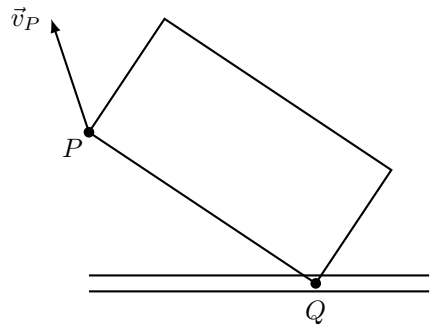
(B) $6 \text{ m/s} \leq v_Q < 8 \text{ m/s}$

(C) $0 \text{ m/s} \leq v_Q < 2 \text{ m/s}$

(D) $8 \text{ m/s} \leq v_Q$

(E) $4 \text{ m/s} \leq v_Q < 6 \text{ m/s}$

4. (5 points) A rigid body is moving in 2D as shown below with angular velocity $\vec{\omega} = \omega\hat{k}$. A pin at point Q constrains that point to move in a horizontal slot.



Point P on the body has:

$$\vec{r}_{PQ} = 3\hat{\imath} - 2\hat{\jmath} \text{ m}$$
$$\vec{v}_P = -\hat{\imath} + 3\hat{\jmath} \text{ m/s}.$$

What is ω?

(A) $0 \text{ rad/s} < \omega < 2 \text{ rad/s}$

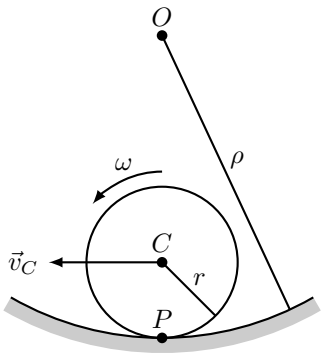
(B) $\omega = 0 \text{ rad/s}$

(C) $2 \text{ rad/s} \leq \omega$

(D) $\omega < -2 \text{ rad/s}$

(E) $-2 \text{ rad/s} \leq \omega < 0 \text{ rad/s}$

5. (5 points) A circular rigid body with radius $r = 2$ m is rolling without slipping on a curved surface with radius of curvature ρ in 2D as shown. The angular velocity of the body is a constant $\vec{\omega} = 2\hat{k}$ rad/s. Point P is fixed to the edge of the body and, at the instant shown, is the contact point. The magnitude of acceleration of P is $a_P = 16$ m/s^2.



What is the radius of curvature ρ of the surface?

(A) $9 \text{ m} \leq \rho < 12 \text{ m}$

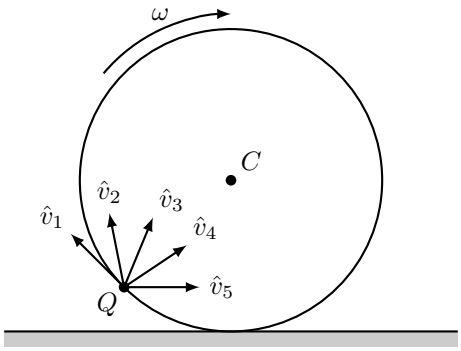
(B) $0 \text{ m} \leq \rho < 3 \text{ m}$

(C) $6 \text{ m} \leq \rho < 9 \text{ m}$

(D) $12 \text{ m} \leq \rho$

(E) $3 \text{ m} \leq \rho < 6 \text{ m}$

6. (5 points) A circular rigid body is rolling without slipping on a flat surface in 2D in a clockwise direction as shown.



What is the direction of the velocity $\vec{v}_Q$ of point Q?

(A) $\hat{v}_5$

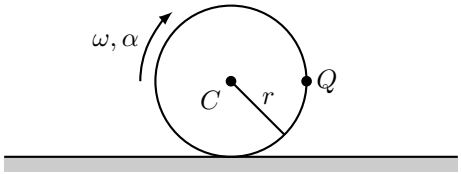
(B) $\hat{v}_3$

(C) $\hat{v}_4$

(D) $\hat{v}_2$

(E) $\hat{v}_1$

7. (5 points) A circular rigid body with radius $r = 2$ m is rolling without slipping with angular velocity $\vec{\omega} = -2\hat{k}$ rad/s on a flat surface in 2D as shown. The body is speeding up and has angular acceleration $\vec{\alpha} = -\alpha\hat{k}$. Point Q is at the right edge of the body and has acceleration $\vec{a}_Q = -6\hat{\imath} - 2\hat{\jmath}$ m/s^2.



What is α?

(A) 1 rad/s^2 $\leq \alpha <$ 1.5 rad/s^2

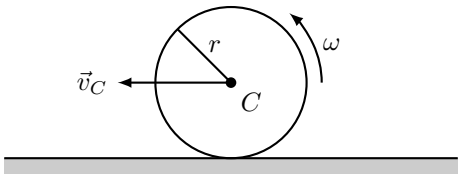
(B) 1.5 rad/s^2 $\leq \alpha <$ 2 rad/s^2

(C) 2 rad/s^2 $\leq \alpha$

(D) 0 rad/s^2 $\leq \alpha <$ 0.5 rad/s^2

(E) 0.5 rad/s^2 $\leq \alpha <$ 1 rad/s^2

8. (5 points) A circular rigid body with radius $r = 3$ m is rolling without slipping on a flat surface in 2D as shown. The speed of the center is $v_C = 7$ m/s.



What is the angular velocity ω?

