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MODULE-01.

Fourier Series.

Periodic function (Formula)

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

where,

$$a_0 = \frac{1}{l}\int_{c}^{c+2l} f(x).dx$$

$$a_n = \frac{1}{l}\int_{c}^{c+2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$b_n = \frac{1}{l}\int_{c}^{c+2l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

Defination:

A real valued function $f(x)$ is said to be periodic function of period T if $f(x+T) = f(x)$.

Fourier series:

The fourier series of a periodic function is given by,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx .$$

Remarks:

If $f(x)$ is discontinuous at x, then the series converges to $\frac{1}{2}[f(x^+) + f(x^-)]$

where,

$f(x^+)$ & $f(x^-)$ are right hand and left hand limits of $f(x)$.

Note:

1. Bernoulli's rule of integration

$\int uv = u\int v - u'\iint v + u''\iiint v - - .$

2. $\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2}[a\sin bx - b\cos bx]$

3. $\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2+b^2}[a\cos bx + a\sin bx]$

4. $\sin n\pi = 0 = \cos \frac{\pi}{2}$, $\cos 0 = 0$ $\cos n\pi = (-1)^n$

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1. Obtain the fourier series of $f(x) = \frac{\pi - x}{2}$ in $0 < x < 2\pi$. Hence deduce that

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} = \frac{\pi}{4}$$

Soln: $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$ — (1)

where,

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x)\,dx$$

$$= \frac{1}{\pi} \int_0^{2\pi} \left(\frac{\pi - x}{2}\right) dx$$

$$= \frac{1}{2\pi} \int_0^{2\pi} (\pi - x)\,dx$$

$$= \frac{1}{2\pi} \left[\pi x - \frac{x^2}{2}\right]_0^{2\pi}$$

$$a_0 = \frac{1}{2\pi} \left\{ \left[\pi(2\pi) - \frac{(2\pi)^2}{2}\right] - 0 - 0 \right\}$$

$$a_0 = \frac{1}{2\pi} \left[2\pi^2 - \frac{4\pi^2}{2}\right]$$

$$\boxed{a_0 = 0}$$

Further,

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\, dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} \left(\frac{\pi - x}{2}\right)\cos nx\, dx.$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \underset{(i)}{(\pi - x)}\ \underset{(ii)}{\cos nx}\, dx.$$

we have by bernoulli's rule of integraty

$$a_n = \frac{1}{2\pi}\left\{(\pi - x)\frac{\sin nx}{n} - (0-1)\left(\frac{-\cos nx}{n^2}\right)\right\}_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[\frac{1}{n}(0-0) - \frac{1}{n^2}\left[\cos 2n\pi - \cos 0\right]\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{n^2}[1-1]\right] \qquad \left[\begin{array}{c}\because \sin 2n\pi = 0 \\ \sin 0 = 0\end{array}\right]$$

$$\boxed{a_n = 0}$$

lly $$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\, dx.$$

$$= \frac{1}{\pi}\int_0^{2\pi}\left(\frac{\pi - x}{2}\right)\sin nx\, dx$$

$$\frac{1}{2\pi}\int_0^{2\pi} \underset{(I)}{(\pi - x)}\ \underset{(II)}{\sin nx}\ dx$$

By bernoli's rule,

$$bn = \frac{1}{2\pi}\left\{ (\pi - x)\left[\frac{-\cos nx}{n}\right] - (0-1)\left(\frac{-\sin nx}{n^2}\right)\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left\{ \left(\frac{-1}{n}\right. (\pi - 2\pi)\ (\cos 2n\pi - (\pi - 0)\cos 0) - \frac{1}{n^2}[0-0]\right\}$$

$\sin 2n\pi = 0$, $\cos 2n\pi = 1$

$$bn = \frac{1}{2\pi}\left\{\frac{-1}{n}\ (-\pi(1) - \pi - 0\right\}$$

$$= \frac{1}{2\pi}\left[\frac{-1}{n}\ -2\pi\right]$$

$$\boxed{bn = \frac{1}{n}}$$

Now, sub a_0, a_n, bn in Eqn ①

$$f(x) = 0 + 0 + \Sigma \frac{1}{n}\sin nx.$$

$$\boxed{f(x) = \sum_{n=1}^{\infty} \frac{1}{n} \sin nx} \quad \text{--- (2)}$$

To deduce the required series,
we put $x = \pi/2$
Eqn (2) =>

$$f(\pi/2) = \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi}{2}\right)$$

put $n = 1, 2, 3 \ldots$

$$\frac{\pi - \pi/2}{2} = \sin(\pi/2) + \frac{\sin(2\pi/2)}{2} + \frac{\sin(3\pi/2)}{3} + \frac{\sin(4\pi/2)}{4} + \frac{\sin(5\pi/2)}{5}$$

$$\frac{2\pi - \pi}{2 \cdot 2} = 1 + 0 - \frac{1}{3} + 0 + \frac{1}{5} - \ldots$$

$$\boxed{\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots}$$

(2) If $f(x) = x(2\pi - x)$ in $0 \le x \le 2\pi$ then
S.T $f(x) = \frac{2\pi^2}{3} - 4\left[\frac{\cos x}{1^2} + \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2}\right]$

Soln: $f(x) = x(2\pi - x)$
we have a f.s of periodic function,
of period 2π as,

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$$f(x) = \frac{a_0}{2} + \Sigma a_n \cos nx + \Sigma b_n \sin nx \quad -(1)$$

where,

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx \quad \left(\int_c^{c+2\pi} = \int_0^{2\pi} \right.$$

$$= \frac{1}{\pi}\int_0^{2\pi} (2\pi x - x^2)\,dx$$

$$= \frac{1}{\pi}\left[\cancel{2}\pi \times \frac{x^2}{\cancel{2}} - \frac{x^3}{3} \right]_0^{2\pi}$$

$$a_0 = \frac{1}{\pi}\left[\pi(2\pi)^2 - \frac{(2\pi)^3}{3} \right] - 0$$

$$a_0 = \frac{1}{\pi}\left[4\pi^3 - \frac{8\pi^3}{3} \right]$$

$$= \frac{1}{\pi}\left[\frac{12\pi^3 - 8\pi^3}{3} \right]$$

$$= \frac{1}{\cancel{\pi}}\left[\frac{4\pi^3}{3} \right]$$

$$a_0 = \frac{4\pi^2}{3}$$

$$a_0 = \frac{2 \times 2\pi^2}{3} \Rightarrow \boxed{\frac{a_0}{2} = \frac{2\pi^2}{3}}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx \, dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} (2\pi x - x^2)\cos nx \, dx$$

