# Soln Q1: Accuracy Question

## Q1 (i): Check for significant difference

To check if there is any significant difference between the vendor's and the agency's data, we perform a paired t-test.

**Step 1: State the Hypotheses**

- Null Hypothesis ($H_0$): $\mu_d = 0$ (The mean difference between the agency and vendor measurements is zero).
- Alternative Hypothesis ($H_1$): $\mu_d \neq 0$ (The mean difference ist not zero).

  Thus, it is a Two-tailed t-test.

**Step 2: Calculate the Differences**

The difference between the vendor and agency measurements is given in Table 1.

**Step 3: Calculate the Mean and Standard Deviation of Differences**

The differences are given in Table 1. To find the mean difference ($\bar{d}$) and the standard deviation ($s_d$):

$$\bar{d} = \frac{\sum d_i}{n} = \frac{-0.2 + 0.1 - 0.1 + 0.1 + 0.3 + 0.3 + 0.2}{19}$$
$$+ \frac{0.4 + 0.0 + 0.0 + 0.2 - 0.2 + 0.0 + 0.0 + 0.3 + 0.0 + 0.3 + 0.2 + 0.1}{19}$$
$$\approx 0.105$$

$$s_d = \sqrt{\frac{\sum(d_i - \bar{d})^2}{n-1}} \approx 0.175$$

**Step 4: Compute the t-statistic**

$$t = \frac{\bar{d}}{s_d/\sqrt{n}} = \frac{0.105}{0.175/\sqrt{19}} \approx 2.615$$

**Step 5: Critical t-value and Conclusion**

| S.No | IRI_Agency | IRI_Vendor | Difference ($d_i$) |
|---|---|---|---|
| 1 | 1.6 | 1.4 | -0.2 |
| 2 | 6.4 | 6.5 | 0.1 |
| 3 | 8.0 | 7.9 | -0.1 |
| 4 | 3.2 | 3.3 | 0.1 |
| 5 | 4.1 | 4.4 | 0.3 |
| 6 | 3.4 | 3.7 | 0.3 |
| 7 | 3.4 | 3.6 | 0.2 |
| 8 | 2.2 | 2.6 | 0.4 |
| 9 | 5.0 | 5.0 | 0.0 |
| 10 | 4.6 | 4.6 | 0.0 |
| 11 | 2.4 | 2.6 | 0.2 |
| 12 | 5.6 | 5.4 | -0.2 |
| 13 | 4.1 | 4.1 | 0.0 |
| 14 | 2.0 | 2.0 | 0.0 |
| 15 | 3.1 | 3.4 | 0.3 |
| 16 | 4.6 | 4.6 | 0.0 |
| 17 | 2.8 | 3.1 | 0.3 |
| 18 | 4.9 | 5.1 | 0.2 |
| 19 | 5.0 | 5.1 | 0.1 |

Table 1: Differences between Agency and Vendor Measurements

For a two-tailed test at a 95% confidence level ($\alpha = 0.05$), with 18 degrees of freedom, the critical t-value is $t_{\alpha/2} = 2.101$. Since $t = 2.615$ is greater than 2.101, we reject the null hypothesis. There is a significant difference between the vendor's and agency's measurements.

## Q1 (ii): Check for potential bias

To check for bias, we compute the confidence interval for the mean difference. If the reference value ($\mu_d = 0$) lies within the confidence interval, we conclude there is no bias; otherwise, there is bias.

**Step 1: Calculate the Confidence Interval**

The confidence interval for the mean difference ($\bar{d}$) is calculated as:

$$CI = \bar{d} \pm t_{\alpha/2} \left( \frac{s_d}{\sqrt{n}} \right)$$

Where:

- $\bar{d} = 0.105$ is the mean difference,
- $s_d = 0.175$ is the standard deviation of differences,
- $n = 19$ is the sample size,
- $t_{\alpha/2} = 2.101$ is the critical t-value for a 95% confidence level with 18 degrees of freedom.

Thus, the confidence interval is:

$$CI = 0.105 \pm 2.101 \left(\frac{0.175}{\sqrt{19}}\right)$$

$$CI = 0.105 \pm 0.084$$

$$CI = [0.021, 0.189]$$

**Step 2: Conclusion**

Since the reference value ($\mu_d = 0$) does not lie within the confidence interval $[0.021, 0.189]$, we conclude that there is a significant bias between the vendor's and the agency's measurements.

To determine whether the vendor's measurements tend to be higher or lower than the agency's measurements, we look at the sign of the mean difference ($\bar{d}$). Since $\bar{d} = 0.105$ is positive, this suggests that, on average, the vendor's measurements are higher than those of the agency. This indicates a tendency toward overestimation by the vendor's equipment.

## Q1 (iii): Accuracy Check (±5% Rule)

The agency's requirement states that the vendor's measurements should be within ±5% of the agency's measurements.

To check if the vendor's equipment is within the acceptable accuracy range, we calculate the percentage error for each data point as follows:

$$\text{Percentage Error} = \left|\frac{\text{IRI\_Vendor} - \text{IRI\_Agency}}{\text{IRI\_Agency}}\right| \times 100$$

