Turbulence Modelling for Smoothed Particle Hydrodynamics

K T Prajwal Prathiksh

Dual Degree Project (Stage I) - Presentation

Supervisor: Prof. Prabhu Ramachandran

Department of Aerospace Engineering
Indian Institute of Technology Bombay

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Introduction

- **Turbulent Flow**
  - Characterization
    - High Reynolds number flows
    - Random spatial & temporal velocity fluctuations
    - Rotational & 3D velocity field
    - Large mixing capacity of the flow
    - Chaotic nature of solutions
  - Modelling
    - Lacks analytical solutions except for simple cases
    - CFD simulations required for complex flows
    - Typical modelling techniques
      - Governing equations → Averaged or filtered
      - Closure problem → Fluctuating components modelled using mean flow properties
      - Stochastic methods
    - Models mostly based on Eulerian framework

- **Lagrangian Modelling – SPH**
  - No background mesh
  - Handles large deformations, complex boundary dynamics
  - Simplified model implementation
  - Highly & efficiently parallelizable

Project Objectives

- Turbulence Modelling in SPH
  - Lacks robust & accurate models
  - Shortcomings of current models:
    - Cannot be generalized to various types of turbulence-based problems
    - Issues in scaling to 3D turbulent flows
      - Accuracy & computational constraints
    - Boundary conditions & treatment not well established

Existing Models

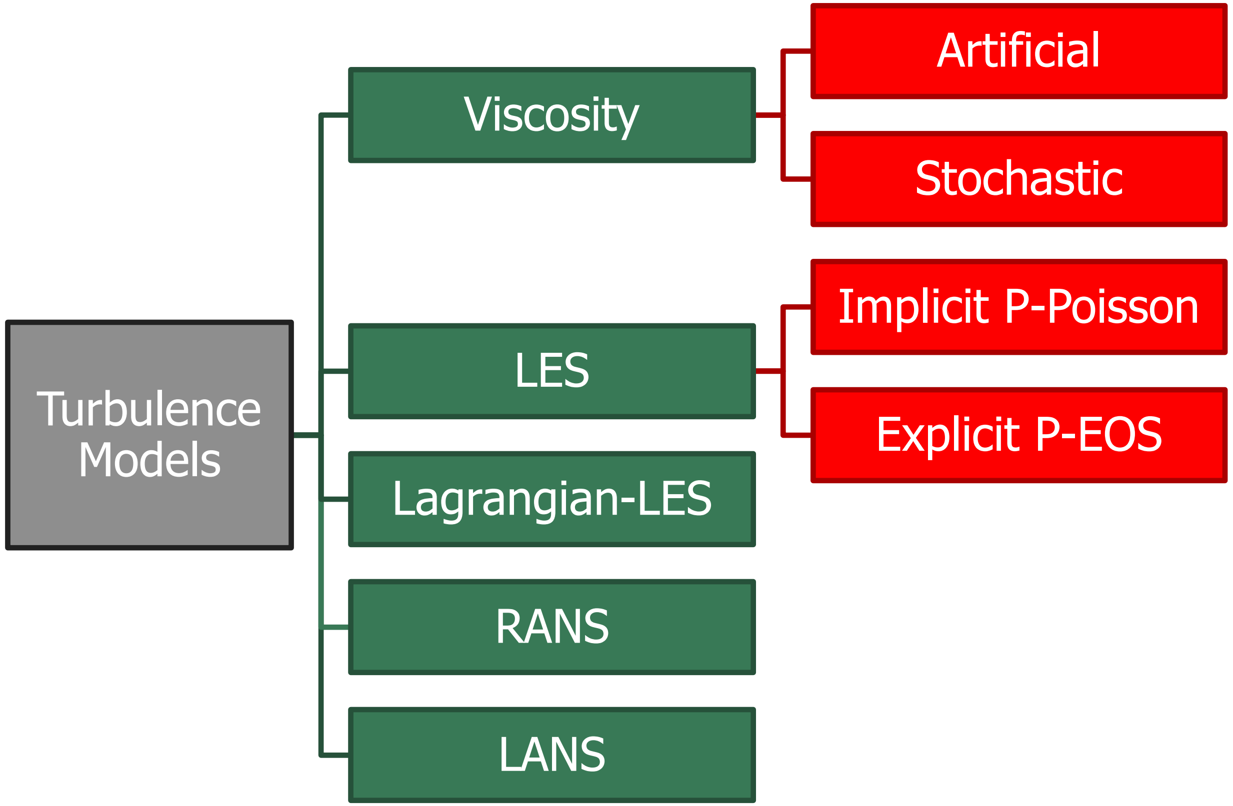
- Project Objective
  - Review state of the art turbulence models in SPH
  - Provide advantages & disadvantages of the major categories of models
  - Extend promising models to robust & accurate SPH schemes[1]
  - Analyze & study its performance in bounded turbulent flow
  - Improve overall scheme using recent developments of SPH

Expected Outcome

[1] P. Negi and P. Ramachandran, "How to train your solver: A method of manufactured solutions for weakly compressible smoothed particle hydrodynamics," Phys. Fluids, vol. 33, no. 12, 2021, doi: 10.1063/5.0072383.

Project's Progress

- Surveyed research papers detailing work on turbulence models for SPH *(c. 2000 – 2022)*
- Classified 5 major categories of turbulence models
- Identified a systematic, model-evaluation method
  - Benchmark problems feasible with SPH
  - Post-simulation analysis techniques



Turbulence Modelling

Viscosity-based Models

- Eddy Viscosity Model[1]
  - One of the first classes of SPH turbulence models
  - Velocities are Reynolds-averaged
  - First-order closure model:
    - Reynolds stress tensor & mean velocity gradients
  - Turbulent eddy viscosity considered
  - Simulated Poiseuille flow $(Re = 6.4 \times 10^4)$

- Comments
  - Reproduced log-law profile near walls
  - Appropriate for shear flows

