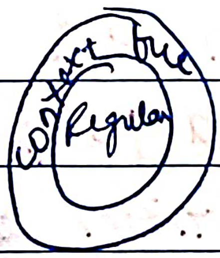
17/2

Context free languages



grammar - expresses string in context free languages

non terminals - uppercase
terminals - alphabet

$S \rightarrow A01$ (production)

set of productions - grammar

$G = (V, T, P, S)$ — V: set of non terminals; T: set of terminals; P: set of productions; S: starting non-terminal

Palindrome strings

$P \to \epsilon$

$P \to 0$

$P \to 1$

$P \to 0P0$

$P \to 1P1$

Q. 010010

$P \Rightarrow 0P0 \Rightarrow 0P1P10$

$010010 \Leftarrow 0P10P01P0$

Q/ $L = \{0^n 1^n\}$ $n \geq 1$

$S \to 0S1 \mid 01$

$S \to 01$

Q/ atleast 3 1s.

$P \Rightarrow S1S1S1S$

$S \to \epsilon$

$S \to 0S$

$S \to 1S$

Q/ $0^*1(0+1)^*$

$S \to P1Q$

$P \to 0P \mid \epsilon$

$Q \to 0Q \mid 1Q \mid \epsilon$

$E \to (E)$
$E \to E * E$
$E \to E + E$
$E \to a \mid b$

Q. $((a*b) + b)$ left most derivation

$E \Rightarrow (E) \xRightarrow{lm} (E + E) \xRightarrow{lm} ((E) + E)$

$\Downarrow lm$

$((a * E) + E) \xLeftarrow{lm} ((E*E) + E)$

$\Downarrow lm$

$((a * b) + E) \xRightarrow{lm} ((a * b) + b)$

Sentential form

$L = \{a^i b^j c^k \mid i = j + k\}$

$S \to$

Q. $S \to AS \mid \epsilon$
$A \to aa \mid ab \mid ba \mid bb$

aabbba

$S \to AS$ (leftmost)
$\to aaS$

capital letters - non terminals
small letters - ~~no~~ terminals

$\rightarrow aaAS$

$\rightarrow aabbS$

$\rightarrow aabbAS$

$\rightarrow aabbbaS$

$\rightarrow aabbba$

Intermediate forms - sentential forms

$S \rightarrow AS$

$\rightarrow AAS$

$\rightarrow AAAS$

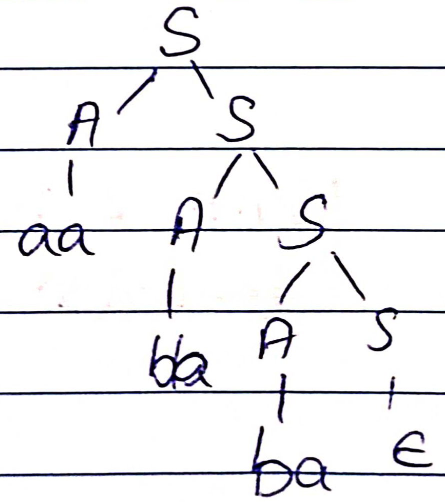
$\rightarrow AAA$

$\rightarrow AAba$

$\rightarrow Abbba$

$\rightarrow aabbba$

Parse tree



Ambiguity-

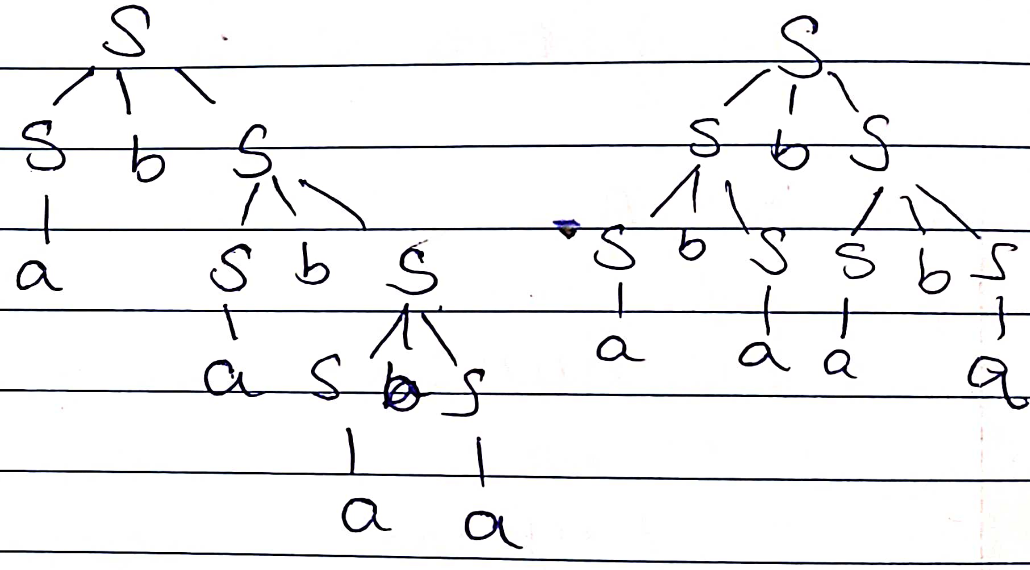
$E \rightarrow (E) \mid E+E \mid E*E \mid I$

