Numerical simulation of the self-propulsive motion of a fish-like swimmer using δ^+ - SPH scheme

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Supervised Learning Project - Presentation

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Outline

Problem Statement

- Reproduce the work done on self-propulsive fish-like swimming hydrodynamics by Sun, P.N, et al.[1], and replicate the results
- Implement their work systematically, based on the two approaches that are laid out

[1] Peng-Nan Sun, Andrea Colagrossi, and A-Man Zhang. "Numerical simulation of the self-propulsive motion of a fishlike swimming foil using the δ -SPH model". In: *Theoretical and Applied Mechanics Letters* 8.2 (2018), pp. 115–125. DOI: 10.1016/j.taml.2018.02.007.

Approach 1

1. The flapping swimmer is fixed (*as is the traditional approach*)
2. A uniform flow is set-up (*of velocity U*)
3. Measure the forces acting on the swimmer
4. Force in the upstream direction implies the capability of the swimmer to swim forward

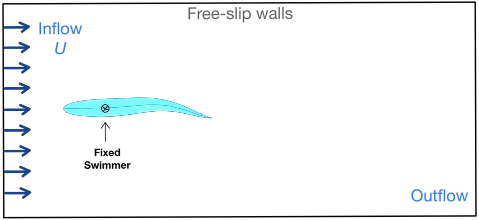


Figure 1: Approach - 1

Approach 1

1. The numerical validation of the simulation is performed at a flow regime of $Re = 5000$
2. *The solution is compared against that of a traditional Navier-Stokes solver*
3. The solution based on the δ^+ - SPH scheme, is performed at different particle resolutions:
   $L/\Delta x = \{50, 100, 200, 400\}$
4. The simulations are checked for the convergence of the swimming velocity
   - Based on these results and the trade-offs (computational time, numerical accuracy), a suitable particle resolution for the entire experiment is decided

Approach 2

1. The flapping swimmer is free (*a more realistic model*)
2. The fluid is still (*has no initial velocity*)
3. Measure the forces acting on the swimmer at each time-instant
4. Solve the equations of motion of the swimmer (based on the forces), to propagate it spatially as well

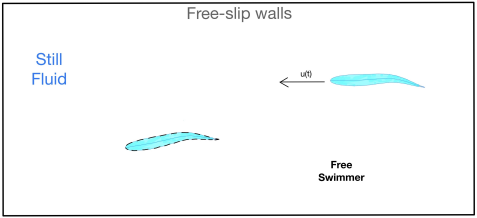


Figure 2: Approach - 2

Approach 2

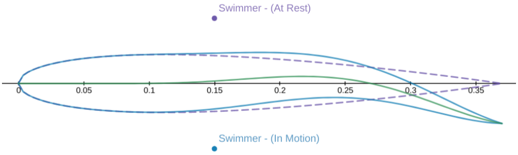
1. The simulations are checked for the convergence of the swimming velocity as well as the swimmer's position
2. The vorticity contours are studied:

   Transient State: Vorticity generated by the flapping foil deflects to the upper part of the flow region

   Steady State: The distribution of vorticity is located in a straight line behind the foil

Swimmer

(a) *Pollachius virens* - Saithe



(b) Swimmer model

Outline

Motivation

1. To explore the optimum hydrodynamic performance of the motion of a fish underwater
2. Current man-made underwater propulsion systems have drawbacks and are inefficient
3. Can lead to developments in field of stealth propulsion
4. Further our understanding of the bionic propulsion of the natural world

Figure 4: Cavitation Erosion

Outline

Preliminary Reading

1. Studied the basics of Smoothed Particle Hydrodynamics[2][3]
2. Solved problems provided by Prof. Prabhu Ramachandran to better understand the concept
3. Solved the Dam-Break problem using the WCSPH scheme on:
   - A custom implementation using Python
   - The PySPH framework

[2] Gui-Rong Liu and Moubin B Liu. *Smoothed particle hydrodynamics: a meshfree particle method*. World scientific, 2003.

[3] Joe J Monaghan. "Smoothed particle hydrodynamics". In: *Annual review of astronomy and astrophysics* 30.1 (1992), pp. 543–574.