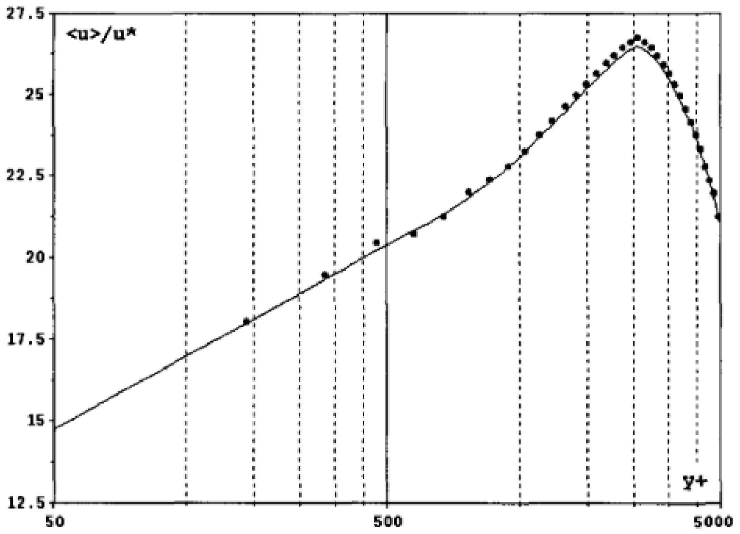


Fig: Computed mean velocity profile of turbulent Poiseuille flow in a pipe (Rep: Violeau et al)

$$\frac{\mathrm{D}\,\mathbf{v}_i}{\mathrm{D}\,t} = -\sum_j m_j \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} + \widetilde{\Pi}_{ij} \right) \nabla_i W_{h,ij} + \mathbf{F}_i$$

$$\widetilde{\Pi}_{ij} = -8 \frac{\nu_{t,i} + \nu_{t,j}}{\rho_i + \rho_j} \frac{\langle \mathbf{v} \rangle_{ij} \cdot \mathbf{r}_{ij}}{|\, \mathbf{r}_{ij} \,|^2 + \boldsymbol{\xi}^2}$$

$$\nu_t = L_m^2 ||\underline{\boldsymbol{S}}||_F = L_m^2 \sqrt{\langle \underline{\boldsymbol{S}}, \underline{\boldsymbol{S}} \rangle}$$

$$\nabla \langle \mathbf{v} \rangle_i = -\frac{1}{\rho_i} \sum_j m_j \langle \mathbf{v} \rangle_{ij} \otimes \nabla_i W_{h,ij}$$

[1] D. VIOLEAU, S. PICCON, and J.-P. CHABARD, “TWO ATTEMPTS OF TURBULENCE MODELLING IN SMOOTHED PARTICLE HYDRODYNAMICS,” in Advances in Fluid Modeling and Turbulence Measurements, Jul. 2002, vol. 1, no. 5, pp. 339–346, doi: 10.1142/9789812777591_0041.

Viscosity-based Models

- Generalized Langevin Model[1]
  - Prescribed the particle velocity as a random process
  - Acceleration → Function of a random vector *(non-correlated with velocity)*
  - Simulated Poiseuille flow $(Re = 6.4 \times 10^4)$
  - Comments
    - Mean operator appeared to behave like a LES filter
    - Large deviations observed in mean velocity profile $(\propto k^{0.5})$
    - Can be generalized to various types of turbulent flows, & not only shear flows

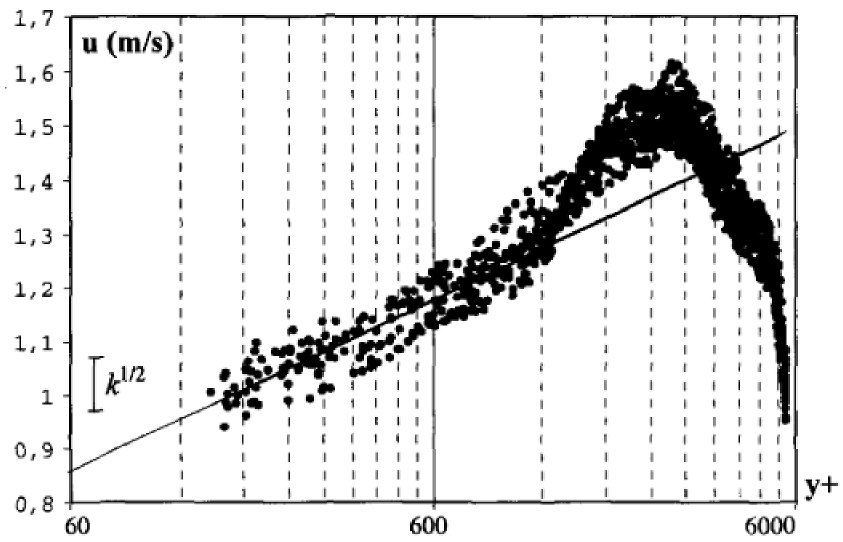


Fig: Computed mean velocity profile of turbulent Poiseuille flow in a pipe (Rep: Violeau et al)

$$\frac{\mathrm{D}\,\mathbf{v}_i}{\mathrm{D}\,t} = -\sum_j m_j \left(\frac{\langle P \rangle_i}{\rho_i^2} + \frac{\langle P \rangle_j}{\rho_j^2} \right) \nabla_i W_{h,ij} - \frac{1}{2} C_1 \frac{\epsilon_i}{k_i} \mathbf{v}'_i + C_2 \nabla \langle \mathbf{v} \rangle_i \cdot \mathbf{v}'_i + \sqrt{\frac{C_0 \epsilon_i}{\Delta t}} \vec{\xi}_i$$

$$\epsilon_i = 2\nu_{t,i} + ||\boldsymbol{S_i}||_F^2$$

$$k_i = \frac{\epsilon_i \nu_{t,i}}{C_\mu} \quad , \quad C_\mu = 0.009$$

[1] D. VIOLEAU, S. PICCON, and J.-P. CHABARD, "TWO ATTEMPTS OF TURBULENCE MODELLING IN SMOOTHED PARTICLE HYDRODYNAMICS," in Advances in Fluid Modeling and Turbulence Measurements, Jul. 2002, vol. 1, no. 5, pp. 339–346, doi: 10.1142/9789812777591_0041.

