

# Normalization

# Motivation

- We have designed ER diagram, and translated it into a relational db schema  $R = \text{set of } R_1, R_2, \dots$
- Now what?
- We can do the following
  - specify all relevant constraints over  $R$
  - implement  $R$  in SQL
  - start using it, making sure the constraints always remain valid
- However,  $R$  may not be well-designed, thus causing us a lot of problems

# Example of Bad Design

Persons with several phones:

Name	SSN	Phone Number
Fred	123-321-99	(201) 555-1234
Fred	123-321-99	(206) 572-4312
Joe	909-438-44	(908) 464-0028
Joe	909-438-44	(212) 555-4000

## Problems (also called "Anomalies"):

- Redundancy = repetition of data
- update anomalies = update one item and forget others
- 
- = inconsistencies
- deletion anomalies = delete many items,  
                          delete one item, loose other information
- insertion anomalies = can't insert one item without inserting others<sup>3</sup>

# Better Designs Exist

**Break the relation into two:**

SSN	Name
123-321-99	Fred
909-438-44	Joe

SSN	Phone Number
123-321-99	(201) 555-1234
123-321-99	(206) 572-4312
909-438-44	(908) 464-0028
909-438-44	(212) 555-4000

# How do We Obtain a Good Design?

- Start with the original db schema  $R$
- Transform it until we get a good design  $R^*$
- Desirable properties for  $R^*$ 
  - must preserve the information of  $R$
  - must have minimal amount of redundancy
  - must be dependency-preserving
    - if  $R$  is associated with a set of constraints  $C$ , then it should be easy to also check  $C$  over  $R^*$
  - (must also give good query performance)

# OK, But ...

- How do we recognize a good design  $R^*$ ?
- How do we transform  $R$  into  $R^*$ ?
- Answers: use normal forms

# Normal Forms

- DB gurus have developed many normal forms
- Most important ones
  - Boyce-Codd, 3rd, and 4th normal forms
- If  $R^*$  is in one of these forms, then  $R^*$  is guaranteed to achieve certain good properties
  - e.g., if  $R^*$  is in Boyce-Codd NF, it is guaranteed to not have certain types of redundancy
- DB gurus have also developed algorithms to transform  $R$  into  $R^*$  that is in some of these normal forms

# Normal Forms (cont.)

- DB gurus have also discussed trade-offs among normal forms
- Thus, all we have to do is
  - learn these forms
  - transform  $R$  into  $R^*$  in one of these forms
  - carefully evaluate the trade-offs
- Many of these normal forms are defined based on various constraints
  - functional dependencies and keys

# Functional Dependencies and Keys

# Functional Dependencies

- A form of constraint (hence, part of the schema)
- Finding them is part of the database design
- Used heavily in schema refinement

Definition:

If two tuples agree on the attributes

$A_1, A_2 \dots A_n$

then they must also agree on the attributes

$B_1, B_2, \dots B_m$

Formally:  $A_1, A_2 \dots A_n \longrightarrow B_1, B_2, \dots B_m$

# Examples

EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E1847	John	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234	Lawyer

- EmpID → Name, Phone, Position
- Position → Phone
- but Phone ~~→~~ Position

# In General

- To check  $A \rightarrow B$ , erase all other columns

...	A	...	B	
	X1		Y1	
	X2		Y2	
	...		...	

- check if the remaining relation is many-one  
(called *functional* in mathematics)

# Example

EmpID	Name	Phone	Position
E0045	Smith	1234 ←	Clerk
E1847	John	9876 ←	Salesrep
E1111	Smith	9876 ←	Salesrep
E9999	Mary	1234 ←	lawyer

More examples:

Product: name → price, manufacturer

Person: ssn → name, age

Company: name → stock price, president

# More about FDs

- $A \rightarrow B$ , we say “A functionally determines B”
- When creating a DB schema, we should list all FDs we believe are valid
- These FDs should be valid on ALL DB database instances conforming to our schema
  - can't just be valid on one database instance
  - and not valid on another database instance

