22-06-22

Module - III

Numerical method - I

In this module the 1st order 1st degree diff eqn of the form,

$$\frac{dy}{dx} = f(x, y)$$

if the initial condn $y(x_0) = y_0$. which is also called initial value problems.

There are 5 methods which are as follows:

1. Taylor's series method
2. Modified Euler's method
3. Range - kutta method of order 4
4. Milne's method
5. Adams - Bashforth method.

1. Taylor's series method:

Consider, the initial value problem,

$\frac{dy}{dx} = f(x, y)$ & $y(x_0) = y_0$. Then,

the taylor's series expansion of $y(x)$ about the point x_0 is given by,

$$y(x) = y(x_0) + (x - x_0) y'(x_0) + \frac{(x - x_0)^2}{2} y''(x_0) + \frac{(x - x_0)^3}{3} y'''(x_0) + - -$$

Assign

1. Use Taylor's series method to find y at $x = 0.1, 0.2, 0.3$, consider terms upto the third degree given that,

$$\frac{dy}{dx} = x^2+y^2 \quad \& \quad y(0) = 1$$

Sol: Given that,

$$\frac{dy}{dx} = y' = x^2+y^2 \quad \& \quad y(0) = 1$$

$\boxed{x_0 = 0}$, $\boxed{y_0 = 1}$

we have,

$$y(x) = y(x_0) + (x-x_0)\,y'(x_0) + \frac{(x-x_0)^2}{2!}\,y''(x_0) + \frac{(x-x_0)^3}{3!}\,y'''(x_0) + \cdots \quad (0)$$

$$y(x) = y(0) + x\,y'(0) + \frac{x^2}{2!}\,y''(0) + \frac{x^3}{3!}\,y'''(0) \quad (1)$$

we have,

$$y' = x^2+y^2 \Rightarrow y'(0) = x_0^2 + y_0^2 = 0+1 = 1$$

$\boxed{y'(0) = 1}$

Diff w.r.t x

$$y'' = 2x + 2yy'$$

$$y''(0) = 2(0) + 2(1)(1)$$

$\boxed{y''(0) = 2}$

Again diff w.r.t x

$y''' = 2 + 2[yy'' + y' \cdot y']$

$= 2 + 2[1(2) + 2(2)]$

$= 2 + 2[2 + 4]$

$= 2 + 2(3)$

$= 2 + 6$

$\boxed{y'''(0) = 8}$

Sub values in (1)

$y(x) = 1 + x(1) + \frac{x^2}{2!}(2) + \frac{x^3}{3!}(8)$

$= 1 + x + x^2 + \frac{x^3 \times 8}{6}$

$\boxed{y(x) = 1 + x + x^2 + \frac{4x^3}{3}}$

required Taylor's series.

Put $x = 0.1, 0.2, 0.3$

$y(0.1) = 1 + 0.1 + (0.1)^2 + \frac{4(0.1)^3}{3}$

$= 1 + 0.1 + 0.01 + \frac{4(0.001)}{3}$

$= 1 + 0.1 + 0.01 + 0.00133$

$\boxed{y(0.1) = 1.1113}$

$$y(0.2) = 1+0.2+\frac{(0.2)^2}{\cancel{2}}(2)+\frac{4(0.2)^3}{3}$$

$$= 1+0.2+0.04+\frac{0.008\times 4}{3}$$

$$= 1+0.2+0.04+0.01066$$

$$\boxed{y(0.2) = 1.2507}$$

$$y(0.3) = 1+0.3+(0.3)^2+\frac{4(0.3)^3}{3}$$

$$= 1+0.3+0.09+0.036$$

$$\boxed{y(0.3) = 1.426}$$

23-06-22

2. Find y at $x=1.02$ correct to 5 decimal places. given that, $dy=(xy-1)\,dx$ & $y=2$ at $x=1$ By applying Taylor's series method.

Soln) we have Taylor's series expansion as,

$$y(x) = y(x_0)+(x-x_0)\,y'(x_0)+\frac{(x-x_0)^2}{2!}\,y''(x_0)$$

$$+ \cdots \quad \text{--- (1)}$$

Given that,

$$dy = (xy-1)\,dx$$

$$\frac{dy}{dx} = y \quad \Rightarrow \cancel{8}\ \cancel{x}$$

$$y' = (xy-1)$$

$y_0 = 2$ $\quad x_0 = 1$ $\quad \boxed{y(1) = 2}$

we have,

$$y(x) = y(1) + (x-1)\,y'(1) + \frac{(x-1)^2}{2!}\,y''(1)$$

$$+ \frac{(x-1)^3}{3!}\,y'''(1) \quad \text{—— (1)}$$

we have,

$y' = xy - 1 \Rightarrow y'(1) = x_0 y_0 - 1$

$y'(1) = 1(2) - 1$

$\boxed{y'(1) = 1}$

Diff w.r.t x

$y'' = (xy' + y) - 0$

$y'' = xy' + y$

$y''(1) = x_0 y'(1) + y(1)$

$= 1(1) + 2$

$\boxed{y''(1) = 3}$

again diff w.r.t x

$y''' = xy'' + y'.1 + y'$

$= 1(3) + 1 + 1$

$= 3 + 2$

$\boxed{y'''(1) = 5}$

sub all values in (i)

$$y(x) = 2 + (x-1)(1) + \frac{(x-2)^2}{2} \times 3 + \frac{(x-1)^3}{6} \times 5$$

$$= \cancel{2+(x-1)} + \frac{x^2 \cancel{+1-2x} \times 3}{\cancel{2}}$$

$$= 2 + (1.02-1) + \frac{(1.02-1)^2}{2} \times 3 + \frac{(1.02-1)^3}{6} \times 5$$

$$= 2 + 0.02 + 2\times10^{-4} \times 3 + \frac{8\times10^{-6} \times 5}{6}$$

$$= 2 + 0.02 + 0.0002 \times 3 + 6.666 \times 10^{-6}$$

$$= 2 + 0.02 + 0.0006 + 0.00000666$$

$\boxed{y(1.02) = 2.0206066}$ $\simeq$ $y(1.02) = 2.02061$

3. use Taylor's series method, to obtain a power series in $(x-4)$ for the Eqn $5x\frac{dy}{dx} + y^2 - 2 = 0$; $x_0 = 4$, $y_0 = 1$

& use it to find y at $x = 4.1, 4.2, 4.3$ correct to 4 decimal places.

