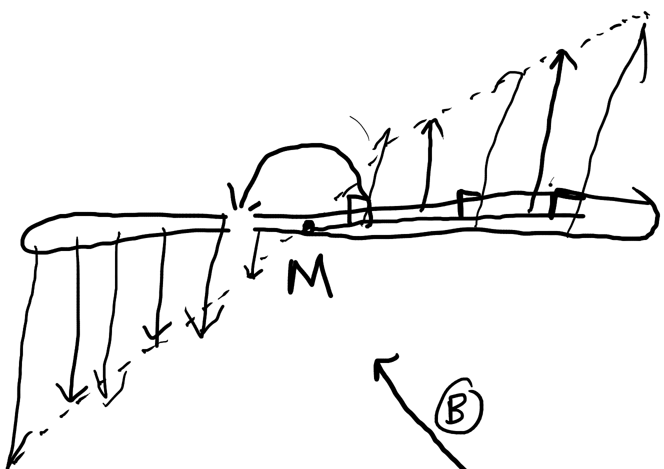
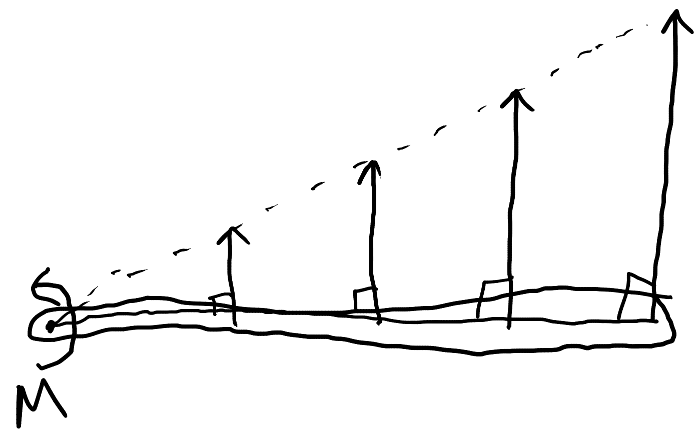
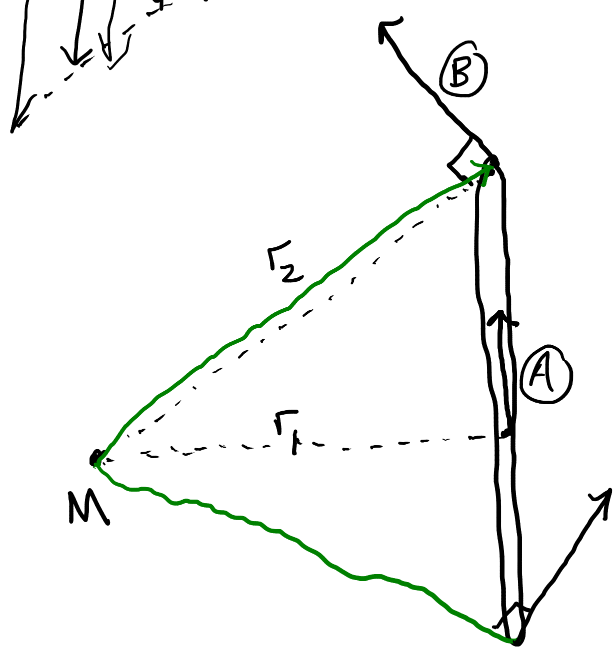
TAM 212

Instantaneous Center of ZERO Velocity "M"

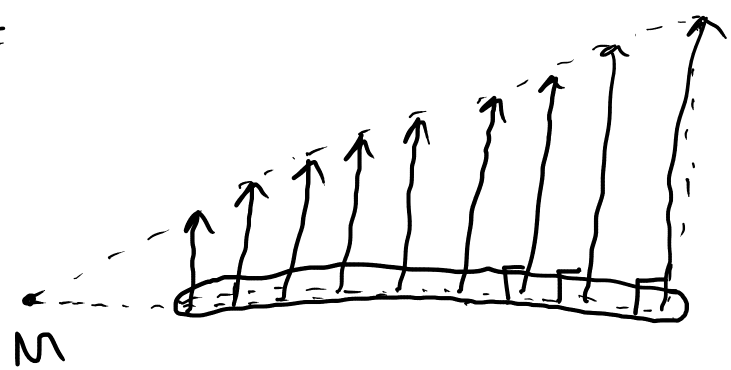
* point that a rigid body is rotating about at a given instant in time