(A) $2 \text{ rad/s} \leq \omega < 3 \text{ rad/s}$

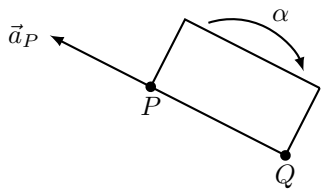
(B) $1 \text{ rad/s} \leq \omega < 2 \text{ rad/s}$

(C) $3 \text{ rad/s} \leq \omega < 4 \text{ rad/s}$

(D) $0 \text{ rad/s} \leq \omega < 1 \text{ rad/s}$

(E) $4 \text{ rad/s} \leq \omega$

9. (5 points) A rigid body is moving in 2D as shown below, with a clockwise angular acceleration and points P and Q on the body (α is positive in the direction shown). We know that $a_P = \omega^2\, r_{PQ} = \alpha\, r_{PQ}$.



What is the direction of the acceleration $\vec{a}_Q$?

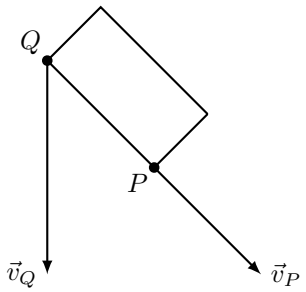
(A) $\leftarrow$

(B) $\uparrow$

(C) $\downarrow$

(D) $\rightarrow$

10. (5 points) A rigid body is moving in 2D as shown below.

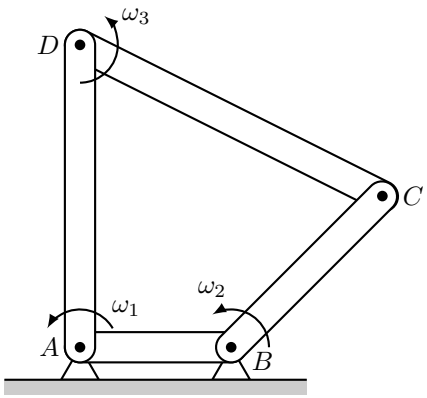


What is the direction of the angular velocity of the body?

(A) ↻ (clockwise)

(B) ↺ (counterclockwise)

11. (5 points) A four-bar linkage has rigid rods connecting pins at A, B, C, and D, as shown. The angular velocities are $\vec{\omega}_1 = 2\hat{k}$ for rod AD, $\vec{\omega}_2 = \omega_2\hat{k}$ for rod BC, and $\vec{\omega}_3 = \omega_3\hat{k}$ for rod DC.



At the current instant the positions are:

$$\vec{r}_{AB} = \hat{\imath} \text{ m}$$
$$\vec{r}_{BC} = \hat{\imath} + \hat{\jmath} \text{ m}$$
$$\vec{r}_{AD} = 2\hat{\jmath} \text{ m}$$
$$\vec{r}_{DC} = 2\hat{\imath} - \hat{\jmath} \text{ m}.$$

What is ω_2?

(A) $1 \text{ rad/s} \leq \omega_2 < 1.5 \text{ rad/s}$

(B) $0 \text{ rad/s} \leq \omega_2 < 0.5 \text{ rad/s}$

(C) $1.5 \text{ rad/s} \leq \omega_2 < 2 \text{ rad/s}$

(D) $2 \text{ rad/s} \leq \omega_2$

(E) $0.5 \text{ rad/s} \leq \omega_2 < 1 \text{ rad/s}$

12. (5 points) A rigid body is moving in 2D with angular velocity $\vec{\omega} = \hat{k}$ rad/s and angular acceleration $\vec{\alpha} = -\hat{k}$ rad/s^2. Points P and Q on the body have:

$$\vec{r}_{PQ} = \hat{\imath} - \hat{\jmath} \text{ m}$$
$$\vec{a}_P = 3\hat{\imath} \text{ m/s}^2.$$

What is the $\hat{\jmath}$ component a_{Qy} of the acceleration $\vec{a}_Q$ of point Q?

(A) $0 \text{ m/s}^2 < a_{Qy} < 3 \text{ m/s}^2$

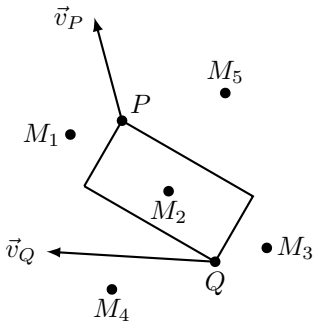
(B) $-3 \text{ m/s}^2 \leq a_{Qy} < 0 \text{ m/s}^2$

(C) $a_{Qy} = 0 \text{ m/s}^2$

(D) $3 \text{ m/s}^2 \leq a_{Qy}$

(E) $a_{Qy} < -3 \text{ m/s}^2$

13. (5 points) A rigid body is moving in 2D as shown below.



Which point M_i is the instantaneous center?

(A) M_2

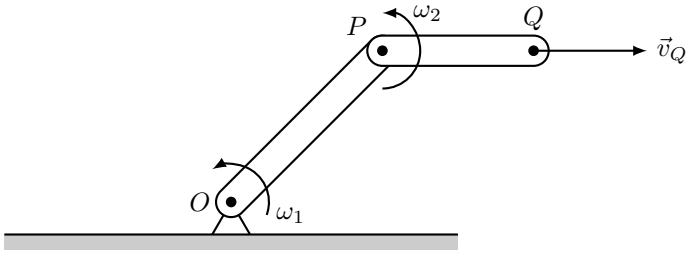
(B) M_1

(C) M_5

(D) M_4

(E) M_3

14. (5 points) Two rods are connected with pin joints at O, P, and Q as shown. Rod OP has angular velocity $\vec{\omega}_1 = \omega_1 \hat{k}$ and rod PQ has angular velocity $\vec{\omega}_2 = \omega_2 \hat{k}$.



