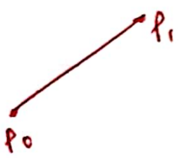
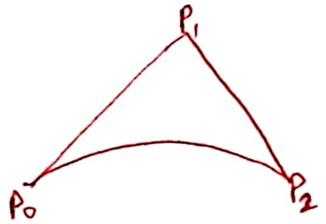
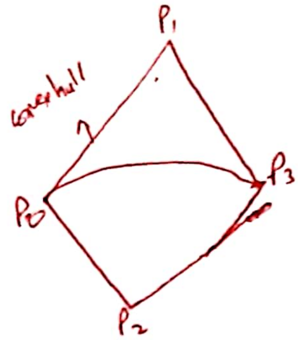
Imp.

⟨Bezier Curve⟩

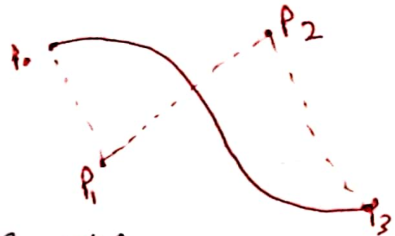
- A bezier curve is particularly a kind of spline generated from a set of control points by forming a set of polynomial function.
- These functions are computed from the coordinates of the control points.
- A bezier curve is defined by the defining polygon. It has number of properties that makes them highly & convenient for curve & surface design.
- It is an approximate spline curve.





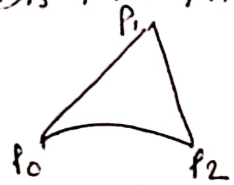


Spline → Bezier, B-spline

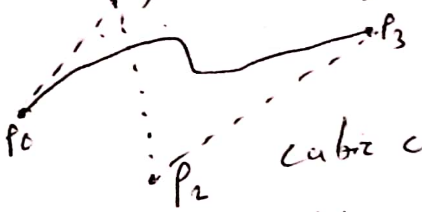


→ **Properties of Bezier Curve:**

1. Bezier curve is a polynomial of degree one less than the number of control points.



Quadratic

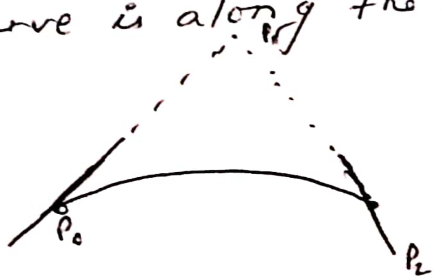


cubic curve.

2. Bezier curve always passes through the first & last points i.e.

$$P(0) = P_0 \quad , \quad P(1) = P_n .$$

3. The slope at the beginning of the curve is along the line joining the first two control points and the slope at the end of the curve is along the line joining the last 2 points.



4. Bezier blending function are all positive and sum is always 1.

$$\left[\sum_{i=0}^{n} Bez_{n,i}(u) = 1 \right]$$

5) The curve follows the shape of the defining polygon

6) The curve lies entirely within the convex hull formed by the four control points.

7) The convex hull property of the Bezier curve ensures that the polynomial smoothly follows the control points.

* Blending function:

$$Bez_{i,n}(u) = {}^{n}C_{i} \cdot u^{i} (1-u)^{n-i}$$

$$= \frac{n!}{i!\,(n-i)!} \cdot u^{i}(1-u)^{n-i}$$

For Individual Coordinates

$$x(u) = \sum_{i=0}^{n} x_i \, Bez_{i,n}(u)$$

$$y(u) = \sum_{i=0}^{n} y_i \, Bez_{i,n}(u)$$

$$z(u) = \sum_{i=0}^{n} z_i \, Bez_{i,n}(u)$$

Let suppose we are given $(n+1)$ control points positions. then $P_i = (x_i, y_i, z_i)$ $0 \leq i \leq n$

These co-ordinates points can be blended to produce the following position vector $P(u)$ which describes the path of an approximation. So, Bezier polynomial function between P_0 to P_n is

$$\left[P(u) = \sum_{i=0}^{n} P_i \, Bez_{i,n}(u)\right] \quad 0 \leq u \leq 1$$

Bezier polynomial function

where $P_i \rightarrow$ control points.

$Bez_{i,n} \rightarrow$ Blending function or Bezier function or Barstein function.