# Relation Keys

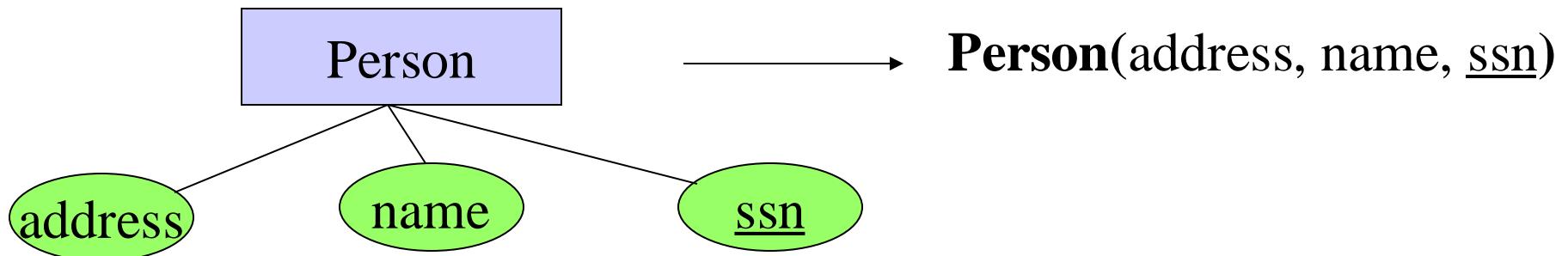
- Key of a relation R is a set of attributes that
  - functionally determines all attributes of R
    - this creates a FD  $A \rightarrow B$
  - none of its subsets determines all attributes of R
- Superkey
  - a set of attributes that contains a key
  - so a key is also a superkey

# Finding the Keys of a Relation

Given a relation constructed from an E/R diagram, what is its key?

Rules:

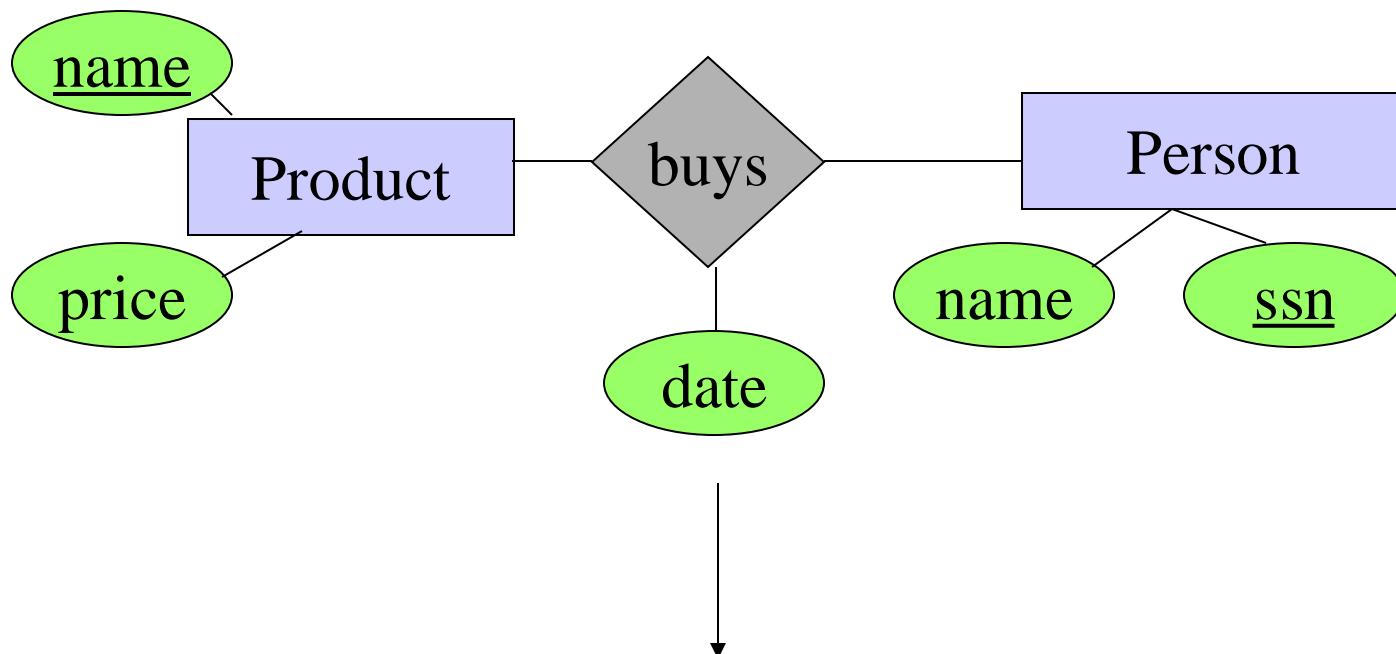
1. If the relation comes from an entity set,  
the key of the relation is the set of attributes which is the  
key of the entity set.



