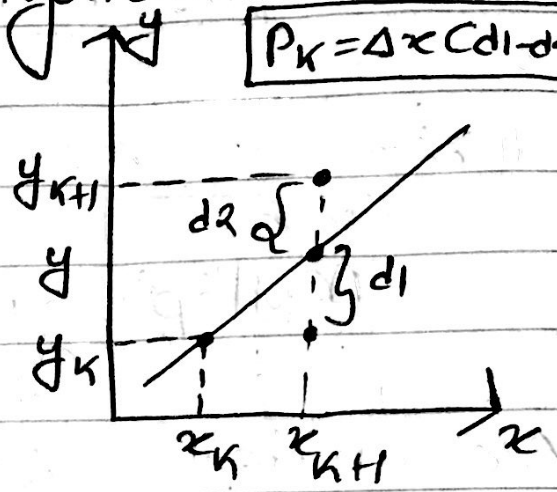
Bresenham's line drawing algorithm ($|m| \leq 1$)

$\boxed{P_K = \Delta x (d_1 - d_2)}$



- Let us assume that pixel (x_k, y_k) is already plotted assuming that the sampling direction is along x- direction.
- Now we have to decide which point to plot for x_{k+1} i.e (x_{k+1}, y_k) or (x_{k+1}, y_{k+1}).
- We have the equation of line as

$$y = mx + c \quad \text{— (1)}$$

for point x_{k+1},

$$y = m(x_{k+1} + c)$$
$$= m(x_k + 1 + c) \text{ — (2)} \quad [x_{k+1} = x_k + 1]$$

- Now from above figure,

$$d_1 = y - y_k \quad \text{and} \quad d_2 = y_{k+1} - y$$

Then
$$d_1 - d_2 = y - y_k - y_{k+1} + y$$
$$= 2y - y_k - (y_k + 1) \quad [y_{k+1} = y_k + 1]$$
$$= 2y - y_k - y_k - 1$$
$$= 2y - 2y_k - 1$$
$$= 2[m(x_k + 1) + c] - 2y_k - 1 \quad [\text{from (2)}]$$
$$= 2\left[\frac{\Delta y}{\Delta x}(x_k + 1) + c\right] - 2y_k - 1 \quad \left[m = \frac{\Delta y}{\Delta x}\right]$$

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$$= \frac{2[\Delta y(x_k+1)+\Delta x c] - 2\Delta x y_k - \Delta x}{\Delta x}$$

or $$d1-d2 = \frac{2[\Delta y(x_k+1)+\Delta x_c] - 2\Delta x y_k - \Delta x}{\Delta x}$$

Multiplying both side by Δx

$$\Delta x(d1-d2) = 2\Delta y x_k + 2\Delta y + 2\Delta x c - 2\Delta x y_k - \Delta x$$

or $$\boxed{P_k = 2\Delta y x_k + 2\Delta y - 2\Delta x y_k + \Delta x(2c-1)} \quad \text{--- (3)}$$

Now initial decision parameter $P_0 = ?$

From equation 3, at initial point (x_k, y_k)

$$P_0 = 2\Delta y x_k + 2\Delta y - 2\Delta x y_k + \Delta x(2(y-mx)-1) - \Delta x$$

$$[y = mx+c]$$

or $$P_0 = 2\Delta y x_k + 2\Delta y - 2\Delta x y_k + \Delta x(2y - 2mx - 1) - \Delta x$$

or $$P_0 = 2\Delta y x_k + 2\Delta y - 2\Delta x y_k + 2y_k\Delta x - 2\cdot\cancel{\Delta x}\cdot x_k \cdot \frac{\Delta y}{\cancel{\Delta x}} - \Delta x$$

$$\left[y = y_k, x = x_k, m = \frac{\Delta y}{\Delta x}\right]$$

or $$P_0 = \cancel{2\Delta y x_k} + 2\Delta y - \cancel{2\Delta x y_k} + \cancel{2y_k\Delta x} - \cancel{2\Delta y x_k} - \Delta x$$

or $$\boxed{P_0 = 2\Delta y - \Delta x} \quad \text{--- (4)}$$

Successive decision parameter $P_{k+1} = ?$

Again from 3

$$P_{k+1} = 2\Delta y x_{k+1} + 2\Delta y - 2\Delta x y_{k+1} + \Delta x(2c-1) \quad -(5)$$

