Continuous system simulation

-A continuous system is the system in which the activities of the main elements of the system cause smooth changes in the attributes of the entities of the system.

On mathematical modeling, the attributes of the system are controlled by a continuous functions.

In such system, the relationships depicts the rates at which the attributes changes.

The continuous system is modeled using the differential equations.

The complex continuous system with non-linearity can be simulated by showing the application to models for linear differential equations to obtain constant coefficients and then generalize to more complex equations.

Differential and Partial Differential Equations

The equation that consists of the higher order derivatives of the dependent variable is known as differential equations.

-The differential equation is said to be linear if any of the Dependent variables and its derivatives have power of one and are multiplied by the constant.

Eg: M x’’ + D x’ + K x = K F(t)

where, M, D and K are constants; F(t) is the input to the system depending upon the independent variable t; x’’ and x’ are second and first order derivatives of dependent variable x.

-The differential equation is said to be non-linear if the dependent variable or any of its derivatives are raised to a power or are combined in other way like multiplication.

$$y\left(\frac{dy}{dx}\right)^2 + (x-y)\frac{dy}{dx} - x = 0.$$

-The differential equation is said to be partial if more than one independent variable occurs in a differential equation.
-Eg: Equation of flow of heat in three-dimensional body. It consists of four independent variables (three dimensions and time) and one dependent variable (temperature).

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Necessity of differential equations:

1.Most physical and chemical process occurring in the nature involves rate of change, which requires differential equations to provide mathematical model.
2.It can be used to understand general effects of growth trends as differential equations can represent a growth rate.

Continuous system

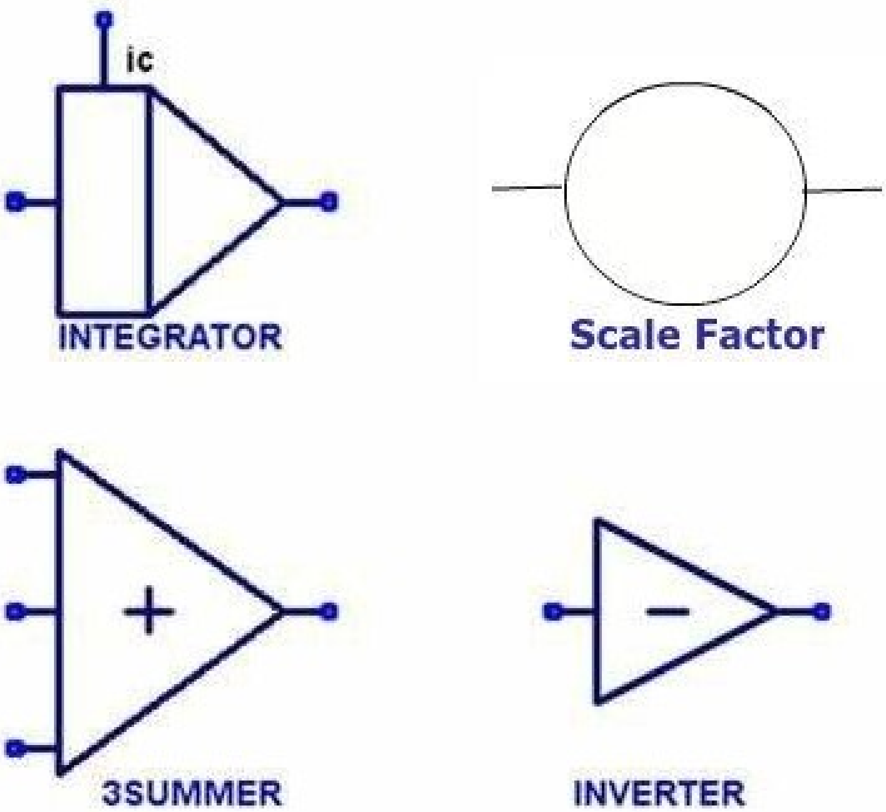
- If the dependent variable or any of its derivatives appear in any other form, such as being raised to a power, or are combined in any other way for example, by being multiplied together, the differential equation is said to be nonlinear.
- When more than one independent variable occurs in a differential equation, the equation is said to be partial differential equation.
- It can involve the derivatives of the same dependent variable with respect to each of the independent variables.

