19-07-22

Module - IV.

Range-Kutta method.

we have to compute, $y(x_0+h)$ & if required $y'(x_0+h)$ & is given by,

$$y(x_0+h) = y_0 + \frac{1}{6}(k_1+2k_2+2k_3+k_4)$$

$$y'(x_0+h) = z(x_0+h) = z_0 + \frac{1}{6}(l_1+2l_2+2l_3+l_4)$$

where,

$$k_1 = hf(x_0, y_0, z_0) \;;\; l_1 = hg(x_0, y_0, z_0)$$

$$k_2 = hf\left(x_0+\frac{h}{2}, y_0+\frac{k_1}{2}, z_0+\frac{l_1}{2}\right);$$

$$l_2 = hg\left(x_0+\frac{h}{2}, y_0+\frac{k_1}{2}, z_0+\frac{l_1}{2}\right)$$

$$k_3 = hf\left(x_0+\frac{h}{2}, y_0+\frac{k_2}{2}, z_0+\frac{l_2}{2}\right);$$

$$l_3 = hg\left(x_0+\frac{h}{2}, y_0+\frac{k_2}{2}, z_0+\frac{l_2}{2}\right)$$

$$k_4 = hf(x_0+h, y_0+k_3, z_0+l_3)$$

$$l_4 = hg(x_0+h, y_0+k_3, z_0+l_3)$$

1.

Given $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1$; $y(0)=1$; $y'(0)=0$. Evaluate $y(0.1)$ using R-K method of order 4.

soln Given that,

$\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1$ & $x_0 = 0$; $y_0 = 1$; $y_0' = 0$

put $\frac{dy}{dx} = z$.

diff w.r.t x

$$\frac{d^2y}{dx^2} = \frac{dz}{dx}$$

$\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1$ —— (1)

sub above values in (1)

$\frac{dz}{dx} - x^2 z - 2xy = 1$

$$\boxed{\frac{dz}{dx} = 1 + x^2 z + 2xy}$$

Now, we have the system of eqn's as,

$\frac{dy}{dx} = z$ & $\frac{dz}{dx} = 1 + x^2 z + 2xy$.

$f(x,y,z) = z$,
$g(x,y,z) = 1 + x^2 z + 2xy$ with
$x_0 = 0,\ y_0 = 1,\ z_0 = 0$.

we need to compute
$y(x_0+h) = y(0.1) = ?$ $\boxed{h = 0.1}$

we have,
$$y(x_0+h) = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

where,
$$k_1 = hf(x_0, y_0, z_0)$$
$$= (0.1) f(0,1,0)$$
$$= (0.1)\,0$$
$$\boxed{k_1 = 0}$$

$$l_1 = hg(x_0, y_0, z_0)$$
$$= (0.1)\, g(0,1,0)$$
$$= (0.1)[1 + (0)^2(0) + 2(0)(1)]$$
$$l_1 = (0.1)(1)$$
$$\boxed{l_1 = 0.1}$$

$$k_2 = hf\left[x_0 + \frac{h}{2},\ y_0 + \frac{k_1}{2},\ z_0 + \frac{l_1}{2}\right]$$
$$= (0.1) f\left[0 + \frac{0.1}{2},\ 1 + \frac{0}{2},\ 0 + \frac{0.1}{2}\right]$$

$= (0.1)\, f\, (0.05, 1, 0.05)$

$= (0.1)\, (0.05)$

$k_2 = 0.005$

$$l_2 = hg\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right]$$

$$= (0.1)\, g\left[\frac{(0.1)}{2}, 1, \frac{0.1}{2}\right]$$

$$= (0.1)\, g\, [0.05, 1, 0.05]$$

$$= (0.1)\, [1 + (0.05)^2 (0.05) + 2(1)(0.05)]$$

$$= (0.1)\, [1 + 0.000000125 + 0.1]$$

$$= 0.11$$

$$k_3 = hf\left[x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right]$$

$$= (0.1)\, f\left[\frac{0+0.1}{2}, 1 + \frac{0.005}{2}, 0 + \frac{0.11}{2}\right]$$

$$= (0.1)\, f\, (0.05, 0.0025+1, 0.055)$$

$$= (0.1)\, f\, (0.05, 1.0025, 0.055)$$

$k_3 = (0.1)\, (0.055) \Rightarrow 0.0055$

Date
Page

$$l_3 = hg\left[x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right]$$

$$= (0.1)\, g\left[\frac{0.1}{2}, \frac{1 + 0.005}{2}, \frac{0.11}{2}\right]$$

$$= (0.1)\, g\,[0.05, 1.0025, 0.055)$$

$$= (0.1)\,[1 + (0.05)^2 (0.055) + 2(0.05)(1.0025)]$$

$$= (0.1)\,(1.1004)$$

$$\boxed{l_3 = 0.11004}$$

$$k_4 = hf\,(x_0 + h, y_0 + k_3, z_0 + l_3]$$

$$= 0.1\, f\,(0 + 0.1, 1 + 0.0055, 0 + 0.1104]$$

$$= 0.1\, f\,(0.1, 1.0055, 0.11004)$$

$$= (0.1)(0.11004)$$

$$\boxed{k_4 = 0.011004}$$

SUB all value in main FORM:-

$$y(x_0 + h) = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= 1 + \frac{1}{6}(0 + 2(0.005) + 2(0.0055) + 0.011)]$$

20-7-22

$= 1 + \frac{1}{6} [0.01 + 0.011 + 0.011]$

$= 1 + \frac{0.032}{6}$

$= 1 + 0.00533$

$\boxed{y(0.1) = 1.00533}$

2. Given $y'' - xy' - y = 0$ with the initial conditions $y(0) = 1$, $y'(0) = 0$. compute $y(0.2)$ & $y'(0.2)$ using fourth order R-K method.

soln:

$y'' - xy' - y = 0$; $y(0) = 1$
$y(x_0) = y_0$

$\therefore x_0 = 0$, $y_0 = 1$, $y_0'(x_0) = 0$.

Put $\frac{dy}{dx} = \boxed{y' = z}$

diff w.r.t x.
$\boxed{y'' = z'}$

Now the given eqn reduces to

$z' - xz - y = 0$
$\boxed{z' = xz + y}$

Now we have the system of eqns as,

$$\frac{dy}{dx} = z \quad \& \quad \frac{dz}{dx} = xz + y$$

$f(x, y, z) = z \qquad g(x, y, z) = xz + y.$

with,

$x_0 = 0; \quad y_0 = 1; \quad z_0 = 0; \quad h = 0.2.$

we need to find,

$y(x_0 + h) = y(0.2) = ?$;

$y'(x_0 + h) = y'(0.2) = ?$

we have,

$$y(x_0 + h) = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4] \quad \text{--- (1)}$$

$$y'(x_0 + h) = z_0 + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4) \quad \text{--- (2)}$$

