

Chapter 4 Understanding Money and Its Management

Nominal and Effective Interest Rates

4.1

- (a) Monthly interest rate: $i = 12.5\% \div 12 = 1.04\%$
Annual effective rate: $i_a = (1 + 0.0104)^{12} - 1 = 13.22\%$

(b) $\$2,000(1 + 0.0104)^2 = \$2,041.82$

4.2

- Nominal interest rate:
 $r = 1.05\% \times 12 = 12.6\%$
- Effective annual interest rate:
 $i_a = (1 + 0.0105)^{12} - 1 = 13.35\%$

4.3 Assuming a weekly compounding

$$r = 6.89\%$$
$$i_a = \left(1 + \frac{0.0689}{52}\right)^{52} - 1 = 0.07128$$

4.4

- Interest rate per week

Given : $P = \$500, A = 40, N = 16$ weeks

$$\$500 = \$40(P / A, i, 16)$$

$$i = 3.06\% \text{ per week}$$

- Nominal annual interest rate:

$$r = 3.06\% \times 52 = 159.12\%$$

- Effective annual interest rate:

$$i_a = (1 + 3.06\%)^{52} - 1 = 379.39\%$$

- 4.5 Interest rate per week: (**Correction:** In the first printing, the lending amount was stated as \$40. It should be \$400.)

$$\$450 = \$400(1 + i)$$

$$i = 12.5\% \text{ per week}$$

- (a) Nominal interest rate:

$$r = 12.5\% \times 52 = 650\%$$

- (b) Effective annual interest rate

$$i_a = (1 + 0.125)^{52} - 1 = 45,601\%$$

4.6

- 24-month lease plan:

$$\begin{aligned} P &= (\$2,500 + \$520 + \$500) + \$520(P/A, 0.5\%, 23) - \$500(P/F, 0.5\%, 24) \\ &= \$14,347 \end{aligned}$$

- Up-front lease plan:

$$\begin{aligned} P &= \$12,780 + \$500 - \$500(P/F, 0.5\%, 24) \\ &= \$12,836 \end{aligned}$$

\therefore Select up-front lease plan

- 4.7 No. Since the debt interest rates are higher than the return on the investment funds, it is better to pay off the debt.

Compounding More Frequent than Annually

4.8

- (a) Nominal interest rate:

$$r = 1.8\% \times 12 = 21.6\%$$

- (b) Effective annual interest rate

$$i_e = (1 + 0.018)^{12} - 1 = 23.87\%$$

(c)

$$\begin{aligned}
 3P &= P(1+0.018)^N \\
 \log 3 &= N \log 1.018 \\
 N &= 61.58 \text{ months} \\
 \therefore 61.58 / 12 &= 5.13 \text{ years}
 \end{aligned}$$

(d)

$$\begin{aligned}
 3 &= e^{0.018N} \\
 \ln(3) &= 0.018N \\
 N &= 61.03 \text{ months} \\
 \therefore 61.03 / 12 &= 5.08 \text{ years}
 \end{aligned}$$

4.9

$$F = \$10,000\left(1 + \frac{0.09}{4}\right)^4 = \$10,930.83$$

4.10

$$(a) F = \$5,635\left(1 + \frac{0.05}{2}\right)^{20} = \$9,233.60$$

$$(b) F = \$7,500\left(1 + \frac{0.06}{4}\right)^{60} = \$18,324.15$$

$$(c) F = \$38,300\left(1 + \frac{0.09}{12}\right)^{84} = \$71,743.64$$

4.11

(a) Quarterly interest rate = 2.25%

$$\begin{aligned}
 3P &= P(1+0.0225)^N \\
 \log 3 &= N \log 1.0225 \\
 N &= 49.37 \text{ quarters} \\
 \therefore 49.37 / 4 &= 12.34 \text{ years}
 \end{aligned}$$

(b) Monthly interest rate = 0.75%

$$\begin{aligned}
 3P &= P(1+0.0075)^N \\
 \log 3 &= N \log 1.0075 \\
 N &= 147.03 \text{ months} \\
 \therefore 147.03 / 12 &= 12.25 \text{ years}
 \end{aligned}$$