The positions and velocities at the current instant are:

$$\vec{r}_{OP} = 2\hat{\imath} + 2\hat{\jmath} \text{ m}$$
$$\vec{r}_{PQ} = 2\hat{\imath} \text{ m}$$
$$\vec{v}_Q = 2\hat{\imath} \text{ m/s}.$$

What is ω_2?

(A) $0 \text{ rad/s} < \omega_2 < 1 \text{ rad/s}$

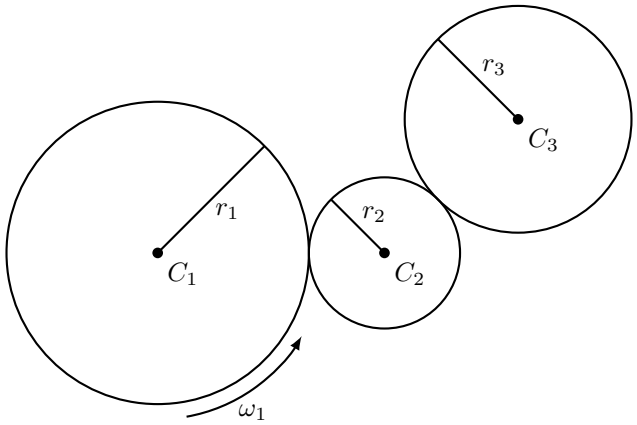
(B) $\omega_2 < -1 \text{ rad/s}$

(C) $-1 \text{ rad/s} \leq \omega_2 < 0 \text{ rad/s}$

(D) $\omega_2 = 0 \text{ rad/s}$

(E) $1 \text{ rad/s} \leq \omega_2$

15. (5 points) Three meshed gears rotate about fixed centers as shown. The radii are $r_1 = 4$ m, $r_2 = 2$ m, and $r_3 = 3$ m and the corresponding angular velocities are $\vec{\omega}_1 = 2\hat{k}$ rad/s, $\vec{\omega}_2 = \omega_2\hat{k}$, and $\vec{\omega}_3 = \omega_3\hat{k}$.



What is ω_3?

(A) 2 rad/s $\leq \omega_3 <$ 3 rad/s

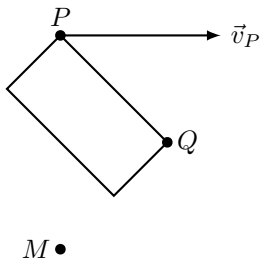
(B) 3 rad/s $\leq \omega_3 <$ 4 rad/s

(C) 4 rad/s $\leq \omega_3$

(D) 0 rad/s $\leq \omega_3 <$ 1 rad/s

(E) 1 rad/s $\leq \omega_3 <$ 2 rad/s

16. (5 points) A rigid body is moving in 2D as shown below with points P and Q attached to the body. The instantaneous center of the body is at point M.



What is the direction of the velocity $\vec{v}_Q$ of point Q?

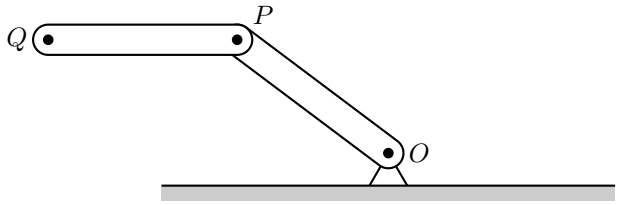
(A) $\searrow$

(B) $\swarrow$

(C) $\nearrow$

(D) $\nwarrow$

17. (5 points) Two rods are connected with pin joints at O, P, and Q as shown. The angular velocity and acceleration for rod OP are $\vec{\omega}_1$ and $\vec{\alpha}_1$, while the angular velocity and acceleration for rod PQ are $\vec{\omega}_2$ and $\vec{\alpha}_2$.



The positions and angular velocities of the rods at the current instant are:

$$\vec{r}_{OP} = -4\hat{\imath} + 3\hat{\jmath} \text{ m} \qquad \vec{r}_{PQ} = -5\hat{\imath} \text{ m}$$
$$\vec{\omega}_1 = 0 \qquad \vec{\omega}_2 = -\hat{k} \text{ rad/s}$$
$$\vec{\alpha}_1 = -2\hat{k} \text{ rad/s}^2 \qquad \vec{\alpha}_2 = 2\hat{k} \text{ rad/s}^2.$$

What is the $\hat{\jmath}$ component a_{Qy} of the acceleration $\vec{a}_Q$ of point Q?

(A) $a_{Qy} = 0$ m/s^2

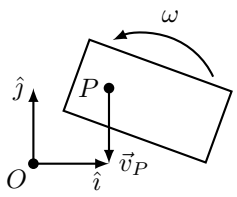
(B) $a_{Qy} < -2$ m/s^2

(C) -2 m/s$^2 \leq a_{Qy} < 0$ m/s^2

(D) 0 m/s$^2 < a_{Qy} < 2$ m/s^2

(E) 2 m/s$^2 \leq a_{Qy}$

18. (5 points) A rigid body is moving in 2D as shown below with angular velocity $\vec{\omega} = \hat{k}$ rad/s.