Subtracting 3 from 5,

$$P_{k+1} - P_k = 2\Delta y x_{k+1} + \cancel{2\Delta y} - 2\Delta x y_{k+1} + \cancel{\Delta x(2c-1)}$$
$$- 2\Delta y x_k - \cancel{2\Delta y} + 2\Delta x y_k - \cancel{\Delta x(2c-1)}$$

or $P_{k+1} - P_k = 2\Delta y(x_{k+1} - x_k) - 2\Delta x(y_{k+1} - y_k)$

or $P_{k+1} - P_k = 2\Delta y(\cancel{x_k} + 1 - \cancel{x_k}) - 2\Delta x(y_{k+1} - y_k)$

$(x_{k+1} = x_k + 1)$

or $\boxed{P_{k+1} = P_k + 2\Delta y - 2\Delta x(y_{k+1} - y_k)} \quad -(6)$

Algorithm in short

- Decision parameter (P_k)

initial : $P_0 = 2\Delta y - \Delta x$

Successive : $P_{k+1} = P_k + 2\Delta y - 2\Delta x(y_{k+1} - y_k)$

- if $(P_k < 0)$ i.e d_2 is greater

$x_{k+1} = x_k + 1$

$y_{k+1} = y_k$

$P_{k+1} = P_k + 2\Delta y$

else

$x_{k+1} = x_k + 1$

$y_{k+1} = y_k + 1$

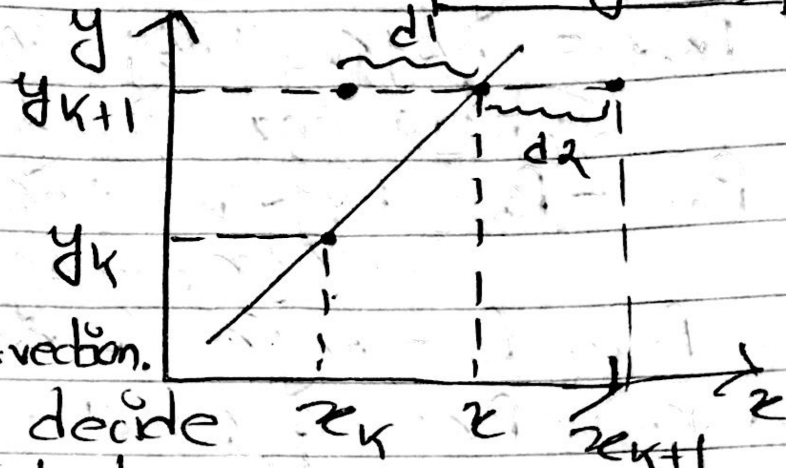
$P_{k+1} = P_k + 2\Delta y - 2\Delta x$

- Repeat Δx times.

Bresenhams line drawing algorithm ($|m| \geq 1$)

$\boxed{P_k = \Delta y (d_1 - d_2)}$

- Let us assume that pixel (x_k, y_k) is already plotted assuming that the sampling direction is along y direction.



- Now we have to decide which point to plot for y_{k+1} i.e (x_k, y_{k+1}) or (x_{k+1}, y_{k+1})
- we have the equation of line as

$$y = mx + c \quad \text{--- (1)}$$

At point y_{k+1},

$$y_{k+1} = mx + c$$

$$x = \frac{y_{k+1} - c}{m} \quad \text{--- (2)}$$

From above figure,

$d_1 = x - x_k$ and $d_2 = x_{k+1} - x$

Then $d_1 - d_2 = x - x_k - x_{k+1} + x$

$$= x - x_k - (x_k + 1) + x$$

$$= 2x - 2x_k - 1$$

$$= 2\left(\frac{y_{k+1} - c}{m}\right) - 2x_k - 1$$

[from - 2]

or $d_1 - d_2 = 2(y_{k+1} - c) \cdot \frac{\Delta x}{\Delta y} - 2x_k - 1$

or $d_1 - d_2 = \frac{2\Delta x y_{k+1} - 2\Delta x c - 2\Delta y x_k - \Delta y}{\Delta y}$

or $\Delta y(d_1 - d_2) = 2\Delta x(y_{k+1}) - 2\Delta x c - 2\Delta y x_k - \Delta y$

or $P_k = 2\Delta x(y_k + 1) - 2\Delta x c - 2\Delta y x_k - \Delta y$

$= 2\Delta x y_k + 2\Delta x - 2\Delta x c - 2\Delta y x_k - \Delta y$

$\boxed{P_k = 2\Delta x y_k - 2\Delta y x_k - 2\Delta x(c-1) - \Delta y}$ — 3

Now initial decision parameter, $p_0 = ?$

From equation 3 at initial point (x_k, y_k)

