Problems on End-to-End Delay

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Question 1: 100 Tera Bytes of data is to be transferred between two hosts, Host A in Bengaluru and Host B in Chennai, separated by a distance of 350 km. The transmission speed at Host A is 1 Gbps and the propagation speed of the dedicated link is $2 \times 10^8\ m/s$. Which one of these is better:

- **to transfer the data over the link or**
- **to drive to Chennai to deliver the data and drive back to Bengaluru, if a one-way trip takes 7 hours**

Which component is dominant in the total time taken for the data transmission? Assume negligible processing and queuing delays.

The end-to-end delay in transferring the data is given by

$$d_{end-end} = d_{trans} + d_{prop}$$

$$d_{end-end} = \frac{L}{R} + \frac{d}{s}$$

$$= \frac{100 \times 10^{12} \times 8}{10 \times 10^9} + \frac{350 \times 10^3}{2 \times 10^8}$$

$$= 80000 + 0.00175$$

$$d_{end-end} = 80000.00175\ seconds \approx 22.2\ hours$$

It approximately takes 22 hours to transfer all the data between the hosts. On the other hand, it only takes 14 hours (2×7 hours) to deliver the data by hand.

Question 2: Host A is connected to Host B by two dedicated links, with transmission speeds 1 Gbps and 100 Mbps respectively. What is the minimum amount of time required for Host A to transfer 5.5 Giga Bytes of Data to Host B? Assume negligible queuing and propagation delays.

Host B can transfer the data efficiently by using both the links. Given the transmission speeds of the links, the amount of data transferred through each link must be divided in the ratio of their transmission speeds, which is 10 : 1. The total data to be transmitted through each link is

$$D_1 = \frac{10}{11} \times 5.5 = 5\, GB$$

$$D_2 = 5.5 - 5 = 0.5\, GB$$

The time taken to transmit the data through each link is

$$t_1 = \frac{5 \times 10^9 \times 8}{1 \times 10^9} = 40\, seconds$$

$$t_2 = \frac{0.5 \times 10^9 \times 8}{100 \times 10^6} = 40\, seconds$$

Hence, the minimum amount of time taken to transmit all of the data is 40 seconds (maximum of t_1 and t_2).

Question 3: Two hosts are separated by 10 nodes over a distance of 10,000 km. Starting from from the first node, every other node has no queuing delay but has a processing delay of $0.5\mu s$ for each packet. Starting from the second node every other node has no processing delay but has a queuing delay of $1.5\mu s$ for each packet. There are no processing or queuing delays at the hosts. Assume the transmission speeds of the hosts and all the nodes is 100 Gbps. Also assume the propagation speed of all dedicated links is $2 \times 10^8\ m/s$. What is the total time taken to transmit 10,000 packets each of size 64 Kilo Bytes between the hosts?

The total end-to-end delay is given by

$$d_{end-end} = d_{proc} + d_{que} + d_{trans} + d_{prop}$$

$$d_{proc} = 5 \times (0.5 \times 10^{-6}) \times 10000 = 25\ ms$$
$$d_{que} = 5 \times (1.5 \times 10^{-6}) \times 10000 = 75\ ms$$
$$d_{trans} = \frac{L}{R} = \frac{10000 \times 64 \times 10^3 \times 8}{100 \times 10^9} = 51.2\ ms$$
$$d_{prop} = \frac{d}{s} = \frac{10000 \times 10^3}{2 \times 10^8} = 0.05\ s = 50\ ms$$
$$d_{end-end} = 25 + 75 + 51.2 + 50 = 201.2\ ms$$

Question 4: Suppose a huge file of size 500 Giga Bytes is to be transferred over a distance of 100,000 km. The file is broken into packets of size 64 Bytes each. If the transmission speed at the host is 8.192 Gbps and the propagation speed of the dedicated link is $2 \times 10^8\ m/s$, find the number of packets the link will contain at any point in time.

As the propagation speed of the link is $2 \times 10^8 m/s$, for a packet to reach the destination, it will take

$$d_{prop} = \frac{d}{s} = \frac{100000 \times 10^3}{2 \times 10^8} = 0.5\ s = 500\ ms$$

In 500 ms, the host can transmit

$$D = 8.192 \times 0.5 = 4.096\ Gb$$

4.096 Giga bits of data can be broken down into

$$\frac{4.096 \times 10^6}{64 \times 8} = 8 \times 10^6\ packets$$

Therefore, the link will contain 8×10^6 packets at any given time.

