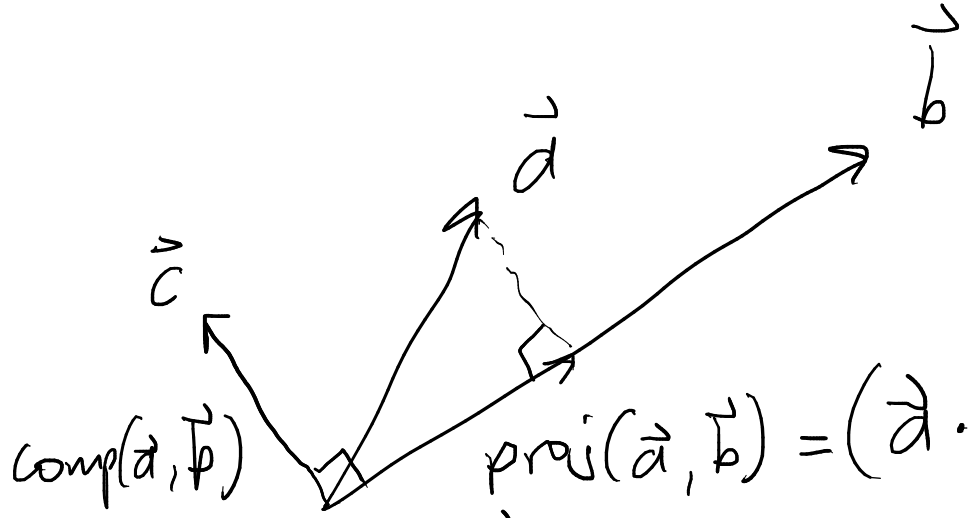
Projections



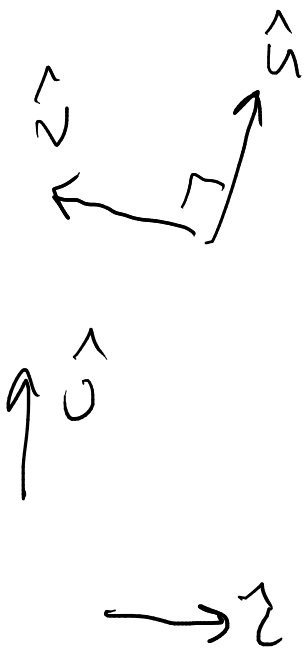
$\text{comp}(\vec{a},\vec{b})$ $\quad \text{proj}(\vec{a},\vec{b}) = (\vec{a}\cdot\hat{b})\,\hat{b}$

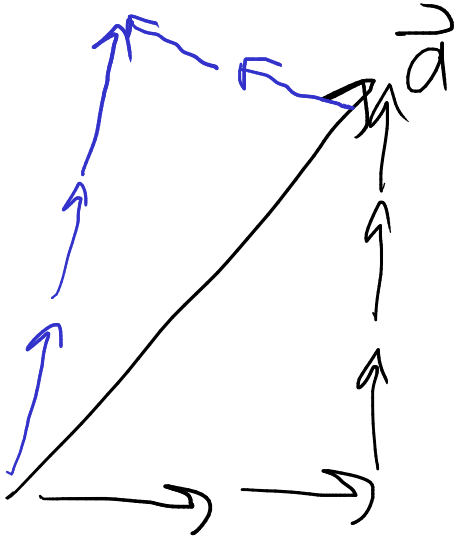
$= \vec{a} - \text{proj}(\vec{a},\vec{b})$

$$\text{proj}(\vec{a},\vec{b}) + \text{comp}(\vec{a},\vec{b}) = \vec{a}$$

$\vec{c}\cdot\vec{b} = 0$ $\quad$ $\vec{c}$ has no component in the $\vec{b}$ direction.

Bases



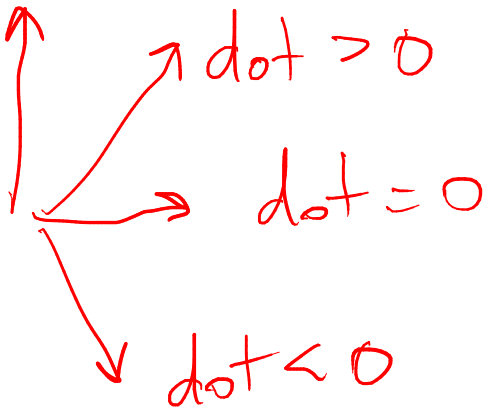


$$\vec{a} = 2\hat{\imath} + 3\hat{\jmath}$$

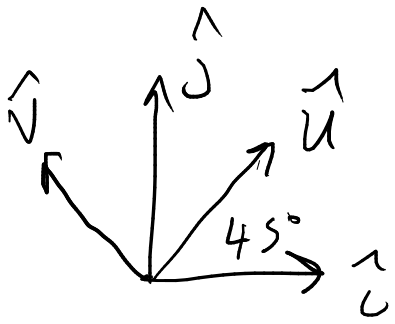
$$\vec{a} = 3\hat{u} - 2\hat{v}$$

$$= a_u \hat{u} + a_v \hat{v}$$

$a_u = \vec{a} \cdot \hat{u}$ $\quad$ $\vec{a} \cdot \hat{v}$

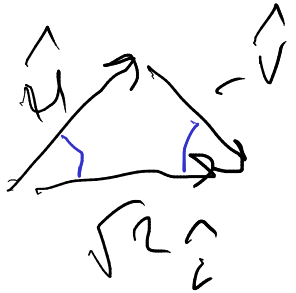


Changing Basis



$$\vec{a} = 3\hat{i} + 2\hat{j}$$

$$= ?\,\hat{u} + ?\,\hat{v}$$

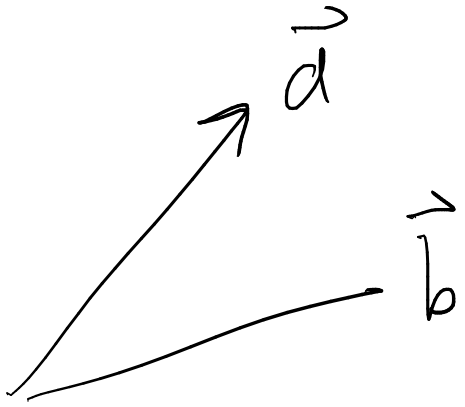


$$\hat{i} = \frac{1}{\sqrt{2}}\hat{u} - \frac{1}{\sqrt{2}}\hat{v}$$

$$\hat{j} = \frac{1}{\sqrt{2}}\hat{u} + \frac{1}{\sqrt{2}}\hat{v}$$

$$\vec{a} = 3\left(\frac{1}{\sqrt{2}}\hat{u} - \frac{1}{\sqrt{2}}\hat{v}\right) + 2\left(\frac{1}{\sqrt{2}}\hat{u} + \frac{1}{\sqrt{2}}\hat{v}\right)$$

$$= \frac{5}{\sqrt{2}}\hat{u} - \frac{1}{\sqrt{2}}\hat{v}$$



$$\vec{a} \cdot \vec{b} = a_i b_i + a_j b_j$$

$$= a_u b_u + a_v b_v$$

$$= \cancel{a_i b_i + a_v b_v}$$

all vector operations are independent of basis choice.