Simulation and Modeling

Unit-2

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2. System Simulation

- Monte Carlo methods, or Monte Carlo experiments, are **a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results**. The underlying concept is to use randomness to solve problems that might be deterministic in principle.

monte carlo method

- A Monte Carlo simulation is used to tackle a range of problems in many fields including investing, business, physics, and engineering.
- It is also referred to as a multiple probability simulation.
- A particular numerical computation method called the ***monte carlo method*** consists of experimental sampling with random numbers.
- For example, the integral of a single variable over a given range corresponds to finding the area under the graph representing the function. Suppose the function *f(x)* is positive and has lower and upper bounds a and b. Also the function is bounded above by the value c.

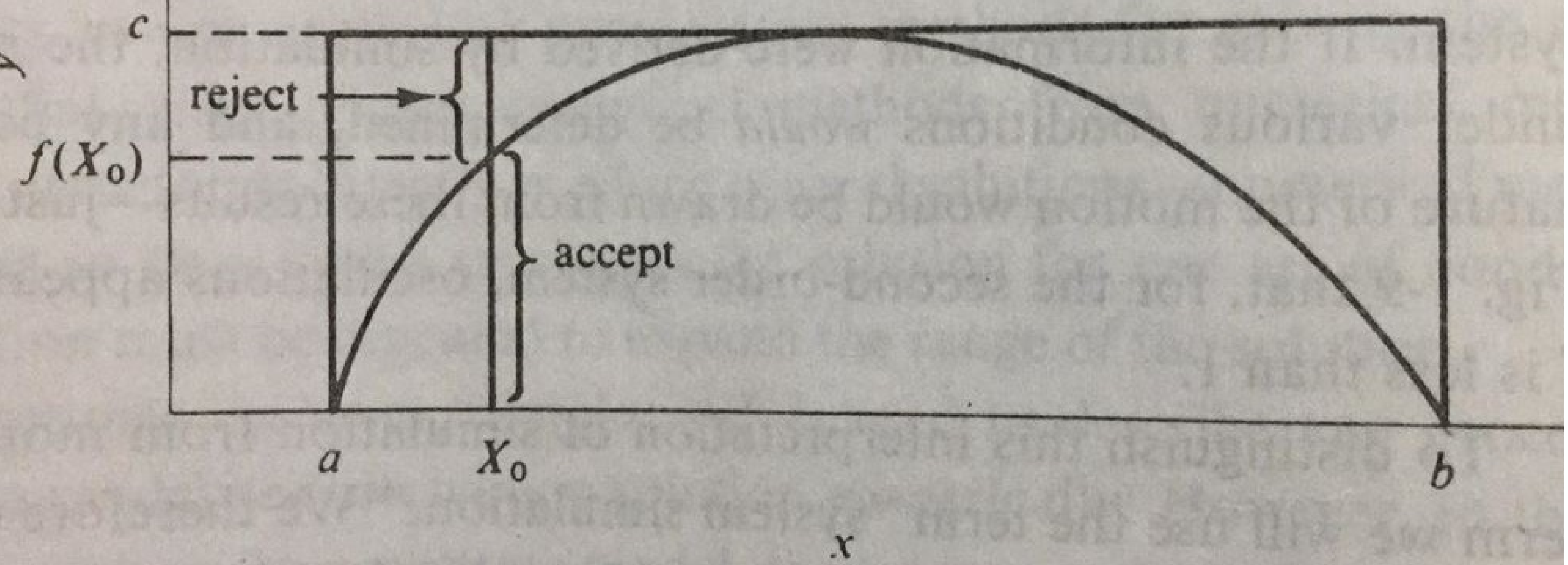


Figure 3-1. The Monte Carlo method.

- If N points are used and n of them fall under the curve then approximately,

$$\frac{n}{N} = \int_a^b \frac{f(x)\,dx}{c(b-a)}$$

- The accuracy improves as the number N increases. When it is decided that sufficient points have been taken the value of the integral is estimated by multiplying n/N by the area of the rectangle. C(b-a).

Comparison of Simulation and Analytical Methods

- Simulation gives specification solution than general solution.
- An analytical solution gives all the conditions.
- Eg. of automobile wheel motion: analytical gives all the condition that can oscillation but simulation tells only whether a particular set of condition did or did not cause oscillation.
- To find all such condition simulation must be repeated under many different conditions.

- To judge which method to apply, it is necessary to consider carefully what questions need to be answered in the system study and to what degree of accuracy the answer need to be known.
- The amount of details in the model need to be limited to minimum level in simulation, as level increases to maximum the amount of computation increases vary rapidly.
- The ideal way of using simulation is an extension of mathematical solutions.

