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8 queen problem.

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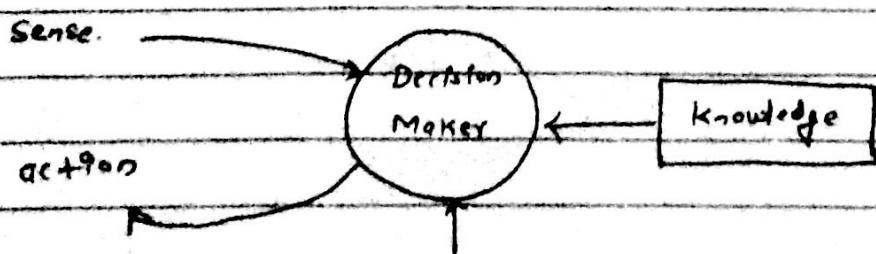
4 queen problems.

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9 queen problems

for \rightarrow goose \rightarrow grain

Knowledge Representation



Chapter : 2

Intelligence:

Ability of a system to calculate, reason, perceive relationships.

Types

- Linguistic
- Musical
- Logical - Mathematical
- Spatial
- Bodily - Kinesthetic
- ~~Intra~~-personal
- Inter-personal.

propositional logic :-

It is the simpler form of a logic. Proposition is a statement. for example, Ram is intelligent, Ram is hardworking, If Ram is intelligent and hardworking then Ram scores high marks.

intelligent(Ram)

hardworking(Ram)

Syntax :-

let, P stands for intelligent(Ram)

q stands for hardworking(Ram)

P AND q are called compound proposition.

Syntactic elements of propositional logic

• Vocabulary :-

It is a set of propositional symbols (p, q, r, etc.) which can be true or false.

- Set of logical operators:-

$\text{AND}(\wedge)$, $\text{OR}(\vee)$, $\text{NOT}(\neg)$, $\text{implies}(\rightarrow)$

- Parenthesis :-

Parenthesis is used for grouping.

Two special symbols :- T and F.

How to form propositional Sentence ?

Each symbol (a proposition or a constant) is a sentence.

If (P) is a sentence and (Q) is a sentence, then

1) $(P), (Q)$ is a sentence

2) $P \wedge Q, P \vee Q$ is a sentence.

3) $\neg P, \neg Q$ is a sentence.

4) $P \rightarrow Q, Q \rightarrow P$ is a sentence.

Sentence is also known as well formed formula (WFF)

Implication

$$P \rightarrow Q$$

if P is true then Q is true.

Bijection or Equivalence

$$P \leftrightarrow Q \quad \text{or} \quad P \Leftrightarrow Q$$

e.g. if two sides of a triangle is equal then ^{two} base angles of the triangle are equal.

$$P \rightarrow Q$$

$$Q \rightarrow P$$

P : two sides of a triangle is equal.

Q : two base angles of a triangle are equal.

then,

$$P \leftrightarrow Q$$

$$P \rightarrow Q$$

$$Q \rightarrow P$$

How do we get the meaning?

- A sentence can be compound by using connective like and, implies or, not, etc.
- Interpret each atomic proposition in the same world.
- Assign truth values to each interpretation.
- Compute the truth value of compound proposition.

p : likes (Sumit, Sunil).

q : knows (Jyoti, Sudhir).

World: Sumit and Sunil are friends and Jyoti and Sudhir are known to each other.

$$p \wedge q = T$$

$$p \wedge (\neg q) = F$$

Validity of a sentence

If a propositional sentence always evaluates to true under all possible interpretation, it is valid.

Tautology

$$p \vee (\neg p)$$

Quiz :-

Express the following English statement in the language of propositional logic

- ① It rains in July.
- ② The book is not costly.
- ③ If it rains today and one does not carry an umbrella, one will be drenched.

- Rains (July)
- \neg Costly (Book)
- Rains (today) \wedge \neg carry (umbrella) $\xrightarrow{\text{Rains}}$ get drenched (~~Rom~~)
 \neg carry (Rom, umbrella).

Quiz 2 :-

If P is true and Q is true, then are the following sentences true or false

1. $P \rightarrow Q$ T.
2. $(\neg P) \vee Q \rightarrow Q$. T
3. $(\neg P) \vee \neg Q \rightarrow P$. \blacksquare T
4. $P \vee (\neg P) \rightarrow T$ T

Truth Table :-

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

De-morgan's Theorem

$$\neg(P \wedge Q) = \neg P \vee \neg Q$$

$$\neg(P \vee Q) = \neg P \wedge \neg Q$$

$$\textcircled{E} \quad \neg(P \wedge Q) = \neg P \vee \neg Q$$

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg P \vee \neg Q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	F	T

$$\neg(\neg P \vee \neg Q) = \neg\neg P \wedge \neg\neg Q$$

P	Q	$\neg(\neg P \vee \neg Q)$	$\neg(\neg P \vee \neg Q)$	$\neg\neg P$	$\neg\neg Q$	$\neg\neg P \wedge \neg\neg Q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Reasoning in terms of propositional logic

P: It is the month of July.

Q: It rains.

R: $P \rightarrow Q$ [Rule]

If it is the month of July then it rains.

Given,

It is the month of July conclude it rains.

Modus Ponens (Inference Rule)

$$\frac{P \rightarrow Q, P}{Q}$$

$$P \rightarrow Q \Rightarrow \neg(\neg P \vee \neg Q)$$

$$P \rightarrow Q \Rightarrow \neg(\neg P \vee \neg Q)$$

(conjoining with P)

$$P \wedge (\neg(\neg P \vee \neg Q))$$

$$(P \wedge \neg(\neg P \vee \neg Q))$$

$$F \vee Q$$

$$Q //$$

Modus ponens is an inference rule that allows us to deduce the truth of a consequent depending on the truth of the

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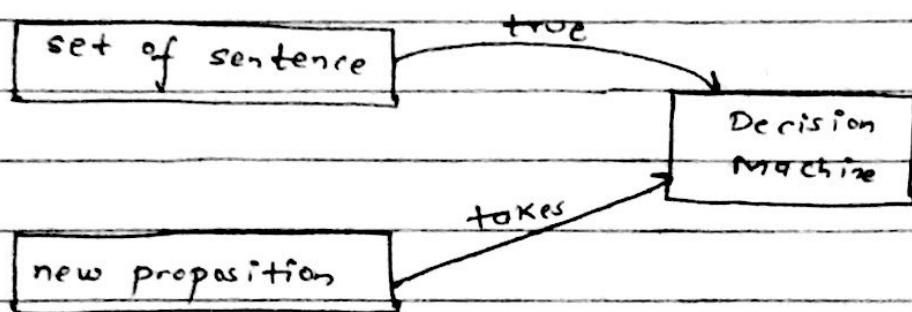
antecedent.

Satisfiability

A sentence is satisfiable by an interpretation if under that interpretation the sentence evaluates to true. If no interpretation makes all the sentences in the set to be true then the set of sentences is unsatisfiable or inconsistent.

Entailment (\models)

$P \models Q$, Q logically follows P.



Resolution

Clause :-

Clause is a special form of a sentence. Basically, ~~and~~ ^{not} and or are only used. It is a disjunction of literals.

morgan's law:-

$$(A \wedge B) \vee (\neg C \wedge A)$$

II) convert to conjunctive normal form by using distributive and associative laws.

$$= (A \vee (\neg C \wedge A)) \wedge (\neg B \vee \neg C \wedge A)$$

$$= (A \vee \neg C) \wedge (A \vee \neg B) \wedge (A \vee \neg C \wedge \neg B)$$

III) get set of clauses.

$$(A \vee \neg C) \quad (\neg B \vee \neg C \wedge A)$$

Clausal form allows us to use Resolution.

Resolution by Refutation

Procedure

Procedure for Resolution :-

I) Convert the given proposition into clausal form.

II) Convert the negation of sentence to be proved into clausal form.

III) combine the clause into set.

IV) Iteratively apply resolution to the set and add resolved resolvent to the set.

V) Continue until no further resolvents can be obtained or a null clause is obtained.

VI)

Q. One mammal ~~drinks~~ ^{drinks} milk

1. Mammals drinks milk.

2. Man is mortal

3. Man is a mammal.

4. Tom is a Man.

prove :- A) Tom drinks milk.

Q) Tom is mortal.

A: Mammals drink milk

B: Man is mortal

C: Man is mammal

D: Tom is a man.

$\text{mammal}(\text{Tom}) \rightarrow \text{drinks}(\text{Tom}, \text{milk})$ P

$\text{Man}(\text{Tom}) \rightarrow \text{mortal}(\text{Tom})$ Q

$\text{Man}(\text{Tom}) \rightarrow \text{mammal}(\text{Tom})$. R

$\text{man}(\text{Tom})$ S.

$\text{mammal}(\text{Tom}) \rightarrow \text{drinks}(\text{Tom}, \text{milk})$

conjoining with $\text{mammal}(\text{Tom})$

$\text{mammal}(\text{Tom}) \wedge (\neg \text{mammal}(\text{Tom}) \vee \text{drinks}(\text{Tom}, \text{milk}))$

$((\text{mammal}(\text{Tom}) \wedge \neg \text{mammal}(\text{Tom})) \vee \text{drinks}(\text{Tom}, \text{milk}))$

$\text{drinks}(\text{Tom}, \text{milk})$.