where,

$k_1 = hf(x_0, y_0, z_0)$

$= 0.2\, f(0, 1, 0)$

$\boxed{k_1 = 0}$

$l_1 = hg(x_0, y_0, z_0)$

$= 0.2g[0, 1, 0)$

$= 0.2[0(0) + 1] \Rightarrow \boxed{l_1 = 0.2}$

Date
Page

$$k_2 = hf\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right]$$

$$= 0.2\, f\left[0 + \frac{0.2}{2}, 1 + 0, 0 + \frac{0.2}{2}\right]$$

$$= 0.2\, f\,[0.1, 1, 0.1]$$

$$= 0.2\,(0.1)$$

$$\boxed{k_2 = 0.02}$$

$$l_2 = hg\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right]$$

$$= (0.2)\, g\left[\frac{0.2}{2}, 1, \frac{0.2}{2}\right]$$

$$= (0.2)\, g\,[0.1, 1, 0.1]$$

$$= (0.2)\left[(0.1)(0.1) + 1\right]$$

$$= (0.2)\,(0.01 + 1)$$

$$\boxed{l_2 = 0.202}$$

$$k_3 = hf\left[x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right]$$

$$= (0.2)\, f\left(\frac{0.2}{2}, 1 + \frac{0.02}{2}, \frac{0.202}{2}\right]$$

$$= (0.2)\, f\,(0.1, 1.01, 0.101)$$

$$= (0.2)\,(0.101)$$

$$\boxed{k_3 = 0.0202}$$

$$l_3 = hg\left[x_0+\frac{h}{2}, y_0+\frac{k_2}{2}, z_0+\frac{l_2}{2}\right]$$

$$=(0.2)g\left[\frac{0.2}{2}, 1+\frac{0.02}{2}, \frac{0.202}{2}\right]$$

$$=(0.2)g[0.1, 1.01, 0.101]$$

$$=(0.2)[(0.1)(0.101)+1.01]$$

$$=(0.2)[1.0201]$$

$$\boxed{l_3 = 0.20402}$$

$$k_4 = hf[x_0+h, y_0+k_3, z_0+l_3]$$

$$=(0.2)f[0.2, 1+0.0202, 0.20402]$$

$$=(0.2)[0.20402]$$

$$\boxed{k_4 = 0.040804}$$

$$l_4 = hg[x_0+h, y_0+k_3, z_0+l_3]$$

$$=(0.2)g[0.2, 0.0202+1, 0.20402]$$

$$=(0.2)g[0.2, 1.0202, 0.20402]$$

$$=(0.2)[(0.2)(0.20402)+1.0202]$$

$$=(0.2)[0.040804+1.0202]$$

$$=(0.2)[1.061004]$$

$$\boxed{l_4 = 0.2122008} \Rightarrow 0.2122008.$$

Sub values in 1 & 2

Date ________
Page ________

$$y(x_0+h) = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= 1 + \frac{1}{6}[0 + 2(0.02) + 2(0.0202) + 0.004804]$$

$$= 1 + \frac{1}{6}[0.04 + 0.0404 + 0.004804]$$

$$= 1 + \cancel{0.085204}\ \frac{0.121204}{6}$$

$$= 1 + 0.0202$$

$$\boxed{y(0.2) = 1.0202}$$

$$y'(x_0+h) = z_0 + \frac{1}{6}[l_1 + 2l_2 + 2l_3 + l_4]$$

$$= 0 + \frac{1}{6}[0.2 + 2(0.202) + 2(0.204) + 0.2122]$$

$$= \frac{1}{6}[0.2 + 0.404 + 0.408 + 0.2122]$$

$$= \frac{1.2242}{6}$$

$$\boxed{y'(0.2) = 0.204033}$$

3. Compute $y(0.1)$, given, $\frac{d^2y}{dx^2} = y^3$ & $y = 10$, $\frac{dy}{dx} = 5$ at $x = 0$ by R-K of 4th order.

Soln:

Put $\frac{dy}{dx} = z$

$$\frac{d^2y}{dx^2} = \frac{dz}{dx}.$$

The given eqn becomes,

$$\frac{dz}{dx} = y^3.$$

we have,

$$\frac{dy}{dx} = z, \quad \frac{dz}{dx} = y^3.$$

$f(x, y, z) = z$, $g(x, y, z) = y^3$.

$\boxed{h = 0.1}$ $x_0 = 0$ $y_0 = 10$ $z_0 = 5$.

$$y(x_0 + h) = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4] \longleftarrow (1)$$

Date
Page

where,

$k_1 = hf(x_0, y_0, z_0)$

$= hf(0, 10, 5)$

$= (0.1)(5)$

$\boxed{k_1 = 0.5}$

$l_1 = hg[x_0, y_0, z_0)$

$= (0.1)g[0, 10, 5)$

$= (0.1)(10)3$

$= 0.1(1000)$

$\boxed{l_1 = 100}$

$k_2 = \cancel{g}\, hf\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right]$

$= (0.1)f\left[0 + \frac{0.1}{2}, 10 + \frac{0.5}{2}, 5 + \frac{100}{2}\right]$

$= (0.1)f[0.05, 10.25, 55)$

$= (0.1)(55)$

$\boxed{k_2 = 5.5}$

$l_2 = hg\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right]$

$= (0.1)g[0.05, 10.25, 55)$

$= (0.1)(10.25)^3$

$$\boxed{l_2 = 107.69}$$

$$k_3 = hf\left[x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right]$$

$$= (0.1)\, f\left[\frac{0.1}{2}, 10 + \frac{5.5}{2}, 5 + \frac{107.69}{2}\right]$$

$$= (0.1)\, f\left[0.05, 12.75, 58.844\right)$$

$$= (0.1)\,(58.844)$$

$$\boxed{k_3 = 5.88445}$$

$$l_3 = hg\left[x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right]$$

$$= (0.1)\, g\left[\frac{0.1}{2}, 10 + \frac{5.5}{2}, 5 + \frac{107.69}{2}\right]$$

$$= (0.1)\, g\left[0.05, 12.75, 58.845\right]$$

$$= (0.1)\,(12.75)^3$$

$$\boxed{l_3 = 207.27}$$

$$k_4 = hf\,(x_0 + h, y_0 + k_3, z_0 + l_3)$$

$$= (0.1)\, f\,(0.1, 10 + 5.88, 5 + 207.27)$$

$$= (0.1)\, f\,[0.1, 15.88, 212.27)$$

$$k_4 = (0.1)\,(212.27) \Rightarrow \boxed{21.227 = k_4}$$

Sub value in (1)

$$y(x_0+h) = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= 10 + \frac{1}{6}[0.5 + 2(5.5) + 2(5.8845) + 21.227]$$

$$= 10 + \frac{1}{6}[0.5 + 11 + 11.769 + 21.227]$$

$$= 10 + \frac{44.496}{6}$$

$$= 10 + 7.416$$

$$y(0.1) = \boxed{17.416}$$

4. By R-K method, solve $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ for $x = 0.2$ correct to 4 decimal places. using the initial conditions $y = 1$ & $y' = 0$ when $x = 0$.

Given,

$$\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2 \quad \& \quad y_0 = 1,\ y_0' = 0,\ x_0 = 1$$

put $\boxed{\frac{dy}{dx} = z}$

22-07-22

Date
Page

diff w.r.t x.

Module I.

1. $\frac{d^2y}{dx^2} = \frac{dz}{dx}$

Now, the given eqn reduces to

$$\boxed{\frac{dz}{dx} = xz^2 - y^2.}$$

we have the system of eqn as,

$\frac{dy}{dx} = z$ & $\frac{dz}{dx} = xz^2 - y^2$

$f(x,y,z) = z$. $g(x,y,z) = xz^2 - y^2$

& $x_0 = 0$, $y_0 = 1$, $z_0 = 0$.

$\therefore \boxed{h = 0.2}$

we need to find

$y(x_0 + h) = y(0.2) = ?$

we have,

$y(x_0+h) = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$ —(1)

where,

$k_1 = hf(x_0, y_0, z_0)$

$= (0.2)\, f(0,1,0)$

$\boxed{k_1 = 0}$

$l_1 = hg(x_0, y_0, z_0)$

$= (0.2)\, g[0,1,0]$

Date
Page

$$l_1 = 0.2\,[0-1]$$

$$\boxed{l_1 = -0.2}$$

$$k_2 = hf\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right]$$

$$= (0.2)\,f\left[\frac{0.2}{2}, 1+0, 0-\frac{0.2}{2}\right]$$

$$= (0.2)\,f\,[0.1, 1, -0.1]$$

$$= (0.2)(-0.1)$$

$$\boxed{k_2 = -0.02}$$

$$l_2 = hg\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right]$$

$$= (0.2)\,g\left[\frac{0.2}{2}, 1, \frac{-0.2}{2}\right]$$

$$= (0.2)\,g\,[0.1, 1, (-0.1)]$$

$$= (0.2)\,[(0.1)(0.1)^2 - 1]$$

$$= (0.2)\,[-0.999]$$

$$\boxed{l_2 = -0.1998}$$

$$k_3 = hf\left[x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right]$$

$$= (0.2)\,f\left[\frac{0.2}{2}, 1 - \frac{0.02}{2}, 0 - \frac{0.1998}{2}\right]$$

$$= (0.2)\,f\,[0.1, 0.99, -0.0999]$$

NOTES

$= (0.2)(-0.0999)$

$\boxed{k_3 = -0.01988}$

$l_3 = hg\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right)$

$= (0.2)\, g\,[0.1, 0.99, -0.0999]$

$= (0.2)\,[(0.1)(-0.0999)^2 - (0.99)^2]$

$= (0.2)\,[-0.9791]$

$\boxed{l_3 = -0.19582}$

$k_4 = hf\,[x_0 + h, y_0 + k_3, z_0 + l_3]$

$= (0.2)\, f\,[0.2, 1 + -0.01988, -0.19582]$

$= (0.2)\,[-0.19582]$

$\boxed{k_4 = -0.03916}$

Sub values in (1)

$y(0.2) = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= 1 + \frac{1}{6}[0 + 2(-0.02) + 2(-0.01988) - 0.03916]$

$= 1 + \frac{-0.118924}{6}$

$y(0.2) = 1 - 0.01982$

$\boxed{y(0.2) = 0.98017}$

21-07-22.

