SHARNBASVA UNIVERSITY

Faculty of Engineering and Technology

Engineering Mathematics-IV(18MAT41)

Module I : Fourier Series

Neither Even nor Odd

1. Definition of periodic function of period 2π
2. Obtain the Fourier Series of $f(x) = \frac{\pi - x}{2}$ in $0 < x < 2\pi$. Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ------- = \frac{\pi}{4}$
3. If $f(x) = x(2\pi - x)$ in $0 \leq x \leq 2\pi$ show that $(x) = \frac{2\pi^2}{3} - 4\left(\frac{cosx}{1^2} + \frac{cos2x}{2^2} + \frac{cos3x}{3^2} + ---\right)$
4. Obtain the Fourier Series for the function $(x) = \begin{cases} -\pi & in -\pi < x < 0 \\ x & in \quad 0 < x < \pi \end{cases}$.

Fourier Series of Even & odd functions of $(\mathbf{0}, \boldsymbol{\pi})$ *and* $(\mathbf{0}, \mathbf{2}\boldsymbol{\pi})$& arbitrary

5. Find Fourier Series of $f(x) = \begin{cases} 1 + \frac{2x}{\pi} & in -\pi < x \leq 0 \\ 1 - \frac{2x}{\pi} & in \ 0 \leq x < \pi \end{cases}$ & hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ----$.
6. Obatin the Fourier series expansion of the function $f(x) = \begin{cases} x & in \ 0 < x < \pi \\ x - 2\pi & i \quad < x < 2\pi \end{cases}$ Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ---- = \frac{\pi}{4}$
7. Obtain the F.S for the function $(x) = 2x - x^2$ in $0 \leq x \leq 2$.

Half Range Fourier Series

1. Find the cosine half range series for the function $f(x) = lx - x^2$ in $0 < x < l$
2. Expand $(x) = 2x - 1$ as a cosine half range Fourier series in $0 < x < 1$.
3. Find sine half range series of $(x) = x^2$ in $0 < x < \pi$.
4. If $f(x) = \begin{cases} x & in \ 0 < x < \frac{\pi}{2} \\ \pi - x & in \ \frac{\pi}{2} < x < \pi \end{cases}$ Show that $f(x) = \frac{4}{\pi}\left[sinx - \frac{sin3x}{3^2} + \frac{sin5x}{5^2} - ------\right]$

Practical Harmonic Analysis:

1. Determine the constant term and the first cosine and sine terms of the Fourier series expansion of y from the following data

X_0	0	45	90	135	180	225	270	315
y	2	3/2	1	½	0	½	1	3/2

2. The turning moment T on the crank shaft of a steam engine for the crank angle θ is given as follows.

$\theta°$	0	30	60	90	120	150	180	210	240	270	300	330
T	0	2.7	5.2	7	8.1	8.3	7.9	6.8	5.5	4.1	2.6	1.2

3. Express y as F.S upto the 2nd harmonic given that

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
Y	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

4. Obtain the constant term and the coefficients of the first cosine and sine terms in the Fourier expansion of y from the table

X	0	1	2	3	4	5
Y	9	18	24	28	26	20

Module III : Numerical Methods

Numerical solution of ODE of first order and first degree.

Taylor's series method:

1. Use Taylor's series method to find y at $x = 0.1, 0.2, 0.3$ considering terms upto third degree given that $\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 1$
2. Find y at $x = 1.02$ correct to five decimal places given $dy = (xy - 1)x$ and $y = 2$ at $x = 1$ applying Taylor's series method.
3. Use Taylor's series method to obtain a power series in $(x - 4)$ for the equation $5x\frac{dy}{dx} + y^2 - 2 = 0$; $x_0 = 4, y_0 = 1$ and use it to find y at $x = 4.1, 4.2, 4.3$ correct to four decimal places.

Modified Euler's Method:

1. Given $\frac{dy}{dx} = 1 + \frac{y}{x}$; $y_0 = 2$ at $x_0 = 1$, find the approximate value of y at $x = 1.4$ by taking step size $h = 0.2$ applying modified Euler's method
2. Using Modified Euler's Method to find $y(20.2)$ given that $\frac{dy}{dx} = \log_{10}\left(\frac{x}{y}\right)$ with $y(20) = 5$ taking $h = 0.2$.