$\text{man}(\text{Tom}) \rightarrow \text{mortal}(\text{Tom})$

conjoining with $\text{man}(\text{Tom})$

$\text{man}(\text{Tom}) \wedge (\neg \text{man}(\text{Tom}) \vee \neg \text{mortal}(\text{Tom}))$

$((\text{man}(\text{Tom}) \wedge \neg \text{man}(\text{Tom})) \vee \text{mortal}(\text{Tom}))$

$\text{mortal}(\text{Tom})$

$\neg \text{mortal}(\text{Tom}) \rightarrow \text{drinks}(\text{Mammal}, \text{milk})$

-P

$\neg \text{Man}(\text{Tom}) \rightarrow \text{mortal}(\text{Man}) \Rightarrow \neg \text{Man}(\text{Tom}) \rightarrow \text{mortal}(\text{Tom})$. -Q

$\neg \text{Man}(\text{Tom}) \rightarrow \text{mammal}(\text{Man}) \Rightarrow \neg \text{Man}(\text{Tom}) \rightarrow \text{mammal}(\text{Tom})$. -R

$\neg \text{Man}(\text{Tom})$.

-S

① Applying modus ponens on R and S, we get

$\text{mammal}(\text{Tom})$ -S

② Applying modus ponens on S and P.

$\text{drinks}(\text{Tom}, \text{milk})$.

Applying modus ponens on Q and S
 $\text{mortal}(\text{Tom})$.

Converting to clausal form,
 $\text{mammal}(\text{Tom}) \rightarrow \text{drinks}(\text{Tom}, \text{milk})$

(1) Eliminate implication sign

$\neg \text{mammal}(\text{Tom}) \vee \text{drinks}(\text{Tom}, \text{milk})$

$\neg \text{man}(\text{Tom}) \vee \text{mortal}(\text{Tom})$

$\neg \text{man}(\text{Tom}) \vee \text{mammal}(\text{Tom})$

$\text{man}(\text{Tom})$

Q. Number of propositions

1. If a triangle is equilateral then it is isosceles.

2. If a triangle is isosceles, then two sides AB and AC are equal.

3. If AB and AC is equal, then angle B and angle C are equal.

4. Triangle ABC is an equilateral triangle.

prove :-

$\angle B = \angle C$.

Solution:-

Steps:-

I) Converting the given propositions into clausal form as follows

equilateral (Triangle) \rightarrow isosceles (Triangle) - ①

isosceles (Triangle) \rightarrow equal (Triangle, two sides AB and AC)

equal (Triangle, two sides AB and AC) \rightarrow equal (Triangle, two angles B and C)

equilateral (Triangle ABC) - ④

- II) Applying modus ponens on ① and ④, we get
isoreles (Triangle) — ⑤
- III) Applying modus ponens on ⑤ and ②
equal (Triangle, two sides AB and BC) — ⑥
- IV) Applying modus ponens on ③ and ⑥
equal (Triangle, two angles B and C).

Hence, we get

Two angles B and C are equal
i.e. $\angle B = \angle C$.

- ① equilateral ($\triangle ABC$) \longrightarrow isoreles ($\triangle ABC$)
- ② isoreles ($\triangle ABC$) \longrightarrow equal (AB, AC)
- ③ equal (AB, AC) \longrightarrow equal ($\angle B, \angle C$)
- ④ equilateral ($\triangle ABC$)
- ⑤ equal ($\angle B, \angle C$) — prove.

Converting these sentences into clausal form.

$\neg \text{equilateral}(\triangle ABC) \vee \text{isoreles}(\triangle ABC)$

$\neg \text{isoreles}(\triangle ABC) \vee \text{equal}(AB, AC)$

$\neg \text{equal}(AB, AC) \vee \text{equal}(\angle B, \angle C)$

$\text{equilateral}(\triangle ABC)$

-②

-③ Let. us disprove ⑤

$\neg \text{equal}(AB, AC)$

$\neg \text{equal}(\Delta AB, \Delta AC) \quad \text{--- } ⑦$

from ⑦ and ②

$\neg \text{equal}(\Delta AB, \Delta AC) \vee \neg \text{isoreles}(\Delta ABC) \vee \text{equal}(\Delta AB, \Delta AC)$
 $\neg \text{isoreles}(\Delta ABC) \quad \text{--- } ⑧$

from ⑧ and ①

$\neg \text{isoreles}(\Delta ABC) \vee \neg \text{equilateral}(\Delta ABC) \vee \text{isoreles}(\Delta ABC)$
 $\neg \text{equilateral}(\Delta ABC) \quad \text{--- } ⑨$

from ⑨ and ④

$\neg \text{equilateral}(\Delta ABC) \vee \text{equilateral}(\Delta ABC)$
 $\emptyset [\text{null}]$.

Since, we arrived at the contradictory situation i.e. null clause. Therefore, our assumption $\neg \text{equal}(\Delta B, \Delta C)$ is false. Therefore, the given statement $\text{equal}(\Delta B, \Delta C)$ is true.

- ①. Mammal (Tom) \rightarrow drinks (Tom, milk)
- ②. Men (Tom) \rightarrow Mortal (Tom).
- ③. Men (Tom) \rightarrow Mammal (Tom)
- ④. Men (Tom).

we prove

Converting these into classical form.

- ①. $\neg \text{Mammal}(\text{Tom}) \vee \text{drinks}(\text{Tom}, \text{milk})$
- ②. $\neg \text{Men}(\text{Tom}) \vee \text{Mortal}(\text{Tom})$
- ③. $\neg \text{Men}(\text{Tom}) \vee \text{Mammal}(\text{Tom})$
- ④. $\text{Men}(\text{Tom})$
- ⑤. $\text{drinks}(\text{Tom}, \text{milk})$

we prove: ~~$\neg \text{drinks}(\text{Tom}, \text{milk})$~~ — ⑤

from ⑤ and ①

$\neg \text{Mammal}(\text{Tom}) \vee \text{drinks}(\text{Tom}, \text{milk}) \vee \neg \text{drinks}(\text{Tom}, \text{milk})$

$\neg \text{Mammal}(\text{Tom})$ — ⑥

from ② and ③

$\neg \text{Men}(\text{Tom}) \vee \text{Mammal}(\text{Tom}) \vee \neg \text{Mammal}(\text{Tom})$

$\neg \text{Men}(\text{Tom})$ — ⑦

from ⑥ and ⑦

$\neg \text{Men}(\text{Tom}) \vee \neg \text{Mammal}(\text{Tom})$

0 [contradiction]

Since, we arrived at the contradictory situation i.e. null clauses. Therefore our assumption $\text{drinks}(\text{Tom}, \text{milk})$ is false. Therefore given statement $\text{drinks}(\text{Tom}, \text{milk})$ is true.

First order Logic / First order predicate Logic (FOL or FOPL)

Limitations of Propositional Logic :

* Consider following arguments

A) All dogs are faithful.

B) Tommy is a dog.

Therefore, Tommy is faithful.

Suppose,

P : all dogs are faithful.

Q : Tommy is a dog.

$P \wedge Q \Rightarrow$ Tommy is faithful ?

We can't infer such in a propositional logic.

* Tom is hardworking.

hardworking (Tom).

Tom is an intelligent student.

intelligent (Tom)

If Tom is hardworking and Tom is an intelligent student,
then Tom scores high marks.

hardworking (Tom) \wedge intelligent (Tom) $\xrightarrow{\text{scores}}$ highmarks (Tom) scored

Converting to the classical form.

$\neg(\text{hardworking}(Tom) \wedge \text{intelligent}(Tom)) \vee \text{scores highmarks}(Tom)$.

$\neg(\text{hardworking}(Tom)) \vee \neg(\text{intelligent}(Tom)) \vee \text{scores highmarks}(Tom)$.

We cannot depict these into so such relationship for
plurality of instances. But, we could write instead :-

All students who are hardworking and intelligent scores high marks.

For, All x such that, x is a student and x is intelligent and x is hardworking then x scores highmarks.

Therefore, FOL is generalization of propositional logic.

The problem of infinite model of propositional logic is addressed by predicate logic.

Syntax of FOL

FOL can be defined in terms of :- A & B

- 1) Terms
- 2) Predicates
- 3) Quantifiers

1) Terms :-

It denotes some objects other than true or false (constants). e.g. Tommy is a dog. All men are mortal.

Terms can be constants or variables.

- * Constants of type W_1 is a name that denotes a particular objects in set W_1 . e.g. 5, Tommy ; etc.
- * Variable of type W_1 is a name that denotes any element in the set W_1 . e.g. $X \in N$. N is a set of Natural numbers.

$X \in D$, D belongs to set of dogs.

Function :- A functional term of arity n takes an object of type W_1, W_2, \dots, W_n as input and returns an object W_1 . e.g. $F(W_1, W_2, \dots, W_n)$.

Sum(5, 6).

2) Predicates :-

Pred Like function but return type is either true or false. e.g. greater than (X, Y), is true \Leftrightarrow ($X > Y$ if and only if)

iff $x > y$

Types of predicates :-

- 1) A predicate with no variable is a proposition. e.g. Tommy is a dog.
- 2) A predicate with one variable is called property. e.g. X is a dog.

$\text{dog}(x)$, iff x is dog.
 \downarrow

Formulation of Predicates.

Let, $P(x_1, y_1, \dots)$ and $Q(x_2, y_2, \dots)$ be two predicates, then
 $P \vee Q$, $P \wedge Q$, $\neg P$, $\neg Q$, $P \rightarrow Q$, $P \leftrightarrow Q$ are valid predicates.

Predicate examples:-

If Man is x . then x is Mortal.

$$\text{Man}(x) \rightarrow \text{Mortal}(x)$$

$$\neg \text{Man}(x) \vee \text{Mortal}(x)$$

If n is a natural number, then n is either odd or even.

$$\text{natural number}(n) \rightarrow \text{even}(n) \vee \text{odd}(n)$$

$$\neg \text{natural number}(n) \vee \text{even}(n) \vee \text{odd}(n)$$

Quantifier

It is to represent the number. There are two types of quantifier.