Viscosity-based Models

- Modified-SPH (mSPH)[1]
  - Observation: Absence of viscosity → Noisy particle motion
  - Finite viscosity → Over-prediction of dissipation
  - Modified EOS & MOM eqs. → Particle distribution homogenised
  - Simulated:
    - 2D Taylor-Green Vortex (TGV) $\left(Re = \infty,\ N_i = 64^2\right)$
    - 3D Taylor-Green Vortex (TGV) $\left(Re = 400,\ N_i = 64^3\right)$
  - Comments
    - Merging of vortices observed
    - Energy cascade (Kolmogorv scale) slopes reproduced
      - mSPH (-3); standard SPH (1)
    - Transitional flow simulated with reasonable dissipation rate
    - mSPH ~ Eddy viscosity model below numerical resolution

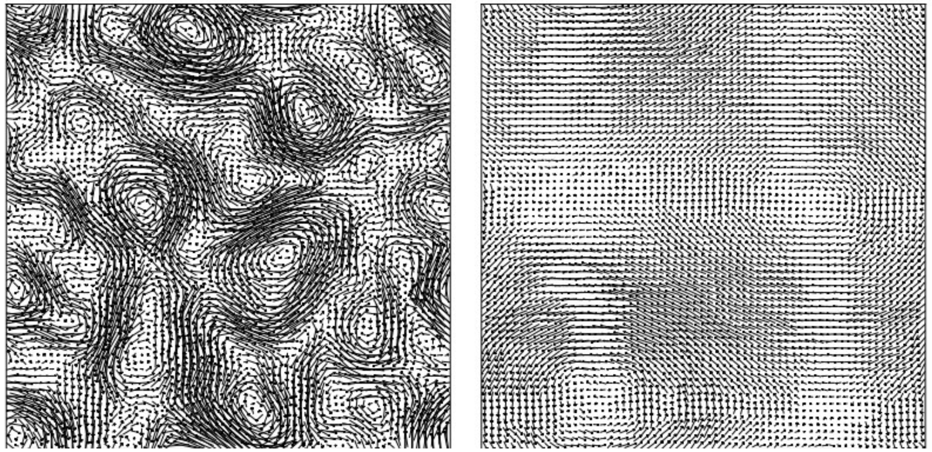


Fig: Velocity vector plot at t=2 (left) and t=30 (right). (Rep: Adami et al)

[1] S. Adami, X. Y. Hu, and N. A. Adams, "Simulating three-dimensional turbulence with SPH," 2012.

Large-Eddy Simulation-based Models

- Implicit Pressure Poisson-based Models[1,2]
  - Assumption: Incompressible flow
  - Navier-Stokes (NS) eqs. → Spatially filtered
  - Stress tensor → Closed using Boussinesq's hypothesis
  - Eddy viscosity → Estimated using modified Smagorinsky model
    - Model modified to include wall effects
  - Predictive-corrective time integrator
    - Requires implicit solution of pressure poisson eq
  - Simulated: $(Re \approx 10^6,\ N_i \approx 10^4)$
    - 2D wave interaction with partially immersed breakwater[1]
    - 2D wave breaking on a beach[2]

  - Comments
    - Accurately tracked free surfaces with large deformation
    - Surfaces did not exhibit numerical diffusion
    - Scheme's accuracy: $O(\Delta t + \Delta x^{1.25})$

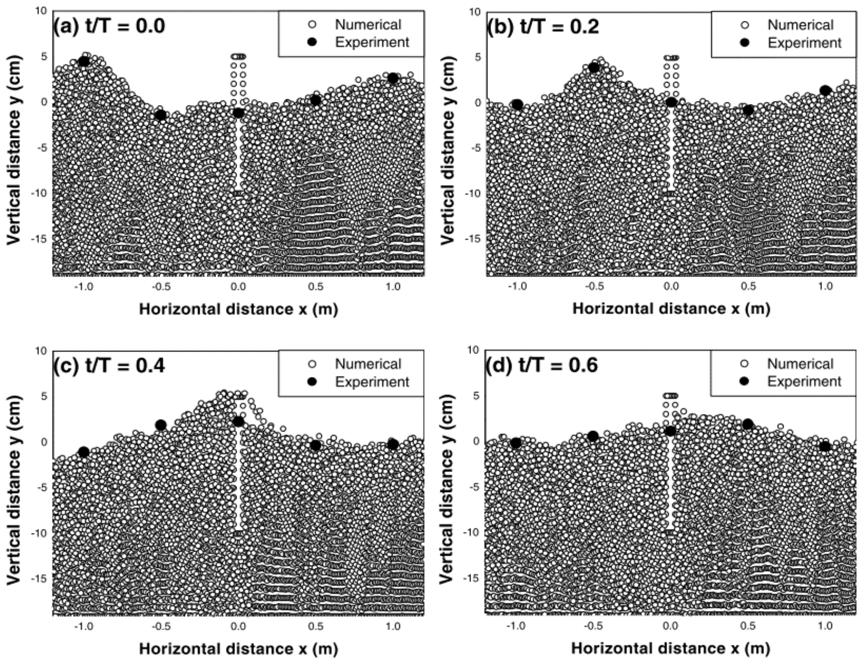


Fig: Time sequences of wave profile.
(Rep: Gotoh et al)

$$\frac{D\,\overline{\mathbf{v}}}{D\,t} = -\frac{1}{\rho}\nabla\overline{P} + \nu\Delta(\overline{\mathbf{v}}) + \frac{1}{\rho}\nabla\cdot\underline{\boldsymbol{\tau}} + \mathbf{F}$$

$$\frac{1}{\rho}\underline{\boldsymbol{\tau}} = 2\nu_t\underline{\boldsymbol{S}} - \frac{2}{3}k\underline{\boldsymbol{I}}$$

$$\nu_t = \min(C_s\Delta x, \kappa d_{wall})^2\sqrt{2\langle\underline{\boldsymbol{S}},\underline{\boldsymbol{S}}\rangle}$$

$$\nabla\cdot\left(\frac{1}{\rho_*}\nabla\overline{P}_{t+1}\right) = \frac{\rho_o - \rho_*}{\rho_o\Delta t^2}$$

[1] H. Gotoh, S. Shao, and T. Memita, "SPH-LES model for numerical investigation of wave interaction with partially immersed breakwater," Coast. Eng. J., vol. 46, no. 1, pp. 39–63, 2004, doi: 10.1142/S0578563404000872.
[2] S. Shao and H. Gotoh, "Turbulence particle models for tracking free surfaces," J. Hydraul. Res., vol. 43, no. 3, pp. 276–289, May 2005, doi: 10.1080/00221680509500122.

