Mechanical Engineering-2007

AI24BTECH11005 - Bhukya Prajwal Naik

1) A circular disc of radius a on the xy plane has a surface charge density $\sigma = \frac{\sigma_0 r \cos\theta}{a}$. The electric dipole moment of this charge distribution is

   a) $\frac{\sigma_0 \pi a^4}{4}\hat{x}$ b) $\frac{\sigma_0 \pi a^3}{4}\hat{x}$ c) $-\frac{\sigma_0 \pi a^3}{4}\hat{x}$ d) $-\frac{\sigma_0 \pi a^4}{4}\hat{x}$

2) At time $t = 0$, a charge distribution $\rho(\mathbf{r}, 0)$ exists within an ideal homogeneous conductor of permittivity ε and conductivity σ. At a later time $\rho(\mathbf{r}, t)$ is given by

   a) $\rho(\mathbf{r}, t) = \rho(\mathbf{r}, 0)\exp\left(-\frac{\sigma t}{\varepsilon}\right)$
   b) $\rho(\mathbf{r}, t) = \frac{\rho(\mathbf{r},0)}{1+(\sigma t/\varepsilon)^2}$
   c) $\rho(\mathbf{r}, t) = \rho(\mathbf{r}, 0)\exp\left[-\left(\frac{\sigma t}{\varepsilon}\right)^2\right]$
   d) $\rho(\mathbf{r}, t) = \rho(\mathbf{r}, 0)\frac{\varepsilon}{\sigma t}\sin\left(\frac{\sigma t}{\varepsilon}\right)$

3) A nonrelativistic charged particle moves along the positive x-axis with a constant positive acceleration $a\hat{x}$. The particle is at the origin at $t = 0$. Radiation is observed at $t = 0$ at a distant point $(0, d, 0)$ on the y-axis. Which one of the following statements is correct?
   a) The radiation is unpolarized.
   b) The radiation is plane polarized with polarization parallel to the x-axis.
   c) The radiation is plane polarized with polarization parallel to the xy plane along a line inclined to the x axis.
   d) The radiation is elliptically polarized.

4) For a physical system, two observables $\emptyset_1$ and O_2 are known to be compatible. Choose the correct implication from amongst those given below:
   a) Every eigenstate of O_1 must necessarily be an eigenstate of O_2.
   b) Every non-degenerate eigenstate of O_1 must necessarily be an eigenstate of O_2.
   c) When an observation of O_1 is carried out on an arbitrary state $|\Psi\rangle$ of the physical system, a subsequent observation of O_2 leads to an unambiguous result.
   d) Observation of O_1 and O_2, carried out on an arbitrary state $|\Psi\rangle$ of the physical system, lead to the identical results irrespective of the order in which the observations are made.

5) An exact measurement of the position of a simple harmonic oscillator (SHO) is made with the result $x = x_0$. [The SHO has energy levels $E_n(n = 0, 1, 2, ...)$ and associated normalized wavefunctions ψ_n]. Subsequently, an exact measurement of energy E is made. Using the general notation $Pr(E = E')$ denoting the probability that a result E' is obtained for this measurement, the following statements are written. Which one of the following statements is correct?

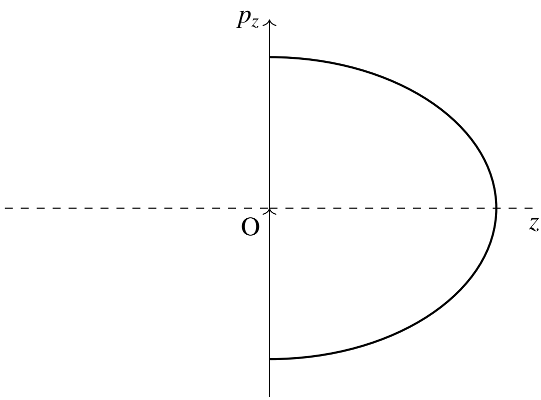
a) $Pr(E = E_0) = 0$
b) $Pr(E = E_n) = 1$ for some value of n.
c) $Pr(E = E_n) \propto \psi_n(x)$
d) $Pr(E > E'') > 0$ for any E''.