Soln: We have Taylor's series expansion as,

$$y(x) = y(x_0) + (x - x_0) y'(x_0) + \frac{(x-x_0)^2}{2!} y''(x_0)$$

$$+ \dots$$

Given:

$5xy' + y^2 - 2 = 0$, $x_0 = 4$, $y_0 = 1$

$\boxed{y(4) = 1}$

$$y(x) = y(4) + (x - x_0) y'(4) + \frac{(x-x_0)^2}{2!} y''(4)$$

$$+ \frac{(x-x_0)^3}{3!} y'''(4) \quad \text{——— (1)}$$

$$y' = \frac{2 - y_0^2}{5x_0}$$

$$= \frac{2-1}{5(4)}$$

$$= \frac{1}{20}$$

$\boxed{y'(4) = 0.05}$.

Diff w.r.t x

$y''(4) = 0 \Rightarrow 5[xy'' + y' \cdot 1] + 2yy' = 0$

$\Rightarrow 5[xy'' + y''] + 2yy' = 0$

$$5xy'' + 5y'' = -2yy'$$

$$5xy'' = -5y'' - 2yy'$$

$$y'' = \frac{-5y'' - 2yy'}{5x}$$

$$= \frac{-5[0.05] - 2(1)(0.05)}{5(4)}$$

$$y''(4) = \frac{-0.025 - 0.1}{20}$$

$$= \frac{-0.35}{20}$$

$$\boxed{y''(4) = -0.0175}$$

Sub values in (1)

$$y(4.1) = y(4) + (4.1-4)y'(4) + \frac{(4.1-4)^2}{2!} y''(4)$$

$$= 1 + (0.1)(0.05) + 0.005(-0.0175)$$

$$= 1 + 0.005 - 0.0000875$$

$$\boxed{y(4.1) = 1.0049125}$$

$$y(4.2) = y(4.4) + (4.2-4)y'(4) + \frac{(4.2-4)^2}{2!} y''(4)$$

$$= 1 + 0.2(0.05) + 0.02(-0.0175)$$

$= 1 + 0.01 + -0.00035$

$\boxed{y(4.2) = 1.00965} \approx 1.0097$

$y(4.3) = y(4) + (4.3-4)\,y'(4) + \frac{(4.3-4)^2}{2!}$

$y''(4)$

$= 1 + 0.3\,(0.05) + \frac{0.09}{2} \times (-0.0175)$

$= 1 + 0.015 - 0.0007875$

$= 1.015 - 0.0007875$

$\boxed{y(4.3) = 1.0142125}$ //.

24-06-22

Runge-Kutta method of fourth order.

$\frac{dy}{dx} = f(x_0, y_0) \qquad y(x_0) = y_0$

$y(x_0 + h) = ?$

$y[(x_0+h)] = y_0 + \frac{1}{6}\,[k_1 + 2k_2 + 2k_3 + k_4]$

where, $k_1 = hf(x_0, y_0)$

$k_2 = hf\left[x_0 + \frac{h}{2},\; y_0 + \frac{k_1}{2}\right]$

22-06-22

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$k_4 = hf\left(x_0 + h, y_0 + k_3\right)$$

1. consider the initial value problem $\frac{dy}{dx} = f(x, y)$ with an initial condn

$y(x_0) = y_0$. we need to find, $y(x_0 + h)$. where, h is step size. and is given by,

$$y(x_0+h) = y_0 + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$$

where,

$$k_1 = hf(x_0, y_0)$$

$$k_2 = hf\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right]$$

$$k_3 = hf\left[x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right]$$

$$k_4 = hf\left[x_0 + h, y_0 + k_3\right].$$

1. Given $\frac{dy}{dx} = \frac{3x+y}{2}$; $y(0) = 1$.

compute $y(0.2)$ by taking $h = 0.2$ using R-K method of fourth order.

Soln:

$$\frac{dy}{dx} = f(x, y) = \frac{3x+y}{2} \qquad y(0) = 1$$

$$x_0 = 0$$

$$y_0 = 1$$

$\boxed{h = 0.2}$ $\qquad y(x_0+h) = y(0.2) = ?$

$$y(x_0+h) = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4] \quad \text{--- (1)}$$

where,

$$k_1 = hf(x_0, y_0)$$

$$= (0.2) f(0, 1)$$

$$= 0.2 \left[3(0) + \frac{1}{2}\right]$$

$$k_1 = \frac{0.2}{2}$$

$$\boxed{k_1 = 0.1}$$

$$k_2 = hf\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right]$$

$$= (0.2)\, f\left[\frac{0+0.2}{2}, \frac{1+0.1}{2}\right]$$

$$= (0.2)\, f\,(0.1, 1.05)$$

$$= (0.2)\left[3(0.1) + \frac{1.05}{2}\right]$$

$$\boxed{k_2 = 0.165}$$

$$k_3 = hf\left[x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right]$$

$$= (0.2)\, f\left[\frac{0+0.2}{2}, 1 + \frac{0.165}{2}\right]$$

$$= 0.2\, f\left[0.1, 1 + 0.0825\right]$$

$$= 0.2\, f\left[0.1, 1.0825\right]$$

$$= 0.2\left[3(0.1) + \frac{1.0825}{2}\right]$$

$$\boxed{k_3 = 0.16825}$$

$$k_4 = hf\,(x_0 + h, y_0 + k_3)$$

$$= 0.2\, f\,(0+0.2, 1+0.16825)$$

$$= 0.2\, f\,(0.2, 1.16825)$$

$$= 0.2\left[3(0.2) + \frac{1.6825}{2}\right]$$

$$\boxed{k_4 = 0.2368}$$

sub k_1, k_2, k_3, k_4 in Eqn (1)

$$y(x_0+h) = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= 1 + \frac{1}{6}[0.1 + 2(0.165) + 2(0.16825) + 0.2368]$$

$$= 1 + \frac{1}{6}[0.1 + 0.33 + 0.3365 + 0.2368]$$

$$= 1 + \frac{1}{6}[1.003]$$

$$= 1 + 0.\cancel{1833}\ 0.1672$$

$$\boxed{y(x_0+h) = 1.1834} \Rightarrow 1.16721$$

2. use fourth order R-K method to solve $(x+y)\frac{dy}{dx} = 1$; $y(0.4) = 1$ at $x = 0.5$ correct to 4 decimal places.

