

18. Definite Integrals and Applications of Integrals

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I. MCQs with One Correct Answer

14)

$$If f(x) = \begin{cases} e^{\cos x} \sin x, & (for |x| \leq 2) \\ 2, & (otherwise) \end{cases} \quad (1)$$

then $\int_{-2}^3 f(x) dx =$ (2000S)

- a) 0
- b) 1
- c) 2
- d) 3

15) The value of the integral $\int_{e^{-1}}^{e^2} \left| \frac{\log_e x}{x} \right| dx$ is: (2000S)

- a) $\frac{3}{2}$
- b) $\frac{5}{2}$
- c) 3
- d) 5

16) The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx, a > 0$ (2001S)

- a) π
- b) $a\pi$
- c) $\frac{\pi}{2}$
- d) 2π

17) The area bounded by the curves $y = -|x|$ and $y = -|x| + 1$ is (2002S)

- a) 1
- b) 2
- c) $\frac{2}{\sqrt{2}}$
- d) 4

18) Let $f(x) = \int_1^x \sqrt{2-t^2} dt$. Then the real roots of the equation $x^2 - f'(x) = 0$ are (2002S)

- a) 1
- b) $1\sqrt{2}$
- c) $\frac{1}{2}$
- d) 0 and 1

19) Let $T > 0$ be a real number. Suppose f is continuous function such that for all $x \in R, f(x+T) = f(x)$. If $I = \int_0^T f(x) dx$ then the value of $\int_3^{3+3T} f(2x) dx$ is (2002S)

- a) $\frac{3}{2}I$
- b) $2I$

c) 3I

d) 6I

20) The integral $\int_{-1/2}^{1/2} ([x] + \ln(\frac{1+x}{1-x})) dx$ equal to (2002S)

- a) $-\frac{1}{2}$
- b) 0
- c) 1
- d) $2\ln(\frac{1}{2})$

21) If $l(m, n) = \int_0^1 t^m (1+t)^n dt$, then the expression for $l(m, n)$ in terms of $l(m+1, n-1)$ is (2002S)

- a) $\frac{2^n}{m+1} - \frac{n}{m+1} l(m+1, n-1)$
- b) $\frac{n}{m+1} l(m+1, n-1)$
- c) $\frac{2^n}{m+1} + \frac{n}{m+1} l(m+1, n-1)$
- d) $\frac{m}{n+1} l(m+1, n-1)$

22) If $f(x) = \int_{x^2}^{x^2-1} e^{-t^2} dt$, then $f(x)$ increases in (2003S)

- a) $(-2, 2)$
- b) no value of x
- c) $(0, \infty)$
- d) $(-\infty, 0)$

23) The area bounded by the curves $y = \sqrt{x}$, $2y + 3 = x$ and x -axis in the 1st quadrant is (2003S)

- a) 9
- b) $\frac{27}{4}$
- c) 36
- d) 18

24) If $f(x)$ is differentiable and $\int_0^x xf(x) dx = \frac{2}{5}x^5$, then $f(\frac{4}{25})$ (2004S)

- a) $\frac{2}{5}$
- b) $\frac{5}{2}$
- c) 1
- d) $\frac{1}{2}$

25) The value of the integral $\int_0^1 \sqrt{\frac{1-x}{1+x}} dx$ is (2004S)

- a) $\frac{\pi}{2} + 1$
- b) $\frac{\pi}{2} - 1$
- c) -1
- d) 1

26) The area enclosed between the curves $y = ax^2$ and $x = ay^2$ and the line $y = 1/4$ is (2004S)

- a) $\frac{1}{\sqrt{3}}$
- b) $\frac{1}{2}$
- c) 1
- d) $\frac{1}{3}$

27) $\int_{-2}^0 \{x^3 + 3x^2 + 3x + 3 + (x + 1) \cos(x + 1)\} dx$
(2005S)

- a) -4
- b) 0
- c) 4
- d) 6

28) The area bounded by the parabolas $y = (x + 1)^2$ and $y = (x - 1)^2$ and the line $y = \frac{1}{4}$ is (2005S)

- a) 4 sq units
- b) $\frac{1}{6}$ sq units
- c) $\frac{4}{3}$ sq units
- d) $\frac{1}{3}$ sq units

29) The area of the region between the curves $y = \sqrt{\frac{1+\sin x}{\cos x}}$ and $y = \sqrt{\frac{1-\sin x}{\cos x}}$ bounded by the lines $x = 0$ and $x = \frac{\pi}{4}$ is (2008S)

- a) $\int_0^{\sqrt{2}-1} \frac{t}{1+t^2 \sqrt{1-t^2}} dt$
- b) $\int_0^{\sqrt{2}-1} \frac{4t}{1+t^2 \sqrt{1-t^2}} dt$
- c) $\int_0^{\sqrt{2}+1} \frac{4t}{1+t^2 \sqrt{1-t^2}} dt$
- d) $\int_0^{\sqrt{2}+1} \frac{t}{1+t^2 \sqrt{1-t^2}} dt$

30) Let f be a non negative function defined on the interval $[0, 1]$. If $\int_0^x \sqrt{1 - (f'(t))^2} dt = \int_0^x f(t) dt, 0 \leq x \leq 1$, and $f(0) = 0$, then (2009S)

- a) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$
- b) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$
- c) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$
- d) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$