18. Definite Integrals and Applications of Integrals

ai24btech11005,Bhukya Prajwal Naik

I. MCQs with One Correct Answer

14)
$$\text{If } f(x) = \begin{cases} e^{\cos x}\sin x, & \text{for } |x| \le 2) \\ 2, & \text{otherwise} \end{cases} \quad (1)$$
then $\int_{-2}^{3} f(x)\,dx=$ (2000S)
a) 0
b) 1
c) 2
d) 3

15) The value of the integral $\int_{e^{-1}}^{e^2} \left|\frac{\log_e x}{x}\right| dx$ is: (2000S)
a) $\frac{3}{2}$
b) $\frac{5}{2}$
c) 3
d) 5

16) The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx, a > 0$ (2001S)
a) π
b) $a\pi$
c) $\frac{\pi}{2}$
d) 2π

17) The area bounded by the curves $y = |x|\, and y = -|x| + 1$ is (2002S)
a) 1
b) 2
c) $\frac{2}{\sqrt{2}}$
d) 4

18) Let$f(x) = \int_1^x \sqrt{2-t^2}dt$.Then the real roots of the equation $x^2 - f('x) = 0$ are (2002S)
a) 1
b) $1\sqrt{2}$
c) $\frac{1}{2}$
d) 0 and 1

19) Let $T > 0$ be a real number. Suppose f is continuous function such that for all $x \in R$,f$(x+T) = f(x)$. If I=$\int_0^T f(x)\,dx$ then the value of $\int_3^{3+3T} f(2x)\,dx$ is (2002S)
a) $\frac{3}{2}I$
b) 2I
c) 3I
d) 6I

20) The integral $\int_{-1/2}^{1/2} \left(\lfloor x\rfloor + \ln\left(\frac{1+x}{1-x}\right)\right) dx$ equal to (2002S)
(a) $\frac{-1}{2}$
(b) 0
(c) 1
(d) $2ln\left(\frac{1}{2}\right)$

21) If $l(m,n) = \int_0^1 t^m (1+t)^n dt$,then the expression for l(m,n) in terms of l$(m+1, n-1)$ is (2002S)
a) $\frac{2^n}{m+1} - \frac{n}{m+1} l(m+1, n-1)$
b) $\frac{n}{m+1} l(m+1, n-1)$
c) $\frac{2^n}{m+1} + \frac{n}{m+1} l(m+1, n-1)$
d) $\frac{m}{n+1} l(m+1, n-1)$

22) If $f(x) = \int_{x^2}^{x^2-1} e^{-t^2}dt$, then f$(x)$ increases in (2003S)
a) $(-2, 2)$
b) no value of x
c) $(0, \infty)$
d) $(-\infty, 0)$

23) The area bounded by the curves $y = \sqrt{x}, 2y + 3 = x$ and x-axis in the1^{st} quadrant is (2003S)
a) 9
b) $\frac{27}{4}$
c) 36
d) 18

24) If f(x) is differentiable and $\int_o^{t^2} xf(x)\,dx = \frac{2}{5}t^5$, then f$\left(\frac{4}{25}\right)$ (2004S)
a) $\frac{2}{5}$
b) $\frac{-5}{2}$
c) 1
d) $\frac{1}{2}$

25) The value of the integral $\int_0^1 \sqrt{\frac{1-x}{1+x}dx}$ is (2004S)
a) $\frac{\pi}{2}$+1
b) $\frac{\pi}{2}$-1
c) -1
d) 1

26) The area enclosed between the curves $y = ax^2$ and $x = ay^2$ and the line $y = 1/4$ is (2004S)
a) $\frac{1}{\sqrt{3}}$

b) $\frac{1}{2}$
c) 1
d) $\frac{1}{3}$

27) $\int_{-2}^{0}\{x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1)\}dx$ (2005S)

a) -4
b) 0
c) 4
d) 6

28) The area bounded by the parabolas $y = (x+1)^2$ and $y = (x-1)^2$ and the line $y = \frac{1}{4}$ is (2005S)

a) 4 sq units
b) $\frac{1}{6}$ sq units
c) $\frac{4}{3}$ sq units
d) $\frac{1}{3}$ sq units

29) The area of the region between the curves $y = \sqrt{\frac{1+\sin x}{\cos x}}$ and $y = \sqrt{\frac{1-\sin x}{\cos x}}$ bounded by the lines $x = 0$ and $x = \frac{\pi}{4}$ is (2008S)

a) $\int_0^{\sqrt{2}-1} \frac{t}{1+t^2\sqrt{1-t^2}dt}$
b) $\int_0^{\sqrt{2}-1} \frac{4t}{1+t^2\sqrt{1-t^2}dt}$
c) $\int_0^{\sqrt{2}+1} \frac{4t}{1+t^2\sqrt{1-t^2}dt}$
d) $\int_0^{\sqrt{2}+1} \frac{t}{1+t^2\sqrt{1-t^2}dt}$

30) Let f be a non negative function defined on the interval $[0,1]$. If $\int_0^x \sqrt{1-(f'(t))^2}dt = \int_0^x f(t)\,dt, 0 \leq x \leq 1$,and $f(0) = 0$, then (2009S)

a) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$
b) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$
c) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$
d) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$