(c)

$$3 = e^{0.09N}$$

$$\ln(3) = 0.09N$$

$$N = 12.21 \text{ years}$$

4.12

(a) Quarterly interest rate = 2.25%

$$P = \$5,000(P / A, 0.0225\%, 48) = \$145,848$$

(b) Quarterly effective interest rate = 2.2669%

$$P = \$5,000(P / A, 0.022669\%, 48) = \$145,360$$

(c) Quarterly effective interest rate = 2.2755%

$$P = \$5,000(P / A, 0.022755\%, 48) = \$145,112$$

4.13

$$F = A(F / A, i, 20) = \$3,000(F / A, 2.02\%, 20) = \$73,037$$

4.14

(a) Quarterly interest rate = 1.5%

$$F = \$2,000(F / A, 0.015\%, 60) = \$192,429$$

(b) Quarterly effective interest rate = 1.5075%

$$F = \$2,000(F / A, 0.015075\%, 60) = \$192,912$$

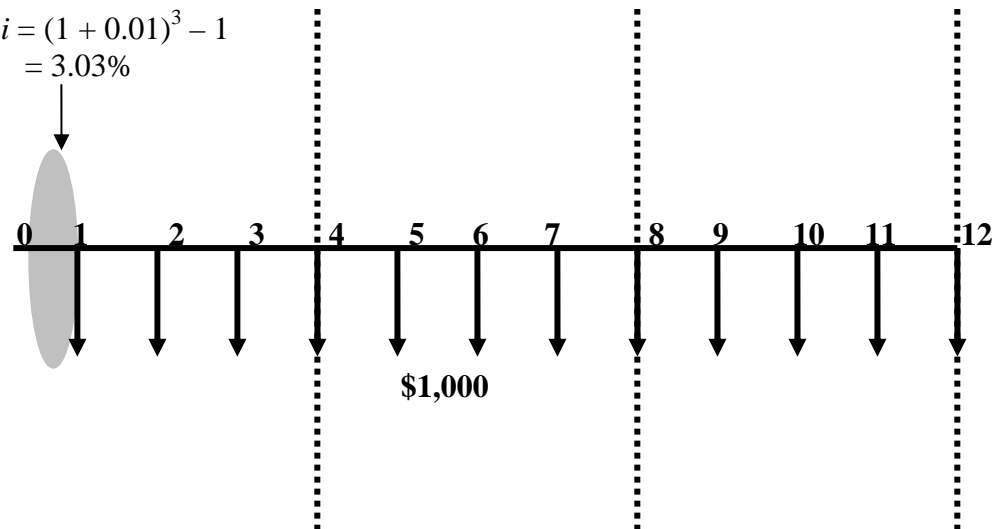
(c) Quarterly effective interest rate = 1.5113%

$$F = \$2,000(F / A, 0.015113\%, 60) = \$193,157$$

4.15 (d)

Effective interest rate per
payment period

$$i = (1 + 0.01)^3 - 1 = 3.03\%$$



4.16

$$i = e^{0.085/4} - 1 = 2.1477\%$$

$$A = \$12,000(A/P, 2.1477\%, 20) = \$744$$

4.17

(a) Monthly effective rate = 0.9902%

$$F = \$2,500(F/A, 0.9902\%, 96) = \$397,670.75$$

(b) Monthly effective rate = 1.00%

$$F = \$2,500(F/A, 1\%, 96) = \$399,818.25$$

(c) Monthly effective interest rate = 1.005%

$$F = \$2,500(F/A, 1.005\%, 96) = \$400,909.25$$

4.18 (b)

4.19 Equivalent present worth of the series of equal quarterly payments of \$2,000 over 15 years at 8% compounded continuously:

$$i = e^{0.02} - 1 = 2.02013\%$$

$$P = \$2,000(P / A, 2.02013\%, 60) = \$69,184$$

Equivalent future worth of \$69,184 at the end of 10 years:

$$F = \$69,184(F / P, 2.02013\%, 40) = \$153,972$$

4.20 Nominal interest rate per quarter = 1.95%

$$\text{Effective interest rate per quarter} = e^{0.0195} - 1 = 1.9691\%$$

$$A = \$32,000(A / P, 1.9691\%, 20) = \$1,952$$

4.21 Nominal interest rate per quarter = 2.1875%

$$\text{Effective interest rate per quarter} = e^{0.021875} - 1 = 2.2116\%$$

$$P = \$4,000(A / P, 2.2116\%, 12) = \$41,756.8$$

4.22

$$(a) F = \$6,000(F / A, 3\%, 12) = \$85,152$$

$$(b) F = \$42,000(F / A, 2\%, 48) = \$3,332,847$$

$$(c) F = \$75,000(F / A, 0.75\%, 96) = \$10,489,215$$

4.23

$$(a) A = \$21,000(A / F, 3.225\%, 20) = \$764.4$$

$$(b) A = \$9,000(A / F, 2.3375\%, 60) = \$70.2$$

$$(c) A = \$24,000(A / F, 0.5458\%, 60) = \$338.4$$

4.24

$$i = e^{\frac{0.05}{365}} - 1 = 0.0136996\%$$

$$F = \$2.5(P / A, 0.0136996\%, 10950) = \$63,536.61$$

4.25

$$\$24,000 = \$543.35(P / A, i, 48)$$

$$i = 0.3446\%$$

$$i_a = (1 + 0.003446)^{12} - 1 = 4.21\%$$

4.26

$$\$12,000 = \$445(P / A, i, 30)$$

$$i = 0.7021\%$$

$$r = 0.007021 \times 12 = 8.43\%$$

4.27

$$A = \$22,000(A / F, 0.5\%, 24)$$

$$= \$865.05$$

4.28

$$(a) P = \$1,500(P / A, 4\%, 24) = \$22,870.5$$

$$(b) P = \$2,500(P / A, 2\%, 32) = \$58,670.75$$

$$(c) P = \$3,800(P / A, 0.75\%, 60) = \$183,058.92$$

4.29

- Equivalent future worth of the receipts:

$$\begin{aligned} F_1 &= \$1,500(F / P, 2\%, 4) + \$2,500 \\ &= \$4,123.60 \end{aligned}$$

- Equivalent future worth of deposits:

$$\begin{aligned} F_2 &= A(F / A, 2\%, 8) + A(F / P, 2\%, 8) \\ &= 9.7546A \end{aligned}$$

\therefore Letting $F_1 = F_2$ and solving for A yields $A = \$422.73$

4.30

- The balance just before the transfer:

$$\begin{aligned}
 F_9 &= \$15,000(F / P, 0.5\%, 108) + \$14,000(F / P, 0.5\%, 72) \\
 &\quad + \$12,500(F / P, 0.5\%, 48) \\
 &= \$61,635.22
 \end{aligned}$$

Therefore, the remaining balance after the transfer will be \$30,817.61. This remaining balance will continue to grow at 6% interest compounded monthly. Then, the balance 6 years after the transfer will be

$$F_{15} = \$30,817.61(F / P, 0.5\%, 72) = \$44,132.18$$

- The funds transferred to another account will earn 8% interest compounded quarterly. The resulting balance six years after the transfer will be:

$$F_{15} = \$30,817.61(F / P, 2\%, 24) = \$49,568.19$$

- 4.31 Establish the cash flow equivalence at the end of 25 years. Let's define A as the required quarterly deposit amount. Then we obtain the following:

$$\begin{aligned}
 A(F / A, 1.5\%, 100) &= \$60,000(P / A, 6.136\%, 10) \\
 228.8038A &= \$438,774 \\
 A &= \$1,917.69
 \end{aligned}$$

4.32

- Monthly installment amount:

$$A = \$15,000(A / P, 1\%, 48) = \$394.5$$

- The lump-sum amount for the remaining:

$$P_{20} = \$394.5(P / A, 1\%, 28) = \$9,592.82$$

4.33

$$\begin{aligned}
 \$100,000 &= \$1,000(P / A, 0.75\%, N) \\
 (P / A, 0.75\%, N) &= 100 \\
 N &= 186.16 \text{ months or } 15.5 \text{ years}
 \end{aligned}$$

4.34

$$\begin{aligned}
 \$20,000 &= \$922.90(P/A, i, 24) \\
 (P/A, i, 24) &= 21.6708 \\
 i &= 0.8333\% \\
 APR &= 0.8333\% \times 12 = 10\%
 \end{aligned}$$

4.35 Given $r = 6\%$ per year compounded monthly, the effective annual rate is 6.186%.

Now consider the four options:

1. Buy 3 single-year subscriptions at \$66 each.
2. Buy a single-year (\$66) subscription now, and buy a two-year (\$120) subscription next year
3. Buy a two-year (\$120) subscription now, and buy a single-year (\$66) subscription at its completion
4. Buy a three-year subscription (\$160) now.

To find the best option, compute the equivalent PW for each option.

$$\begin{aligned}
 \circ \quad P_{\text{option 1}} &= \$66 + \$66(P/A, 6.186\%, 2) = \$186.69 \\
 \circ \quad P_{\text{option 2}} &= \$66 + \$120(P/F, 6.186\%, 1) = \$179.01 \\
 \circ \quad P_{\text{option 3}} &= \$120 + \$66(P/F, 6.186\%, 2) = \$178.53 \\
 \circ \quad P_{\text{option 4}} &= \$160
 \end{aligned}$$

\therefore Option 4 is the best option.