$P_0 = 2\Delta x y_k - 2\Delta y x_k - 2\Delta x(y - mx - 1) - \Delta y$

$[y = mx + c]$

$= 2\Delta x y_k - 2\Delta y x_k - 2\Delta x(y_k - \frac{\Delta y}{\Delta x} \cdot x_k - 1) - \Delta y$

$[y = y_k, x = x_k, m = \frac{\Delta y}{\Delta x}]$

$= \cancel{2\Delta x y_k} - \cancel{2\Delta y x_k} - \cancel{2\Delta x y_k} + \cancel{2\Delta y x_k} + 2\Delta x - \Delta y$

or $\boxed{P_0 = 2\Delta y - \Delta y}$ — 4

Successive decision parameter, $P_{k+1} = ?$

Again from 3,

$P_{k+1} = 2\Delta x y_{k+1} - 2\Delta y x_{k+1} - 2\Delta x(c-1) - \Delta y$

— 5

Subtracting 3 from 5

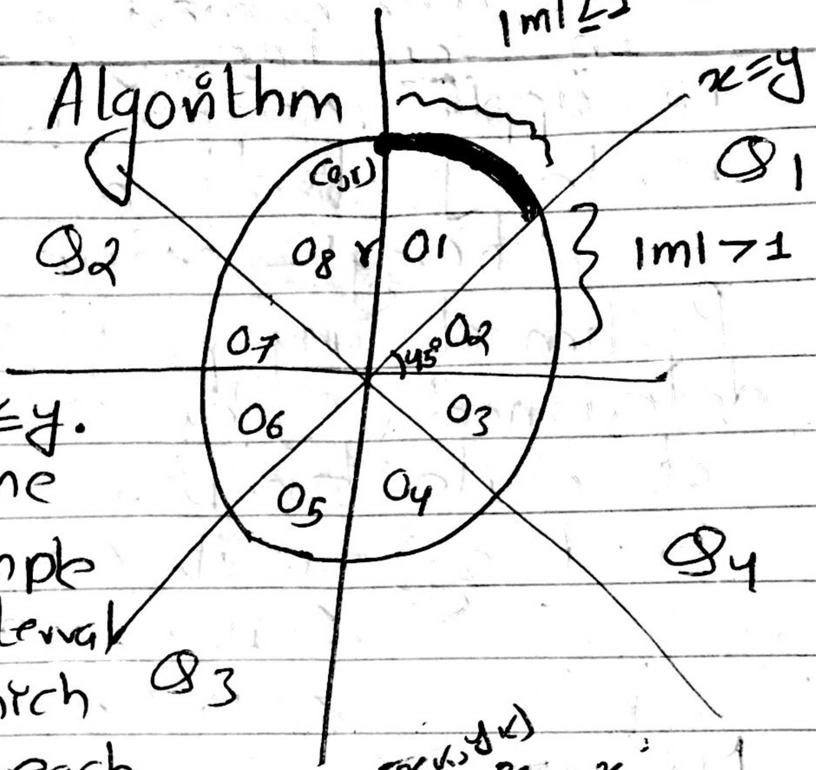
$$P_{K+1} - P_K = 2\Delta x y_{K+1} - 2\Delta y x_{K+1} - 2\Delta x(\cancel{c-1}) - \cancel{\Delta y}$$
$$- 2\Delta x y_K + 2\Delta y x_K + 2\Delta x(\cancel{c-1}) + \cancel{\Delta y}$$
$$= 2\Delta x(y_K + 1) - 2\Delta y x_{K+1} - 2\Delta x y_K + 2\Delta y x_K$$
$$= \cancel{2\Delta x y_K} + 2\Delta x - 2\Delta y x_{K+1} - \cancel{2\Delta x y_K} + 2\Delta y x_K$$
$$= 2\Delta x - 2\Delta y(x_{K+1} - x_K)$$

$$\boxed{\therefore P_{K+1} = P_K + 2\Delta x - 2\Delta y(x_{K+1} - x_K)} \quad \text{— } 6$$

