2016 spring

1(a) Why simulation is essential in depicting real world problem. Differentiate continous and discrete system.

Simulation is a representation of real life system by another system, which inherits the important characteristics of real system and allows experimentation on it. simulation is used when conducting experiments on a real system.

Simulation is essential in depicting real world because it may be too dificult or expensive to observe a real operational system, parts of system may not be observable (e.g internals of a ~~system~~ silicon chips.

1. Analyze the system before they build
2. Reduce risk: how do you know they are right changes?
3. Minimize your spending: cash is tight. You cannot afford to waste a single dollars
4. Reduces no. of design mistakes
5. Optimize design
6. Analyze operational system
7. Create virtual environment for training

Continous and Discrete system

→ System such as aircraft, in which the changes are ~~peri~~ predominantly smooth, are called continous system

→ System like the factory, in which the changes are predominantly discontinous, called discrete systems.

→ Few systems are wholly continous or discrete

→ aircraft may make discrete adjustment to to its trim as altitude changes. while in factory, machining proceeds continously, even though Start and finish of a job are discrete changes.

→ The complete aircraft system might even be regarded as a discrete system.

→ In factory system, if the number of parts is sufficiently large, there may be no point in treating the no. as a discrete variable.

→ Instead, the no. of parts might be represented by a continous variable with the machining activity controlling the rate at which parts flow from one state to another.

→ System that are intrinsically continous but information about them is only available at discrete points in time.

→ General programming methods used to simulate continous system and ~~discrete~~ discrete system differently.

Qn. 21a) What are the steps used in simulation study? Explain with neat block.

There are four phases in simulation study:

Phase I

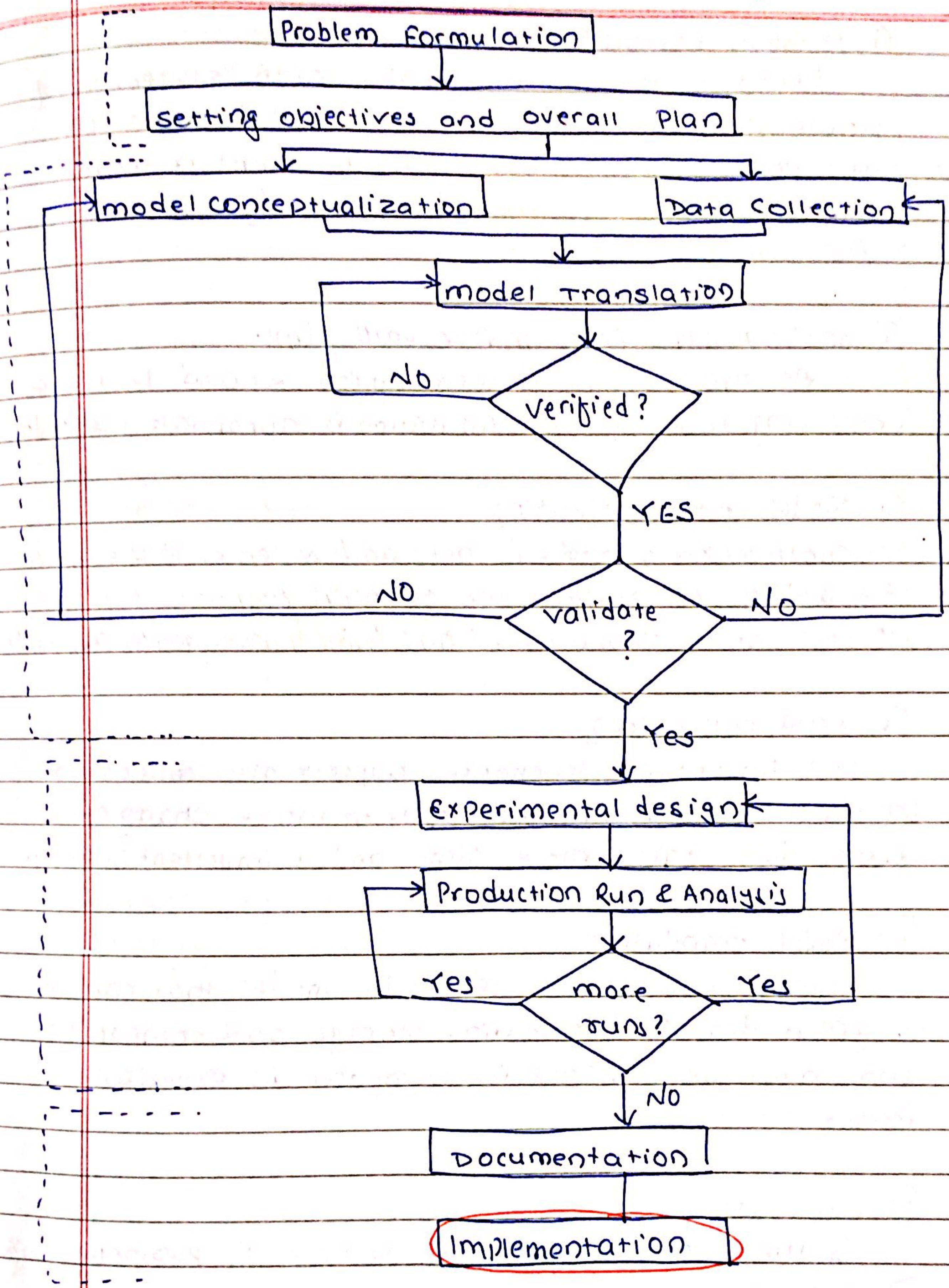
- Problem formulation
- Setting objectives and overall plan

Phase II

- Model Conceptualization
- Data collection
- Model translation
- Verification / validation

Phase III — Experimental design
- Production runs and Analysis
- Additional runs

Phase IV — (i) Documentation and reporting (ii) implementation.


Problem formulation
setting objectives and overall Plan
model conceptualization
Data Collection
model Translation
verified?
NO
YES
validate ?
NO
NO
Yes
Experimental design
Production Run & Analysis
more runs?
Yes
Yes
NO
Documentation
Implementation

(i) Problem formulation:

Every study should begin with 'statement of problem'. If the problem statement is provided by policy maker, then analyst must assume that problem being developed by the analysis for ~~problem~~ solution

(ii) Setting objectives and overall plans.

At this point, determination should be made concerning whether the simulation is apropriate methodology.

(iii) Model conceptualization

Constrcxtion of model is art and science. It is enhanced by ability to abstract the essential feature. It is best to start with simple model and build toward greater complexity.

