2022-July-27 Shift-2

AI24BTECH11005 - Prajwal Naik

16) Let X have a binomial distribution $B(n, p)$ such that the sum and product of the mean and variance of X are 24 and 128 respectively. If $P(X > n-3) = \frac{k}{2^n}$

a) 528 b) 529 c) 629 d) 630

17) A six faced die is biased such that $3 \times P(\text{a prime number}) = 2 \times P(1) = 6 \times P(\text{a composite number})$. Let X be a random variable that counts the number of times one gets a perfect square on some throws of this die. If the die is thrown twice, then the mean of X is :

a) $\frac{3}{11}$
b) $\frac{5}{11}$
c) $\frac{7}{11}$
d) $\frac{8}{11}$

18) The angle of elevation of the top P of a vertical tower PQ of height 10 from point A on the horizontal ground is 45°. Let R be a point on AQ and from a point B, the angle of elevation of P is 60°. If $\angle BAQ$=30°, $AB = d$ and the area of the trapezium PQRB is α, then the ordered pair (d, α) is :

a) $\left(10\left(\sqrt{3}-1\right), 25\right)$
b) $\left(10\left(\sqrt{3}-1\right), \frac{25}{2}\right)$
c) $\left(10\left(\sqrt{3}+1\right), 25\right)$
d) $\left(10\left(\sqrt{3}+1\right), \frac{25}{2}\right)$

19) Let $S = \{\theta \in \left(0, \frac{\pi}{2}\right) : \sum_{m=1}^{9} \sec\left(\theta + (m-1)\frac{\pi}{6}\right)\sec\left(\theta + \frac{m\pi}{6}\right)\}$

a) $\{\frac{\pi}{6}\}$
b) $\{\frac{2\pi}{3}\}$
c) $\sum_{\theta \in S} \theta = \frac{\pi}{2}$
d) $\sum_{\theta \in S} \theta = \frac{3\pi}{4}$

20) If the truth value of the statement $(P \wedge (\neg R)) \rightarrow ((\neg R) \wedge Q)$ is F, then the truth value of which is of the following is F ?

a) $P \vee Q \rightarrow \neg R$
b) $R \vee Q \rightarrow \neg P$
c) $\neg(P \vee Q) \rightarrow \neg R$
d) $\neg(R \vee Q) \rightarrow \neg P$

21) A= $\begin{pmatrix} 4 & -2 \\ \alpha & \beta \end{pmatrix}$. If $A^2 + \gamma A + 18I = 0$, then $\det(A)$ is equal to :

22) The number of functions f, from the set $A = \{x \in N : x^2 - 10x + 9 \leq 0\}$ to the set $B = \{n^2 : n \in N\}$ such that $f(x) \leq (x-3)^2 + 1$, for every $x \in A, is$

23) Let for the 9^{th} term in the binomial expansion of $(3 + 6x)^n$, in the increasing powers of $6x$, to be the greatest for $x = \frac{3}{2}$, the least value of n is n_0. If K is the ratio of the

coefficient of x^6 to the coefficient of x^3, then $k + n_0$ is equal to:

24) $\frac{2^3-1^3}{1\times7} + \frac{4^3-3^3+2^3-1^3}{2\times11} + ... + \frac{6^3-5^3+4^3-3^3+2^3-1^3}{3\times15} + \frac{30^3-29^3+28^3-27^3+.....+2^3-1^3}{15\times63}$ is equal to .

25) A water tank has the shape of a right circular cone with axis vertical and vertex downwards. Its semi-vertical angle is $\tan^{-1}\frac{3}{4}$. Water is poured in at a constant rate of 6 cubic meter per hour. The rate, at which wet curved surface area of the tank is increasing, when the depth of the tank is 4 meters, is:

26) For the curve $C : \left(x^2 + y^2 - 3\right) + \left(x^2 - y^2 - 1\right)^5 = 0$, the value of $3y' - y^3y''$, at the point (α, α), $\alpha \geq 0$, on C, is equal to :

27) Let $f(x) = min\{[x-1], [x-2], ..., [x-10]\}$ where $[t]$ denotes the greatest integer $\leq t$. Then $\int_0^{10} f(x)\,dx + \int_0^{10} (f(x))^2 dx + \int_0^{10} |f(x)|dx$ is equal to :

28) Let f be a differentiable function satisfying $f(x) = \frac{2}{\sqrt{3}}\int_0^{\sqrt{3}} f\left(\frac{\lambda^2 x}{3}\right)d\lambda$, $x \geq 0$ and $f(1) = \sqrt{3}$. If $y = f(x)$ passes through the point $(\alpha, 6)$, then α is equal to :

29) A common tangent T to the curves $C_1 : \frac{x^2}{4} + \frac{y^2}{9} = 1$ and $C_2 : \frac{x^2}{42} - \frac{y^2}{143} = 1$ does not pass through the fourth quadrant. If T touches C_1 at (x_1, y_1) and C_2 at (x_2, y_2), then $|2x_1 + x_2|$ is equal to :

30) Let, $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors such that $\vec{a}\times\vec{b} = 4\vec{c}$, $\vec{b}\times\vec{c} = 9\vec{a}$, and $\vec{c}\times\vec{d} = \alpha\vec{b}$, $\alpha > 0$ If $\left|\vec{a}\right| + \left|\vec{b}\right| + \left|\vec{c}\right| = \frac{1}{36}$, then the α is equal to :