2023-April-10 Shift-1

AI24BTECH11005 - Prajwal Naik

1) An arc PQ of a circle subtends a right angle at the centre O. The midpoint of the arcPq is R.If $\overrightarrow{OP} = \vec{u}$ and $\overrightarrow{OR} = \vec{v}$ and $\overrightarrow{OQ} = \alpha\vec{u} + \beta\vec{v}$, then α, β^2 are the roots of the equation :

a) $3x^2 - 2x - 1 = 0$
b) $3x^2 + 2x - 1 = 0$
c) $x^2 - x - 2 = 0$
d) $x^2 + x - 2 = 0$

2) A square piece of tin of side 30 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. If the volume of the box is maximum, then its surface area is equal to :

a) 800
b) 1025
c) 900
d) 675

3) Let O be the origin and the position vector of the point P be $-\hat{i} - 2\hat{j} + 3\hat{k}$. If the position vectors of the A, B and C are $-2\hat{i} + \hat{j} - 3\hat{k}, 2\hat{i} + 4\hat{j} - 2\hat{k}$ and $-4\hat{i} + 2\hat{j} - \hat{k}$ respectively, then the projection of the vector $\overrightarrow{OP}$ on a vector perpendicular to the vectors $\overrightarrow{AB}, \overrightarrow{AC}$ is

a) $\frac{10}{3}$
b) $\frac{8}{3}$
c) $\frac{7}{3}$
d) 3

4) If A is a 3×3 matrix and $|A| = 2$, then $\left|3adj\left(|3A|\,A^2\right)\right|$ is equal to :

a) 1
b) 2
c) 3^{10}
d) None of the above

5) Let two vertices of a triangle ABC be $(2, 4, 6)$ and $(0, -2, -5)$, and its centroid be $(2, 1, -1)$. If the image of the third vertex in the plane $x + 2y + 4z = 11$ is (a, b, c), then $ab + bc + ca$ is equal to:

a) 76
b) 74
c) 70
d) 72

6) The negation of the statement : $(p \vee q) \wedge (q \vee (\neg r))$ is

a) $((\neg p) \vee r)) \wedge (\neg q)$
b) $((\neg p) \vee (\neg q) \wedge (\neg r)$
c) $((\neg p) \vee (\neg q) \vee (\neg r)$
d) $(p \vee r) \wedge (\neg q)$

7) The shortest distance between the lines $\frac{x+2}{1} = \frac{y}{-2} = \frac{z-5}{2}$ and $\frac{x-4}{1} = \frac{y-1}{2} = \frac{z+3}{0}$ is :

a) 8
b) 7
c) 6
d) 9

8) If the coefficient of x^7 in $\left(ax - \frac{1}{bx^2}\right)^{13}$ and the coefficient of x^{-5} in $\left(ax + \frac{1}{bx^2}\right)^{13}$ are equal, then a^3b^4 is equal to :

a) 22
b) 44
c) 11
d) 33

9) A line segment AB of length λ moves such that the points A and B remain on the periphery of a circle of radius λ. The locus of the point, that divides the line segment AB in the ratio 2:3, is a circle of radius :

a) $\frac{2}{3}\lambda$
b) $\frac{\sqrt{19}}{7}\lambda$
c) $\frac{3}{5}\lambda$
d) $\frac{\sqrt{19}}{5}\lambda$

10) For the system of linear equations
$2x - y + 3z = 5$
$3x + 2y - z = 7$
$4x + 5y + \alpha z = \beta$
Which of the following is not correct?

a) The system is inconsistent for $\alpha = -5$, $\beta = 8$
b) The system has infinitely many solutions for $\alpha = -6$, $\beta = 9$
c) The system has a unique solution for $\alpha = -5$, $\beta = 8$
d) The system has infinitely many solutions for $\alpha = -5$, $\beta = 8$

11) Let the first term a and the common ratio r of a geometric progression be positive integers. If the sum of squares of its first three is 33033, then the sum of these terms is equal to :

a) 210
b) 220
c) 231
d) 241

12) Let P be the point of intersection of the line $\frac{x+3}{3} = \frac{y+2}{1} = \frac{z-1}{-2}$ and the plane $x+y+z = 2$. If the distance of the point P from the plane $3x - 4y + 12z = 32$ is q, then q and 2q are the roots of the equation :

a) $x^2 + 18x - 72 = 0$
b) $x^2 + 18x + 72 = 0$
c) $x^2 - 18x - 72 = 0$
d) $x^2 - 18x + 72 = 0$

13) Let f be a differentiable function such that $x^2 f(x) - x = 4\int_0^4 tf(t)dt$. $f(1) = \frac{2}{3}$. Then 18f(3) is equal to :

a) 180
b) 150
c) 210
d) 160

14) Let N denote the sum of the numbers obtained when two dice are rolled. If the probability that $2^{N!} < N!$ is $\frac{m}{n}$. Where $(m, n) = 1$, then $4m - 3n$ equal to:

a) 180 b) 150 c) 210 d) 160

15) If $I(x) = \int_e^{\sin x^2} (\cos x \sin 2x - \sin x)dx$ and $I(0) = 1$, then $I(\frac{\pi}{3})$ is equal to :

a) $e^{\frac{3}{4}}$ b) $-e^{\frac{3}{4}}$ c) $\frac{1}{2}e^{\frac{3}{4}}$ d) $-\frac{1}{2}e^{\frac{3}{4}}$