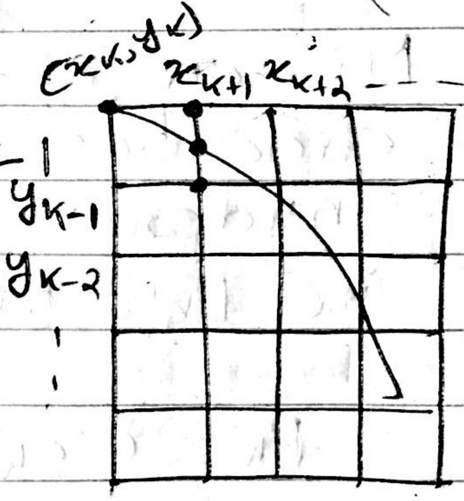
In short

- Decision parameter (P_K)
  Initial (P_0) $= 2\Delta x - \Delta y$
  Successive $P_{K+1} = P_K + 2\Delta x - 2\Delta y(x_{K+1} - x_K)$
- if ($P_K < 0$) i.e d_2 is greater
  $x_{K+1} = x_K$
  $y_{K+1} = y_K + 1$
  $P_{K+1} = P_K + 2\Delta x$
  else
  $x_{K+1} = x_K + 1$
  $y_{K+1} = y_K + 1$
  $P_{K+1} = P_K + 2\Delta x - 2\Delta y$
- Repeat Δy times.

Midpoint Circle Algorithm



- let us consider Q2 the first octant from $x=0$ to $x \le y$.
- As in raster line algorithm, we sample at unit x-interval and determine which pixel to plot for each successive y i.e y_k or y_{k-1}



- we need to select, which of these pixels is closer to the true circle, for this we find midpoint betn two pixels.

i.e. midpoint $m = \left(x_{k+1}, \frac{y_k + y_{k-1}}{2}\right)$

$$= \left(x_{k+1}, \frac{y_k + y_k - 1}{2}\right)$$

$$m = \left(x_k + 1, y_k - \frac{1}{2}\right)$$

- So it depends on midpoint which pixel to select.

to apply midpoint, we define a circle function,
$$f(x, y) = x^2 + y^2 - r^2$$
position of any point (x, y) can be determined by checking the sign of circle function.

i.e $f(x, y) = 0$ if (x, y) is in circle boundary
< 0 if (x, y) is inside " "
> 0 if (x, y) is outside " "

Now our decision parameter is the circle function evaluated at midpoint

i.e $P_k = f(x_k + 1, y_k - \frac{1}{2})$

$$\boxed{P_k = (x_k + 1)^2 + (y_k - \frac{1}{2})^2 - r^2} \quad \text{--- } 1$$

So if $P_k < 0$, then this midpoint is inside circle boundary and the pixel on scan line y_k is closer otherwise the midpoint is outside or on the circle boundary and we select pixel on scan line $y_k - 1$.

Now initial decision parameter $P_0 = ?$

from equation 1 at starting point $(0, r)$

$$P_0 = (0+1)^2 + (r - \frac{1}{2})^2 - r^2$$

$$= 1 + \cancel{r^2} - r + \frac{1}{4} - \cancel{r^2}$$

$$= \frac{4 - 4r + 1}{4}$$

$$= \frac{5 - 4r}{4}$$

$$= \frac{5}{4} - r$$

or $\boxed{P_0 = 1 - r}$ — (2)

Successive decision parameter, $P_{k+1} = ?$

Again from 1,

$$P_{k+1} = (x_{k+1} + 1)^2 + (y_{k+1} - \frac{1}{2})^2 - r^2 \quad -(3)$$

Subtracting 1 from 3

$$P_{k+1} - P_k = (x_{k+1} + 1)^2 + (y_{k+1} - \frac{1}{2})^2 - \cancel{r^2}$$
$$- (x_k + 1)^2 + (y_k - \frac{1}{2})^2 + \cancel{r^2}$$

$$= (\underbrace{x_k + 1}_{a} \underbrace{+1}_{b})^2 + \left(y_{k+1} - \frac{1}{2}\right)^2$$
$$- (x_k + 1)^2 + (y_k - \frac{1}{2})^2$$

or $P_{k+1} - P_k = \cancel{(x_k+1)^2} + 2(x_k+1) + 1 + y^2_{k+1} - y_{k+1} + \cancel{\frac{1}{4}} - \cancel{(x_k+1)^2} - y_k^2 + y_k - \cancel{\frac{1}{4}}$