Ajay

4. obtain the value of x & $\frac{dx}{dt}$. when

$t=0.1$, given that x satisfy the equation $\frac{d^2x}{dt^2} = t\frac{dx}{dt} - 4x$ & $x=3$, $\frac{dx}{dt}=0$ when

$t=0$ initially. use 4th order R-K method.

soln:-

$$\frac{d^2x}{dt^2} = t\frac{dx}{dt} - 4x \; ;$$

$x_0=3$, $x_0'=0$, $t_0=0$

put,

$$\frac{dx}{dt} = z$$

diff w.r.t t

$$\frac{d^2x}{dt^2} = \frac{dz}{dt}$$

Now, the given eqn reduces to,

$$\boxed{\frac{dz}{dt} = tz - 4x}$$

Now, we have the system of eqn as,

$$\frac{dx}{dt} = z \;, \quad \frac{dz}{dt} = tz - 4x.$$

Ajay

Date

Page

$f(t, x, z) = z$ & $g(t, x, z) = tz - 4x$.

$t_0 = 0, \quad x_0 = 3, \quad z_0 = 0$

$\therefore \boxed{h = 0.1}$

Now, we need to find

$x(t_0+h) = x(0.1) = ?$ &

$z(t_0+h) = z(0.1) = ?$

$x(t_0+h) = x_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$ — (1)

$z(t_0+h) = z_0 + \frac{1}{6}[l_1 + 2l_2 + 2l_3 + l_4]$ — (2)

where,

$k_1 = hf(t_0, x_0, z_0)$

$= (0.1) f(0, 3, 0)$

$\boxed{k_1 = 0}$

$l_1 = hg(t_0, x_0, z_0)$

$= (0.1) g[0, 3, 0]$

$= (0.1)[0 - 4(3)]$

$= (0.1)[-12]$

$\boxed{l_1 = -1.2}$

$$k_2 = hf\left[t_0 + \frac{h}{2}, x_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right]$$

$$= (0.1)\, f\left[\frac{0+0.1}{2}, 3+(0), -\frac{1.2}{2}\right]$$

$$= (0.1)\, f\left[0.05, \overset{3}{\cancel{0}}, -0.6\right]$$

$$= (0.1)(-0.6)$$

$$\boxed{k_2 = -0.06}$$

$$l_2 = hg\left[t_0 + \frac{h}{2}, x_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right]$$

$$= (0.1)\, g\left[\frac{0.1}{2}, 3+0, -\frac{1.2}{2}\right]$$

$$= (0.1)\, g\left[0.05, \overset{3}{\cancel{0}}, -0.6\right]$$

$$= (0.1)\left[(0.05)(-0.6) - 4(3)\right]$$

$$= (0.1)\left[-12.03\right]$$

$$\boxed{l_2 = -1.203}$$

$$k_3 = hf\left[t_0 + \frac{h}{2}, x_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right]$$

$$= (0.1)\, f\left[\frac{0.1}{2}, 3 - \frac{0.06}{2}, -\frac{1.203}{2}\right]$$

$$= (0.1)\, f\left[0.05, 2.97, -0.6015\right]$$

$$= (0.1)(-0.6015) \Rightarrow \boxed{k_3 = -0.06015}$$

Ajay

$$l_3 = hfg\left[t_0 + \frac{h}{2}, x_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right]$$

$$= (0.1)g\left[\frac{0.1}{2}, 3\frac{-0.06}{2}, \frac{-1.203}{2}\right]$$

$$= (0.1)g\,[0.05, 2.97, -0.6015]$$

$$= (0.1)\left[(0.05)(-0.6015) - 4(2.97)\right]$$

$$= (0.1)(-11.910)$$

$$\boxed{l_3 = -1.191}$$

$$k_4 = hf\,[t_0+h, x_0+k_3, z_0+l_3]$$

$$= (0.1)f\,[0.1, 3-0.06015, -1.191]$$

$$= (0.1)(-1.191)$$

$$\boxed{k_4 = -0.1191}$$

$$l_4 = hg\,[t_0+h, x_0+k_3, z_0+l_3]$$

$$= (0.1)g\,[0.1, 3-0.06015, -1.191]$$

$$= (0.1)g\,[0.1, 2.93985, -1.191]$$

$$= (0.1)\left[(0.1)(-1.191) - 4(2.93985)\right]$$

$$= (0.1)\left[-0.1191 - 11.7594\right.$$

$$= (0.1)(-11.8785)$$

$$\boxed{l_4 = -1.18785}$$

Ajay

Date
Page

sub values in ① & ②

$$x(t_0+h) = x_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= 3 + \frac{1}{6}[0 + 2(-0.06) + 2(-0.06015) - 0.1191]$$

$$= 3 + \frac{1}{6}[-0.3594]$$

$$= 3 - 0.0599$$

$$\boxed{x(0.1) = 2.9401}$$

$$z(t_0+h) = z_0 + \frac{1}{6}[l_1 + 2l_2 + 2l_3 + l_4]$$

$$= 0 + \frac{1}{6}[-1.2 + 2(-1.203) + 2(-1.191) - 1.18785]$$

$$= \frac{-7.17585}{6}$$

$$= -1.1959$$

$$\boxed{z(0.1) = -1.196}$$

22-07-22

Ajay

Date
Page

Milne's prediction & correction formula:-

$$y_4^{(p)} = y_0 + \frac{4h}{3}[2y_1' - y_2' + 2y_3']$$

$$z_4^{(p)} = z_0 + \frac{4h}{3}[2z_1' - z_2' + 2z_3']$$

$$y_4^{(c)} = y_2 + \frac{h}{3}[z_2 + 4z_3 + z_4]$$

$$z_4^{(c)} = z_2 + \frac{h}{3}[z_2' + 4z_3' + z_4'].$$

1. Apply Milne's method to solve, $\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}$. Given, the following table of initial values.

x	0	0.1	0.2	0.3
y	1	1.1103	1.2427	1.399
y'	1	1.2103	1.4427	1.699

compute $y(0.4)$ numerically.

Ajay

soln:

Given that,

$$\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}.$$

Put, $\frac{dy}{dx} = z$.

diff w.r.t x.

$$\frac{d^2y}{dx^2} = \frac{dz}{dx}$$

Now,

the given eqn reduces to,

$$\frac{dz}{dx} = 1+z$$

$$\boxed{z' = 1+z}$$

	x	y	$y' = z$	$z' = 1+z$
x_0:	0	1	1	$z_0' = 1 + z_0 = 1+1 = 2$
x_1:	0.1	1.1103	1.2103	$z_1' = 1 + 1.2103 = 2.2103$
x_2:	0.2	1.2427	1.4427	$z_2' = 1 + 1.4427 = 2.4427$
x_3:	0.3	1.399	1.699	$z_3' = 1 + 1.699 = 2.699$
x_4:	0.4	?	?	?