soln: given:

$$\frac{dy}{dx} = f(x+y) = \frac{1}{x+y} \qquad y(0.4) = 1$$

$$y(x_0) = y_0$$

$$y(x_0+h) = y(0.5) = ? \qquad x_0 = 0.4$$

$x_0+h = 0.5$

$h = 0.5 - \cancel{x}_0$

$= 0.5 - 0.4$

$\boxed{h = 0.1}$

w.k.t

$$y(x_0+h) = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

where,

$k_1 = h f(x_0, y_0)$

$= 0.1\, f(0.4, 1)$

$= 0.1 \left[\frac{1}{0.4+1}\right]$

$= 0.1\, [0.\cancel{6}6714]$

$k_1 = 0.07143$

$$k_2 = h f\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right]$$

$$= 0.1 f\left[0.4 + \frac{0.1}{2}, 1 + \frac{0.0714}{2}\right]$$

$= 0.1 f[0.4 + 0.05, 1 + 0.0357]$

$= 0.1 f[0.45, 1.0357]$

$$k_2 = 0.1\left[\frac{1}{0.45 + 1.0357}\right]$$

$$= 0.1\left[\frac{1}{1.4857}\right]$$

$$= 0.1\,[0.67308]$$

$$\boxed{k_2 = 0.0673}$$

$$k_3 = hf\left[x_0 + \frac{h}{2},\ y_0 + \frac{k_2}{2}\right]$$

$$= 0.1\,f\left[0.4 + \frac{0.1}{2},\ 1 + \frac{0.0673}{2}\right]$$

$$= 0.1\,f\,[0.4 + 0.05,\ 1 + 0.0336]$$

$$= 0.1\,f\,[0.45,\ 1.0336]$$

$$= 0.1\left[\frac{1}{0.45 + 1.0336}\right]$$

$$= \frac{0.1}{1.4836}$$

$$\boxed{k_3 = 0.0674}$$

$$k_4 = hf(x_0 + h,\ y_0 + k_3)$$

$$= 0.1\,f(0.4 + 0.1,\ 1 + 0.0674)$$

$$= 0.1\,f(0.5,\ 1.0674)$$

$$= 0.1 \left[\frac{1}{0.5 + 1.0674} \right]$$

$$= \frac{0.1}{1.5674}$$

$$\boxed{k_4 = 0.0638}$$

Sub k_1, k_2, k_3, k_4 in ①

$$y(x_0+h) = 1 + \frac{1}{6} \left[0.07143 + 2(0.0673) + 2[0.0674] + 0.0638 \right]$$

$$= 1 + \frac{1}{6} \left[0.0714 + 0.1346 + 0.1348 + 0.0638 \right]$$

$$= 1 + \frac{0.4046}{6}$$

$$= 1 + 0.0674$$

$$\boxed{y(x_0+h) = 1.0674}$$

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28/06/22

3. use Fourth order Runge - kutta method, to find y at $x = 0.1$ given that $\frac{dy}{dx} = 3e^x + 2y$; $y(0) = 0$. & $h = 0.1$.

Soln Given that,

$\frac{dy}{dx} = f(x,y) = 3e^x + 2y$; $y(0) = 0$

$x_0 = 0$; $y_0 = 0$

$h = 0.1$

$y(x_0+h) = y(0+0.1) = y(0.1) = ?$

we have,

$y(x_0+h) = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$ ⟵ (1)

where,

$k_1 = hf(x_0, y_0)$

$= 0.1 f(0.0)$

$= 0.1 (3e^0 + 2(0))$

$= 0.3$

$\boxed{k_1 = 0.3}$

lly $k_2 = hf\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right]$

$= 0.1 f\left[0 + \frac{0.1}{2}, 0 + \frac{0.3}{2}\right]$

$= (0.1) f (0.05, 0.15)$

$= 0.1 [3e^{0.05} + 2(0.15)]$

$= 0.1 (3.153 + 0.3)$

$\boxed{k_2 = 0.3454}$

$k_3 = hf \left[x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right]$

$= 0.1 f \left[\frac{0 + 0.1}{2}, \frac{0 + 0.3454}{2}\right]$

$= 0.1 f [0.05, 0.1727]$

$= (0.1) [3e^{0.05} + 2 \times (0.1727)]$

$= 0.1 [3(0.15) + 0.3454]$

$= 0.1 [3.153 + 0.3454]$

$= 0.1 [3.4991]$

$\boxed{k_3 = 0.3498}$

$k_4 = hf [x_0 + h, y_0 + k_3]$

$= (0.1) f [0.1, 0.3498]$

$= (0.1) [3e^{0.1} + 2 \times 0.3498]$

$= (0.1) [3.3155 + 0.6996]$

$= 0.1 \times 4.0151$

$\boxed{k_4 = 0.4015}$

Date
Page

Sub all values in (1)

$y(x_0+h) = y_0 + \frac{1}{6}[k_1+2k_2+2k_3+k_4]$

$y(0.1) = 0 + \frac{1}{6}[0.3 + 2(0.345) + 2(0.3498) + 0.40153]$

$= \frac{1}{6}[0.3 + 0.6908 + 0.6996 + 0.4015]$

$= \frac{1}{6}[2.0919]$

$\boxed{y(0.1) = 0.3486}$

4. Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} + y = 2x$ at $x=1.1$, given that $y=3$ at $x=1$ initially.