$= 2(x_k+1) + y^2_{k+1} - y_k^2 - y_{k+1} + y_k + 1$

or $\boxed{P_{k+1} = P_k + 2(x_k+1) + (y^2_{k+1} - y^2_k) - (y_{k+1} - y_k) + 1}$ — (4)

In short

- Decision parameter (P_k)

Initial $P_0 = 1 - r$

Successive $P_{k+1} = P_k + 2(x_k+1) + (y^2_{k+1} - y^2_k) - (y_{k+1} - y_k) + 1$

- if ($P_k < 0$)

$x_{k+1} = x_k + 1$

$y_{k+1} = y_k$

$P_{k+1} = P_k + 2(x_{k+1}) + 1$

else

$x_{k+1} = x_k + 1$

$y_{k+1} = y_k - 1$

$P_{k+1} = P_k + 2(x_{k+1}) - 2(y_{k+1}) + 1$

- Continue until $x \leq y$ holds.

Note: [in else part]

$$P_{K+1} = P_K + 2(x_K+1) + ((y_K-1)^2 - y^2_K) - (\cancel{y_K} - 1 - \cancel{y_K}) + 1$$

$$= P_K + 2(x_K+1) + (\cancel{y^2_K} - 2y_K + 1 - \cancel{y^2_K}) + 1 + 1$$

$$= P_K + 2(x_{K+1}) - 2y_K + 1 + 1 + 1$$

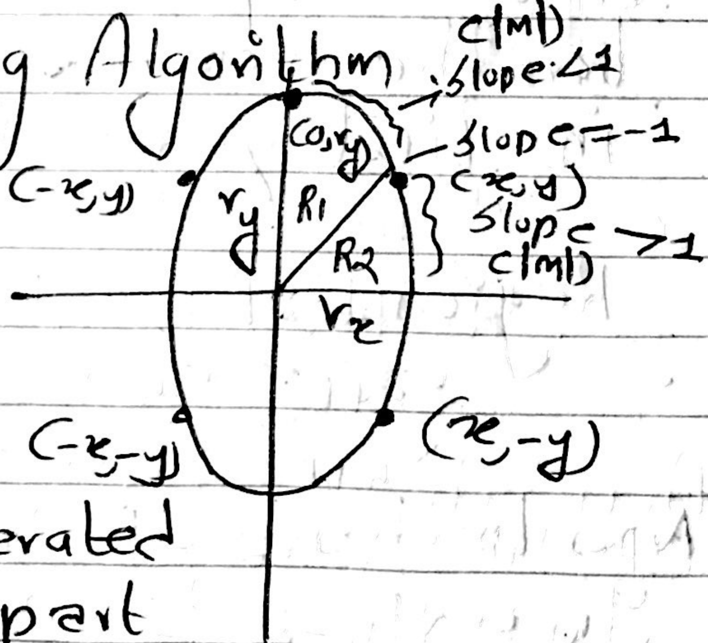
$$= P_K + 2(x_{K+1}) - 2y_K + 2 + 1$$

$$= P_K + 2(x_{K+1}) - 2(y_K - 1) + 1$$

$$= P_K + 2(x_{K+1}) - 2(y_{K+1}) + 1$$

$[\because y_{K+1} = y_K - 1]$

Ellipse Dracing Algorithm



- An ellipse is divided into four equal symmetrical parts, so if one quadrant is generated then other three part can be easily replicated using symmetry property.

