TAM 212

Energy $E = T + V$

Total Energy

Kinetic Energy

potential Energy

Kinetic Energy

$$T = \frac{1}{2} m v_c^2 + \frac{1}{2} I_c \omega^2$$

about center of mass C

(König's Theorem)

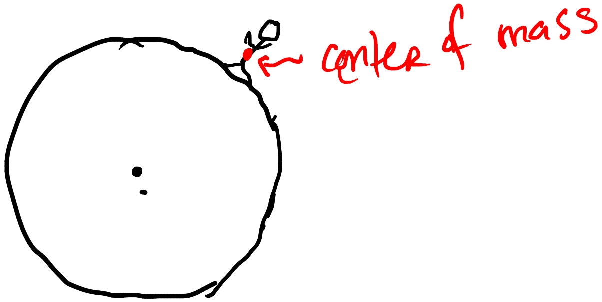
$$T = \frac{1}{2} I_{I} \omega^2$$

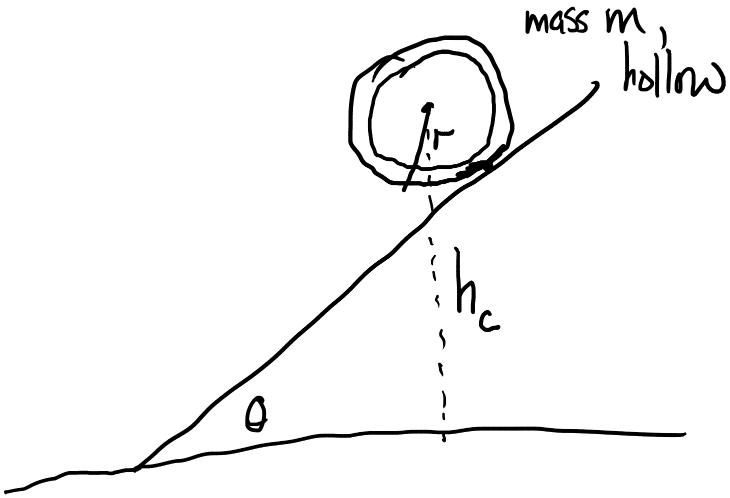
about instantaneous center I

$$T = \frac{1}{2} I_O \omega^2$$ about a fixed pt O

Potential Energy gravity applies at the center of mass

$$V = mgh_c$$





Both cyl. are released from rest at the same height.
In both cases, roll w/o slipping.
Which one reaches the bottom first? What is v_c for both cases at the bottom?

Rolling w/o slip $\Rightarrow$ friction does no work on the cylinders
$\Rightarrow$ no dissipation
$\Rightarrow$ Energy is constant for the process

$$E_i = E_f$$

initial $E_i = \cancel{T_i}^{0} + \cancel{V_i}^{mgh_c}$

final $E_f = T_f + \cancel{V_f}^{0} = T_f = \frac{1}{2} m v_c^2 + \frac{1}{2} I_c \omega^2$

$$= \frac{1}{2} m (r\omega)^2 + \frac{1}{2} I_c \omega^2$$

hollow: $I_c = mr^2$

$$E_f = T_f = m\omega^2 r^2$$
$$= m v_c^2$$

solid $I_c = \frac{1}{2} m r^2$

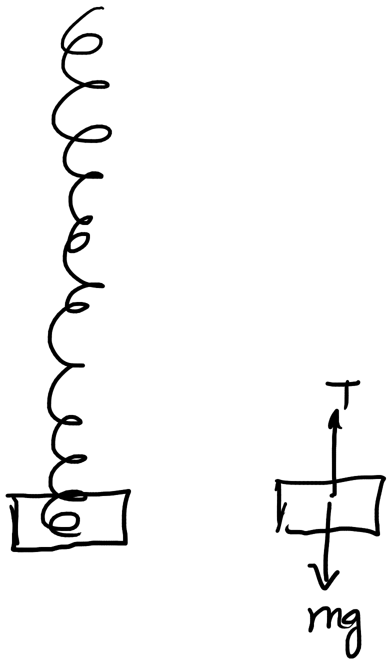
$$E_f = \frac{3}{4} m \omega^2 r^2$$
$$= \frac{3}{4} m v_c^2$$

applying conservation of energy:

$$E_i = E_f$$
$$mgh_c = m v_c^2$$
$$v_c = \sqrt{gh_c}$$

$$E_i = E_f$$
$$mgh_c = \frac{3}{4} m v_c^2$$
$$v_c = \sqrt{\frac{4}{3} g h_c}$$

$\Rightarrow$ solid cylinder is $\sqrt{\frac{4}{3}} \approx 1.15$ times faster than hollow cyl.;
it reaches the bottom first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