δ^+ - SPH Scheme

1. Worked on implementing the δ^+ - SPH Scheme, consisting of the:
    1. Continuity Equation - 2, 3, 4
    2. Momentum Equation - 5
    3. Equation of State - 1
    4. Particle Shifting Technique - 6
2. Roadblocks:
    1. Did not thoroughly explore the PySPH framework - *WCSPH scheme*
    2. Formulations that were required and written from scratch, had already been implemented in the framework
    3. More time could have been devoted to analyzing the results of the benchmark problems, and working towards the original problem statement

Outline

Taylor-Green Vortex

1. The Taylor-Green Vortex (TGV) problem was taken up to study the δ^{+} - SPH scheme on a simple boundary-less problem
2. The solution was compared with the other available schemes - WCSPH, TVF, & EDAC

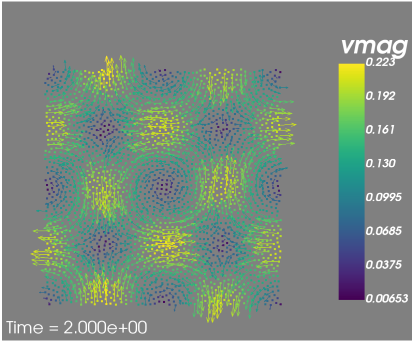
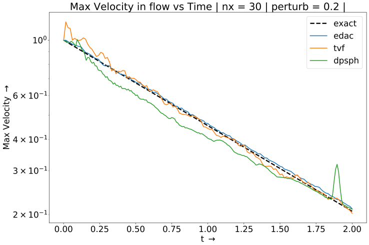
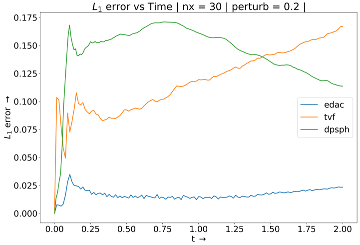
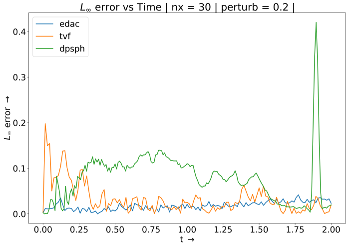


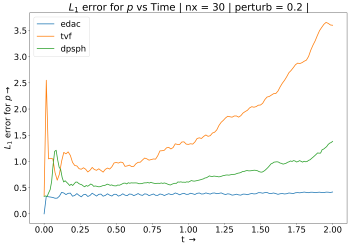
Figure 5: Taylor-Green Vortex

TGV - Results *(Low resolution, perturbed particles)*

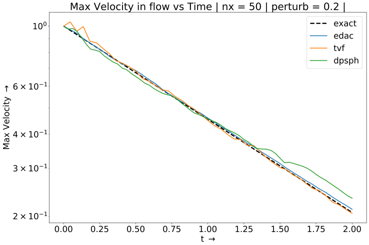


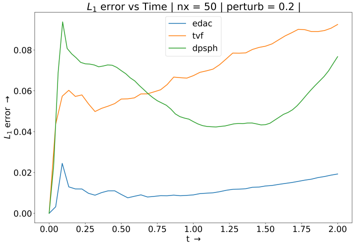


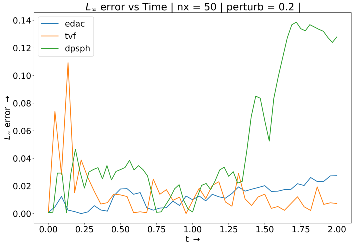


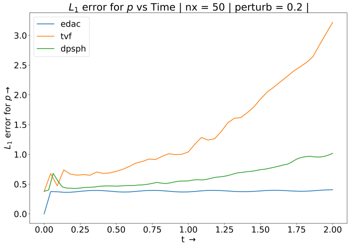


TGV - Results *(High resolution, perturbed particles)*









TGV - Runtime

1. The average of three run-times, of each scheme was taken
2. δ^+ had the longest run-time when the simulation was run at a lower resolution
3. δ^+ had a run-time lower than EDAC at higher resolution, while maintaining accuracy which was slightly below-par compared to EDAC - *needs further analysis*

