Real World Analytics

Assignment 3 - Linear Programming and Game Theory

Name: Prajith Raj Suresh
Student ID: S220156351
Deakin University (2022-2024)

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Unit Chair: Prof. Maia Angelova Turkedjieva

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Question 1: <u>The Cheese Factory Model using Linear Programming - Graphical Method</u>

<u>Solution</u>:

1.a)

In general, **Linear programming** is the commonly used **mathematical model** in terms of **optimizing a real-world problem into a viable solution**. LP (Linear Programming) always involves a **decision variable** for which the optimisation needs to be done, **an objective function** which indicates whether the solution needs to be **maximized or minimized** and all of these are **subject to multiple constraints**.

In the same way, the problem that is poised on us has the necessity to **minimize the cost of producing cheese** which is obtained by mixing 2 products A and B whilst also being **subject to constraints like having at least 45 litres of cow milk**, so on and so forth in the mixture while also dealing with **minimum production quantities of cheese**. Thus, all of this puts it in a plate for us into using **an LP model with the decision variables, objective function, constraints**.

1.b)

<u>Formulating an LP model</u>

<u>Decision Variables:</u>

- Let **'a'** be the product of A.
- Let **'b'** be the product of B.

<u>Objective Function:</u>

- To minimize the cost of production
- Cost of A - **$5/kg**
- Cost of B - **$8/kg**

Therefore, the objective function is **Min z = 5a + 8b**

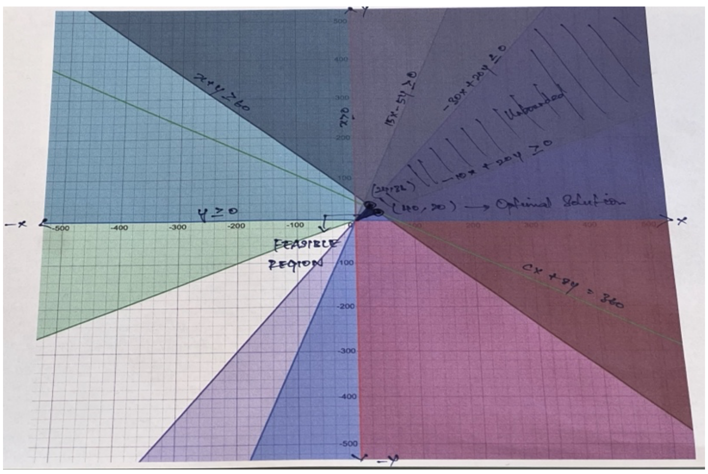
<u>Constraints:</u>

1. ***<u>Ratio for recipe constraint:</u>***
   i) Cow Milk: **(60a + 40b)/(a+b) >= 45**
   ii) Goat Milk: **(40a + 70b)/(a+b) >= 50**
   iii) Sheep Milk: **(30a + 80b)/(a+b) <= 60**

2. ***<u>Production constraint:</u>***
   A minimum of 60 kgs of cheese needs to be produced per week.
   i) **(a+b)>=60**

<u>Non-Negativity constraint:</u> a,b >=0

1.c)



Optimal Solution: The optimal cost for the minimum production of cheese is **$360**. The optimal point of the decision variables is: **(a,b) = (40,20)**

Refer to R code for the solution of First Question.

1.d)

The range for the cost of A that can be change without affecting the optimal point of (40,20) is **$0-$8**. This is obtained by performing sensitivity analysis on the equations by replacing the cost of **A** which is 5 with a Constant **C** and checking for values of A that produce the same optimal solution. The aforementioned sensitivity analysis can be performed by taking the **slope** of the **Objective function** and checking for slope for the **outer constraint (a+b) >= 60**, which provides the slope value range of **8>= C >= -24.** Thus, we ignore the negation value of -24 as we are dealing with cost and consider the range as 0 to 8.

Question 2: The Food Factory Model using LP with more than 2 variables

Solution:

2.a) ***Formulating an LP model***

Decision Variables:
Let x_{ij} be the decision variable that denotes the **number of tons of ingredients** that is **i**, where i could be **Oats, Apricots, Coconuts, Hazelnuts** that are being used to produce **cereal j** , here j is one of **A,B, and C (in boxes)**.

- Let x_{11} be the number of tons of Oats in Cereal A
- Let x_{21} be the number of tons of Apricots in Cereal A
- Let x_{31} be the number of tons of Coconuts in Cereal A
- Let x_{41} be the number of tons of Hazelnuts in Cereal A
- Let x_{12} be the number of tons of Oats in Cereal B
- Let x_{22} be the number of tons of Apricots in Cereal B
- Let x_{32} be the number of tons of Coconuts in Cereal B
- Let x_{42} be the number of tons of Hazelnuts in Cereal B
- Let x_{13} be the number of tons of Oats in Cereal C
- Let x_{23} be the number of tons of Apricots in Cereal C
- Let x_{33} be the number of tons of Coconuts in Cereal C
- Let x_{43} be the number of tons of Hazelnuts in Cereal C

Number of Cereals using the ingredients:
- A : $x_{11} + x_{21} + x_{31} + x_{41}$
- B: $x_{12} + x_{22} + x_{32} + x_{42}$
- C: $x_{13} + x_{23} + x_{33} + x_{43}$

Number of ingredients used to make the cereals:
- Oates: $x_{11} + x_{12} + x_{13}$
- Apricots: $x_{21} + x_{22} + x_{23}$
- Coconuts: $x_{31} + x_{32} + x_{33}$
- Hazelnuts: $x_{41} + x_{42} + x_{43}$

Note: I have converted the **kgs(box) into tons** for uniformity.