Analog Computer

- The most widely used form of analog computer is the electronic analog computer, based on the use of high gain dc (direct current) amplifiers, called operational amplifiers.
- Voltages in the computer are equated to mathematical variables, and the operational amplifiers can add and integrate the voltages.
- With appropriate circuits, an amplifier can be made to add several input voltages, each representing a variable of the model, to produce a voltage representing the sum of the input variables.

Components of Analog Computer

- Different scale factors can be used on the inputs to represent coefficients of the model equations.
- Such amplifiers are called summers.
- Another circuit arrangement produces an integrator for which the output is the integral with respect to time of a single input voltage or the sum of several input voltages.
- All voltages can be positive or negative to correspond to the sign of the variable represented.
- To satisfy the equations of the model, it is sometimes necessary to use a sign inverter, which is an amplifier designed to cause the output to reverse the sign of the input.


ic
INTEGRATOR
Scale Factor
3SUMMER
INVERTER

Analog Computer

- Electronic analog computers are limited in accuracy for several reasons.
- It is difficult to carry the accuracy of measuring a voltage beyond a certain point.
- Secondly, a number of assumptions are made in deriving the relationships for operational amplifiers, none of which is strictly true; so, amplifiers do not solve the mathematical model with complete accuracy.
- A particularly troublesome assumption is that there should be zero output for zero input.
- Another type of difficulty is presented by the fact that the operational amplifiers have a limited dynamic range of output, so that scale factors must be introduced to keep within the range.

Analog Computer

- As a consequence, it is difficult to maintain an accuracy better than 0.1% in an electronic analog computer.
- Other forms of analog computers have similar problems and their accuracies are not significantly better.
- A digital computer is not subject to the same type of inaccuracies.
- Virtually any degree of accuracy can be programmed and, with the use of floating-point representation of numbers, an extremely wide range of variations can be tolerated.
- Integration of variables is not a natural capability of a digital computer, as it is in an analog computer, so that integration must be carried out by numerical approximations.

Analog Computer

- However, methods have been developed which can maintain a very high degree of accuracy.
- A digital computer also has the advantage of being easily used for many different problems.
- An analog computer must usually be dedicated to one application at a time, although time-sharing sections of an analog computer has become possible.
- In spite of the widespread availability of digital computers, many users prefer to use analog computers. There are several considerations involved.

Analog Computer

- The analog representation of a system is often more natural in the sense that it directly reflects the structure of the system; thus simplifying both the setting—up of a simulation and the interpretation of the results.

- Under certain circumstances, an analog computer is faster than a digital computer, principally because it can be solving many equations in a truly simultaneous manner; whereas a digital computer can be working only on one equation at a time, giving the appearance of simultaneity by interfacing the equations.

- On the other hand, the possible disadvantages of analog computers, such as limited accuracy and the need to dedicate the computer to one problem, may not be significant.

Analog methods

- The general method by which analog computers are applied can be demonstrated using the second—order differential equation that has already been discussed:

- Solving the equation for the highest order derivative gives

$$M\frac{d^2x}{dy}+D\frac{dx}{dy}+Kx=KF(t)$$

$$M\frac{d^2x}{dy}=KF(t)-D\frac{dx}{dy}-Kx$$

Analog methods

- Suppose a variable representing the input F(t) is supplied, and assume for the time being that there exist variables representing —x and $\frac{dx}{dy}$
- These three variables can be scaled and added with a summer to produce a voltage representing MX.
- Integrating this variable with a scale factor of 1/M produces $\frac{dx}{dy}$
- Changing the sign produces —x, which supplies one of the variables initially assumed; and a further integration produces —x, which was the other assumed variable.

Analog methods

- For convenience, a further sign inverter is included to produce $+x$ as an output.
- A block diagram to solve the problem in this manner is shown in following Figure.