1) Universal quantifier (\forall)

Read as for all.

2) Existential quantifier (\exists)

Read as there exists.

example :-

x is a set of days of a week.

$\exists x : \text{day } x \text{ is days of a week}$.

Universal Quantifier :-

* All dogs are faithful.

$\forall x : \text{dog}(x) \rightarrow \text{faithful}(x)$.

* All mammal drink milk

$\forall x : \text{mammal}(x) \rightarrow \cancel{\text{drink milk}}(x), \text{drink}(x, \text{milk})$

* All man are mortal

$\forall x : \text{man}(x) \rightarrow \text{mortal}(x)$

* All birds cannot fly.

$\neg (\forall x : \text{bird}(x) \rightarrow \text{fly}(x))$

* All man are mammal.

$\forall x : \text{man}(x) \rightarrow \text{mammal}(x)$.

Existential Quantifier :-

Represents some numbers in the set.

* At least one planet has a life in it.

$\exists x : (\text{planet}(x) \wedge \text{has life}(x))$

* There exists a bird, that can't fly.

$$\exists x : (\text{bird}(x) \wedge \neg \text{fly}(x))$$

Duality of a Quantifier

All men are mortal \Leftrightarrow No man is immortal.

$$\forall x : \text{man}(x) \rightarrow \text{mortal}(x) \Leftrightarrow \exists x (\text{man}(x) \wedge \neg \text{mortal}(x))$$

Sentences

A predicate is a sentence.

If P and $\neg P$ are sentences and x is a variable then
 $P, \neg P, \exists x(P), P \wedge \neg P, P \vee \neg P, P \rightarrow \neg P$ are sentences.

Quiz

- 1) Some dog bark.
- 2) All dogs have four legs.
- 3) All barking dogs are irritating.
- 4) No dogs roar.
- 5) Fathers are male parents with children.
- 6) Students are people who are enrolled in a course.
- 7) Markers was a man.
- 8) Markus was a Pompeian.
- 9) All pompeian were Roman.
- 10) Caesar was a Ruler.
- 11) All Roman were either loyal to Coeser or hated him.
- 12) Everyone is loyal to someone.
- 13) People only try to assassinate rulers they are not loyal to.
- 14) Markus tried to assassinate Caesar.
- 15) All men are people.
- 16) There is a person who loves everyone in the world.

- 17) Everyone in this world is loved by at least one person.
 18) John likes all kind of food.
 19) Stephen likes easy courses.
 20) Everybody loves somebody.
 21) Abraham is the father of Isaac.
 22) John gave the book to Cena.
 23) Science courses are hard.
 24) Anyone passing his history exam and winning the lottery is happy.
 25) Anyone who studies or is lucky can pass all his exams.
 26) John did not study but he is lucky.
 27) Anyone who is lucky wins the lottery.
 28) Apples are food.
 29) Chicken is food.
 30) Anything anyone eats and is not killed by is food.
 31) Bill eats peanuts and is still alive.
 32) Sue eats everything Bill eats.

- 1) $\exists x : (\text{dog}(x) \wedge \text{bark}(x))$
 2) $\forall x : (\text{dogs}(x) \rightarrow \text{fourlegs}(x))$
 3) $\forall x : (\overset{\text{dog}(x) \wedge \text{bark}(x)}{\text{barking dogs}(x)} \rightarrow \text{irritating}(x))$
 4) $[\exists x : \text{dog}(x) \wedge \text{purr}(x)] / \forall x : \text{dog}(x) \rightarrow \text{purr}(x)$
 5) $\forall x : (\text{father}(x) \rightarrow \text{male}(x) \wedge \text{has_children}(x))$
 6) $\exists x : (\text{people}(x) \wedge \text{enrolled_in_course}(x) \rightarrow \text{student}(x))$
 7) $\text{Markus}(x) \rightarrow \text{Mark}(x) \text{ Max(Markus)}$
 8) $\text{Pompeii}(Markus)$
 9) $\text{Ruler}(\text{Caesar}) \quad \forall x : \text{Roman}(x) \rightarrow \text{Pompeian}$
 10) $\text{ruler}(\text{Caesar})$
 11) $\forall x : \text{Romans}(x) \rightarrow \text{Loyal_to}(x, \text{Caesar}) \vee \text{Hate}(x, \text{Caesar})$
 12) $\forall x \exists y : \text{loyal}(x, y)$

- 17) Everyone in this world is loved by at least one person.
 18) John likes all kind of food.
 19) Stephen likes easy courses.
 20) Everybody loves somebody.
 21) Abram is the father of Isaac.
 22) John gave the book to Cena.
 23) Science courses are hard.
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 30) Anything anyone eats and is not killed by is food.
 31) Bill eats peanuts and is still alive.
 32) Sue eats everything Bill eats.

- 1) $\exists x : (\text{dog}(x) \wedge \text{bark}(x))$
- 2) $\forall x : (\text{dogs}(x) \rightarrow \text{four legs}(x))$
- 3) $\forall x : (\text{barking dogs}(x) \rightarrow \text{irritating}(x))$
- 4) $[\exists x : \text{dog}(x) \wedge \text{furry}(x)] / \forall x : \text{dog}(x) \rightarrow \text{furry}(x)$
- 5) $\forall x : (\text{father}(x) \rightarrow \text{male}(x) \wedge \text{has children}(x))$
- 6) $\exists x : (\text{people}(x) \wedge \text{enrolled in course}(x) \rightarrow \text{student}(x))$
- 7) $\text{Markus}(x) \rightarrow \text{Mark} \text{ and } \text{Markus}(x)$
- 8) $\text{Pompeian}(Markus)$
- 9) ~~Ruler(Caesar)~~ $\forall x : \text{Ruler}(x) \rightarrow \text{Pompeian}$
- 10) $\forall x : \text{Pompeian}(x) \rightarrow \text{Roman}(x)$.
- 11) $\forall x : \text{Ruler}(x) \rightarrow \text{Loyal-to}(x, \text{Caesar}) \vee \text{Hate}(x, \text{Caesar})$
- 12) $\forall x \forall y : \text{loyal}(x, y)$

13)

14) Tryassinate (Merius, Caesar).

15) $\forall x : \text{man}(x) \rightarrow \text{people}(x)$.16) $\exists x \forall y : \text{person}(x) \rightarrow \text{loves everyone loves}(x, y)$.17) $\forall x \exists y : \exists x \forall y : \text{loves}(x, y)$ 18) $\forall x \exists y : \text{Loves}(x, y)$ 19) $\forall x : \text{food}(x) \rightarrow \text{likes}(\text{John}, x)$.20) $\exists x : \text{easy course}(x) \rightarrow \text{likes}(\text{Steve}, x)$

$\text{course}(x) \rightarrow \text{likes}(\text{Steve}, x) \wedge \text{easy}(x)$
 $\text{course}(\text{science}) \rightarrow \text{hard}(\text{science})$
 $\text{course}(x) \wedge \text{easy}(x) \rightarrow \text{likes}(\text{Steve}, x)$

21) Father (Abraham, Isaac).

22) gave book to (John, Cena).

23) $\text{course}(\text{science}) \rightarrow \text{hard}(\text{science})$ 24) $\forall x : (\text{pass}(x, \text{history}) \wedge \text{win}(x, \text{lottery})) \rightarrow \text{happy}(x)$ 25) $\exists x \text{study}(\text{John}) \wedge \text{lucky}(\text{John})$. $\forall x \forall y \text{study}(x) \vee \text{lucky}(x) \rightarrow \text{pass}(x, y)$ 26) $\exists x \text{study}(\text{John}) \wedge \text{lucky}(\text{John})$ 27) $\forall x \text{lucky}(x) \rightarrow \text{win}(x, \text{lottery})$

28) food (Apple)

29) food (chicken)

30)

Resolution in first order predicate logic

All people who are graduating are happy.

All happy people smile.

Someone is graduating.

prove :

Is someone smiling?

Rules

Encoding / Converting into predicate logic.

$\forall x : \text{graduating}(x) \rightarrow \text{happy}(x)$

$\forall x : \text{happy}(x) \rightarrow \text{smile}(x)$.

$\exists x : \text{graduating}(x)$

To prove:-

$\exists x : \text{smile}(x)$

Eliminate implication operator.

$\forall x : \neg \text{graduating}(x) \vee \text{happy}(x)$

$\forall x : \neg \text{happy}(x) \vee \text{smile}(x)$

$\exists x : \text{graduating}(x)$

$\exists x : \text{smile}(x)$

(Convert it into Canonical or normal form / Reduce scope of negation).

$\forall x : \neg \text{graduating}(x) \vee \text{happy}(x)$

$\forall x : \neg \text{happy}(x) \vee \text{smile}(x)$

$\exists x : \text{graduating}(x)$

$\exists x : \neg \text{smile}(x) \quad \forall x : \neg \text{smile}(x)$

Standardize variable apart.

$\forall z \text{ graduating}(z) \vee \text{happy}(z)$

$\forall y \text{ : } \exists z \text{ happy}(y) \vee \text{smile}(y)$

$\exists z \text{ : } \text{graduating}(z)$

$\exists w \text{ : } \exists z \text{ smile}(w)$

Skolemization (remove existential quantifier)

The process of finding constant or function for existentially quantified variables that will make the clause to be true.

$\forall x \text{ : } \exists z \text{ graduating}(x) \vee \text{happy}(x)$

$\forall y \text{ : } \exists z \text{ happy}(y) \vee \text{smile}(y)$

$\text{graduating}(\text{name}_1)$

$\exists w \text{ : } \exists z \text{ smile}(w)$

5) drop universal quantifier.