4.36 Given $r = 6\%$ per year compounded quarterly, the quarterly interest rate is 1.5% and the effective annual rate is 6.186%. To find the amount of quarterly deposit (A), we establish the following equivalence relationship:

$$\begin{aligned}
 A(F/A, 1.5\%, 60) &= \$50,000 + \$50,000(P/A, 6.136\%, 3) \\
 A &= \$183,314/96.2147 \\
 A &= \$1,905.26
 \end{aligned}$$

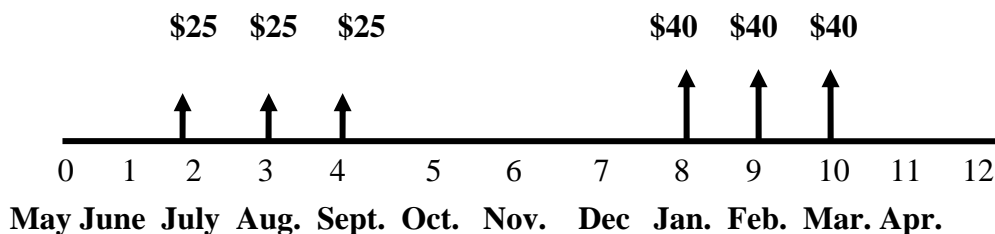
4.37 Setting the equivalence relationship at the end of 15 years gives

$$\begin{aligned} A(F/A, 2\%, 60) &= \$25,000(P/A, 4.04\%, 10) \\ 114.0515A &= \$202,369 \\ A &= \$1,774.37 \end{aligned}$$

4.38 Given $i = \frac{6\%}{12} = 0.5\%$ per month

$$\begin{aligned} A &= \$250,000(A/P, 0.5\%, 120) \\ &= \$2,775 \end{aligned}$$

4.39 First compute the equivalent present worth of the energy cost savings during the first operating cycle:



$$\begin{aligned} P &= \$25(P/A, 0.75\%, 3)(P/F, 0.75\%, 1) + \$40(P/A, 0.75\%, 3)(P/F, 0.75\%, 7) \\ &= \$185 \end{aligned}$$

Then, compute the total present worth of the energy cost savings over 5 years.

$$\begin{aligned} P &= \$185 + \$185(P/F, 0.75\%, 12) + \$185(P/F, 0.75\%, 24) \\ &\quad + \$185(P/F, 0.75\%, 36) + \$185(P/F, 0.75\%, 48) \\ &= \$779.37 \end{aligned}$$

Continuous Payments with Continuous Compounding

4.40 Given $i = 12\%$, $N = 12$ years, and $\bar{A} = \$63,000 \times 365 = \$22,995,000$

- Daily payment with daily compounding:

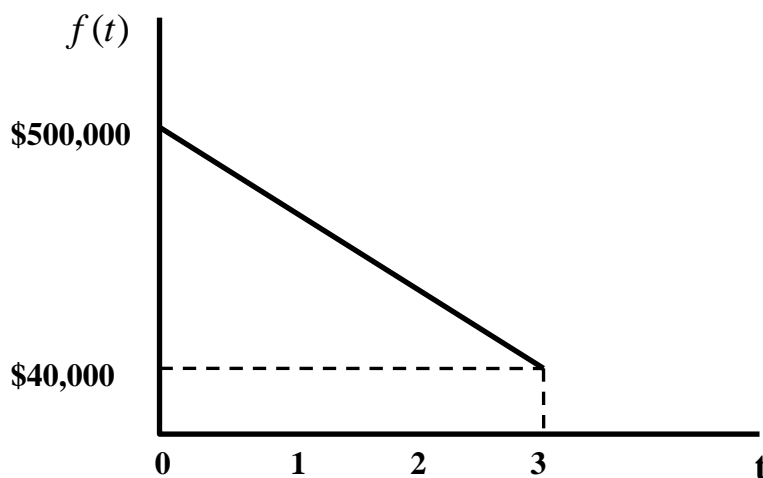
$$P = \$63,000(P/A, 12\% / 365, 4380) = \$146,212,973$$

- Continuous payment and continuous compounding:

$$\begin{aligned}
 P &= \int_0^{12} \bar{A} e^{rt} dt \\
 &= \$22,995,000 \left[\frac{e^{(0.12)(12)} - 1}{0.12 e^{(0.12)(12)}} \right] \\
 &= \$146,223,718
 \end{aligned}$$

\therefore The difference between the two compounding schemes is only \$10,745.

4.41 Given $i = 11\%$, $N = 3$, $F_0 = \$500,000$, $F_N = \$40,000$, and $0 \leq t \leq 3$,



$$\begin{aligned}
 f(t) &= 500,000 - \frac{460,000}{3}t \\
 P &= \int_0^3 (500,000 - \frac{460,000}{3}t) e^{-rt} dt \\
 &= 500,000 \left[\frac{1 - e^{-rN}}{r} \right] - \frac{F_0 - F_N}{Nr^2} [-rNe^{-rN} - e^{-rN} + 1] \\
 &= \$1,277,619.39 - \$555,439.67 \\
 &= \$722,179.72
 \end{aligned}$$