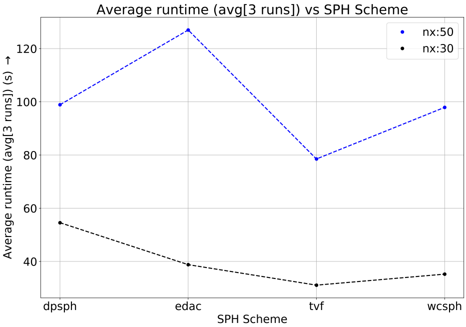


Figure 8: Taylor-Green Vortex

Outline

Conclusion

1. The δ^+ - SPH scheme is observed to perform better in terms of accuracy than TVF and WCSPH, falling short of EDAC
2. These results are valid for only the simple boundary-less problem, that is the Taylor-Green Vortex
3. The δ^+ scheme needs to be implemented on complex benchmark problems, involving:
    - Boundaries
    - Fluid-Solid interactions

Future Work

1. The currently implemented δ^+ scheme can be optimized, as well as validated by making use of the already available PySPH implementations
2. The adaptive particle refinement (APR) was not explored, which should be included in future studies
3. The differences between the Monaghan formulation[4] and, Bouscasse, B, et al. formulation[5] of the forces and torques that act on a fluid-solid system should be studied
4. Once the underlying scheme is in place, some of the open-ended questions from the original paper can be tackled

[4] Joseph J Monaghan. "Smoothed particle hydrodynamics and its diverse applications". In: *Annual Review of Fluid Mechanics* 44 (2012), pp. 323–346.

[5] B. Bouscasse et al. "Nonlinear water wave interaction with floating bodies in SPH". In: *Journal of Fluids and Structures* 42 (2013), pp. 112–129. DOI: 10.1016/j.jfluidstructs.2013.05.010.

Thank You

Equation of State

$$p_i = c_o^2(\rho_i - \rho_o) \tag{1}$$

where,

1. ρ_o - The reference density when pressure is zero initially
2. c_o - The artificial speed that is based on the weakly-compressible hypothesis [11]. Here, c_o is a constant in the whole simulation and it is determined as: $15U$, where U is the reference velocity [10]

Renormalization Tensor

$$\mathbb{L}_i = \left[\sum_j \mathbf{r_{ji}} \otimes \nabla_{\mathbf{i}} \mathbf{W_{ij}} V_j \right]^{-1} \tag{2}$$

$$\lambda_i = \min\big(\text{eigenvalue}(\mathbb{L}_i^{-1})\big) \tag{3}$$

Refer - [5, 11]

Continuity Equation

$$\frac{D\rho_i}{Dt} = \sum_j \rho_i \mathbf{u_{ij}}.\nabla_\mathbf{i} \mathbf{W_{ij}} V_j + \delta h c_o \Psi_{ij} \frac{\mathbf{r_{ji}}.\nabla_\mathbf{i} \mathbf{W_{ij}}}{|\mathbf{r_{ji}}|^2} V_j \tag{4}$$

where,

$$\Psi_{ij} = 2(\rho_j - \rho_i) \tag{5}$$

$$V_j = \frac{m_j}{\rho_j} \tag{6}$$

1. δ - The density diffusion parameter $(= 0.1)$ [10]

Refer - [1]

Continuity Equation - RDGC

$$\frac{D\rho_i}{Dt} = \sum_j \rho_i \mathbf{u_{ij}}.\nabla_\mathbf{i} \mathbf{W_{ij}} V_j + \delta h c_o \Psi_{ij} \frac{\mathbf{r_{ji}}.\nabla_\mathbf{i}\mathbf{W_{ij}}}{|\mathbf{r_{ji}}|^2} V_j \tag{7}$$

where,

$$\Psi_{ij} = 2(\rho_j - \rho_i) - (\langle\nabla\rho\rangle_\mathbf{i}^\mathbf{L} + \langle\nabla\rho\rangle_\mathbf{j}^\mathbf{L}).\mathbf{r_{ji}} \tag{8}$$

$$\langle\nabla\rho\rangle_\mathbf{i}^\mathbf{L} = \sum_j (\rho_j - \rho_i)\mathbb{L}_i.\nabla_\mathbf{i}\mathbf{W_{ij}} V_j \tag{9}$$