Soln)

$\frac{dy}{dx} = f(x,y) = 2x-y;$

$x_0 = 1;$

$y_0 = 3;$

$y(x_0+h) = y(1.1) = ?$

$x_0+h = 1.1$

$h = 1.1 - x_0$

$h = 1.1 - 1$

$\boxed{h = 0.1}$

we have,

$$y(x_0 + h) = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4] \quad \text{--- (1)}$$

where,

$k_1 = hf(x_0, y_0)$

$= (0.1) f(0.4, 1)$

$= (0.1)\left[\frac{1}{0.4+1}\right]$

$= (0.1)(0.7142)$

$\boxed{k_1 = 0.07142}$

$$k_2 = hf\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right]$$

$$= (0.1) f\left[0.40 + \frac{0.1}{2}, 1 + \frac{0.0714}{2}\right]$$

$$= (0.1) f(\underset{x}{0.45}, \underset{y}{1.0357})$$

$$= (0.1)\left[\frac{1}{0.45 + 1.0357}\right]$$

$$= (0.1)\left[\frac{1}{1.4857}\right]$$

$$= (0.1)\ (0.6730)$$

$$\boxed{k_2 = 0.0673}$$

$$k_3 = hf\left[x_0 + \frac{h}{2},\ y_0 + \frac{k_2}{2}\right]$$

$$= (0.1) f\left[0.4 + \frac{0.1}{2},\ 1 + \frac{0.0673}{2}\right]$$

$$= (0.1) f\ (\underset{x}{0.45},\ \underset{y}{1.03365})$$

$$= 0.1\left[\frac{1}{0.45 + 1.03365}\right].$$

$$= (0.1)\ (0.6740)$$

$$\boxed{k_3 = 0.0674}$$

$$k_4 = hf\ (x_0 + h,\ y_0 + k_3)$$

$$= (0.1)\ f\ (0.4 + 0.1,\ 1 + 0.0674)$$

$$= (0.1)\ f\ (0.5,\ 1.0674)$$

$$= (0.1)\left[\frac{1}{0.5+1.0674}\right]$$

$$= (0.1)\left[\frac{1}{1.5674}\right]$$

$$= (0.1)\,(0.6379)$$

$$\boxed{k_4 = 0.0638}$$

Sub all values in (1)

$$y(x_0+h) = y_0 + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$$

$$= 1 + \frac{1}{6}\left[0.0714 + 2(0.0673) + 2(0.0674) + 0.0638\right]$$

$$= 1 + \frac{1}{6}\left[0.0714 + 0.1346 + 0.1348 + 0.0638\right]$$

$$= 1 + \frac{1}{6}(0.4046)$$

$$= 1 + 0.0674$$

$$\boxed{y(0.5) = 1.0674}$$

27-06-22.

Date
Page

Modified Euler's method.

Consider the initial value problem

$\frac{dy}{dx} = f(x, y)$; $y(x_0) = y_0$.

We need to find y at x_1 where, $x_1 = x_0 + h$.

We first obtain $y(x_1) = y_1$ by applying Euler's formula & is also called as initial approximation for y_1 & is denoted by, $y_1^{(0)}$ & is given by,

$$y_1^{(0)} = y_0 + h f(x_0, y_0)$$

Since, the accuracy is ~~poor~~ in this formula, so this value of y_1 is improved by applying the corrected formula also known as modified Euler's formula & is given by,

$$y_1^{(1)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(0)})]$$

$$y_1^{(2)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(1)})]$$

$$y_1^{(3)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(2)})]$$

- - - so on

1. Given $\frac{dy}{dx} = \frac{1+y}{x}$; $y=2$ at $x=1$
$\quad y_0 \quad x$

Find the approximate value of y at $x=1.4$ by taking stepsize $h=0.2$ applying modified euler's method.

Soln: This problem has to be solved in two steps:

Step 1: Given,

$$\frac{dy}{dx} = f(x,y) = \frac{1+y}{x}$$

$$x_0 = 1, \quad y_0 = 2, \quad h = 0.2$$

$$f(x_0, y_0) = \frac{1+y_0}{x_0}$$

$$= \frac{1+2}{1}$$

$$\boxed{f(x_0, y_0) = 3}$$

$$x_1 = x_0 + h$$

$$x_1 = 1 + 0.2$$

$$\boxed{x_1 = 1.2}$$

We have, Euler's formula as,

$$y_1^{(0)} = y_0 + h f(x_0, y_0)$$

$$= 2 + 0.2 (3)$$

$$\boxed{y_1^{(0)} = 2.6}$$

Now, we have modified Euler's formula as,

$$y_1^{(1)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(0)})]$$

$$= 2 + \frac{0.2}{2} \left[f(3) + 1 + \frac{y_1^{(0)}}{x_1} \right]$$

$$= 2 + 0.1 \left[3 + \frac{2.6}{1.2} \right]$$

$$\boxed{y_1^{(1)} = 2.6167}$$

Similarly,

$$y_1^{(2)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(1)})]$$

$$= 2 + \frac{0.2}{2} \left[3 + 1 + \frac{y_1^{(1)}}{x_1} \right]$$

$$= 2 + 0.1 \left[3 + 1 + \frac{2.6167}{1.2} \right]$$

$$= 2 + 0.1 \left[4 + \frac{2.6167}{1.2} \right] \Rightarrow 2.6181$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$$

$$= 2 + \frac{0.2}{2}\left[3 + 1 + \frac{y_1^{(2)}}{x_1}\right]$$

$$= 2 + 0.1\left[4 + \frac{2.6181}{1.2}\right]$$

$$\boxed{y_1^{(3)} = 2.6181}$$

$$\therefore\ y(1.2) = 2.6181$$

Step 2: Consider $y(1.2) = 2.6181$ as an initial approximation i.e,

$$x_0 = 1.2;\quad y_0 = 2.6181$$

$$f(x_0, y_0) = 1 + \frac{y_0}{x_0}$$

$$x = x_0 + h$$

$$= 1 + \frac{2.6181}{1.2}$$

$$x_1 = 1.2 + 0.2$$

$$\boxed{x_1 = 1.4}$$

$$\boxed{f(x_0, y_0) = 3.1817}$$

we have Euler's formula as,

$$y_1^{(0)} = y_0 + hf(x_0, y_0)$$

$$= 2.6181 + 0.2\,(3.1817)$$

$$\boxed{y_1^{(0)} = 3.2544}$$

Now, we have modified euler's formula as,

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

$$= 2.6181 + \frac{0.2}{2}\left[3.1817 + 1 + \frac{y_1^{(0)}}{x_1}\right]$$

$$= 2.6181 + 0.1\left[4.1817 + \frac{3.2544}{1.4}\right]$$

$$\boxed{y_1^{(1)} = 3.2687}$$

$$\text{11ly}, \quad y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$= 2.6181 + \frac{0.2}{2}\left[3.1817 + 1 + \frac{y_1^{(1)}}{x_1}\right]$$