$\exists z \text{ graduating}(z) \vee \text{happy}(z)$

$\exists y \text{ happy}(y) \vee \text{smile}(y)$

$\text{graduating}(\text{name}_1)$

$\exists z \text{ smile}(z)$

From ④ and ②

$\exists z \text{ smile}(z)$

$\exists y \text{ happy}(y) \vee \text{smile}(y)$

(y/z)

$\exists y \text{ happy}(y)$

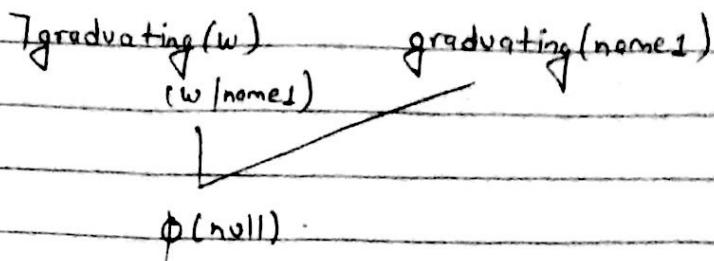
from ①

$\exists y \text{ happy}(y)$

$\exists z \text{ graduating}(z) \vee \text{happy}(z)$

(z/w)

$\exists z \text{ graduating}(z)$



Hence, contradiction of goal is disproved. So, goal is proved.

Q3:

I) >

1. $\forall x : \text{poss}(x, \text{history}) \wedge \text{win}(x, \text{lottery}) \rightarrow \text{happy}(x)$.
2. $\forall x \forall y : \text{study}(x) \vee \text{lucky}(x) \rightarrow \text{poss}(x, y)$.
3. $\neg \text{study}(\text{John}) \wedge \text{lucky}(\text{John})$.
4. $\forall x \text{lucky}(x) \rightarrow \text{win}(x, \text{lottery})$.

To prove:-

5. $\neg \text{happy}(\text{John})$

II) Eliminate implication operator.

- ① $\forall x : \neg (\text{poss}(x, \text{history}) \vee \neg \text{win}(x, \text{lottery})) \vee \text{happy}(x)$
- ② $\forall x \forall y : \neg (\text{study}(x) \vee \text{lucky}(x)) \vee \text{poss}(x, y)$
- ③ $\neg \text{study}(\text{John})$
- ④ $\text{lucky}(\text{John})$
- ⑤ $\forall x : \neg \text{lucky}(x) \vee \text{win}(x, \text{lottery})$.
- ⑥ $\neg \text{happy}(\text{John})$.

III) Reduce scope of negation

- ① $\forall x : \neg \text{poss}(x, \text{history}) \vee \neg \text{win}(x, \text{lottery}) \vee \text{happy}(x)$
- ② $\forall x \forall y : [\neg \text{study}(x) \wedge \neg \text{lucky}(x)] \vee \text{poss}(x, y)$
- ③ $\neg \text{study}(\text{John})$
- ④ $\text{lucky}(\text{John})$
- ⑤ $\forall x : \neg \text{lucky}(x) \vee \text{win}(x, \text{lottery})$
- ⑥ $\neg \text{happy}(\text{John})$.

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$\forall z : \exists x \text{ pass}(x, \text{history}) \vee \exists x \text{ win}(x, \text{lottery}) \vee \text{happy}(x)$

$\forall x \forall y : (\exists x \text{ study}(x) \vee \text{pass}(x, y)) \wedge (\exists x \text{ study}(x) \rightarrow \exists x \text{ lucky}(x) \vee \text{pass}(x, y))$

$\exists x \text{ study}(\text{john})$

$\exists x \text{ lucky}(\text{john})$

$\forall x : \exists x \text{ lucky}(x) \vee \text{win}(x, \text{lottery})$

$\exists x \text{ happy}(\text{john})$

Skelemization:-

$\forall x : \exists x \text{ pass}(x, \text{history}) \vee \exists x \text{ win}(x, \text{lottery}) \vee \text{happy}(x)$

$\forall x \forall y : \exists x \text{ study}(x) \vee \text{pass}(x, y)$

$\forall u \forall v : \exists u \text{ lucky}(u) \vee \text{pass}(u, v)$

$\exists x \text{ study}(\text{john})$

$\exists x \text{ lucky}(\text{john})$

$\forall w : \exists w \text{ lucky}(w) \vee \text{win}(w, \text{lottery})$

$\exists x \text{ happy}(\text{john})$

drop universal quantifier

$\exists x \text{ pass}(x, \text{history}) \vee \exists x \text{ win}(x, \text{lottery}) \vee \text{happy}(x)$

$\exists x \text{ study}(x) \vee \text{pass}(x, y)$

$\exists x \text{ lucky}(x) \vee \text{pass}(u, v)$

$\exists x \text{ study}(\text{john})$

$\exists x \text{ lucky}(\text{john})$

$\exists x \text{ lucky}(x) \vee \text{win}(w, \text{lottery})$

$\exists x \text{ happy}(\text{john})$

from (B) and (C)

from (8) and (6)

$\neg \text{pass}(\text{john}, \text{history}) \vee \neg \text{win}(\text{john}, \text{lottery})$: $\neg \text{lucky}(w) \vee \text{win}(w, \text{lottery})$
(w/john)

$\neg \text{pass}(\text{john}, \text{history}) \vee \neg \text{lucky}(\text{john})$ — (9)

from (9) and (5)

$\neg \text{pass}(\text{john}, \text{history}) \vee \neg (\text{lucky } \text{john})$ $\text{lucky } \text{john}$

$\neg \text{pass}(\text{john}, \text{history})$ — (10)

from (10) and (3)

$\neg \text{pass}(\text{john}, \text{history})$ $\neg \text{lucky}(u) \vee \text{pass}(u, v)$
(u/john , $v/\text{history}$)

$\neg \text{pass}(\text{john}, \text{history})$

$\neg \text{lucky}(\text{john}) \vee \text{pass}(\text{john}, \text{history})$

$\neg \text{lucky}(\text{john})$ — (11)

from (11) and (5)

$\neg \text{lucky}(\text{john})$

$\text{lucky } \text{john}$

ϕ_{CAVILZ}

Hence Contradiction of the goal is disproved. So, goal i.e. john is happy is proved.

Q5)

1. Steve only like easy courses.
2. Science courses are hard.
3. all courses in basket-weaving are easy.
4. BK301 is a basket weaving course.
5. Which course would Steve likes?

(a) $\forall x : \text{easy}(x) \rightarrow \text{likes}(\text{Steve}, x)$

(b) $\forall x : \text{science}(x) \rightarrow \neg \text{easy}(x)$

(c) $\forall x : \text{basket-weaving}(x) \rightarrow \text{easy}(x)$.

(d) basketweaving(BK301).

(e) likes(Steve, x).

(f) Converting to clausal form.

(d) Basketweaving (BK301)

(e) Likes (stone, x).

a, e \rightarrow 7 easy (x) — (f)

f, c \rightarrow 7 BK301 Basketweaving (x) — (g)
(x / BK301)

g, d \rightarrow \emptyset

Q).

A) Ram likes all kind of food.

B) Oranges are food.

C) Rice is a food.

D) Krishna eats popcorn and is still alive.

E) Radha eats anything that Krishna eats.

F) Anything anyone eats and is not killed by is food.

G) Prove : Ram likes popcorn.

I) Converting to clausal form.

A) $\forall x : \text{food}(x) \rightarrow \text{Re}^{\text{eats}}(x, \text{Ram})$

B) food (orange).

C) food (Rice).

D) eats (Krishna, popcorn) \wedge ~~eats (Krishna, popcorn)~~^{7 killed}

E) $\forall x : \text{eats}(\text{Krishna}, x) \rightarrow \forall x : \text{eats}(\text{Krishna}, x) \rightarrow \text{eats}(\text{Radha}, x)$.

F) $\forall x \exists y : (\text{eats}(x, y) \wedge \neg \text{killed}(y)) \rightarrow \text{food}(y)$

G) $\exists x : \text{eats}(\text{Ram}, \text{popcorn})$.

II) Eliminate implication operator.

A) $\forall x : \neg \text{food}(x) \vee \text{eats}(\text{Ram}, x)$.

B) food (orange)

C) food (Rice)

D) $\neg \text{eats}(\text{Krishna}, \text{popcorn}) \wedge \neg \text{killed}(\text{Krishna}), \text{popcorn}$.

E) $\forall x : \neg \text{eats}(\text{Krishna}, x) \vee \text{eats}(\text{Radha}, x)$.

F) $\forall x \exists y [eats(x, y) \wedge \neg killed(y)] \vee food(y)$

G) $\exists eats(Ram, popcorn)$

III) Reduce scope of negation.

A) $\forall x : \exists food(x) \vee eats(Ram, x)$

B) $food(orange)$

C) $food(Rice)$

D) $\exists eats(Krishna, popcorn) \wedge \neg killed(Krishna, popcorn)$

E) $\forall x : \exists eats(Krishna, x) \vee eats(Radha, x)$

F) $\forall x \forall y : \exists eats(x, y) \wedge \neg killed(y) \vee food(y)$

G) $\exists eats(Ram, popcorn)$.

IV) Standardize variable apart.

A) $\forall x : \exists food(x) \vee eats(Ram, x)$

B) $food(orange)$

C) $food(Rice)$

D) $\exists eats(Krishna, popcorn) \wedge \neg killed(Krishna, popcorn)$

E) $\forall y : \exists eats(Krishna, y) \vee eats(Radha, y)$.