# Finding the Keys

Rules:

2. If the relation comes from a many-many relationship, the key of the relation is the set of all attribute keys in the relations corresponding to the entity sets



**buys(name, ssن, date)**

# Finding the Keys

More rules:

- Many-one, one-many, one-one relationships
- Multi-way relationships
- Weak entity sets

Note that if you say the set of attributes A is a key, you are basically saying certain FDs are true.

# Reasoning with FDs

- 1) closure of FD sets
- 2) closure of attribute sets

# Closure of FD sets

- Given a relation schema  $R$  & a set  $S$  of FDs
  - is the FD  $f$  logically implied by  $S$ ?
- Example
  - $R = \{A, B, C, G, H, I\}$
  - $S = A \rightarrow B, A \rightarrow C, CG \rightarrow H, CG \rightarrow I, B \rightarrow H$
  - would  $A \rightarrow H$  be logically implied?
  - yes (you can prove this, using the definition of FD)
- Closure of  $S$ :  $S^+ = \text{all FDs logically implied by } S$
- How to compute  $S^+$ ?
  - we can use Armstrong's axioms

# Armstrong's Axioms

- Reflexivity rule
  - $A_1A_2\dots A_n \rightarrow$  a subset of  $A_1A_2\dots A_n$
- Augmentation rule
  - $A_1A_2\dots A_n \rightarrow B_1B_2\dots B_m$ , then  
 $A_1A_2\dots A_n C_1C_2\dots C_k \rightarrow B_1B_2\dots B_m C_1C_2\dots C_k$
- Transitivity rule
  - $A_1A_2\dots A_n \rightarrow B_1B_2\dots B_m$  and  
 $B_1B_2\dots B_m \rightarrow C_1C_2\dots C_k$ , then  
 $A_1A_2\dots A_n \rightarrow C_1C_2\dots C_k$

# Inferring $S^+$ using Armstrong's Axioms

- $S^+ = S$
- Loop
  - for each  $f$  in  $S$ , apply reflexivity and augment. rules
  - add the new FDs to  $S^+$
  - for each pair of FDs in  $S$ , apply the transitivity rule
  - add the new FD to  $S^+$
- Until  $S^+$  does not change any further

# Additional Rules

- Union rule
  - $X \rightarrow Y$  and  $X \rightarrow Z$ , then  $X \rightarrow YZ$
  - ( $X, Y, Z$  are sets of attributes)
- Decomposition rule
  - $X \rightarrow YZ$ , then  $X \rightarrow Y$  and  $X \rightarrow Z$
- Pseudo-transitivity rule
  - $X \rightarrow Y$  and  $YZ \rightarrow U$ , then  $XZ \rightarrow U$
- These rules can be inferred from Armstrong's axioms

# Closure of a Set of Attributes

Given a set of attributes  $\{A_1, \dots, A_n\}$  and a set of FDs S.