4.42 Given $r = 9\%$, $\bar{A} = \$80,000$, $N_s = 2$, $N_e = 7$,

$$\begin{aligned}
 P &= \int_2^7 80,000 e^{-rt} dt \\
 &= \$80,000 \left[\frac{e^{-0.09(2)} - e^{-0.09(7)}}{0.09} \right] \\
 &= \$269,047.48
 \end{aligned}$$

4.43

$$\begin{aligned}
y_t &= 5e^{-0.25t}, & u_t &= \$55(1 + 0.09t) \\
y_t u_t &= 275e^{-0.25t} + 24.75te^{-0.25t} \\
P &= \int_0^{20} 275e^{-0.25t} e^{-0.12t} dt + \int_0^{20} 24.75te^{-0.25t} e^{-0.12t} dt \\
&= 275 \left[\frac{e^{0.37(20)} - 1}{0.37e^{0.37(20)}} \right] + \frac{24.75}{0.37^2} (1 - e^{-0.37(20)}) - \frac{24.75}{0.37} (20e^{-0.37(20)}) \\
&= \$742.79 + \$179.86 = \$922.65
\end{aligned}$$

Changing Interest Rates

4.44 Given $r_1 = 6\%$ compounded quarterly, $r_2 = 10\%$ compounded quarterly, and $r_3 = 8\%$ compounded quarterly, indicating that $i_1 = 1.5\%$ per quarter, $i_2 = 2.5\%$ per quarter, and $i_3 = 2\%$ per quarter.

(a) Find P :

$$\begin{aligned}
P &= \$2,000(P / F, 1.5\%, 4) + \$2,000(P / F, 1.5\%, 8) \\
&\quad + \$3,000(P / F, 2.5\%, 4)(P / F, 1.5\%, 8) + \$2,000(P / F, 2.5\%, 8)(P / F, 1.5\%, 8) \\
&\quad + \$2,000(P / F, 2\%, 4)(P / F, 2.5\%, 8)(P / F, 1.5\%, 8) \\
&= \$8,875.42
\end{aligned}$$

(b) Find F :

$$\begin{aligned}
F &= P(F / P, 1.5\%, 8)(F / P, 2.5\%, 8)(F / P, 2\%, 4) \\
&= \$13,186
\end{aligned}$$

(c) Find A , starting at 1 and ending at 5:

$$\begin{aligned}
F &= A + A(F / P, 2\%, 4) + A(F / P, 2.5\%, 4)(F / P, 2\%, 4) \\
&\quad + A(F / P, 2.5\%, 8)(F / P, 2\%, 4) + A(F / P, 1.5\%, 4)(F / P, 2.5\%, 8)(F / P, 2\%, 4) \\
&= 5.9958A
\end{aligned}$$

$$A = \frac{\$13,186}{5.9958} = \$2,194.22$$

4.45

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(a)

$$\begin{aligned}
 P &= \$300(P/F, 0.5\%, 12) + \$300(P/F, 0.75\%, 12)(P/F, 0.5\%, 12) \\
 &\quad + \$500(P/F, 0.75\%, 24)(P/F, 0.5\%, 12) \\
 &\quad + \$500(P/F, 0.5\%, 12)(P/F, 0.75\%, 24)(P/F, 0.5\%, 12) \\
 &= \$1,305.26
 \end{aligned}$$

(b)

$$\begin{aligned}
 \$1,305.26 &= \$300(P/A, i, 2) + \$500(P/A, i, 2)(P/F, i, 2) \\
 i &= 7.818\% \text{ per year}
 \end{aligned}$$

4.46 Since payments occur annually, you may compute the effective annual interest rate for each year.

$$i_1 = \left(1 + \frac{0.09}{365}\right)^{365} - 1 = 9.416\%, \quad i_2 = e^{0.09} - 1 = 9.417\%$$

$$\begin{aligned}
 F &= \$400(F/P, 9.416\%, 2)(F/P, 9.417\%, 2) + \$250(F/P, 9.416\%, 1)(F/P, 9.417\%, 2) \\
 &\quad + \$100(F/P, 9.417\%, 2) + \$100(F/P, 9.417\%, 1) + \$250 \\
 &= \$1,379.93
 \end{aligned}$$

Amortized Loans

4.47 Loan repayment schedule for the first 6 months:

End of month (<i>n</i>)	Interest Payment	Repayment of Principal	Remaining Balance
1	\$150.00	\$347.7	\$19,652.30
2	\$147.39	\$350.31	\$19,301.99
3	\$144.76	\$352.94	\$18,949.05
4	\$142.12	\$355.58	\$18,593.47
5	\$139.45	\$358.25	\$18,235.22
6	\$136.76	\$360.94	\$17,874.28