(iv) Data collection

It is important to choose appropriate collection of data input as the complexity of model changes. since, data takes more time and is important in early.

(v) Model translation

since, real system results in model that require a great deal of information storage and computation and must be entered in computer in recognizable format.

(vi) Verification

is the program performing properly. i/p parameter are right?

(vii) validation

determination that model is an accurate representation of real system usually.

(viii) experimental design:
- the real system ~~[illegible]~~ may be simulated correctly by determining alternative.
- concerning with alternative to simulate may be a function of runs that have been completed and Analyzed.

(ix) Production run and Analysis:
It is used to estimate measure of performance for the system design that are being simulated

(x) Additional runs
Based on analysis of runs that have been completed the analyst determines if additional runs are needed and what design those additional experiment should follow.

(xi) Documentation and Reporting
program documentation is necessary for the various reason if program is re-used by same or different analyst progress reporting provides the important written history of simulation.

(xii) Implementation
The success of implementation phase depends on how well the previous steps have been performed.

2016 fall

1(a) Diff betⁿ continous and discrete system in detail. What do you mean by verification process.

First part done.

It is concerned with building the model right. It is utilized in the comparision of conceptual model to the computer representation that implements that conception

Verification is the process of determining that a model implementation ~~that a model imt~~ and its associated data accurately represent the developer's conceptual description and specification

Validation is the process of determining the degree to which a model and its associated data are an accurate representation of the real world from the ~~ver~~ perspective of the intended use of the model.

verification: system ले हामीले सोचे जस्तो काम गर्छ?
validation: system ले real world ले जस्तो काम गर्छ?

Verification phase focus on comparing the element of a simulation model of the system with the description of what the requirements and capabilities of model were to be. Verification is a iterative process aimed at determining whether the difference are acceptable given the intented use of the model.

Why monte-carlo method is best for computing static model? Derive the value of Pi using Monte-carlo simulation method.

Monte-carlo simulation or probability simulation, is a technique used to understand the impact of risk and uncertainty in financial, project management and cost.

In general, Montecarlo method are used in mathematics to solve various problem by generating suitable random numbers and observing that fraction of numbers that obey some properties.

For static model:

- Useful when we need to estimate the value of function too complicated to deal with mathematically.
- In static, we deal with repeated random trials
-

Code for pi in Python

```
from random import random
from math import sqrt
#Number of random points
N = 10000
# Counter of points inside
I = 0
for i in range(N):
    #Generate random point
    # in the 1x1 square:
    x = random()
    y = random()
    #Is it inside the circle ?
    r = sqrt(x**2 + y**2)
    If r<1: I += 1
#calculate Pi:
print(4*(I/N))
```

Output :

3.136
3.04
3.1768
3.1496

The Technique of Simulation

2016 Spring

(1b) $I = \int_0^5 \frac{x^3}{x^4+16}$

$\rightarrow$ let $y = \int_0^5 \frac{x^3}{x^4+16}$

$f(x) = \frac{x^3}{x^4+16}$

The To find max^m value of y.

$y'(x) = 0$

$\frac{d}{dx}\left(\frac{x^3}{x^4+16}\right) = 0$

$\left[\frac{d(u/v)}{dx} = \frac{v\,du - u\,dv}{v^2}\right]$

$\frac{d}{dx}\ \frac{(x^4+16)\cdot 3x^2 - x^3 \cdot 4x^3}{(x^4+16)^2}$

$3x^6 + 48x^2 - 4x^6 = 0$

$x^2(48 - x^4) = 0$

$\therefore x = 0$ and $x^4 = 48$

$x = \sqrt[4]{48}$

c is ~~maximum~~ optimum at $x = \sqrt[4]{48}$

$\therefore c = f(x) = \frac{x^3}{x^4+16} = 0.2849$

By monte carlo,

$$\frac{n}{N} = \frac{\int_a^b f(x)\,dx}{c*(b-a)}$$

b = 5 by qn)
a = 0

$$\therefore \int_a^b f(x)\,dx = \frac{n}{N} * c*(b-a)$$
$$= \frac{n}{N} * 0.2859 * (5-0)$$
$$= \frac{n}{N} * 1.4295$$

~~Suppose, let~~ choose N = total no. of Sample
n = Sample falls under the f(x)

S.N	N	n	(S) standard	(O) Observe	error = $\left(\frac{S-O}{S}\right) * 100\%$
1	1000	500	0.9296	0.7147	23.11 %
2	1000	700	0.9296	1.00	7.64 %
3	1000	600	0.9296	0.8577	7.73 %

Extra:
use calculator, to find observe value
ie Observe = 0.9296

trick, ~~multiply~~ divide it by (1.4295) we get 0.6502

$$\left[\frac{n}{N} * 1.4295\right]$$

~~so~~ multiply by 1000 (N), 650
Choose n around 650

Qn 2(b) Explain Analog Computer. Draw suitable analog computer model for following set of diff. eqn?

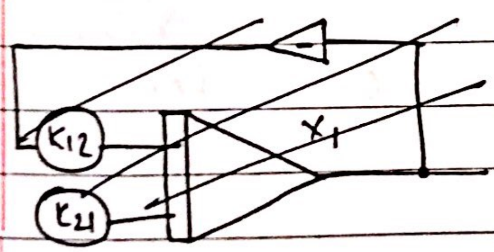
x' $\quad \frac{dx_1}{dt} = -k_{12}x_1 + k_{21}x_2$

