

18. Definite Integrals and Applications of Integrals

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I. MCQs WITH ONE CORRECT ANSWER

14)

$$If f(x) = \begin{cases} e^{\cos x} \sin x, & (for |x| \leq 2) \\ 2, & (otherwise) \end{cases} \quad (1)$$

then $\int_{-2}^3 f(x) dx =$

- a) 0
- b) 1
- c) 2
- d) 3

2000S

15) The value of the integral $\int_{e^{-1}}^{e^2} \left| \frac{\log_e x}{x} \right| dx$ is:

- a) $\frac{3}{2}$
- b) $\frac{5}{2}$
- c) 3
- d) 5

2000S

16) The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx, a > 0$

- a) π
- b) $a\pi$
- c) $\frac{\pi}{2}$
- d) 2π

2001S

17) The area bounded by the curves $y = |-1|$ and $y = -|x| + 1$ is

- a) 1
- b) 2
- c) $2\sqrt{2}$
- d) 4

2002S

18) Let $f(x) = \int_1^x \sqrt{2-t^2} dt$. Then the real roots of the equation $x^2 - f'(x) = 0$ are

- a) 1
- b) $1\sqrt{2}$
- c) $\frac{1}{2}$
- d) 0 and 1

2002S

19) Let $T > 0$ be a real number. Suppose f is continuous function such that for all $x \in R, f(x+T) = f(x)$. If $I = \int_0^T f(x) dx$ then the value of $\int_3^{3+3T} f(2x) dx$ is

- a) $\frac{3}{2}I$
- b) $2I$
- c) $3I$
- d) $6I$

2002S

20) The integral $\int_{-1/2}^{1/2} ([x] + \ln(\frac{1+x}{1-x})) dx$ equal to

- a) $-\frac{1}{2}$
- b) 0
- c) 1
- d) $2\ln(\frac{1}{2})$

2002S

21) If $l(m, n) = \int_0^1 t^m (1+t)^n dt$, then the expression for $l(m, n)$ in terms of $l(m+1, n-1)$ is

- a) $\frac{2^n}{m+1} - \frac{n}{m+1} l(m+1, n-1)$
- b) $\frac{n}{m+1} l(m+1, n-1)$
- c) $\frac{2^n}{m+1} + \frac{n}{m+1} l(m+1, n-1)$
- d) $\frac{m}{n+1} l(m+1, n-1)$

22) If $f(x) = \int_{x^2}^{x^2-1} e^{-t^2} dt$, then $f(x)$ increases in

- a) $(-2, 2)$
- b) no value of x
- c) $(0, \infty)$
- d) $(-\infty, 0)$

2003S

23) The area bounded by the curves $y = \sqrt{x}$, $2y + 3 = x$ and x -axis in the 1st quadrant is

- a) 9
- b) $\frac{27}{4}$
- c) 36
- d) 18

2003S

24) If $f(x)$ is differentiable and $\int_0^{t^2} xf(x) dx = \frac{2}{5}t^5$, then $f(\frac{4}{25})$

- a) $\frac{2}{5}$
- b) $\frac{5}{2}$
- c) 1
- d) $\frac{1}{2}$

2004S

25) The value of the integral $\int_0^1 \sqrt{\frac{1-x}{1+x}} dx$ is

- a) $\frac{\pi}{2} + 1$
- b) $\frac{\pi}{2} - 1$
- c) -1
- d) 1

2004S

26) The area enclosed between the curves $y = ax^2$ and $x = ay^2$ and the line $y = 1/4$ is

- a) $\frac{1}{\sqrt{3}}$
- b) $\frac{1}{2}$
- c) 1

d) $\frac{1}{3}$ 2004S

27) $\int_{-2}^0 \{x^3 + 3x^2 + 3x + 3 + (x + 1) \cos(x + 1)\} dx$

a) -4

b) 0

c) 4

d) 6

2005S

28) The area bounded by the parabolas $y = (x + 1)^2$ and $y = (x - 1)^2$ and the line $y = \frac{1}{4}$ is

a) 4 sq units

b) $\frac{1}{4}$ sq units

c) $\frac{5}{4}$ sq units

d) $\frac{7}{3}$ sq units

2005S

29) The area of the region between the curves $y = \sqrt{\frac{1+\sin x}{\cos x}}$ and $y = \sqrt{\frac{1-\sin x}{\cos x}}$ bounded by the lines $x = 0$ and $x = \frac{\pi}{4}$ is

a) $\int_0^{\sqrt{2}-1} \frac{t}{1+t^2 \sqrt{1-t^2}} dt$

b) $\int_0^{\sqrt{2}-1} \frac{4t}{1+t^2 \sqrt{1-t^2}} dt$

c) $\int_0^{\sqrt{2}+1} \frac{4t}{1+t^2 \sqrt{1-t^2}} dt$

d) $\int_0^{\sqrt{2}+1} \frac{t}{1+t^2 \sqrt{1-t^2}} dt$

2008

30) Let f be a non negative function defined on the interval $[0, 1]$. If $\int_0^x \sqrt{1 - (f'(t))^2} dt = \int_0^x f(t) dt, 0 \leq x \leq 1$, and $f(0) = 0$, then

a) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

b) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

c) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

d) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

2009S