3. Using Euler's predictor and corrector formula compute $y(1.1)$ correct to five decimal places given that $\frac{dy}{dx}+\frac{y}{x}=\frac{1}{x^2}$ $and\ y=1\ at\ x=1.$

Runge Kutta Method of 4th order:

1. Use fourth order Runge Kutta method to solve $(x+y)\frac{dy}{dx}=1, y(0.4)=1\ at\ x=0.5$ correct to four decimal places.
2. Use fourth order Runge Kutta method to find $y\ at\ x=0.1$ given that $\frac{dy}{dx}=3e^x+2y$, $(0)=0\ and\ h=0.1.$
3. using Runge Kutta Method of 4th order Solve $\frac{dy}{dx}+y=2x\ at\ x=1.1$ given that $y=3\ at$ $x=1$ initially.

Milne's and Adams – Bashforth predictor and corrector method :

1. Apply Milne's method to compute $y(1.4)$ correct to four decimal places given $\frac{dy}{dx}=x^2+\frac{y}{2}$ and the following data : $y(1)=2, y(1.1)=2.2156, y(1.2)=2.4649, y(1.3)=2.7514.$
2. The following table gives the solution of $5xy'+y^2-2=0$. Find the value of y at $x=4.5$ using Milne's predictor and corrector formulae. Use the corrector formula twice.

X	4	4.1	4.2	4.3	4.4
Y	1	1.0049	1.0097	1.0143	1.0187

3. Apply Adams – Bashforth method to solve the equation $(y^2+1)dy-x^2dx=0\ at\ x=1$ given $y(0)=1, y(0.25)=1.0026, y(0.5)=1.0206, y(0.75)=1.0679.$
4. Solve the differential equation $y'+y+xy^2=0$ with the initial values of y: $y_0=1, y_1=0.9008, y_2=0.8066,\ y_3=0.722$ corresponding to the values of $x: x_0=0, x_1=0.1, x_2=0.2, x_3=0.3$ by computing the value of y corresponding to x=0.4 applying Adams – Bashforth predictor and corrector formula.

Module – IV : Numerical Methods:

Numerical solution of Second order ODE by Runge-Kutta method of 4th order:

1. Given $y''-xy'-y=0$ with the initial conditions $y(0)=1, y'(0)=0$, compute $y(0.2)$ and $y'(0.2)$ using fourth order Runge-Kutta method.
2. Compute $y(0.1)$ given $\frac{d^2y}{dx^2}=y^3$ and $y=10, \frac{dy}{dx}=5$ at $x=0$ by Runge-Kutta method of fourth order.
3. Obtain the values of x & $\frac{dx}{dt}$ when t=0.1 given that x satisfies the equation $\frac{d^2x}{dt^2}=t\frac{dx}{dt}-4x$ & $x=3, \frac{dx}{dt}=0\ when\ t=0$ initially. Use 4th order Runge Kutta Method.

Milne's Method

1. Apply Milne's method solve $\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}$ given the following table of initial values.

X	0	0.1	0.2	0.3
Y	1	1.1103	1.2427	1.399
Y'	1	1.2103	1.4427	1.699

Compute (0.4) numerically and also theoretically.

2. Given the ODE $y'' + xy' + y = 0$ and the following table of initial values, compute (0.4) by applying Milne's method.

x	0	0.1	0.2	0.3
Y	1	0.995	0.9801	0.956
Y'	0	-0.0995	-0.196	-0.2867

3. Apply Milne's method to compute y(0.4) given the equation $y'' + y' = 2e^x$ and the following table of initial values. Compare the result with theoretical value

X	0	0.1	0.2	0.3
y	2	2.01	2.04	2.09
y'	0	0.2	0.4	0.6

Numerical solution of Heat equation

1. Solve $u_{xx} = 32u_t$ subject to the conditons $(0, t) = 0, u(1, t) = t \text{ and } u(x, 0) = 0$. Find the values of u upto t=5 by Schmidt's process taking h=1/4. Also extract the following values: a) u(0.75, 4) b) u(0.5, 5) c) u(0.24, 4)
2. Find the numerical solution of the parabolic equation $\frac{\partial^2 u}{\partial x^2} = 2\frac{\partial u}{\partial t}$ when $u(0, t) = 0 = (4, t)$ and $u(x, 0) = x(4 - x)$ by taking $h = 1$ Find the values upto t=5.