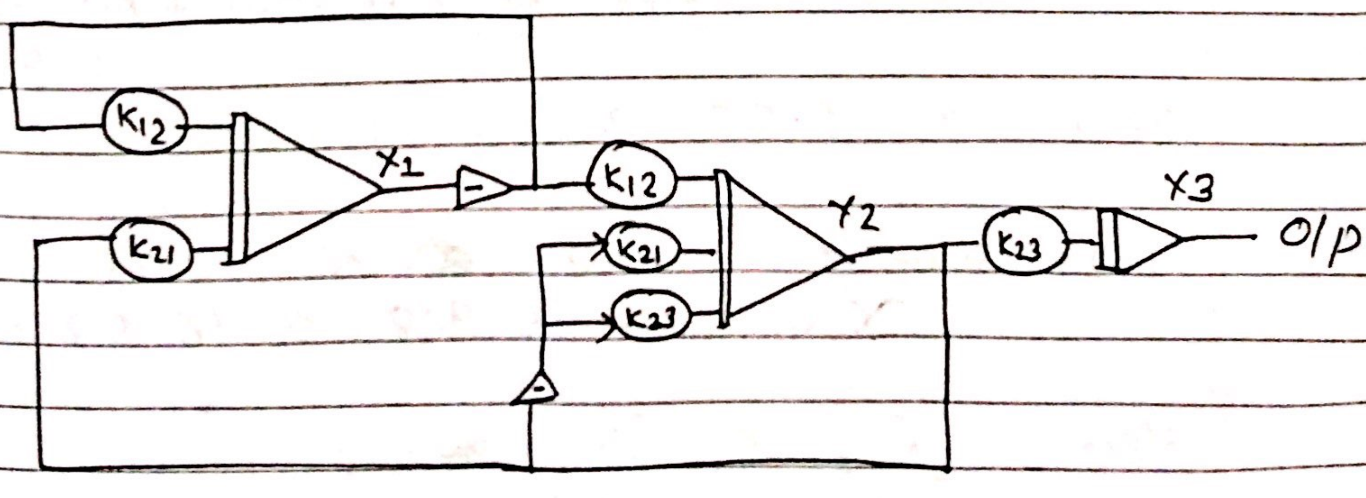
x'' $\quad \frac{dx_2}{dt} = -k_{12}x_1 - (k_{21}+k_{23})x_2$

x''' $\quad \frac{dx_3}{dt} = k_{23}x_2$

→ An analog computer is a computer which is used to process analog data. Analog data are stored in continous form of physical quantities and perform calculation with the help of measures. Analog computer are excellent for situations which require data to be measured directly without converting into numerals or codes.

Advantage: real time operation
simultaneous computation





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Use Chi-Square test with $\alpha = 0.05$ to test whether data shown below are uniformly distributed. Critical value for $n = 10$ is 16.9

$$X_{calc} = \frac{\sum O_i - e_i}{e_i}$$

S.N	interval	observed (O_i)	expected e_i	$(O_i - e_i)^2$	$\frac{(O_i - e_i)^2}{e_i}$
1	0 - 0.10	8	10	4	0.4
2	0.11 - 0.20	8	10	4	0.4
3	0.21 - 0.30	10	10	0	0
4	0.31 - 0.40	9	10	1	0.1
5	0.41 - 0.50	12	10	4	0.4
6	0.51 - 0.60	8	10	4	0.4
7	0.61 - 0.70	10	10	0	0
8	0.71 - 0.80	14	10	16	1.6
9	0.81 - 0.90	10	10	0	0
10	0.91 - 1.00	11	10	1	0.1

$X^2_{calc} = 3.4$

$e_i = \frac{\sum O_i}{n} = \frac{100}{10} = 10$ (for each)

~~$\sum O_i = 100$~~ degree of freedom $= N - 1 = 10 - 1$
$= 9$

and $\alpha = 0.05 \rightarrow$ critical value

Using table for 5% at $N = 9$ is
$X^2_{table} = 16.919 \approx 16.92$

Here $X^2_0 < X^2_{0.5}$
$3.8 < 16.92$

$\therefore$ value is accepted [yes distributed]

How to get Observed — Count each & mark in range

0 – 0.10	IIIIIIII	8
0.11 – 0.2	IIIIIIII	8
0.21 – 0.3	IIIIIIIIII	10
0.31 – 0.4	IIIIIIIII	9
0.41 – 0.5	IIIIIIIIIIII	12
0.51 – 0.6	IIIIIIII	8
0.61 – 0.7	IIIIIIIIII	10
0.71 – 0.8	IIIIIIIIIIIIII	14
0.81 – 0.9	IIIIIIIIII	10
0.91 – 1.00	IIIIIIIIIII	11

100 (correct if no. of data)

2015 spring

Draw cobweb model (in graph) for:

(i) fluctuation of market price

(ii) cobweb model for market economy graph from given data.

$D = 12.4 - 1.2P$ Demand

$S = 8.0 - 0.6P_{-1}$ Supply

$D = S$

$P_0 = 1.0$ market price

Cobweb model

A particularly simple, but never thless useful, distributed lag model can be constructed from the static market model. ~~the~~ To be more realistic, the supply should be dependent upon the price from the previous marketing period since that is only figure available to the supplier at the time of making future plans. The demand however, will respond to current price

$D = 12.4 - 1.2P$

$S = 8.0 - 0.6P_{-1}$

$D = S$

Given an intial value of Price P_0, the value of S at the end of the first interval can be derived. This determines the value of D, since the market is cleared, and from this the new value of P can be derived. This becomes the value of P_{-1} used to calculate the values for the second interval and so on.

Since $D = S$

$$12.4 - 1.2P = 8.0 - 0.6P$$

$$12.4 - 8.0 = 1.2P - 0.6P$$

$$4.4 = 0.6P$$

$$P = \frac{4.4}{0.6}$$

$$P = 7.33$$

Practice

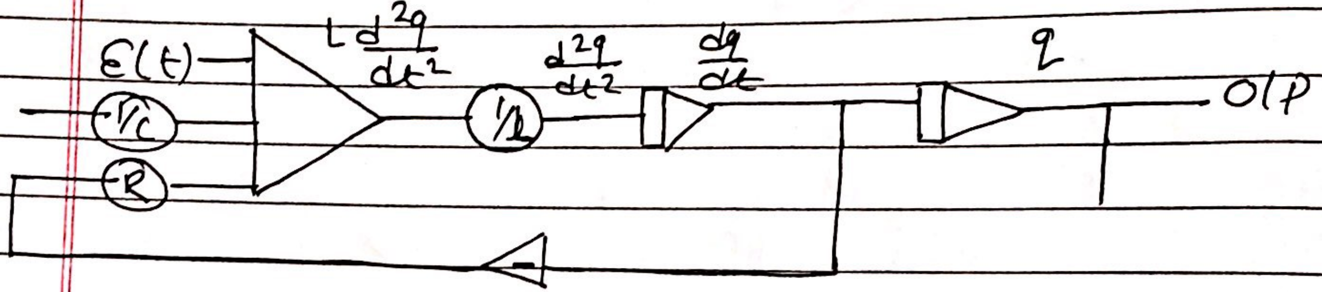
~~Digital~~ Analog computer

(i) $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + q/C = \mathcal{E}(t)$

Soln :

$$L\frac{d^2q}{dt^2} = \mathcal{E}(t) + \left(-R\frac{dq}{dt}\right) + \left(-q/C\right)$$