$I \rightarrow a \mid b$

$a+b*a$

Q. $S \rightarrow SbS \mid a$

ababaBa



To convert to unambiguous

$E \rightarrow T \mid E+T$

$T \rightarrow F \mid T*F$

$F \rightarrow I \mid (E)$

Q. S ababaBa   Write 2 left most derivations

$S \rightarrow \underline{S}bS$

$\rightarrow ab\underline{S}$

$\rightarrow ab\underline{S}bS$

$\rightarrow ababS$

$\rightarrow ababSbS$

$\rightarrow abababS$

$\rightarrow abababa$

$S \rightarrow SbS$

$\rightarrow SbSbS$

$\rightarrow absbS$

$\rightarrow ababS$

$\rightarrow ababSbS$

$\rightarrow abababS$

$\rightarrow abababa$

Chomsky Normal form

$A \rightarrow BC$ 2 non-terminals OR

$A \rightarrow a$ 1 terminal

1. NO ϵ productions
2. Useless symbols → non generating, non reachable
3. Unit productions

} remove

Q. $S \rightarrow aX \mid bX$ Convert to chomsky

$X \rightarrow a \mid b \mid \epsilon$

$S \rightarrow aX \mid bX \mid a \mid b$

$X \rightarrow a \mid b \mid \epsilon$

Q. $S \to AB$ (sub ϵ in S and put more in S $\to$)

$A \to aAA \mid \epsilon$

$B \to bBB \mid \epsilon$

$S \to AB \mid B \mid A$

$A \to aAA \mid aA \mid a$

$B \to bBB \mid bB \mid b$

Q. $S \to ABC$

$A \to BC \mid a$

$B \to bAC \mid \epsilon$

$C \to cAB \mid \epsilon$

$S \to aSd \mid A$

$A \to aAb \mid \epsilon$

$S \to AC \mid AB \mid A \mid ABC \mid BC \mid B \mid C$

$A \to BC \mid B \mid C \mid a \mid \epsilon$

$B \to bAC \mid bA \mid bC \mid b$

$C \to cAB \mid cA \mid cB \mid c$

Q. Elimination of unit productions

$S \to aX \mid Yb \mid Y$

$X \to S$

$Y \to Yb \mid b$

(S, S) (X, X) (Y, Y)

(X, S), (S, Y),

(X, Y)

unit pair - 2 non terminals

(A, B) add to P', $A \to \alpha$ where $B \to \alpha$ is a non unit production

$(S,S) \Rightarrow S \to aX \mid Yb \mid b$

$\cancel{X} \longrightarrow X \to aX \mid Yb \mid b$ (S cannot be added - unit)

$Y \to Yb \mid b$

Q. $I \to a \mid b$

$F \to I \mid (E)$

$T \to F \mid T * F$

$E \to T \mid E + T$

$(I, I), (F, F), (T, T), (E, E), (F, I), (T, F), (E, T) (T, I) (E, F), (E, I)$

(I, I) $I \to a \mid b$

(F, F) $F \to (E)$

(T, T) $T \to T * F$

(E, E) $E \to E + T$

(F, I) $F \to a \mid b$

(T, F) $T \to (E)$

(T, I) $T \to a \mid b$

(E, T) $E \to T * F$

(E, I) $E \to a \mid b$

(E, F) $E \to (E)$

$I \to a \mid b$

$F \to a \mid b \mid (E)$

$E \to E + T \mid T * F \mid a \mid b \mid (E)$

$T \to T * F \mid (E) \mid a \mid b$

Q $S \to AB \mid a$
$A \to b$

Remove
1. Non generating B
2. Non reachable

Q $S \to AB \mid \underline{AC}$ → non reachable
(gen) $A \to aA \mid bAa \mid a$
(gen) $B \to \underline{bbA} \mid aB \mid AB$ — bbA - all symbols gen
non gen → circular
$C \to aCa \mid aD$
$D \to aD \mid bc$

Chomsky Normal Form (CNF)

$S \to ABC \mid BaB$
$A \to aA \mid BaC \mid aaa$
$B \to bBb \mid a \mid D$
$C \to CA \mid AC$
$D \to \epsilon$

