A3) Let Y denote the noisy image & X be the original image (which we want)

$$Y = [Y^R \quad Y^G \quad Y^B] \longrightarrow [Y^Y \quad Y^{C_B} \quad Y^{C_R}]$$

RGB space (correlated) → $Y C_B C_R$ space statistically most uncorrelated

We take the R, G, B value of each pixel and build an eigenspace of that - you get 3 eigenvector each corresponding to 3 eigenvalue.

Using those eigenvalue, we can construct almost uncorrelated basis of color (we call it $Y\ C_b\ C_R$).

Generally, for most RGB images, it is found that

$$Y = 16 + (65.481R + 128.553G + 24.966B)$$
$$C_B = 128 + (-37.79R + 112B - 74.203G)$$
$$C_R = 128 + (112R - 93.786G - 18.214B)$$

$$\Rightarrow \underbrace{\begin{bmatrix} Y \\ C_B \\ C_R \end{bmatrix}}_{I_N} = \underbrace{\begin{bmatrix} 16 \\ 128 \\ 128 \end{bmatrix}}_{\delta} + \underbrace{\begin{bmatrix} 65.481 & 128.553 & 24.966 \\ -37.79 & -74.203 & 112 \\ 112 & -93.786 & -18.214 \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} R \\ G \\ B \end{bmatrix}}_{I}$$

$\therefore$ we get $I_N = \delta + MI$

Now since values of I_N are statistically uncorrelated (Y, C_B, C_R) we can either denoise those channels individually or together

However note that human eye is most sensitive to luminescence (Y), hence denoising that seperately will be better, which is not prominently useful for C_B, C_R channels.

Hence, we denoise Y channel seperately from C_B, C_R channel (which we do together)

① Channel Y : This is the luminescence channel where we specifically want to preserve edges hence we use Discontinuity-adaptive function : $g(u) = r|u| - r^2 \log(1 + |u|/r)$

where u is the difference of Y in 4 neighbour system.

As we are told in slides that noise model for microscopy images is typically assumed to be poisson. Hence we write the Bayesian formulation for denoising as follows:

$$\max_{x_i} (x|y, \theta) = \max_{x_i} \left(\log P(y_i | x_i, \theta) + \log P(x_i | x_{N_i}, \theta) \right)$$

(where x, y denotes Y component of X, Y image respectively)

$$\Rightarrow \max_{x_i} (x|y, \theta) = \max_{x_i} \left(\log_e \left[\frac{\lambda^{(y_i - x_i)} e^{-\lambda}}{(y_i - x_i)!} \right] + \sum_{i \in C_2} g(u_i) \right)$$

u_i = Neighbour difference

$$\max_{x_i} p(x|y,\theta) = \max\Big(-\lambda + (y_i - x_i)\ln\lambda - \ln[(y_i - x_i)!]$$

$$+ \sum g(u_i)\Big)$$

For solving this, differentiate :-

$$\frac{\partial p(x|y,\theta)}{\partial x_i} = -\ln\lambda - \frac{\partial}{\partial x_i}\ln[(y_i - x_i)!] + \frac{\partial}{\partial x_i}\sum g(u_i)$$

$$\ln(y_i - x_i)! = \sum_{i=0}^{y_i - x_i - 1} \ln[(y_i - x_i) - i]$$

$$\therefore \frac{\partial}{\partial x_i}\ln(y_i - x_i)! = \frac{-1}{y_i - x_i} - \frac{1}{y_i - x_i - 1} - \cdots - 1$$

$$= -\sum_{i=0}^{-x_i + y_i - 1} \frac{1}{y_i - x_i - i}$$

$$\& \ \frac{\partial}{\partial x_i}\sum g(u_i) = \sum \frac{\partial}{\partial x_i} g(u_i) = \sum \frac{\partial}{\partial u_i} g(u_i) \cdot \underbrace{\frac{\partial u_i}{\partial x_i}}_{=1}$$

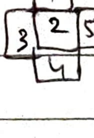
$$\& \ \frac{\partial}{\partial u} g(u) = \frac{2ur}{2(r+|u|)} = \frac{ur}{r+|u|}$$

$$\therefore \frac{\partial p(x|y;\theta)}{\partial x_i} = -\ln\lambda + \sum_{i=0}^{y_i - x_i - 1} \frac{1}{y_i - x_i - i} + \sum \frac{u_i r}{r + |u_i|}$$

u_i are the neighbour distances in 4-Neighbour

& update rule for Y becomes:

$$\boxed{Y_{new} = Y_{old} - \eta \frac{\partial}{\partial x_i} p(x|y,\theta)}$$



$u_1 = I_1 - I_2$

$u_2 = I_3 - I_2$

and so on

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② Channel C_B, C_R: consider these both together as y_i, x_i (2-D vectors) and follow similar steps:

$$\cancel{\frac{\partial}{\partial x_i} P(\quad)}$$

$$\frac{\partial}{\partial x_i} P(x|y,\theta) = -\ln\lambda - \frac{\partial}{\partial x_i} \ln(y_i - x_i)! - \frac{\partial}{\partial x_i} \sum g(u_i)$$

let us take $(y_i - x_i)$ as the L1 norm between 2 vectors then,

$$\frac{\partial}{\partial x_i} \ln(y_i - x_i) = \cancel{\sum \frac{\quad}{-\|x_i - y_i\|_1 + i}} = \cancel{\sum_{i=0}^{\|y_i - x_i\|_1 - 1} \frac{1}{\|y_i - x_i\|_1 - i}}$$

$$\underset{(\text{let})}{A} = \begin{bmatrix} \sum_{i=0}^{y_i^1 - x_i^1 - 1} \frac{-1}{y_i^1 - x_i^1 - i} \\ \sum_{i=0}^{y_i^2 - x_i^2 - 1} \frac{-1}{y_i^2 - x_i^2 - i} \end{bmatrix} \qquad y_i = \begin{bmatrix} y_i^1 \\ y_i^2 \end{bmatrix} \quad x_i = \begin{bmatrix} x_i^1 \\ x_i^2 \end{bmatrix}$$

& $$\frac{\partial}{\partial x_i} \sum g(u_i) = \sum \frac{\partial}{\partial u_i} g(u_i) \cdot \underbrace{\frac{\partial u_i}{\partial x_i}}_{=1 \text{ (for L1 Norm)}}$$

$$= \sum \frac{u_i r}{r + \|u_i\|_1} \quad \text{(sum of func. of neighbours)}$$

$$\therefore \frac{\partial}{\partial x_i} P(x|y,\theta) = -\ln\lambda + A - \sum \frac{u_i r}{r + \|u_i\|_1}$$

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$$\left[\begin{matrix} C_B \\ C_R \end{matrix}\right]_N = \left[\begin{matrix} C_B \\ C_R \end{matrix}\right] - \eta_2 \frac{\partial p(x|y,\theta)}{\partial x_i}$$

& we can calculate R, G, B channel individually

by: $I_N = \delta + M I$ equation $\Rightarrow$ $\boxed{I = M^{-1}(I_N - \delta)}$

(we can also use L_2 norm instead of L_1 norm for C_B, C_R channels).