Gordon Book 103, 5-12

3.16: Predator-Prey Model:

Let $x(t)$: Numbers of individual in the prey at time t.
$y(t)$ = " " " " , predator " t.

Suppose Now, there is sufficient ~~prey~~ food (grass) for prey animals. & in the absence of predator,
~~r(x~~ $rx(t)$ = rate of growth of pray animals.
(we can think r as natural birth rate − natural death race).
The death rate of the prey due to interaction with predator can be assumed to be propotitioned to the product of two population sizes, $x(t)\,y(t)$.
∴ The overall rate of change of the prey population is given by,
$$\frac{dx}{dt} = rx(t) - ax(t)\cdot y(t) \quad \text{--- (1)}$$
a = positive constant of proportionality.
The preddtors depends upon prey, and in the absence of prey, their rate of change is $-sy(t)$. s is +ve
Due to interaction betn the population the predators increase at a rate, which is proportionate to $x(t)y(t)$.
∴ The overall rate of change of predator population is
$$\frac{dy}{dt} = -sy(t) + b\,x(t)y(t) \quad \text{--- (2)}$$ b is a +ve constant

Given initial condition $x(0) > 0$, $y(0) > 0$, the solⁿ of the model given by equⁿ ① & ② has the interesting property that $x(t) > 0$ & $y(t) > 0$ for all $t \geq 0$.

Eg: Given: $r = 0.001$, $a = 2 * 10^{-6}$, $s = 0.01$, $b = 10^{-6}$ & initial population size $x(0) = 12{,}000$, $y(0) = 600$.

for a period of 500 time units.