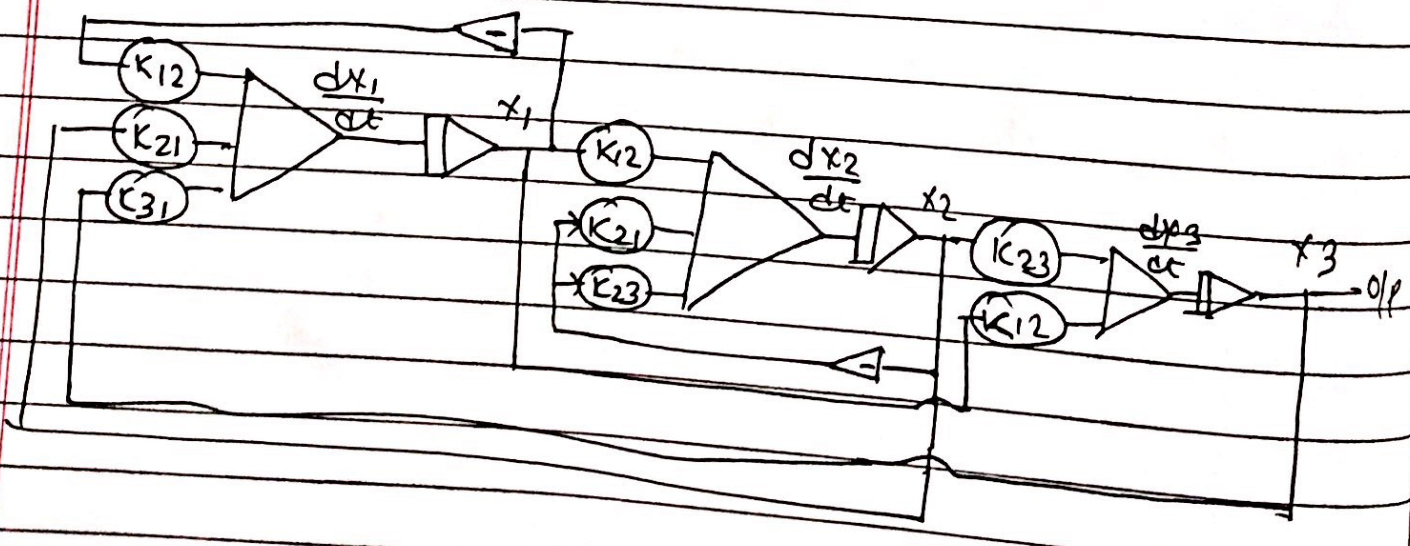
target: find q as output



(ii) $\frac{dx_1}{dt} = -k_{12}x_1 + k_{21}x_2 + k_{31}x_3$

$\frac{dx_2}{dt} = k_{12}x_1 - (k_{21}+k_{23})x_2$

$\frac{dx_3}{dt} = k_{23}x_2 - k_{12}x_1$



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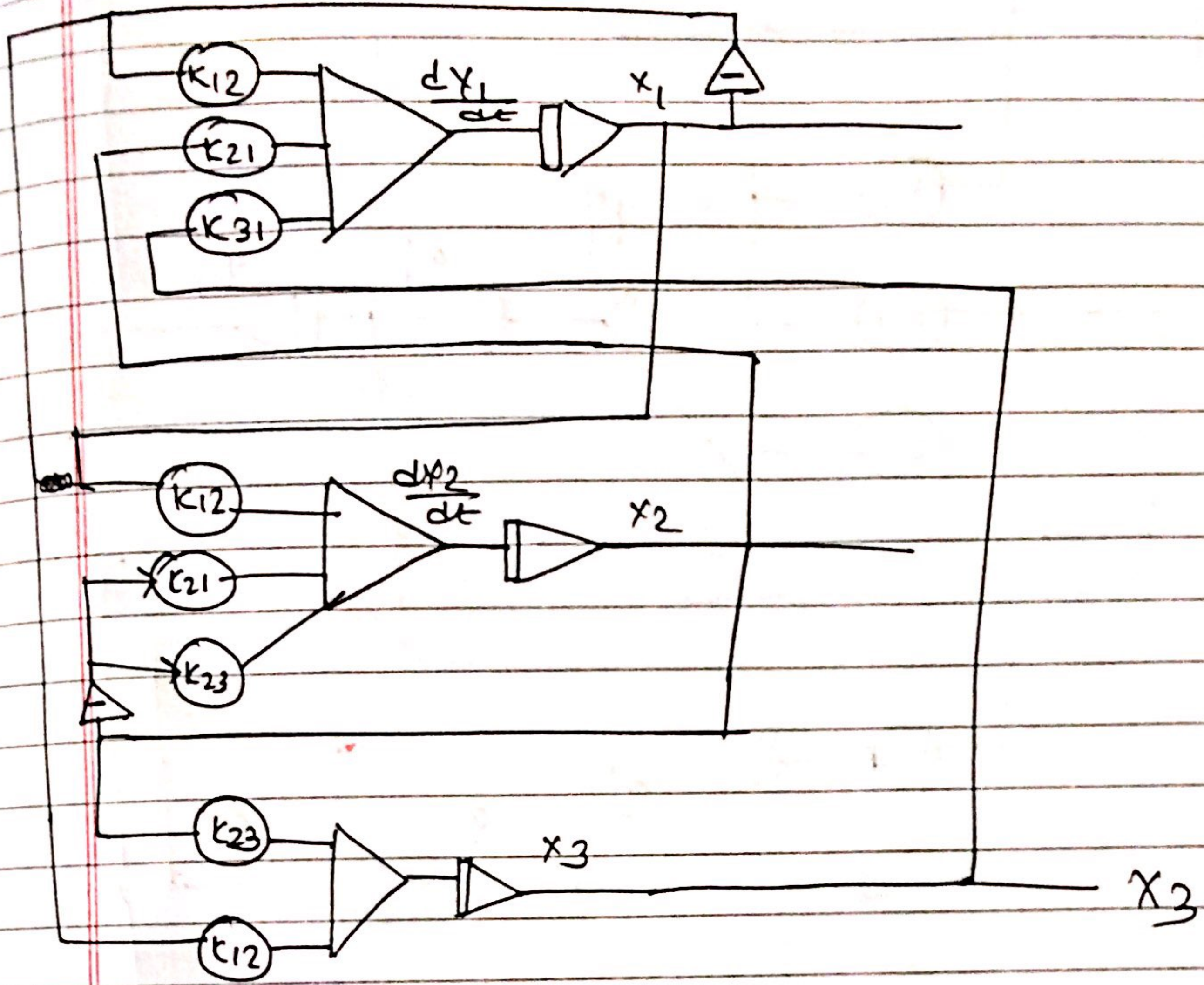
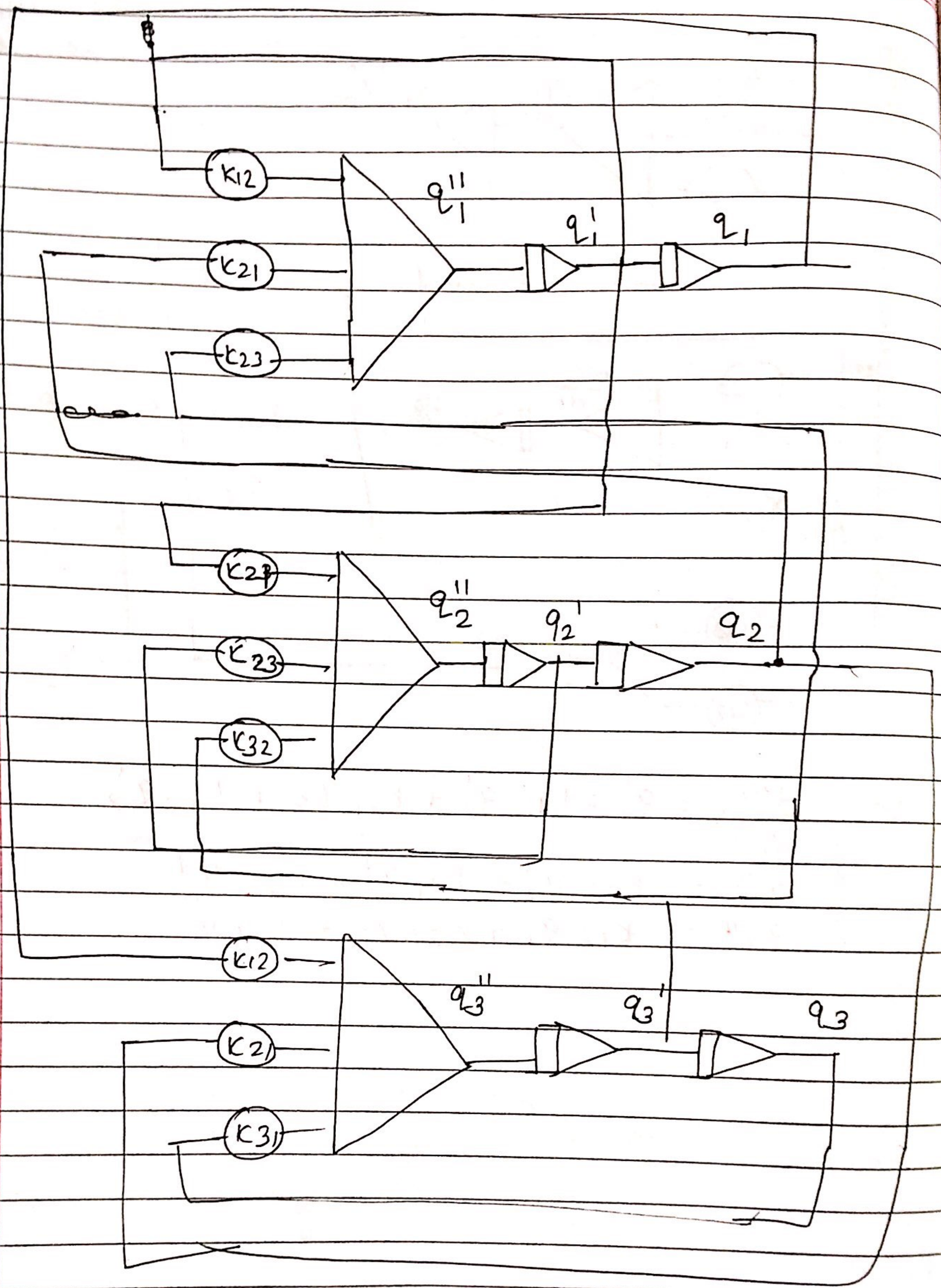


(Qn) $\frac{d^2q_1}{dt^2} = q_1'' = k_{12}\, q_1' + k_{21}\, q_2 + k_{23}\, q_3'$

$q_2'' = k_{23}\, q_2' + k_{21}\, q_1' + k_{32}\, q_3'$

$q_3'' = k_{12}\, q_1 + k_{21}\, q_2 + k_{31}\, q_3$



K12
q_1''
K21
q_1'
q_1
K23
K23
q_2''
K23
q_2'
q_2
K32
K12
q_3''
q_3'
q_3
K21
K31

2012 fall

(7b) Mention the system, entities attributes and activities of the following table:

System	Entities	Attributes	Activities
Bank	customers	Balance, credit ~~Depositing~~ status	Depositing
Traffic	cars	speed, distance	driving
Supermarket	customers	shopping List	Checking point
Communication	Messages	length Priority	transmitting
College	Teachers Student	Education cource	Teaching Practical

find the value of π using Monte Carlo method

let us consider a circle having radius 'r'