Apply bernoli's rule of integration,

$$a_n = \frac{1}{\pi}\Big[(2\pi x - x^2)\frac{\sin nx}{n} - (2\pi - 2x)\left(\frac{-\cos nx}{n^2}\right) + (0-2)\left(\frac{-\sin nx}{n^3}\right)\Big]_0^{2\pi}$$

$$= \frac{1}{\pi}\Big\{\Big[2\pi(2\pi) - (2\pi)^2 \frac{\sin n(2\pi)}{n} - [2\pi - 2(2\pi)]\left[\frac{-\cos n(2\pi)}{n^2}\right] + (-2)\left[\frac{-\sin n(2\pi)}{n^3}\right] - 0\Big\}$$

$$(\sin 2n\pi = 0 \quad \cos 2n\pi = 1)$$

$$= \frac{1}{\pi}\left[0 - 2\pi\right] * \frac{1}{n^2} \quad -2\pi \times 1\Big]$$

$$= \frac{1}{\pi}\left[\frac{1}{n^2}(-4\pi)\right]$$

$$\boxed{a_n = \frac{-4}{n^2}}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\, dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} (2\pi x - x^2)\sin nx\, dx.$$

$$= \frac{1}{\pi}\left[(2\pi x - x^2)\left(\frac{-\cos nx}{n}\right) - (2\pi - 2x)\left(\frac{-\sin nx}{n^2}\right) + (0-2)\left[\frac{\cos nx}{n^3}\right]\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{-1}{n}\left[2\pi(2\pi) - (2\pi)^2 - 0\right] + \frac{1}{n^2}\left[-2\pi\times 0 - \cancel{2\pi(2\pi)}\right) - \left[\frac{2}{n^3} - 0\right]\right]$$

$$= \frac{1}{\pi}\left\{0 + 0 - \frac{2}{n^3}(0)\right\}$$

$<2\pi ...$

$\boxed{b_n = 0}$

sub a_0, a_n, b_n in ①

$$f(x) = \frac{a_0}{2} + \sum a_n \cos nx + \sum b_n \sin nx.$$

$$= \frac{2\pi^2}{3} + \sum_{n=1}^{\infty}\left(\frac{-4}{n^2}\right)\cos nx + 0$$

$$f(x) = \frac{2\pi^2}{3} - 4\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$$

expand the series by putting $n = 1, 2, 3, 4 \ldots$

$$f(x) = \frac{2\pi^2}{3} - 4\left[\cos x + \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} + \cdots\right]$$

3 obtain the f.s for the function.

$$f(x) = \begin{cases} -\pi & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$$

we have,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \, dx + \sum_{n=1}^{\infty} b_n \sin nx \, dx$$

where,

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\,dx + \int_{0}^{\pi} f(x)\,dx\right]$$

$$= \frac{1}{\pi}\left[-\pi\,(x)\Big|_{-\pi}^{0} + \int_{0}^{\pi} x\,dx\right]$$

$$= \frac{1}{\pi}\left[-\pi - (-\pi) + \left[\frac{x^2}{2}\right]_0^{\pi}\right]$$

$$= \frac{1}{\pi}\left[-\pi^2 + \pi^2/2\right]$$

$\sin \pi = 1$
$\cos \pi = 0$

$$= \frac{1}{\pi}\left[\frac{-2\pi^2 + \pi^2}{2}\right]$$

$$= \frac{1}{\pi}\left[\frac{-\pi^2}{2}\right]$$

$$\boxed{a_0 = \frac{-\pi}{2}}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} f(x)\cos nx\,dx + \int_{0}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} -\pi\cos nx\,dx + \int_{0}^{\pi} x\cos nx\,dx$$

$$= \frac{1}{\pi}\left\{\left(-\pi\frac{\sin nx}{n}\right)_{-\pi}^{0} + \left[\frac{x}{n}\sin nx + \frac{\cos nx}{n^2}\right]_0^{\pi}\right.$$

$$= \frac{1}{\pi}\left[\frac{-\pi}{n}\left[\sin n0 - \sin n(-\pi)\right) + \frac{1}{n}\right.$$

$$\left[\pi\sin n\pi - 0\right] + 1/n^2\left[\cos n\pi - \cos n\times 0\right]$$

$$= \frac{1}{\pi}\left[\frac{-\pi}{n}(0+1) + \frac{1}{n}(\pi - 0) + \frac{1}{n^2}((-1)^n - 1)\right)$$

$$= \frac{1}{\pi}\left(\frac{-\pi}{n} + \frac{\pi}{n} + \frac{1}{n^2}((-1)^n - 1)\right)$$

$$a_n = \frac{1}{\pi n^2}\left\{(-1)^n - 1\right\}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx \sin nx\, dx.$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} -\pi \sin nx\, dx + \int_{0}^{\pi} x \sin nx\, dx$$

$$= \frac{1}{\pi}\left\{\left[\frac{\pi \cos nx}{n}\right]_{-\pi}^{0} + \left[\frac{-x \cos nx}{n} + \frac{\sin nx}{n^2}\right]_{0}^{\pi}\right\}$$

$$= \frac{1}{\pi}\left[\frac{\pi}{n}\left[1-(-1)^n\right]. + \left[\frac{-\pi(-1)^n}{n} - 0\right] + \frac{1}{n^2}(0)\right.$$

$$= \frac{1}{\pi}\left[\frac{\pi}{n}\left(1-(-1)^n\right) - \frac{\pi}{n}(-1)^n\right]$$

$$= \frac{\pi}{\pi n}\left[1-(-1)^n - (-1)^n\right]$$

$$b_n = \frac{1}{n}\left[1 - 2(-1)^n\right]$$

$< \cos 0 = 1$

sub a_0, a_n, b_n in (1)

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$= \frac{-\pi}{4} + \sum_{n=1}^{\infty} \frac{1}{\pi n^2} ((-1)^n - 1) \cos nx +$$

$$\sum_{n=1}^{\infty} \frac{1}{n} [1 - 2(-1)^n] \sin nx.$$

Fourier Series of even or odd functions.

Defn:

A function $f(x)$ is said to be even, if $f(-x) = f(x)$ and a function $f(x)$ is said to be odd $f(-x) = -f(x)$

for ex: $x^2, x^4, x^6, \cos x$ are even fun.

: $x^3, x^5, x^7, \sin x$ are odd fun.

Property:

1. The product of 2 even fun & the product of 2 odd fun is always even. whereas, the product of an even & an odd functions is always odd.

