Exponential distribution.

let $\alpha > 0$, then the continuous probability distribution for which

$$E(\alpha, x) = \begin{cases} \alpha e^{-\alpha x} & , 0 \le x < \infty \\ 0 & , \text{elsewhere} \end{cases} = \begin{cases} 0 & , -\infty < x \le 0 \\ \alpha e^{-\alpha x} & , 0 \le x < \infty \end{cases}$$

is the Probability density function is called the negative exponential distribution or just the exponential distribition.

In $E(\alpha, x)$, x is called the exponential variate, α is called the parameter of the distributon.

(i) $E(\alpha, x) \ge 0$ (ii) $\int_{-\infty}^{\infty} E(\alpha, x) dx = 1$.

S.T. the mean and standard deviation of an exponential distribution are equal.

P.D.F of Exponential distribution is $E(\alpha, x) = \begin{cases} 0 & , -\infty < x \le 0 \\ \alpha e^{-\alpha x} & , 0 \le x < \infty \end{cases}$

$$\text{Mean} = \mu = E[X] = \int_{-\infty}^{\infty} x \, E(\alpha, x) \, dx$$

$$= \int_{-\infty}^{0} x \cdot 0 \cdot dx + \int_{0}^{\infty} x \cdot \alpha e^{-\alpha x} dx = \alpha \int_{0}^{\infty} x e^{-\alpha x} dx.$$

By using Bernoulli's rule.

$$= \alpha \left[x \left(\frac{e^{-\alpha x}}{-\alpha} \right) - 1 \left(\frac{e^{-\alpha x}}{(-\alpha)^2} \right) \right]_0^{\infty}$$

$$= \alpha \left[-\frac{1}{\alpha} (x e^{-\alpha x})_0^{\infty} - \frac{1}{\alpha^2} (e^{-\alpha x})_0^{\infty} \right]$$

$$= \alpha\left[\cancel{\frac{-1}{\alpha}}(0-0) - \frac{1}{\alpha^2}(e^{-\infty} - e^0)\right] \quad \text{But } e^{-\infty} = \frac{1}{e^{\infty}} = 0$$

$$= \alpha\left[-\frac{1}{\alpha^2}(-1)\right] = \frac{\alpha}{\alpha^2} \quad \therefore \boxed{\mu = \frac{1}{\alpha}}.$$

$$E[x^2] = \int_{-\infty}^{\infty} x^2 E(\alpha, x)\,dx = \int_{-\infty}^{0} x^2 \cdot 0\,dx + \int_0^{\infty} x^2 \alpha e^{-\alpha x}\,dx$$

$$= \alpha\int_0^{\infty} x^2 e^{-\alpha x}\,dx = \alpha\left[\cancel{x^2\left(\frac{e^{-\alpha x}}{-\alpha}\right)} - \cancel{(2x)\left(\frac{e^{-\alpha x}}{(-\alpha)^2}\right)} + 2\left(\frac{e^{-\alpha x}}{(-\alpha)^3}\right)\right]_0^{\infty}$$

$$= \alpha\left(-\frac{2}{\alpha^3}\right)\left[e^{-\alpha x}\right]_0^{\infty} = -\frac{2}{\alpha^2}\left[\cancel{e^{-\infty}} - e^0\right] = \frac{2}{\alpha^2}$$

$$\text{Variance} = \sigma^2 = E[x^2] - \mu^2 = \frac{2}{\alpha^2} - \frac{1}{\alpha^2} = \frac{1}{\alpha^2}.$$

$$\text{Standard deviation} = \sigma = \frac{1}{\alpha}$$

Here $\boxed{\mu = \sigma = \frac{1}{\alpha}}$.

Find the Cummulative distribution function of the exponential distribution.

$$F(t) = P(-\infty < x < t) = \int_{-\infty}^{t} E(\alpha, x)\,dx.$$

$$E(\alpha, x) = \begin{cases} 0 & , -\infty < x < 0 \\ \alpha e^{-\alpha x} & , 0 < x < \infty \end{cases}$$

For $-\infty < t < 0$, $F(t) = \int_{-\infty}^{t} E(\alpha, x)\,dx = \int_{-\infty}^{t} 0\,dx = 0.$

For $0 < t < \infty$, $F(t) = \int_{-\infty}^{t} \epsilon(\alpha, x)\,dx = \cancel{\int_{-\infty}^{0} 0\,dx} + \int_0^t \alpha e^{-\alpha x}\,dx$

$$= \alpha \left[\frac{e^{-\alpha x}}{-\alpha}\right]_0^t = -\left[e^{-\alpha x}\right]_0^t = -\left[e^{-\alpha t} - e^0\right] = 1 - e^{-\alpha t}.$$

$\therefore$ C.D.F is $F(t) = \begin{cases} 0, & -\infty < t < 0 \\ 1 - e^{-\alpha t}, & 0 \le t < \infty \end{cases}$

1) In a certain Town, the duration of a shower is exponentialy distributed with mean equal to 5 minutes. What is the probability that a shower will last for (i) 10 minutes or more? (ii) less than 10 minutes? (iii) between 10 minutes and 12 minutes.

Sol: $\mu = 5$ $\therefore \alpha = \frac{1}{5} = 0.2$.

$\therefore$ P.D.F is $\epsilon(x) = \epsilon(\alpha, x) = \epsilon(0.2, x) = \begin{cases} 0, & -\infty < x < 0 \\ 0.2 e^{-0.2x}, & 0 < x < \infty \end{cases}$

(i) Probability that a shower will last for 10 minutes or more $= P(x > 10) = P(10 < x < \infty) = \int_{10}^{\infty} \epsilon(\alpha, x)\,dx$

$$= \int_{10}^{\infty} 0.2 e^{-0.2x}\,dx = \cancel{0.2}\left[\frac{e^{-0.2x}}{\cancel{-0.2}}\right]_{10}^{\infty} = \frac{e^{-\infty} - e^{-2}}{(-1)}$$

$$= \frac{0 - e^{-2}}{-1} = e^{-2} = \frac{1}{e^2} = 0.1353$$

(ii) Probability that a shower will last for less that 10 minutes $= P(x < 10) = P(-\infty < x < 10)$

$$= \int_{-\infty}^{10} \epsilon(x)\,dx = \int_{\infty}^{0} 0\,dx + \int_0^{10} \epsilon(x)\,dx.$$

Scanned with Car

$$= \int_0^{10} 0.2\, e^{-0.2x}\, dx = 0.2 \left[\frac{e^{-0.2x}}{-0.2}\right]_0^{10} = \frac{e^{-2} - e^{0}}{-1}$$

$$= 1 - e^{-2} = 1 - \frac{1}{e^2} = \frac{e^2 - 1}{e^2} = 0.8647$$

(iii) Probability that shower will last between 10 minutes and 12 minutes.

$$= P(10 < x < 12) = \int_{10}^{12} E(\alpha, x)\, dx = \int_{10}^{12} 0.2\, e^{-0.2x}\, dx$$

$$= 0.2 \left[\frac{e^{-0.2x}}{-0.2}\right]_{10}^{12} = \frac{e^{-2.4} - e^{-2}}{(-1)} = e^{-2} - e^{-2.4}$$

$$= \frac{1}{e^2} - \frac{1}{e^{2.4}}$$

=

(2). The duration of a telephone conversation has found to have an exponential distribution with mean 3 mins. Find the probability that the conversation may lass (i) more that 1 min (ii) less than 3 mins.

Ans: (i) $e^{-\frac{1}{3}}$ (ii) $1 - e^{-1}$

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