

2022-July-27 Shift-2

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AI24BTECH11005 - Prajwal Naik

- 16) Let X have a binomial distribution $B(n, p)$ such that the sum and product of the mean and variance of X are 24 and 128 respectively. If $P(X > n - 3) = \frac{k}{2^n}$
- a) 528 b) 529 c) 629 d) 630
- 17) A six faced die is biased such that $3 \times P(\text{a prime number}) = 2 \times P(1) = 6 \times P(\text{a composite number})$. Let X be a random variable that counts the number of times one gets a perfect square on some throws of this die. If the die is thrown twice, then the mean of X is :
- a) $\frac{3}{11}$ c) $\frac{7}{11}$
b) $\frac{5}{11}$ d) $\frac{8}{11}$
- 18) The angle of elevation of the top P of a vertical tower PQ of height 10 from point A on the horizontal ground is 45° . Let R be a point on AQ and from a point B, the angle of elevation of P is 60° . If $\angle BAQ = 30^\circ$, $AB = d$ and the area of the trapezium PQRB is α , then the ordered pair (d, α) is :
- a) $\left(10(\sqrt{3} - 1), 25\right)$ c) $\left(10(\sqrt{3} + 1), 25\right)$
b) $\left(10(\sqrt{3} - 1), \frac{25}{2}\right)$ d) $\left(10(\sqrt{3} + 1), \frac{25}{2}\right)$
- 19) Let $S = \{\theta \in (0, \frac{\pi}{2}) : \sum_{m=1}^9 \sec\left(\theta + (m-1)\frac{\pi}{6}\right) \sec\left(\theta + \frac{m\pi}{6}\right)\}$
- a) $\{\frac{\pi}{6}\}$ c) $\sum_{\theta \in S} \theta = \frac{\pi}{2}$
b) $\{\frac{2\pi}{3}\}$ d) $\sum_{\theta \in S} \theta = \frac{3\pi}{4}$
- 20) If the truth value of the statement $(P \wedge (\neg R)) \rightarrow ((\neg R) \wedge Q)$ is F, then the truth value of which is of the following is F ?
- a) $P \vee Q \rightarrow \neg R$ c) $\neg(P \vee Q) \rightarrow \neg R$
b) $R \vee Q \rightarrow \neg P$ d) $\neg(R \vee Q) \rightarrow \neg P$
- 21) $A = \begin{pmatrix} 4 & -2 \\ \alpha & \beta \end{pmatrix}$. If $A^2 + \gamma A + 18I = 0$, then $\det(A)$ is equal to :
- 22) The number of functions f , from the set $A = \{x \in N : x^2 - 10x + 9 \leq 0\}$ to the set $B = \{n^2 : n \in N\}$ such that $f(x) \leq (x-3)^2 + 1$, for every $x \in A$, is
- 23) Let for the 9^{th} term in the binomial expansion of $(3 + 6x)^n$, in the increasing powers of $6x$, to be the greatest for $x = \frac{3}{2}$, the least value of n is n_0 . If K is the ratio of the

coefficient of x^6 to the coefficient of x^3 , then $k + n_0$ is equal to:

- 24) $\frac{2^3-1^3}{1 \times 7} + \frac{4^3-3^3+2^3-1^3}{2 \times 11} + \dots + \frac{6^3-5^3+4^3-3^3+2^3-1^3}{3 \times 15} + \frac{30^3-29^3+28^3-27^3+\dots+2^3-1^3}{15 \times 63}$ is equal to .
- 25) A water tank has the shape of a right circular cone with axis vertical and vertex downwards. Its semi-vertical angle is $\tan^{-1} \frac{3}{4}$. Water is poured in at a constant rate of 6 cubic meter per hour. The rate, at which wet curved surface area of the tank is increasing, when the depth of the tank is 4 meters, is:
- 26) For the curve $C : (x^2 + y^2 - 3) + (x^2 - y^2 - 1)^5 = 0$, the value of $3y' - y^3y''$, at the point (α, α) , $\alpha \geq 0$, on C , is equal to :
- 27) Let $f(x) = \min\{[x-1], [x-2], \dots, [x-10]\}$ where $[t]$ denotes the greatest integer $\leq t$. Then $\int_0^{10} f(x) dx + \int_0^{10} (f(x))^2 dx + \int_0^{10} |f(x)| dx$ is equal to :
- 28) Let f be a differentiable function satisfying $f(x) = \frac{2}{\sqrt{3}} \int_0^{\sqrt{3}} f\left(\frac{\lambda^2 x}{3}\right) d\lambda$, $x \geq 0$ and $f(1) = \sqrt{3}$. If $y = f(x)$ passes through the point $(\alpha, 6)$, then α is equal to :
- 29) A common tangent T to the curves $C_1 : \frac{x^2}{4} + \frac{y^2}{9} = 1$ and $C_2 : \frac{x^2}{42} - \frac{y^2}{143} = 1$ does not pass through the fourth quadrant. If T touches C_1 at (x_1, y_1) and C_2 at (x_2, y_2) , then $|2x_1 + x_2|$ is equal to :
- 30) Let, $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors such that $\vec{a} \times \vec{b} = 4\vec{c}$, $\vec{b} \times \vec{c} = 9\vec{a}$, and $\vec{c} \times \vec{a} = \alpha\vec{b}$, $\alpha > 0$ If $|\vec{a}| + |\vec{b}| + |\vec{c}| = \frac{1}{36}$, then the α is equal to :