TEST – 1

TEST FOR A POPULATION PROPORTION

Aim

To test the population proportion, P be regarded as P_0, based on a random sample. That is, to investigate the significance of the difference between the observed sample proportion p and the assumed population proportion P_0.

Source

If X is the number of occurrences of an event in n independent trials with constant probability P of occurrences of that event for each trial, then $E(X) = nP$ and $V(X) = nPQ$, where $Q = 1 - P$, is the probability of non-occurrence of that event. It has proved that for large n, the binomial distribution tends to normal distribution. Hence, the normal test can be applied. In a random sample of size n, let X be the number of persons possessing the given attribute. Then the observed proportion in the sample be

$$\frac{X}{n} = p, \text{ (say), then } E(p) = P \text{ and } S.E(p) = \sqrt{Var(p)} = \sqrt{\frac{P(1-P)}{n}}.$$

Assumption

The sample size must be sufficiently large (*i.e.*, $n > 30$) to justify the normal approximation to binomial.

Null Hypothesis

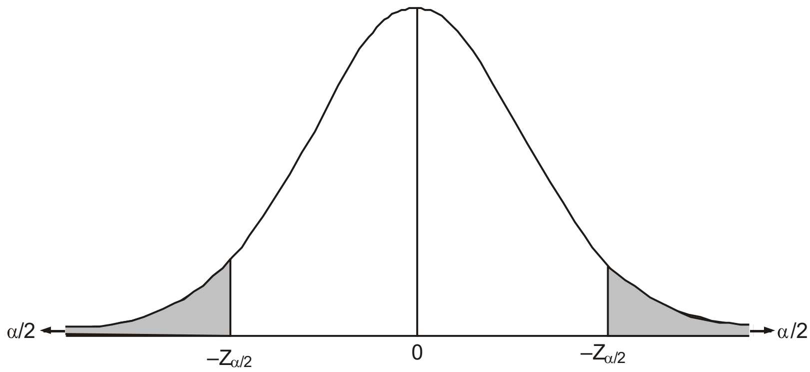
H_0: The population proportion (P) is regarded as P_0. That is, there is no significant difference between the observed sample proportion p and the assumed population proportion P_0. *i.e.*, $H_0 : P = P_0$.

Alternative Hypotheses

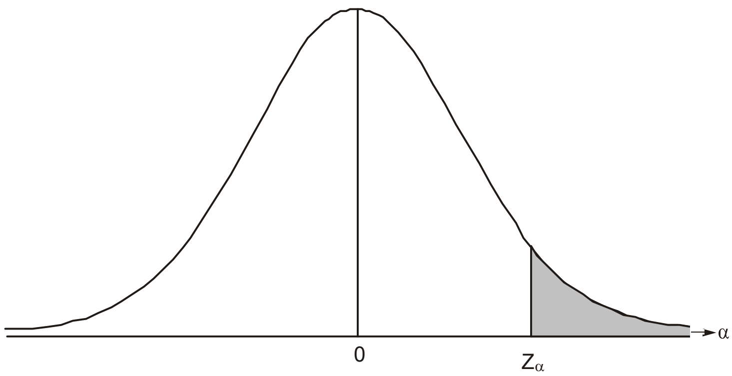
$H_1(1) : P \neq P_0$
$H_1(2) : P > P_0$
$H_1(3) : P < P_0$

Level of Significance (α) and Critical Region

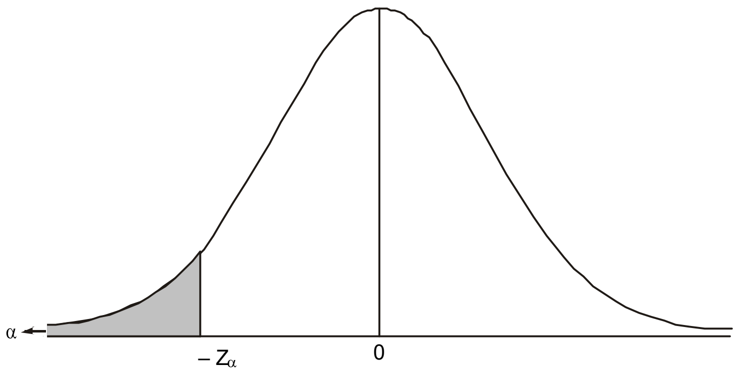
(1) $|Z| > |Z_\alpha|$ such that $P\{|Z| > |Z_\alpha|\} = \alpha$



(2) $Z > Z_\alpha$ such that $P\{Z > Z_\alpha\} = \alpha$



(3) $Z < -Z_\alpha$ such that $P\{Z < -Z_\alpha\} = \alpha$



Critical Values (Z_α)