2. $\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_{0}^{a} f(x)\,dx & \text{if } f(x) \text{ is even} \\ 0 & \text{if } f(x) \text{ is odd.} \end{cases}$

Note: If $f(x) = \begin{cases} \phi(x) & \text{in } -\pi < x < 0 \\ \psi(x) & \text{in } 0 < x < \pi \end{cases}$ then,

$\phi(-x) = \psi(x) \rightarrow$ even

$\phi(-x) = -\psi(x) \rightarrow$ odd.

For $(0, 2\pi)$ interval,

$f(2\pi - x) = f(x) \rightarrow$ even

$f(2\pi - x) = -f(x) \rightarrow$ odd.

	Nature & condition	a_0	a_n	b_n
$(-\pi, \pi)$ $(0, 2\pi)$	Even fun $f(-x) = f(x)$ / $f(2\pi - x) = f(x)$	$\frac{2}{\pi}\int_{0}^{\cancel{x}\,\pi} f(x)\,dx$	$\frac{2}{\pi}\int_{0}^{\pi} f(x)\cos nx\,dx$	
$(-\pi, \pi)$ $(0, 2\pi)$	odd fun $f(-x) = -f(x)$ $f(2\pi - x) = -f(x)$	0	0	$\frac{2}{\pi}\int_{0}^{\pi} f(x)\sin nx\,dx$

1. Find the fourier series of

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi} & \text{in } -\pi < x \leq 0 \\ 1 - \frac{2x}{x} & \text{in } 0 \leq x < \pi \end{cases}$$

Hence, deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

Soln;

let $f(x) = 1 + \frac{2x}{\pi}$ & $\varphi(x) = 1 - \frac{2x}{\pi}$

Put $x = -x$

$\phi(-x) = 1 - \frac{2x}{-\pi} = \varphi(x) \longrightarrow$ even

Hence, consider $\boxed{b_n = 0}$

& a_0, a_n is given by,

$$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x)\,dx$$

$$= \frac{2}{\pi} \int_0^{\pi} 1 - \frac{2x}{\pi}\,dx.$$

$$= \frac{2}{\pi} \left[x - \frac{\cancel{2}x^2}{\cancel{2}\pi} \right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\pi - \frac{\pi^2}{\pi}\right](0)$$

$$\boxed{a_0 = 0}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx \, dx$$

$$= \frac{2}{\pi}\int_0^{\pi} 1 - \frac{2x}{\pi} \cdot \cos nx \, dx$$

Apply bernoli's rule of integration.

$$= \frac{2}{\pi}\left[1 - \frac{2x}{\pi}\,\frac{\sin nx}{n} - \left(0 - \frac{2}{\pi}\right)\frac{-\cos nx}{n^2} + 0 \right]_0^{\pi}$$

$$= \frac{2}{\pi}\left\{ \frac{1}{n}(0-0) - \frac{2}{\pi n^2}(\cos n\pi - \cos 0) \right\}$$

$$\boxed{a_n = \frac{4}{\pi^2 n^2}\left[1 - (-1)^n\right]} \longrightarrow (3)$$

sub a_0, a_n & b_n in the F.S as, $f(x) = \frac{a_0}{2}$

$+ \sum a_n \cos nx + \sum b_n \sin nx$.

$= 0 + \sum \frac{4}{\pi^2 n^2} \left[1-(-1)^n\right] \cos nx + 0$

$$\boxed{f(x) = \frac{4}{\pi^2} \sum \frac{(1-(-1)^n)}{n^2} \cos nx} \rightarrow$$

required F.S.

To reduce to the required F.S series we put $x=0$

Eqn ② $\Rightarrow f(0) = \frac{4}{\pi^2} \sum_{n=1}^{\infty} \left[\frac{1-(-1)^n}{n^2}\right]$

$[\because \cos 0 = 1]$

But $1-(-1)^n = \begin{cases} 2 & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even} \end{cases}$

$f(0) = \frac{4}{\pi^2} \sum_{n=1,3,5}^{\infty} \frac{2}{n^2}$

put $n = 1, 3, 5 \ldots$

$1 - \frac{2(0)}{\pi} = \frac{4}{\pi^2} \times 2 \left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\right]$

$$1 = \frac{8}{\pi^2}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} - - -\right]$$

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} - - -$$

required series.

2. Obtain a fourier series for a function x^2 in $-\pi < x \le \pi$ & hence deduce that,

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} - - - = \frac{\pi^2}{12}$$

given,

$f(x) = x^2$

Put $x = -x$

$f(-x) = (-x)^2 = x^2 = f(x)$

Since $f(x)$ is even function,

$b_n = 0$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x^2\,dx$$

$$= \frac{2}{\pi}\left[\frac{x^3}{3}\right]_0^{\pi}$$

$$a_0 = \frac{2}{\pi}\left[\frac{\pi^3}{3} - 0\right]$$

$$a_0 = \frac{2\pi^3}{3\pi}$$

$$\boxed{a_0 = \frac{2\pi^2}{3}} \quad \text{—— (1)}$$

$$a_n = \frac{2}{\pi}\int_0^\pi f(x)\cos nx\,dx$$

$$= \frac{2}{\pi}\int_0^\pi x^2\cos nx\,dx$$

Apply B, rule of integration

$$= \frac{2}{\pi}\left[x^2\frac{\sin nx}{n} - \left[2x \times \frac{-\cos nx}{n^2}\right] + 2\frac{-\sin nx}{n^3}\right]_0^\pi$$

$$= \frac{2}{\pi}\left[\frac{1}{n}\left[\pi^2(0) - 0\right] + \frac{2}{n^2}\left[\pi(-1)^n - 0\right] - \frac{2}{n^3}\left[0 - 0\right]\right\}$$

$$= \frac{2}{\pi}\left\{0 + \frac{2}{n^2}\left[\pi(-1)^n\right]\right. = \frac{4}{n^2}(-1)^n$$

(2)

Sub the values in fourier series,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$= \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2} \cos nx + 0$$

$$\boxed{= \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n \cos nx}{n^2}}$$

required F.S.

