SUNDAR COACHING CENTRE

DESIGN AND ANALYSIS OF ALGORITHMS, SEMESTER-4, 2023

PRE-END SEMESTER REVISION EXAMINATION, May06

This Paper consists of 5 Sections.

Section-A convers Introduction of Algorithms. It spans Q1 and Q2, with mark split-up as 2+3, totalling 5 MARKS.

Section-B convers Greedy Approach. It spans Q3 to Q9, with mark split-up as 6+5+5+5+5+5+6, totalling 37 MARKS.

Section-C covers Dynamic Programming. It spans Q10 to Q17, with mark split-up as 5+5+5+5+5+5+5+5, totalling 40 MARKS.

Section-D convers Amortized Analysis. It spans Q18 and Q19, with mark split-up as 6+5, totalling 11 MARKS.

Section-E covers NP-Problems. It spans Q20, with mark split-up as 7, totalling 7 MARKS.

Therefore, in total, this paper is of 5+37+40+11+7 = 100 MARKS.

SECTION-A: INTRODUCTION TO ALGORITHMS

Question-1

Using tournament method, find the maximum element of the array {3, 2, 8, 1, 4, 9, 10, 1}.

[2]

Question-2

Find out the time complexity of the following C++ Code Snippet using the Master's Theorem. Also calculate the value of foo (6).

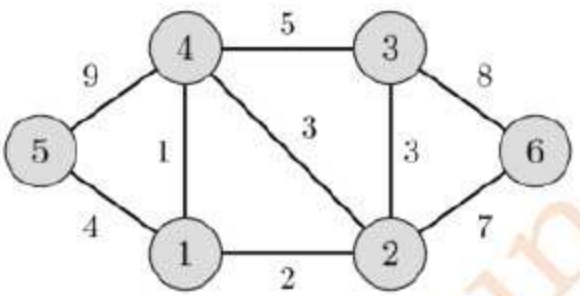
```
int foo(int n) {
    if (n <= 1) {
        return 1;
    }

    int count = 0;
    for (int i = 0; i < n; i++) {
        count++;
    }

    return foo(n / 2) + foo(n / 2) + count;
}
```

[3]

SECTION-B: GREEDY APPROACH

Question-3

Using Kruskal's algorithm, find the minimal spanning tree and using Prim's algorithm, find the maximal spanning tree for the following graph:



[3+3]

Question-4

Using Strassen's Matrix Multiplication, multiply the following matrices:

$$\begin{bmatrix} 5 \\ 4 \end{bmatrix} \times \begin{bmatrix} 4 & 5 \end{bmatrix}$$

[5]

Question-5

Solve the Fractional Knapsack Problem using Greedy Approach. The knapsack capacity is 15 kg. The weights of the items and their corresponding profits are given by {10, 5, 8, 2} and {60, 30, 45, 10}.

[5]

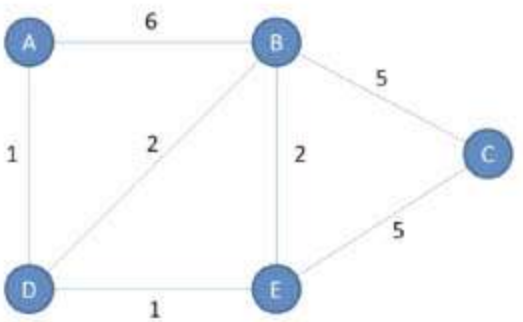
Question-6

Consider a text document with the characters {A, B, C, D, E} and their corresponding frequencies as {10, 15, 12, 3, 4}. Using Huffman coding, encode the string DCBABED.

[5]

Question-7

Using Dijkstra's Algorithm, find the shortest distance and corresponding path to each vertex starting from A:



[5]

Question-8

You are given a set of 10 jobs, each with a specific deadline and profit. Each job takes one unit of time to complete. You can only work on one job at a time. The objective is to maximize the total profit by selecting a subset of jobs to complete, considering their deadlines. The deadlines are given by {2, 1, 2, 3, 1, 4, 2, 3, 1, 4} and the profits are given by {100, 50, 75, 90, 60, 110, 45, 80, 55, 70}. Using the Job Sequencing with Deadlines algorithm, determine the maximum profit achievable and the sequence of jobs to be completed.

[5]

Question-9

Show the steps involved in sorting the array {6, 5, 12, 10, 9, 1} using merge sort and quick sort (taking last element as pivot).

[3+3]

SECTION-C: DYNAMIC PROGRAMMING

Question-10

Solve the given matrix chain multiplication problem using dynamic programming. The matrix dimension sequence is given by {2, 4, 4, 5, 2, 6}. Find the optimal order of multiplication to maximize the number of scalar multiplications required.