$$= 2.6181 + 0.1\left[4.1817 + \frac{3.2687}{1.4}\right]$$

$$= 2.6181 + 0.1\left[4.1817 + 2.3347\right]$$

$$\boxed{y_1^{(2)} = 3.2698}$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$$

$$= 2.6181 + \frac{0.2}{2}\left[4.1817 + \frac{y_1^{(2)}}{x_1}\right]$$

$$= 2.6181 + 0.1\left[\frac{4.1817 + 1 + 3.2698}{1.4}\right]$$

$$\boxed{y_1^{(3)} = 3.2698}$$

$$\boxed{\therefore y(1.4) = 3.2698}$$

30-06-22

1. Using modified Euler's method find $y(20.2)$, given that, $\frac{dy}{dx} = \log_{10}\left(\frac{x}{4}\right)$ with $y(20) = 5$. [taking $h = 0.2$]

Soln:-

$$\frac{dy}{dx} = f(x,y) = \log_{10}\left(\frac{x}{4}\right);$$

$y(20) = 5$ $\quad x_0 = 20, \; y_0 = 5.$

$h = 0.2$

we need to complete $y[20.2] = ?$

$x_1 = x_0 + h$

$= 20 + 0.2$

$$\boxed{x_1 = 20.2}$$

$$f(x_0, y_0) = \log_{10}\left(\frac{x_0}{y_0}\right)$$

$$= \log_{10}\left(\frac{20}{5}\right)$$

$$= \log_{10} 4$$

$$\boxed{f(x_0, y_0) = 0.6021}$$

we have Euler's formula,

$$y_1^{(0)} = y_0 + h f(x_0, y_0)$$

$$= 5 + (0.2)(0.6021)$$

$$\boxed{y_1^{(0)} = 5.1204}$$

we have by modified Euler's formula,

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

$$= 5 + \frac{0.2}{2}\left[0.6021 + \log_{10}\left(\frac{x_1}{y_1^{(0)}}\right)\right]$$

$$= 5 + 0.1\left[0.6021 + \log_{10}\left(\frac{20.2}{5.1204}\right)\right]$$

$$y_1^{(1)} = 5.1198$$

IIIly,

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$= 5 + \frac{0.2}{2}\left[0.6021 + \log_{10}\left(\frac{x}{y(1)}\right)\right]$

$= 5 + 0.1\left[0.6021 + \log_{10}\left[\frac{20.2}{5.1198}\right]\right]$

$\boxed{y_1^{(2)} = 5.1198}$

$\therefore y(20.2) = 5.1198$

3. Using Euler's predictor & corrector formula, compute $y(1.1)$ correct to 5 decimal places given that,

$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$ & $y = 1$ at $x = 1$.

Soln:

Given that,

$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2} \Rightarrow \frac{dy}{dx} = \frac{1}{x^2} - \frac{y}{x}$

$\boxed{f(x, y) = \frac{1 - yx}{x^2}}$

$y_0 = 1$

$x_0 = 1.$

let us take $h=0.1$ & $x_1 = x_0 + h$

$= 1 + 0.1$

$\boxed{x_1 = 1.1}$

we need to find $y(x_1) = y(1.1) = y_1 = ?$

$$f(x_0, y_0) = \frac{1 - x_0 y_0}{x_0^2}$$

$$= \frac{1-1}{1}$$

$$= 0$$

we have by euler's formula,

$$y_1^{(0)} = y_0 + h f(x_0, y_0)$$

$$= 1 + 0.1(0)$$

$$y_1^{(0)} = 1$$

we have by modified euler's formula,

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

$$= 1 + \frac{0.1}{2}\left[0 + \frac{1 - x_1 y_1^{(0)}}{x_1^2}\right]$$

$$= 1 + \frac{0.1}{2}\left[\frac{1 - 1.1(1)}{1}\right]$$

$$y_1^{(1)} = 0.9959$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f(x_1,y_1^{(1)})\right]$$

$$= 1 + \frac{0.1}{2}\left[0 + \frac{1 - x_1 y_1^{(1)}}{x_1^2}\right]$$

$$= 1 + \frac{0.1}{2}\left[\frac{1 - (1.1)(0.9959)}{(1.1)^2}\right]$$

$$y_1^{(2)} = 0.99605$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f(x_1,y_1^{(2)})\right]$$

$$= 1 + \frac{0.1}{2}\left[\frac{1 - x_1 y_1^{(2)}}{x_1^2}\right]$$

$$= 1 + \frac{0.1}{2}\left[\frac{1 - (1.1)(0.99605)}{(1.1)^2}\right]$$

$$y_1^{(3)} = 0.99605$$

$$\boxed{\therefore y(1.1) = 0.99605}$$

5-06-22

Milen's Predictor & corrector method.

consider the diff eqn,

$$\bar{y} = \frac{dy}{dx} = f(x,y)$$

which, a set of 4 pre-determined values of y.

$y(x_0) = y_0$

$y(x_1) = y_1$

$y(x_2) = y_2$

$y(x_3) = y_3$.

Here, x_0, x_1, x_2, x_3 are equally spaced values of x with width h.