Question 5: Hosts A and B are connected by a satellite, at distances 6400 km and 8000 km respectively from it. The uplink and downlink transmission speeds are 1 Mbps and 8 Mbps respectively and the propagation speed in air is $2.5 \times 10^8\ m/s$. If A and B send each other a packet that is 64 Kilo Bytes in size, calculate the extra end-to-end delay B experiences. Assume there are no processing and queuing delays.

As both A and B must transmit and receive one packet, the total transmission speed is the same for both of them. The only difference arises due to the fact that B is further away from the satellite than A is. Therefore, the difference in propagation delay is

$$\Delta d_{prop} = \frac{\Delta d}{s} = \frac{(8000 - 6400) \times 10^3}{2.5 \times 10^8} = \frac{1600 \times 10^3}{2.5 \times 10^8} = 6.4\ ms$$

Question 6: Suppose you are watching a cricket match live on your computer. The provider offers a 1080p stream (1920×1080 pixels) at 24 frames per second. Each pixel requires 3 Bytes to represent the RGB color levels. If the processing, queuing and the propagation delays are absent and the data for the audio is negligible, what is the minimum transmission speed required at the server to have less than 1 second end-to-end delay for the stream?

The total amount of data being transmitted in one second is

$$D = 1920 \times 1080 \times 3 \times 8 \times 24 \approx 1.2 \; Giga \; bits$$

We know

$$d_{trans} = \frac{L}{R}$$

To find R

$$R = \frac{L}{d_{trans}} = \frac{1.2 \times 10^9}{1} = 1.2 \; Gbps$$

Therefore, the server must have a minimum transmission speed of 1.2 Gbps for an end-to-end delay of less than 1 second.

Question 7: An ISP offers internet connections with transmission speeds of 10 Mbps. To attract customers, it promises less than 5 ms of end-to-end delay while playing online games, if the other players are also their customers and are no more than 10,000 km away. Assume an idle network with propagation speed of $2.5 \times 10^8 \; m/s$ and no processing and queuing delays. If the average amount of data generated per second is 100 Kilo Bytes, find out if the ISP can deliver on their promise.

The end-to-end delay in transferring the data is given by

$$d_{end-end} = d_{trans} + d_{prop}$$

$$d_{end-end} = \frac{L}{R} + \frac{d}{s}$$

$$= \frac{100 \times 10^3 \times 8}{10 \times 10^6} + \frac{10000 \times 10^3}{2.5 \times 10^8}$$

$$= 0.08 + 0.04$$

$$d_{end-end} = 0.12 \; seconds = 120 \; ms$$

In the worst-case scenario, the end-to-end delay is 120 ms. Therefore, the ISP is lying.

Question 8: Suppose there are two networks, Network A and Network B, in mesh and star topology respectively. In Network A, the transmission speed of the hosts is 10 Mbps and the queuing delay of each packet is 5 ms. In Network B, the transmission speeds of the hosts and the hub are 100 Mbps and 1 Gbps respectively and the queuing delays are 5 ms and 1 ms respectively. If a packet of size 64 Kilo Bytes is to be sent from one host to another, which network has the least end-to-end delay? Assume negligible processing and propagation delays.

In network A, the end-to-end delay in transferring the data is given by

$$d_{end-end(A)} = d_{que(host)} + d_{trans(host)}$$

$$d_{end-end(A)} = (5 \times 10^{-3}) + \frac{64 \times 10^3 \times 8}{10 \times 10^6}$$

$$= 5 + 51.2$$

$$d_{end-end(A)} = 56.2 \; ms$$

In network B, the end-to-end delay in transferring the data is given by

$$d_{end-end(B)} = d_{que(host)} + d_{trans(host)} + d_{que(hub)} + d_{trans(hub)}$$

$$d_{end-end(B)} = (5 \times 10^{-3}) + \frac{64 \times 10^3 \times 8}{100 \times 10^6} + (1 \times 10^{-3}) + \frac{64 \times 10^3 \times 8}{1 \times 10^9}$$

$$= 5 + 5.12 + 1 + 0.512$$

$$d_{end-end(B)} = 11.632 \; ms$$

Hence, packet transmission is faster in Network B (star topology).

Question 9: The Iridium constellation is a constellation of low-earth-orbit satellites, with 12 satellites orbiting along the circumference of the earth at any given time. If a call, with one packet of 64 Kilo bits in size generated every second, is to be made from the South Pole to the North Pole, what is the end-to-end delay experienced? Assume no delays at the hosts, queuing delay of 5 ms per packet and transmission speed of 10 Mbps at the satellites and propagation speed of $3 \times 10^8\ m/s$. Also assume the radius of the earth is 6400 km.

