

- 30) The donor concentration in a sample of n-type silicon is increased by a factor of 100. The shift in the position of the Fermi level at 300 K, assuming the sample to be non degenerate is _____ meV .

$$(k_B T = 25 \text{ meV at } 300 \text{ K})$$

- 31) A particle of mass m is subjected to a potential,

$$V(x, y) = \frac{1}{2} m \omega^2 (x^2 + y^2), -\infty \leq x \leq \infty, -\infty \leq y \leq \infty$$

The state with energy $4\hbar\omega$ is g -fold degenerate. The value of g is _____ .

- 32) A hydrogen atom is in the state $\Psi = \sqrt{\frac{8}{21}}\psi_{200} - \sqrt{\frac{3}{7}}\psi_{310} + \sqrt{\frac{4}{21}}\psi_{321}$ where n, l, m in ψ_{nlm} denote the principal, orbital and magnetic quantum numbers, respectively. If \mathbf{L} is the angular momentum operator, the average value of L^2 is () \hbar^2

- 33) A planet of mass m moves in a circular orbit of radius r_0 in the gravitational potential $V(r) = -\frac{k}{r}$, where k is a positive constant. The orbital angular momentum of the planet is

- a) $2r_0 km$ b) $\sqrt{2r_0 km}$ c) $r_0 km$ d) $\sqrt{r_0 km}$

- 34) The moment of inertia of a rigid diatomic molecule A is 6 times that of another rigid diatomic molecule B . If the rotational energies of the two molecules are equal, then the corresponding values of the rotational quantum numbers J_A and J_B are

- a) $J_A = 2, J_B = 1$ b) $J_A = 3, J_B = 1$ c) $J_A = 5, J_B = 0$ d) $J_A = 6, J_B = 1$

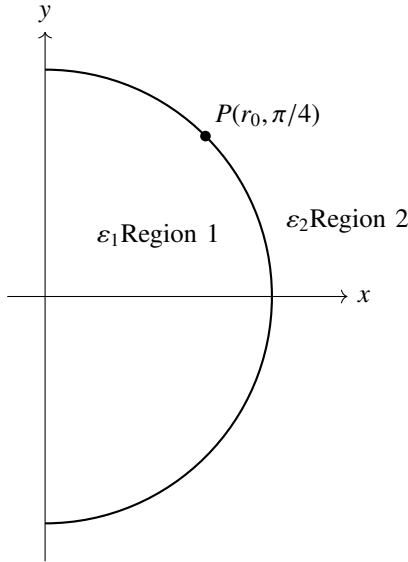
- 35) The value of the integral

$$\oint_C \frac{z^2}{e^z + 1} dz$$

where C is the circle $|z| = 4$, is

- a) $2\pi i$
 b) $2\pi^2 i$
 c) $4\pi^2 i$
 d) $4\pi^2 i$
- 36) A ray of light inside Region 1 in the xy -plane is incident at the semicircular boundary that carries no free charges. The electric field at the point $P(r_0, \pi/4)$ in plane polar coordinates is $\mathbf{E}_1 = 7\hat{e}_r - 3\hat{e}_\varphi$, where \hat{e}_r and \hat{e}_φ are the unit vectors. The

emerging ray in Region 2 has the electric field \mathbf{E}_2 parallel to x - axis. If ϵ_1 and ϵ_2 are the dielectric constants of Region 1 and Region 2 respectively, then $\frac{\epsilon_2}{\epsilon_1}$ is



- 37) The solution of the differential equation

$$\frac{d^2y}{dt^2} - y = 0$$

subject to the boundary conditions $y(0) = 1$ and $y(\infty) = 0$, is

- a) $\cos t + \sin t$ b) $\cosh t + \sinh t$ c) $\cos t - \sin t$ d) $\cosh t - \sinh t$

- 38) Given that the linear transformation of a generalized coordinate q and the corresponding momentum p ,

$$Q = q + 4ap$$

$$P = q + 2p$$

is canonical, the value of the constant a is _____.

- 39) The value of the magnetic field required to maintain non-relativistic protons of energy 1 MeV in a circular orbit of radius 100 mm is _____ Tesla. (Given: $m_p = 1.67 \times 10^{-27}$ kg, $e = 1.6 \times 10^{-19}$ C)

- 40) For a system of two bosons, each of which can occupy any of the two energy levels 0 and ϵ , the mean energy of the system at a temperature T with $\beta = \frac{1}{k_B T}$ is given by

- a) $\frac{\epsilon e^{-\beta\epsilon} + 2\epsilon e^{-2\beta\epsilon}}{1 + 2e^{-\beta\epsilon} + e^{-2\beta\epsilon}}$ b) $\frac{1 + \epsilon e^{-\beta\epsilon}}{2e^{-\beta\epsilon} + e^{-2\beta\epsilon}}$ c) $\frac{2\epsilon e^{-\beta\epsilon} + \epsilon e^{-2\beta\epsilon}}{2 + e^{-\beta\epsilon} + e^{-2\beta\epsilon}}$ d) $\frac{\epsilon e^{-\beta\epsilon} + 2\epsilon e^{-2\beta\epsilon}}{2 + e^{-\beta\epsilon} + e^{-2\beta\epsilon}}$

- 41) In an interference pattern formed by two coherent sources, the maximum and the minimum of the intensities are $9I_0$ and I_0 , respectively. The intensities of the individual waves are

- a) $3I_0$ and I_0 b) $4I_0$ and I_0 c) $5I_0$ and $4I_0$ d) $9I_0$ and I_0

42) ψ_1 and ψ_2 are two orthogonal states of a spin $\frac{1}{2}$ system. It is given that

$$\psi_1 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \sqrt{\frac{2}{3}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

where $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ represent the spin-up and spin-down states, respectively. When the system is in the state ψ_2 , its probability to be in the spin-up state is .