Analog methods

Automobile Suspension Problem: Example

The model is defined by the second order differential equation as:

$$M x'' + D x' + K x = K F(t)$$

$$\text{Or, } M x'' = K F(t) - D x' - K x$$

The diagram analog method modeling is shown below:

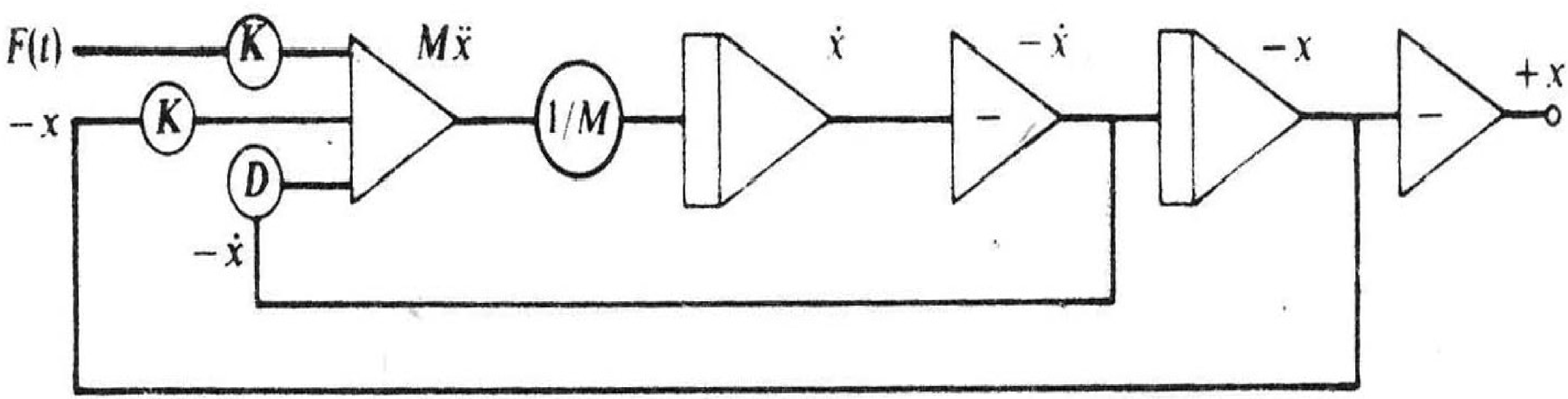


Diagram for the automobile suspension problem

Analog methods

- The symbols used in the figure are standard symbols for drawing block diagrams representing analog computer arrangements.
- The circles indicate scale factors applied to the variables.
- The triangular symbol at the left of the figure represents the operation of adding variables.
- The triangular symbol with a vertical bar represents an integration, and the one containing a minus sign is a sign changer.

Analog methods

- The addition on the left, with its associated scaling factors, corresponds to the addition of the variables representing the three forces on the wheel, producing a variable representing M .
- The scale is changed to produce $\frac{d^2x}{dy}$ and the result is integrated twice to produce both $\frac{dx}{dy}$ and x.
- Sign changers are introduced so that variables of the correct sign can be fed back to the adder, and the output can be given in convenient form.
- With an electronic analog computer, the variables that have been described would be voltages, and the symbols would represent operational amplifiers arranged as adders, integrators, and sign changers.