F) $\forall u, \forall v : \exists eats(u, v) \wedge \neg killed(u) \vee food(v)$

G) $\exists eats(Ram, popcorn)$.

V) drop universal quantifier.

A) $\exists food(r) \vee eats(Ram, r)$

B) $food(orange)$

C) $food(Rice)$

D) $\exists eats(Krishna, popcorn) \wedge \neg killed(Krishna, popcorn)$

E) $\exists eats(Krishna, y) \vee eats(Radha, y)$.

F) $\exists eats(v, v) \wedge \neg killed(v) \vee food(v)$.

G) $\exists eats(Ram, popcorn)$.

from ④ and ⑤

$$\text{Rati} : 7 \text{ food}(x) \vee \text{eats}(Rami, x) \\ (\neg \exists \text{ popcorn})$$

\swarrow

$$7 \text{ food(popcorn)} \longrightarrow \textcircled{E}$$

from ⑥ and ⑦

$$7 \text{ food(popcorn)}$$

\swarrow

$$7 \text{ eats}(u, v) \vee 7 \text{ killed}(u) \vee \text{food}(v)$$

$\neg \exists \text{ popcorn}$

$$7 \text{ eats}(u, \text{popcorn}) \vee 7 \text{ killed } \text{popcorn} \longrightarrow \textcircled{T}$$

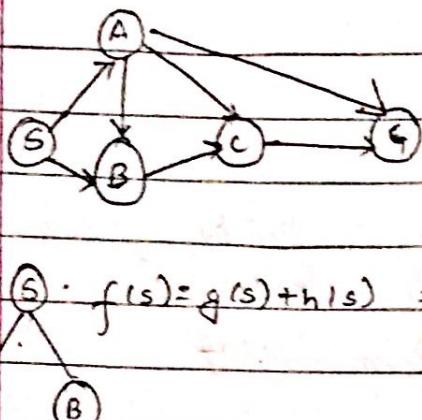
from ⑧ and ⑨

$$7 \text{ eats}(u, \text{popcorn}) \vee 7 \text{ killed } \text{popcorn} \quad 7 \text{ eats}(krishna, v) \vee \text{eats}(v, u)$$
$$7 \text{ eats}(u, \text{popcorn}) \vee 7 \text{ killed } \text{popcorn}$$

\swarrow

$$7 \text{ killed } \text{popcorn} \longrightarrow \textcircled{K}$$

from ⑩ and ⑪

FramesA* Search

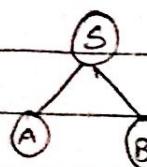
	S	A	B	C	G	state A[7]
S		1	4			S 7
A			2	5	12	A 6
B			.	2		B 2
C				3		C 1
G						G 0

$$f(S) = g(S) + h(S) = 0 + 7 = 7$$

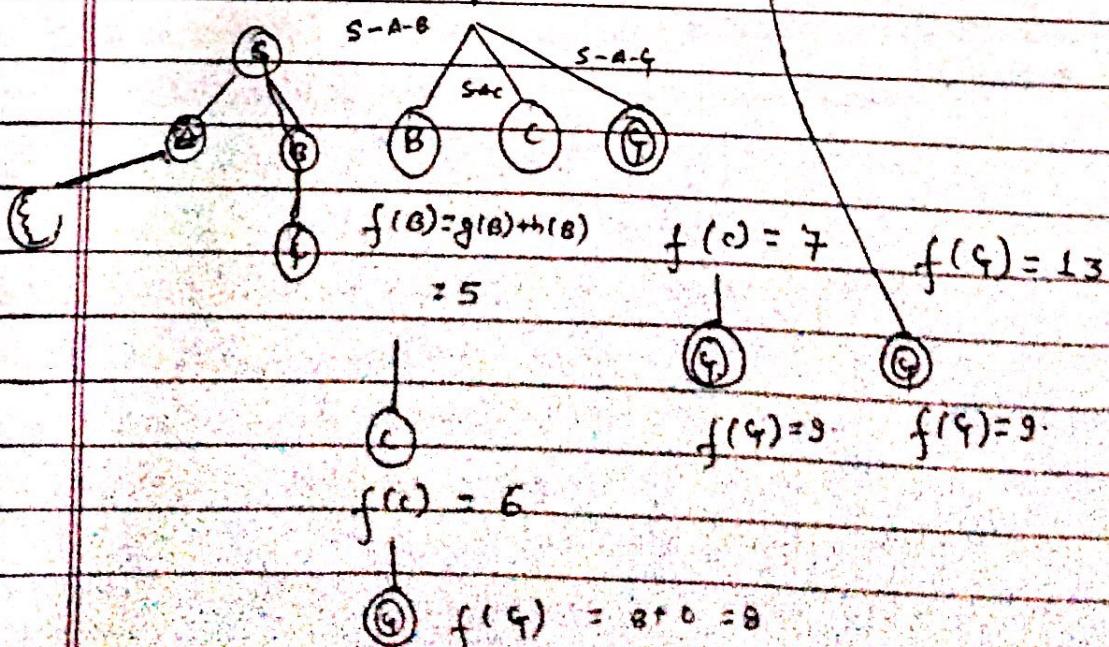
(A) (B)

$$f(A) = g(A) + h(A) = 1 + 6 = 7$$

$$f(B) = g(B) + h(B) = 4 + 9 = 13$$



$$\begin{aligned} f(C) &= g(C) + h(C) \\ &= [(S-B) + (B-C)] + 1 \\ &= 7 \end{aligned}$$

Alphabetical order

In a lattice, using A* Search algorithm we can reach the goal in 4 different ways

for $S - A - B - C - G$

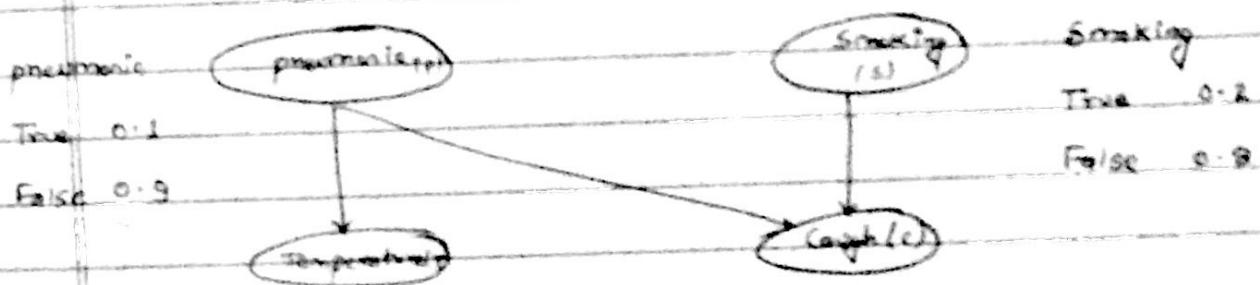
$S - A - C - G$

$S - A - G$

$S - B - C - G$

whose evaluation function ($f(n)$) equals to 9, 9, 13, 9 respectively.

Among these path $S - A - B - C - G$ is the shortest and hence this path is chosen.



Pneumonia	Temperature	Pneumonia	Smoking	Cough	True	False
yes	yes	no	true	yes	True	False
No	0.3	0.1	false	No	0.95	0.05
	0.2	0.8		Yes	0.8	0.2
			False	No	0.6	0.4
					0.05	0.95

$$\textcircled{1} \quad P(c|s \wedge p) = 0.95$$

$$\textcircled{2} \quad P(s|c) = \frac{P(c|s) * P(s)}{P(c)}$$

We have to calculate,

$$\begin{aligned}
 P(c|s) * P(s) &= P(s) * [P(c|s \wedge p) * P(p) + P(c|s \wedge \neg p) * P(\neg p)] \\
 &= 0.2 [0.95 * 0.1 + 0.6 * 0.9] \\
 &= 0.127
 \end{aligned}$$

$$\begin{aligned}
 P(c) &= P(c|p \wedge s) * P(p \wedge s) + P(c|p \wedge \neg s) * P(p \wedge \neg s) + \\
 &\quad P(c|\neg p \wedge s) * P(\neg p \wedge s) + P(c|\neg p \wedge \neg s) * P(\neg p \wedge \neg s) \\
 &= 0.95 * 0.1 * 0.2 + 0.9 * 0.1 * 0.3 + 0.6 * 0.9 * 0.2 \\
 &\quad + 0.05 * 0.9 * 0.8
 \end{aligned}$$

Unit 5 - Machine Learning

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Learning

Types of Learning :-

- 1> Rote Learning
- 2> Learning by Examples.
- 3> Explanation based Learning.
- 4> Learning by taking advice.
- 5> Learning by analogy.
- 6> Inductive Learning
- 7> Deductive Learning

459 - 461 of AI, Rich and Knight.

Genetic Algorithms by Elaine Rich, Kevin Knight, Shivasankar B. Nair.

Search procedure based on a simple model of evolution

Uses a "random" process to explore search space.

Has been applied in many domains.

Begin with a population of individuals. Each individual represents a solution to the problem we are trying to solve.

A data structure describes the genetic structure of the individual.

(Assume for initial discussion that this is a string of 0's and 1's).

- In genetics, the strings are called chromosomes and the bits are called genes.
- The string associated with each individual is its genotype.
- Selection is based on fitness of individuals.
- Each evolving population of individuals is called a generation.
- Given a population of individuals corresponding to one generation the algorithm simulates natural selection and reproduction in order to obtain the next generation.

Three basic operations:-

1) Reproduction

→ Individuals from one generation are selected for the next generation.

2) Crossover

→ Genetic material from one individual is exchanged with genetic material from another individual.