To reduce the required series we put $x = 0$

Eqn (2) $\Rightarrow$ $0 = \frac{\pi^2}{3} + 4 \sum \frac{(-1)^n}{n^2}$.

$$= \frac{-\pi^2}{12} = \sum \frac{(-1)^n}{n^2}.$$

Put $n = 1, 2, 3 \cdots$

$$\frac{-\pi^2}{12} = \frac{-1}{1^2} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2} + \cdots$$

$$\frac{\cancel{-}\pi^2}{12} = \cancel{-}\left[\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2}\right]$$

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2}.$$

3. obtain the fourier series expansion of the function $f(x) = \begin{cases} x & 0<x<\pi \\ x-2\pi & \pi<x<2\pi \end{cases}$

& hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \cdots$

$\Rightarrow$ let $\phi(x) = x$ & $\psi(x) = x - 2\pi$

Put $x = 2\pi - x$

$\phi(2\pi - x) = 2\pi - x$

$= -(x - 2\pi)$

$\phi(2\pi - x) = -\psi(x)$

$\therefore f(x)$ is an odd function

$a_0, a_n = 0$ & $b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$

$= \frac{2}{\pi}\int_0^{\pi} x\sin nx\,dx$

Apply Bernoli's rule of integration, we get

$b_n = \frac{2}{\pi}\left[-x\frac{\cos nx\,dx}{n} + 1\frac{\sin nx}{n^2}\right]_0^{\pi}$

$= \frac{2}{\pi}\left[\frac{-1}{n}\left[\pi(-1)^n - 0\right] + \frac{1}{n^2}(0)\right]$

$$= \frac{2}{\pi}\left\{\frac{-1}{n}\pi(-1)^n\right\}$$

$$= \frac{-2(-1)^n}{n} = \frac{2(-1)^{n+1}}{n}$$

Sub a_0, a_n, B_n in F.S Eq^n.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx\, dx + \sum_{n=1}^{\infty} b_n \sin nx\, dx$$

$$= 0 + 0 + \frac{2(-1)^{n+1}}{n}\sin nx \longrightarrow \text{required F.S.}$$

$$f(x) = \sum \frac{2(-1)^{n+1}}{n}\sin nx$$

To reduce F.S put $x = \pi/2$

$$\frac{\pi}{2} = 2\sum_{n=1,3,5}^{\infty} \frac{(-1)^{n+1}}{n}\sin\frac{n\pi}{2}$$

$$\frac{\pi}{4} = \left\{(-1)^{1+1}\sin\pi/2 + \frac{(-1)^{2+1}}{2}\sin 2\pi/2 + \frac{(-1)^{3+1}}{3}\sin 3\pi/2 + \cdots\right\}$$

$$\frac{\pi}{4} = \left\{ \frac{1}{1} + 0 + \frac{1}{3} + 0 + \frac{1}{5} \cdots \right\}$$

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Fourier Series of Orbitary Period.

The f.s of Orbitary Period $2l$ is given by,

$$f(x) = \frac{a_0}{2} + \Sigma a_n \cos\left(\frac{n\pi x}{l}\right) + \Sigma b_n \sin\left(\frac{n\pi x}{l}\right)$$

where,

$$a_0 = \frac{1}{l} \int_{c}^{c+2l} f(x)\,dx$$

$$a_n = \frac{1}{l} \int_{c}^{c+2l} f(x) \cos\left(\frac{n\pi x}{l}\right) dx$$

$$b_n = \frac{1}{l} \int_{c}^{c+2l} f(x) \sin\left(\frac{n\pi x}{l}\right) dx$$

Interval	Nature & condition	a_0
$(-l, l)$	$f(-x) = f(x)$	$\frac{2}{l}\int_0^l f(x)\,dx$
$(0, 2l)$	$f(2l-x) = f(x)$	
	odd function	
$(-l, l)$	$f(-x) = f(x)$	0
$(0, 2l)$	$f(2l-x) = -f(x)$	

	a_n	b_n
odd	0	$\frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx$
even	$\frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx$	0

Working procedure:

If the period of the given function other than 2π, we first equate the given period to $2l$. and obtain the value of l.

we then write a appropriate series and compute a_0, a_n & b_n associate with it.

1. obtain the f.s for the function $f(x) = 2x - x^2$ in $0 \le x \le 2$

Soln: Comparing the given interval $(0, 2)$ to $(0, 2l)$ we get

$$2l - 2 = 0 \Rightarrow 2l = 2$$
$$\Rightarrow l = 1$$

we have the f.s for the arbitrary period $2l$ is,

$$f(x) = \frac{a_0}{2} + \sum a_n \cos\left(\frac{n\pi x}{l}\right) + \sum b_n \sin\left(\frac{n\pi x}{l}\right) \quad \text{——— (i)}$$

Given, $f(x) = 2x - x^2$

put $x = 2l - x = 2 - x$

$$= 2[2l - x] - [2 - x]^2$$
$$f(2l - x) = 4 - 2x - [4 + x^2 - 4x]$$
$$= 4 - 2x - 4 - x^2 + 4x$$
$$f(2 - x) = 2x - x^2 = f(x)$$

since we have $f(x)$ is even

$$\boxed{b_n = 0}$$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$$

$$= 2\int_0^1 (2x - x^2)\,dx$$

$$= 2\left[2\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= 2\left[(1 - 1/3)\right] - 0$$

$$= 2\left[\frac{2}{3}\right]$$

$$\boxed{\frac{a_0}{2} = \frac{2}{3}}$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{2}{1}\int_0^1 (2x - x^2)\cos(n\pi x)\,dx$$

Apply Bernoli's rule of integration

$$= 2\left\{(2x - x^2)\frac{\sin(n\pi x)}{n\pi} - (2 - 2x)\left(\frac{-\cos(n\pi x)}{n^2\pi^2}\right)\right.$$

$$\left. + \left[(0-2) - \frac{\sin(n\pi x)}{n^3\pi^3}\right]\right\}_0^1$$

$$= 2\left\{ \frac{\sin n\pi x}{n\pi} + \frac{1}{n^2\pi^2}\left[0-(2-0)\cos 0\right) - \frac{1}{n^3\pi^3}(0-0)\right\}$$

$$a_n = \frac{2(-2)}{n^2\pi^2}$$

$$\boxed{a_n = \frac{-4}{n^2\pi^2}}$$

sub a_0, a_n & b_n in f.s Eqn ①

$$f(x) = \frac{2}{3} + \sum \frac{-4}{n^2\pi^2}\cos\left(\frac{n\pi x}{1}\right) + 0$$

$$\boxed{f(x) = \frac{2}{3} - \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos\left(\frac{n\pi x}{1}\right)}$$

2. Obtain the f.s to represent $f(x) = x - x^2$ in $-1 < x < 1$

Soln: comparing the given interval $(-1, 1)$ to $(0, 2l)$

we get,

$2l = 1-(-1) \Rightarrow 2l = 2$

$\therefore \boxed{l = 1}$

we have f.S for the arbitrary Period $2l$ is,

$$f(x) = \frac{a_0}{2} + \Sigma a_n \cos\left(\frac{n\pi x}{l}\right) + \Sigma b_n \sin\left(\frac{n\pi x}{l}\right) \quad \text{---} (1)$$