Large-Eddy Simulation-based Models

- Explicit Pressure EOS-based Models[1,2]
  - Assumption: Compressible flow
  - NS eqs → Favre averaged
  - Observation: Density variations → Unphysical behaviour on free surfaces
    - Shepard filtering of density performed periodically
  - Simulated: $(Re \approx 10^6,\ N_i \approx 10^4)$
    - Weakly plunging breaker in 2D & 3D[1]
    - 2D green-water overtopping, 2D & 3D wave breaking, 3D dam break[2]

$$\frac{\mathrm{D}\,\overline{\rho}}{\mathrm{D}\,t} = -\overline{\rho}\nabla\cdot\widetilde{\mathbf{v}}$$

$$\frac{\mathrm{D}\,\widetilde{\mathbf{v}}}{\mathrm{D}\,t} = -\frac{1}{\overline{\rho}}\nabla\overline{P} + \frac{1}{\overline{\rho}}(\nabla\cdot\overline{\rho\nu}\nabla)\widetilde{\mathbf{v}} + \frac{1}{\overline{\rho}}\nabla\cdot\underline{\boldsymbol{\tau}} + \mathbf{F}$$

$$\underline{\boldsymbol{\tau}} = \overline{\rho}\left(2\nu_t\underline{\boldsymbol{S}} - \frac{2}{3}\,\mathrm{tr}[\underline{\boldsymbol{S}}]\underline{\boldsymbol{I}}\right) - \frac{2}{3}\overline{\rho}C_I\overline{\Delta}^2\underline{\boldsymbol{I}}$$

$$\rho_i = \frac{\sum_j \rho_j W_{h,ij}\mathcal{V}_j}{\sum_j W_{h,ij}\mathcal{V}_j}$$

- Comments
  - Model predicted regions of high vorticity in 2D
  - Model captured vertically oriented eddies in 3D
  - Accurate for flow separation or splash-based problems
  - Requires large N_i & very small Δt
  - Uncertain regarding scalability to large-scale problems

[1] B. D. ROGERS and R. A. DALRYMPLE, "SPH MODELING OF BREAKING WAVES," in Coastal Engineering 2004, Apr. 2005, pp. 415–427, doi: 10.1142/9789812701916_0032.
[2] R. A. Dalrymple and B. D. Rogers, "Numerical modeling of water waves with the SPH method," vol. 53, pp. 141–147, 2006, doi: 10.1016/j.coastaleng.2005.10.004.

Large-Eddy Simulation-based Models

- Explicit Pressure EOS-based Models[1]
  - Observation: Standard Smagorinsky model cannot enforce
    - Wall conditions
    - Non-vanishing stresses with laminar flows
  - Devised Wall-adapting local eddy viscosity model (WALE)
  - Simulated an array of cylinders in 2D flow ($Re \approx 10^2,\ N_i \approx 10^4$)
  - Comments
    - LCS captured reasonably well
    - Observed merging of vortices to larger structures
    - Interactions of vortices with opposing strengths → Vorticity cancellation
    - Claim: Complex vortex interactions → Possible difficulty in interpreting the energy spectrum

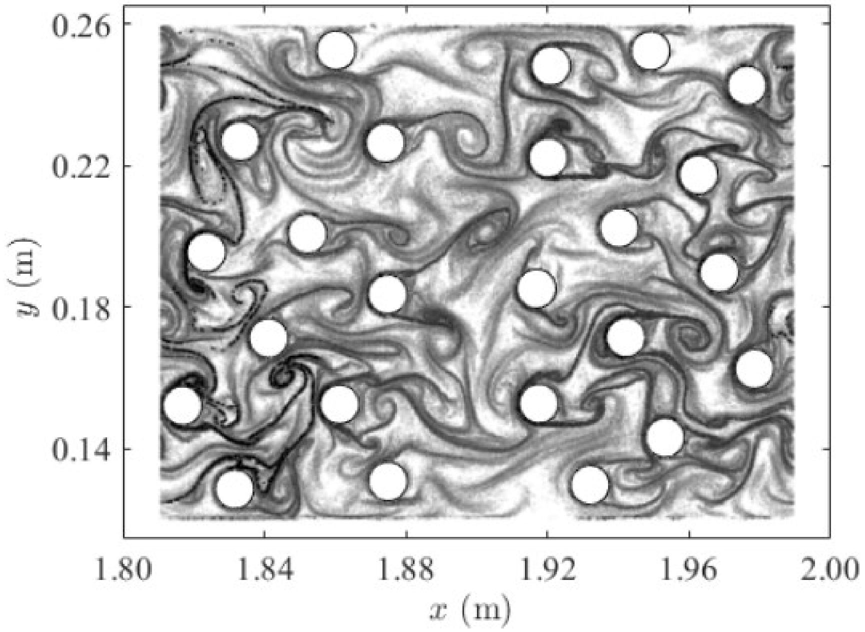


Fig: Backward-in-time FTLE field. (Rep: Canelas et al)

$$\nu_t = \rho (C_w \Delta x)^2 \frac{\langle \underline{\boldsymbol{S^d}}, \underline{\boldsymbol{S^d}} \rangle^{3/2}}{\langle \underline{\boldsymbol{S}}, \underline{\boldsymbol{S}} \rangle^{5/2} + \langle \underline{\boldsymbol{S^d}}, \underline{\boldsymbol{S^d}} \rangle^{5/4}}$$

$$\underline{\boldsymbol{S^d}} = \frac{1}{2}\left((\nabla \mathbf{v})^2 + \left((\nabla \mathbf{v})^T \right)^2 \right) - \frac{1}{3} \mathrm{tr}[(\nabla \mathbf{v})^2] \underline{\boldsymbol{I}}$$

[1] R. B. Canelas, A. M. Ricardo, R. M. L. Ferreira, J. M. Domínguez, and A. J. C. Crespo, "Hunting for Lagrangian Coherent Structures : SPH-LES turbulence simulations with Wall-adapting Local Eddy Viscosity (WALE) model," 11th SPHERIC, no. March 2017, 2016, [Online].