6) Consider the combined system of proton and electron in the hydrogen atom in its (electronic) ground state. Let I denote the quantum number associated with the total angular momentum and let α denote the magnitude of the expectation value of the net magnetic moment in the state. Which of the following pairs represents a possible state of the system (μ_B is Bohr magneton)?

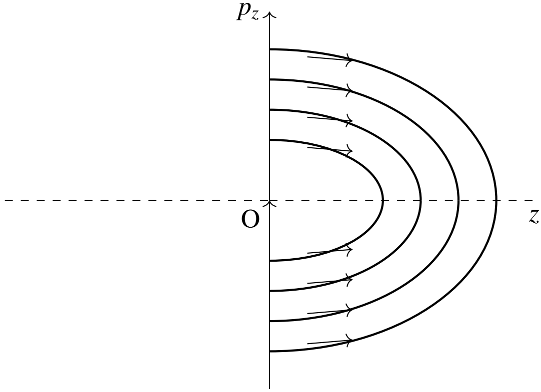
a) $I = 0, \alpha = 0$
b) $I = \frac{1}{2}, \alpha = 1\mu_B$
c) $I = 1, \alpha = 1\mu_B$
d) $I = 0, \alpha = 2\mu_B$

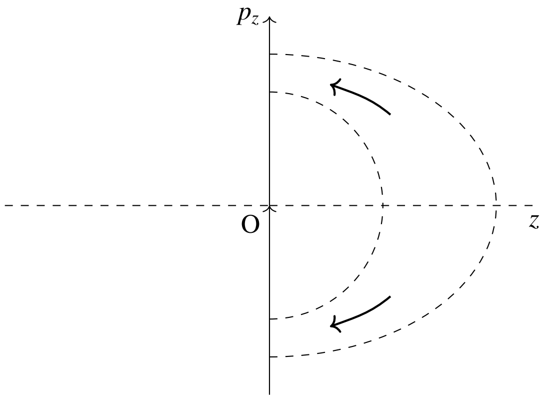
7) A particle is placed in a one dimensional box of size L along the x-axis ($0 < x < L$). Which of the following is true?
   a) In the ground state, the probability of finding the particle in the interval $(\frac{L}{4}, \frac{3L}{4})$ is half.
   b) In the first excited state, the probability of finding the particle in the interval $(\frac{L}{4}, \frac{3L}{4})$ is half. This also holds for states with $n = 4, 6, 8, ...$
   c) For an arbitrary state $|\Psi\rangle$, the probability of finding the particle in the left half of the well is half.
   d) In the ground state, the particle has a definite momentum.

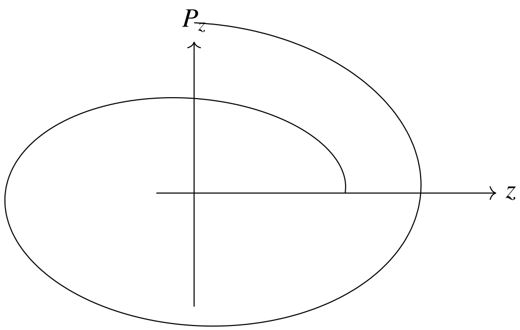
8) A calculator has accuracy up to 8 digits after decimal place. The value of $\int_0^{2\pi} \sin x dx$ when evaluated using this calculator by trapezoidal method with 8 equal intervals, to 5 significant digits is



a)

b) 

c) 

d) 

9) A system containing N non-interacting localized particles of spin $\frac{1}{2}$ and magnetic moment μ each is kept in constant external magnetic field B and in thermal equilibrium at temperature T. The magnetization of the system is,

a) $N\mu coth\frac{\mu B}{k_BT}$ b) $N\mu\tanh\frac{\mu B}{k_BT}$ c) $N\mu\sinh\frac{\mu B}{k_BT}$ d) $N\mu\cosh\frac{\mu B}{k_BT}$

10) Two identical particles have to be distributed among three energy levels. Let r_B, r_F and r_C represent the ratios of probability of finding two particles to that of finding one particle in a given energy state. The subscripts B, F and C correspond to whether the

particles are bosons, fermions and classical particles, respectively. Then, $r_B : r_F : r_C$ is equal to

a) $\frac{1}{2} : 0 : 1$ b) $1 : \frac{1}{2} : 1$ c) $1 : \frac{1}{2} : \frac{1}{2}$ d) $1 : 0 : \frac{1}{2}$