As the North Pole and South Pole are at opposite ends of the earth, the distance between them is half the circumference of the earth. Therefore, there are 6 satellites between them (on average) and the distance is

$$d = \pi \times 6400 \approx 20106\ km$$

The end-to-end delay in transferring the packet is given by

$$d_{end-end} = d_{que} + d_{trans} + d_{prop}$$

$$d_{que} = 6 \times 5 = 30\ ms$$

$$d_{trans} = \frac{L}{R} = \frac{64 \times 10^3 \times 8}{10 \times 10^6} = 51.2\ ms$$

$$d_{prop} = \frac{d}{s} = \frac{20106 \times 10^3}{3 \times 10^8} \approx 67\ ms$$

$$d_{end-end} = 30 + 51.2 + 67 = 148.2\ ms$$

The end-to-end delay experienced is 142.8 ms.

Question 10: Two hosts are separated by a distance of 5,000 km. Two packets that are 4 Mega Bytes in size must be sent from one host to the other. The queuing delay at the host is 10 ms and the the propagation speed of the link is $2.5 \times 10^8 m\ /s$. Find the transmission speed at the source such that it finishes transmitting the second packet when the first packet just arrives at the destination.

The propagation speed is given by

$$d_{prop} = \frac{d}{s} = \frac{5000 \times 10^3}{2.5 \times 10^8} = 20\ ms$$

Given,

$$d_{que} + d_{trans} = d_{prop}$$

To find d_{trans}

$$d_{trans} = d_{prop} - d_{que} = 20 - 10 = 10\ ms$$

We know

$$d_{trans} = \frac{L}{R}$$

To find R

$$R = \frac{L}{d_{trans}} = \frac{4 \times 10^4 \times 8}{20 \times 10^{-3}} = 3.2\ Gbps$$

Question 11: A network has 4 hosts arranged in the ring topology. The queuing delay at a host is 10 ms per packet and the transmission speed is 1 Gbps. If a packet of size 4 Mega Bytes is to be transmitted between any two hosts, what is the average end-to-end delay? Assume there are no processing and propagation delays.

Let the hosts in the network be A, B, C and D, such that B and C are adjacent to A and D is opposite A. The average end-to-end delay is given by

$$d_{end-end(avg)} = \frac{d_{end-end(A-B)} + d_{end-end(A-C)} + d_{end-end(A-D)}}{3}$$

$$d_{end-end(A-B)} = d_{end-end(A-C)} = d_{que} + d_{trans}$$
$$= (10 \times 10^{-3}) + \frac{4 \times 10^6 \times 8}{1 \times 10^9}$$
$$d_{end-end(A-B)} = 10 + 32 = 42\ ms$$
$$d_{end-end(A-D)} = d_{que(A-B)} + d_{trans(A-B)} + d_{que(B-D)} + d_{trans(B-D)}$$
$$= (10 \times 10^{-3}) + \frac{4 \times 10^6 \times 8}{1 \times 10^9} + (10 \times 10^{-3}) + \frac{4 \times 10^6 \times 8}{1 \times 10^9}$$
$$d_{end-end(A-D)} = 10 + 32 + 10 + 32 = 84\ ms$$

Therefore,

$$d_{end-end(avg)} = \frac{42 + 42 + 84}{3} = 56\ ms$$

Question 12: 100 packets each of size 16 Kilo Bytes must be transmitted from a server to two hosts, A and B, each at a distance of 5,000 km from the server. Transmission to host A occurs first and after its completion, transmission to host B starts. The link connecting the server and host A has a packet loss probability of 20%, while that connecting host B has a packet loss probability of 10%, and each lost packet is re-transmitted. If each link has propagation speed $2 \times 10^8\ m/s$ and the transmission speed at the server is 1 Gbps, at what times are the transmissions to each host completed? Assume negligible processing and queuing delays.

Since the link connecting host A and the server has a packet loss probability of 20%, the total number of packets transmitted to host A is

$$100 + (20\%\ of\ 100) = 120$$

The end-to-end delay in transferring the packets to host A is given by

$$d_{end-end} = d_{trans} + d_{prop}$$
$$d_{end-end} = \frac{L}{R} + \frac{d}{s}$$
$$= \frac{120 \times 16 \times 10^3 \times 8}{10 \times 10^9} + \frac{5000 \times 10^3}{2 \times 10^8}$$
$$d_{end-end} = 15.36 + 25 = 40.36\ ms$$

As the link connecting host B and the server has a packet loss probability of 10%, the total number of packets transmitted to host A is

$$100 + (10\%\ of\ 100) = 110$$

The end-to-end delay in transferring the packets to host B is given by

$$d_{end-end} = d_{trans} + d_{prop}$$
$$d_{end-end} = \frac{L}{R} + \frac{d}{s}$$
$$= \frac{110 \times 16 \times 10^3 \times 8}{10 \times 10^9} + \frac{5000 \times 10^3}{2 \times 10^8}$$
$$d_{end-end} = 14.08 + 25 = 39.08\ ms$$

Transmissions to hosts A and B are completed after 40.36 ms and 79.44 ms (40.36 + 39.08) respectively.