Objective Function:
- To **maximize** the profit.
- **Profit = Sales - Purchase Price - Production Cost**
- Sales equation is : $2500(x_{11} + x_{21} + x_{31} + x_{41}) + 2000(x_{12} + x_{22} + x_{32} + x_{42}) + 3500(x_{13} + x_{23} + x_{33} + x_{43})$

- Purchase price equation is : **$100(x_{11} + x_{12} + x_{13}) + 120(x_{21} + x_{22} + x_{23}) + 80(x_{31} + x_{32} + x_{33}) + 200(x_{41} + x_{42} + x_{43})$**

- Production cost equation is : **$4.00(x_{11} + x_{21} + x_{31} + x_{41}) + 2.80(x_{12} + x_{22} + x_{32} + x_{42}) + 3.00(x_{13} + x_{23} + x_{33} + x_{43})$**

- Profit = **$2500(x_{11} + x_{21} + x_{31} + x_{41}) + 2000(x_{12} + x_{22} + x_{32} + x_{42}) + 3500(x_{13} + x_{23} + x_{33} + x_{43}) - (100(x_{11} + x_{12} + x_{13}) + 120(x_{21} + x_{22} + x_{23}) + 80(x_{31} + x_{32} + x_{33}) + 200(x_{41} + x_{42} + x_{43})) - (4.00(x_{11} + x_{21} + x_{31} + x_{41}) + 2.80(x_{12} + x_{22} + x_{32} + x_{42}) + 3.00 (x_{13} + x_{23} + x_{33} + x_{43}))$**

<u>Constraints:</u>

a) ***Minimum Demand Constraint***
b) ***Maximum Availability Constraint***
c) ***Proportion Constraint***

1. ***<u>Minimum Demand Constraint:</u>***

- $x_{11} + x_{21} + x_{31} + x_{41} \geq 1$
- $x_{12} + x_{22} + x_{32} + x_{42} \geq 0.7$
- $x_{13} + x_{23} + x_{33} + x_{43} \geq 0.75$

2. ***<u>Maximum Availability Constraint:</u>***

- $x_{11} + x_{12} + x_{13} \leq 10$
- $x_{21} + x_{22} + x_{23} \leq 5$
- $x_{31} + x_{32} + x_{33} \leq 2$
- $x_{41} + x_{42} + x_{43} \leq 2$

3. ***<u>Proportion Constraint:</u>***

- $x_{11} / x_{11} + x_{21} + x_{31} + x_{41} = 0.8$
  - **Simplifying: $0.2x_{11} - 0.8x_{21} - 0.8x_{31} - 0.8x_{41} = 0$**
- $x_{21} / x_{11} + x_{21} + x_{31} + x_{41} = 0.1$
  - **Simplifying: $- 0.1x_{11} + 0.9x_{21} - 0.1x_{31} - 0.1x_{41} = 0$**
- $x_{31} / x_{11} + x_{21} + x_{31} + x_{41} = 0.05$
  - **Simplifying: $- 0.0.5x_{11} - 0.05x_{21} + 0.95x_{31} - 0.05x_{41} = 0$**
- $x_{41} / x_{11} + x_{21} + x_{31} + x_{41} = 0.0.5$
  - **Simplifying: $- 0.05x_{11} - 0.05x_{21} - 0.05x_{31} + 0.95x_{41} = 0$**
- $x_{12} / x_{12} + x_{22} + x_{32} + x_{42} = 0.65$
  - **Simplifying: $0.35x_{12} - 0.65x_{22} - 0.65x_{32} - 0.65x_{42} = 0$**
- $x_{22} / x_{12} + x_{22} + x_{32} + x_{42} = 0.2$
  - **Simplifying: $- 0.2x_{12} + 0.8x_{22} - 0.2x_{32} - 0.2x_{42} = 0$**
- $x_{32} / x_{12} + x_{22} + x_{32} + x_{42} = 0.05$
  - **Simplifying: $- 0.05x_{12} - 0.05x_{22} + 0.95x_{32} - 0.05x_{42} = 0$**
- $x_{42} / x_{12} + x_{22} + x_{32} + x_{42} = 0.1$
  - **Simplifying: $- 0.1x_{12} - 0.1x_{22} - 0.1x_{32} + 0.9x_{42} = 0$**
- $x_{13} / x_{13} + x_{23} + x_{33} + x_{43} = 0.5$
  - **Simplifying: $0.5x_{13} - 0.5x_{23} - 0.5x_{33} - 0.65x_{43} = 0$**
- $x_{23} / x_{13} + x_{23} + x_{33} + x_{43} = 0.1$
  - **Simplifying: $- 0.1x_{13} + 0.9x_{23} - 0.1x_{33} - 0.1x_{43} = 0$**
- $x_{33} / x_{13} + x_{23} + x_{33} + x_{43} = 0.1$
  - **Simplifying: $- 0.1x_{13} - 0.1x_{23} + 0.9_{33} - 0.1x_{43} = 0$**
- $x_{43} / x_{13} + x_{23} + x_{33} + x_{43} = 0.3$
  - **Simplifying: $- 0.3x_{13} - 0.3x_{23} - 0.3x_{33} + 0.7x_{43} = 0$**