Eliminate ϵ, then unit, then useless

$S \to \cancel{ABC} \mid BaB \mid \cancel{AC} \mid a \mid Ba \mid aB$
$A \to aA \mid \cancel{BaC} \mid \cancel{aC} \mid aaa$
$B \to bb \mid bBb \mid a$
$\cancel{C \to CA \mid AC}$

no - unit
useless - C

~~A~~ $S \to BaB \mid a \mid Ba \mid aB$

~~$A \to aA \mid aaa$~~ (non reachable)

$B \to bb \mid bBb \mid a$

(eliminate terminals of length>1)

$X \to a$ $\quad$ $S \to BXB \mid a \mid BX \mid XB$

$Y \to b$ $\quad$ $B \to YBY \mid YY \mid a$

$M \to XB$

$N \to BY$

$S \to BM \mid a \mid BX \mid M$

$B \to YN \mid YY \mid a$

Q// $S \to aAa \mid bBb \mid \epsilon$

$A \to C \mid a$

$B \to c \mid b$

$C \to CDE \mid \epsilon$

$D \to A \mid B \mid ab$

Eliminate ϵ

$S \to aAa \mid bBb \mid aa \mid bb$

$A \to \cancel{C} \mid a$

$B \to \cancel{C} \mid b$

~~$C \to CDE \mid DE \mid CE \mid E$~~ nongen

$D \to A \mid B \mid ab$ $\to$ non reachable

Remove unit

$S \to aAa \mid bBb \mid aa \mid bb$

$A \to a$ $\quad\quad M \to AA$

$B \to b$ $\quad\quad N \to BB$

$S \to MA \mid NB \mid M \mid N$

$M \to AA$

$N \to BB$

$A \to a$

$B \to b$

Q. $S \to aS \mid aSbS \mid \epsilon$

aab

$S \to aS$

$\to aaSbS$

$\to aabS$

$\to aab$

$S \to aSbS$

$\to aaSbS$

$\to aabS$

$\to aab$

ambiguous

Q. $L \to \{a^n b^n c^m d^m\} \mid \{a^n b^m c^m d^n\}$

$S \to AB$

$A \to aAb$

$B \to cBd$

$C \to$

$L = \{a^n b^n c^m d^m\} \cup \{a^n b^m c^m d^n\}$ $n \geq 1, m \geq 1$

$S \rightarrow AB \mid C$

$A \rightarrow aAb \mid ab$

$B \rightarrow cBd \mid cd$

$C \rightarrow aCd \mid aDd$

$D \rightarrow bDc \mid bc$

Greibach Normal form

$A \rightarrow a\alpha$ Convert to CNF

α is a string of zero or more variables (non terminals)

Q) $S \rightarrow XA \mid BB$

$B \rightarrow b \mid SB$

$X \rightarrow b$

$A \rightarrow a$

$A_1 \rightarrow A_2 A_3 \mid A_4 A_4$

$A_4 \rightarrow b \mid A_1 A_4$

$A_2 \rightarrow b$

$A_3 \rightarrow a$

$A_i \rightarrow A_j$ $i < j \rightarrow$ fine, otherwise substitution

$A_4 \rightarrow A_2 A_3 A_4 \mid A_4 A_4 A_4$

$\underbrace{A_4}_{A} \rightarrow \underbrace{b A_3 A_4}_{\beta} \mid \underbrace{A_4}_{A} \underbrace{A_4 A_4}_{\alpha}$

$A \to A\alpha \mid \beta$
$A \to \beta A' \mid \beta$
$A' \to \alpha A' \mid \beta$

$A \to b A_3 A_4$

1. convert to CNF
2. rename non terminals
3. $A_i \to A_j \alpha \quad i<j$
4. $A_i \to A_i \alpha$ eliminate left recursion

$A \to A\alpha \mid \beta$
$A \to \beta A' \mid \beta$
$A' \to \alpha A' \mid \alpha$

Q $A_1 \to A_2 A_3 \mid A_4 A_4 \qquad i<j$
$A_4 \to b A_3 A_4 B_1 \mid b B_1 \mid b \mid b A_3 A_4$
$B_1 \to A_4 A_4 B_1 \mid A_4 B_4$
$A_2 \to b$
$A_3 \to a$

Q
Let L be a CFL. there exists a constant n such that if z is any string $|z| \geq n$ then $z = uvwxy$