- The midpoint method is applied throughout the first quadrant into two part i.e Region 1 (R1) and Region 2 (R2)
- If the slope of the curve is less than 1, we are in R1 otherwise R2. ($|m| < 1$)
- In R1, we take unit step in x direction until we reach boundary betn R1 and R2.
- For region 2 ($|m| > 1$), we take unit step in y direction for the remainder of the curve in first quadrant

Region 1 : R1 ($|m| <= 1$)

- $x_{k+1} = x_k + 1$

  $y_{k+1} = y_k$ or y_{k-1}

Now to choose which pixel i.e y_k or y_{k-1} to choose for y_{k+1}, we take mid point

$$\text{midpoint} = \left(x_k+1, \frac{y_k+y_k-1}{2}\right)$$

$$= \left(x_k+1, y_k-\frac{1}{2}\right)$$

As we know the ellipse function is given as

$$\frac{x^2}{r_x^2} + \frac{y^2}{r_y^2} = 1$$

$$x^2 r_y^2 + y^2 r_x^2 = r_x^2 r_y^2$$

i.e $\boxed{f(x,y) = x^2 r_y^2 + y^2 r_x^2 - r_x^2 r_y^2}$ — (1)

Such that for any point (x,y)

$f(x,y) = 0$ if (x,y) lie on ellipse boundary

> 0 if (x,y) lie outside ellipse boundary

< 0 if (x,y) lie inside " "

So our decision parameter is ellipse function evaluated at midpoint

i.e $P_{1k} = f\left(x_k+1, y_k-\frac{1}{2}\right)$

$$\boxed{P_{1k} = (x_k+1)^2 r_y^2 + \left(y_k-\frac{1}{2}\right)^2 r_x^2 - r_x^2 r_y^2}$$ (2)

So if $P_{1k} < 0$, midpoint lie inside ellipse hence we select y_k otherwise, the midpoint is outside the ellipse, so we select y_{k-1}.

Now initial decision parameter $P_{10} = ?$
From eqn 2 at starting point $(0, r_y)$

$$P_{10} = (0+1)^2 r_y^2 + (r_y - \frac{1}{2})^2 r_x^2 - r_x^2 r_y^2$$

$$= r_y^2 + \cancel{r_y^2 r_x^2} - r_y r_x^2 + \frac{1}{4} r_x^2 - \cancel{r_x^2 r_y^2}$$

$$\boxed{P_{10} = r_y^2 + \frac{r_x^2}{4} - r_y r_x^2} \quad — (3)$$

Now successive decision parameter $P_{1k+1} = ?$
Again from (2),

$$P_{1k+1} = (x_{k+1}+1)^2 r_y^2 + (y_{k+1} - \frac{1}{2})^2 r_x^2 - r_x^2 r_y^2 \quad — (4)$$

Subtracting 3 from 4.

$$P_{1k+1} - P_{1k} = (x_{k+1}+1)^2 r_y^2 + (y_{k+1} - \frac{1}{2})^2 r_x^2 - \cancel{r_x^2 r_y^2} - (x_k+1)^2 r_y^2 - (y_k - \frac{1}{2})^2 r_x^2 + \cancel{r_x^2 r_y^2}$$

or $P_{1k+1} - P_{1k} = (\underbrace{x_k+1}_{a} + \underbrace{1}_{b})^2 r_y^2 + (y_{k+1} - \frac{1}{2})^2 r_x^2$

$- (x_k+1)^2 r_y^2 - (y_k - \frac{1}{2})^2 r_x^2$

$= \cancel{(x_k+1)^2 r_y^2} + 2(x_k+1) r_y^2 + r_y^2$

$+ r_x^2 y_{k+1}^2 - y_{k+1} r_x^2 + \cancel{\frac{1}{4} r_x^2}$

$- \cancel{(x_k+1)^2 r_y^2} - y_k^2 r_x^2 + y_k r_x^2$

$\cancel{-\frac{1}{4} r_x^2}$

or $\boxed{P_{1k+1} = P_{1k} + r_y^2 + 2(x_k+1) r_y^2 + r_x^2 (y_{k+1}^2 - y_k^2) - r_x^2 (y_{k+1} - y_k)}$

— (5)

Similarly for Region 2 : R2 (|m| > 1)

— After completing region 1, we have to move to region 2.