Numerical solution of Wave equation

1. Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ subject to $u = (0, t) = 0$, $u(4, t) = 0$, $u_t(x, 0) = 0$ and $u(x, 0) = x(4 - x)$ by taking $h = 1, k = 0.5$ upto four steps.

2. Solve the wave equation $u_{xx} = 0.0625u_{tt}$ subject to the conditions $u(0, t) = 0 = u(5, t)$, $u(x, 0) = x^2(x - 5)$&$u_t(x, 0) = 0$ by taking $h = 1$ for $0 \leq t \leq 1$.
3. Solve : $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ given that $u(0, t) = 0$, $u(x, 0) = 0$, $u_t(x, 0) = 0$ and $(1, t) = 100sin(\pi t)$ in the range $0 \leq t \leq 1$ by taking $h = 1/4$.

Module V: Joint Probability Distribution

1. The joint distribution of two random variable x & y is as follows

X \ Y	-4	2	7
1	1/8	¼	1/8
5	¼	1/8	1/8

compute the following $a)\ E(X) \& E(Y)$ $b)\ E(XY)$ $c) \sigma_x \& \sigma_y$ $d) COV(X,Y)$ $e) \rho(X,Y)$.

2. X And Y are Independent random variables.X take values 2,5,7 with probability ½,1/4,1/4 respectively Y take the values 3,4,5 with probability 1/3,1/3,1/3.Find the joint probability distribution of X and Y.
3. Suppose X and Y are independent random variables with the following respective distribution find the joint distribution of X and Y also verify that COV(X,Y)=0

x_i	1	2
$f(x_i)$	0.7	0.3

y_j	-2	5	8
$g(y_j)$	0.3	0.5	0.2

4. The joint distribution table for two random variables X and Y is as follows

x \ Y	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Also compute E(X),E(Y),E(XY),COV(XY),S.D. of X,Y

5. If X & Y are Independent random variables, Prove the following results.
   a) E(XY)= E(X).E(Y) b) COV(X,Y)=0 c) $\sigma^2_{x+y} = \sigma^2_x + \sigma^2_y$.

Stochastic process:

1. Definition: i) Stochastic process ii) Stochastic matrix and iii) Regular stochastic matrix.
2. If $A = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}$ is a stochastic matrix & $V = [v_1, v_2]$ is a probability vector, show that VA is also a probability vector.
3. Prove with reference to two second order stochastic matrices that their product is also a stochastic matrix.
4. If A is a square matrix of order n whose rowsare each the same vector $a = (a1, a2, \ldots an)$ and if $v = (v1, v2, \ldots . vn)$is a probability vector P.T $vA = a$
5. Find the Unique fixed probability vector of the regular stochastic matrix $A = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$
6. Find the Unique fixed probability vector of the regular stochastic matrix

$$P = \begin{bmatrix} 0 & 1/2 & 1/4 & 1/4 \\ 1/2 & 0 & 1/4 & 1/4 \\ 1/2 & 1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \end{bmatrix}$$

Markov Chains :

1. Definition :Markov Chains
2. The transition matrix P of a Markov Chains is given by $\begin{bmatrix} 1/2 & 1/2 \\ 3/4 & 1/4 \end{bmatrix}$ with the initial probability distribution $p^{(0)} = (\frac{1}{4}, \frac{3}{4})$. Define & find the following $i)\ _{21}^{(2)}$ $\quad ii)\ p_{12}^{(2)}$ $\quad iii)\ p^2$ $\quad iv) p_1^{(2)}$ $\quad v) the\ vector\ p^{(0)}p^n$ approaches.

Transition probability matrix (t.p.m)

1. P.T the markov chain whose t.p.m.is $P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ is irreduciable find the corresponding stationary probability vector.
3. A student study habits are as fallows. If he studies one night , he is 70% sure not to study the next night. on the other hand if he does not study one night ,he is 60% sure not to study the next night. In the long run how often does he study.