Large-Eddy Simulation-based Models

- Explicit Pressure EOS-based Models[1]
  - Reinterpret SPH → Lagrangian quadrature technique for explicit LES
    - Kernel scale limits SPH's physical resolution → Unsuitable DNS alternative
  - Navier-Stokes (NS) eqs. → Spatially filtered
  - Compared multiple Smagorinsky models
    - (standard, σ, MCG-form)
  - Simulated 3D TGV $(Re \approx 10^4,\ N_i = 200^3 - 500^3)$

  - Comments
    - Smagorisnky models reduced averaged kinetic energy
    - Dissipation rates not predicted accurately
    - SPH can capture turbulence up to kernel scale *(high cost)*
    - Explicit SGS models remove kinetic energy → Increases energy deficit of standard SPH
    - "SGS models in SPH framework only degrade the quality of the subsonic turbulent flow approximation"[2]

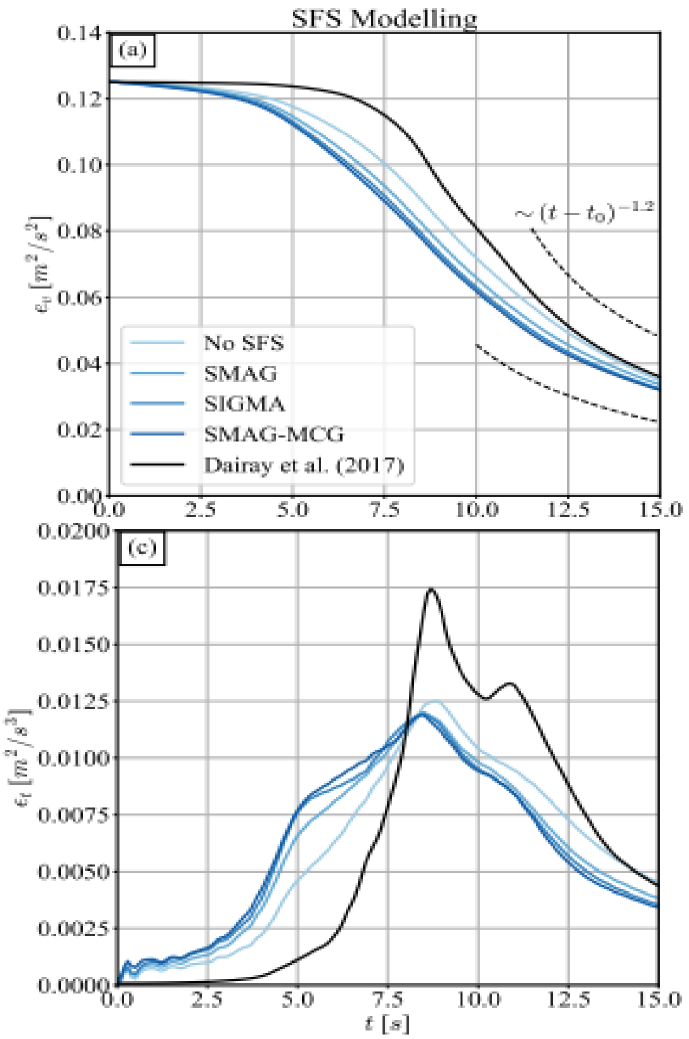


Fig: Density-weighted averaged kinetic energy (top), averaged dissipation rate (bottom). (Rep: Okraschevski et al)

[1] M. Okraschevski, N. Bürkle, R. Koch, and H.-J. Bauer, "Smoothed Particle Hydrodynamics Physically Reconsidered -- The Relation to Explicit Large Eddy Simulation and the Issue of Particle Duality," 2022, [Online].
[2] D. Rennehan, "Mixing matters," Mon. Not. R. Astron. Soc., vol. 506, no. 2, pp. 2836–2852, 2021.

Lagrangian LES-based Models

- Lagrangian form of LES[1,2]
  - Assumption: Weakly compressible flow
  - Lagrangian filter - ϕ (compact support, even) introduced
  - ϕ decomposed into independent spatial & temporal components
  - Governing eqs. written using spatially filtered terms
    - Temporal terms of higher order neglected
    - Continuity eq. closed using Fick-like diffusion term
    - MOM eq. closed using Yoshizawa-model for the stress tensor
    - Considered δ-SPH formulation
  - Simulated:
    - 2D TGV ($Re \approx 10^3 - 10^6,\ N_i \approx 10^3 - 10^6$)
    - 3D homogenous turbulence ($Re \approx 10^3,\ N_i = 64^3 - 256^3$)

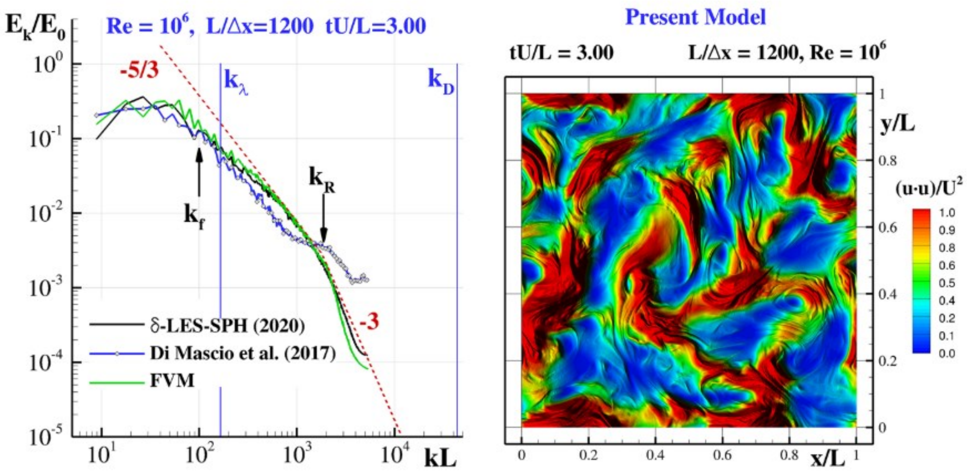


Fig: Energy spectrum (left), k field (right). (Rep: Colagrossi2021)