Relative to the origin O, the point P has:

$$\vec{r}_P = \hat{\imath} + \hat{\jmath} \text{ m}$$
$$\vec{v}_P = -\hat{\jmath} \text{ m/s}.$$

What is the x coordinate M_x of the instantaneous center M of the body?

(A) $1 \text{ m} \leq M_x$

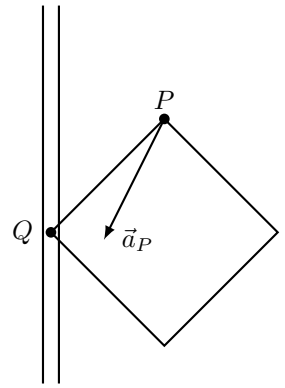
(B) $M_x = 0$ m

(C) $0 \text{ m} < M_x < 1$ m

(D) $-1 \text{ m} \leq M_x < 0$ m

(E) $M_x < -1$ m

19. (5 points) A rigid body is moving in 2D as shown below with angular velocity $\vec{\omega} = \omega\hat{k}$ and zero angular acceleration. A pin at point Q constrains that point to move in a vertical slot.



Point P on the body has:

$$\vec{r}_{PQ} = -\hat{\imath} - \hat{\jmath}\ \text{m}$$
$$\vec{a}_P = -\hat{\imath} - 2\hat{\jmath}\ \text{m/s}^2.$$

What is the magnitude a_Q of the acceleration $\vec{a}_Q$ of point Q?

(A) $3\ \text{m/s}^2 \leq a_Q < 4\ \text{m/s}^2$

(B) $0\ \text{m/s}^2 \leq a_Q < 1\ \text{m/s}^2$

(C) $1\ \text{m/s}^2 \leq a_Q < 2\ \text{m/s}^2$

(D) $2\ \text{m/s}^2 \leq a_Q < 3\ \text{m/s}^2$

(E) $4\ \text{m/s}^2 \leq a_Q$

20. (5 points) A rigid body is moving in 2D with angular velocity $\vec{\omega} = -\hat{k}$ rad/s. Points P and Q on the body have:

$$\vec{r}_{PQ} = 3\hat{\imath} - 2\hat{\jmath} \text{ m}$$
$$\vec{v}_P = \hat{\imath} + 2\hat{\jmath} \text{ m/s}.$$

What is the $\hat{\imath}$ component v_{Qx} of the velocity $\vec{v}_Q$ of point Q?

(A) $2 \text{ m/s} \leq v_{Qx}$

(B) $v_{Qx} < -2 \text{ m/s}$

(C) $-2 \text{ m/s} \leq v_{Qx} < 0 \text{ m/s}$

(D) $0 \text{ m/s} < v_{Qx} < 2 \text{ m/s}$

(E) $v_{Qx} = 0 \text{ m/s}$

TAM 212. Midterm 2. Apr 4, 2013.

- There are 20 questions, each worth 5 points.
- You must not communicate with other students during this test.
- No electronic devices allowed.
- This is a 2 hour exam.
- Do not turn this page until instructed to do so.
- There are several different versions of this exam.
- The notation $\vec{r}_{PQ}$ denotes the position vector from P to Q.

1. Fill in your information:

Full Name: _____

UIN (Student Number): _____

NetID: _____

2. Circle your discussion section:

	Monday	**Tuesday**	**Wednesday**	**Thursday**
8–9		ADI (260) Karthik		
9–10		ADC (260) Venanzio		ADK (260) Aaron
10–11		ADD (256) Aaron ADQ (344) Jan	ADS (252) Ray	ADT (243) Aaron ADU (344) Jan
11–12		ADE (252) Jan		ADL (256) Kumar
12–1	ADA (243) Ray ADP (135) Seung	ADF (335) Seung ADG (336) Kumar	ADJ (256) Ray ADR (252) Lin	ADN (260) Kumar
1–2				
2–3				
3–4				
4–5	ADV (252) Karthik		ADO (260) Mazhar ADW (252) Lin	
5–6	ADB (260) Mazhar	ADH (260) Karthik	ADM (243) Mazhar	

3. Fill in the following answers on the Scantron form:

94. C

95. A

96. E

1. (5 points) A rigid body is moving in 2D with angular velocity $\vec{\omega} = \hat{k}$ rad/s. Two points P and Q are fixed to the body and the offset between them is in the direction $\hat{r}_{PQ} = \frac{1}{5}(-3\hat{\imath} - 4\hat{\jmath})$ (note that this is the unit vector in the direction of the offset vector $\vec{r}_{PQ}$, not the actual offset vector $\vec{r}_{PQ}$). The velocities are:

$$\vec{v}_P = -3\hat{\imath} + 3\hat{\jmath} \text{ m/s}$$
$$\vec{v}_Q = 5\hat{\imath} - 3\hat{\jmath} \text{ m/s}.$$

What is the distance r_{PQ} between P and Q?