3) Mutation

Selection, crossover and mutation operations:

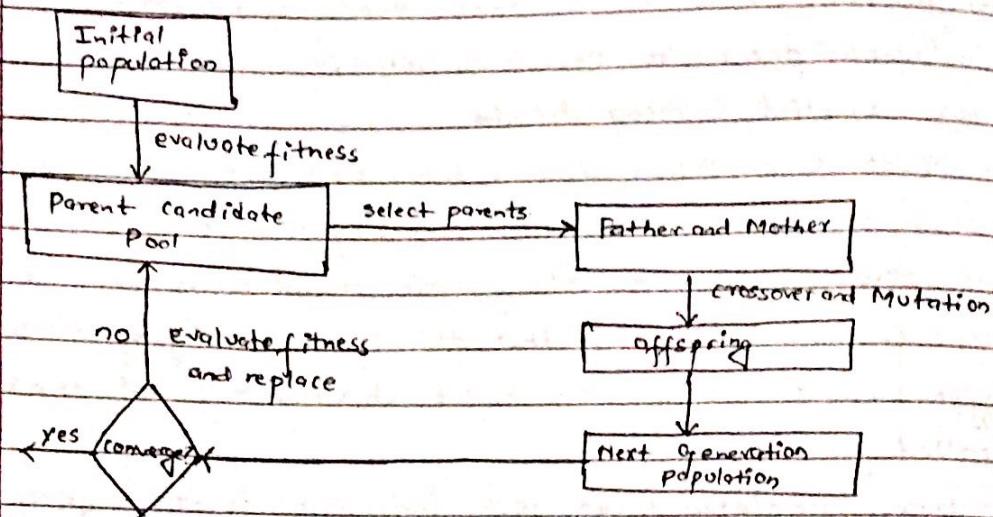
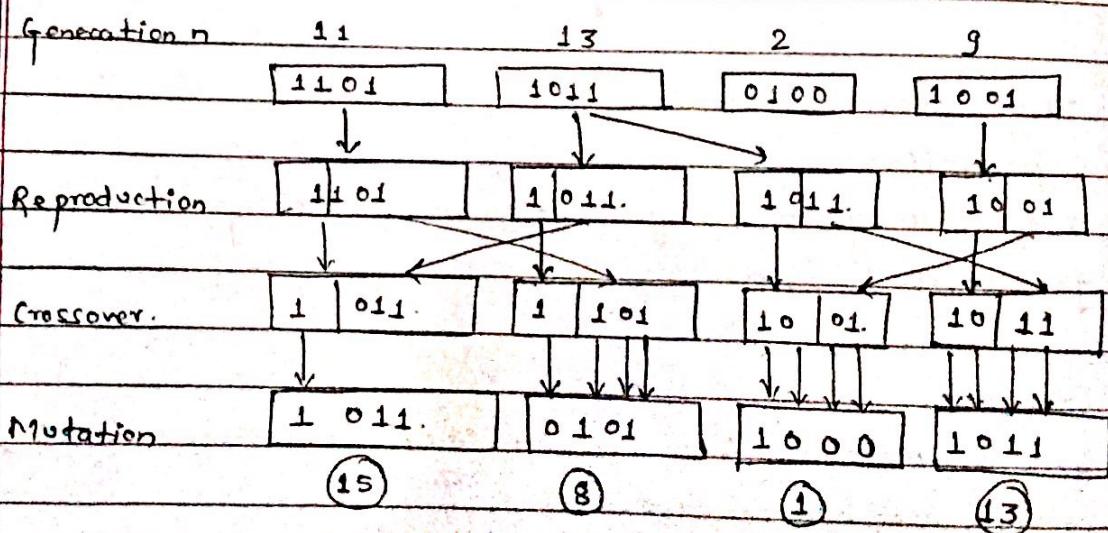


fig: Selection, crossover and mutation process

Example of General GA Procedure



Two keys to the success of a GA

- Data structure for
- Genes
- Chromosomes
- Population
- Fitness evaluation function

Example 7 :- While watching a game of champions League football in a cafe, you observe someone who is clearly supporting Manchester United in the game.

What is probability that they were actually born within 25 miles of Manchester?

Assume that

the probability that a randomly selected person in a typical local bar environment is born within 25 miles of Manchester is $\frac{1}{20}$ and

the chance that a person born within 25 miles of Manchester actually supports United is $\frac{7}{10}$.

the probability that a person not born within 25 miles of Manchester supports United with probability $\frac{1}{10}$.

Solution,

$$P(B) = \frac{1}{20}$$

$$P(U|B) = \frac{7}{10}$$

Now,

$$P(B|U) = \frac{P(U|B) * P(B)}{P(U)}$$

$$= \frac{\frac{7}{10} * \frac{1}{20}}{}$$

$$P(U|B) * P(B) + P(U|B^c) * P(B^c)$$

$$= \frac{\frac{7}{10} * \frac{1}{20} + \frac{1}{10} * \frac{19}{20}}{}$$

$$= \frac{\frac{7}{200} + \frac{19}{200}}{26} = 0.26$$

$$f(x) = 15x - x^2$$

[0-31]

$$n = 6$$

Solution:-

Steps:

→ Generate initial population at random. They are chromosomes or genotypes. e.g.

$$01101 \rightarrow 13$$

$$01001 \rightarrow 9$$

$$01111 \rightarrow 15$$

$$01011 \rightarrow 13$$

$$10011 \rightarrow 19$$

$$10010 \rightarrow 18$$

→ Calculate the fitness value for each individual.

(a) Decode the individual into an integer (called phenotypes).

$$01101 \rightarrow 13$$

$$01011 \rightarrow 13$$

$$01001 \rightarrow 9$$

$$10011 \rightarrow 19$$

$$01111 \rightarrow 15$$

$$10010 \rightarrow 18$$

(b) Evaluate the fitness according to $f(x) = 15x - x^2$.

$$13 \rightarrow 26$$

$$13 \rightarrow 26$$

$$9 \rightarrow 54$$

$$19 \rightarrow -76$$

$$15 \rightarrow 250$$

$$18 \rightarrow 324 - 54$$

→ Select parents (two individuals) for crossover based on their fitness in P_i . Out of many methods for selecting the best chromosomes, if roulette wheel selection is used, then the probability of the i^{th} string in the population is $p_i = F_i / (\sum_{j=1}^n F_j)$,

String NO	Initial pop	X-value	Fitness F_i $f(x) = 15x - x^2$	P_i	Expected count $N \times p_{nb}$
1	01101	13	26	0.04 - 0.10	- 0.04 - 0.60
2	01011	13	44	0.07 - 0.17	- 1.03
3	01001	9	54	0.09 - 0.21	- 1.26
4	10011	19	-76	-0.12 - 0.29	1.78
5	01111	15	-230	0.00	5.85
6	10010	18	-324	-0.04 - 0.21	1.26
Sum			-296	1.00	6.00

Average

-43 0.17 1

Max.

54 -0.21 -1.26

The string no 3 has maximum chance of selection.

→ Produce a new generation of solutions by picking from the existing pool of solutions with a preference for solutions which are better suited than others:

We divide the range into six bins, sized according to the relative fitness of the solutions which they represent.

Strings	Prob i	Associated Bin
01101	-0.10	0.0 ----- -0.10
01011	-0.17	-0.10 ----- -0.27
01001	-0.21	-0.27 ----- -0.48
10011	0.29	-0.48 ----- -0.19
01111	0.98	-0.19 ----- 0.79
10010	0.21	0.79 ----- 1.00

By generating 6 uniform (0,1) random values and seeing which bin they fall into we pick the ~~six~~ string that will form the basis for the next generation.

Random No	Falls into bin	Chosen string
-0.08	0.0 ----- -0.10	01101
-0.24	-0.10 ----- -0.27	01011
-0.35	-0.27 ----- -0.48	01001
-0.70	-0.10 ----- -0.27	01011 01111
0.40	-0.27 ----- 0.48	01001
0.87	0.79 ----- 1.00	10010

→ Randomly pair the members of the new generation.

For the first pair of strings: 01101, 01011

→ We randomly select the crossover point to be after ~~fourth~~^{third} digit

$$01101 \Rightarrow 01|01 \Rightarrow 01111$$

$$01011 \Rightarrow 01|011 \Rightarrow 01001$$

For the second pair of strings: 01001, 01011

→ We randomly select the crossover point to be after second digit

$$01|001 \Rightarrow 01001 \Rightarrow 01011$$

$$01|011 \Rightarrow 01|011 \Rightarrow 01001$$

For the last pair of strings: 01001, 10010

→ We randomly select the crossover point to be after ~~fourth~~^{fifth} digit.

$$0100|1 \Rightarrow 01000$$

$$1001|0 \Rightarrow 10011$$

→ Randomly mutate a very small fraction of genes in population.

→ Go back and re-evaluates fitness of the population.

String No	Initial pop ⁿ (chromosomes)	X-value: (phenotypes)	Fitness $f(x) = 15x - x^2$	prob i (fraction of total)	Expected amount
1	01111				
2	01001				
3	01011				
4	01001				
5	10000				

$$15x - x^2$$

[0 - 15]

① 0000 . . . 1111 .

0001

0010

0011 .

