Example: Poker Test

A sequence of 1000 four-digit numbers has been generated and an analysis indicates the following combinations & frequencies:

Combination Distribution (i)	***Observed Frequency (O_i)***
4 different digits	540
3 like digits	50
4 like digits	20
1 pair	320
2 pairs	70

Use Poker's test to determine if these random numbers are independent, $\alpha = 0.05$.

Solution:

In 4-digit numbers, there are only five possibilities- four different, 3 of a kind, 4 of a kind, 1 pair and 2 pairs. Let's calculate the probabilities for each of these cases:

Case-I: P(4 different digits)

= P(2^{nd} diff. from 1^{st}) * P(3^{rd} diff. from 1^{st} & 2^{nd}) * P(4th diff. from 1^{st}, 2^{nd} & 3^{rd})

= 0.9 * 0.8 * 0.7 = 0.504

Case-II: P(3 like digits)

= P(2^{nd} digit same as 1^{st}) * P(3^{rd} digit same as 1^{st}) * P(4^{th} digit diff. from 1^{st}) * ${}^4C_3$

= 0.1 * 0.1 * 0.9 * 4= 0.036

Case-III: P(4 like digits)

= P(2^{nd} digit same as 1^{st}) * P(3^{rd} digit same as 1^{st}) * P(4^{th} digit same as 1^{st})

= 0.1 * 0.1 * 0.1 = 0.001

Case-IV: P(1 pair)

= No. of possible combinations for a pair from 4 digits * P(2^{nd} digit same as 1^{st} in the pair) * P(3^{rd} digit diff. from 1^{st}) * P(4^{th} digit diff. from 1^{st} & 3^{rd})

= ${}^4C_2$ * 0.1 * 0.9 * 0.8 = 0.432

Case-V: P(2 pairs)

= 1 - P(4 different digits) - P(3 like digits) - P(4 like digits) - P(1 pair)

= 1 – 0.504 – 0.036 – 0.001 – 0.432 = 0.027

With N = 1000, let's summarize the results for Poker's test in the following table:

Combination Distribution (i)	***Observed Frequency O_i***	***Expected Frequency E_i = Prob.*N***	***$(O_i - E_i)^2/E_i$***
4 different digits	540	504	2.571
3 like digits	50	36	5.44
4 like digits	20	1	361
1 pair	320	432	29.037
2 pairs	70	27	68.48
	1000	**1000**	**466.528**

Calculated χ^2 value = 466.528

Degree of freedom = n – 1 = 5 – 1 = 4

At $\alpha = 0.05$, acceptable value of χ^2 from table = 9.49

Since, the calculated value of χ^2 is greater than the acceptable value at four degree of freedom, the independence of the random numbers is rejected on the basis of this test.