(A) $6 \text{ m} \leq r_{PQ} < 8 \text{ m}$

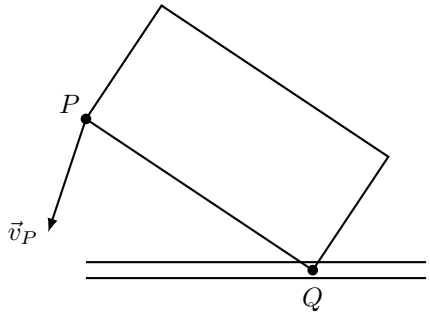
(B) $2 \text{ m} \leq r_{PQ} < 4 \text{ m}$

(C) $4 \text{ m} \leq r_{PQ} < 6 \text{ m}$

(D) $8 \text{ m} \leq r_{PQ}$

(E) $0 \text{ m} \leq r_{PQ} < 2 \text{ m}$

2. (5 points) A rigid body is moving in 2D as shown below with angular velocity $\vec{\omega} = \omega\hat{k}$. A pin at point Q constrains that point to move in a horizontal slot.



Point P on the body has:

$$\vec{r}_{PQ} = 3\hat{\imath} - 2\hat{\jmath} \text{ m}$$
$$\vec{v}_P = -2\hat{\imath} - 6\hat{\jmath} \text{ m/s}.$$

What is ω?

(A) $0 \text{ rad/s} < \omega < 2 \text{ rad/s}$

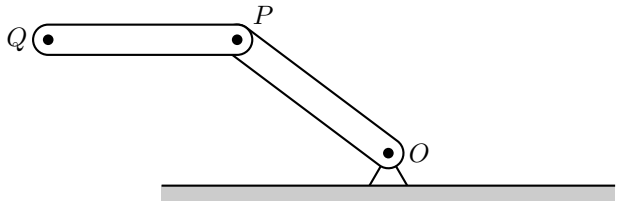
(B) $2 \text{ rad/s} \leq \omega$

(C) $\omega < -2 \text{ rad/s}$

(D) $-2 \text{ rad/s} \leq \omega < 0 \text{ rad/s}$

(E) $\omega = 0 \text{ rad/s}$

3. (5 points) Two rods are connected with pin joints at O, P, and Q as shown. The angular velocity and acceleration for rod OP are $\vec{\omega}_1$ and $\vec{\alpha}_1$, while the angular velocity and acceleration for rod PQ are $\vec{\omega}_2$ and $\vec{\alpha}_2$.



The positions and angular velocities of the rods at the current instant are:

$$\vec{r}_{OP} = -4\hat{\imath} + 3\hat{\jmath} \text{ m} \qquad \vec{r}_{PQ} = -5\hat{\imath} \text{ m}$$
$$\vec{\omega}_1 = 0 \qquad \vec{\omega}_2 = \hat{k} \text{ rad/s}$$
$$\vec{\alpha}_1 = 2\hat{k} \text{ rad/s}^2 \qquad \vec{\alpha}_2 = -\hat{k} \text{ rad/s}^2.$$

What is the $\hat{\jmath}$ component a_{Qy} of the acceleration $\vec{a}_Q$ of point Q?

(A) $0 \text{ m/s}^2 < a_{Qy} < 2 \text{ m/s}^2$

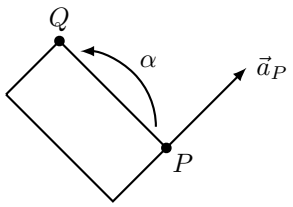
(B) $2 \text{ m/s}^2 \leq a_{Qy}$

(C) $a_{Qy} < -2 \text{ m/s}^2$

(D) $-2 \text{ m/s}^2 \leq a_{Qy} < 0 \text{ m/s}^2$

(E) $a_{Qy} = 0 \text{ m/s}^2$

4. (5 points) A rigid body is moving in 2D as shown below, with a counterclockwise angular acceleration and points P and Q on the body (α is positive in the direction shown). We know that $a_P = \omega^2 r_{PQ} = \alpha r_{PQ}$.



What is the direction of the acceleration $\vec{a}_Q$?

(A) ↙

(B) ↖

(C) ↘

(D) ↗

5. (5 points) A rigid body is moving in 2D with angular velocity $\vec{\omega} = -\hat{k}$ rad/s and angular acceleration $\vec{\alpha} = -2\hat{k}$ rad/s^2. Points P and Q on the body have:

$$\vec{r}_{PQ} = 2\hat{\imath} + \hat{\jmath} \text{ m}$$
$$\vec{a}_P = \hat{\imath} + 3\hat{\jmath} \text{ m/s}^2.$$

What is the $\hat{\jmath}$ component a_{Qy} of the acceleration $\vec{a}_Q$ of point Q?

(A) $3 \text{ m/s}^2 \leq a_{Qy}$

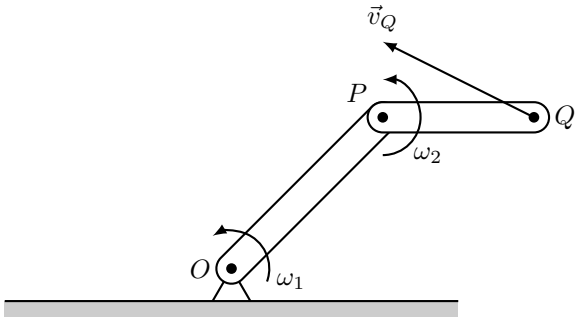
(B) $a_{Qy} = 0 \text{ m/s}^2$

(C) $0 \text{ m/s}^2 < a_{Qy} < 3 \text{ m/s}^2$

(D) $-3 \text{ m/s}^2 \leq a_{Qy} < 0 \text{ m/s}^2$

(E) $a_{Qy} < -3 \text{ m/s}^2$

6. (5 points) Two rods are connected with pin joints at O, P, and Q as shown. Rod OP has angular velocity $\vec{\omega}_1 = \omega_1 \hat{k}$ and rod PQ has angular velocity $\vec{\omega}_2 = \omega_2 \hat{k}$.