11) A photon gas is at thermal equilibrium at temperature T. The mean number of photons in an energy state $\varepsilon = \hbar\omega$ is

a) $\exp(\frac{\hbar\omega}{k_BT}) + 1$ b) $\exp(\frac{\hbar\omega}{k_BT}) - 1$ c) $(\exp(\frac{\hbar\omega}{k_BT}) + 1)^{-1}$ d) $(\exp(\frac{\hbar\omega}{k_BT}) - 1)^{-1}$

12) Consider a system of N atoms of an ideal gas of type A at temperature T and volume V. It is kept in diffusive contact with another system of N atoms of another ideal gas of type B at the same temperature T and volume V. Once the combined system reaches equilibrium,

a) the total entropy of the final system is the same as the sum of the entropy of the individual system always.
b) the entropy of mixing is $2Nk_B \ln 2$.
c) the entropy of the final system is less than that of sum of the initial entropies of the two gases.
d) $theentropyofmixingisnon-zerowhentheatoms\mathrm{A}and\mathrm{B}areofthesametype.$

13) Consider a system of two non-interacting classical particles which can occupy any of the three energy levels with energy values $E = 0, \varepsilon$ and 2ε having degeneracies $g(E) = 1, 2$ and 4 respectively. The mean energy of the system is

a) $\varepsilon\frac{4\exp(-\varepsilon/k_BT)+8\exp(-2\varepsilon/k_BT)}{1+2\exp(-\varepsilon/k_BT)+4\exp(-2\varepsilon/k_BT)})$
b) $\varepsilon(\frac{2\exp(-\varepsilon/k_BT)+4\exp(-2\varepsilon/k_BT)}{1+2\exp(-\varepsilon/k_BT)+4\exp(-2\varepsilon/k_BT)})^2$
c) $\varepsilon(\frac{2\exp(-\varepsilon/k_BT)+8\exp(-2\varepsilon/k_BT)}{1+2\exp(-\varepsilon/k_BT)+4\exp(-2\varepsilon/k_BT)})$
d) $\varepsilon(\frac{\exp(-\varepsilon/k_BT)+2\exp(-2\varepsilon/k_BT)}{1+\exp(-\varepsilon/k_BT)+\exp(-2\varepsilon/k_BT)})$

14) Three consecutive absorption lines at $64.275\ cm^{-1}, 77.130\ cm^{-1}$ and $89.985\ cm^{-1}$ have been observed in a microwave spectrum for a linear rigid diatomic molecule. The moments of inertia I_A and I_B are (I_A is with respect to the bond axis passing through the centre of mass and I_B is with respect to an axis passing through the centre of mass and perpendicular to bond axis)

a) both equal to $\frac{\hbar^2}{12.855hc}gmcm^2$
b) zero and $\frac{\hbar^2}{12.855hc}gmcm^2$
c) both equal to $\frac{\hbar^2}{6.427hc}gmcm^2$
d) zero and $\frac{\hbar^2}{6.427hc}gmcm^2$

15) A pure rotational Raman spectrum of a linear diatomic molecule is recorded using electromagnetic radiation of frequency v_e. The frequency of two consecutive Stokes lines are

a) $v_e - 10B, v_e - 14B$
b) $v_c - 2B, v_e - 4B$
c) $v_e + 10B, v_e + 14B$
d) $v_e + 2B, v_e + 4B$

16) Which one of the following statement is Incorrect in vibrational spectroscopy with anharmonotonicity?
   a) The selection rule for vibrational spectroscopy is $\triangle v = 1, 2, 3, ...$
   b) Anharmonicity leads to multiple absorption lines
   c) The intensies of hot band lines are stronger than the fundamental absorption
   d) The frequencies of hot band lines are smaller than the fundamental absorption

17) The molecular spectra of two linear molecules O-C-O and O-C-S are recorded in the micro wave region. Which one of the following is correct?
   a) Both would show absorption lines
   b) Both would not show absorption lines
   c) O-C-O would show absorption lines, but not O-C-S.
   d) O-C-S would show absorption lines , but not O-C-O.