— for this a condition is needed to find when region 1 ends and

region 2 starts.
- for this we have the eqn of ellipse as

$$\frac{x^2}{r_x^2} + \frac{y^2}{r_y^2} = 1$$

$$x^2 r_y^2 + y^2 r_x^2 = r_x^2 r_y^2 \quad \text{— } 6$$

Now slope $(m) = \frac{dy}{dx}$

Differentiating 6 wrt x

$$r_y^2 \cdot 2x + r_x^2 \cdot \frac{dy^2}{dy} \cdot \frac{dy}{dx} = 0$$

$$2x r_y^2 + r_x^2 \, 2y \frac{dy}{dx} = 0$$

or $\frac{dy}{dx} = -\frac{2x r_y^2}{2y r_x^2}$

At the boundary betn region 1 and region 2, slope $= -1$
hence we move out of region 1. when $\boxed{2x r_y^2 \geq 2y r_x^2}$

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for Region 2

$y_{k+1} = y_{k-1}$

$x_{k+1} = x_k$ or x_{k+1}

Now to choose which pixel x_k or x_{k+1} to choose for x_{k+1}, we take midpoint

i.e midpoint $= \left(\frac{x_k + x_{k+1}}{2}, y_{k-1} \right)$

$$= \left(\frac{x_k + x_k + 1}{2}, y_k - 1 \right)$$

$$= \left(x_k + \frac{1}{2}, y_k - 1 \right)$$

The decision parameter is calculated as

$P_{2k} = f\left(x_k + \frac{1}{2}, y_k - 1 \right)$

$$\boxed{P_{2k} = \left(x_k + \frac{1}{2}\right)^2 r_y^2 + (y_k - 1)^2 r_x^2 - r_x^2 r_y^2}$$

$\rightarrow$

so if $P_{2k} < 0$, then midpoint lie inside ellipse hence we select x_{k+1} otherwise the midpoint is outside or on boundary of ellipse, so we select x_k.

Now initial decision parameter $p_{20} = ?$

This can be calculated using equation 7 for the point when

$$\boxed{2r_y^2 x \geq 2r_x^2 y}$$

condition is true

Successive decision parameter, $p_{k+1} = ?$

Again from eqn 7

$$P_{2k+1} = (x_{k+1} + \tfrac{1}{2})^2 \cdot r_y^2 + (y_{k+1} - 1)^2 r_x^2 - r_x^2 r_y^2 \quad (8)$$

Subtracting 7 from 8

$$P_{2k+1} - P_{2k} = (x_{k+1} + \tfrac{1}{2})^2 r_y^2 + (y_{k+1} - 1)^2 r_x^2 - \cancel{r_x^2 r_y^2} - (x_k + \tfrac{1}{2})^2 r_y^2 - (y_k - 1)^2 r_x^2 + \cancel{r_x^2 r_y^2}$$

$$= (x_{k+1} + \tfrac{1}{2})^2 r_y^2 + (\underbrace{y_k - 1}_{a} \underbrace{- \tfrac{1}{2}}_{b})^2 r_x^2 - (x_k + \tfrac{1}{2})^2 r_y^2 - (y_k - 1)^2 r_x^2$$

$$\cancel{x_{k+1}^2 r_y^2} + x_{k+1} r_y^2 + \cancel{\frac{1}{4} r_y^2} +$$

$$\cancel{(y_k - 1)^2 r_x^2} - 2(y_k - 1) r_x^2 + r_x^2$$

$$- x_k^2 r_y^2 - x_k r_y^2 - \cancel{\frac{r_y^2}{4}} -$$

$$\cancel{(y_k - 1)^2 r_x^2}$$

$$\boxed{\therefore P_{2k+1} = P_{2k} + r_x^2 - 2r_x^2 (y_k - 1) + r_y^2 (x_{k+1}^2 - x_k^2) + r_y^2 (x_{k+1} - x_k)}$$

— 9

In short

$\boxed{\text{For Region 1}}$

- Decision parameter P_{1k}

Initial $P_{10} = r_y^2 + \frac{r_x^2}{4} - r_y r_x^2$

Successive $P_{1k+1} = P_{1k} + r_y^2 + 2(x_k + 1) r_y^2 + r_x^2 (y_{k+1}^2 - y_k^2) - r_x^2 (y_{k+1} - y_k)$

- if $(P_{1k} \geq 0)$

$x_{k+1} = x_k + 1$

$y_{k+1} = y_k - 1$

$P_{1k+1} = P_{1k} + r_y^2 + 2(x_{k+1})r_y^2 - 2r_x^2 y_{k+1} + r_y^2$

else

$x_{k+1} = x_k + 1$

$y_{k+1} = y_k$

$P_{1k+1} = P_{1k} + r_y^2 + 2r_y^2 (x_k + 1)$

- for each generated point check

$2r_y^2 x_{k+1} \geq 2r_x^2 y_{k+1}$

if the above condition is met start region 2.