The positions and velocities at the current instant are:

$$\vec{r}_{OP} = 2\hat{\imath} + 2\hat{\jmath} \text{ m}$$
$$\vec{r}_{PQ} = 2\hat{\imath} \text{ m}$$
$$\vec{v}_Q = -4\hat{\imath} + 2\hat{\jmath} \text{ m/s}.$$

What is ω_2?

(A) $0 \text{ rad/s} < \omega_2 < 1 \text{ rad/s}$

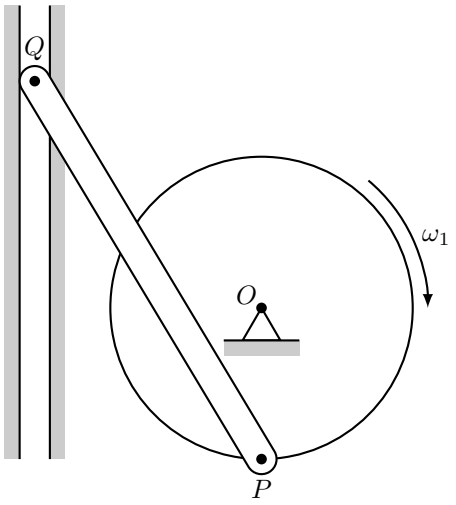
(B) $\omega_2 < -1 \text{ rad/s}$

(C) $-1 \text{ rad/s} \leq \omega_2 < 0 \text{ rad/s}$

(D) $\omega_2 = 0 \text{ rad/s}$

(E) $1 \text{ rad/s} \leq \omega_2$

7. (5 points) A circular rigid body rotates about the fixed center O with angular velocity $\vec{\omega}_1 = -10\hat{k}$ rad/s as shown. A rigid rod connects pins P and Q, and point Q is constrained to only move vertically.



At the current instant the positions are:

$$\vec{r}_{OP} = -2\hat{\jmath} \text{ m}$$
$$\vec{r}_{PQ} = -3\hat{\imath} + 5\hat{\jmath} \text{ m}.$$

What is the speed v_Q of point Q?

(A) $8 \text{ m/s} \leq v_Q$

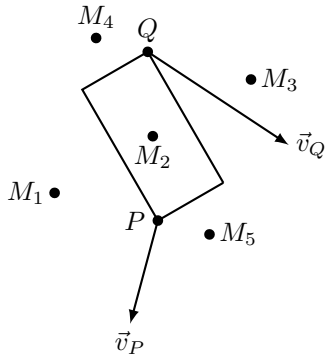
(B) $0 \text{ m/s} \leq v_Q < 2 \text{ m/s}$

(C) $4 \text{ m/s} \leq v_Q < 6 \text{ m/s}$

(D) $2 \text{ m/s} \leq v_Q < 4 \text{ m/s}$

(E) $6 \text{ m/s} \leq v_Q < 8 \text{ m/s}$

8. (5 points) A rigid body is moving in 2D as shown below.



Which point M_i is the instantaneous center?

(A) M_2

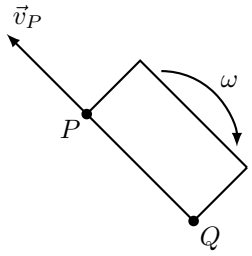
(B) M_4

(C) M_1

(D) M_5

(E) M_3

9. (5 points) A rigid body is moving in 2D as shown, with a clockwise rotation (ω is positive in the direction indicated). The angular velocity ω, distance r_{PQ}, and speed v_P satisfy $\omega r_{PQ} = v_P$.



What is the direction of $\vec{v}_Q$?

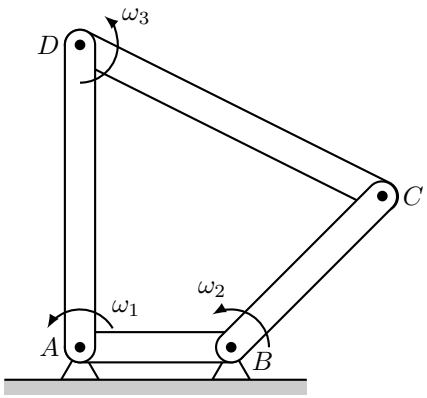
(A) $\downarrow$

(B) $\uparrow$

(C) $\leftarrow$

(D) $\rightarrow$

10. (5 points) A four-bar linkage has rigid rods connecting pins at A, B, C, and D, as shown. The angular velocities are $\vec{\omega}_1 = 3\hat{k}$ for rod AD, $\vec{\omega}_2 = \omega_2\hat{k}$ for rod BC, and $\vec{\omega}_3 = \omega_3\hat{k}$ for rod DC.



At the current instant the positions are:

$$\vec{r}_{AB} = \hat{\imath} \text{ m}$$
$$\vec{r}_{BC} = \hat{\imath} + \hat{\jmath} \text{ m}$$
$$\vec{r}_{AD} = 2\hat{\jmath} \text{ m}$$
$$\vec{r}_{DC} = 2\hat{\imath} - \hat{\jmath} \text{ m}.$$

What is ω_2?