- The limitation in mathematical model like physical stops, finite time delay, non-linear forces, etc. are easily removed by simulation methods.
- Simulation method provides a quicker or more convenient way of deriving results than analytical methods.
- It is more convenient to use simulation to use simulation to obtain results directly from a model with specific values rather than to perform the numerical evaluation of the analytical solutions.

Simulation Technique Method	Analytical Technique Method
Simulation gives specific solutions rather than general solutions. Each execution of simulation tells only whether a particular set of conditions did or did not meet the goal.	Where, analytical solution gives general solution.
Many simulation runs may be needed to find a maximum.	Mathematical solution is preferable, when the solution being sought is maximizing condition.
Various types of complex problems can be solved through simulation.	The range of problems that can be solved mathematically is limited. Mathematical techniques require that the model be expressed in some particular format.
There need a little abstraction for no abstraction to apply simulation methods.	Sometimes, the degree of abstraction required to apply analytical method is too severe. It reduces the degree of accuracy.
The ideal way of using simulation is an execution of mathematical solutions that might have been obtained at the cost of too much simplification.	Sometimes, it needs too much simplification to form a model for analytical solution.
Simulation easily removes many limitations on a system, such as physical stop, finite time delays, nonlinear forces, etc.	These limitations make solvable mathematical model in solution.
Simulation will provide a quicker or more convenient way of deriving results.	Many analytical results occur in the form of complex series or integrals that require extension evolution.

Experimental nature of Simulation

- Simulation is essentially an experimental problem solving technique. Many simulation runs have to be maid to understand the relationships involved in the system.

- So, the use of simulation in a study must be planned as a series of experiments.

Types of System Simulation

- We differ between continuous and discrete systems.
- The distinction between continuous and discrete model was not made in the classification of models, because the difference does not determine whether analytical or numerical techniques will be applied to the models.
- The distinction between continuous and discrete model becomes becomes important when it is decided to use simulation particularly when the simulation is to be carried out on a computer.
- The general calculation or executing techniques used with the two kinds of model differ significantly.

Distributed Lag Model

- Models that have the property of changing only at fixed interval of time and based on current values of variables on other current values of variables are called distributed lag model.
- In economic studies some economic data are collected over uniform time interval such as a month or year.
- This model consists of linear algebraic equations that represent continuous system but data are available at fixed points in time.

- **For example: Mathematical model of national economy**
- Let
  C=consumption
- I=investment
- T=Taxes
- G=government expenditures
- Y=national income
- $C=20+0.7(Y-T)$
  $I=2+0.1Y$
- $T=0.2Y$
- $Y=C+I+G$

- All the equation are expressed in billions of rupees.
- This is static model and can be made dynamic by lagging all the variables as follows
- $C=20+0.7(Y_{-1}-T_{-1})$
- $I=2+0.\ 1Y_{-1}$
- $T=0.2Y_{-1}$
- $Y=C_{-1}+I_{-1}+G_{-1}$
- Any variable that can be expressed in the form of its current value and one or more previous value is called lagging variable. And hence this model is given the name distributed lag model. the variable in a previous interval is denoted by attaching –n suffix to the variable. Where -n indicate the nthe interval.