Problem: find all attributes  $B$  such that:

any relation which satisfies S also satisfies:

$$A_1, \dots, A_n \rightarrow B$$

The **closure** of  $\{A_1, \dots, A_n\}$ , denoted  $\{A_1, \dots, A_n\}^+$ ,  
is the set of all such attributes  $B$

→ *basically all attributes that are functionally determined  
by  $A_1, \dots, A_n$*

We will discuss the motivations for attribute closures soon

# Algorithm to Compute Closure

Start with  $X = \{A_1, \dots, A_n\}$ .

**Repeat until  $X$  doesn't change do:**

if  $B_1, B_2, \dots, B_n \longrightarrow C$  is in  $S$ , **and**

$B_1, B_2, \dots, B_n$  are all in  $X$ , **and**

$C$  is not in  $X$

**then**

add  $C$  to  $X$ .

# Example

$$\begin{array}{l} A \ B \longrightarrow C \\ A \ D \longrightarrow E \\ B \longrightarrow D \\ A \ F \longrightarrow B \end{array}$$

Closure of {A,B}:  $X = \{A, B, C, D, E\}$

Closure of {A, F}:  $X = \{A, F, B, D, C, E\}$

# Usage for Attribute Closure

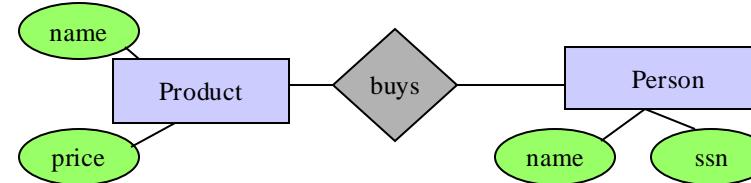
- Test if  $X$  is a superkey
  - compute  $X^+$ , and check if  $X^+$  contains all attrs of  $R$
- Check if  $X \rightarrow Y$  holds
  - by checking if  $Y$  is contained in  $X^+$
- Another way to compute closure  $S^+$  of FDs
  - for each subset of attributes  $X$  in relation  $R$ , compute  $X^+$
  - for each subset of attributes  $Y$  in  $X^+$ , output the FD  $X \rightarrow Y$

# Desirable Properties of Schema Refinement

- 1) minimize redundancy
- 2) avoid info loss
- 3) preserve dependency
- 4) ensure good query performance

# Relational Schema Design (or Logical Design)

Conceptual Model:



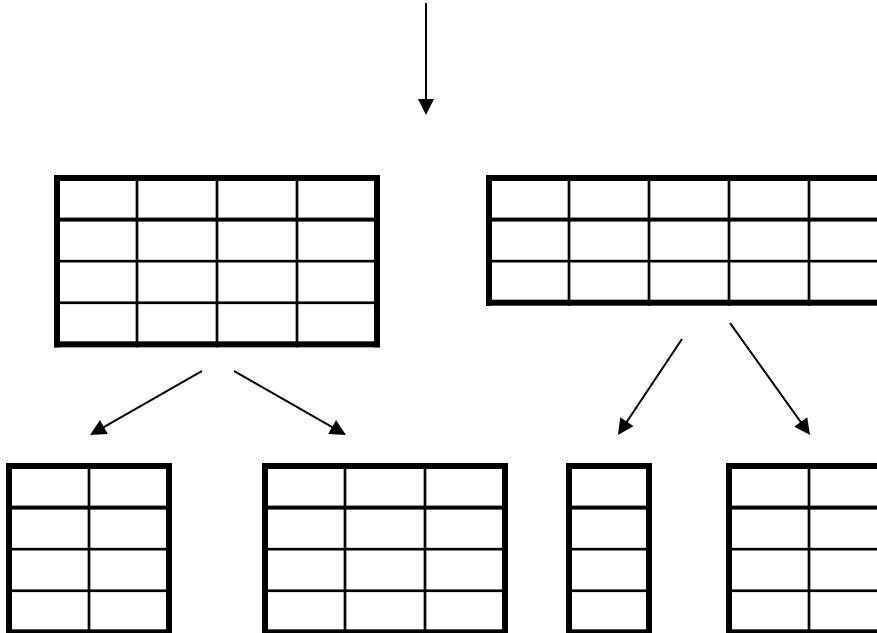
Relational Model:

- create tables
- specify FD's
- find keys

Normalization

- use FDs to  
**decompose** tables

to achieve better design



# Recall: Relation Decomposition

The original relation schema

Name	SSN	Phone Number
Fred	123-321-99	(201) 555-1234
Fred	123-321-99	(206) 572-4312
Joe	909-438-44	(908) 464-0028
Joe	909-438-44	(212) 555-4000

# Relation Decomposition (cont.)

**Break the relation into two:**

SSN	Name
123-321-99	Fred
909-438-44	Joe

SSN	Phone Number
123-321-99	(201) 555-1234
123-321-99	(206) 572-4312
909-438-44	(908) 464-0028
909-438-44	(212) 555-4000

- Desirable Property #1: Minimize redundancy

# Decompositions in General

Let  $R$  be a relation with attributes  $A_1, A_2 \dots A_n$

Create two relations  $R1$  and  $R2$  with attributes

$B_1, B_2 \dots B_m$

$C_1, C_2 \dots C_l$

Such that:

$$B_1, B_2 \dots B_m \cup C_1, C_2 \dots C_l = A_1, A_2 \dots A_n$$

And

- $R1$  is the projection of  $R$  on  $B_1, B_2 \dots B_m$
- $R2$  is the projection of  $R$  on  $C_1, C_2 \dots C_l$

# Certain Decomposition May Cause Problems

Name	Price	Category
Gizmo	19.99	Gadget
OneClick	24.99	Camera
DoubleClick	29.99	Camera

Decomposition : **Name, Category** and **Price, Category**

Name	Category	Price	Category
Gizmo	Gadget	19.99	Gadget
OneClick	Camera	24.99	Camera
DoubleClick	Camera	29.99	Camera

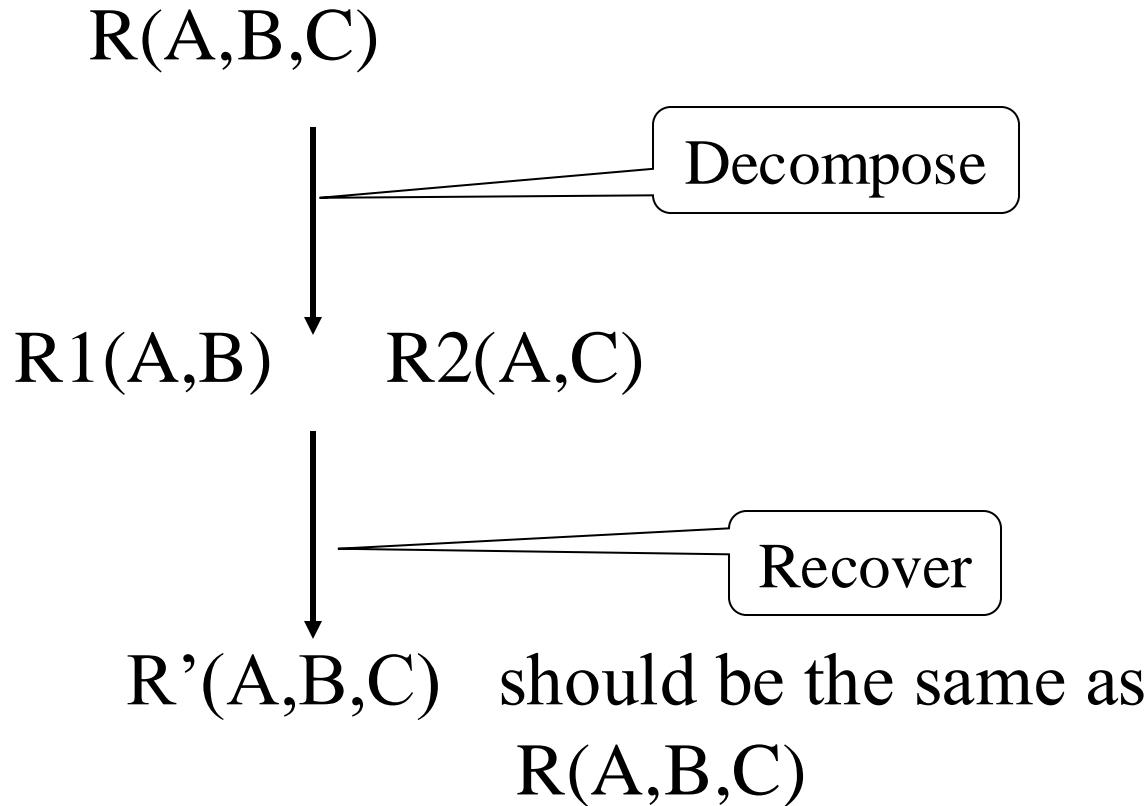
When we put it back:

Cannot recover information

Name	Price	Category
Gizmo	19.99	Gadget
OneClick	24.99	Camera
OneClick	29.99	Camera
DoubleClick	24.99	Camera
DoubleClick	29.99	Camera

# Lossless Decompositions

A decomposition is *lossless* if we can recover:



$R'$  is in general larger than  $R$ . Must ensure  $R' = R$

- Desirable Property #2: Lossless decomposition

# Put Another Way: "Lossless" Joins

- The main idea: if you decompose a relation schema, then join the parts of an instance via a natural join, you might get more rows than you started with, i.e., spurious tuples
  - This is bad!
  - Called a "lossy join".
- Goal: decompositions which produce only "lossless" joins
  - "non-additive" join is more descriptive

# Dependency Preserving

- Given a relation  $R$  and a set of FDs  $S$
- Suppose we decompose  $R$  into  $R_1$  and  $R_2$
- Suppose
  - $R_1$  has a set of FDs  $S_1$
  - $R_2$  has a set of FDs  $S_2$
  - $S_1$  and  $S_2$  are computed from  $S$
- We say the decomposition is dependency preserving if by enforcing  $S_1$  over  $R_1$  and  $S_2$  over  $R_2$ , we can enforce  $S$  over  $R$

# An Example

Unit	Company	Product

FD's:  $\text{Unit} \rightarrow \text{Company}$ ;  $\text{Company}, \text{Product} \rightarrow \text{Unit}$

So, there is a BCNF violation, and we decompose.

Unit	Company

$\text{Unit} \rightarrow \text{Company}$

Unit	Product

No FDs

# So What's the Problem?

Unit	Company	Unit	Product
Galaga99	UI	Galaga99	databases
Bingo	UI	Bingo	databases

No problem so far. All *local* FD's are satisfied.

Let's put all the data back into a single table again:

Unit	Company	Product
Galaga99	UW	databases
Bingo	UW	databases

**Violates the dependency: company, product  $\rightarrow$  unit!**

# Preserving FDs

- What if, when a relation is decomposed, the X of an  $X \rightarrow Y$  ends up only in one of the new relations and the Y ends up only in another?
- Such a decomposition is not “dependency-preserving.”
- Desirable Property #3: always have FD-preserving decompositions
- We will talk about "Desirable Property #4: Ensure Good Query Performance" later

# Review

- When decomposing a relation  $R$ , we want to decomposition to
  - minimize redundancy
  - avoid info loss
  - preserve dependencies (i.e., constraints)
  - ensure good query performance
- These objectives can be conflicting
- Various normal forms achieve parts of the objectives

# Normal Forms

**First Normal Form** = all attributes are atomic

**Second Normal Form (2NF)** = old and obsolete

**Boyce Codd Normal Form (BCNF)** 

**Third Normal Form (3NF)**

**Fourth Normal Form (4NF)**

Others...