Ajay

we have milne's Prediction formula,

$y_4^{(P)} = y_0 + \frac{4h}{3}[2z_1 - z_2 + 2z_3]$

$= 1 + \frac{4(0.1)}{3}[2(1.2103) - 1.4497 + 2(1.699)]$

$= 1 + \frac{0.4}{3}[4.3759]$

$= 1 + \frac{1.75036}{3}$

$= 1 + 0.58345$

$\boxed{y_4^{(P)} = 1.5835}$

llly,

$z_4^{(P)} = z_0 + \frac{4h}{3}[2z_1' - z_2' + 2z_3']$

$= 1 + \frac{4(0.1)}{3}[2(2.2103) - 2.4497 + 2(2.699)]$

$= 1 + \frac{0.4}{3}(7.3759)$

$= 1 + 0.98345$

$\boxed{z_4^{(P)} = 1.9835}$

Ajay

Now, we have milne's corrector formula,

$$y_4^{(c)} = y_2 + \frac{h}{3}\left[z_2 + 4z_3 + z_4\right]$$

$$= 1.2427 + \frac{0.1}{3}\left[1.4427 + 4(1.699) + 1.9835\right]$$

$$= 1.2427 + \frac{1.02222}{3}$$

$$= 1.2427 + 0.34074$$

$$\boxed{y_4^{(c)} = 1.58344}$$

$$z_4^{(c)} = z_2 + \frac{h}{3}\left[z_2' + 4z_3' + z_4'\right]$$

$= 1.4427$

Next, $z_4' = 1 + z_4 \Rightarrow 1 + 1.9835$

$z_4' \Rightarrow 2.9835$

$$z_4^{(c)} = 1.4427 + \frac{0.1}{3}\left[2.4427 + 4(2.699) + 2.9835\right]$$

$$= 1.4427 + \frac{1.6222}{3}$$

$$= 1.4427 + 0.54074$$

$$\boxed{z_4^{(c)} = 1.98344}$$

Ajay

Date ____
Page ____

Applying the corrector formula again, we get,

$$y_4^{(c)} = y_2 + \frac{h}{3}\left[z_2 + 4z_3 + z_4^{(c)}\right]$$

$$= 1.2427 + \frac{0.1}{3}\left[1.4427 + 4(1.699) + 1.98344\right]$$

$$= 1.2427 + \frac{1.022214}{3}$$

$$= 1.2427 + 0.340738$$

$$\boxed{y_4^{(c)} = 1.58344}$$

$$\therefore y_4 = y(0.1) = 1.58344$$

2. Given the ODE $y'' + xy' + y = 0$ & the following table of initial values, compute $y(0.4)$ by applying Milne's method.

x	0	0.1	0.2	0.3
y	1	0.995	0.9801	0.956
y'	0	-0.0995	-0.196	-0.2867

Soln

Given, $y'' + xy' + y = 0$.

Put $\frac{dy}{dx} = z$

Ajay

diff w.r.t x

$$\frac{d^2y}{dx^2} = z'$$

Now,

the given eqn reduces to,

$z' + xz + y = 0$

$z' = -(xz+y)$

	x	y	$y' = z$	$z' = -(xz+y)$
x_0	0	1	$z_0 = 0$	$z_0' = -1$
x_1	0.1	0.995	$z_1 = -0.0995$	$z_1' = 0.98005$
x_2	0.2	0.9801	$z_2 = -0.196$	$z_2' = \cancel{-1.01953}$ -0.9400
x_3	0.3	0.956	$z_3 = -0.2867$	$z_3' = -0.86999$
x_4	0.4	?		

we have Milne's Prediction formula,

$$y_4^{(p)} = y_0 + \frac{4h}{3}[2z_1 - z_2 + 2z_3]$$

$$= 1 + \frac{4(0.1)}{3}[2(-0.0995) + 0.196 + 2(-0.2867)]$$

$$= 1 + \frac{0.4}{3}(-0.5769)$$

$$= 1 - 0.07685 \Rightarrow \boxed{y_4^{(p)} = 0.9231}$$

Ajay

Date ____
Page ____

IIIly,

$$z_4^{(P)} = z_0 + \frac{4h}{3}\left[2z_1' - z_2' + 2z_3'\right]$$

$$= 0 + \frac{4(0.1)}{3}\left[2(-0.98505) + 0.9409 + 2(-0.86999)\right]$$

$$= \frac{0.4}{3}\left[-2.76918\right]$$

$$\boxed{z_4^{(P)} = -0.3692}$$

Now, we have milne's correction formula,

$$y_4^{(C)} = y_2 + \frac{h}{3}\left[z_2 + 4z_3 + z_4\right]$$

$$= 0.9801 + \frac{0.1}{3}\left[-0.196 + 4(-0.2867) + (-0.3692)\right]$$

$$= 0.9801 + \frac{0.1}{3}\left[-1.712\right]$$

$$= 0.9801 - 0.057066$$

$$\boxed{y_4^{(C)} = 0.9230}$$

$$z_4^{(C)} = y_2 + \frac{h}{3}\left[z_2' + 4z_3' + z_4'\right]$$

$$z_4' = -(xz + y) = -\left[(0.4)(-0.3692) + 0.923\right]$$

$z_4' = -0.77532$.

$y_4(c) = y_2 + \frac{h}{3}[2z_2' + 4z_3' + z_4']$

$= 0.9801 + \frac{0.1}{3}[2(-0.9409) + 4(-0.8699) - 0.77532]$

$= \cancel{0.9801} + \frac{0.1}{3}(\cancel{-6.13672})$

-0.196

$= -0.196$

$z_4' = -0.77532$.

$z_4^{(c)} = z_2 + \frac{h}{3}[z_2' + 4z_3' + z_4']$

$= -0.196 + \frac{0.1}{3}[-0.9409 + 4(-0.8699) - 0.77532)]$

$= -0.196 + \frac{0.1}{3}[-5.19582]$

$= -0.196 - 0.173194$

$\boxed{z_4^{(1)} = -0.3692}$

$\therefore y(0.4) = 0.9230$

26-07-22.

Ajay

Date
Page

3. Apply Milne's method to compute $y(0.4)$. Given the equation, $y'' + y' = 2e^x$. & the following table of initial values.

x	0	0.1	0.2	0.3
y	2	2.01	2.04	2.09
y'	0	0.2	0.4	0.6

Given, $y'' + y' = 2e^x$.

put $y' = z$

diff w.r.t x

$\boxed{y'' = z'}$.

Now, the given eqn reduces to

$z' + z = 2e^x$.

$z' = 2e^x - z$

x	y	$y' = z$	$z' = 2e^x - z$
0	2	$y_0' = 0$	$z_0' = 2$
0.1	2.01	$y_1' = 0.2$	$z_1' = 2.0103$
0.2	2.04	$y_2' = 0.4$	$z_2' = 2.043$
0.3	2.09	$y_3' = 0.6$	$z_3' = 2.09971$
0.4			

Ajay

Date
Page

we have milne's predictor formula,

$$y_4^{(p)} = y_0 + \frac{4h}{3}[2z_1 - z_2 + 2z_3]$$

$$= 2 + \frac{4(0.1)}{3}[2(0.2) - 0.4 + 2(0.6)]$$

$$= 2 + \frac{0.4}{3}[1.2]$$

$$\boxed{y_4^{(p)} = 2.16}$$

llly,

$$z_4^{(p)} = z_0 + \frac{4h}{3}[2z_1' - z_2' + 2z_3']$$

$$= 0 + \frac{4(0.1)}{3}[2(2.0103) - 2.043 + 2(2.09971)]$$

$$= \frac{0.4}{3}[6.17702]$$

$$\boxed{z_4^{(p)} = 0.8236}$$

now, milne's corrector formula,

$$y_4^{(c)} = y_2 + \frac{h}{3}[z_2 + 4z_3 + z_4]$$

$$= 2.04 + \frac{0.1}{3}[0.4 + 4(0.6) + \cancel{2.16}\ 0.8236]$$

$$= 2.04 + \frac{0.12}{3}[14.46]$$

Ajay

$$= 2.04 + 0.1957$$

$$\boxed{y_4^{(c)} = 2.16}$$

$$z_4^{(c)} = \cancel{y_2}^{z_2} + \frac{h}{3}\left[z_2' + 4z_3' + z_4'\right]$$

$$z_4' = 2e^x - z$$

$$= 2e^{0.4} - 0.8936$$

$$\boxed{z_4' = 2.16}$$

$$= 0.4 + \frac{0.1}{3}\left[2.043 + 4(2.09971) + 2.16\right]$$

$$= 0.4 + 0.42006$$

$$\boxed{z_4^{(c)} = 0.82}$$

$$\boxed{\therefore y(0.4) = 2.16}$$

27-07-99

Numerical solution of Heat equation

working procedure:

i) Based on the given steps sizes h & k we form the rectangular network of sides h & k.

ii) If only h is given, we can obtain k by using $h^2/2c^2$.

The points of division of x are, $x_0, x_1, x_2, x_3, x_4 \dots x_n$ & t are, $t_0, t_1, t_2, t_3, \dots t_n$

iii) we form the basic table as follows.