  - Comments
    - The model overcomes issues of spurious high-frequency noise & onset of tensile instability (High Re flows)
    - The energy spectra agrees well with theoretical decay rate
    - Wall functions need to be incorporated to deal with boundaries
    - Higher-order approach can significantly improve the performance

$$\frac{D\tilde{\rho}}{Dt} = -\tilde{\rho}\langle\nabla\cdot\tilde{\mathbf{v}}\rangle + C_1 + C_2$$

$$\frac{D\tilde{\mathbf{r}}}{Dt} = \tilde{\mathbf{v}}$$

$$\tilde{P} = F(\tilde{\rho})$$

$$C_2 \approx \nabla\cdot(\nu_\delta\nabla\tilde{\rho})$$

$$\frac{D\tilde{\mathbf{v}}}{Dt} = -\frac{\langle\nabla\tilde{P}\rangle}{\tilde{\rho}} + \nu\langle\Delta(\tilde{\mathbf{v}})\rangle + (\lambda' + \nu)\langle\nabla(\nabla\cdot\tilde{\mathbf{v}})\rangle + M_1 + M_2$$

$$M_2 = \nabla\cdot\underline{\boldsymbol{T}}_l = \nabla\cdot\left(-\frac{k^2}{3}\underline{\boldsymbol{I}} - \frac{2}{3}\nu_t\,\mathrm{Tr}[\underline{\tilde{\boldsymbol{S}}}]\underline{\boldsymbol{I}} + 2\nu_t\underline{\tilde{\boldsymbol{S}}}\right)$$

[1] A. Di Mascio, M. Antuono, A. Colagrossi, and S. Marrone, "Smoothed particle hydrodynamics method from a large eddy simulation perspective," Phys. Fluids, vol. 29, no. 3, 2017, doi: 10.1063/1.4978274.
[2] A. Colagrossi, "Smoothed particle hydrodynamics method from a large eddy simulation perspective . Generalization to a quasi-Lagrangian model Smoothed particle hydrodynamics method from a large eddy simulation perspective . Generalization to a quasi-Lagrangian model," vol. 015102, no. December 2020, 2021, doi: 10.1063/5.0034568.

RANS-based $k - \epsilon$ Models

- $k - \epsilon$ Models[1,2]
  - Considered incompressible, unsteady RANS equations
  - Predictive-corrective time integrator
    - Requires implicit solution of pressure poisson eq
  - Simulated:
    - 2D wave breaking and overtopping of sloping wall[1] $(Re \approx 10^6,\ N_i = 6000)$
    - 2D solitary wave propagating over a bottom-mounted barrier[2] $(Re \approx 10^6,\ N_i \approx 1.3 \times 10^5)$

  - Comments
    - Accurately tracks free-surfaces
    - $k - \epsilon$ coefficients (empirically derived from quasi-steady state) perform sub-optimally in transient flow
    - Model requires a sensitivity analysis for the turbulence model & spatial resolution for improved performance
    - Underpredicts max k & is sensitive to initial seeding of k
    - Effects of viscous dissipation & numerical dissipation need to be balanced

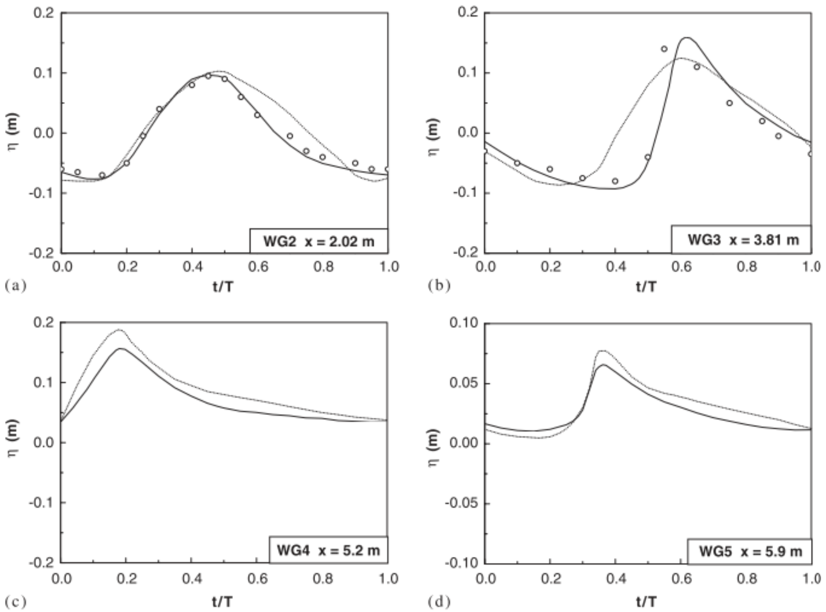


Fig: Water surface elevations. (Rep: Shao et al)c

$$\nu_t = c_d \frac{k^2}{\epsilon}$$

$$\frac{\mathrm{D}\, k}{\mathrm{D}\, t} = \nabla \cdot \left(\frac{\nu_t}{\sigma_k} \nabla k \right) + P_k - \epsilon$$

$$\frac{\mathrm{D}\, \epsilon}{\mathrm{D}\, t} = \nabla \cdot \left(\frac{\nu_t}{\sigma_\epsilon} \nabla \epsilon \right) + c_{1\epsilon} \frac{\epsilon}{k} P_k - c_{2\epsilon} \frac{\epsilon^2}{k}$$

$$P_k = 2\nu_t \langle \underline{\boldsymbol{S}}, \underline{\boldsymbol{S}} \rangle$$

[1] S. Shao, "Incompressible SPH simulation of wave breaking and overtopping with turbulence modelling," no. May 2005, pp. 597–621, 2006.
[2] D. Wang and P. L. F. Liu, "An ISPH with k–ε closure for simulating turbulence under solitary waves," Coast. Eng., vol. 157, no. July 2019, p. 103657, 2020, doi: 10.1016/j.coastaleng.2020.103657.