- $I=2+0.1Y_{-1}$
- $Y=45.45+2.27(I+G)$
- $T=0.2Y$
- $C=20+0.7(Y-T)$

2015 - Fall

Q 2 b

$I = 2 + 0.1\, Y_{-1}$

$Y = 45.45 + 2.27(I + G)$

$T = 0.2Y$

$C = 20 + 0.7(Y - T)$

$Y_{-1} = 80$

Find Growth in nation in 5 years

Years	G
1	20
2	25
3	30
4	35
5	40

⇒ Given $Y_{-1} = 80$ For 1^{st} year,

$I = 2 + (0.1 * 80) = 10$

$Y = 45.45 + 2.27(10 + 20) = 113.55$

$T = 0.2 \cdot Y = 0.2 * 113.55 = 22.71$

$C = 20 + 0.7(113.55 - 22.71) = 83.588$

For 2^{nd} year,

New $Y_{-1} = 113.55$

$I = 2 + (0.1 * 113.55) = 13.355$

$Y = 45.45 + 2.27(13.355 + 25) = 132.51$

$T = 0.2 * (132.51) = 26.503$

$C = 20 + 0.7(132.51 - 26.503) = 94.204$

For 3^{rd} year,

New $Y_{-1} = 132.51$

Cob web model

$Q = a - bP$

$S = c + dP_{-1}$

$Q = S$

Do numerical

Steps of simulation Study

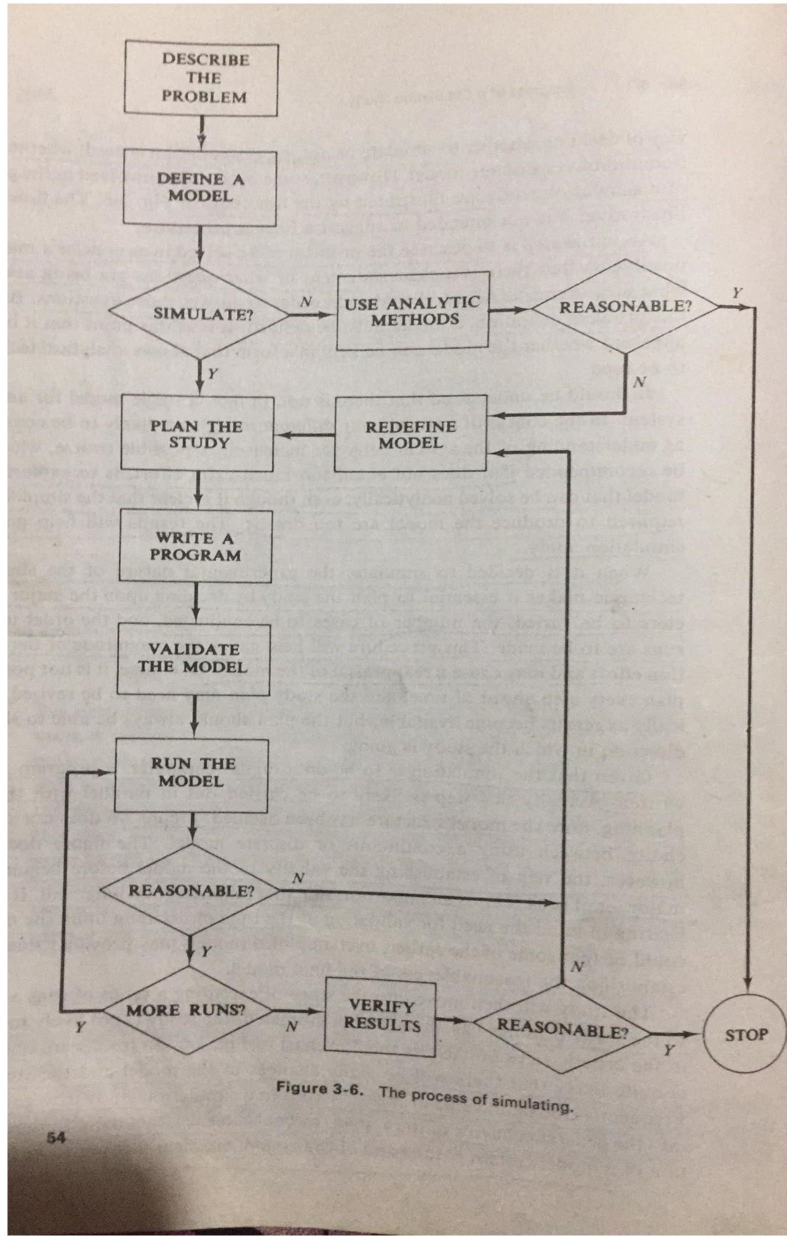


Figure 3-6. The process of simulating.

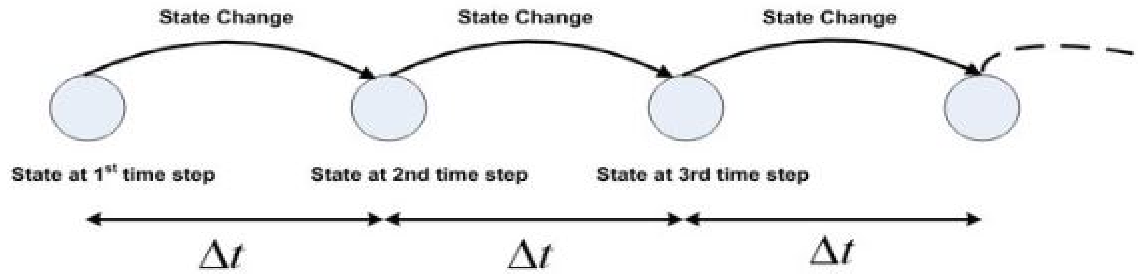
- An initial step is to describe the problem to be solved in as concise a manner as possible so that there is a clear statement of what questions are being asked and what measurement need to be taken in order to answer those questions.
- Based on this problem definition a model must be defined.
- It should be understood that there is not , in fact, a single model for any given system.
- When it is decided to simulate, the experimental nature of the simulation technique makes it essential to plan the study by deciding upon the major parameter to be varied.

- Given the simulation is to be on a digital computer, a program must be written.
- The model should be validate before beginning the major sets of runs.
- The study will then move into the stage of executing a series of runs according to the study plan.
- A major factor that need to be considered in the presence of random variables is the problem of statistically verifying the results.
- It is thus essential to repeat the run with different sets of random numbers so that more then one sample is available.