$t \backslash x$		x_0	x_1	x_2	x_3	---	x_n
t		0	1	2	3	---	n
t_0	0	$u_{0,0}$	$u_{1,0}$	$u_{2,0}$	$u_{3,0}$		$u_{n,0}$
t_1	1	$u_{0,1}$	$u_{1,1}$	$u_{2,1}$	u_{3,u_1}		$u_{n,1}$
t_2	2	$u_{0,2}$	$u_{1,2}$	u_{2,u_1}	u_{3,u_2}		$u_{n,2}$
⋮	⋮	⋮	⋮	⋮	⋮		⋮
t_n	n	$u_{0,n}$	$u_{1,n}$	u_{2,u_n}	u_{3,u_n}		$u_{n,n}$

Ajay

Date
Page

4) we write values of $u_{i,j}$ with reference to the given boundary conditions $u(0,t)=0$ & $u(4,t)=0$ (i.e, values along first & last coloumn are zero)

v) The values along the first row are obtained by using the initial condition $u(x,0)=f(x)$.

vi) The values along second row onwards are obtained by using Bendre-schmidth formula.

$$u_{i,j+1} = \frac{1}{2}[u_{i-1,j} + u_{i+1,j}]$$

① Find the numerical solution of Parabolic eqn (heat equation), $\frac{\partial^2 u}{\partial x^2} = 2\frac{\partial u}{\partial t}$.

when $u(0,t)=0=u(4,t)$ & $u(x,0)=x(4-x)$ by taking $h=1$. find the values upto $t=5$.

Soln: Consider the given eqn,

$$\frac{\partial u}{\partial t} = \frac{1}{2}\frac{\partial^2 u}{\partial x^2}$$

standard heat eqn $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$.

$c^2 = \frac{1}{2}$

$\therefore k = \frac{h^2}{2c^2} = \frac{1}{2 \times \frac{1}{2}} = 1$

$\boxed{k=1}$, $\boxed{h=1}$

The points of division for x with $h=1$ are: 0, 1, 2, 3, 4.

The points of division for t upto $t=5$ with $k=1$ are,

t: 0, 1, 2, 3, 4, 5

Consider the following basic table

	t\x	0	1	2	3	4	
t_0	0	$u_{0,0}$	$u_{1,0}$	$u_{2,0}$	$u_{3,0}$	$u_{4,0} = 0$	$\cancel{=0}$
t_1	1	$u_{0,1}$	$u_{1,1}$	$u_{2,1}$	$u_{3,1}$	$u_{4,1} = 0$	$\cancel{=0}$
t_2	2	$u_{0,2}$	$u_{2,2}$	$u_{2,2}$	$u_{3,3}$	$u_{4,2} = 0$	$\cancel{=0}$
t_3	3	$u_{0,3}$	$u_{1,3}$	$u_{2,3}$	$u_{3,3}$	$u_{4,3} = 0$	
t_4	4	$u_{0,4}$	$u_{1,4}$	$u_{2,4}$	$u_{3,4}$	$u_{4,4} = 0$	
t_5	5	$u_{0,5}$	$u_{1,5}$	$u_{2,5}$	$u_{3,5}$	$u_{4,5} = 0$.	

we have from the boundary condition,
$u(0,t)=0$ & $u(4,t)=0$.

we have,
$u(x,0) = x(4-x)$
$u_{1,0} = u(1,0) = 1[4-1] = 3$
$u_{2,0} = U(2,0) = 2(4-2) = 2(2) = 4$
$u_{3,0} = U(3,0) = 3(4-3) = 3(1) = 3$
(First row completed)

consider,
$u_{i,j+1} = \frac{1}{2}[u_{i-1,j} + u_{i+1,j}]$

$i=1, j=0$ $u_{1,1} = \frac{1}{2}[u_{0,0} + u_{2,0}]$

$u_{1,1} = \frac{1}{2}[0+4]$

$u_{1,1} = 2$

$i=2, j=0$ $u_{2,1} = \frac{1}{2}[u_{1,0} + u_{3,0}]$

$u_{2,1} = \frac{1}{2}[3+3] = \frac{6}{2} = 3.$

Ajay

Date

Page

$i=3, j=0$ $u_{3,1} = \frac{1}{2}[u_{2,0} + u_{4,0}]$

$= \frac{1}{2}[4+0]$

$u_{3,1} = 2$

(Second row completed)

consider,

$i=1, j=1$ $= \frac{1}{2}[u_{0,1} + u_{2,1}]$

$(u_{1,2}) = \frac{1}{2}[0+3]$

$(u_{1,2}) = 1.5$

$i=2, j=1$, $u(2),2 = \frac{1}{2}[u_{1,1} + u_{3,1}]$

$= \frac{1}{2}[2+2]$

$u_{2,2} = \cancel{1.5}\,2$

$(u_{3,2}) = \frac{1}{2}[u_{2,1} + u_{4,1}]$

$= 1/2[3+0] = 1.5$

Ajay

Date ______
Page ______

$i=1, \; j=2 \quad u_{i,j+1} = \frac{1}{2}\left[u_{i-1,j} + u_{i+1,j}\right]$

$u_{1,3} = \frac{1}{2}\left[u_{0,2} + u_{2,2}\right]$

$= \frac{1}{2}\left[0+2\right]$

$u_{1,3} = 1$

$i=2, \; j=2 \quad u_{2,3} = \frac{1}{2}\left[u_{1,2} + u_{3,2}\right]$

$= \frac{1}{2}\left[1.5+1.5\right]$

$= \frac{1}{\cancel{2}} \cancel{2}(1.5)$

$u_{2,3} = 1.5$

$i=3, \; j=2 \quad u_{3,3} = \frac{1}{2}\left[u_{2,2} + u_{4,2}\right]$

$= \frac{1}{2}\left[2+0\right]$

$u_{3,3} = 1$

(fourth row completed)

$i=1$, $j=3$ $\quad u_{1,4} = \frac{1}{2}[u_{0,3} + u_{2,3}]$

$$= \frac{1}{2}[0 + 1.5]$$

$$= 0.75$$

$i=2$, $j=3$ $\quad u_{2,4} = \frac{1}{2}[u_{1,3} + u_{3,3}]$

$$= \frac{1}{2}[1+1]$$

$$u_{2,4} = \cancel{0}\,1$$

$i=3$, $j=3$ $\quad u_{3,4} = \frac{1}{2}[u_{2,3} + u_{4,3}]$

$$= \frac{1}{2}[1.5 + 0]$$

$$= \frac{1}{2}[1.5]$$

$$= 0.75$$

(fifth row completed)

$i=1$, $j=4$ $\quad u_{1,5} = \frac{1}{2}[u_{0,4} + u_{2,4}]$

$$= \frac{1}{2} [0+1]$$

$$= 0.5$$

$i=2,\ j=4$ $\quad u_{2,5} = \frac{1}{2} [u_{1,4} + u_{3,4}]$

$$= \frac{1}{2} [0.75+0.75]$$

$$= \frac{1}{2} [0.5]$$

$$= 0.75$$

$i=3,\ j=4$ $\quad u_{3,5} = \frac{1}{2} [u_{2,4} + u_{4,4}]$

$$= \frac{1}{2} [1+0]$$

$$= \frac{1}{2} (1)$$

$$u_{3,5} = 0.5$$

28-07-22

Ajay

9. Solve $u_{xx} = 32\, u_t$ subject to the condns $u(0,t) = 0$; $u(1,t) = t$ & $u(x,0) = 0$. find the values of u upto $t = 5$. By Schmidt Process by taking $h = 1/4$ Also extract the following values (a) $u(0.75, 4)$ (b) $u(0.5, 5)$ (c) $(0.25, 4)$

Soln:

comparing the given eqn $u_t = \frac{1}{32} u_{xx}$.

with heat eqn $u_t = c^2 u_{xx}$, we get

$$c^2 = \frac{1}{32}$$

$$k = \frac{h^2}{2c^2} = \frac{(1/4)^2}{2 \times \frac{1}{32}} = \frac{16}{16} = 1$$

$$\boxed{k = 1}$$

The points of division for x with $h = 0.25$ are $x: 0, 0.25, 0.5, 0.75, 1$.

The points of division for t with $k = 1$ upto $t = 5$ are,

$t: 0, 1, 2, 3, 4, 5$.