Given,

$$f(x) = x - x^2$$

put $x = -x$

$$f(-x) = -x - x^2$$

The function $f(x)$ is neither even nor odd we have,

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx$$

$$= \frac{1}{1}\int_{-1}^{+1} (x - x^2)\,dx$$

$$= 1\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{-1}^{+1}$$

$$= \left[\cancel{\frac{1}{2}} - \cancel{\frac{1}{2}}\right] - \frac{1}{3} - \frac{1}{3}\Big]$$

$$a_0 = \frac{-2}{3}$$

$$\boxed{\frac{a_0}{2} = \frac{-1}{3}}$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx.$$

$$= \frac{1}{1}\int_{-1}^{1} f(x)\cos\left(\frac{n\pi x}{1}\right)dx$$

$$= 1\int_{-1}^{+1} (x-x^2)\cos\left(\frac{n\pi x}{1}\right)dx$$

$$= \left\{ (x-x^2)\frac{\sin n\pi x}{n\pi} + (1-2x)\left(\frac{-\cos n\pi x}{n^2\pi^2}\right) + \frac{2\sin n\pi x}{n^3\pi^3} \right|_{-1}^{+1}$$

$$= \left\{\left[0 - \frac{2\sin n\pi}{n\pi}\right] \frac{-1}{n^2\pi^2}\left[\cos n\pi + 3(\cos n\pi)\right.\right.$$

$$+ \frac{2}{n^3\pi^3}\left[\sin n\pi + 0\right]$$

$$= 0 + \left[\frac{1}{n^2\pi^2} \quad 1(-1)^n - 3(-1)^n \right]$$

$$= \frac{-4(-1)^n}{n^2\pi^2} - \frac{4(-1)^n}{n^2\pi^2}$$

$$a_n = \frac{4 \cdot (-1)^{n+1}}{n^2\pi^2}$$

10-06-22

Half range fourier series.

f(x) in	Required series	series	co-efficients.
$(0,\pi)$	cosine half range series	$\frac{a_0}{2} + \sum a_n \cos nx$	$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x)\,dx$ $a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx\,dx.$
$(0,\pi)$	sine half range series	$\sum b_n \sin nx$	$b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx\,dx$
$(0,l)$	cosine half range series	$\frac{a_0}{2} + \sum a_n \cos\left(\frac{n\pi x}{l}\right)$	$a_0 = \frac{2}{l} \int_0^{l} f(x)\,dx$ $a_n = \frac{2}{l} \int_0^{l} f(x) \cos\left(\frac{n\pi x}{l}\right) dx$
$(0,l)$	$\sum b_n \sin\left(\frac{n\pi x}{l}\right)$ sine half range	$\sum b_n \sin\left(\frac{n\pi x}{l}\right)$	$b_n = \frac{2}{l} \int_0^{l} f(x) \sin\left(\frac{n\pi x}{l}\right) dx.$

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obtain sine half range series of
$f(x) = x^2$ in $0 < x < \pi$

The sine half range series is given by,

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx \quad \text{---} (1)$$

where,

$$b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx \, dx$$

$$= \frac{2}{\pi} \int_0^{\pi} \underset{(i)}{x^2} \cdot \underset{(ii)}{\sin nx} \, dx$$

$$b_n = \frac{2}{\pi} \left\{ \left[(x^2) \left[\frac{-\cos nx}{n} \right] - 2x \left[\frac{-\sin nx}{n^2} \right] + 2 \left[\frac{+\cos nx}{n^3} \right] \right]_0^{\pi} \right.$$

$$= \frac{2}{\pi} \left\{ \pi^2 \cdot \left[\frac{-\cos n\pi}{n} \right] - 2\pi \left[\frac{-\sin n\pi}{n^2} \right] + 2 \left[\frac{-\cos n\pi}{n^3} \right] - \right.$$

$$\left[\frac{x^2}{n} - a - \cancel{2/n3} \right] \left[\cancel{x} \right.$$

$\cos n\pi = (-1)^n$.

$$b_n = \frac{2}{\pi}\left\{\frac{\pi^2 (-1)^n \cancel{\pi}}{n} + \frac{2}{n^3}\left[(-1)^n - 1\right]\right\}$$

Or

$$b_n = \frac{2}{\pi}\left\{\frac{-1}{n}\left(\pi^2 \cos n\pi - 0\right] + \frac{2}{n^2}\left[0-0\right] + \frac{2}{n^3}\left[\cos n\pi - \cos 0\right]\right.$$

$\sin n\pi = 0$ $\cos n\pi = (-1)^n$

$\cos 0 = 1$ $\sin 0 = 0.$

$$= \frac{2}{\pi}\left\{\frac{-\pi^2 (-1)^n}{n} + \frac{2}{n^3}(-1)^n - 1\right]$$

$$b_n = \frac{2}{\pi}\left\{\frac{\pi^2 (-1)^n}{n} + \frac{2}{n^3}\left[(-1)^n - 1\right]\right\}$$

sub in (1)

$$f(x) = \Sigma \frac{2}{\pi}\left\{\frac{\pi^2 (-1)^{n+1}}{n} + \frac{2}{n^3}\left[(-1)^n - 1\right]\right\} \sin nx.$$

required sol. of series.

2. Expand $f(x) = 2x-1$ as a cosine half range series in $0<x<1$

soln:

The cosine half range series is given by,

$$f(x) = \frac{a_0}{2} + \Sigma a_n \cos\left(\frac{n\pi x}{l}\right) \rightarrow (1)$$

comparing the given interval $(0,1)$ with $(0,l)$ $l=1$

we have,

$$a_0 = \frac{2}{l}\int_0^1 f(x)dx$$

$$= \frac{2}{1}\int_0^1 (2x-1)dx$$

$$= \left[\frac{2x^2}{2} - x\right]_0^1$$

$$= 1-1$$

$$\boxed{a_0 = 0}$$

$$a_n = \frac{2}{1}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$= 2\int_0^l (2x-1)\cos\left(\frac{n\pi x}{l}\right)dx$$

Apply Bernoli's rule.

$$= 2\left\{(2x-1)\left(\frac{\sin n\pi x}{n\pi}\right) - (2-0)\left(\frac{-\cos n\pi x}{(n\pi)^2}\right\}_0^1\right.$$

$$= 2\left[0 - \left[\frac{2(-1)^n - 1}{n^2\pi^2}\right]\right] \text{ or } \left[\frac{2}{n^2\pi^2}[\cos n\pi - \cos 0]\right]$$

$$= \frac{4}{n^2\pi^2}\left[(-1)^n - 1\right]$$

Sub in Eqn (1)

$$f(x) = \sum \frac{4}{n^2\pi^2}\left[(-1)^n - 1\right]\cos(n\pi x)$$

?