then area of quadrature of circle is given by

$\therefore$ Area of quadrature of circle (A) $= \frac{\pi r^2}{4}$ —— (i)

as shown in figure below

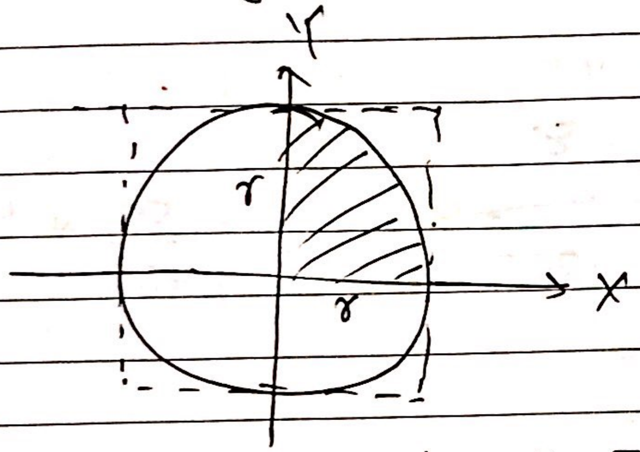


fig: determining the value of π using MC method

Now, the area square having length 'r' in fig above

$\therefore$ Area of square of quadrature (a) $= r^2$ —— (ii)

let 'N' be the total no. of points (sample) which falls with the area of square out of which 'n' be the no. of points falling within area of circle.
Then by monte-carlo

$$\frac{\text{Area of quadrature circle (A)}}{\text{area of quadrature square (a)}} = \frac{n}{N}$$

$$\frac{\left(\frac{\pi \cancel{r^2}}{4}\right)}{\cancel{r^2}} = \frac{n}{N}$$

$$\boxed{\pi = 4 * \frac{n}{N}}$$

where n, N be the random sample points of corresponding circle and sequence

let $N = 12$ then $\pi = \frac{10}{12} * 4 = 3.33$

$n = 10$

$$\boxed{\pi = 3.33}$$

let $N = 100$

$n = 80$

then ~~$\pi = \frac{11}{12} * 4$~~ $\pi = \frac{80}{100} * 4 = 3.2$

$$\boxed{\pi = 3.2}$$

Hence the value of Pi (π) using monte carlo simulation method can be determined using sampling random points.