Ajay

Date ______
Page ______

let us constract the basic table,

x		x_0	x_1	x_2	x_3	x_4
t		0	0.25	0.5	0.75	1
t_0	0	$u_{0,0}$	$u_{1,0}$	$u_{2,0}$	$u_{3,0}$	$u_{4,0}$
t_1	1	$u_{0,1}$	$u_{1,1}$	$u_{2,1}$	$u_{3,1}$	$u_{4,1}$
t_2	2	$u_{0,2}$	$u_{1,2}$	$u_{2,2}$	$u_{3,2}$	$u_{4,2}$
t_3	3	$u_{0,3}$	$u_{1,3}$	$u_{2,3}$	$u_{3,3}$	$u_{4,3}$
t_4	4	$u_{0,4}$	$u_{1,4}$	$u_{2,4}$	$u_{3,4}$	$u_{4,4}$
t_5	5	$u_{0,5}$	$u_{1,5}$	$u_{2,5}$	$u_{3,5}$	$u_{4,5}$

consider the given boundary condn's.

$u(0,t)=0$

The first column values will be 0.

lly, we have $u(1,t)=t$.

we get last coloumn values

we have,

$u(x,0)=0$, the first values will be zero.

we have,

$$u_{i,j+1} = \frac{1}{2}[u_{i-1,j} + u_{i+1,j}]$$

$i=1, \; j=0 \quad u_{1,1} = \frac{1}{2}[u_{0,0} + u_{2,0}]$

$= \frac{1}{2}[0+0]$

$= 0$

$i=2 \quad j=0 \quad u_{2,1} = \frac{1}{2}[u_{1,0} + u_{3,0}]$

$= \frac{1}{2}[0+0]$

$= 0$

$i=3, \; j=0 \quad u_{3,1} = \frac{1}{2}[u_{2,0} + u_{4,0}]$

$= \frac{1}{2}[0+0] = 0$

(second row completed)
put in eqn (1)

$i=1, \; j=1 \quad u_{1,2} = \frac{1}{2}[u_{0,1} + u_{2,1}]$

$= \frac{1}{2}[0+0] = 0.$

$i=2,\ j=1 \quad u_{2,2} = \frac{1}{2}[u_{1,1} + u_{3,1}]$

$= \frac{1}{2}[0+0] = 0.$

$i=3,\ j=1 \quad u_{3,2} = \frac{1}{2}[u_{2,1} + u_{4,1}]$

$= \frac{1}{2}[0+1] = 0.5$

(Third row completed)

$i=1,\ j=2 \quad u_{1,3} = \frac{1}{2}[u_{0,2} + u_{2,2}]$

$= \frac{1}{2}[0+0] = 0$

$i=2,\ j=2 \quad u_{2,3} = \frac{1}{2}[u_{0,3} + u_{3,2}]$

$= \frac{1}{2}[0+0.5] = 0.25$

$i=3,\ j=2 \quad u_{3,3} = \frac{1}{2}[u_{0,4} + u_{4,2}]$

Ajay

$= \frac{1}{2} [0+2]$

$= 1.$

(fourth row completed)

$i=1 \quad j=3 \quad u_{1,4} = \frac{1}{2} [u_{0,3} + u_{2,3}]$

$= \frac{1}{2} [0+0.25] = 0.125$

$i=2 \quad j=3 \quad u_{2,4} = \frac{1}{2} [u_{1,3} + u_{3,3}]$

$= 1/2 [0+1] = 0.5.$

$i=3 \quad j=3 \quad u_{3,4} = \frac{1}{2} [u_{2,3} + u_{4,3}]$

$= \frac{1}{2} [0.25+3]$

$= 1.625$

(fifth row completed)

$i=1 \quad j=4 \quad u_{1,5} = \frac{1}{2} [u_{0,4} + u_{2,4}]$

Ajay

$= 1/2\ [0 + 0.5]$

$= 0.25.$

$i = 2 \quad j = 4 \quad u_{2,5} = \frac{1}{2}\ [u_{1,4} + u_{3,4}]$

$= \frac{1}{2}\ [0.125 + 1.625]$

$= 0.875$

$i = 3 \quad j = 4 \quad u_{3,5} = \frac{1}{2}\ [u_{2,4} + u_{4,4}]$

$= 1/2\ [0.5 + 4] = 2.25.$

The required values of $u_{i,j}$ are tabulated as follows.

x		x_0	x_1	x_2	x_3	x_4
t		0	0.25	0.5	0.75	1
t_0	0	0	0	0	0	0
t_1	1	0	0	0	0	1
t_2	2	0	0	0	0.5	2
t_3	3	0	0	0.25	1	2
t_4	4	0	0.125	0.5	1.625	4
t_5	5	0	0.25	0.875	2.25	5

Ajay

Also we have

a. $u(0.75, 4) = 1.625$

b. $u(0.5, 5) = 0.875$

c. $u(0.25, 4) = 0.125$

3. Solve: $u_t = u_{xx}$ subject to the conditions $u(0,t) = 0$, $u(1,t) = 0$, $u(x,0) = \sin(\pi x)$ for $0 \le t \le 0.1$ by taking $h = 0.2$. Also write down the following values from the table.

a. $u(0.2, 0.04)$ b. $u(0.40, 0.08)$

c. $u(0.6, 0.06)$

comparing the given eqn $u_t = c^2 u_{xx}$ we get $c^2 = 1$

$$k = \frac{h^2}{2c^2} = \frac{(0.2)^2}{2} = \frac{0.04}{2} = 0.02.$$

$k = 0.02$.

The points of x with $h = 0.2$ are

$x : 0, 0.2, 0.4, 0.6$

The points of division for t, with $k = 0.02$ upto $t = 5$ are

Ajay

t: 0, 0.02, 0.04, 0.06, 0.08

let us construct the basic table

x		x_0	x_1	x_2	x_3	x_4	x_5
t							
t_0	0	$u_{0,0}$	$u_{1,0}$	$u_{2,0}$	$u_{3,0}$	$u_{4,0}$	$u_{5,0}$
t_1	0.02	$u_{0,1}$	$u_{1,1}$	$u_{2,1}$	$u_{3,1}$	$u_{4,1}$	$u_{5,1}$
t_2	0.04	$u_{0,2}$	$u_{1,2}$	$u_{2,2}$	$u_{3,2}$	$u_{4,2}$	$u_{5,2}$
t_3	0.06	$u_{0,3}$	$u_{1,3}$	$u_{2,3}$	$u_{3,3}$	$u_{4,3}$	$u_{5,3}$
t_4	0.08	$u_{0,4}$	$u_{1,4}$	$u_{2,4}$	$u_{3,4}$	$u_{4,4}$	$u_{5,4}$
t_5	0.1	$u_{0,5}$	$u_{1,5}$	$u_{2,5}$	$u_{3,5}$	$u_{4,5}$	$u_{5,5}$

consider the given boundary condition $u(0,t)=0$ the first coloumn values will be zero.

Hy, we have $u(1,t)=0$
we get largest column values
we have,

$u(x,0) = \sin(\pi x)$

$u(1,0) = \sin(180 \times 0.2) = 0.59$

$u(2,0) = \sin(180 \times 0.4) = 0.95$

$u(3,0) = \sin(180 \times 0.6) = 0.95$

Ajay

$u(4,0) = \sin(180 \times 0.8) = 0.59$

(first row completed)

put $i=1$, $j=0$

$$u_{i,j+1} = \frac{1}{2}[u_{i-1,j} + u_{i+1}]$$

$$u_{1,1} = \frac{1}{2}[u_{0,0} + u_{1,0}]$$

$$= \frac{1}{2}[0 + 0.59]$$

$$= 0.295$$

$i=2$, $u_{2,1} = \frac{1}{2}[u_{1,0} + u_{3,0}]$

$$= \frac{1}{2}[0 + 0.95]$$

$$= 0.475$$

$i=3$, $u_{3,1} = \frac{1}{2}[u_{2,0} + u_{4,0}]$

$$= \frac{0.59}{2}$$

$= 0.295$ (2^{nd} row completed)

Ajay

put $i=1$ $j=1$ $u_{1,2} = \frac{1}{2}[u_{0,1} + u_{2,1}]$

$= \frac{1}{2}[0+0.475]$

$= 0.2375$

$i=2$ $j=1$ $u_{2,2} = \frac{1}{2}[u_{1,1} + u_{3,1}]$

$= \frac{1}{2}[0.295+0.295]$

$= 0.295$

$i=3$ $j=1$ $u_{3,2} = \frac{1}{2}[u_{2,1} + u_{4,1}]$

$= \frac{1}{2}[$

29-07-22

Ajay

Date
Page

Numerical solution of 1-D wave equation.