Critical value (Z_α)	*Level of Significance* (α)		
	1%	5%	10%
1. Two-sided test	$\lvert Z_\alpha \rvert = 2.58$	$\lvert Z_\alpha \rvert = 1.96$	$\lvert Z_\alpha \rvert = 1.645$
2. Right-sided test	$Z_\alpha = 2.33$	$Z_\alpha = 1.645$	$Z_\alpha = 1.28$
3. Left-sided test	$Z_\alpha = -2.33$	$Z_\alpha = -1.645$	$Z_\alpha = -1.28$

Test Statistic

$$Z = \frac{p - P}{\sqrt{\dfrac{P(1-P)}{n}}} \quad \text{(Under } H_0\text{: } P = P_0\text{)}$$

The statistic Z follows Standard Normal Distribution.

Conclusions

1. If $|Z| \leq Z_\alpha$, we conclude that the data do not provide us any evidence against the null hypothesis H_0 . Hence, it may be accepted at α% level of significance. Otherwise reject H_0 or accept H_1 (1).
2. If $Z \leq Z_\alpha$, we conclude that the data do not provide us any evidence against the null hypothesis H_0 and hence it may be accepted at α% level of significance. Otherwise reject H_0 or accept H_1 (2).
3. If $|Z| \leq |Z_\alpha|$, we conclude that the data do not provide us any evidence against the null hypothesis H_0 and hence it may be accepted at α% level of significance. Otherwise reject H_0 or accept H_1 (3).

***Example* 1**

Hindustan Lever Ltd. Company expects that more than 30% of the households in Delhi city will consume its product if they manufacture a new face cream. A random sample of 500 households from the city is surveyed, 163 are favorable in manufacturing the product. Examine whether the expectation of the company would be met at 2% level.

Solution

Aim: To test the HLL Company's manufacture of a new product of face cream will be consumed by 30% of the households in New Delhi or more.

H_0: The HLL Company's manufacture of a new product of face cream will be consumed by 30% of the households in New Delhi. *i.e.*, H_0: $P = 0.3$.

H_1: The HLL Company's manufacture of a new product of face cream will be consumed by more than 30% of the households in New Delhi. *i.e.*, H_1: $p > 0.3$

Level of Significance: $\alpha = 0.05$ and *Critical Value*: $Z_\alpha = 1.645$

Based on the above data, we observed that, $n = 500$, $p = (163/500) = 0.326$

Test Statistic: $Z = \dfrac{p - P}{\sqrt{\dfrac{P(1-P)}{n}}}$ (Under H_0: $P = 0.3$) $= \dfrac{0.326 - 0.3}{\sqrt{\dfrac{(0.3)(0.7)}{500}}} = 1.27$

Conclusion: Since $Z < Z_\alpha$, we conclude that the data do not provide us any evidence against the null hypothesis H_0. Hence, accept H_0 at 5% level of significance. That is, the HLL Company's manufacture of a new product of face cream will be consumed by 30% of the households in New Delhi.

Example 2

A plastic surgery department wants to know the necessity of mesh repair of hernia. They think that 15% of the hernia patients only need mesh. In a sample of 250 hernia patients from hospitals, 42 only needed mesh. Test at 2% level of significance that the expectation of the department for mesh repair of hernia patients is true.

Solution

Aim: To test the necessity of hernia repair with mesh is 15% or not.

H_0: The necessity of mesh repair of hernia is 15%. *i.e.*, H_0: $P = 0.15$

H_1: The necessity of mesh repair of hernia is not 15%. *i.e.*, H_1: $P \neq 0.15$

Level of Significance: $\alpha = 0.02$ and *Critical Value*: $Z_\alpha = 2.33$

Based on the above data, we observed that, $n = 250$, $p = (42/250) = 0.326$

Test Statistic: $Z = \dfrac{p - P}{\sqrt{\dfrac{P(1-P)}{n}}}$ (Under H_0: $P = 0.15$) $= \dfrac{0.168 - 0.15}{\sqrt{\dfrac{(0.15)(0.85)}{250}}} = 0.80$

Conclusion: Since $|Z| < Z_\alpha$, we conclude that the data do not provide us any evidence against the null hypothesis H_0. Hence, accept H_0 at 2% level of significance. That is, the necessity of mesh repair of hernia as expected by the plastic surgery department 15% is true.

EXERCISES

1. A random sample of 400 apples was taken from large consignment and 35 were found to be bad. Examine whether the bad items in the lot will be 7% at 1% level.
2. 150 people were attacked by a disease of which 5 died. Will you reject the hypothesis that the death rate, if attacked by this disease is 3% against the hypothesis that it is more, at 5% level?