14-06-22

3. Find the cosine half range series for $f(x) = lx - x^2$; $0 \leq x \leq l$

Soln: The cosine half range series is given by,

$$f(x) = \frac{a_0}{2} + \sum a_n \cos\left(\frac{n\pi x}{l}\right)$$

$$a_0 = \frac{2}{\cancel{\pi}l}\int_0^l f(x)\,dx$$

$$= \frac{2}{l}\int_0^l (lx - x^2)\,dx$$

$$= \frac{2}{l}\left[l \times \frac{x^2}{2} - \frac{x^3}{3}\right]_0^l$$

$$= \frac{2}{l}\left[l \times \frac{l^2}{2} - \frac{l^3}{3}\right] \quad [0-0]$$

$$= \frac{2}{l}\left[\frac{l^3}{2} - \frac{l^3}{3}\right]$$

$$= \frac{2}{l}\left[\frac{3l^3 - 2l^3}{6}\right]$$

$$= \frac{\cancel{2}}{\cancel{l}} \times \frac{l^{\cancel{3}}}{\cancel{6}\,3}$$

$$a_0 = \frac{l^2}{3} \quad \underline{\underline{\text{or}}} \quad \boxed{\frac{a_0}{2} = \frac{l^2}{6}}$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{2}{l}\int_0^l (lx - x^2)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{2}{l}\left\{\left[(lx - x^2)\frac{\sin(n\pi x)}{nl} - (l\cancel{x^2} - 2x)\left(\frac{-\cos n\pi x}{n^2 l}\right) + (0-2)\left[\frac{-\sin n\pi x}{n^3 l}\right]\right]_0^l\right\}$$

$$\int uv = u\int v - d(u)\iint v + d''(u)\iiint v$$

$$a_n = \frac{2}{l}\left\{ (lx - x^2)\frac{\sin\left(\frac{n\pi x}{l}\right)}{n\pi/l} - (l-2x)\left[\frac{-\cos\left(\frac{n\pi x}{l}\right)}{(n\pi/l)^2}\right] + (0-2)\left[\frac{-\sin\left(\frac{n\pi x}{l}\right)}{(n\pi/l)^3}\right]\right.$$

$$+ 0 \Big|_0^l$$

$$\frac{2}{l}\left\{ (l(l) - l^2)\frac{\sin\left(\frac{n\pi l}{l}\right)}{n\pi/l} - (l-2l)\left[\frac{-\cos\left(\frac{n\pi l}{l}\right)}{(n\pi/l)^2}\right] + (0-2)\left[\frac{-\sin\left(\frac{n\pi l}{l}\right)}{(n\pi/l)^3}\right]\right.$$

-0

$$-\left\{ 0 - (-l)(-1)^n - 2 \right]$$

$$\Rightarrow \frac{2}{l}\left\{ \frac{l}{2\pi}[0-0] + \frac{l^2}{n^2\pi^2}\left\{ (l-2l)\cos\frac{n\pi l}{l} - (l-0)\cos 0 + \frac{2l^3}{n^3\pi^3}[0-0] \right\}\right.$$

$\because \sin n\pi = 0$

$\sin 0 = 0$

$$= \frac{2}{l}\left\{\frac{l^2}{n^2\pi^2}\left[(-l)\cos n\pi\right] - l\cos 0\right\}$$

$$= \frac{2}{l} \times \frac{l^2}{n^2\pi^2} \times (-l)\left[(-1)^n + 1\right]$$

$$\boxed{a_n = \frac{-2l^2\left[(-1)^n + 1\right]}{n^2\pi^2}}$$

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Sub a_0, a_n in (1)

$$f(x) = \frac{l^2}{6} + \sum \frac{-2l^2}{n^2\pi^2}\left[(-1)^n + 1\right] \cos\left(\frac{n\pi x}{l}\right)$$

$$f(x) = \frac{l^2}{6} - \frac{2l^2}{\pi^2} \sum \frac{\left[(-1)^n + 1\right]}{n^2} \cos\left(\frac{n\pi x}{l}\right)$$

This is required cosine series.

Q5) (2) If $f(x) = \begin{cases} x & \text{in } 0 < x < \pi/2 \\ \pi - x & \text{in } \pi/2 < x < \pi \end{cases}$

s.t

$$f(x) = \frac{4}{\pi}\left\{\sin x - \frac{\sin 3x}{3^2} + \frac{\sin 5x}{5^2} + \cdots\right.$$

soln: we need to find the sine half range series for the interval $(0, \pi)$ is given by,

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx \quad \text{--- (1)}$$

where,

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x) \sin(nx)\,dx$$

$$= \frac{2}{\pi}\left\{\int_0^{\pi/2} x \sin nx + \int_{\pi/2}^{\pi} (\pi - x) \sin nx \, dx\right.$$

Apply Bernoulli's rule of integration

$$= \frac{2}{\pi}\left\{ x\left[\frac{-\cos nx}{n}\right] - 1\left[\frac{-\sin nx}{n^2}\right]\right\}_0^{\pi/2}$$

$$+ \left\{(\pi - x)\left[\frac{-\cos nx}{n}\right] - (-1)\left[\frac{-\sin nx}{n^2}\right]\right\}_{\pi/2}^{\pi}$$

$$= \frac{2}{\pi}\left\{\frac{1}{n}\left[\pi/2 \cdot \cos n(\pi/2)\right] + \frac{1}{n^2}\left[\sin n(\pi/2)\right.\right.$$

$$\left.\right\} - \{0 - 0\} + \left\{\frac{1}{n}\left[0 - (-\cos n\pi) + \frac{1}{n^2}\right.\right.$$

$$\left[\sin n\pi - \sin \pi/2 n\right]\cancel{\}}$$

$$= \frac{2}{\pi}\left\{\frac{1}{n}\left[\pi/2\,(-\cos(n(\pi/2)) - 0\right] + \frac{1}{n^2}\right.$$

$$\left[\sin(n\pi/2) - 0\right] + 1/n\left[-0 - (\pi - \pi/2)\right.$$

$$\left(-\cos(n\pi/2)\right] + \frac{1}{n^2}\left[-\sin n\pi + \sin(n\pi/2)\right.$$

$$= \frac{2}{\pi}\left\{\left[0 - 0\right] + \frac{1}{n^2}\left[\sin(n\pi/2) + 0\right]\right.$$

$$+ \frac{1}{n^2}\left[0 + \sin(n\pi/2)\right]$$

$$= \frac{2}{\pi n^2}\left[\sin(n\pi/2) + \sin(n\pi/2)\right]$$

$$= \frac{2}{\pi n^2}\left[2\sin(n\pi/2)\right] = \frac{4}{\pi n^2}\left[\sin(n\pi/2)\right]$$