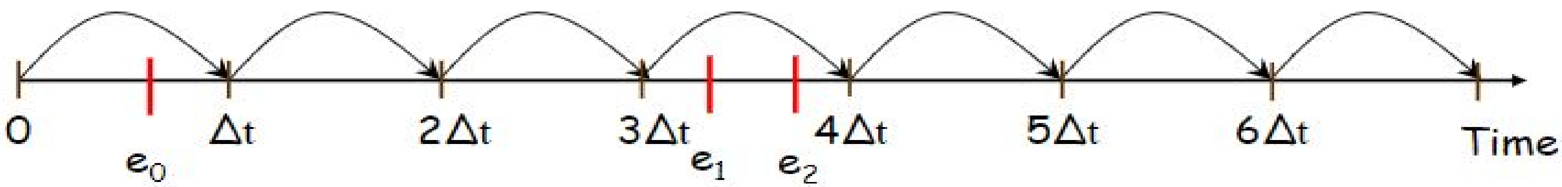
Time advancement Mechanism

- Next event time advance is a classification of time advances mechanism and formally called discrete event simulation (DES) model.
- Time advances mechanism is a science of dynamic system which depicts the time dependence of a point and it can be studied by two categories;
- the first one is discrete event simulation (DES) model (next event time) and
- the second one is discrete time simulation (DTS) model (time-step).
- Models under time advances mechanism are dynamic and facilitate stimulates time for the values.

- Fixed time increment Method:
- Also called time oriented Simulation.
- Under this approach stimulation clock advances a specified unit of time t for representing exact increment.
- While up gradation in the further process in stimulation clock, it examines the event list to identify the possible occurrence of any event in past period of time t.



Where, st_i is state transition i



- Events occur at a fixed increment
- Events occurring between time increments must be moved to an increment boundary
- Simple to implement, but not an accurate realization of occurrence of events

- Next event time advances:
- Also called event oriented simulation.
- Timer is advanced from event to event.
- State of the system is updated at each events.
- Next event time advances mechanism estimates the time of futuristic events that are going to happen on the basis of a list of events (in terms of arrival state or departure state).
- Under this approach, the mechanism is started along with locating the stimulation clock at zero.

- Next-event time advance for the single-server queue

  t_i = time of arrival of ith customer ($t_0 = 0$)

  $A_i = t_i - t_{i-1}$ = interarrival time between (i-1)st and ith customers (usually assumed to be a random variable from some probability distribution)

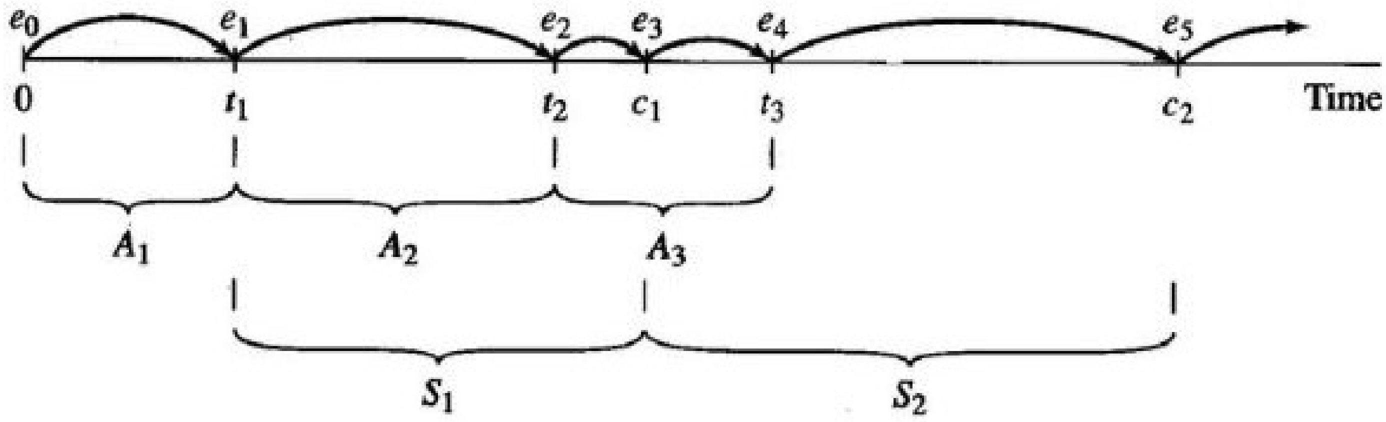
  S_i = service-time requirement of ith customer (another random variable)

  D_i = delay in queue of ith customer

  $C_i = t_i + D_i + S_i$ = time ith customer completes service and departs

  e_j = time of occurrence of the jth event (of any type), $j = 1, 2, 3, \ldots$

  – Possible trace of events (detailed narrative in text)



- Now Queuing system