As the no. of sampling point increases, the accuracy of value for π will increase.

classmate

Date

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2012 Fall

(2 b) Design analog computer from following electrical circuit system.

$$Mx'' + Dx' + Kx = f(t)$$

(target find x)

→

$$Mx'' + Dx' + Kx = f(t)$$

$$Mx'' = \underline{f(t)} + (-\underline{D}x') + (-\underline{K}x)$$

Constant: $f(t)$, K, D

target: to find x as O/P

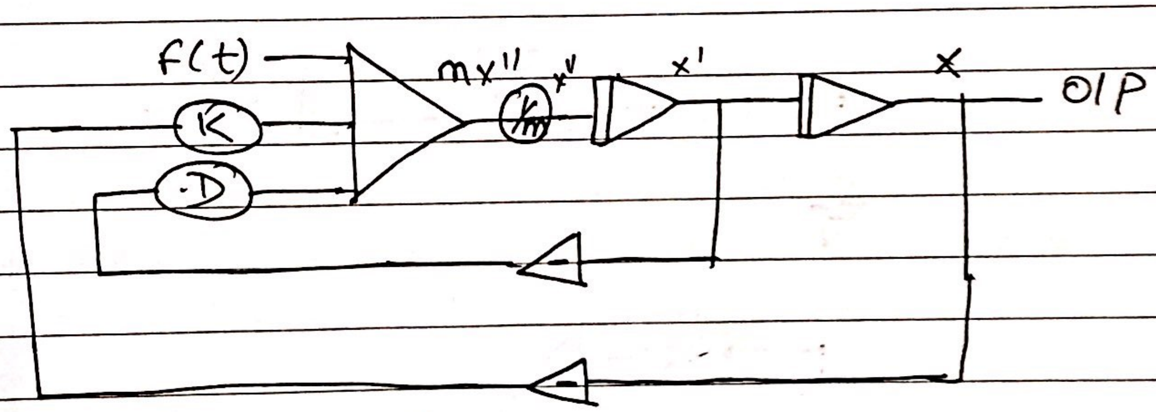


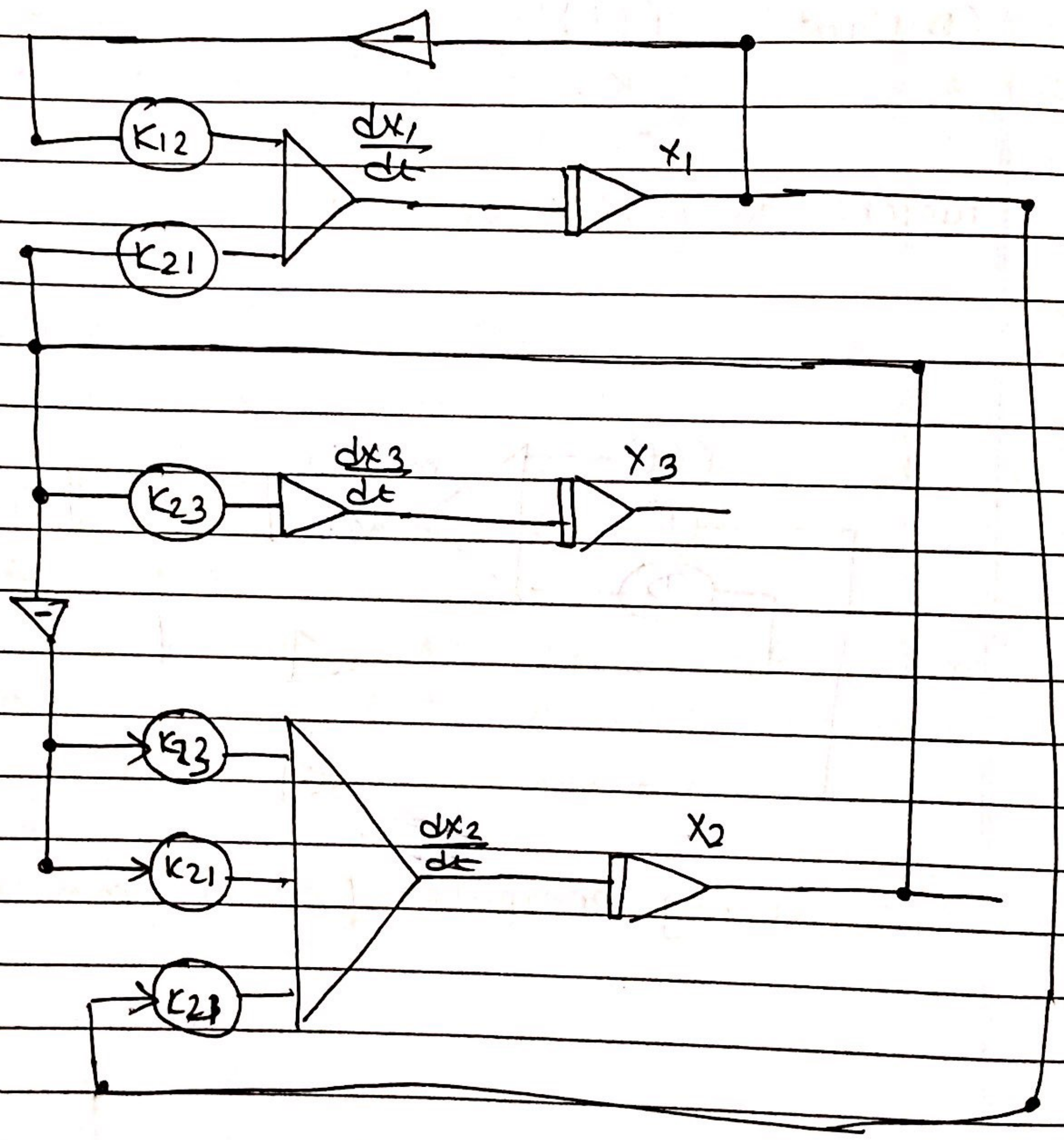
fig: analog computer model from given equation