# Recall: What We Want to Do with Normal Forms

- Take a relation schema...
- Test it against a normalization criterion...
- If it passes, fine!
  - Maybe test again with a higher criterion
- If it fails, decompose into smaller relations
  - Each of them will pass the test
  - Each can then be tested with a higher criterion

# Boyce-Codd Normal Form

A simple condition for removing anomalies from relations:

A relation R is in BCNF if and only if:

Whenever there is a nontrivial FD  $A_1, A_2 \dots A_n \rightarrow B$  for  $R$ , it is the case that  $\{ A_1, A_2 \dots A_n \}$  is a super-key for  $R$ .

In English (though a bit vague):

Whenever a set of attributes of  $R$  is determining another attribute, it is a super-key, and thus should determine all attributes of  $R$ . (A key is also a super-key)

# Example

Name	SSN	Phone Number
Fred	123-321-99	(201) 555-1234
Fred	123-321-99	(206) 572-4312
Joe	909-438-44	(908) 464-0028
Joe	909-438-44	(212) 555-4000

What are the FDs?

$$\text{SSN} \rightarrow \text{Name}$$

Does this FD satisfy the BCNF condition?

Is the relation in BCNF?

# Decompose it into BCNF

SSN	Name
123-321-99	Fred
909-438-44	Joe

SSN → Name

SSN	Phone Number
123-321-99	(201) 555-1234
123-321-99	(206) 572-4312
909-438-44	(908) 464-0028
909-438-44	(212) 555-4000

# What About This?

Name	Price	Category
Gizmo	\$19.99	gadgets
OneClick	\$24.99	camera

Name → Price, Category

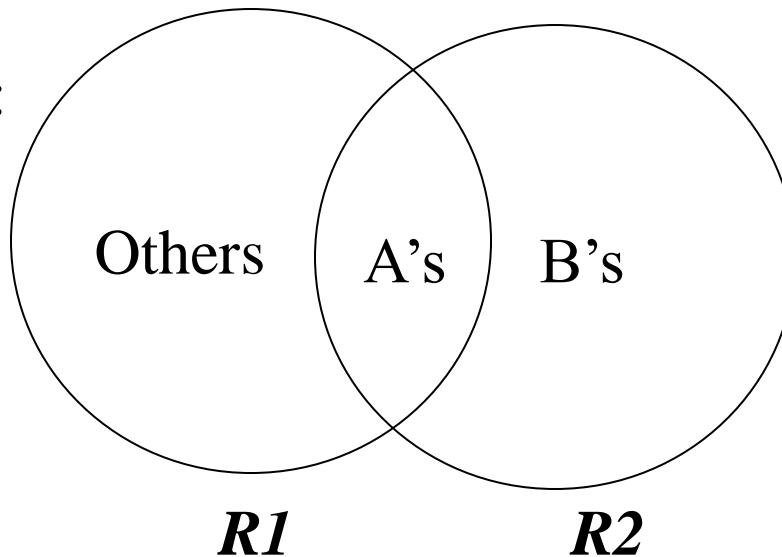
# BCNF Decomposition

Find a dependency that violates the BCNF condition:

$$A_1, A_2, \dots, A_n \longrightarrow B_1, B_2, \dots, B_m$$

Heuristics: choose  $B_1, B_2, \dots, B_m$  “as large as possible”

Decompose:



Any  
2-attribute  
relation is  
in BCNF.

Continue until  
there are no  
BCNF violations  
left.