(i) =>

$$f(x) = \sum_{n=1}^{\infty} \frac{4}{\pi n^2} \sin(n\pi/2)$$

$$f(x) = \frac{4}{\pi \cancel{x}^2} \sum_{1}^{\infty} \frac{\sin(n\pi/2)}{n^2} \sin(nx)\,dx$$

$$= \frac{4}{\pi}\left\{ \sin x - \frac{\sin 3x}{3\cancel{x}^2} + \frac{\sin 5x}{5^2} + \cdots \right\}$$

15-06-22

Practical Harmonil Analysis.

$$f(x) = \frac{a_0}{2} + \sum a_n \cos nx + \sum b_n \sin nx$$

$$a_0 = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x) \cos nx\,dx$$

$$b_n = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x) \sin nx\,dx .$$

15.06.21

Practical Harmonic Analysis

Formula's:-

f(x) Harmonic Analysis is a Process of finding the constant term and the first few cosine & sine terms numerically.

The f.s of (f(x)) Period 2π is given by,

$$f(x) = \frac{a_0}{2} + \Sigma a_n \cos nx + \Sigma b_n \sin nx.$$

where, the fourier co-efficients are given by,

$$a_0 = \frac{2}{N} \Sigma y$$

$$a_n = \frac{2}{\cancel{2}N} \Sigma y \cos nx$$

$n = 1, 2, 3 --$

$$b_n = \frac{2}{N} \Sigma y \sin nx$$

Here, $a_0/2$ is called the constant term & the group of terms $(a_1 \cos x + b_1 \sin x)$, $(a_2 \cos 2x + b_2 \sin 2x)$ etc are called the 1st harmonic, 2nd harmonic - - - - -

Remarks:-

Suppose we have a set of N values of $y = f(x)$ having a period 2π at equidistant points of x in the interval ($c \le x < c+2\pi$ or $c < x \le c+2\pi$) (if the values of y at $x=c$ & $x=c+2\pi$ are given we must neglect one of them as $(y)_{x=c} = (y)_{x=c+2\pi}$

working Procedure: (Imp)

i) we have to first write down the Period of $y = f(x)$ from the given range of x.

ii) if the Period is 2π depending on the harmonics required we Prepare the relevent table along with the summations of y; $y\cos x$; $y\cos 2x$; ----,
$y\sin x$; $y\sin 2x$; $y\sin 3x$; ---

iii) if the period is not 2π, we equate it with $2l$ to obtain the value of l.

iv) ~~acc~~ The Σ's of y, $y\cos\theta$, $y\cos 2\theta$, --- ; $y\sin\theta$, $y\sin 2\theta$ ---
where, $\theta = \pi x/l$ will be required

to compute the desired harmoni-
-cs.

~~assign four~~

Imp

① determine the constant term and the first cosine & sine terms of the fourier series expansion of y from the following table.

$x°$	0	45	90	135	180	225	270	315
y	2	3/2	1	1/2	0	1/2	1	3/2

Soln:

Here the interval $0 \le x < 360°$ &
Here we need to find a_0, a_1 & b_1 & the corresponding formula,

$$a_0 = \frac{2}{N} \Sigma y$$

$$a_1 = \frac{2}{N} \Sigma y \cos x$$

$$b_1 = \frac{2}{N} \Sigma y \sin x$$

$$\boxed{N = 8}$$

$$f(x) = \frac{a_0}{2} + \Sigma a_1 \cos x + \Sigma b_1 \sin x.$$

x°	y	$\cos x$	$y\cos x$	$\sin x$	$y\sin x$
0	2	1	2	0	0
45	3/2	0.707	1.0605	0.707	1.0605
90	1	0	0	1	1
135	1/2	-0.707	-0.3535	0.707	0.3535
180	0	-1	0	0	0
225	1/2	-0.707	-0.3535	-0.707	-0.3535
270	1	0	0	-1	-1
315	3/2	0.707	1.0605	-0.707	-1.0605
	$\Sigma y = 8$		$\Sigma y\cos x =$ 3.4142		$\Sigma y\sin x =$ 0

$$a_0 = \frac{2}{N}\Sigma y \qquad a_1 = \frac{2}{N}\Sigma y\cos x$$

$$= \frac{2}{8} \times 8 \qquad = \frac{\cancel{2}}{\cancel{8}4} \times 3.4142$$

$$\boxed{a_0 = 2} \text{ or } \boxed{\frac{a_0}{2} = 1} \qquad \boxed{a_1 = 0.85355}$$

$$b_1 = \frac{2}{N}\Sigma y\sin x$$

$$= \frac{2}{8} \times 0$$

$$\boxed{b_1 = 0}$$

we have f.s up to 1st harmonic is,

$$f(x) = \frac{a_0}{2} + (a_1 \cos x + b_1 \sin x)$$

$$= 1 + (0.8535 \cancel{40}) \cos x$$

$$\boxed{f(x) = 1 + (0.8535) \cos x}$$

16-06-22

2. The turning moment T on the crank shaft of a steam engine for the crank angle θ is given as follows:

θ°	0	30	60	90	120	150	180	210	240	270	300	330
T	0	2.7	5.2	7	8.1	8.3	7.9	6.8	5.5	4.1	2.6	1.2

Expand T as a Fourier series upto first harmonics.

Soln: Here the interval is $0 \le \theta \le 2\pi$ & we need to find a_0, a_1 & b_1 and corres-ponding formulas are given by,

$$a_0 = \frac{2}{N} \Sigma y \qquad a_1 = \frac{2}{N} \Sigma T \cos\theta$$

$$b_1 = \frac{2}{N} \Sigma T \sin\theta$$

$$\boxed{N = 12}$$

Consider the following table.