② 0010, 0110, 1100, 1010

③ $f(x) = 15x - x^2$

$f(2) = 26$

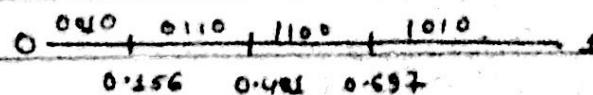
$f(6) = 54$

$f(12) = 36$

$f(18) = 50$

S-N	Chromosome	decimal	$f(x)$	P_i
1	0010	2	26	0.156
2	0110	6	54	0.325
3	1100	12	36	0.216
4	1010	10	50	0.301.
			166	1.00
Sum			41.5	
avg				

⑤



$$0 \dots 0.156 \rightarrow 0010$$

$$0.156 \dots 0.481 \rightarrow 0110$$

$$0.481 \dots 0.697 \rightarrow 1100$$

$$0.697 \dots 1.000 \rightarrow 1010$$

6 0 0 1 0 ⑤ 1 9 1 0
2 0 1 1 0 ⑦ 1 2 1 0

0 1 1 0 3' 1 0 0 0
0 0 1 0 4' 1 1 1 0

S.N.	Chromosomes	decimal	freq	pli
1	0 1 1 0	6	54	0.56
2	0 0 1 0	2	26	0.123

Assignments

- ① Explanation Based Learning
- ② Boltzmann machine

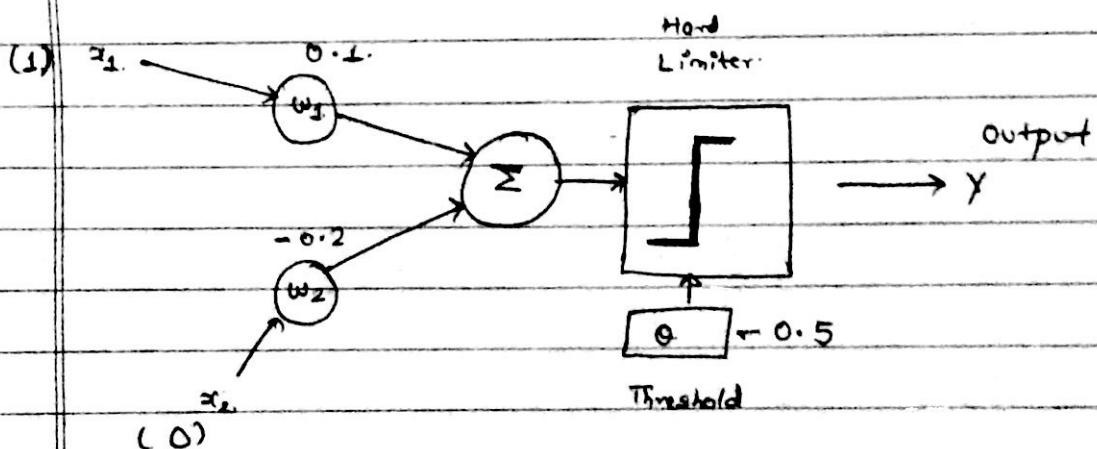
Neural NetworksPerception

Step 1: Initialization.

Step 2: Activation

Step 3: Weight Training

Step 4: Iteration



Realize AND operation using Perceptron training algorithm, perform two iterations

$$1 \text{ AND } 0 = 0 \quad \therefore y_d(1) = 0$$

$$y_a(p) = \text{Sigmoid} \left[\sum_{i=1}^2 w_i(p) x_i(p) - \theta \right].$$

$$y_a(1) = \text{Sigmoid} [1 \times 0.1 + 0 \times (-0.2) + 0.5]$$

$$y_d(1) = \text{Sigmoid} [0.6].$$

$$\frac{1}{1+e^{-0.6}}$$

$$= 0.14887 \quad 0.6456$$

$$e(z) = y_d(z) - y(z)$$

$$= 0 - 0.6456$$

$$= -0.6456$$

$$w_i(p+1) = w_i(p) + \Delta w_i(p)$$

$$= w_i(p) + \alpha \cdot x_i(p) \cdot e(p)$$

$$w_1(2) = w_1(1) + 0.01 * 1 \neq (-0.6456)$$

$$= 0.09354$$

$$w_2(2) = -0.2 + 0.01 * 0 \neq (-0.6456)$$

$$= -0.2$$

Iteration 2nd

$$y_d(z) = \text{sigmoid} \left[\sum_{i=1}^2 x_i(p) \cdot w_i(p) - \theta \right]$$

$$= \text{sigmoid} [1 \cdot 0.09354 + 0 \cdot (-0.2) + 0.5]$$

$$= \text{sigmoid} [0.59354]$$

$$= \frac{1}{1+e^{-0.59354}}$$

$$= 0.64417$$

$$e(2) = y_d(z) - y(z)$$

$$= 0 - 0.64417$$

$$= -0.64417$$

$$w_i(p+1) = w_i(p) + \Delta w_i(p)$$

$$\begin{aligned} w_1(3) &= 0.09359 + 0.01 \times 1 \times (-0.64417) \\ &= 0.08709 \end{aligned}$$

$$\begin{aligned} w_2(3) &= -0.2 + 0.01 \times 0 \times (-0.64417) \\ &= -0.2 \end{aligned}$$

$$\begin{array}{c} - \\ \hline -0.6456 \quad -0.64417 \quad 0 \end{array}$$

Adaline (Adaptive Linear Element)

Madaline (Multiple Adaline) \rightarrow (Always Sigmoid function)

Multi-layer Perceptron

- Q. A factory production line is manufacturing bolts using three machines, A, B, C. 60% of the total output, machine A is responsible for 25%, B for 35% and C for the rest. It is known from the previous experience with the machines that 5% of the output from machine A is defective, 4% from machine B, and 2% from machine C. A bolt is chosen at random from the production line and found to be defective. What is the probability that it came from

- (A) Machine A
- (B) Machine B
- (C) Machine C.

Soln:-

$$P(A) = \frac{25}{100} = 0.25 \quad P(B) = \frac{35}{100} = 0.35 \quad P(C) = 0.40$$

$$P(D/A) = \frac{5}{100} = 0.05$$

$$P(D/B) = \frac{4}{100} = 0.04$$

$$P(D/C) = 0.02$$

$$\begin{aligned} P(D) &= P(A) \times P(D/A) + \\ &\quad P(B) \times P(D/B) + \\ &\quad P(C) \times P(D/C) \\ &= 0.0345. \end{aligned}$$

Now,

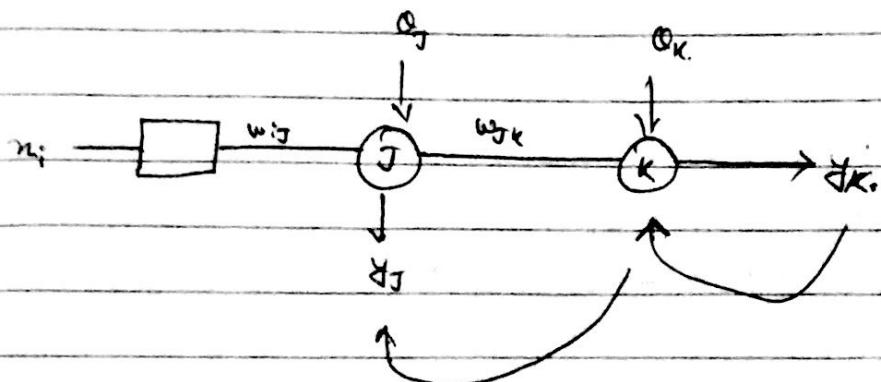
$$\begin{aligned} \textcircled{1} \quad P(A/D) &= \frac{P(D/A) \times P(A)}{P(D)} \\ &= \frac{0.05 \times 0.25}{0.0345} \\ &= 0.3623 \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad P(B/D) &= \frac{P(D/B) \times P(B)}{P(D)} \\ &= 0.406 \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad P(C/D) &= \frac{P(D/C) \times P(C)}{P(D)} \\ &= \frac{0.02 \times 0.40}{0.0345} \\ &= 0.232. \end{aligned}$$

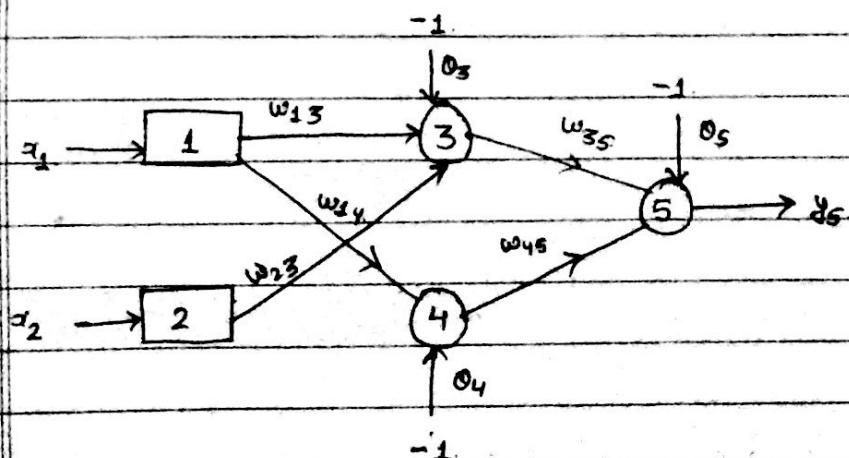
Back Propagation

$$\hat{y}_j(p) = \text{sigmoid} \left[\sum_{i=1}^2 x_i(p) \cdot w_{ij}(p) - \theta_j \right]$$

Imp

Kohonen Network

- Self organized maps.
- competitive Learning (neurons compete among themselves to be activated)
- Learning of Neural network without the presence of teacher.



$$w_{13} = 0.5, w_{14} = 0.9, w_{23} = 0.4, w_{45} = 1.0, w_{35} = -1.2,$$

$$w_{45} = 1.1, \theta_3 = 0.8, \theta_4 = -0.5 \text{ and } \theta_5 = 0.3$$

We consider a training set where input x_1 and x_2 are equal to 1 and desired output $y_{4,5}$ is 0. The actual output of neurons 3 and 4 in the hidden layers are calculated as:

$$\begin{aligned}y_3 &= \text{sigmoid}(x_1 w_{13} + x_2 w_{23} - \theta_3) \\&= \text{sigmoid}(1 \times 0.5 + 1 \times 0.4 - 0.8) \\&= \text{sigmoid}(0.1) \\&= \frac{1}{1 + e^{-0.1}} \\&= 0.5250\end{aligned}$$

$$\begin{aligned}y_4 &= \text{sigmoid}(x_1 w_{14} + x_2 w_{24} - \theta_4) \\&= \text{sigmoid}(1 \times 0.9 + 1 \times 1.0 + 0.1) \\&= \text{sigmoid}(2) \\&= \frac{1}{1 + e^{-2}} \\&= 0.8808\end{aligned}$$

Now,

Actual output of neuron 5 in the output layer is determined as:

$$\begin{aligned}y_5 &= \text{sigmoid}(y_3 w_{35} + y_4 w_{45} - \theta_5) \\&= \text{sigmoid}(0.5250 \times (-1.2) + 0.8808 \times 1.1 - 0.3) \\&= \text{sigmoid}(0.0388) \\&= \frac{1}{1 + e^{-0.0388}} \\&= 0.5097\end{aligned}$$

Thus, the following error is obtained as.

$$e = y_{d,5} - y_5 = 0 - 0.5097 = 0.5097$$

The next step is weight training. To update the weights and threshold levels in our network, we propagate the error, e from the output layer backward to the input layer.