4.48

(a)

$$(i) \$10,000(A/P, 0.75\%, 24)$$

(b)

$$(iii) B_{12} = A(P/A, 0.75\%, 12)$$

4.49 Given information:

$$i = 8.48\% / 365 = 0.02323\% \text{ per day}, N = 36 \text{ months.}$$

- Effective monthly interest rate, $i = (1 + 0.0002323)^{30} - 1 = 0.6993\% \text{ per month}$
- Monthly payment, $A = \$10,000(A/P, 0.6993\%, 36) = \315 per month
- Total interest payment, $I = \$315 \times 36 - \$10,000 = \$1,340$

4.50 Given Data: Purchase price = \$18,000, Down payment = \$1,800, Monthly payment = \$421.85, $N = 48$ end of month payments.

(a) Using the bank loan at 11.75% compound monthly

$$A = \$16,200(A/P, (11.75/12)\%, 48) = \$424.62$$

(b) Using the dealer's financing, find the effective interest rate:

$$\$421.85 = \$16,200(A/P, i, 48)$$

$$i = 0.95\% \text{ per month}$$

$$r = 0.95\% \times 12 = 11.40\%$$

4.51 Given Data: $P = \$25,000$, $r = 10\%$ compounded monthly, $N = 36$ month $i = 0.8333\% \text{ per month}$.

- Required monthly payment:

$$A = \$25,000(A/P, 0.8333\%, 36) = \$807.5$$

- The remaining balance immediately after the 20th payment:

$$B_{20} = \$807.5(P/A, 0.8333\%, 16) = \$12,048.87$$

4.52 Given Data: $P = \$200,000 - \$20,000 = \$180,000$.

- Option 1:

$$N = 20 \text{ years} \times 12 = 240 \text{ months}$$

$$\text{APR} = 9\%$$

$$\therefore A = \$180,000(A/P, 9\% / 12, 240) = \$1,619.46$$

- Option 2:

$$N = 30 \text{ years} \times 12 = 360 \text{ months}$$

$$\text{APR} = 10\%$$

$$\therefore A = \$180,000(A/P, 10\%/12, 360) = \$1,579.68$$

$$\therefore \text{Difference} = \$1,619.46 - \$1,579.68 = \$39.78$$

4.53

- The monthly payment to the bank: Deferring the loan payment for 6 months is equivalent to borrowing

$$\$4,800(F/P, 1\%, 6) = \$5,095.30$$

To payoff the bank loan over 36 months, the required monthly payment is

$$A = \$5,095.30(A/P, 1\%, 36) = \$169.24 \text{ per month}$$

- The remaining balance after making the 16th payment:

$$\$169.24(P/A, 1\%, 20) = \$3,054.03$$

- The loan company will pay off this remaining balance and will charge \$104 per month for 36 months. The effective interest rate for this new arrangement is:

$$\$3,054.03 = \$104(P/A, i, 36)$$

$$(P/A, i, 36) = 29.3657$$

$$i = 1.1453\% \text{ per month}$$

$$\therefore r = 1.1453\% \times 12 = 13.74\% \text{ per year}$$

4.54

(a)

$$A = \$200,000(A/P, \frac{8.5\%}{12}, 180) = \$1,969.4$$

(b) Remaining balance at the end of 6th payment:

$$B_6 = \$1,969.4(P/A, \frac{8.5\%}{12}, 174) = \$196,615.84$$

- Interest for the 7th payment = $(8.5\%/12) \times B_6 = \$1,392.70$

- Principal payment for the 7th payment = $\$1,969.4 - \$1,392.70 = \$576.7$

4.55

$$A = \$350,000(A/P, \frac{9\%}{12}, 240) = \$3,148.95$$

- Total payments over the first 5 years (60 months) = $\$3,148.95 \times 60 = \$188,937$
- Remaining balance at the end of 5 years:

$$B_{60} = \$3,148.95(P/A, 0.75\%, 180) = \$310,465.69$$

- Reduction in principal = $\$350,000 - \$310,465.69 = \$39,534.31$
- Total interest payments = $\$188,937 - \$39,534.31 = \$149,402.69$

4.56 The amount to finance = $\$400,000 - \$60,000 = \$340,000$

$$A = \$340,000(A/P, 0.75\%, 360) = \$2,735.64$$

Then, the minimum acceptable monthly salary (S) should be,

$$S = \frac{A}{0.25} = \frac{\$2,735.64}{0.25} = \$10,942.56$$

4.57 Given Data: purchase price = $\$150,000$, down payment (sunk equity) = $\$30,000$, interest rate = 0.75% per month, $N = 360$ months,