Working Procedure

i) Based on the given sub size h & k we form the rectangular network of size h & k.

ii) If only h is given and find the value of k using $k = \frac{h}{c}$ then, obtain the points of division for x & t.

iii) Form the basic information table.

iv) We write the values of $u_{i,j}$ with reference to the conditions $u(0,t)=0$ and $u(l,t)=0$.

v) The values of first row are obtained by using the condition $u(x,0) = f(x)$

vi) The values along second row are obtained by using the relation

$$u_{i,1} = \frac{1}{2}[u_{i-1,0} + u_{i+1,0}]$$

vii) Rest of the values are obtained using the formula,

$$u_{i,j+1} = u_{i-1,j} + u_{i+1,j} - u_{i,j-1}$$

Ajay

Date

Page

1. solve the wave equation $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$

subject to $u(0,t)=0$, $u(4,t)=0$, $u_t(x,0)=0$ & $u(x,0)=x(4-x)$ by taking $h=1$, $k=0.5$ upto 4 steps.

soln:-

The points of divisions for x with $h=1$ are, $x: 0, 1, 2, 3, 4$

The points of division for t with $k=0.5$ upto 4 steps are, $t: 0, 0.5, 1, 1.5, 2$

let us consider the following table.

x		x_0	x_1	x_2	x_3	x_4
t						
t_0	0	$u_{0,0}$	$u_{1,0}$	$u_{2,0}$	$u_{3,0}$	$u_{4,0}$
t_1	0.5	$u_{0,1}$	$u_{1,1}$	$u_{2,1}$	$u_{3,1}$	$u_{4,1}$
t_2	1	$u_{0,2}$	$u_{1,2}$	$u_{2,2}$	$u_{3,2}$	$u_{4,2}$
t_3	1.5	$u_{0,3}$	$u_{1,3}$	$u_{2,3}$	$u_{3,3}$	$u_{4,3}$
t_4	2	$u_{0,4}$	$u_{1,4}$	$u_{2,4}$	$u_{3,4}$	$u_{4,4}$

By given Boundary conditions $u(0,t)=0$ & $u(4,t)=0$.

Ajay

we get first and last column as zero.

First row values are obtained by using,

$$u(x,0) = x(4-x)$$

$$u_{1,0} = u(1,0) = 1(4-1) = 3$$

$$u_{2,0} = u(2,0) = 2(4-2) = 4$$

$$u_{3,0} = u(3,0) = 3(4-3) = 3$$

Second row values are obtained by using

$$u_{i,1} = \frac{1}{2}[u_{i-1,0} + u_{i+1,0}]$$

$$i=1 \quad u_{1,1} = \frac{1}{2}[u_{0,0} + u_{2,0}]$$

$$= \frac{1}{2}[0+4]$$

$$\boxed{u_{i,1} = 2}$$

$$i=2 \quad u_{2,1} = \frac{1}{2}[u_{1,0} + u_{3,0}]$$

$$= \frac{1}{2}[3+3] = 3$$

Ajay

Date
Page

$i=3$ $u_{3,1} = \frac{1}{2}[u_{2,0} + u_{4,0}]$

$= \frac{1}{2}[4+0] = 2$

(second row completed)

Rest of the values are obtained by using the explicit formula,

$u_{i,j+1} = u_{i-1,j} + u_{i+1,j} - u_{i,j-1}$ —— (1)

$j=1 \Rightarrow u_{i,2} = u_{i-1,1} + u_{i+1,1} - u_{i,0}$

put $i=1 \Rightarrow u_{1,2} = u_{0,1} + u_{2,1} - u_{1,0}$

$= 0+3-3$

$=0$

$i=2$ $u_{2,2} = u_{1,1} + u_{3,1} - u_{2,0}$

$= 2+2-4 = 0$

$i=3$ $u_{3,2} = u_{2,1} + u_{4,1} - u_{3,0}$

$= 3+0-3 = 0$

(Third row completed)

Ajay

$j = 2$

put $i = 1$ $\quad u_{1,3} = u_{0,2} + u_{2,2} - u_{1,0}$

$= 0 + 2 - 2 = 0 - 2$

$i = 2$ $\quad u_{2,3} = u_{1,2} + u_{3,2} - u_{2,0}$

$= 2 + 2 - 3 = 0 - 3$

$i = 3$ $\quad u_{3,3} = u_{2,2} + u_{4,2} - u_{3,0}$

$= 0 + 2 - 2 = 0 - 2$

$j = 3$ $\quad u_{i,4} = u_{i-1,3} + u_{i+1,3} - u_{i,2}$

put $i = 1$ $\quad u_{1,4} = u_{0,3} + u_{2,3} - u_{1,2}$

$= 0 - 3 - 0 = -3$

$i = 2$ $\quad u_{2,4} = u_{1,4} + u_{3,3} - u_{2,2}$

$= -2 - 2 - 0 = -4$

$i = 3$ $\quad u_{3,4} = u_{2,3} + u_{4,3} - u_{3,2}$

$= -3 + 0 - 3$

$= -3$

(last row completed)

Ajay

Date ______
Page ______

The values of $u_{i,j}$ are tabulated as follows.

x		x_0	x_1	x_2	x_3	x_4
t		0	1	2	3	4
t_0	0	0	3	4	3	0
t_1	0.5	0	2	3	2	0
t_2	1	0	0	0	0	0
t_3	1.5	0	-2	-3	-2	0
t_4	2	0	-3	-4	-3	0

03-08-22

2. Solve numerically $u_{xx} = 0.0625\, u_{tt}$ subject to the conditions $u(0,t)=0$, $u(5,t)=0$; $u(x,0) = x^2(x-5)$ and $u_t(x,0) = 0$ by taking $h=1$ for $0 \le t \le 1$

→ comparing the given eqn

$$\frac{1}{0.0625} u_{xx} = u_{tt} \text{ with}$$

Standard wave of eqn $u_{tt} = c^2 u_{xx}$

$$c^2 = \frac{1}{0.0625} = 16$$

Ajay

$c^2 = 16$

$\boxed{c = 4}$

$\therefore k = \frac{h}{c} = \frac{1}{4} = 0.25$

The points of division for x with $h=1$ are, $x = 0, 1, 2, 3, 4, 5$

The points of division for t with $k = 0.25$ are, $0, 0.25, 0.5, 0.75, 1$

Consider the following basic table.

$t \backslash x$		x_0	x_1	x_2	x_3	x_4	x_5
t_0	0	$u_{0,0}$	$u_{1,0}$	$u_{2,0}$	$u_{3,0}$	$u_{4,0}$	$u_{5,0}$
t_1	0.25	$u_{0,1}$	$u_{1,1}$	$u_{2,1}$	$u_{3,1}$	$u_{4,1}$	$u_{5,1}$
t_2	0.5	$u_{0,2}$	$u_{1,2}$	$u_{2,2}$	$u_{3,2}$	$u_{4,2}$	$u_{5,2}$
t_3	0.75	$u_{0,3}$	$u_{1,3}$	$u_{2,3}$	$u_{3,3}$	$u_{4,3}$	$u_{5,3}$
t_4	1	$u_{0,4}$	$u_{1,4}$	$u_{2,4}$	$u_{3,4}$	$u_{4,4}$	$u_{5,4}$

The given conditions $u(0,t) = 0 = u(5,t)$ first & last column values becomes zero.