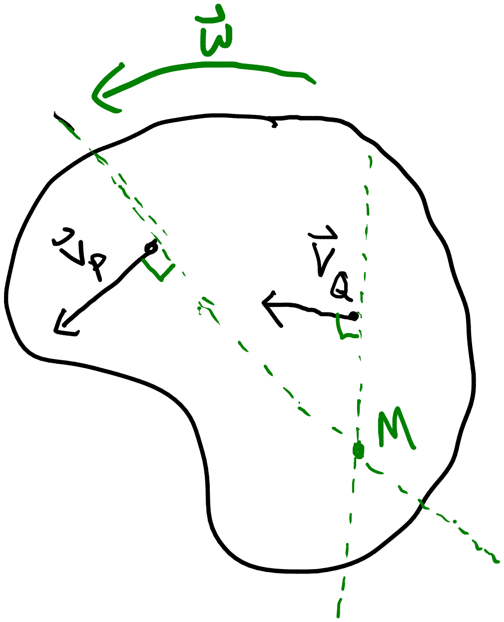


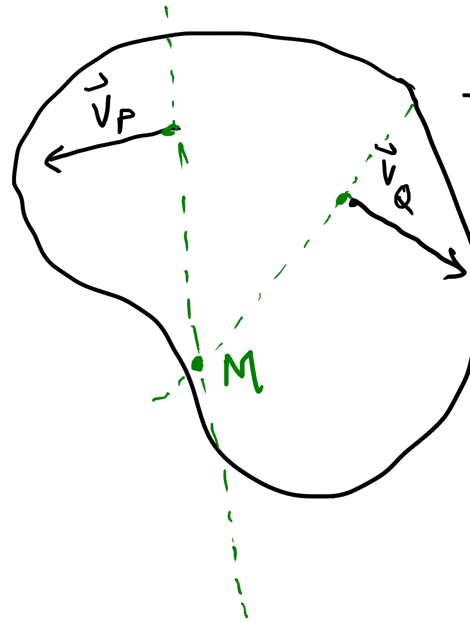


$$\underline{\underline{v}} = \omega \underline{\underline{r}}$$

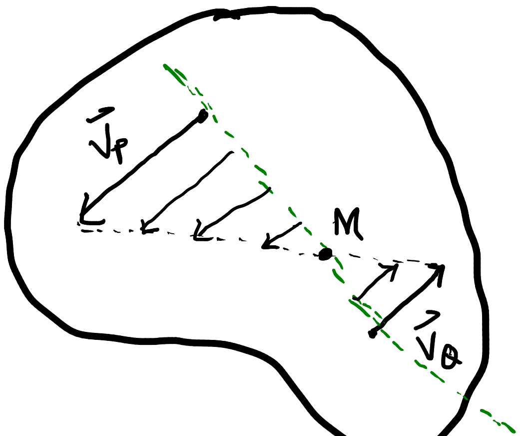


if a rigid body has $\vec{\omega} \neq 0$, there exists an instantaneous center

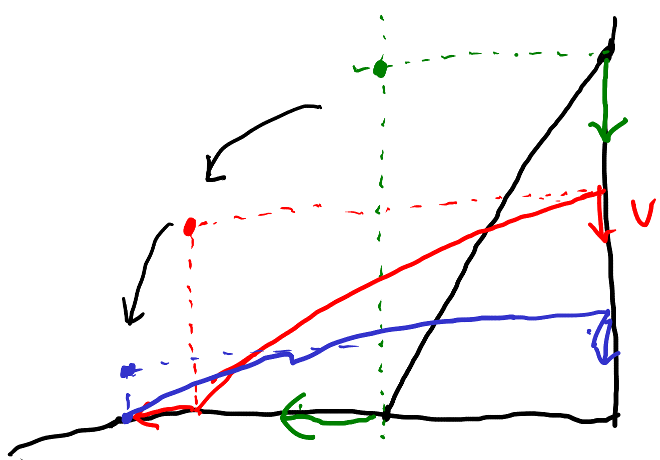




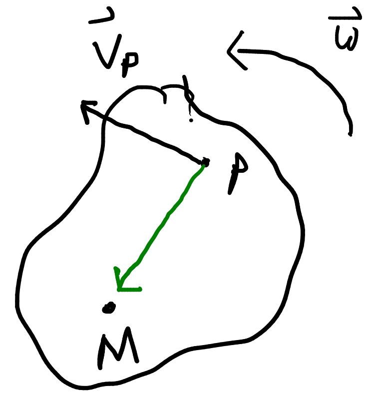
TRICK QUESTION!



Note: The instantaneous center of zero velocity may not stay fixed over the course of movement



Equation For Locating The Instantaneous Center



$\vec{r}_{PM}$?

Use the fact that $\vec{V}_M = 0$

$$0 = \vec{V}_M = \vec{V}_P + \vec{\omega} \times \vec{r}_{PM}$$

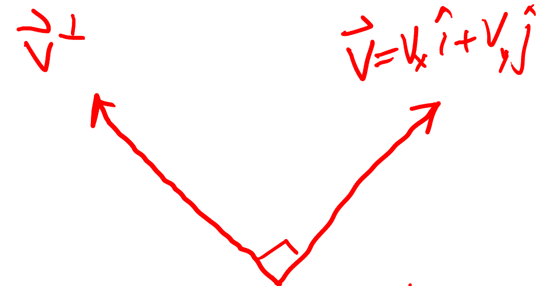
cross multiply by $\vec{\omega}$:

$$0 = \vec{\omega} \times \left(\vec{V}_P + \vec{\omega} \times \vec{r}_{PM} \right)$$

$$= \vec{\omega} \times \vec{V}_P + \underline{\vec{\omega} \times} \left(\vec{\omega} \times \vec{r}_{PM} \right)$$

$$0 = \vec{\omega} \times \vec{V}_P - \omega^2 \vec{r}_{PM}$$

$$\boxed{\vec{r}_{PM} = \frac{1}{\omega^2} \vec{\omega} \times \vec{V}_P}$$



$$\hat{k} \times \vec{V} = \vec{V}^{\perp}$$
$$= -V_y \hat{i} + V_x \hat{j}$$

$$\vec{\omega} = \omega \hat{\omega}$$

$$\vec{r}_{PM} = \frac{1}{\omega^2} \omega \hat{\omega} \times \vec{V}_P$$
$$= \frac{1}{\omega} \boxed{\hat{\omega} \times \vec{V}_P} = \frac{1}{\omega} \boxed{\vec{V}_P^{\perp}}$$

Example: Rolling