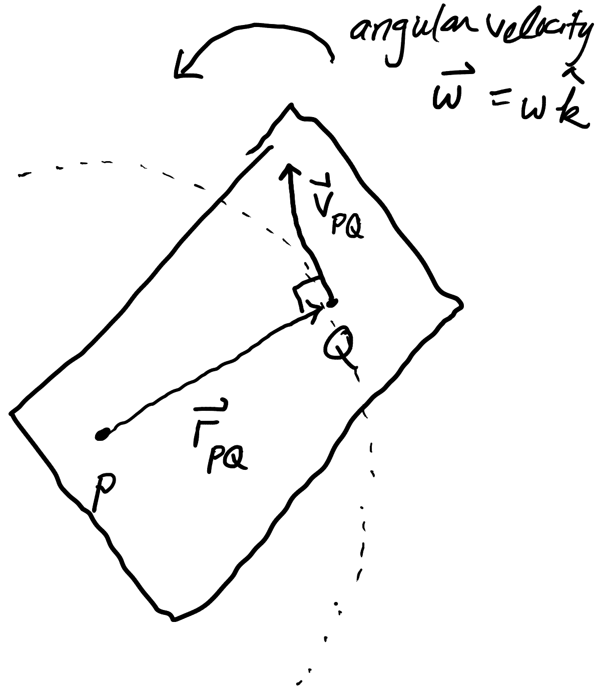
TAM 212

Rigid Bodies

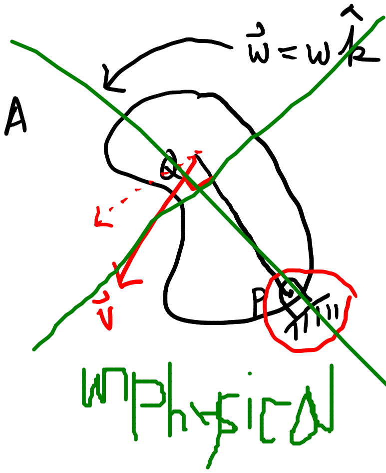
$$\vec{r}_Q = \vec{r}_P + \vec{r}_{PQ}$$

$$\vec{v}_Q = \vec{v}_P + \underbrace{\vec{\omega} \times \vec{r}_{PQ}}_{\vec{v}_{PQ}}$$

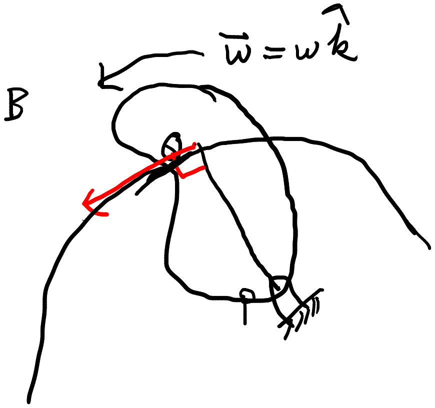


Ex)

A



B



$\vec{\omega} \times \vec{r}_{PQ}$ is $\perp$ to $\vec{r}_{PQ}$ for $\vec{\omega} = \omega\hat{k}$

$$\vec{a}_Q = \vec{a}_P + \underbrace{\vec{\alpha} \times \vec{r}_{PQ}}_{\text{direction?}} + \underbrace{\vec{\omega} \times (\vec{\omega} \times \vec{r}_{PQ})}_{\text{direction?}}$$

