18. Definite Integrals and Applications of Integrals

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I. MCQs with One Correct Answer

14)

$$If f(x) = \begin{cases} e^{\cos x}\sin x, & (for\,|x| \le 2) \\ 2, & (otherwise) \end{cases} \quad (1)$$

then $\int_{-2}^{3} f(x)\,dx$=

a) 0
b) 1
c) 2
d) 3 2000S

15) The value of the integral $\int_{e^{-1}}^{e^2} \left|\frac{\log_e x}{x}\right| dx$ is:

a) $\frac{3}{2}$
b) $\frac{5}{2}$
c) 3
d) 5 2000S

16) The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}dx$,$a > 0$

a) π
b) $a\pi$
c) $\frac{\pi}{2}$
d) 2π 2001S

17) The area bounded by the curves $y = |-1|\,and y = -|x| + 1$ is

a) 1
b) 2
c) $\frac{2}{\sqrt{2}}$
d) 4 2002S

18) Let$f(x) = \int_1^x \sqrt{2-t^2}dt$.Then the real roots of the equation $x^2 - f(')x) = 0$ are

a) 1
b) 1 $\sqrt{2}$
c) $\frac{1}{2}$
d) 0 and 1 2002S

19) Let $T > 0$ be a real number. Suppose f is continuous function such that for all $x \in R$,f$(x+T) = f(x)$. If I=$\int_0^T f(x)\,dx$ then the value of $\int_3^{3+3T} f(2x)\,dx$ is

a) $\frac{3}{2}I$
b) 2I
c) 3I
d) 6I 2002S

20) The integral $\int_{-1/2}^{1/2} \left(\lfloor x \rfloor + \ln\left(\frac{1+x}{1-x}\right)\right) dx$ equal to

(a) $\frac{-1}{2}$
(b) 0
(c) 1
(d) $2ln\left(\frac{1}{2}\right)$ 2002S

21) If $l(m,n) = \int_0^1 t^m (1+t)^n\,dt$,then the expression for l(m,n) in terms of l$(m+1,n-1)$ is

a) $\frac{2^n}{m+1} - \frac{n}{m+1}l(m+1,n-1)$
b) $\frac{n}{m+1}l(m+1,n-1)$
c) $\frac{2^n}{m+1} + \frac{n}{m+1}l(m+1,n-1)$
d) $\frac{m}{n+1}l(m+1,n-1)$

22) If $f(x) = \int_{x^2}^{x^2-1} e^{-t^2}dt$, then f$(x)$ increases in

a) $(-2,2)$
b) no value of x
c) $(0,\infty)$
d) $(-\infty,0)$ 2003S

23) The area bounded by the curves $y = \sqrt{x}, 2y + 3 = x$ and x-axis in the1^{st} quadrant is

a) 9
b) $\frac{27}{4}$
c) 36
d) 18 2003S

24) If f(x) is differentiable and $\int_o^{t^2} xf(x)\,dx = \frac{2}{5}t^5$, then f$\left(\frac{4}{25}\right)$

a) $\frac{2}{5}$
b) $\frac{-5}{2}$
c) 1
d) $\frac{1}{2}$ 2004S

25) The value of the integral $\int_0^1 \sqrt{\frac{1-x}{1+x}}dx$ is

a) $\frac{\pi}{2}$+1
b) $\frac{\pi}{2}$-1
c) -1
d) 1 2004S

26) The area enclosed between the curves $y = ax^2$ and $x = ay^2$ and the line $y = 1/4$ is

a) $\frac{1}{\sqrt{3}}$
b) $\frac{1}{2}$

c) 1
d) $\frac{1}{3}$ 2004S

27) $\int_{-2}^{0}\{x^3+3x^2+3x+3+(x+1)\cos(x+1)\}dx$
a) -4
b) 0
c) 4
d) 6 2005S

28) The area bounded by the parabolas $y=(x+1)^2$ and $y=(x-1)^2$ and the line $y=\frac{1}{4}$ is
a) 4 sq units
b) $\frac{1}{6}$ sq units
c) $\frac{4}{3}$ sq units
d) $\frac{1}{3}$ sq units 2005S

29) The area of the region between the curves $y=\sqrt{\frac{1+\sin x}{\cos x}}$ and $y=\sqrt{\frac{1-\sin x}{\cos x}}$ bounded by the lines $x=0$ and $x=\frac{\pi}{4}$ is
a) $\int_0^{\sqrt{2}-1}\frac{t}{1+t^2\sqrt{1-t^2}dt}$
b) $\int_0^{\sqrt{2}-1}\frac{4t}{1+t^2\sqrt{1-t^2}dt}$
c) $\int_0^{\sqrt{2}+1}\frac{4t}{1+t^2\sqrt{1-t^2}dt}$
d) $\int_0^{\sqrt{2}+1}\frac{t}{1+t^2\sqrt{1-t^2}dt}$ 2008S

30) Let f be a non negative function defined on the interval $[0,1]$. If $\int_0^x\sqrt{1-(f'(t))^2}dt=\int_0^x f(t)\,dt, 0\le x\le 1$,and $f(0)=0$, then
a) $f\left(\frac{1}{2}\right)<\frac{1}{2}$ and $f\left(\frac{1}{3}\right)>\frac{1}{3}$
b) $f\left(\frac{1}{2}\right)>\frac{1}{2}$ and $f\left(\frac{1}{3}\right)>\frac{1}{3}$
c) $f\left(\frac{1}{2}\right)<\frac{1}{2}$ and $f\left(\frac{1}{3}\right)<\frac{1}{3}$
d) $f\left(\frac{1}{2}\right)>\frac{1}{2}$ and $f\left(\frac{1}{3}\right)<\frac{1}{3}$ 2009S