Here we need to find $y(x_4) = y_4 = ?$ and the corresponding formula are given by,

y_4 Milne's Predictor formula.

$$y_4^{(p)} = y_0 + \frac{4h}{3}\left[2y_1' - y_2' + 2y_3'\right]$$

Milne's corrector formula.

$$y_4^{(c)} = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right].$$

1. Given that $\frac{dy}{dx} = x - y^2$ & the data

$y(0) = 0$; $y(0.2) = 0.02$; $y(0.4) = 0.0795$; $y(0.6) = 0.1762$ compute y at $x = 0.8$ by applying Milne's method.

Soln:-

let us prepare a table for given data, $h = 0.2$

x	y	$y' = x - y^2$
0	$y_0 = 0$	$y' = x_0 - y_0^2 = 0$
0.2	$y_1 = 0.02$	$y_1' = x_1 - y_1^2 = 0.1996$
0.4	$y_2 = 0.0795$	$y_2^2 = (0.4) - (0.0795)^2 = 0.3936$
0.6	$y_3 = 0.1762$	$y_3^2 = (0.6) - (0.1762)^2 = 0.5689$
0.8	$y_4 = ?$	?

we have milne's Predictor formula as,

$$y_4^{(P)} = y_0 + \frac{4h}{3} [2y_1' - y_2' + 2y_3']$$

$$= 0 + \frac{4(0.2)}{3} [2(0.1996) - 0.3936 + 2(0.5689)]$$

$$y_4^{(P)} = 0.3049$$

Next,

$y_4' = x_4 - y_4^2 = (0.8) - (0.3049)^2$
$= 0.707$

Now, we have milne's corrector formula as,

$y_4^{(c)} = y_2 + \frac{h}{3} [y_2' + 4y_3' + y_4']$

$= 0.0795 + \frac{0.2}{3} [0.3936 + 4(0.5689) + 0.707]$

$\boxed{y_4^{(c)} = 0.3046}$

Next,

$y_4' = x_4 - (y_4)^2 = 0.8 - (0.3046)^2$
$= 0.7072.$

Again apply corrector formula,

$y_4^{(c)} = y_2 + \frac{h}{3} [y_2' + 4y_3' + y_4']$

$= 0.0795 + \frac{0.2}{3} [0.3936 + 4(0.5689) + 0.7072]$

$\boxed{y_4^{(c)} = 0.3046}$ $\therefore y_4(0.8) = 0.3046$

6.07.22

2. Apply Milne's method to compute $y(1.4)$ correct to 4 decimal places given that, $\frac{dy}{dx} = \frac{x^2 + y}{2}$ & the following data., $(y(1) = 2$; $y(1.1) = 2.2156$, $y(1.2) = 2.469$; $y(1.3) = 2.7514$.

Soln:

let us prepare a table for given data, $\boxed{h = 0.1}$

x	y	$y' = x^2 + y/x$
$x_0 = 1$	$y_0 = 2$	$y_0' = 1 + 2/2 \Rightarrow 1+0.5 = 1.5$ 2
$x_1 = 1.1$	$y_1 = 2.2156$	$y_1' = (1.1)^2 + 2.2156/1.1 = 3.2241$
$x_2 = 1.2$	$y_2 = 2.4696$	$y_2' = (1.2)^2 + 2.4696/1.2 = 3.498$
$x_3 = 1.3$	$y_3 = 2.7514$	$y_3' = (1.3)^2 + 2.7514/1.3 = 3.8064$
$x_4 = 1.4$	?	

x	y	$y' = x^2 + \frac{y}{2}$
$x_0 = 1$	$y_0 = 2$	$y'_0 = 1^2 + 2/2 = 2$
$x_1 = 1.1$	$y_1 = 2.2156$	$y'_1 = (1.1)^2 + \frac{2.2156}{2} = 2.3178$
$x_2 = 1.2$	$y_2 = 2.4696$	$y_2^2 = 2.675$
$x_3 = 1.3$	$y_3 = 2.7514$	$y_3' =$
$x_4 = 1.4$	$y_4 = ?$	

$$y_1' = (1.1)^2 + \frac{2.2156}{2}$$
$$= 1.21 + 1.1078$$
$$y_1' = 2.3178$$

$$y_2^2 = (1.2)^2 + \frac{2.4696}{2}$$
$$= 1.44 + 1.2348$$
$$y_2' = 2.6725$$

$$y_3' = (1.3)^2 + \frac{2.7514}{2}$$
$$1.69 + 1.3757$$
$$y_3' = 3.0657$$

$$y_4' = ?$$

we have the milne's Predictor formula as,

$$y_4^{(P)} = y_0 + \frac{4h}{3}\left[2y_1' - y_2' + 2y_3'\right]$$

$$= 2 + \frac{4(0.1)}{3}\left[2(2.3178) - 2.6225 + 2(3.0657)\right]$$

$$= 2 + 0.1333\left[8.092\right]$$

$$= 2 + 1.078663$$

$$\boxed{y_4^{(P)} = 3.07983}$$

Next,

$$y_4' = x_4^2 + \frac{y_4}{2}$$

$$= (1.4)^2 + \frac{3.0793}{2}$$

$$= 1.96 + 1.5396$$

$$\boxed{y_4' = 3.49965}$$

we have by milne's corrector formula,

$$y_4^{(C)} = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right]$$

$$= 2.4649 + \frac{0.1}{3}[2.6725 + 4(3.0657) + 3.49965]$$

$$= 2.4649 + 0.0333[18.43495]$$

$$= 2.4649 + 0.61\cancel{38}4498$$

$$y_4^{(c)} = 3.0794$$

next,

$$y_4' = x_4^2 + \frac{y_4}{2}$$

$$= (1.4)^2 + \frac{3.0794}{2}$$

$$= 1.96 + 1.5397$$

$$\boxed{y_4' = 3.4997}$$

we have by milne's corrector formula,

$$y_4^{(c)} = y_2 + \frac{h}{3}[y_2' + 4y_3' + y_4']$$

$$= 2.4649 + \frac{0.1}{3}[2.6725 + 4(3.0657) + 3.4997)$$

$$= 2.4649 + 0.0333(18.435)$$

$$\boxed{y_4^{(c)} = 3.0794}$$

3. If $\frac{dy}{dx} = 2e^{x} - y$; $y(0) = 2$; $y(0.1) = 2.010$; $y(0.2) = 2.040$ & $y(0.3) = 2.090$

Find $y(0.4)$ correct to 4 decimal places using milne's predictor corrector method.