$\vec{\omega} = \omega \hat{k}$

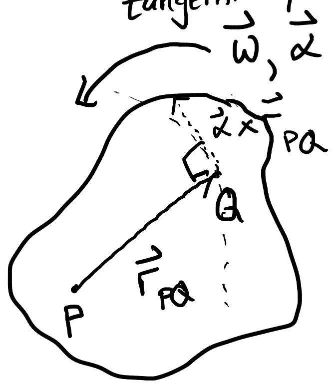
$\vec{\alpha} = \alpha \hat{k}$

$\perp$ to $\vec{r}_{PQ}$

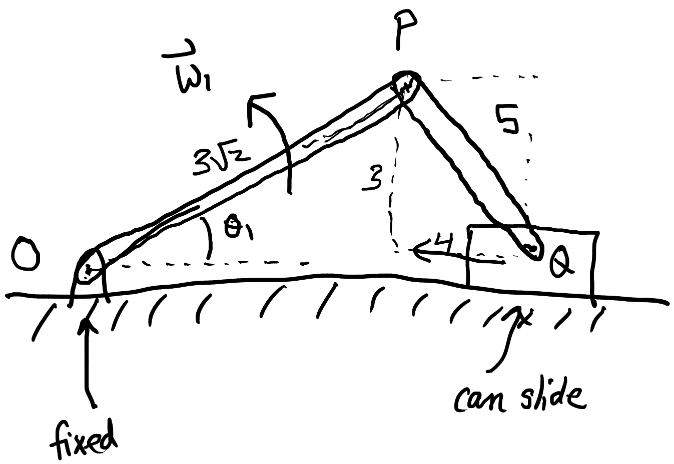
tangential part of $\vec{a}_{PQ}$

normally inwards, along $-\vec{r}_{PQ}$

changing direction of $\vec{r}_{PQ}$



Example: Coupled Rigid Bodies



<u>Given</u>

$OP = 3\sqrt{2}\ m$; $QP = 5m$

$\theta_1 = 45°$

$\vec{\omega}_1 = 4\hat{k}$ rad/sec

<u>Find</u>

$\vec{\omega}_2 = \omega_2 \hat{k}$

$\vec{V}_Q$

strategy:

1- work your way along the rigid bodies one at a time

2- use constraints to solve for variables

$$\vec{V}_P = \underline{\underline{\vec{V}_O}} + \vec{\omega}_1 \times \vec{r}_{OP}$$

$$\underline{\vec{V}_Q} = \underline{\underline{\vec{V}_P}} + \underline{\vec{\omega}_2 \times \vec{r}_{PQ}}$$

$$\boxed{\vec{V}_Q = \cancel{\vec{V}_O} + \vec{\omega}_1 \times \vec{r}_{OP} + \vec{\omega}_2 \times \vec{r}_{PQ}}$$

i, j-component
$\Rightarrow$ 2 equations

constraints: 2 variables

$\vec{V}_O = 0$

$\vec{V}_Q = V_Q \hat{i} + 0\hat{j}$

$\vec{\omega}_2 = \omega_2 \hat{k}$

$$\vec{V}_Q = \cancel{\vec{V}_O} + \vec{\omega}_1 \times \vec{r}_{OP} + \vec{\omega}_2 \times \vec{r}_{PQ}$$

$\vec{\omega}_1 = 4\hat{k}$ $\qquad$ $\vec{\omega}_2 = \omega_2 \hat{k}$ $\qquad$ $\vec{V}_Q = V_Q \hat{i}$

$\vec{r}_{OP} = 3\hat{i} + 3\hat{j}$ $\qquad$ $\vec{r}_{PQ} = 4\hat{i} - 3\hat{j}$

$$\vec{V}_Q = 4\hat{k} \times (3\hat{i} + 3\hat{j}) + \omega_2 \hat{k} \times (4\hat{i} - 3\hat{j}) = V_Q \hat{i} + 0\hat{j}$$

$12\,\hat{k} \times \hat{i} = 12\hat{j}$ $\qquad$ $12\hat{k} \times \hat{j} = -12\hat{i}$ $\qquad$ $4\omega_2 \hat{j}$ $\qquad$ $+3\omega_2 \hat{i}$

$i \; j \; k \; i \; j \; k$

$$V_Q \hat{i} = 12\hat{j} - 12\hat{i} + 4\omega_2 \hat{j} + 3\omega_2 \hat{i}$$

i-components: $V_Q = -12 + 3\omega_2$

j-components: $0 = 12 + 4\omega_2$

$V_Q = -21$ m/s

$\omega_2 = -3$ rad/sec

$$\boxed{\begin{aligned} \vec{V}_Q &= -21\hat{i} \text{ m/s} \\ \vec{\omega}_2 &= -3\hat{k} \text{ rad/s} \end{aligned}}$$