# Example Decomposition

Person:

Name	SSN	Age	EyeColor	PhoneNumber

Functional dependencies:

$$\text{SSN} \longrightarrow \text{Name, Age, Eye Color}$$

BNCF: Person1(SSN, Name, Age, EyeColor),  
Person2(SSN, PhoneNumber)

What if we also had an attribute Draft-worthy, and the FD:

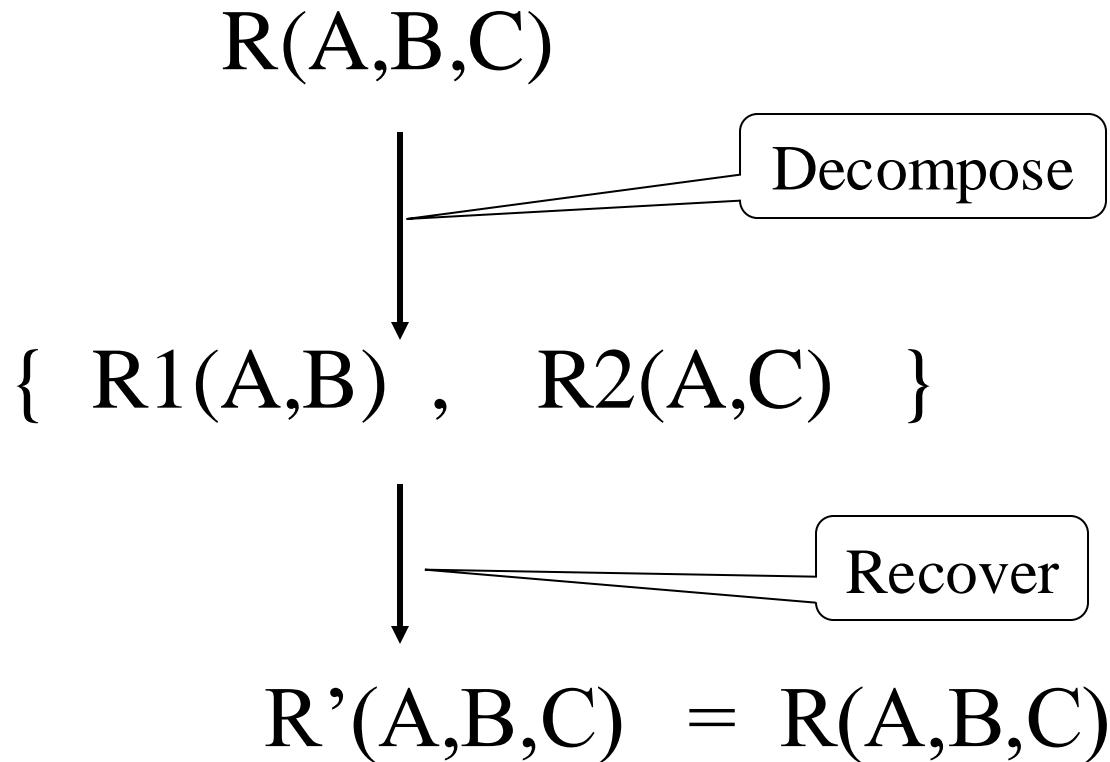
$$\text{Age} \longrightarrow \text{Draft-worthy}$$

Thus,

- BCNF removes certain types of redundancy
- For examples of redundancy that it cannot remove, see "multivalued redundancy"
- BCNF avoids info loss

# Recall: Lossless Decompositions

A decomposition is *lossless* if we can recover:



$R'$  is in general larger than  $R$ . Must ensure  $R' = R$

# Decomposition Based on BCNF is Necessarily Lossless

$R(A, B, C)$ ,       $A \rightarrow C$

BCNF:  $R1(A,B)$ ,  $R2(A,C)$

Some tuple  $(a,b,c)$  in  $R$                            $(a,b',c')$  also in  $R$   
decomposes into  $(a,b)$  in  $R1$                            $(a,b')$  also in  $R1$   
     $(a,c')$  also in  $R2$   
    and  $(a,c)$  in  $R2$

Recover tuples in  $R$ :  $(a,b,c)$ ,                           $(a,b,c')$ ,  $(a,b',c)$ ,  $(a,b',c')$  also in  $R$  ?

Can  $(a,b,c')$  be a bogus tuple? What about  $(a,b',c')$  ?

# However,

- BCNF is not always dependency preserving
- In fact, some times we cannot find a BCNF decomposition that is dependency preserving
- Can handle this situation using 3NF
- Not covered in this course, see book for examples