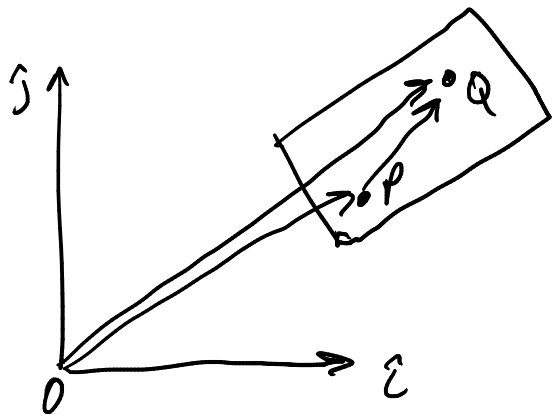
Rigid Bodies and rotation

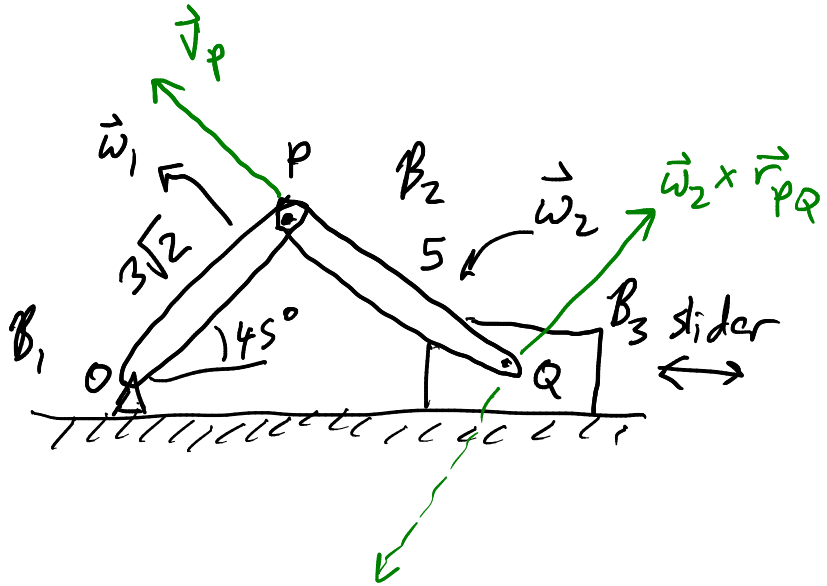


$$\vec{r}_Q = \vec{r}_P + \vec{r}_{PQ}$$

$$\vec{v}_Q = \vec{v}_P + \vec{\omega} \times \vec{r}_{PQ}$$

$\dot{\vec{r}} = \vec{\omega} \times \vec{r}$ if $\vec{r}$ is only rotating (no length change).

ex



$\vec{\omega}_1 = 2\hat{k}$ rad/s $\quad \vec{\omega}_2$?

$$\vec{r}_{OP} = 3\hat{\imath} + 3\hat{\jmath}$$

$$\vec{r}_{PQ} = 4\hat{\imath} - 3\hat{\jmath}$$

$$\vec{v}_P = \cancel{\vec{v}_O}^{\,0} + \vec{\omega}_1 \times \vec{r}_{OP}$$

$$2\hat{k} \times (3\hat{\imath} + 3\hat{\jmath})$$

$$= -6\hat{\imath} + 6\hat{\jmath}$$

$$\vec{v}_Q = \vec{v}_P + \vec{\omega}_2 \times \vec{r}_{PQ}$$

$$= -6\hat{i} + 6\hat{j} + \omega_2 \hat{k} \times (4\hat{i} - 3\hat{j})$$

$$= -6\hat{i} + 6\hat{j} + 3\omega_2 \hat{i} + 4\omega_2 \hat{j}$$

$$= (-6 + 3\omega_2)\hat{i} + (6 + 4\omega_2)\hat{j}$$

OR $\vec{v}_Q \cdot \hat{j} = 0$

(Q does not move in $\hat{j}$ dir.)

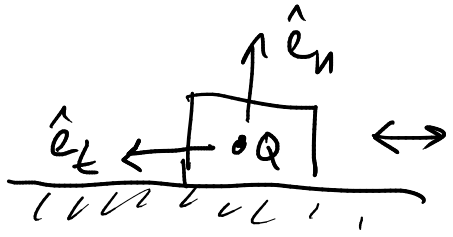
$$[(-6 + 3\omega_2)\hat{i} + (6 + 4\omega_2)\hat{j}] \cdot \hat{j} = 0$$

$$6 + 4\omega_2 = 0$$

we know $\hat{j}$ component of $\vec{v}_Q$ is zero $\Rightarrow$ $6 + 4\omega_2 = 0$

$\omega_2 = -1.5$ rad/s.

Constraints on allowed directions



Q moves horiz. $\Rightarrow$ Q does not move vertically

$\Rightarrow$ Q does not move in $\hat{j}$ direction.

Q does not move in $\hat{e}_n$ direction

$$\vec{v}_Q \cdot \hat{e}_n = 0$$

$$\vec{r}_P = \vec{r}_Q + \vec{r}_{PQ}$$

$$\vec{v}_P = \vec{v}_Q + \vec{\omega} \times \vec{r}_{PQ}$$

$$\vec{a}_P = \vec{a}_Q + \frac{d}{dt}(\vec{\omega} \times \vec{r}_{PQ})$$

$$\dot{\vec{\omega}} \times \vec{r}_{PQ} + \vec{\omega} \times \dot{\vec{r}}_{PQ}$$

$$\vec{a}_P = \vec{a}_Q + \vec{\alpha} \times \vec{r}_{PQ} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{PQ})$$

$$\dot{\vec{r}} = \vec{\omega} \times \vec{r} \quad \text{for rotation}$$

$$\dot{\vec{\omega}} = \vec{\alpha}$$

angular acc.

A: true (equal)

B: false (diff).

$\vec{\omega} \times \vec{\omega}$ → always zero.

$$(\vec{\omega} \times \vec{\omega}) \times \vec{r}_{PQ}$$