Ajay

Date ______
Page ______

next, consider the condition :

$u(x,0) = x^2(x-5)$

$u_{1,0} = u(x_1, t_0) = u(1,0) = (1)^2 \cdot (1-5) = -4$

$u_{2,0} = (2)^2 \cdot (2-5) = -12$

$u_{3,0} = (3)^2 (3-5) = -18$

$u_{4,0} = (4)^2 (4-5) = -16$

(first row completed)

next consider,

$u_{i,1} = \frac{1}{2} [u_{i-1,0} + u_{i+1,0}]$

put $i = 1$

$u_{1,1} = \frac{1}{2} [u_{0,0} + u_{2,0}]$

$= \frac{1}{2} [0 + (-12)]$

$= \frac{-12}{2}$

$u_{1,1} = -6$

$i = 2$

$u_{2,1} = \frac{1}{2} [u_{1,0} + u_{3,0}]$

Ajay

$$= \frac{1}{2}[-9-18]$$

$u_{2,1} = 11 (-11)$

$i = 3$

$$u_{3,1} = \frac{1}{2}[u_{2,0} + u_{4,0}]$$

$$= \frac{1}{2}[-12-16]$$

$$= -14$$

$i = 4$

$$u_{4,1} = \frac{1}{2}[u_{3,0} + u_{5,0}]$$

$$= \frac{1}{2}[-18+0]$$

$u_{4,1} = -9$

(second row completed)

Explicit formula,

$$u_{i,j+1} = u_{i-1,j} + u_{i+1,j} - u_{i,j-1} \quad \text{--- (1)}$$

Put $j=1$ in (1)

$u_{i,2} = u_{i-1,1} + u_{i+1,1} - u_{i,0}$

$i=1 \quad u_{1,2} = u_{0,1} + u_{2,1} - u_{1,0}$

$= 0 - 11 + 4$

$= -7$

$i=2 \quad u_{2,2} = u_{1,1} + u_{3,1} - u_{2,0}$

$= -6 - 14 + 12$

$= -8$

$i=3 \quad u_{3,2} = u_{2,1} + u_{4,1} - u_{3,0}$

$= -11 - 9 + 18$

$= -2$

$i=4 \quad u_{4,2} = u_{3,1} + u_{5,1} - u_{4,0}$

$= -14 + 0 + 16$

$= 2$

(Third row completed)

$j=2 \Rightarrow u_{i,3} = u_{i-1,2} + u_{i+1,2} - u_{i,1} \quad — (2)$

$i=1 \quad u_{1,3} = u_{0,2} + u_{2,2} - u_{1,1}$

$= 0 - 8 + 6$

$= -2$

Ajay

Date
Page

$i=2 \quad u_{2,3} = u_{1,2} + u_{3,2} - u_{2,1}$

$= -7 - 2 + 11$

$= 2$

$i=3 \quad u_{3,3} = u_{2,2} + u_{4,2} - u_{3,1}$

$= -8 + 2 + 14$

$= 8$

$i=4 \quad u_{4,3} = u_{3,2} + u_{5,2} - u_{4,1}$

$= -2 + 0 + 9$

$= \cancel{10}\ 7$

$j=3 \Rightarrow u_{i,4} = u_{i-1,3} + u_{i+1,3} - u_{i,2}$ — (3)

$i=1 \quad u_{1,4} = u_{0,3} + u_{2,3} - u_{1,2}$

$= 0 + 2 + 7$

$= 9$

$i=2 \quad u_{2,4} = u_{1,3} + u_{3,3} - u_{2,2}$

$= -2 + 8 + 8$

$= 14$

Ajay

$i=3$ $\quad u_{3,4} = u_{2,3} + u_{4,3} - u_{3,2}$

$u \quad = 2 + 7 + 2$

$= 11$

$i=4$ $\quad u_{4,4} = u_{3,3} + u_{5,3} - u_{4,2}$

$= 8 + 0 - 2$

$= 6$

(last row completed)

The values of $u_{i,j}$ are tabulated below

x		x_0	x_1	x_2	x_3	x_4	x_5
t			1	2	3.	4	0
t_0	0	0	-4	-12	-18	-16	0
t_1	0.25	0	-6	-11	-14	-9	0
t_2	0.5	0	-7	-8	-2	2	0
t_3	0.75	0	-2	2	8	7	0
t_4	1	0	+9	14	11	6	0

3. Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ given that $u(x,0) = 0$;

$u(0,t) = 0$; $\quad u_t(x,0) = 0$ & $u(1,t) = 100\sin(\pi t)$

in the range $0 \le t \le 1$ by taking $h = 1/4$.

Ajay

Soln: comparing $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial^2 x^2}$ given equation to the standard wave eqn $u_{tt} = c^2 u_{xx}$

$\therefore c^2 = 1$

$c = 1$

$k = \frac{h}{c} = \frac{1/4}{1}$

$k = 0.25$

The points of division for x with $h = 0.25$ are, $x: 0, 0.25, 0.5, 0.75, 1$

The points of division for t with $k = 0.25$ are, $t = 0, 0.25, 0.5, 0.75, 1$.

Consider the following basic table.

x / t		x_0	x_1	x_2	x_3	x_4
t_0	0	$u_{0,0}$	$u_{1,0}$	$u_{2,0}$	$u_{3,0}$	$u_{4,0}$
t_1	0.25	$u_{0,1}$	$u_{1,1}$	$u_{2,1}$	$u_{3,1}$	$u_{4,1}$
t_2	0.5	$u_{0,2}$	$u_{1,2}$	$u_{2,2}$	$u_{3,2}$	$u_{4,2}$
t_3	0.75	$u_{0,3}$	$u_{1,3}$	$u_{2,3}$	$u_{3,3}$	$u_{4,3}$
t_4	1	$u_{0,4}$	$u_{1,4}$	$u_{2,4}$	$u_{3,4}$	$u_{4,4}$

From the given condition $u(0,t)=0$ & $u(x,0)=0$. first row and first column values becomes zero.

Consider the condition.

$$u(1,t)=100\sin\pi t$$

$$u_{4,1}=u(x_4,t)$$
$$=u(1.025)$$
$$=100\sin(0.25\pi)$$
$$u_{4,1}=70.71$$

$$u_{4,2}=u(x_4,t_2)=u(1,0.5)=100\sin(0.5\pi)$$
$$=100$$

$$u_{4,3}=u(x_4,t_3)=u(1.075)=100\sin(0.75\pi)$$
$$=70.71$$

$$u_{4,4}=u(x_4,t_4)=u(1,1)=100\sin\pi=0$$
(last coloumn values are obtained)

Next consider,

$$u_{i,1}=\frac{1}{2}[u_{i-1,0}+u_{i+1,0}]$$

Ajay

$i=1 \quad u_{1,1} = \frac{1}{2}[u_{0,0} + u_{2,0}]$

$= \frac{1}{2}(0+0)$

$u_{1,1} = 0$

$i=2 \quad u_{2,1} = \frac{1}{2}[u_{1,0} + u_{3,0}]$

$= \frac{1}{2}[0+0]$

$= 0$

$i=3 \quad u_{3,1} = \frac{1}{2}[u_{2,0} + u_{4,0}]$

$= \frac{1}{2}[0+0]$

$= 0$

Rest of the values are obtained by following formula,

$u_{i,j+1} = u_{i-1,j} + u_{i+1,j} - u_{i,j-1}$ — (1)

Ajay

if $j=1$

① $\Rightarrow u_{i,2} = u_{i-1,1} + u_{i+1,1} - u_{i,0}$

$i=1$ $u_{1,2} = u_{0,1} + u_{2,1} - u_{1,0}$

$= 0+0-0$

$u_{1,2} = 0$

$i=2$ $u_{2,2} = u_{1,1} + u_{3,1} - u_{2,0}$

$= 0+0-0$

$u_{2,2} = 0$

$i=3$ $u_{3,2} = u_{2,1} + u_{4,1} - u_{3,0}$

$= 0 + 70.701$

$= 70.71$

Third row completed.

if $j=2$

① $\Rightarrow u_{i,3} = u_{i-1,2} + u_{i+1,2} - u_{i,1}$

$i=1$ $u_{1,3} = u_{0,2} + u_{2,2} - u_{0,1}$

$= 0+0-0$

$= 0$

$i=2$ $u_{2,3} = u_{1,2} + u_{3,2} - u_{2,1}$

$= 0 + 70.71 - 0 = 70.71$

Ajay

if $j=3$

(1) $\Rightarrow$ $u_{i,4} = u_{i-1,3} + u_{i+1,3} - u_{i,2}$

$i=1$ $\quad u_{1,4} = u_{0,3} + u_{2,3} - u_{2,2}$

$= 0 + 70.71 - 0$

$= 70.71$

$i=2$ $\quad u_{2,4} = u_{1,3} + u_{3,3} - u_{2,2}$

$= 0 + 100 - 0$

$= 100$

$i=3$ $\quad u_{3,4} = u_{2,3} + u_{4,3} - u_{3,2}$

$= 70.71 + 70.71 - 70.71$

$= 70.71$

Last Row completed.

The values of u_{ij} are tabulated as follows.

	x	x_0	x_1	x_2	x_3	x_4
t		0	0.25	0.5	0.75	1
t_0	0	0	0	0	0	0
t_1	0.25	0	0	0	0	70.71
t_2	05	0	0	0	70.71	100
t_3	0.75	0	0	70.71	100	70.71
t_4	1	0	70.71	100	70.71	0