(A) $6 \text{ rad/s} \leq \omega_2 < 9 \text{ rad/s}$

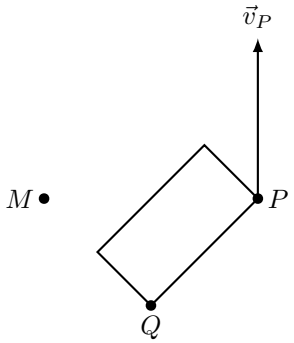
(B) $0 \text{ rad/s} \leq \omega_2 < 3 \text{ rad/s}$

(C) $12 \text{ rad/s} \leq \omega_2$

(D) $3 \text{ rad/s} \leq \omega_2 < 6 \text{ rad/s}$

(E) $9 \text{ rad/s} \leq \omega_2 < 12 \text{ rad/s}$

11. (5 points) A rigid body is moving in 2D as shown below with points P and Q attached to the body. The instantaneous center of the body is at point M.



What is the direction of the velocity $\vec{v}_Q$ of point Q?

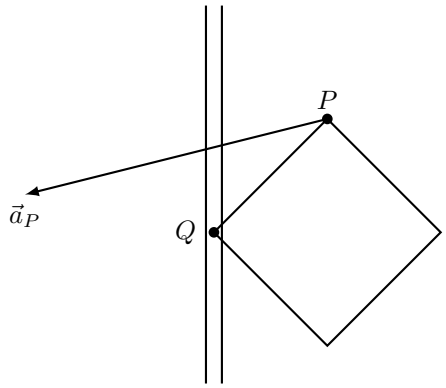
(A) ↖

(B) ↙

(C) ↘

(D) ↗

12. (5 points) A rigid body is moving in 2D as shown below with angular velocity $\vec{\omega} = \omega\hat{k}$ and zero angular acceleration. A pin at point Q constrains that point to move in a vertical slot.



Point P on the body has:

$$\vec{r}_{PQ} = -\hat{\imath} - \hat{\jmath}\ \text{m}$$
$$\vec{a}_P = -4\hat{\imath} - \hat{\jmath}\ \text{m/s}^2.$$

What is the magnitude a_Q of the acceleration $\vec{a}_Q$ of point Q?

(A) $4\ \text{m/s}^2 \leq a_Q$

(B) $1\ \text{m/s}^2 \leq a_Q < 2\ \text{m/s}^2$

(C) $0\ \text{m/s}^2 \leq a_Q < 1\ \text{m/s}^2$

(D) $2\ \text{m/s}^2 \leq a_Q < 3\ \text{m/s}^2$

(E) $3\ \text{m/s}^2 \leq a_Q < 4\ \text{m/s}^2$

13. (5 points) A rigid body is moving in 2D with angular velocity $\vec{\omega} = 2\hat{k}$ rad/s. Points P and Q on the body have:

$$\vec{r}_{PQ} = \hat{\imath} + \hat{\jmath} \text{ m}$$
$$\vec{v}_P = 2\hat{\imath} - \hat{\jmath} \text{ m/s}.$$

What is the $\hat{\imath}$ component v_{Qx} of the velocity $\vec{v}_Q$ of point Q?

(A) $-2 \text{ m/s} \leq v_{Qx} < 0 \text{ m/s}$

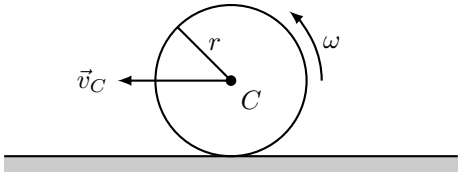
(B) $v_{Qx} = 0 \text{ m/s}$

(C) $2 \text{ m/s} \leq v_{Qx}$

(D) $0 \text{ m/s} < v_{Qx} < 2 \text{ m/s}$

(E) $v_{Qx} < -2 \text{ m/s}$

14. (5 points) A circular rigid body with radius $r = 6$ m is rolling without slipping on a flat surface in 2D as shown. The speed of the center is $v_C = 4$ m/s.



What is the angular velocity ω?

(A) $2 \text{ rad/s} \leq \omega < 3 \text{ rad/s}$

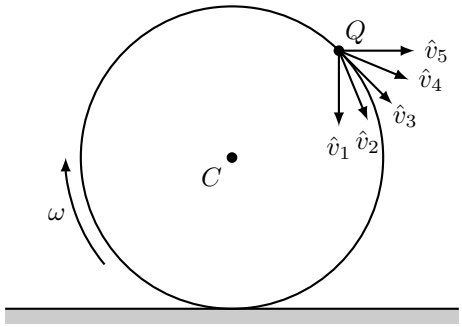
(B) $1 \text{ rad/s} \leq \omega < 2 \text{ rad/s}$

(C) $3 \text{ rad/s} \leq \omega < 4 \text{ rad/s}$

(D) $0 \text{ rad/s} \leq \omega < 1 \text{ rad/s}$

(E) $4 \text{ rad/s} \leq \omega$

15. (5 points) A circular rigid body is rolling without slipping on a flat surface in 2D in a clockwise direction as shown.



What is the direction of the velocity $\vec{v}_Q$ of point Q?