- Monthly payment:

$$A = \$120,000(A/P, 0.75\%, 360) = \$965.55$$

- Balance at the end of 5 years (60 months):

$$B_{60} = \$966.55(P/A, 0.75\%, 300) = \$115,056.50$$

- Realized equity = sales price – balance remaining – sunk equity:

$$\$185,000 - \$115,056.60 - \$30,000 = \$39,943.50$$

4.58 Given Data: interest rate = 0.75% per month, each family has the identical remaining balance prior to their 15th payment, that is, \$80,000:

∴ With equal remaining balances, all will pay the same interest.

$$\$80,000(0.0075) = \$600$$

4.59 Given Data: loan amount = \$130,000, point charged = 3%, $N = 360$ months, interest rate = 0.75% per month, actual amount loaned = \$126,100:

- Monthly repayment:

$$A = \$130,000(A/P, 0.75\%, 360) = \$1,046$$

- Effective interest rate on this loan

$$\$126,100 = \$1,046(P/A, i, 360)$$

$$i = 0.7787\% \text{ per month}$$

$$\therefore i_a = (1 + 0.007787)^{12} - 1 = 9.755\% \text{ per year}$$

4.60

(a)

$$\$35,000 = \$5,250(P/A, i, 5) + \$1,750(P/G, i, 5)$$

$$i = 6.913745\%$$

(b)

$$P = \$35,000$$

$$\text{Total payments} = \$5,250 + \$7,000 + \$8,750 + \$10,500 + \$12,250 = \$43,750$$

$$\text{Interest payments} = \$43,750 - \$35,000 = \$8,750$$

Period (n)	Beginning Balance	Interest Payment	Repayment of Principal	Remaining Balance
1	\$35,000.00	\$2,149.81	\$5,250	\$32,169.81
2	\$32,169.81	\$2,224.14	\$7,000	\$27,393.95
3	\$27,393.95	\$1,893.95	\$8,750	\$20,537.90
4	\$20,537.90	\$1,419.94	\$10,500	\$11,457.85
5	\$11,457.85	\$792.17	\$12,250	0

4.61 Given Data: $r = 7\%$ compounded daily, $N = 360$ years

- Since deposits are made at year end, the effective annual interest rate is

$$i_a = (1 + 0.07 / 365)^{365} - 1 = 7.25\%$$

- Total amount accumulated at the end of 25 years

$$\begin{aligned} F &= \$3,000(F / A, 7.25\%, 25) + \$150(F / G, 7.25\%, 25) \\ &= \$3,000(F / A, 7.25\%, 25) + \$150(P / G, 7.25\%, 25)(F / P, 7.25\%, 25) \\ &= \$280,626 \end{aligned}$$

4.62

(a) The dealer's interest rate to calculate the loan repayment schedule.

(b)

- Required monthly payment under Option A:

$$A = \$26,200(A / P, \frac{1.9\%}{12}, 36) = \$749.29$$

- Breakeven savings rate

$$\begin{aligned} \$24,048 &= \$749.29(P / A, i, 36) \\ i &= 0.6344\% \text{ per month} \\ r &= 0.6344\% \times 12 = 7.6129\% \text{ per year} \end{aligned}$$

As long as the decision maker's APR is greater than 7.6129%, the dealer's financing is a least cost alternative.

(c) The dealer's interest rate is only good to determine the required monthly payments. The interest rate to be used in comparing different options should be based the earning opportunity foregone by purchasing the vehicle. In other words, what would the decision maker do with the amount of \$24,048 if he or she decides not to purchase the vehicle? If he or she would deposit the money in a saving account, then the savings rate is the interest rate to be used in the analysis.

Add-on Loans

4.63

(a)

$$\$3,000 = \$156.04(P/A, i, 24)$$

$$i = 1.85613\% \text{ per month}$$

$$r = 1.85613\% \times 12 = 22.2735\%$$

(b)

$$P = \$156.04(P/A, 1.85613\%, 12) = \$1,664.85$$

4.64

$$\$5,025 = \$146.35(P/A, i, 48)$$

$$i = 1.46\% \text{ per month}$$

$$P = \$146.35(P/A, 1.46\%, 33) = 3,810.91$$

Loans with Variable Payments

4.65

(a) Amount of dealer financing = $\$15,458(0.90) = \$13,912$

$$A = \$13,912(A/P, 0.9583\%, 60) = \$305.96$$

(b) Assuming that the remaining balance will be financed over 56 months,

$$B_4 = \$305.96(P/A, 0.9583\%, 56) = \$13,211.54$$

$$A = \$13,211.54(A/P, 0.875\%, 56) = \$299.43$$

(c) Interest payments to the dealer:

$$I_{\text{dealer}} = \$305.96 \times 4 - (\$13,912 - \$13,211.54) = \$523.38$$

Interest payments to the credit union:

$$I_{\text{union}} = \$299.43 \times 56 - \$13,211.54 = \$3,556.54$$

 \therefore Total interest payments = \$4,079.92

4.66 Given: purchase price = \$155,000, down payment = \$25,000

- Option 1: $i = 7.5\% / 12 = 0.625\%$ per month, $N = 360$ months

- Option 2: For the assumed mortgage, $P_1 = \$97,218$, $i_1 = 0.458\%$ per month, $N_1 = 300$ months, $A_1 = \$597$ per month; For the 2nd mortgage $P_2 = \$32,782$, $i_2 = 9\% / 12 = 0.75\%$ per month, $N_2 = 120$ months