1. $|vwx| \leq n$
2. $vx \neq \epsilon$

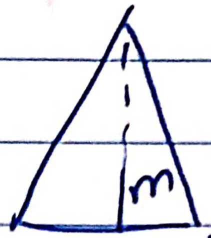
(3) for $i \geq 0$

$uv^i w x^i y$ is in L

Grammar has m non terminals

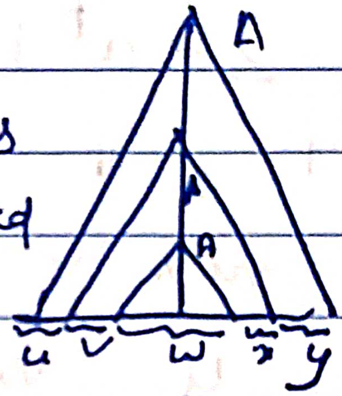
$n = 2^m$ $\quad |z| = n$

parse tree with m non terminals



$m+1 \rightarrow$ some non terminals are repeated

$2^{m-1} = n/2$ not n



$A \overset{*}{\Rightarrow} v\,w\,x$ $\quad \underline{vwx}$

$A \overset{*}{\Rightarrow} w$ $\quad \underline{vwx}$

$\downarrow$

$uv^i w x^i y \rightarrow$ part of string of grammar - pumping lemma is true.

Q. $L = \{0^n 1^n 2^n \quad n \geq 0\}$

1) Assume L is a CFL
2) Assume constant n
3) pick $z \in L$ / $|z| \geq n$

4) Look at all decompositions of

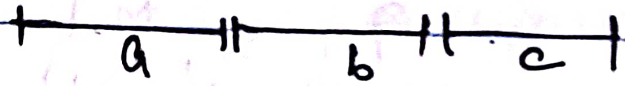
$z = uvwxy \quad |vwx| \leq n$

$vx \neq \epsilon$

Q: $L = \{a^n b^n c^n \quad n \geq 0\}$

L is a CFL constant p

$z = a^p b^p c^p$



case 1: $u \quad vwx \quad \underbrace{\qquad\qquad}_{y}$

case 2: $u \quad vwx \quad y$

case 3: $\underbrace{\qquad\qquad}_{u} \quad vwx$

$a^p \epsilon \epsilon b^p c^p \qquad i = 2$

Q: $L = \{a^i b^j c^k\}$ ST $i <= j, j <= k$

$z = a^{p-1} b^p c^{p+1}$

$a^{p-2} \quad \underline{a b \cancel{b}} \quad b^{p-2} \quad c^{p+1}$

$i=2$

$a^2 \quad b \quad b^{2p-4}$

$p \leq 2p-3 \leq p+1$

X

Q $L_2 = \{ww, \ w \in \{0,1\}^*\}$

$z = 0^p 1^p 0^p 1^p$

$\underset{u}{0^p} \ \underset{v}{\epsilon} \ \underset{w}{1^p} \ \underset{x}{0^p} \ \underset{y}{1^p}$

$i = 2$

$0^p \epsilon^2 1^p 0^{2p} 1^p$

Q $S \rightarrow ABA$

$A \rightarrow aA \mid \epsilon$

$B \rightarrow bB \mid \epsilon$

$S \rightarrow ABA \mid AB \mid BA \mid AA \mid B \mid A$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$

$(A, A) \quad (B, B)$

$X \rightarrow a$

$Y \rightarrow b$

$S \rightarrow ABA \mid AB \mid BA \mid AA$

$A \rightarrow XA \mid X$

$B \rightarrow YB \mid Y$

$M \rightarrow BA$

$N \rightarrow AB$

So

$S \rightarrow AM \mid AB \mid M \mid AA$

$A \rightarrow XA \mid X$

$B \rightarrow YB \mid Y$

$M \rightarrow BA$

$X \rightarrow a$

$Y \rightarrow b$

$A_1 \rightarrow A_2 A_3 \mid A_2 A_4 \mid A_3$

$A_2 A_2$

$A_2 \rightarrow A_5 A_2 \mid A_5$

$A_4 \rightarrow A_6 A_4 \mid A_6$

$A_3 \rightarrow A_4 A_2$

$A_5 \rightarrow a$

$A_6 \rightarrow b$

Q $L_2 = \{ww,\ w \in \{0,1\}^*\}$

$z = 0^p 1^p 0^p 1^p$

$\underset{u}{0^p}\ \underset{v}{\epsilon}\ \underset{w}{1^p}\ \underset{x}{0^p}\ \underset{y}{1^p}$

$i = 2$

$0^p \epsilon^2 1^p 0^p 1^p$

Q $S \to ABA$

$A \to aA \mid \epsilon$

$B \to bB \mid \epsilon$

$S \to ABA \mid AB \mid BA \mid AA \mid B \mid A$

$A \to aA \mid a$

$B \to bB \mid b$

(A, A) (B, B)