Question 13: In the new 5G technology, the peak upload and download speeds are in the ratio 1:2. Find their values if a packet of size 2.5 Mega Bytes experiences an end-to-end delay of 3 ms. Assume the processing, queuing and propagation speeds are negligible.

$$d_{end-end} = d_{trans(upload)} + d_{trans(download)}$$
$$3 \times 10^{-3} = \frac{2.5 \times 10^6 \times 8}{x} + \frac{2.5 \times 10^6 \times 8}{2x}$$

To find x

$$x = \frac{2.5 \times 10^6 \times 8 + 2 \times 2.5 \times 10^6 \times 8}{3 \times 10^{-3}}$$

$$x = \frac{2.5 \times 10^6 \times 8}{3 \times 10^{-3}} = 10 \times 10^9 = 10\ Gbps$$

Therefore, the peak upload and download speeds are 10 Gbps and 20 Gbps respectively.

Question 14: There are two paths to transmit 100 packets each of size 16 Mega Bytes from host A to host B: a lossy channel with loss probability of 50% and transmission speed of 20 Gbps; and a lossless channel with transmission speed of 10 Gbps. If both the channels have a propagation speed of $2 \times 10^8\ m/s$ and the queuing delay at the host is 10 ms per packet, which path is better? Assume all lost packets are re-transmitted with no additional overhead.

Since both paths have the same propagation speed and queuing delay, the only difference arises from the transmission delays. For the lossy channel, the total number of packets that must be transmitted is

$$100 + (50\%\ of\ 100) = 150$$

The transmission delay in the lossy channel is

$$d_{trans} = \frac{L}{R} = \frac{150 \times 16 \times 10^6 \times 8}{20 \times 10^9}$$

$$d_{trans} = 960\ ms$$

For the lossless channel, the total number of packets that must be transmitted is 100. The transmission delay in the lossless channel is

$$d_{trans} = \frac{L}{R} = \frac{100 \times 16 \times 10^6 \times 8}{10 \times 10^9}$$

$$d_{trans} = 1280\ ms$$

Therefore, transfer of the packets is faster in the lossy channel.

Question 15: Suppose a circuit-switched network has a one-time setup delay of 500 ms and a transmission speed of 100 Mbps, with no additional queuing delay. Also, suppose a packet-switched network has a queuing delay of 10 ms per packet and transmission speed of 10 Mbps. After transmission of how many packets, each of size 16 Kilo Bytes, does the circuit-switched network seem like a better choice for data transmission in terms of end-to-end delay? Assume transmission is happening between hosts separated by the same distance and the propagation speed of the links is also the same in both the networks.

As it is given that the hosts in both the cases are separated by the same distance, the propagation time will be same. Hence, it need not be calculated. Let x packets be transmitted. In the circuit-switched network, the end-to-end delay is given by

$$d_{end-end(1)} = d_{setup} + d_{trans}$$

$$d_{end-end(1)} = 0.5 + \frac{L}{R}$$

$$= 0.5 + \frac{x \times 16 \times 10^3 \times 8}{100 \times 10^6}\ seconds$$

In the packet-switched network, the end-to-end delay is given by

$$d_{end-end(2)} = d_{que} + d_{trans}$$

$$d_{end-end(2)} = 0.01x + \frac{L}{R}$$

$$= 0.01x + \frac{x \times 16 \times 10^3 \times 8}{10 \times 10^6}\ seconds$$

For the circuit switched network to be more efficient,

$$d_{end-end(1)} < d_{end-end(2)}$$

$$
\begin{gathered}
0.5 + \frac{x \times 16 \times 10^3 \times 8}{100 \times 10^6} < 0.01x + \frac{x \times 16 \times 10^3 \times 8}{10 \times 10^6} \\
0.5 < 0.01x + \frac{128x}{10^4} - \frac{12.8x}{10^4} \\
100x + 128x - 12.8x > 5000 \\
x > \frac{5000}{215.2} \\
x >= 24
\end{gathered}
$$

When at least 24 packets are transmitted, the circuit-switched network becomes more efficient than the packet-switched network.