[5]

Question-11

Solve the 0/1 Knapsack Problem using dynamic programming. Knapsack capacity is 15 kg. The weights of the items and their corresponding profits are given by {6, 3, 2, 8, 5, 9} and {10, 7, 4, 12, 8, 15}.

[5]

Question-12

Solve the Longest Common Subsequence problem using dynamic programming for the following strings: String 1: "AGGTAB" and String 2: "GXTXAYB". Determine the length of the LCS and provide the actual LCS sequence.

[5]

Question-13

Given a set of keys – {1, 2, 3, 4, 5}. Frequencies of successful searches are given by {5, 2, 3, 9, 7} and unsuccessful searches are given by {1, 3, 2, 5, 7, 4}. Construct an optimal BST for this situation to minimize the total number of comparisons.

[5]

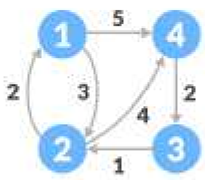
Question-14

A salesperson needs to visit a set of cities A, B, C, D, E and return to the starting city. The salesperson has a list of cities to visit, and the distances between each pair of cities are given by A to B = 10, A to C = 15, A to D = 20, A to E = 25, B to C = 35, B to D = 40, B to E = 45, C to D = 55, C to E = 60, D to E = 70. Find the longest possible route that visits each city exactly once, starting and ending at city A.

[5]

Question-15

Apply the Floyd-Warshall Algorithm on the following graph and find the shortest distance between all possible pairs of vertices:



[5]

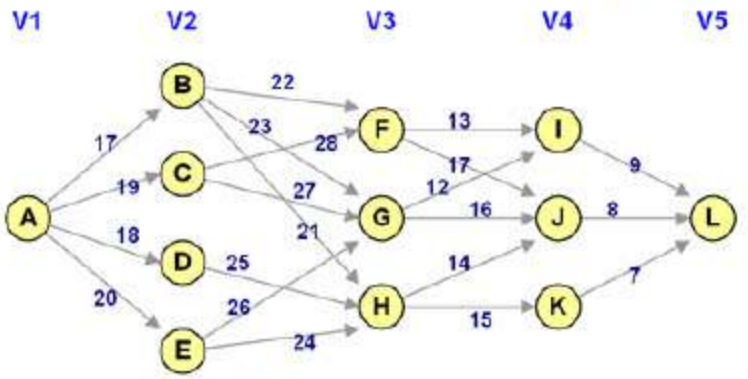
Question-16

You are given an array A of n integers (n ≤ 10^5). You need to find the maximum subarray sum such that no two adjacent elements are included in the subarray. To solve this problem using linear dynamic programming, you can define a state dp[i] as the maximum subarray sum ending at index i, where no two adjacent elements are included. You can then use the recurrence relation: dp[i] = max(dp[i-1], dp[i-2] + A[i]). Find the answer for the array: [1, -2, 3, -1, 2].

[5]

Question-17

Solve the given multi-stage graph and find the path which requires the minimum cost to travel.



[5]

SECTION-D: AMORTIZED ANALYSIS

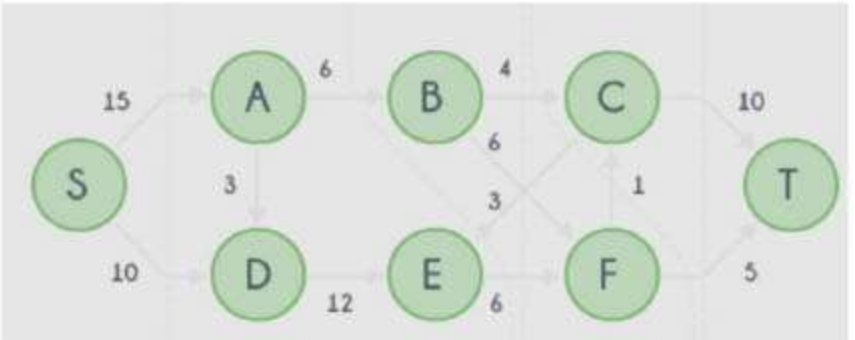
Question-18

Sort the array {8, 1, 6, 4, 0, 3, 9, 5} using randomized quick sort. After sorting, using randomized binary search, search the elements: 4 and 7. Write proper steps and find the time complexities of both operations.

[3+3]

Question-19

What is Amortization? Differentiate between the Las-Vegas and Monte-Carlo type algorithms. Then, find the min cut of the graph below:



[2+3]

SECTION-E: NP PROBLEMS

Question-20

Explain in depth about the classification of problems. Explain Cook's Theorem.

[7]