θ	T	$\cos\theta$	$T\cos\theta$	$\sin\theta$	$T\sin\theta$
0	0	1	0	0	0
30°	2.7	0.866	2.3382	0.5	~~1.317~~ 1.35
60°	5.2	0.5	2.6	0.866	~~2.252~~ 4.503
90°	7	0	0	1	~~0~~ 7
120°	8.1	-0.5	-4.05	0.866	~~-3.507~~ 7.015
150°	8.3	-0.866	-7.187	0.5	~~3.6~~ 4.15
180°	7.9	-1	-7.9	0	0
210°	6.8	-0.866	-5.89	-0.5	-3.4
240°	5.5	-0.5	-2.75	-0.866	-4.763
270°	4.1	0	0	-1	-4.1
330	2.6	0.5	1.3	-0.866	-2.25
330	1.2	0.866	1.04	-0.5	-0.6
	$\Sigma T =$		$\Sigma T\cos\theta =$		$\Sigma T\sin\theta =$
	+59.4		-20.499		8.9032

Now we have from the table,

$$a_0 = \frac{2}{N} \Sigma T$$

$$= \frac{2}{12} \times 59.4$$

$$\boxed{\frac{a_0}{2} = 4.95}$$

$a_1 = \frac{2}{N} \Sigma T \cos\theta$

$= \frac{2}{12} \times -20.499$

$\boxed{a_1 = -3.4165}$

$b_1 = \frac{2}{N} \Sigma T \sin\theta$

$= \frac{2}{12} \times 8.9032$

$\boxed{b_1 = 1.483}$

∴ The F.S upto first harmonic is given by,

$f(x) = \frac{a_0}{2} + (a_1 \cos x + b_1 \cos x)$

$\boxed{f(x) = 4.95 + (-3.4165)x + (1.483)\sin x}$

3. Express y as a F.S upto second harmonics given that

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

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Soln:

Here, the interval is $0 \le x \le 2\pi$
(i.e, the value of y at $x=0$ & $x=2\pi$
are same).

Hence neglate one of them.

let us consider only the values of x as $0°, 60°, 120°, 180°, 240°, 300°$.

we have the F.S upto second harmonic is,

$$f(x) = \frac{a_0}{2} + (a_1 \cos x + b_1 \sin x) + (a_2 \cos 2x + b_2 \sin 2x)$$

where,

$$a_0 = \frac{2}{N} \Sigma y$$

$$a_1 = \frac{2}{N} \Sigma y \cos x, \quad a_2 = \frac{2}{N} \Sigma y \, \cancel{\sin x} \cos 2x$$

$$b_1 = \frac{2}{N} \Sigma y \sin x, \quad b_2 = \frac{2}{N} \Sigma y \sin 2x.$$

$\boxed{N=7}$

x	y	$y\cos x$	$y\sin x$	$y\cos 2x$	$y\sin 2x$
0°	1.98	1.98	0	1.98	0
60°	1.30	0.65	1.126	-0.68	1.126
120°	1.05	-0.525	0.909	-0.525	-0.909
180°	1.30	-1.3	0	1.3	0
240°	-0.88	0.44	0.762	+0.44	-0.762
300°	-0.25	-0.125	0.216	+0.125	0.216
	$\Sigma y =$	$\Sigma y\cos x =$	$\Sigma y\sin x =$	$\Sigma y\cos 2x =$	$\Sigma y\sin 2x =$
	4.5	1.12	3.013	2.64	-0.329

(1)

$$a_0 = \frac{2}{N}\Sigma y$$

$$= \frac{2}{6} \times 4.5$$

$$\frac{a_0}{2} = \cancel{0.64}\ 0.75$$

$$\boxed{\frac{a_0}{2} = 0.75}$$

$$a_1 = \frac{2}{N}\Sigma y\cos x$$

$$= \frac{2}{6} \times 1.12$$

$$\boxed{a_1 = 0.3733}$$

$$b_1 = \frac{2}{N}\Sigma y\sin x$$

$$= \frac{2}{6} \times 3.013$$

$$\boxed{b_1 = 1.0043}$$

$$a_2 = \frac{2}{N} \sum y \cos 2x$$

$$= \frac{2}{6} \times 2.64$$

$$\boxed{a_2 = 0.88}$$

$$b_2 = \frac{2}{N} \sum y \sin 2x$$

$$= \frac{2}{6} \times -0.329$$

$$\boxed{b_2 = -0.10966}$$

$\therefore$ The F.S upto second harmonic is given by,

$$\boxed{f(x) = 0.75 + [(0.3733)\cos x + (1.0043)\sin x] + [(0.88)\cos 2x + (-0.10966)\sin 2x]}$$

4. Obtain the constant term & the co-efficients of the 1st cosine & sine in the F.S expansion of y from the table,

x	0	1	2	3	4	5	
y	9	18	24	28	26	20	

Soln: Here, the interval is $0 \le x \le 6$ &
the F.S upto first harmonic is given by,

$$f(x) = \frac{a_0}{2} + \left[a_1 \cos\left(\frac{\pi x}{l}\right) + b_1 \sin\left(\frac{\pi x}{l}\right)\right] \quad \text{---} (1)$$

comparing the given interval $(0,6)$ to $(0,2l)$ we get,

$$2l = 6 \Rightarrow \boxed{l = 3}$$

$$f(x) = \frac{a_0}{2} + a_1 \cos\left(\frac{\pi x}{3}\right) + b_1 \sin\left(\frac{\pi x}{3}\right)$$

let, $\frac{\pi x}{3} = \theta$

$$f(x) = \frac{a_0}{2} + a_1 \cos\theta + b_1 \sin\theta$$

x	y	$\theta = \pi x/3$	$y\cos\theta$	$y\sin\theta$
0	9	0	9	0
1	18	60°	1.1269	15.58
2	24	120°	-12	20.78
3	28	180°	-28	0
4	26	240°	-13	-22.5
5	20	300°	10	-17.32
	$\Sigma y =$		$\Sigma y\cos\theta =$	$\Sigma y\sin\theta =$
	125		-25	-3.460

$$a_0 = \frac{2}{N}\Sigma y \qquad a_1 = \frac{2}{N}\Sigma y\cos\theta$$

$$= \frac{2}{6}\times 125 \qquad = \frac{2}{6}\times(-25)$$

$$\boxed{\frac{a_0}{2} = 20.833} \qquad \boxed{a_1 = -8.33}$$

$$b_1 = \frac{2}{N}\Sigma y\sin\theta$$

$$= \frac{2}{6}(-3.460)$$

$$\boxed{b_1 = -1.1546}$$

$\therefore$ The F.S is given by,

$$f(x) = 20.833 + (-8.33)\cos\theta + (-1.1546)\sin\theta$$ //.