The percentage errors for each data point are calculated below:

| S.No | IRI_Agency | IRI_Vendor | Percentage Error (%) | Within 5%? |
|---|---|---|---|---|
| 1 | 1.6 | 1.4 | $\left\|\frac{1.4-1.6}{1.6}\right\| \times 100 = 12.5\%$ | No |
| 2 | 6.4 | 6.5 | $\left\|\frac{6.5-6.4}{6.4}\right\| \times 100 = 1.56\%$ | Yes |
| 3 | 8.0 | 7.9 | $\left\|\frac{7.9-8.0}{8.0}\right\| \times 100 = 1.25\%$ | Yes |
| 4 | 3.2 | 3.3 | $\left\|\frac{3.3-3.2}{3.2}\right\| \times 100 = 3.13\%$ | Yes |
| 5 | 4.1 | 4.4 | $\left\|\frac{4.4-4.1}{4.1}\right\| \times 100 = 7.32\%$ | No |
| 6 | 3.4 | 3.7 | $\left\|\frac{3.7-3.4}{3.4}\right\| \times 100 = 8.82\%$ | No |
| 7 | 3.4 | 3.6 | $\left\|\frac{3.6-3.4}{3.4}\right\| \times 100 = 5.88\%$ | No |
| 8 | 2.2 | 2.6 | $\left\|\frac{2.6-2.2}{2.2}\right\| \times 100 = 18.18\%$ | No |
| 9 | 5.0 | 5.0 | $\left\|\frac{5.0-5.0}{5.0}\right\| \times 100 = 0.00\%$ | Yes |
| 10 | 4.6 | 4.6 | $\left\|\frac{4.6-4.6}{4.6}\right\| \times 100 = 0.00\%$ | Yes |
| 11 | 2.4 | 2.6 | $\left\|\frac{2.6-2.4}{2.4}\right\| \times 100 = 8.33\%$ | No |
| 12 | 5.6 | 5.4 | $\left\|\frac{5.4-5.6}{5.6}\right\| \times 100 = 3.57\%$ | Yes |
| 13 | 4.1 | 4.1 | $\left\|\frac{4.1-4.1}{4.1}\right\| \times 100 = 0.00\%$ | Yes |
| 14 | 2.0 | 2.0 | $\left\|\frac{2.0-2.0}{2.0}\right\| \times 100 = 0.00\%$ | Yes |
| 15 | 3.1 | 3.4 | $\left\|\frac{3.4-3.1}{3.1}\right\| \times 100 = 9.68\%$ | No |
| 16 | 4.6 | 4.6 | $\left\|\frac{4.6-4.6}{4.6}\right\| \times 100 = 0.00\%$ | Yes |
| 17 | 2.8 | 3.1 | $\left\|\frac{3.1-2.8}{2.8}\right\| \times 100 = 10.71\%$ | No |
| 18 | 4.9 | 5.1 | $\left\|\frac{5.1-4.9}{4.9}\right\| \times 100 = 4.08\%$ | Yes |
| 19 | 5.0 | 5.1 | $\left\|\frac{5.1-5.0}{5.0}\right\| \times 100 = 2.00\%$ | Yes |

Out of the 19 data points, only 11 have percentage errors within the acceptable range of $\pm 5\%$. Since not all data points meet the accuracy requirement, the vendor's equipment does not fully satisfy the agency's $\pm 5\%$ accuracy rule.

## Soln Q2: Precision Question

The goal is to determine if the standard deviation of the rutting measurements for five sections meets the agency's acceptance criterion of a standard deviation less than or equal to 1.25 mm. We will perform a chi-square test for variance. Assumption given, the data follows a normal distribution.

### Step 1: State the Hypotheses

- Null Hypothesis ($H_0$): The standard deviation of the measurements is less than or equal to 1.25 mm ($\sigma \leq 1.25$).
- Alternative Hypothesis ($H_1$): The standard deviation of the measurements is greater than 1.25 mm ($\sigma > 1.25$).

## Step 2: Calculate the Standard Deviation

The rutting data (in mm) for the five sections is provided in Table 2.

| Section | Rutting (mm) |
|---|---|
| 1 | 10.79 |
| 2 | 13.94 |
| 3 | 12.69 |
| 4 | 8.63 |
| 5 | 11.25 |

Table 2: Rutting Data for Five Sections

The mean ($\bar{x}$) of the data is calculated as:

$$\bar{x} = \frac{10.79 + 13.94 + 12.69 + 8.63 + 11.25}{5} = 11.46 \text{ mm}$$

Next, we calculate the sample variance ($s^2$):

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

$$s^2 = \frac{(10.79 - 11.46)^2 + (13.94 - 11.46)^2 + (12.69 - 11.46)^2 + (8.63 - 11.46)^2 + (11.25 - 11.46)^2}{5-1}$$

$$s^2 \approx \frac{0.4489 + 6.1476 + 1.5121 + 8.0196 + 0.0484}{4} = \frac{16.1766}{4} = 4.0441 \text{ mm}^2$$

The standard deviation ($s$) is:

$$s = \sqrt{4.0441} \approx 2.01 \text{ mm}$$

## Step 3: Chi-square Test Calculation

We now perform a chi-square test to determine if the calculated standard deviation exceeds the agency's threshold of 1.25 mm.

The chi-square statistic is calculated using the formula:

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

Where:

- $n = 5$ is the sample size,
- $s^2 = 4.0441$ mm$^2$ is the sample variance,
- $\sigma_0 = 1.25$ mm is the threshold standard deviation (so $\sigma_0^2 = 1.25^2 = 1.5625$ mm$^2$).

Substituting the values:

$$\chi^2 = \frac{(5-1) \times 4.0441}{1.5625} = \frac{4 \times 4.0441}{1.5625} \approx 10.35$$

The degrees of freedom for this test are $df = n - 1 = 4$.

## Step 4: Critical Value and Conclusion

For a right-tailed test with 4 degrees of freedom at a significance level of 5% ($\alpha = 0.05$), the critical chi-square value ($\chi^2_{0.05,4}$) found from chi-square distribution table is:

$$\chi^2_{0.05,4} = 9.488$$

Since the calculated $\chi^2 = 10.35$ is greater than the critical value of 9.488, we reject the null hypothesis.

The standard deviation of 2.01 mm exceeds the agency's threshold of 1.25 mm. Therefore, the precision of the measurement method does not meet the acceptance criterion.