Final	May 7	Tue	1:30 – 4:30 pm	
Conflict	May 8	Wed	1:30 – 4:30 pm	A
Alternate conflicts		Wed	morning	B
		Tue	morning	C
		Thu	morning	D
		Thu	afternoon	E

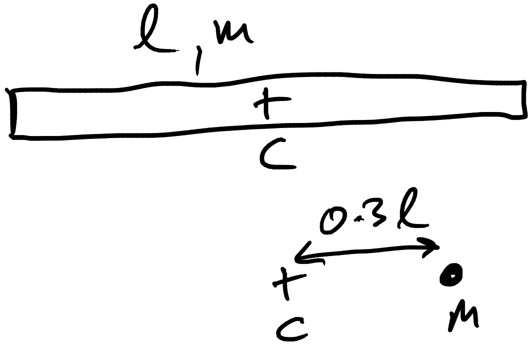
Radius of gyration $r_{g,c}$ or k_c

defined by $r_{g,c} = \sqrt{\dfrac{I_c}{m}}$

$$\Rightarrow \quad I_c = m\, r_{g,c}^2$$

equivalent radius for a point mass to match I.

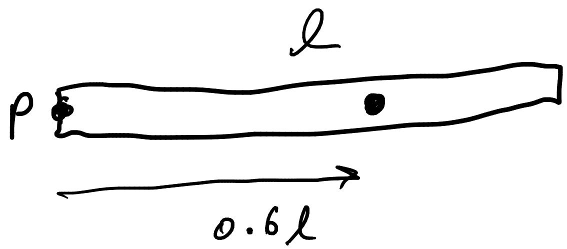
ex



$I_c = \frac{1}{12} m l^2$

$r_{g,c} = \sqrt{\dfrac{\frac{1}{12} m l^2}{m}} = \frac{1}{\sqrt{12}} l \approx 0.3 l$

ex



$I_p = \frac{1}{3} m l^2$

$r_{g,c} = \frac{1}{\sqrt{3}} l \approx 0.6 l$

General forms of Euler's equns for rotating rigid bodies

$\sum \vec{M}_p = 0$ if body is not rotating

$\sum \vec{M}_c = I_c \vec{\alpha}$ where C is center of mass

$\sum \vec{M}_o = I_o \vec{\alpha}$ where O is a fixed point

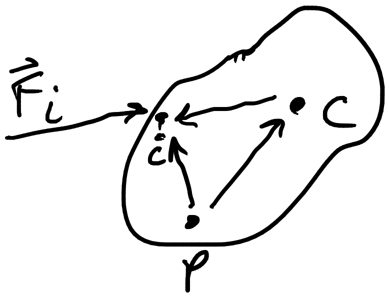
(1) $\sum \vec{M}_p = I_c \vec{\alpha} + \vec{r}_{pc} \times m\vec{a}_c$ where P is any other point on the body

(2) $\sum \vec{M}_p = I_p \vec{\alpha} + \vec{r}_{pc} \times m\vec{a}_p$ P any other point on the body.

(1) $P = C \Rightarrow \vec{r}_{cc} = 0$

(2) $P = O = \text{fixed} \Rightarrow \vec{a}_p = 0$

derivation of (1)



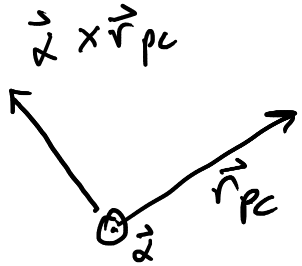
$$\sum_i \vec{M}_{P,i} = \sum_i \vec{r}_{Pi} \times \vec{F}_i$$
$$= \sum_{i=1}^{3} (\vec{r}_{PC} + \vec{r}_{Ci}) \times \vec{F}_i$$
$$= \sum_i \vec{r}_{PC} \times \vec{F}_i + \sum_i \vec{r}_{Ci} \times \vec{F}_i$$
$$= \vec{r}_{PC} \times \left(\sum_i \vec{F}_i \right) + \sum_i \vec{r}_{Ci} \times \vec{F}_i$$
$$= \vec{r}_{PC} \times m\vec{a}_C + \sum_i \vec{M}_{Ci}$$
$$= I_C \vec{\alpha} + \vec{r}_{PC} \times m\vec{a}_C$$