2012 spring

(21b) $\frac{dx_1}{dt} = -k_{12}x_1 + k_{21}x_2$

$\frac{dx_3}{dt} = k_{23}x_2$

$\frac{dx_2}{dt} = k_{12}x_1 - (k_{21}+k_{23})x_2$



(2(b)) Find the growth in national consumption for five years using the model given. Assume the initial income Y_{-1} is 80 and take gov. exp. in 5 yrs to be follow

Year	1	2	3	4	5
G	20	25	30	35	40

C be consumption
I be investment
T be taxes
G be Government Expenditure
Y be national income

Then

$C = 20 + 0.7(Y - T)$

$I = 2 + 0.1Y$

$T = 0 + 0.2Y$

$Y = C + I + G$

This is static model, making dynamic by representing values on previous year called lagged variable. value of previous year denoted by suffix -1

$C = 20 + 0.7(Y_{-1} - T_{-1})$

$I = 2 + 0.1Y_{-1}$

$T = 0.2Y_{-1}$

$Y = C_{-1} + I_{-1} + G_{-1}$

Solve Y

$Y = \cancel{20} C + I + G$

$= 20 + 0.7(Y_{-1} - T_{-1}) + I + G$

$= 20 + 0.7(Y - 0.2Y) + I + G$

$Y = 20 + 0.56Y + I + G$

~~Then, we have~~ $Y = 45.45 + 2.27(I + G)$

$C = 0.2B$: $D = 20 + 0.7(B - C)$

~~ARQ~~:
X! Y! $A = 2.0 + 0.1Y$: $B = 45.45 + 2.27(X + A)$:

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$I = 2.0 + 0.1\,Y_{-1}$ $Y_0 = 80$
$Y = 45.45 + 2.27\,(I + G)$
$T = 0.2Y$
$C = 20 + 0.7\,(Y - T)$

Year	G	$I = 2.0 + 0.1Y_{-1}$	$Y = 45.45 + 2.27(I+G)$	$T = 0.2Y$	$C = 20 + 0.7(Y-T)$
1	20	10	113.55	22.71	83.588
2	25	13.355	132.51	26.5	94.20
3	30	15.251	148.16	29.63	102.975
4	35	16.816	163.07	32.61	111.32
5	40	18.307	177.80	35.56	119.57

(21a) Explain, derive and write the CSMP-III program for simulation of an Auto-pilot.

2014 fall

Chi square for 95% accuracy

See data carefully, all data are in between 1000 − 1050

total no. of sample = 42

i	Class	Observed o_i	expected e_i	$(o_i - e_i)^2$	$\frac{(o_i-e_i)^2}{e_i}$
1	1000−1009	4	8	$4^2 = 16$	2
2	1010−1019	8	8	$0^2 = 0$	0
3	1020−1029	16	8	$8^2 = 64$	8
4	1030−1039	4	8	$4^2 = 16$	2
5	1040−1049	10	8	$2^2 = 4$	0.5
					12.5

$$e_i = \frac{\Sigma o_i}{\text{no. of class}} = \frac{42}{5} = 8.4 \approx 8$$

degree of freedom $(N-1) = 5-1 = 4$

for $\alpha = 95\%$

from calculation $\chi^2_{calc} = 12.5$

from table $\chi^2_{table} = 0.711$

Here, $\chi^2_{calc} > \chi^2_{table}$

$12.5 > 0.711$

hence, not uniformly distributed.

1000 - 1010	|||||
1010 - 1020	|||||||
1021 - 1030	|||||||||||||||||
1031 - 1040	|||||
1041 - 1050	||||||||||||

1000 - 1009	||||	4
1010 - 1019	||||||||	8
1020 - 1029	||||||||||||||||	16
1030 - 1039	||||	4
1040 - 1049	||||||||||	10

classmate

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2013 Fall

(1b) Integrate MC: $\sin x \cos x \, dx$ $0 <= x <= \pi/2$

$$\int_0^{\pi/2} \sin x \cos x \, dx$$

Standard value:

$$= \frac{1}{2} \int_0^{\pi/2} 2 \sin x \cos x \, dx$$

$$= \frac{1}{2} \int_0^{\pi/2} \sin 2x \, dx$$

$$= \frac{1}{2} * - \left(\frac{\cos 2x}{2} \right)_0^{\pi/2}$$

$$= -\frac{1}{4} \left[\cos(2 * \pi/2) - \cos 0 \right]$$

$$= \frac{-1}{4} * (\cos \pi - \cos 0)$$

$$= \frac{-1}{4} * (-2)$$

$$= \frac{1}{2}$$

Or, use calculator

By monte carlo method

$f(x) = \sin x \cos x$

$b = \pi/2$ $a = 0$

$$\frac{d(u,v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

For optimum value of $f(x)$

$f'(x) = 0$

$\frac{d(\sin x \cos x)}{dx} = 0 \Rightarrow \sin x \cdot (-\sin x) + \cos x \cdot \cos x$

$\cos^2 x - \sin^2 x = 0$

$\sin x = \cos x$ OR

$\therefore x = \sin^{-1}(1/\sqrt{2})$

$\cancel{\sin x} = \cancel{\cos x}(1 - \sin^2 x) - \sin^2 x$

$1 - 2\sin^2 x = 0$

$$\sin^2 x = \frac{1}{2}$$

$$\sin x = \frac{1}{\sqrt{2}}$$

$$x = \sin^{-1}\left(1/\sqrt{2}\right)$$

$$x = 45^\circ \text{ or } \frac{\pi}{4}$$

$$\therefore C = f(x) = \sin\frac{\pi}{4} \cdot \cos\frac{\pi}{4} = \frac{1}{2}$$

Now by monte carlo method, we have

$$\frac{n}{N} = \frac{\int_a^b f(x)\,dx}{C * (b-a)}$$

$$\int_a^b f(x)\,dx = \frac{n}{N} * C * (b-a)$$

$$= \frac{n}{N} * 0.5 * \left(\frac{\pi}{2} - 0\right)$$

$$= \frac{n}{N} * \frac{\pi}{4}$$

i	N	n	$\int_a^b f(x)\,dx$	error $= \left(\frac{S_i - O_i}{S_i}\right) \times 100\%$
1	1000	500	$\pi/8$	21.46 %
2	1000	600	$3\pi/20$	5.75 %
3	1000	650	$13\pi/80$	2.10 %
4	10,000	6880	0.540	8.07 %
5	1000	660	$33\pi/200$	3.67 %