* Note: in if part

$P_{1k+1} = P_{1k} + r_y^2 + 2(x_k + 1)r_y^2 + r_x^2 (y_{k+1}^2 - y_k^2) - r_x^2 (y_{k+1} - y_k)$ [from 5]

or $P_{1k+1} = P_{1k} + r_y^2 + 2(x_{k+1}) r_y^2 +$

$r_x^2 \left[(y_k - 1)^2 - y^2_k \right] -$

$r_x^2 (\cancel{y_k} - 1 - \cancel{y_k})$

$= P_{1k} + r_y^2 + 2x_{k+1} r_y^2 + r_x^2 [\cancel{y^2_k} -$

$2y_k + 1 - \cancel{y^2_k}] + r_x^2$

$= P_{1k} + r_y^2 + 2x_{k+1} r_y^2 - 2r_x^2 y_k + r_x^2 + r_x^2$

$= P_{1k} + r_y^2 + 2x_{k+1} r_y^2 - 2r_x^2 (y_k - 1)$

$= P_{1k} + r_y^2 + 2(x_{k+1}) r_y^2 - 2r_x^2 (y_{k+1})$

in else part

$P_{1k+1} = P_{1k} + r_y^2 + 2(x_k + 1) r_y^2 +$

$r_x^2 (y^2_{k+1} - y_k^2) - r_x^2 (y_{k+1} - y_k)$

$= P_{1k} + r_y^2 + 2(x_{k+1}) r_y 2 + 0 - 0$

$= P_{1k} + r_y^2 + 2x_{k+1} r_y^2$

Scanned by CamScanner

For Region 2

- Decision parameter P_{2k}

initial $P_{20} = (x_k + \frac{1}{2})^2 r_y^2 + (y_k - 1)^2 r_x^2 - r_x^2 r_y^2$

such that (x_k, y_k) is a point starting region 2.

i-e point when $\boxed{2x r_y^2 \geq 2y r_x^2}$ holds true.

Successive $P_{2k+1} = P_{2k} + r_x^2 + r_y^2 (x_{k+1}^2 - x_k^2) + r_y^2 (x_{k+1} - x_k) - 2r_x^2 (y_k - 1)$

- if $(P_{2k} > 0)$

$x_{k+1} = x_k$

$y_{k+1} = y_k - 1$

$P_{2k+1} = P_{2k} + r_x^2 - 2r_x^2 (y_{k+1})$

else

$x_{k+1} = x_k + 1$

$y_{k+1} = y_k - 1$

$$P_{2k+1} = P_{2k} + r_x^2 - 2r_x^2(y_{k+1}) + 2r_y^2(x_{k+1})$$

- Repeat till $y > 0$.

Note: in else part

$$P_{2k+1} = P_{2k} + r^2_x + r_y^2(x^2_{k+1} - x^2_k) + r_y^2(x_{k+1} - x_k) - 2r_x^2(y_k - 1)$$

$$= P_{2k} + r_x^2 - 2r_x^2(y_{k+1}) + r_y^2((x_k+1)^2 - x_k^2) + r_y^2(\cancel{x_k} + 1 - \cancel{x_k})$$

$$= P_{2k} + r^2_x + r_y^2(\cancel{x_k^2} + 2x_k + 1 - \cancel{x^2_k}) + r_y^2$$

$$= P_{2k} + r^2_x + r_y^2(2x_k + 1) + r_y^2 - 2r_x^2(y_{k+1})$$

$$= P_{2k} + r^2_x + 2r_y^2 x_k + 2r_y^2 - 2r_x^2(y_{k+1})$$

$$= P_{2k} + r^2_x + 2r_y^2(x_k + 1) - 2r_x^2 y_{k+1}$$

$$= P_{2k} + r^2_x + 2r_y^2(x_{k+1}) - 2r_x^2 y_{k+1}$$

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