LANS-based Models

- Lagrangian-averaged NS eqs. (LANS-$\boldsymbol{\alpha}$) Models[1,2,3,4]
  - Lagrangian-averaging[1]: Performed at the level of variational principle from which NS eqs. are derived
    - $\boldsymbol{\alpha}$ denotes scale of rapid fluctuations in the flow map
  - SPH-$\boldsymbol{\alpha}$ model[2] → Initial models based on LANS-$\boldsymbol{\alpha}$ eqs.
    - Particle transport → Smoothed velocity
    - MOM eq. solved iteratively
  - SPH-$\boldsymbol{\epsilon}$ model[3,4] → Improvement of SPH-$\boldsymbol{\alpha}$ model
    - Explicit MOM eq. with viscous term
  - Simulated 2D flow past a cylinder moving along a Lissajous curve[4] $(Re \approx 10^3,\ N_i \approx 10^4)$

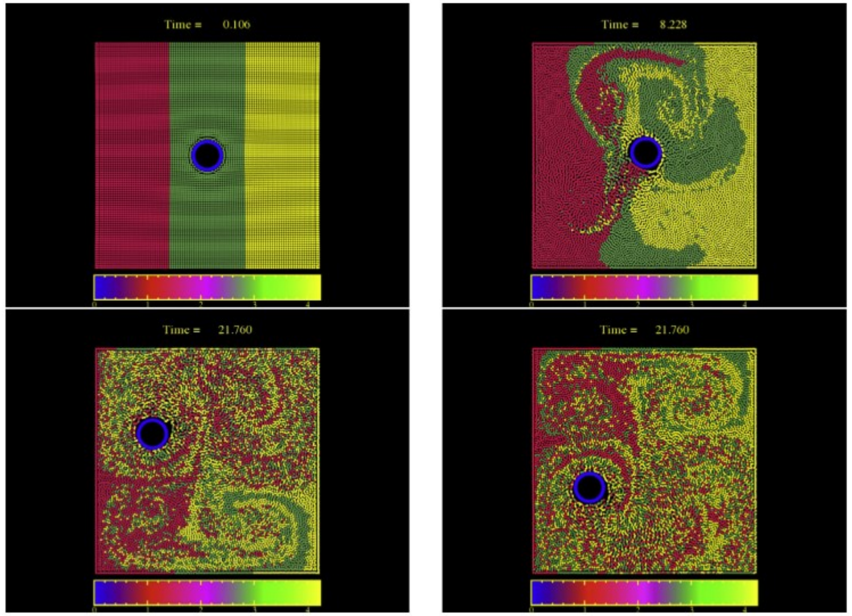


Fig: Stirring of coloured particles. (Rep: Monaghan2017)

$$\widehat{\mathbf{v}}_i = \mathbf{v}_i - \varepsilon \sum_j \frac{m_j}{M_o} \mathbf{v}_{ij} K_{h',ij}$$

- Comments
  - Bounded *(no-slip)* flow implies $\mathbf{v}$ is neither periodic/isotropic → Author prefers correlation functions over energy spectrum
  - Satisfactory results for velocity correlation functions, energy spectrum & mixing with half the particle resolution of DNS
  - Flows with larger Reynolds numbers & other boundary conditions, such as free surfaces need to be studied

[1] J. E. Marsden and S. Shkoller, "Global well-posedness for the Lagrangian averaged Navier-Stokes (LANS-α) equations on bounded domains," Philos. Trans. R. Soc. A Math. Phys. Eng. Sci., vol. 359, no. 1784, pp. 1449–1468, 2001, doi: 10.1098/rsta.2001.0852.
[2] J. J. Monaghan, "SPH compressible turbulence," vol. 852, pp. 843–852, Apr. 2002, doi: 10.1046/j.1365-8711.2002.05678.x.
[3] J. J. Monaghan, "A turbulence model for smoothed particle hydrodynamics," Eur. J. Mech. B/Fluids, vol. 30, no. 4, pp. 360–370, 2011, doi: 10.1016/j.euromechflu.2011.04.002.
[4] J. J. Monaghan, "SPH-ϵ simulation of 2D turbulence driven by a moving cylinder," Eur. J. Mech. B/Fluids, vol. 65, pp. 486–493, 2017, doi: 10.1016/j.euromechflu.2017.03.011.

Miscellaneous Models

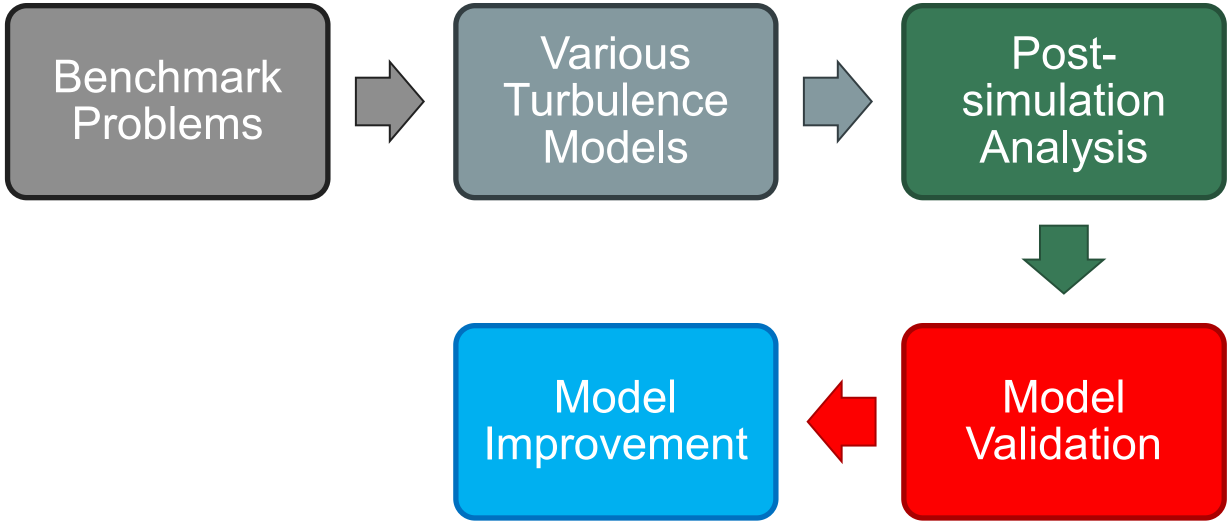
- Viscosity-based vorticity correction model[1]
  - Aimed primarily towards benefiting the CGI community
    - Faster computational time & simplified system of equations
  - Research focuses more on visual artifacts of the flow, not quantified metrics
  - Builds on the fact that rotational kinetic energy ≤ translational kinetic energy