Sol:

let us consider the following ~~data~~, table, $\boxed{h = 0.1}$

	x	y	$y' = 2e^{x} - y$
$x_0 =$	0	$y_0 = 2$	$y_0' = 2(1) - 2 = 0$
$x_1 =$	0.1	$y_1 = 2.010$	$y_1' = 2(1.10517) - 2.010 = 0.20034$
$x_2 =$	0.2	$y_2 = 2.040$	$y_2' = 2(1.2214) - 2.04 = 0.4028$
$x_3 =$	0.3	$y_3 = 2.090$	$y_3' = 2(1.34985) - 2.090 = 0.6097$
$x_4 =$	0.4	$y_4 = ?$	?,

$$y_4' = 2e^{x} - y_4$$
$$= 2e^{(0.4)} - 2.16226$$

$$\boxed{y_4' = 0.821384}$$

we have milne's Predictor formula,

$$y_4^{(p)} = y_0 + \frac{4h}{3}\left[2y_1' - y_2' + 2y_3'\right]$$

$$= 2 + \frac{4(0.1)}{3}\left[2(0.20034) - 0.4028 + 2(0.6097)\right]$$

$$= 2 + 0.1333\left[1.21728\right]$$

$$= 2 + 0.162263$$

$$\boxed{y_4^{(p)} = 2.16226} \approx 2.1623$$

Now, we have, milne's corrector formula,

$$y_4^{(c)} = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right]$$

$$= 2.040 + \frac{0.1}{3}\left[0.4028 + 4(0.6097) + 0.821389\right]$$

$$= 2.040 + 0.03333\left[3.66298\right]$$

$$y_4^{(c)} = 2.1621$$

Next,

$$y_4' = 2e^{x} - y_4$$

$$= 2e^{(0.4)} - 2.1621$$

$$= 2.9836 - 2.1621$$

$$y_4' = 0.8215$$

again apply milne's corrector formula,

$$y_4^{(c)} = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right]$$

$$= 2.040 + \frac{0.1}{3}\left[0.4028 + 4(0.6097) + 0.8215\right]$$

$$= 2.040 + 0.0333\,(3.6631)$$

$$= 2.040 + 0.12198$$

$$\boxed{y_4^{(c)} = 2.1621}$$

2. The following table is the solution of $5xy' + y^2 - 2 = 0$. find the value of y at $x = 4.5$ using milne's P-C formula. use the corrector formula twice.

x	4	4.1	4.2	4.3	4.4
y	1	1.0049	1.0097	1.0143	1.0187

Given,

$$5xy' + y^2 - 2 = 0$$

$$5xy' = 2 - y^2$$

$$\boxed{y' = \frac{2 - y^2}{5x}}$$

let us consider the following table.

x	y	$y' = \dfrac{2-y^2}{5x}$
$x_0 = 4$	$y_0 = 1$	$y_0' = \dfrac{2-(1)^2}{5(4)} = 0.05$
$x_1 = 4.1$	$y_1 = 1.0049$	$y_1' = \dfrac{2-(1.0049)^2}{5(4.1)} = 0.0483$
$x_2 = 4.2$	$y_2 = 1.0097$	$y_2' = \dfrac{2-(1.0097)^2}{5(4.2)} = 0.0467$
$x_3 = 4.3$	$y_3 = 1.0143$	$y_3' = \dfrac{2-(1.0143)^2}{5(4.3)} = 0.0452$
$x_4 = 4.4$	$y_4 = 1.0187$	$y_4' = \dfrac{2-(1.0187)^2}{5(4.4)} = 0.0437$
$x_5 = 4.5$	?	?

$\boxed{h = 0.1}$

we have by milne's predictor formula,

$$y_4^{(P)} = y_0 + \frac{4h}{3}[2y_1' - y_2' + 2y_3']$$

III^{ly},

$$y_5^{(P)} = y_1 + \frac{4h}{3}[2y_2' - y_3' + 2y_4']$$

$$= 1.0049 + \frac{4(0.1)}{3}[2(0.0467) - 0.0452 + 2[0.0437]]$$

$$= 1.0049 + 0.1333[0.1356]$$

$$= 1.0049 + 1.018079$$

$$\boxed{y_5^{(P)} = 1.02298}$$

next,

$$y_5' = \frac{2 - y^2}{5x_5}$$

$$= \frac{2 - (1.02298)^2}{5(4.5)}$$

$$\boxed{y_5' = 0.0424}$$

we have milne's correction formula,

$$y_5^{(c)} = y_3 + \frac{h}{3}[y_3' + 4y_4' + y_5']$$

$$= 1.0143 + \frac{0.1}{3}[0.0452 + 4(0.0437) + 0.0494]$$

$$= 1.0143 + 0.0333[0.2694]$$

$$\boxed{y_5^{(c)} = 1.023}$$

$$y_5' = \frac{2-y^2}{5x_5}$$

$$= \frac{2-(1.023)^2}{5(4.5)}$$

$$\boxed{y_5' = 0.0424}$$

IIIy, again apply milne's correction formula,

$$y_5^{(c)} = y_3 + \frac{h}{3}[y_3' + 4y_4' + y_5']$$

$$= 1.0143 + \frac{0.1}{3}[0.0452 + 4(0.0437) + 0.0424]$$

$$y_5^{(c)} = 1.023$$

$$\boxed{\therefore y(4.5) = 1.023}$$

Adams - Bashforth Prediction - Correction formula.

$$y_4^{(P)} = y_3 + \frac{h}{24}\left[55y_3' - 59y_2' + 37y_1' - 9y_0'\right]$$

$$y_4^{(C)} = y_3 + \frac{h}{24}\left[9y_4' + 19y_3' - 5y_2' + y_1'\right]$$

Problems

1. Apply Adams - Bashforth method to solve the eqn $(y^2+1)dy - x^2dx = 0$ at $x=1$ given, $y(0)=1$; $y(0.25) = 1.0026$; $y(0.5) = 1.0206$; $y(0.75) = 1.0679$.