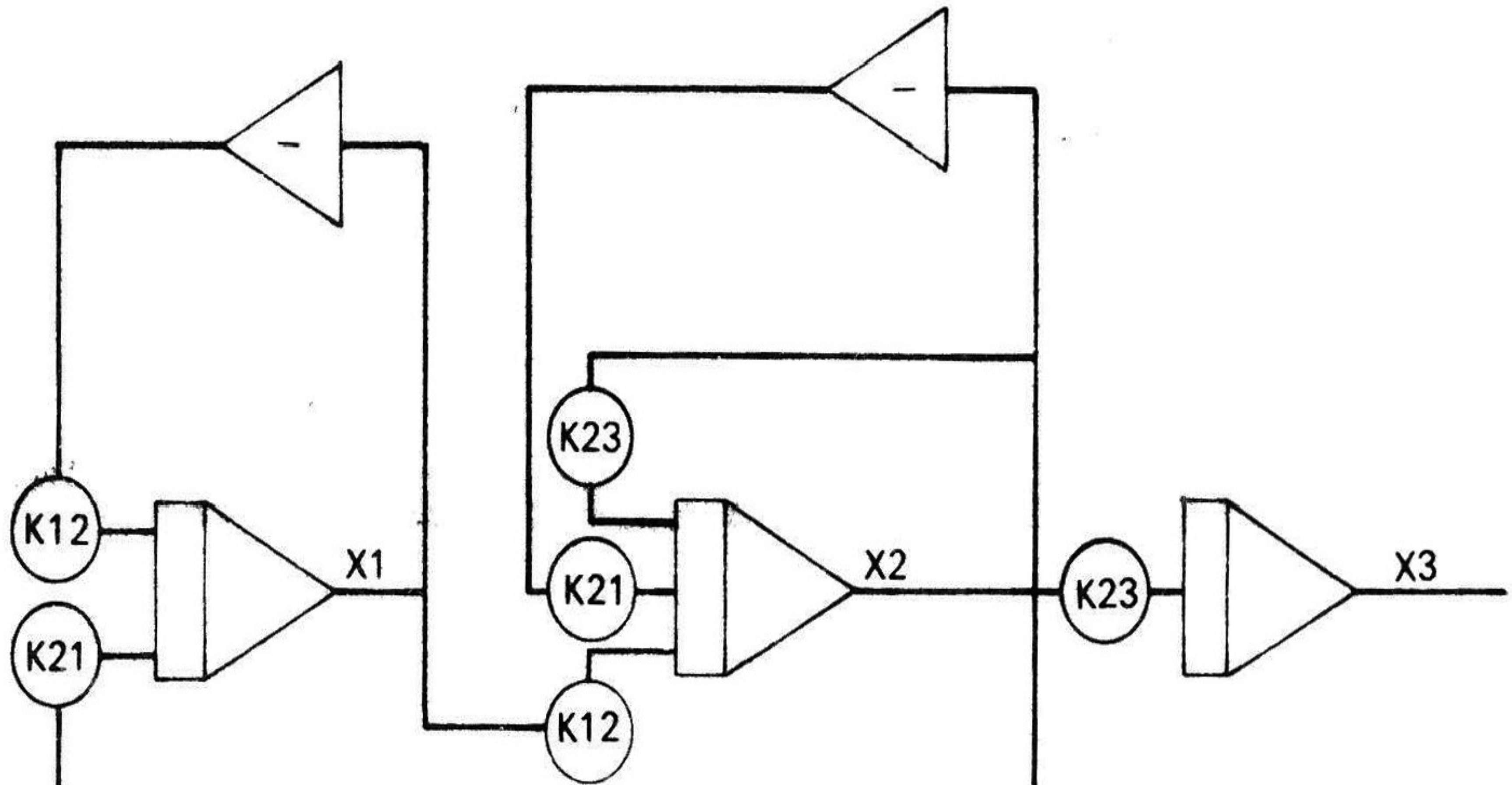
Analog methods

- The above Figure would then represent how the amplifiers are interconnected to solve the equation.
- It should be pointed out, however, that there can be several ways of drawing a diagram for a particular problem, depending upon which variables are of interest, and on the size of the scale factors.
- When a model has more than one independent variable, a separate block diagram is drawn for each independent variable and, where necessary, interconnections are made between the diagrams.

Analog methods

- As an example, the following Figure shows a block diagram for solving the model of the liver.

Analog methods



Analog computer model of the liver

Analog methods

- There are three integrators, shown at the bottom of the figure. Reading from left to right, they solve the equations for X_1, X_2, X_3.
- Interconnections between the three integrators, with sign changers where necessary, provide inputs that define the differential coefficients of the three variables.
- The first integrator, for example, is solving the equation

$$\frac{dx_1}{dy} = -k_{12}x_1 + k_{21}x_2$$

Analog methods

- The second integrator is solving the equation

$$\frac{dx_2}{dy} = k_{12}x_1 - (k_{21} - k_{23})x_2$$

- The last integrator solves the equation

$$\frac{dx_3}{dy} = k_{23}x_2$$