2012 Spring

1(b) find Y using MC

$$Y = \int_2^6 \frac{7x}{5}\, dx$$

→ $f(x) = \frac{7x}{5}$

x	2	3	4	5	6
y	2.8	4.2	5.6	7	8.4

$b = 6 \quad a = 2$

Standard value :

$$\int_a^b f(x)\, dx = \int_2^6 \frac{7x}{5}\, dx$$

$$= \frac{7}{5}\left[\frac{x^2}{2}\right]_2^6$$

$$= \frac{7}{5 \times 2}\left[6^2 - 2^2\right]$$

$$= \frac{112}{5} \approx 22.4$$

By monte-carlo

$b = 6$

$a = 2$

To find c, for maximum value of f(x) or y

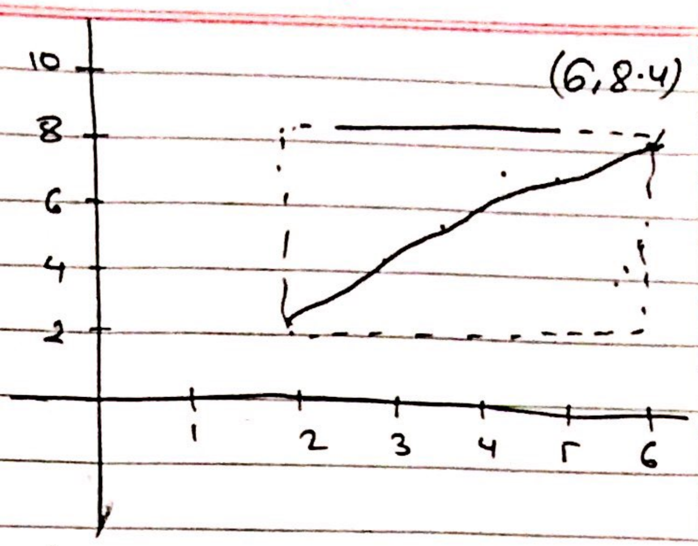
$f'(x) = 0$

$\frac{d}{dx}\left(\frac{7x}{5}\right) = 0$

$x = 0$

$\therefore \; c = f(x) =$

$$y = \int_2^6 \frac{7x}{5}\,dx$$



x	2	3	4	5	6
y	2.8	4.2	5.6	7	8.4

Area of rectangle $= l \times b$

$= (6-2) * (8.4 - 2.8)$

$= 4 * 5.6$

$= 22.4$

By monte carlo

$$\frac{n}{N} = \frac{\cancel{\int_2^6 \frac{7x}{5}\,dx}}{} = \frac{\text{Area of curve}}{\text{Area of rectangle}}$$

$$\frac{n}{N} = \frac{\int_2^6 \frac{7x}{5}\,dx}{22.4}$$

$$\int_2^6 \frac{7x}{5}\,dx = 22.4 * \frac{n}{N}$$

let $N = 1000$

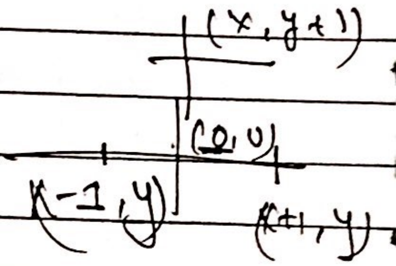
2011

(1/b) A drunkard ....

This type of Problem, where a person walks at random in different direction is called random walk. In this simple case, the position of the drunkard can easily be determined by using the probability theory. But we will simulate the walk using random numbers.

Using simple digit random number, the random number can be allocated to steps in different direction as under

Direction	Probability	Random no.
Forward (F)	0.5	0, 1, 2, 3, 4
Left (L)	0.3	5, 6, 7
Right (R)	0.2	8, 9



initially (0, 0)

Step	Random Number	Direction	Position X, Y
1	6	L	-1, 0
2	2	F	-1, 1
3	0	F	-1 2
4	6	L	-2, 2
5	8	R	-1, 2
6	5	L	-2, 2
7	7	L	-3, 2
8	7	L	-4, 2
9	9	R	-3, 2
10	8	R	-2, 2

F	0·5	0, 1, 2, 3, 4	Y + +
L	0·3	5, 6, 7	X − −
R	0·2	8, 9	X + +

Step	Random Number	Direction	Position (X, Y)
11	4	F	−2, 3
12	8	R	−1, 3
13	2	F	−1, 4
14	6	L	−2, 4
15	2	F	−2, 5
16	1	F	−2, 6
17	3	F	−2, 7
18	0	F	−2, 8
19	8	R	−1, 8
20	4	F	−1, 9
21	9	R	0, 9
22	0	F	0, 10
23	8	R	1, 10
24	6	L	0, 10
25	5	L	−1, 10
26	1	F	−1, 11
27	0	F	−1, 12
28	9	R	0, 12
29	9	R	1, 12
30	8	R	2, 12
31	7	L	1, 12
32	4	F	1, 13
33	3	F	1, 14
34	1	F	1, 15
35	8	R	2, 15
36	5	L	1, 15
37	2	F	1, 16
38	1	F	1, 17
39	0	F	1, 18
40	8	R	2, 18

STEP	Random Number	Direction	Position (X, Y)
41	6	L	1, 18
42	7	L	0, 18
43	8	R	1, 18
44	4	F	1, 19
45	9	R	2, 19
46	3	F	2, 20
47	0	F	2, 21
48	5	L	1, 21
49	1	F	1, 22
50	2	F	1, 23

S/He will be at (1, 23) cordinate position

Comparision of simulation and Analytical method

Both Analytical and Simulation methods are modeling approaches which aim at providing an idea of system performance, in different conditions.

An analytical model is a mathematical abstraction that can be extended to address various ~~tcint~~ working condition. It provides a generic way to get performance results in various conditions through a mathematical formulation. The Accuracy of the model is to be considered through the validity of the assumption to derive the mathematical formulation. Some uncertainties can be handled through a stochastic model to account for modeling and measurement model.

A simulation method such as a monte-carlo also make assumption of a model and some about the behaviour of the process. It is used when it is too complex for analytical formulation (ie - when size of model is too large
 - when no exact solution can be derived)
Simulation model provides specific use case and should be run many times to counterbalance the effect of numerical calculation. A simulation model accepted when result are validated in a number of working condition under various input assumption.

When two approaches can be used, preference should given to analytical approach & Simulation used for validate the assumption