This equation includes the renormalized density gradient correction (RDGC) term as well [4, 2, 11]

Momentum Equation

$$\frac{D\mathbf{u_i}}{Dt} = \frac{1}{\rho_i} \sum_j \left(F_{ij} \nabla_{\mathbf{i}} \mathbf{W_{ij}} V_j + K \mu \pi_{ij} \nabla_{\mathbf{i}} \mathbf{W_{ij}} V_j \right) + \mathbf{f_i} \tag{10}$$

where,

$$F_{ij} = \begin{cases} -(p_j + p_i), & p_i \geq 0 \\ -(p_j - p_i), & p_i < 0 \end{cases} \tag{11}$$

$$K = 2(\text{dim} + 2) \tag{12}$$

$$\pi_{ij} = \frac{\mathbf{u_{ji}} . \mathbf{r_{ji}}}{|\mathbf{r_{ji}}|^2} \tag{13}$$

1. $\mathbf{f_i}$ - Body-forces
2. μ - The dynamic viscosity ($\mu = \rho_o \nu$), where ν is the kinematic viscosity [2, 10]

Particle Shifting Technique

Once the particle positions are advected through the time, a repositioning is performed as follows [9]:

$$\mathbf{r_i}^* = \mathbf{r_i} + \delta\hat{\mathbf{r}}_\mathbf{i} \tag{14}$$

$$\delta\hat{\mathbf{r}}_\mathbf{i} = \begin{cases} 0 & ,\lambda_i \in [0, 0.4) \\ (\mathbb{I} - \mathbf{n_i} \otimes \mathbf{n_i})\delta\mathbf{r_i} & ,\lambda_i \in [0.4, 0.75] \\ \delta\mathbf{r_i} & ,\lambda_i \in (0.75, 1] \end{cases} \tag{15}$$

$$\delta\mathbf{r_i} = -CFL.Ma.(2h_{ij})^2.\sum_j \left[1 + R\left(\frac{W_{ij}}{W(\Delta s_i)}\right)^n\right]\nabla_\mathbf{i}\mathbf{W_{ij}}\varphi_{ij}\left(\frac{m_j}{\rho_i + \rho_j}\right) \tag{16}$$

$$\mathbf{n_i} = \frac{\langle\nabla\lambda_\mathbf{i}\rangle}{|\langle\nabla\lambda_\mathbf{i}\rangle|} \tag{17}$$

$$\langle\nabla\lambda_\mathbf{i}\rangle = -\sum_j(\lambda_j - \lambda_i)\mathbb{L}_i\nabla_\mathbf{i}\mathbf{W_{ij}}V_j \tag{18}$$

Particle Shifting Technique

1. $\Delta s_i =$ Average particle spacing in the neighbourhood of i
2. $R = 0.2, n = 4$, the same values as suggested by Monaghan [8] are used in $\delta+$ SPH scheme as well
3. $\varphi = 1, h_{ij} = h$, for uniform particle resolution
4. $\frac{\delta \mathbf{r_i}}{\Delta x_i} < 0.05$, using a Wendland C2 kernel with $\frac{h}{\Delta x_i} = 2$
5. $\frac{\delta \mathbf{r_i}}{\Delta x_i} \rightarrow 0$ when $\Delta x_i \rightarrow 0$

M. Antuono et al. "Free-surface flows solved by means of SPH schemes with numerical diffusive terms". In: *Computer Physics Communications* 181.3 (2010), pp. 532–549. DOI: 10.1016/j.cpc.2009.11.002.

B. Bouscasse et al. "Nonlinear water wave interaction with floating bodies in SPH". In: *Journal of Fluids and Structures* 42 (2013), pp. 112–129. DOI: 10.1016/j.jfluidstructs.2013.05.010.

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