(A) $\hat{v}_1$

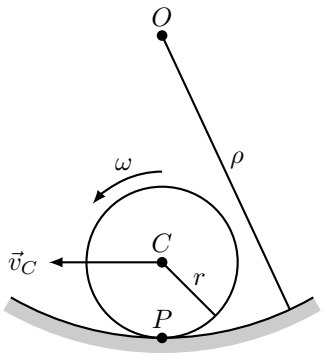
(B) $\hat{v}_2$

(C) $\hat{v}_4$

(D) $\hat{v}_3$

(E) $\hat{v}_5$

16. (5 points) A circular rigid body with radius $r = 3$ m is rolling without slipping on a curved surface with radius of curvature ρ in 2D as shown. The angular velocity of the body is a constant $\vec{\omega} = 2\hat{k}$ rad/s. Point P is fixed to the edge of the body and, at the instant shown, is the contact point. The magnitude of acceleration of P is $a_P = 30$ m/s^2.



What is the radius of curvature ρ of the surface?

(A) 12 m $\leq \rho$

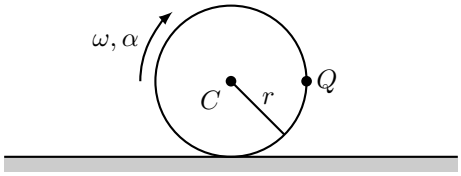
(B) 3 m $\leq \rho <$ 6 m

(C) 6 m $\leq \rho <$ 9 m

(D) 0 m $\leq \rho <$ 3 m

(E) 9 m $\leq \rho <$ 12 m

17. (5 points) A circular rigid body with radius $r = 2$ m is rolling without slipping with angular velocity $\vec{\omega} = -\hat{k}$ rad/s on a flat surface in 2D as shown. The body is speeding up and has angular acceleration $\vec{\alpha} = -\alpha\hat{k}$. Point Q is at the right edge of the body and has acceleration $\vec{a}_Q = \hat{\imath} - 3\hat{\jmath}$ m/s^2.



What is α?

(A) 0.5 rad/s$^2 \leq \alpha < 1$ rad/s^2

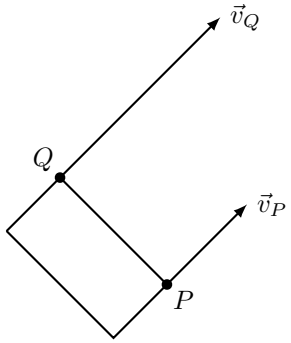
(B) 2 rad/s$^2 \leq \alpha$

(C) 1.5 rad/s$^2 \leq \alpha < 2$ rad/s^2

(D) 1 rad/s$^2 \leq \alpha < 1.5$ rad/s^2

(E) 0 rad/s$^2 \leq \alpha < 0.5$ rad/s^2

18. (5 points) A rigid body is moving in 2D as shown below.

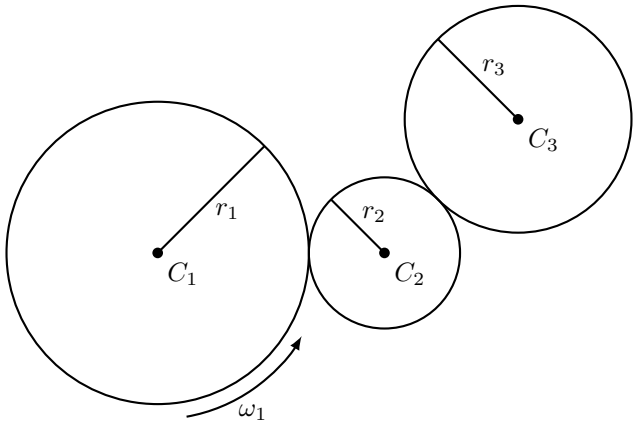


What is the direction of the angular velocity of the body?

(A) ↺ (counterclockwise)

(B) ↻ (clockwise)

19. (5 points) Three meshed gears rotate about fixed centers as shown. The radii are $r_1 = 4$ m, $r_2 = 2$ m, and $r_3 = 3$ m and the corresponding angular velocities are $\vec{\omega}_1 = \hat{k}$ rad/s, $\vec{\omega}_2 = \omega_2\hat{k}$, and $\vec{\omega}_3 = \omega_3\hat{k}$.



What is ω_3?

(A) $2 \text{ rad/s} \leq \omega_3 < 3 \text{ rad/s}$

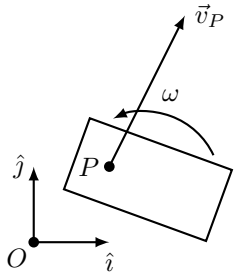
(B) $4 \text{ rad/s} \leq \omega_3$

(C) $3 \text{ rad/s} \leq \omega_3 < 4 \text{ rad/s}$

(D) $0 \text{ rad/s} \leq \omega_3 < 1 \text{ rad/s}$

(E) $1 \text{ rad/s} \leq \omega_3 < 2 \text{ rad/s}$

20. (5 points) A rigid body is moving in 2D as shown below with angular velocity $\vec{\omega} = 2\hat{k}$ rad/s.



Relative to the origin O, the point P has:

$$\vec{r}_P = \hat{\imath} + \hat{\jmath} \text{ m}$$
$$\vec{v}_P = 2\hat{\imath} + 4\hat{\jmath} \text{ m/s}.$$

What is the x coordinate M_x of the instantaneous center M of the body?

(A) $M_x = 0$ m

(B) $M_x < -1$ m

(C) 0 m $< M_x < 1$ m

(D) 1 m $\leq M_x$

(E) -1 m $\leq M_x < 0$ m