Team Anveshak: 4 Degree of Freedom Arm Analytical Solution

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1 Introduction

In this paper we look at the analytical solution for a 4 degree of freedom arm. We use Denavit-Hartenberg algorithm inorder to find the solution. We have assumed a configuration of the arm, that would be used in the actual rover.

2 Arm Configuration

Figure 1 shows the 4 degree of freedom arm's configuration. The arrows show the axes of rotation, for the indivdual joints. Note, O0 and O1 are the same point!, that is the base can rotate in yaw and roll directions. O0 is taken as the base of coordinates, O4 is the final coordinates where we want to transform to. To this arm we shall begin assigning the link and joint parameters (aka the DH parameters).

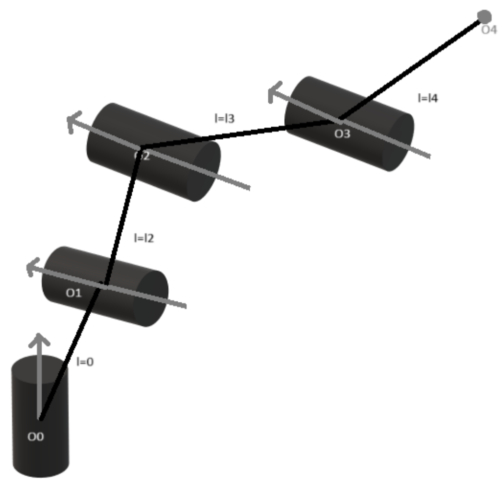

Figure 1: 4 Degree of Freedom Arm Configuration

3 DH parameters

These parameters are essential for the DH algorithm to work. We must be careful while assigning them keeping in mind the assigned coordinate axes for our arm configuration, and the definitions of these individual parameters. Table 1 lists the parameters we obtained for the following configuration.

Coordinates	Joint Angle (θ)	Joint Distance (d)	Link Length (a)	Link Twist Angle (α)
1	θ_1	0	0	-$\pi/2$
2	θ_2	0	l_2	0
3	θ_3	0	l_3	0
4	θ_4	0	l_4	0

Table 1: DH Parameters

4 Transformation Matrices

Now that we know the DH parameters, we can come up with the transformation matrices, such that, $T_0^4 = T_0^1 * T_1^2 * T_2^3 * T_3^4$
The final T_0^4 Matrix is as follows:

$$\begin{bmatrix} \mathrm{C}_1 * C_{234} & \mathrm{C}_1 * (-S_{234}) & \mathrm{S}_1 & \mathrm{C}_1 * (l_3 * C_{23} + l_2 * C_2 + l_4 * C_{234}) \\ \mathrm{S}_1 * C_{234} & \mathrm{S}_1 * (-S_{234}) & \mathrm{C}_1 & \mathrm{S}_1 * (l_3 * C_{23} + l_2 * C_2 + l_4 * C_{234}) \\ \text{-S}_{234} & \text{-C}_{234} & 0 & \text{-l}_3 * S_{23} - l_2 * S_2 - l_4 * S_2 34 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where,
$C_{ijk} = cos(\theta_i + \theta_j + \theta_k)$
$S_{ijk} = sin(\theta_i + \theta_j + \theta_k)$
$C_i = cos(\theta_i)$
$S_i = sin(\theta_i)$

Solutions:
$\theta_1 = \arctan 2(\frac{P_{4y}}{P_{4x}})$

$\theta_2 = \arccos \frac{\sqrt{P_{4x}^2 + P_{4y}^2} - l_4 * \sqrt{x_{4x}^2 + x_{4y}^2} - l_3 * \cos\left(\arccos \sqrt{x_{4x}^2 + x_{4y}^2} - \theta_4\right)}{l_2}$

$\theta_3 = \arccos \frac{0.5 * (P_{4x}^2 + P_{4y}^2 + P_{4z}^2 - l_2^2 - l_3^2 - l_4^2) - l_3 * l_4 * \cos\theta_4 - l_2 * l_4 * \cos(\arccos \sqrt{x_{4x}^2 + x_{4y}^2} - \theta_2)}{l_2 * l_3}$

$\theta_4 = \arccos \sqrt{x_{4x}^2 + x_{4y}^2} - \theta_2 - \theta_3$