$X \to a$

$Y \to b$

$S \to ABA \mid AB \mid BA \mid AA$

$A \to XA \mid X$

$B \to YB \mid Y$

$M \to BA$

$N \to AB$

&

$S \to AM \mid AB \mid M \mid AA$

$A \to XA \mid X$

$B \to YB \mid Y$

$M \to BA$

$X \to a$

$Y \to b$

$A_1 \to A_2A_3 \mid A_2A_4 \mid A_3 \mid A_2A_2$

$A_2 \to A_5A_2 \mid A_5$

$A_4 \to A_6A_4 \mid A_6$

$A_3 \to A_4A_2$

$A_5 \to a$

$A_6 \to b$

4) Look at all decompositions of

$z = uvwxy$ $\quad |vwx| \leq n$

$vx \neq \epsilon$

Q. $L = \{a^n b^n c^n \quad n \geq 0\}$

L is a CFL $\quad$ constant p

$z = a^p b^p c^p$

$\vdash \quad a \quad \| \quad b \quad \| \quad c \quad \dashv$

case 1: $u \quad vwx \quad \underbrace{\qquad\qquad}_{y}$

case 2: $u \quad vwx \quad y$

case 3: $\underbrace{\qquad\qquad}_{u} \quad vwx$

$a^p \epsilon \epsilon b^p c^p \qquad i=2$

Q. $L = \{a^i b^j c^k\}$ ST $i <= j, j <= k$

$z = a^{p-1} b^p c^{p+1}$

$a^{p-2} \quad \underline{a b \cancel{b}} \quad b^{p-2} \quad c^{p+1}$

$i=3$

$a^2 \quad b \quad b^{2p-4}$

$p \leq 2p-3 \leq p+1$

X

Q/ $L_2 = \{ww,\ w \in \{0,1\}^*\}$

$z = 0^p 1^p 0^p 1^p$

$\underset{u}{0^p}\ \underset{v}{\epsilon}\ \underset{w}{1^p}\ \underset{x}{0^p}\ \underset{y}{1^p}$ $i = 2$

$0^p \epsilon^2 1^p 0^{2p} 1^p$

Q/ $S \to ABA$

$A \to aA \mid \epsilon$

$B \to bB \mid \epsilon$

$S \to ABA \mid AB \mid BA \mid AA \mid B \mid A$

$A \to aA \mid a$ (A, A) B, B)

$B \to bB \mid b$

$X \to a$

$Y \to b$

$S \to ABA \mid AB \mid BA \mid AA$

$A \to XA \mid X$ $M \to BA$

$B \to YB \mid Y$

$S \to AM \mid AB \mid M \mid AA$

$A \to XA \mid Y$

$B \to YB \mid Y$

$M \to BA$

$X \to a$

$Y \to b$

$A_1 \to A_2 A_3 \mid A_2 A_4 \mid A_3 \mid A_2 A_2$

$A_2 \to A_5 A_2 \mid A_5$

$A_4 \to A_6 A_4 \mid A_6$

$A_3 \to A_4 A_2$

$A_5 \to a$

$A_6 \to b$

$A_1 \rightarrow A_2 A_3 \mid A_3 \mid A_4 \mid A_2 A_2$
$A_2 \rightarrow A_5 A_2 \mid A_5$
$A_6 \rightarrow A_7 A_6 \mid A_7$
$A_3 \rightarrow A_6 A_2$
$A_4 \rightarrow A_2 A_6$ $\qquad A_4 \rightarrow A_5 A_6 \mid A_5 A_2 A_6$
$A_5 \rightarrow a$
$A_7 \rightarrow b$

$S \rightarrow ABA \mid AB \mid BA \mid B \mid AA \mid A$
$A \rightarrow a \mid aA$
$B \rightarrow b \mid bA$

→ Remove unit prod
$S \rightarrow ABA \mid AB \mid BA \mid bA \mid b \mid AA \mid a \mid aA \mid bB$
$A \rightarrow a \mid aA$
$B \rightarrow b \mid bA$

$X \rightarrow BA$
$S \rightarrow AX \mid AB \mid X \mid bA \mid b \mid AA \mid a \mid aA \mid bB$

$X - A_2 \quad A - A_3$

$A_1 \rightarrow A_3A_2 \mid A_3 A_4 \mid A_2 \mid bA_3 \mid b \mid$
$A_3A_3 \mid b \mid bA_4 \mid a \mid aA_3$

$A_2 \rightarrow A_3 A_4$
$A_3 \rightarrow a \mid aA_2$
$A_4 \rightarrow b \mid bA_4$

should start with terminal
(GNF)

$A_1 \rightarrow aA_2 \mid aA_3A_2 \mid \ldots$

Pushdown Automata (PDA)

ϵ-NFA + stack

$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

Initial stack symbol

Input → Finite State control

L_{ww^r}

i/p sym, stack

$\delta(q_0, 0, Z_0) = (q_0, 0Z_0)$
$\delta(q_0, 1, Z_0) = (q_0, 1Z_0)$
$\delta(q_0, 0, 0) = (q_0, 00)$
$\delta(q_0, 1, 1) = (q_0, 11)$