Soln:

$(y^2+1)dy - x^2dx$.

$(y^2+1)dy = x^2dx$.

$$\frac{dy}{dx} = \frac{x^2}{y^2+1}$$

$$\boxed{y' = \frac{x^2}{y^2+1}}$$

let us consider the following table.

x	y	$y' = \frac{x^2}{y^2+1}$	
$x_0 = 0$	$y_0 = 1$	$y_0' = 0$	
$x_1 = 0.25$	$y_1 = 1.0026$	$y_1' = \frac{(0.25)^2}{(1.0026)^2+1}$	$= 0.0312$
$x_2 = 0.5$	$y_2 = 1.0206$	$y_2' = \frac{(0.5)^2}{(1.0206)^2+1}$	$= 0.1225$
$x_3 = 0.75$	$y_3 = 1.0679$	$y_3' = \frac{(0.75)^2}{(1.0679)^2+1}$	$= 0.2628$
$x_4 = 1$	?		

we have, by A-B predictor formula,

$$y_4^{(p)} = y_3 + \frac{h}{24}\left[55y_3' - 59y_2' + 37y_1' - 9y_0'\right]$$

$$= 1.0679 + \frac{0.25}{24}\left[55(0.2628) - 59(0.1225) + 37(0.0312) - 0\right]$$

$= 1.0679 + 0.0104166\,[8.3809]$

$= 1.0679 + 0.08730$

$\boxed{y_4^{(P)} = 1.1552}$

Now,

$y_4' = \dfrac{x_4^2}{y_4^2+1} = \dfrac{1}{(1.1552)^2+1} = 0.4284$

next, we have by A-B corrector formula,

$y_4^{(C)} = y_3 + \dfrac{h}{24}\,[9y_4' + 19y_3' - 5y_2' + y_1']$

$= 1.0679 + \dfrac{0.25}{24}\,[9(0.4284) + 19(0.2628)$

$-5(0.1225) + 0.0312]$

$= 1.0679 + 0.0104166\,[8.2675]$

$y_4^{(C)} = 1.15401 \simeq 1.154$

Now,

$y_4' = \dfrac{x_4^2}{y_4^2+1} = \dfrac{1}{(1.154)^2+1} = 0.4288$

$$y_4^{(c)} = y_3 + \frac{h}{24}\left[9y_4' + 19y_3' - 5y_2' + y_1'\right]$$

$$= 1.0679 + \frac{0.25}{24}\left[9(0.4288) + 19(0.2698) - 5(0.1995) + 0.0312\right]$$

$$= 1.0679 + 0.01046\,[8.9711]$$

$$\boxed{y_4^{(c)} = 1.154}$$

$$\boxed{\therefore y(1) = 1.154}$$

2. Solve the differential equation, $y' + y + xy^2 = 0$ with the initial values of y: $y_0 = 1$; $y_1 = 0.9008$; $y_2 = 0.8066$; $y_3 = 0.792$ corresponding to the values of x as, $x_0 = 0$, $x_1 = 0.1$, $x_2 = 0.2$, $x_3 = 0.3$; by computing the value of y corresponding to $x = 0.4$ by applying A-B predictor & corrector formula!

soln:

$y' + y + xy^2 = 0$

$y' = -(y + xy^2)$

let us consider the following table.

x	y	$y' = -(y+xy^2)$
$x_0 = 0$	$y_0 = 1$	$y_0' = -[0+1] = -1$
$x_1 = 0.1$	$y_1 = 0.9008$	$y_1' = -[0.9008 + (0.1)(0.9008)^2] = -0.98194$
$x_2 = 0.2$	$y_2 = 0.8066$	$y_2' = -[0.8066 + (0.2)[0.8066]^2] = -0.93672$
$x_3 = 0.3$	$y_3 = 0.722$	$y_3' = -[0.722 + (0.3)(0.722)^2] = -0.8784$
$x_4 = 0.4$	?	?

84.64375

we have by A-B prediction formula,

$$y_4^{(P)} = y_3 + \frac{h}{24}\left[55y_3' - 59y_2' + 37y_1' - 9y_0'\right]$$

$$= 0.722 + \frac{0.1}{24}\left[55(-0.8784) - 59(-0.93672) + 37(-0.98194) - 9(-1)\right]$$

6648

$$= 0.722 + 0.004166\left[-84.64378 + 64.93108\right]$$

$$= 0.722 + 0.004166\left[\cancel{-20.4127}\right]\left[-20.3778\right]$$

$= 0.722 - 0.08489$

$y_4^{(P)} = 0.6371$

$y_4' = -[y_4 + x y_4^2]$

$= -[0.6371 + (0.4)(0.6371)^2]$

$\boxed{y_4' = -0.799456}$

Now, we have A-B corrector formula,

$y_4^{(C)} = y_3 + \frac{h}{24}[9y_4' + 19y_3' - 5y_2' + y_1']$

$= 0.722 + \frac{0.1}{24}[9(-0.799456) + 19(-0.8784) - 5(-0.93672) + (-0.9819$

$= 0.722 + 0.004166 [-29.5502 + 4.6836]$

$= 0.722 - 0.084091$

$\boxed{y_4^{(C)} = 0.6379}$

$y_4' = -[y_4 + x y_4^2]$

$= -[0.6379 + (0.4)(0.6379)^2]$

$y_4' = -[0.80066]$

$$y_4^{(c)} = y_3 + \frac{h}{24}\left[9y_4' + 19y_3' - 5y_2' + y_1'\right]$$

$$= 0.722 + \frac{0.1}{24}\left[9(-0.80066) + 19(-0.8781)\right.$$

$$\left. - 5(-0.93672) + (-0.9819)\right]$$

$$= 0.722 + 0.004166\left[-19.8699 + 4.6836\right]$$

$$24.87744 +$$

$$= 0.722 + 0.004166\left[-8.1879\right]$$

$$-20.19384$$

$$= 0.722 - 0.08412$$

$$\boxed{y_4^{(c)} = 0.6379}$$

$$\boxed{\therefore y(0.4) = 0.6379}$$