18. Definite Integrals and Applications of Integrals

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I. MCQs with One Correct Answer

14)
$$Iff(x) = \begin{cases} e^{cosx}sinx, & (for|x| \leq 2) \\ 2, & (otherwise) \end{cases} \quad (1)$$

then $\int_{-2}^{3} f(x)\, dx$=
a) 0
b) 1
c) 2
d) 3 (2000S)

15) The value of the integral $\int_{e^{-1}}^{e^2} \left| \frac{\log_e x}{x} \right| dx$ is:
a) 3/2
b) 5/2
c) 3
d) 5 (2000S)

16) The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$,$a > 0$
a) π
b) $a\pi$
c) $\pi/2$
d) 2π (2001S)

17) The area bounded by the curves $y = |x| - 1andy = -|x| + 1$ is
a) 1
b) 2
c) $2\sqrt{2}$
d) 4 (2002S)

18) Let$f(x) = \int_1^x \sqrt{2 - t^2} dt$.Then the real roots of the equation $x^2 - f('x) = 0$ are
a) ± 1
b) $\pm 1/\sqrt{2}$
c) $\pm 1/2$
d) 0 and 1 (2002S)

19) Let $T > 0$ be a real number. Suppose f is continuous function such that for all $x \in R$,f$(x + T) = f(x)$. If I=$\int_0^T f(x)\, dx$ then the value of $\int_3^{3+3T} f(2x)\, dx$ is
a) 3/2I
b) 2I
c) 3I
d) 6I (2002S)

20) The integral $\int_{-1/2}^{1/2} \left(\lfloor x \rfloor + \ln\left(\frac{1+x}{1-x}\right)\right) dx$ equal to
(a) -1/2
(b) 0
(c) 1
(d) 2ln(1/2)

21) If $l(m, n) = \int_0^1 t^m (1 + t)^n\, dt$,then the expression for l(m, n) in terms of l$(m + 1, n - 1)$ is
a) $\frac{2^n}{m+1} - \frac{n}{m+1} l(m + 1, n - 1)$
b) $\frac{n}{m+1} l(m + 1, n - 1)$
c) $\frac{2^n}{m+1} + \frac{n}{m+1} l(m + 1, n - 1)$
d) $\frac{m}{n+1} l(m + 1, n - 1)$

22) If $f(x) = \int_{x^2}^{x^2-1} e^{-t^2} dt$, then f$(x)$ increases in
a) $(-2, 2)$
b) no value of x
c) $(0, \infty)$
d) $(-\infty, 0)$ (2003S)

23) The area bounded by the curves $y = \sqrt{x}, 2y + 3 = x$ and x-axis in the1^{st} quadrant is
a) 9
b) 27/4
c) 36
d) 18 (2003S)

24) If f(x) is differentiable and $\int_o^{t^2} xf(x)\, dx = \frac{2}{5}t^5$, then f$\left(\frac{4}{25}\right)$
a) 2/5
b) -5/2
c) 1
d) 5/2 (2004S)

25) The value of the integral $\int_0^1 \sqrt{\frac{1-x}{1+x} dx}$ is
a) $\frac{\pi}{2}$+1
b) $\frac{\pi}{2}$-1
c) -1
d) 1 (2004S)

26) The area enclosed between the curves $y = ax^2$ and $x = ay^2$ and the line $y = 1/4$ is
a) $1/\sqrt{3}$
b) 1/2
c) 1
d) 1/3 (2004S)

27) $\int_{-2}^{0}\{x^3 + 3x^2 + 3x + 3 + (x + 1)\cos(x + 1)\}$dx

a) -4

b) 0

c) 4

d) 6 (2005S)

28) The area bounded by the parabolas $y = \left(x + 1^2\right)$ and $y = \left(x - 1^2\right)$ and the line $y = 1/4$ is

a) 4 sq units

b) 1/6 sq units

c) 4/3 sq units

d) 1/3 sq units (2005S)

29) The area of the region between the curves $y = \sqrt{\frac{1+\sin x}{\cos x}}$ and $y = \sqrt{\frac{1-sinx}{cosx}}$ bounded by the lines $x = 0$ and $x = \frac{\pi}{4}$ is

a) $\int_{0}^{\sqrt{2}-1} \frac{t}{1+t^2\sqrt{1-t^2}dt}$

b) $\int_{0}^{\sqrt{2}-1} \frac{4t}{1+t^2\sqrt{1-t^2}dt}$

c) $\int_{0}^{\sqrt{2}+1} \frac{4t}{1+t^2\sqrt{1-t^2}dt}$

d) $\int_{0}^{\sqrt{2}+1} \frac{t}{1+t^2\sqrt{1-t^2}dt}$ (2008)

30) Let f be a non negative function defined on the interval $[0, 1]$. If $\int_{0}^{x}\sqrt{1 - (f'(t))^2}dt = \int_{0}^{x} f(t)\,dt, 0 \leq x \leq 1$,and $f(0) = 0$, then

a) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

b) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

c) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

d) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$ (2009)