$\delta(q_0, \epsilon, Z_0) = (q_2, \epsilon)$

$\delta(q_0, 0, 1) = (q_0, 01)$

$\delta(q_0, \epsilon, 0) = (q_1, 0)$

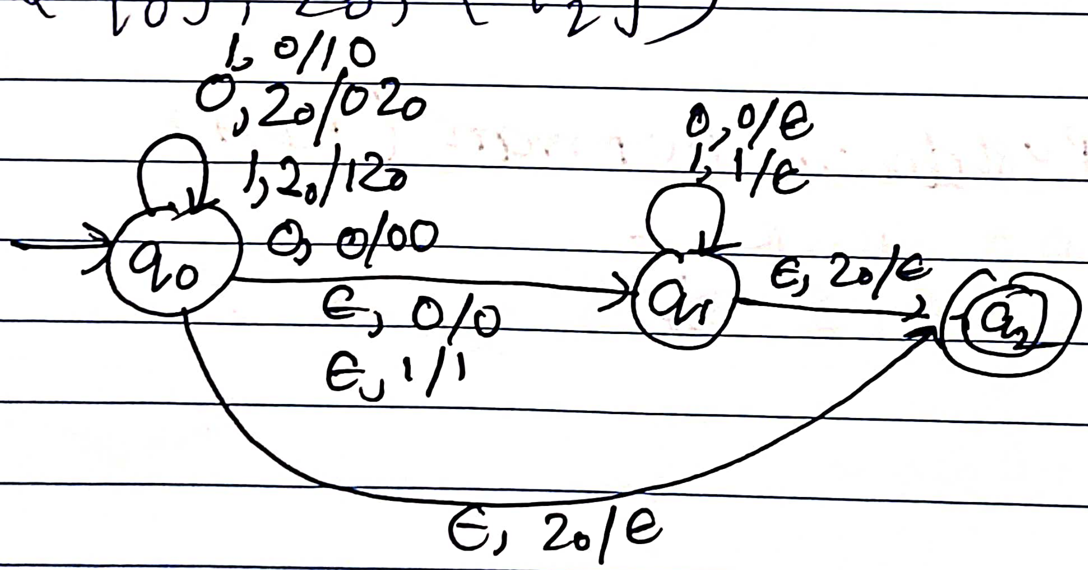
$\delta(q_0, \epsilon, 1) = (q_1, 1)$

$\delta(q_1, 0, 0) = (q_1, \epsilon)$

$\delta(q_1, 1, 1) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, Z_0) = (q_2, \epsilon)$ (accepting state)

$M = (\{q_0, q_1, q_2\}, \{0, 1\}, \{0, 1, Z_0\}, \delta, \{q_0\}, Z_0, \{q_2\})$



$(q, aw, X\beta) \vdash (p, w, \alpha\beta)$

top of stack

contents of stack

push to stack till half the string

Q. $(q_0, \overset{\uparrow}{\underset{}{1111}}, Z_0) \vdash (q_0, 111, 1Z_0) \vdash$

$(q_0, 11, 11Z_0) \vdash (q_1, \underline{11}, \underline{11}Z_0)$

matched

$\vdash^{*} (q_1, \epsilon, Z_0) \vdash (q_2, \epsilon, \epsilon)$

(no. of 0^s > no. of 1^s)

Q. $\delta(q_0, 0, Z) \longrightarrow \{(q_1, ZZ)\}$

pushing

$\rightarrow \{(q_1, \epsilon)\}$ pop

$\delta(q_0, 1, Z) = \{(q_0, XZ)\}$ push

$\delta(q_0, 0, X) = \{(q_0, \epsilon)\}$ pop X

$\delta(q_0, 1, X) = \{(q_0, XX)\}$

$\delta(q_0, 0, Z) = \{(q_1, YZ)\}$

$\delta(q_1, 1, Z) = \{(q_0, Z)\}$

$\delta(q_1, 0, Y) = \{(q_1, YY)\}$

$\delta(q_1, 1, Y) = \{(q_1, \epsilon)\}$

more than 1s

atleast more zero

near state

$(q_0, \underline{0}0011, \underline{Z}) \rightarrow (q_1, 0011, Z)$

$\rightarrow (q_1, \underline{0}11, \underline{Y}Z)$

$\rightarrow (q_1, \underline{1}1, YYZ)$

$\rightarrow (q_1, 1, YZ)$

$\rightarrow (q_1, \epsilon, Z)$

Laws of Regex

$L + \phi \rightarrow$ Identity

$L \cdot \epsilon \longrightarrow$ Identity

$L \cdot \phi = \phi$ $\qquad L^+ = LL^* = L^*L$

$\phi^* = \epsilon$

$\epsilon^* = \epsilon$

$\bar{L} = \Sigma^* - L$

PDA –

Accepts on final state

$\updownarrow$

accepts on empty stack

Q $L = \{0^n 1^n, n \geq 0\}$ $\qquad$ 0011

(accepts on empty stack)