- Hybrid-SPH (hrSPH)[2]
  - Particle data interpolated onto Eulerian mesh grids
  - Rate of change of flow properties computed on mesh grid
  - Updated mesh values interpolated onto particles
  - Advection of particles using the updated properties
  - Remeshing is performed when particles cluster to ensure uniform particle distribution
  - Information from the model cannot help improve traditional SPH schemes

[1] S. Liu, X. Wang, X. Ban, Y. Xu, J. Zhou, and Y. Zhang, "Viscosity-based Vorticity Correction for Turbulent SPH Fluids," in 2019 IEEE Conference on Virtual Reality and 3D User Interfaces (VR), Mar. 2019, pp. 1048–1049, doi: 10.1109/VR.2019.8798224.
[2] A. Obeidat and S. P. A. Bordas, "Three-dimensional remeshed smoothed particle hydrodynamics for the simulation of isotropic turbulence," Int. J. Numer. Methods Fluids, vol. 86, no. 1, pp. 1–19, 2018, doi: 10.1002/fld.4405.

Evaluation of Turbulence Models

- Most of Lagrangian turbulent models → Tested for primarily complex, free-surface flows
- A systematic analysis of isotropic turbulence problems provides better insight on the energy spectrum & its cascade across varying length scales
- Appropriate bounded/periodic test cases must be used when analyzing turbulence
- Analyse results with appropriate metrics, and validate the results with experimental or numerical data from literature



Benchmark Problems

- Taylor-Green Vortex Problem → Periodic, incompressible flow
  - 2D Case: Analytical solutions known
  - 3D Case: Initial flow conditions can be specified

- Thin Double-Shear Layer
  - Flow under-resolved → Spurious structures are produced
  - Generation of the spurious structure depends on the scheme

- 3D Isotropic Turbulence[1]
  - DNS dataset from the JHU Turbulence database Cluster
  - Dataset: Incompressible flow with isotropic and forced turbulence.
  - 1024^3 spatial points & 1024 time samples spanning one large-scale turnover time

- 2D Confined & Driven Turbulence[2]
  - 2D fluid confined to a square box enclosing a cylinder
  - Cylinder moves in a predetermined trajectory

[1] Y. Li et al., "A public turbulence database cluster and applications to study Lagrangian evolution of velocity increments in turbulence," J. turbul., vol. 9, p. N31, 2008.
[2] J. J. Monaghan, "SPH-ϵ simulation of 2D turbulence driven by a moving cylinder," Eur. J. Mech. B/Fluids, vol. 65, pp. 486–493, 2017, doi: 10.1016/j.euromechflu.2017.03.011.

Post-Simulation Analysis

- Energy spectral density
  - Velocity field over a grid generated using suitable interpolation of the particle velocity data[1]
  - Fourier transform of the velocity field → Velocity spectrum as a function of the wave-number
  - Energy spectrum computed from the velocity spectrum
- Velocity gradient-based metrics
  - Iso-vorticity surfaces, Q-criterion, Δ-criterion, etc.
  - Most of the definitions for a vortex → Not objective & suitable for studying the flow (esp. 3D flow)[2]
- Lagrangian coherent structures (LCS)
  - Local velocity fluctuations do not induce noise in LCS
  - FTLE is formulated in the Lagrangian framework → Better identification of LCSs
  - Provides flexibility in its formulation[3]
    - Backward-in-time FTLE can be computed using limited resources during run-time concurrently
    - Forward-in-time FTLE can only be computed during post-processing
    - Easily parallelizable

[1] Y. Shi, X. X. Zhu, M. Ellero, and N. A. Adams, "Analysis of interpolation schemes for the accurate estimation of energy spectrum in Lagrangian methods," Comput. Fluids, vol. 82, pp. 122–131, 2013, doi: 10.1016/j.compfluid.2013.05.003.
[2] G. Haller, "An objective definition of a vortex," J. Fluid Mech., vol. 525, pp. 1–26, 2005.
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Conclusion & Future Work

- Stage – I outcome:
    - A reasonable compilation of past SPH turbulence models
    - The corresponding advantages, limitations & potential areas for refinement are understood

- Entropically Damped Artificial Compressibility (EDAC) scheme[1] → Explicit incompressible SPH method
    - Uses a pressure evolution eq. instead of the continuity equation[2]
    - Produces a smoother & accurate pressure distribution for flows, confined or free
    - Does not require artificial viscosity
- Turbulence Models for EDAC
    - Incompatible with RANS due to incompressibility condition
        - Will consider compressible EDAC scheme to devise a turbulence model
    - Lagrangian LES filtering of the pressure eq → Mixed filtered term obtained; cannot be decoupled further
        - Will investigate potential closure models to decouple/model the mixed term

$$\boxed{\nabla \cdot \mathbf{v} = 0}$$

$$\frac{\mathrm{D}\,P}{\mathrm{D}\,t} = \boxed{-c_s^2 \rho \nabla \cdot \mathbf{v}} + \nu \nabla^2 P$$

$$\frac{\mathrm{D}\,\widetilde{P}}{\mathrm{D}\,t} = -c_s^2 \rho \nabla \cdot \widetilde{\mathbf{v}} + \nu \nabla^2 \widetilde{P} + \widetilde{\mathbf{v}} \cdot \nabla \widetilde{P} - \widetilde{\mathbf{v} \cdot \nabla P}$$

$$f(\widetilde{\mathbf{v}}, \widetilde{P}) \neq \widetilde{f(\mathbf{v}, P)}$$

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Thank you!

Department of Aerospace Engineering
Indian Institute of Technology Bombay

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