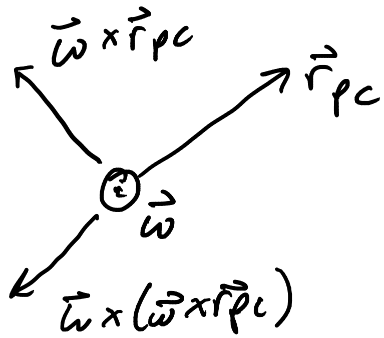
$$(\vec{r}_{PC} + \vec{r}_{C1}) \times \vec{F}_1 + (\vec{r}_{PC} + \vec{r}_{C2}) \times \vec{F}_2 + (\vec{r}_{PC} + \vec{r}_{C3}) \times \vec{F}_3$$
$$= \vec{r}_{PC} \times \vec{F}_1 + \vec{r}_{PC} \times \vec{F}_2 + \vec{r}_{PC} \times \vec{F}_3 + \vec{r}_{C1} \times \vec{F}_1 + \vec{r}_{C2} \times \vec{F}_2 + \dots$$

derivation of (2)

$$\sum_i \vec{M}_{P,i} = I_C \vec{\alpha} + \vec{r}_{PC} \times m\vec{a}_C$$
$$= I_C \vec{\alpha} + \vec{r}_{PC} \times m\left(\vec{a}_P + \vec{\alpha} \times \vec{r}_{PC} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{PC})\right)$$

$$= I_c\vec{\alpha} + \vec{r}_{pc}\times m\vec{a}_p + \underbrace{m\vec{r}_{pc}\times(\vec{\alpha}\times\vec{r}_{pc})}_{m r_{pc}^2\vec{\alpha}} + \underbrace{m\vec{r}_{pc}\times(\vec{\omega}\times(\vec{\omega}\times\vec{r}_{pc}))}_{0}$$

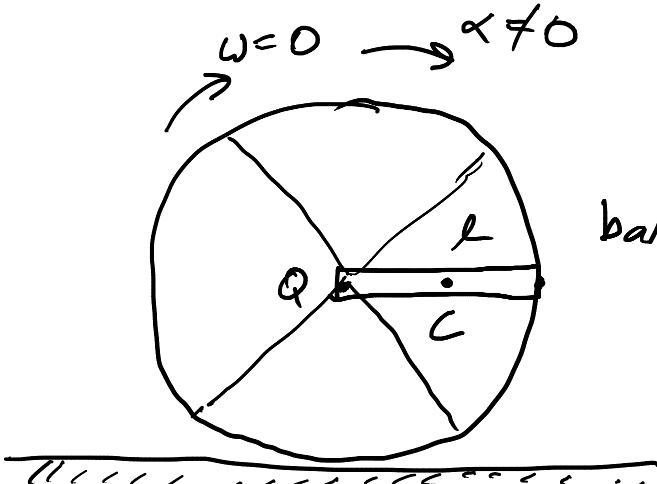




$$= \left(I_c + m r_{pc}^2\right)\vec{\alpha} + \vec{r}_{pc}\times m\vec{a}_p$$

$$= I_p\vec{\alpha} + \vec{r}_{pc}\times m\vec{a}_p$$

ex

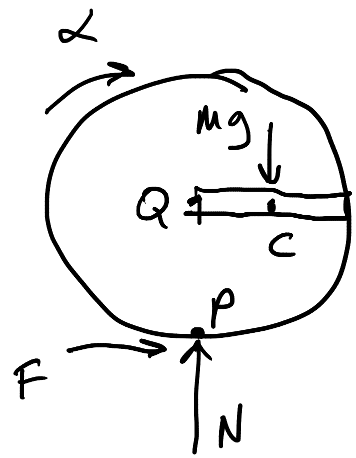


massless wheel

no slip

bar mass m length l

FBD



$$\sum \vec{F} = m\vec{a}_c$$

$$-mg\hat{j} + N\hat{j} = m\vec{a}_c$$

$\Rightarrow \vec{a}_c$ in $\hat{j}$ dir

if no friction

will always be like this.

$$\vec{\alpha} = -\alpha\hat{k}$$

(1) $\sum M_p = I_c\vec{\alpha} + \vec{r}_{pc} \times m\vec{a}_c$

(2) $\sum M_p = I_p\vec{\alpha} + \vec{r}_{pc} \times m\vec{a}_p$