$\delta(q_0, 0, z_0) = (q_0, 0z_0)$

$\delta(q_0, 0, 0) = (q_0, 00)$

$\delta(q_0, 1, 0) = (q_1, \epsilon)$

$\delta(q_1, 1, 0) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$

$\vdash$ turnstile

Q No. of 0s = No. of 1s

0 - push A
1 - push B

$\delta(q, 0, Z) = (q, AZ)$

$\delta(q, 1, Z) = (q, B, Z)$

$\delta(q, 0, A) = (q, \{A, A)$

$\delta(q, 0, B) = (q, \epsilon)$

$\delta(q, 1, A) = (q, \epsilon)$

$\delta(q, 1, B) = (q, BB)$

$\delta(q, \epsilon, Z) = (q, \epsilon)$

(PDA accepts on empty stack)

PDA (accepts on empty stack) $\Rightarrow$ PDA (accepts on final state)

conversion

Initial stack symbol X_0

Instantaneous desc of PDA

$N(P) = \{ w \mid (q_0, w, Z_0) \vdash^*_{N_P} (q, \epsilon, \epsilon) \}$

$(q, 0011, Z) \vdash (q, 011, AZ) \vdash (q, 11, AAZ) \vdash (q, 1, AZ) \vdash (q, \epsilon, Z) \vdash (q, \epsilon)$

$L(P) = \{ w \mid (q_0, w, Z_0) \vdash^* (q, \epsilon, \alpha) \}$

Write a PDA for $0^n 1^m 0^n \mid m, n \geq 1$

$\delta(q_0, 0, z) = (q_0, 0z)$
$\delta(q_0, 0, 0) = (q_0, 00)$
$\delta(q_0, 1, 0) = (q_1, 0)$
$\delta(q_1, 1, 0) = (q_1, 0)$
$\delta(q_1, 0, 0) = \delta(q_1, \epsilon)$
$\delta(q_1, 0, z) = (q_1, \epsilon)$

Let $G = (V, T, Q, S)$ construct a PDA that accepts on empty stack

$P = (\{q\}, T, V \cup T, \delta, Q, S)$ (set of states; initial state)

δ is defined by

1. For each variable A

$\delta(q, \epsilon, A)\ \{(q, \beta) \mid A \to \beta$ is a production of P $\}$

2. For each terminal a, $\delta(q, a, a) = \{(q, \epsilon)\}$

Q. $I \to a \mid b \mid Ia \mid Ib$

$E \to I \mid E*E \mid E+E \mid (E)$

$\delta(q, \epsilon, E) = \{(q, I), (q, E*E), (q, E+E), (q, (E))\}$

$\delta(q, \epsilon, I) = \{(q, a), (q, b),$
$(q, Ia), (q, Ib)\}$

~~non~~ terminals $\Rightarrow a, b, *, +, (,)$

$\delta(q, a, a) = \{(q, \epsilon)\}$
$\delta(q, b, b) = \{(q, \epsilon)\}$
$\delta(q, *, *) = \{(q, \epsilon)\}$
$\delta(q, +, +) = \{(q, \epsilon)\}$
$\delta(q, (, () = \{(q, \epsilon)\}$
$\delta(q,),)) = \{(q, \epsilon)\}$

Q// $((a+b)*a$

$\delta(q, (a+b)*a, E) \vdash \delta(q, (a+b)*a, E*E)$

$\vdash \delta(q, (a+b)*a, (E)*E)$

$\vdash \delta(q, a+b)*a, E)*E)$
$\vdash \delta(q, a+b)*a, E+E)*E)$
$\vdash \delta(q, a+b)*a, \underline{I}+E)*E)$

a

$\vdash \delta(q, \cancel{+}\cancel{b})*a, \cancel{+}\underline{E})*E)$

$\vdash$ $\underline{I}$

$\cancel{b}$

$\vdash \cancel{*} a, \cancel{*}, \underline{E}$ $I\ a$ $\vdash^* (q, \epsilon, \epsilon)$

CFG $\rightarrow$ PDA $\rightarrow$ PDA
ES $\leftarrow$ FS

Q. $S \rightarrow S0S1S0S \mid S0S0S1S \mid S1S0S0S \mid \epsilon$ $\quad \Sigma = \{0, 1\}$

construct a PDA

$\delta(q, \epsilon, S) = \{(q, S0S1S0S), (q, S0S0S1S), (q, S1S0S0S), (q, \epsilon)\}$

$\delta(q, 0, 0) = \{(q, \epsilon)\}$
$\delta(q, 1, 1) = \{(q, \epsilon)\}$

PDA to Grammar

Let $P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0)$ be a PDA
construct $G = (V, \Sigma, R, S)$ set of variables
V consists of

a) start symbol S
b) All symbols of the form $[pXq]$ where p and q are states and X is stack symbol.