First, we calculate the error gradient for neuron 5 in the output layer.

$$\begin{aligned}\delta_5 &= y_5(1-y_5) e \\ &= 0.5097(1-0.5097) \times (-0.5097) \\ &= -0.1274\end{aligned}$$

Then, we determine the weight corrections assuming that the learning rate parameter, α is equal to 0.1.

$$\begin{aligned}\Delta w_{35} &= \alpha \cdot y_3 \cdot \delta_5 = 0.1 \times 0.5250 \times (-0.1274) \\ &= 0.0067\end{aligned}$$

$$\begin{aligned}\Delta w_{45} &= \alpha \cdot y_4 \cdot \delta_5 \\ &= 0.1 \times 0.8808 \times (-0.1274) \\ &= -0.0112\end{aligned}$$

$$\begin{aligned}\Delta \theta_5 &= \alpha \cdot (-1) \cdot \delta_5 \\ &= 0.1 \times (-1) \times (-0.1274) \\ &= 0.0127\end{aligned}$$

Now, we calculate the error gradient for neurons 3 and 4 in hidden layer.

$$\begin{aligned}\delta_3 &= y_3(1-y_3) \cdot \delta_5 \cdot w_{35} \\ &= 0.5250 \times (1-0.5250) \times (-0.1274) \times (-1.2) \\ &= 0.0381\end{aligned}$$

$$\begin{aligned}\delta_4 &= y_4(1-y_4) \cdot \delta_5 \cdot w_{45} \\ &= 0.8808(1-0.8808) \times (-0.1274) \times 1.2 \\ &= -0.0447\end{aligned}$$

We, then determine the weight corrections.

$$\Delta w_{13} = \alpha \cdot x_1 \cdot \delta_3 = 0.1 \times 1 \times 0.0381 = 0.0038$$

$$\Delta w_{23} = \alpha \cdot x_2 \cdot \delta_3 = 0.1 \times 1 \times 0.0381 = 0.0038$$

$$\Delta \theta_3 = \alpha \cdot (-1) \cdot \delta_3 = 0.1 \times (-1) \times 0.0381 = -0.0038$$

$$\Delta w_{14} = \alpha \cdot x_1 \cdot \delta_4 = 0.1 \times 1 \times (-0.0447) = -0.0045$$

$$\Delta w_{13} = \alpha \cdot x_2 \cdot b_w \\ = 0.1 \times 1 \times (-0.0142) \\ = -0.00142$$

$$\Delta c_y = \alpha \cdot (-1) \cdot b_s \\ = 0.1 \times (-1) \times (-0.0142) \\ = 0.00142$$

At last, we update all weights and thresholds.

$$w_{13} = w_{13} + \Delta w_{13} = 0.5 + 0.0038 = 0.5038$$

$$w_{24} = w_{24} + \Delta w_{24} = 0.9 - 0.0015 = 0.8985$$

$$w_{23} = w_{23} + \Delta w_{23} = 0.4 + 0.0038 = 0.4038$$

$$w_{24} = w_{24} + \Delta w_{24} = 1.0 - 0.0015 = 0.9985$$

$$w_{35} = w_{35} + \Delta w_{35} = -1.2 - 0.0067 = -1.2067$$

$$w_{45} = w_{45} + \Delta w_{45} = 1.1 - 0.0112 = 1.0888$$

$$c_3 = c_3 + \Delta c_3 = 0.8 - 0.0038 = 0.7962$$

$$c_y = c_y + \Delta c_y = -0.1 + 0.0015 = -0.0985$$

$$c_5 = c_5 + \Delta c_5 = 0.3 + 0.0122 = 0.3122$$

Note the calling population is finite and consists of 10 machines

(ii) Infinite calling population

In systems with a large population of potential customer the calling population is assumed to be infinite - e.g. the potential customers arrival at ops billing counter.

patient \rightarrow customer

clerk \rightarrow server

The main difference between finite and infinite calling population model is how the arrival rate is defined.

- In infinite population model, the arrival rate is not affected by the no of customers who have left the calling population and joined queuing system when arrival process is homogeneous over time.

\rightarrow The average time rate is usually assumed to be constant
 \rightarrow for finite population model, the arrival rate to the queuing system does not depends on the number of customer being served and waiting.

and they are related as

$$\lambda = \frac{\theta I}{T_a}$$

e.g. for an office working 8 hrs a day 5 day in a week gets about 800 telephone calls in a week. Suppose a model of the office during working hours is to be constructed using time scale of minute; find avg. inter arrival time and arrival rate.

Sol:

$$\text{total working hr} = 8 \times 5 = 40 \text{ hr.}$$

$$\text{Average arrival time} = \frac{40 \times 60}{800} = 3 \text{ minute.}$$

$$\text{i.e. } T_a = 3 \text{ min}$$

$$\text{Annual rate}(\lambda) = \frac{1}{T_a} = \frac{1}{3} = 0.333 \text{ call/min}$$

The calling population

The customers or inputs requesting for service from a system is called calling population. The population of potential customer i.e. the popn or requesting entities for getting a service is known as calling population. It may be finite or infinite.

► finite calling population

The system in which requesting entities are finite are called finite calling population e.g. ATM, lounge lounge service for bank with 10 ATM machine running for money.

After an interval of time money should put by the bank lounge worker should put who will the money into the money box. Here,

ATM machine \rightarrow Customer

Bank lounge work \rightarrow sensor

i) Timely

ii) Random

iii) Batched

⇒ Timely arrival pattern

It is reflected as scheduled arrivals such as patient to a doctor's clinic, students at classes, etc.

⇒ In this case, inter-arrival time may be constant or $(\text{constant} \pm \alpha)$

where, the small random amount of time representing early or late arrival.

ii) Random arrival pattern

In this class, the arrival may occur at any random time. The inter-arrival times are usually characterised by a probability distribution in case of random time.

e.g. arrival of customer at ATM.

iii) Batched arrival pattern :-

In this pattern, customer may arrive one at a time or in batches, the batch may be of constant size or random size.

→ An additional situation of arrival pattern occurs when at least one customer is assumed always be present in the queue so that server is never idle due to lack of customer.

for example, the ~~customer~~ customer may represent raw material for a product and sufficient material is always available.

The most important model for a random arrival are poison arrival pattern.

The following notation are used for describing the arrival patterns

T_a → mean arrival time

λ → mean arrival rate

Generation of Arrival Pattern

Generation of exogenous arrivals are necessary for testing the reliability of a system being simulated in a discrete system simulation e.g.

testing the design of logic circuits such as components of digital computers. A particular sequence of signals might be designed as the simulation input to see if the design reaches as expected or not.

- the sequence of inputs may also be generated from the observation of a system. The approach of generation of arrival pattern is known as trace driven simulation in which system being simulated is tested with the records collected from the running system by program monitors while little or no disturbance in system operation.
- when there is no interaction between the exogenous arrivals and endogenous events, it is permissible to create a sequence of arrivals in preparation for the simulation.
- The process of generating arrival of successor entity by its predecessor is known as bootstrapping.

The arrival process

The arrival process of an infinite population model is usually characterized in terms of inter-arrival time of successive IIP, customers getting for request.

The various arrival pattern process may be of following types

discrete system

The system in which changes are predominantly discontinuous are called discrete systems.

e.g. telephone system, shopping system, etc.

Numerical computation technique for discrete models

In discrete model various parameters at time intervals (t_i) is recorded and performance analysis for the given system is achieved for discrete time interval.

for example, in accounting system an accountant begins his work everyday work with a file of documents to be processed. The time taken to process them may varies. They work through the file, beginning each document as soon as possible, finished the previous job except that he can take a fixed time (5 min) break after finishing a job. If it is an hour or more since he began a work or since he last has a break.

We assume that time to process the document are given. We may also keep count of how many documents are left. The count will be initially set to the no of documents at the beginning time of the day and assume that no additional document arrive on that day. The count will be documented for each complete job and with that if count goes to zero. The computation can be organized as shown in table below:

doc no.	st. time	work time	finish time	Cumulative time	Break time	No of Job
1	0	45	45	45	0	20
2	45	16	61	61	1	19
3	66	5	71	5	0	18
4	71	20	91	25	0	17
5	91	8	99	33	1	16
6	104	10	114	10	0	5