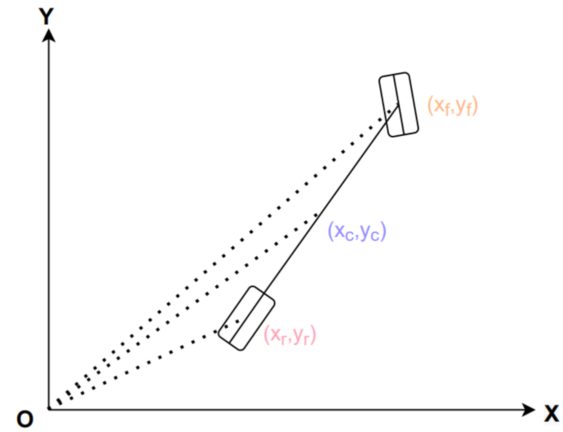
Bicycle Kinematic Model of a Vehicle

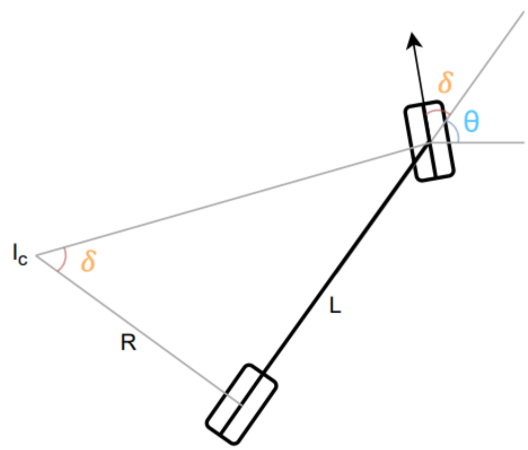
In a x-y Cartesian plane,

Position of center of front wheel axle = (x_f, y_f)

Position of center of rear wheel axle = (x_r, y_r)

Position of center of CG = (x_c, y_c)





I_c = Instantaneous center of rotation

δ = Steering angle

θ= Vehicle positional angle

Since I_c is the instantaneous center of rotation of the vehicle, we get

$\omega = \frac{v}{R} = \frac{v}{L} * \tan(\delta)$

If the center of rear axle is our desired point of consideration:

$$\dot{x_r} = v\cos(\theta)$$

$$\dot{y_r} = v\sin(\theta)$$

$$\dot{\theta} = \frac{v}{L}\tan(\delta)$$

Similarly, if the center of front axle is considered:

$$\dot{x_f} = v\cos(\theta + \delta)$$

$$\dot{y_f} = v\sin(\theta + \delta)$$

$$\dot{\theta} = \frac{v}{L}\sin(\delta)$$

Generally, equations of motion are written for CG. Extension of above idea and geometrical analysis yield:

$$\dot{x_c} = v\cos(\theta + \beta)$$

$$\dot{y_c} = v\sin(\theta + \beta)$$

$$\dot{\theta} = \frac{v}{L}\cos(\beta)\tan(\delta)$$

Where,

$$\beta = \arctan\left(\frac{l_r \tan(\delta)}{L}\right)$$

Having 4 degrees of freedom, the state vector of bicycle kinematic model comprises four state variables and two control inputs.

State = $[x, y, \theta, \delta]^T$

Inputs = $[v, \phi]^T$

State space equations:

$\dot{x_c} = v\cos(\theta + \beta)$

$\dot{y_c} = v\sin(\theta + \beta)$

$\dot{\theta} = \frac{v}{L}\cos(\beta)\tan(\delta)$

$\dot{\delta} = \phi$