Set of productions are
a) For all p, G has $S \rightarrow [q_0 Z_0 p]$
b) Let $\delta(q, a, X)$ contain the pair $(r, Y_1, Y_2 \ldots Y_k)$
where 1. $a \in \Sigma$ $a \cdot \epsilon$
2. $k \geq 0$

For all states $r_1, r_2 \dots r_k$ & has
$[q X r_k] \to a[r Y_1 r_1][r_1 Y_2 r_2] \dots [r_{k-1} Y_k r_k]$

Q $\delta(q, 0, Z) = (q, AZ)$
$\delta(q, 1, Z) = q(BZ)$
$\delta(q, 0, A) = (q, AA)$
$\delta(q, 0, B) = (q, \epsilon)$
$\delta(q, 1, A) = (q, \epsilon)$
$\delta[q, 1, B) = q(BB)$
$\delta(q, \epsilon, Z) = (q, \epsilon)$

$\Gamma = \{A, B, Z\}$
$\Sigma = \{0, 1\}$

States $\Rightarrow q$
$\boxed{[q A q]}$
$[q B q]$
$[q Z q]$

$a = \epsilon \; (q A Z)$

$S \to [q, Z\, q]^A$

$\boxed{\delta(q\ 0Z)} \Rightarrow$
$[q\, Z\, q]^A \to 0\, [q \overset{B}{A} q]\, [q \overset{A}{Z} q] \;|$

$\boxed{\delta(q\ 1\ Z)}$ $\quad 1\, [q\, \overset{C}{B}\, q][q\, \overset{A}{Z}\, q]$

$[q\, \overset{B}{A}\, q] \to 0\, [q \overset{B}{A} q][q\, \overset{B}{A}\, q] \;|\; 1$
$[q\, \overset{C}{B}\, q] \to 0 \;|\; 1\, [q\, \overset{C}{B}\, q][q\, \overset{C}{B}\, q]$

$S \rightarrow A$

$A \rightarrow 0BA \mid 1CA \mid \epsilon$

$B \rightarrow 0BB \mid 1$

$C \rightarrow 0 \mid 1CC$

Q. 1) $\delta(q, 0, Z_0) = (q, XZ_0)$ → 4 productions

2) $\delta(q, 0, X) = (q, XX)$

3) $\delta(q, 1, X) = (p, \epsilon)$

4) $\delta(p, 1, X) = (p, \epsilon)$

5) $\delta(p, \epsilon, X) = (p, \epsilon)$

6) $\delta(p, \epsilon, Z_0) = (p, \epsilon)$

$$S \rightarrow \overset{A}{[q Z_0 q]} \mid \overset{B}{[q Z_0 p]}$$

① $\overset{A}{[q Z_0 q]} \rightarrow 0\overset{C}{[q X p]}\overset{D}{[p Z_0 q]} \mid$

$0\underset{E}{[q X q]}\underset{A}{[q Z_0 q]}$

$(q Z_0) \rightarrow (XZ_0)$

Write $0[q\,X\;\;][\;\;Z_0 q]$

now fill q & p

(2) $\overset{E}{[q\,x\,q]} \rightarrow 0\,\overset{C}{[q\,x\,p]}\,\overset{F}{[p\,x\,q]}\;1$

$\quad 0\,\underset{E}{[q\,x\,q]}\,\underset{E}{[q\,x\,q]}$

$\overset{B}{[q\,2_0\,p]} \rightarrow 0\,\overset{E}{[q\,x\,q]}\,\overset{B}{[q\,2_0\,p]}\;|$

$\quad 0\,\underset{C}{[q\,x\,p]}\,\underset{G}{[p\,2_0\,p]}$

(3) $[q\,x\,p] \rightarrow 1$

$[p\,2_0\,q] \rightarrow$

(?) $[p\,x\,q] \rightarrow 1$

(4) $[p\,x\,p] \rightarrow 1$

$S \rightarrow A \mid B$

$A \rightarrow 0\,C\,D \mid 0\,E\,A$

$E \rightarrow 0\,C\,F \mid 0\,E\,E$

$C \rightarrow 1$

$B \rightarrow 0\,E\,B \mid 0\,G\,G$

$F \rightarrow 1$

(5) $[p\,x\,p] \rightarrow \epsilon$

$[p\,x\,q] \rightarrow \epsilon$

(6) $[p\,2_0\,p] \rightarrow \epsilon$

$[p\,2_0\,q] \rightarrow \epsilon$