* <Bezier Curve for 3 points>

$$[Q(u) = P_0 B_{0,2}(u) + P_1 B_{1,2}(u) + P_2 B_{2,2}(u)]$$

$\rightarrow B_{0,2}(u) = {}^{2}C_0 \, u^0 (1-u)^{2-0} = (1-u)^2$

$\rightarrow B_{1,2}(u) = {}^{2}C_1 \, u^1 (1-u)^{2-1} = 2u(1-u)$

$\rightarrow B_{2,2} = {}^{2}C_2 \, u^2 (1-u)^{2-2} = u^2$

$$Q(u) = P_0(1-u)^2 + P_1\,2u(1-u) + P_2u^2$$

$$x(u) = (1-u)^2x_0 + 2u(1-u)x_1 + u^2x_2$$

$$y(u) = (1-u)^2y_0 + 2u(1-u)y_1 + u^2y_2$$

<Bezier Curve for 4 points>

$$[Q(u) = P_0B_{0,3}(u) + P_1B_{1,3}(u) + P_2B_{2,3}(u) + P_3B_{3,3}(u)]$$

$\rightarrow B_{0,3}(u) = {}^3C_0\,u^0(1-u)^{3-0} = (1-u)^3$

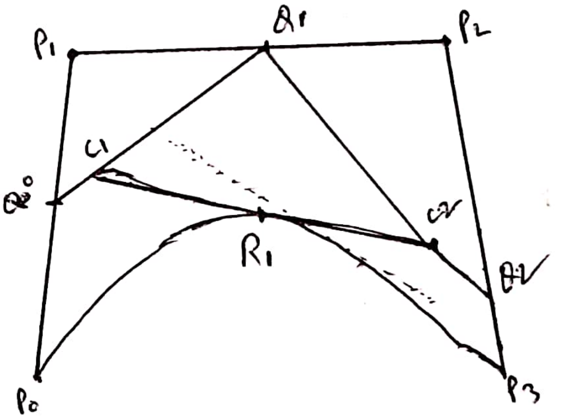
$\rightarrow B_{1,3}(u) = {}^3C_1\,u^1(1-u)^{3-1} = 3u(1-u)^2$

$\rightarrow B_{2,3}(u) = {}^3C_2\,u^2(1-u)^{3-2} = 3u^2(1-u)$

$\rightarrow B_{3,3}(u) = {}^3C_3\,u^3(1-u)^{3-3} = u^3$

$$[Q(u) = P_0(1-u)^3 + 3u(1-u)^2P_1 + 3u^2(1-u)P_2 + u^3P_3].$$

<Derivation>



[Note: In the figure don't give any value to dotted points, they are done by mistake by me while drawing this figure, so ignore it while you draw this figure].

Using a line parametric equation, we will derive the equation for the Bezier curve. No matter how many control points are there.

$Q_0 = (1-u)P_0 + uP_1$ (Q_0 point b/w P_0 & P_1)

$Q_1 = (1-u)P_1 + uP_2$ (Q_1 " " P_1 & P_2)

$Q_2 = (1-u)P_2 + uP_3$ (Q_2 " " P_2 & P_3)

$C_1 = (1-u)Q_0 + uQ_1$ (C_1 " " Q_0 & Q_1)

$C_2 = (1-u)Q_1 + uQ_2$ (C_2 " " Q_1 & Q_2)

$R = (1-u)C_1 + uC_2$ (R " " C_1 & C_2)

Now, $R(u) = (1-u)C_1 + uC_2$ (Replacing value, we get)

$= (1-u)[(1-u)Q_0 + uQ_1] + u[(1-u)Q_0 + uQ_2]$

$= (1-u)^2Q_0 + (1-u)uQ_1 + (1-u)uQ_0 + u^2Q_2$

P.T.O.

$= (1-u)^2 [(1-u)P_0 + uP_1] + 2u(1-u)Q_1 + u^2 Q_2$

$= (1-u)^2 [(1-u)P_0 + uP_1] + 2u(1-u)[(1-u)P_1 + uP_2] + u^2[(1-u)P_2 + uP_3]$

$= (1-u)^3 P_0 + u(1-u)^2 P_1 + 2u(1-u)^2 P_1 + 2u^2(1-u)P_2 + u^2(1-u)P_2 + u^3 P_3$

$= [(1-u)^3 P_0 + 3u(1-u)^2 P_1 + 3u^2(1-u)P_2 + u^3 P_3]$

For Calculating x, y coordinate.

$x(u) = (1-u)x_0 + 3u(1-u)^2 x_1 + 3u^2(1-u)x_2 + u^3 x_3$

$y(u) = (1-u)y_0 + 3u(1-u)^2 y_1 + 3u^2(1-u)y_2 + u^3 y_3$.

[youtube.com/watch?v=grYrC6_LDcy] → watch this once

grYrC6_LDcy

[[MakeIt Easy] → channel]]