Non-Negativity Constraints: $x_{11} \geq 0$, $x_{21} \geq 0$, $x_{31} \geq 0$,$x_{41} \geq 0$,$x_{12} \geq 0$, $x_{22} \geq 0$, $x_{32,} \geq 0$, $x_{42} \geq 0$, $x_{13} \geq 0$, $x_{23} \geq 0$,$x_{33} \geq 0$,$x_{43} \geq 0$

2.b)

Optimal Solution:
The optimal solution for the **maximum profit** is **$39,129**.

The optimal values are :
x_{11} = 7.0641 tons, x_{21} = 0.8830 tons , x_{31} = 0.4415 tons , x_{41} = 0.4415 tons ,
x_{12} = 0.4550 tons , x_{22} = 0.140 tons , x_{32} = 0.0350 tons, x_{42} = 0.070 tons ,
x_{13} = 2.4808 tons, x_{23} = 0.4961 tons, x_{33} = 0.4961 tons , x_{43} = 1.4884 tons

Refer to R code for the solution of Second Question.

Question 3: The Bidding Model using LP and Game Theory with more than 2 strategies

Solution:

3.a)
The game provided is to be considered as a **Two-Player-Zero-Sum** game as we can clearly see that there are only **2** players (companies) involved in the name of **RED** and **BLUE**. Additionally, we all see that there are a number of **strategies (5)** that each player is using which also provides us with the scenario wherein by using either of these strategies, there can only be **ONE sole winner** and the other one will be a **LOSER.** In this scenario, we see that the company that makes the **highest bid is considered as winner** and in the **event of a tie**, the **RED** company will take the spoils of the share and will be considered as winners. Thus, we can confidently say that this is a TWO-PLAYER-ZERO-SUM game.

3.b)
The Payoff matrix is given by:

RED	BLUE				
	15	25	35	45	50
15	+1	+1	-1	-1	-1
25	+1	+1	-1	-1	-1
35	+1	+1	+1	-1	-1
45	1	1	1	+1	-1
50	-1	-1	-1	-1	+1

3.c)

SADDLE POINT: A saddle point is considered as an **optimal solution** using the **Pure strategy** where neither player has an incentive to change their strategy. Thus, a saddle point can be considered as **(A_i , B_j)** wherein **A_i** is the i^th strategy (pure) for Player 1 and **B_j** is the j^th strategy (pure) for Player 2. This is achieved by checking the **Lower Bound (LB)** and **Upper Bound (UB)** values using the **MinMax** technique and checking for similar values. If LB=UB, then that is the **optimal solution**, if it is 0, then it is **FAIR GAME**, and if UB>LB or vice versa, then we will have to perform **Mixed Strategy**.

NO, this game does not have a saddle point as the **LOWER BOUND is -1 and the UPPER BOUND is 1**. Both the values are not identical and hence this needs to be done with mixed strategy.

3.d)

LINEAR PROGRAMMING MODEL FOR BLUE

$$\min \ w = v$$

s.t

$$v - (y_1 - y_2 - y_3 - y_4 - y_5) \geq 0$$

$$v - (y_1 + y_2 - y_3 - y_4 - y_5) \geq 0$$

$$v - (y_1 + y_2 + y_3 - y_4 - y_5) \geq 0$$

$$v - (y_1 + y_2 + y_3 + y_4 - y_5) \geq 0$$

$$v - (-y_1 - y_2 - y_3 - y_4 + y_5) \geq 0$$

$$y_1 + y_2 + y_3 + y_4 + y_5 = 1$$

$$y_i \geq 0, \ \forall i = 1, 2, \ldots\ldots 5$$

v u.r.s

3.e) Refer to R code for Solution of the LP model

3.f)

The interpretation of the solution obtained from the LP model for solving Blue Company is that the value happens to have **'0'** which gives the notion that the bidding is tied. However, since it is explicitly mentioned in the question that in an event of a tie in the bidding, the **winner** will be **RED.** Thus, we'll have to conclude that **BLUE** has **lost** the bidding war. Additionally, we can also say that we are **50%** certain that RED will be the winner in the context of a tie when they bid with the values **15** and **50**. Thus, we conclude that **BLUE has lost the bidding war**.

References:

 # "Wayne L. Winston, Operations Research: Applications and Algorithms, Deakin University Library, Melbourne"
 # "Hamdy A. Taha, Operations Research: An Introduction, Deakin University Library, Melbourne"