(a) For the second mortgage, the monthly payment will be

$$A_2 = P_2(A/P, i_2, N_2) = \$32,782(A/P, 0.75\%, 120) = \$415.27$$

$$\$130,000 = \$597(P/A, i, 300) + \$415.27(P/A, i, 120)$$

$$i = 0.5005\% \text{ per month}$$

$$r = 0.5005\% \times 12 = 6.006\% \text{ per year}$$

$$i_a = 6.1741\%$$

(b) Monthly payment

- Option 1: $A = \$130,000(A/P, 0.625\%, 360) = \908.97
- Option 2: \$1,012.27 for 120 months, then \$ 597 for remaining 180 months,

(c) Total interest payment

- Option 1: $I = \$908.97 \times 360 - \$130,000 = \$197,121.2$
- Option 2: $I = \$228,932.4 - \$130,000 = \$98,932.4$

(d) Equivalent interest rate:

$$\$908.97(P/A, i, 360) = \$597(P/A, i, 300) + \$415.27(P/A, i, 120)$$

$$i = 1.2016\% \text{ per month}$$

$$r = 1.2016\% \times 12 = 14.419\% \text{ per year}$$

$$i_a = 15.4114\%$$

Loans with Variable Payments

4.67

$$\$10,000 = A(P/A, 0.6667\%, 12) + A(P/A, 0.75\%, 12)(P/F, 0.6667\%, 12)$$

$$= 22.05435A$$

$$\therefore A = \$453.43$$

4.68 Given: $i = 0.75\%$ per month , deferred period = 6months, $N = 36$ monthly payments, first payment due at the end of 7th month, the amount of initial loan = \$15,000

(a) First, find the loan adjustment required for the 6-month grace period

$$\$15,000(F / P, 0.75\%, 6) = \$15,687.78 .$$

Then, the new monthly payments should be

$$A = \$15,687.78(A / P, 0.75\%, 36) = \$498.87$$

(b) Since there are 10 payments outstanding, the loan balance after the 26th payment is

$$B_{26} = \$498.87(P / A, 0.75\%, 10) = \$4,788.95$$

(c) The effective interest rate on this new financing is

$$\$4,788.95 = \$186(P / A, i, 300)$$

$$i = 1.0161\% \text{ per month}$$

$$r = 1.0161\% \times 12 = 12.1932\%$$

$$i_a = (1 + 0.010161)^{12} - 1 = 12.90\%$$

4.69 Given: $P = \$120,000$, $N = 360$ months, $i = 0.75\%$ per month

(a)

$$A = \$120,000(A / P, 0.75\%, 360) = \$965.55$$

(b) If $r = 9.75\%$ APR after 5 years, then $i = 0.8125\%$ per month

- The remaining balance after the 60th payment:

$$B_{60} = \$965.55(P / A, 0.75\%, 300) = \$115,056.50$$

- Then, we determine the new monthly payments as

$$A = \$115,056.50(A / P, 0.8125\%, 300) = \$1,025.31$$

Investment in Bonds

4.70 Given: Par value = \$1,000, coupon rate = 12%, paid as \$60 semiannually, $N = 36$ semiannual periods

(a) Find YTM

$$\$1,000 = \$60(P/A, i, 60) + \$1,000(P/F, i, 60)$$

$$i = 6\% \text{ semiannually}$$

$$i_a = 12.36\% \text{ per year}$$

(b) Find the bond price after 5 years with $r = 9\%$: $i = 4.5\%$ semiannually, $N = 2(30 - 5) = 50$ semiannual periods.

$$\begin{aligned} P &= \$60(P/A, 4.5\%, 50) + \$1,000(P/F, 4.5\%, 50) \\ &= \$1,296.43 \end{aligned}$$

(c)

- Sale price after 5 ½ years later = \$922.38, the YTM for the new investors:

$$\$922.38 = \$60(P/A, i, 49) + \$1,000(P/F, i, 49)$$

$$i = 6.5308\% \text{ semiannually}$$

$$i_a = 13.488\%$$

- Current yield at sale = $\$60/\$922.38 = 6.505\%$ semiannually
- Nominal current yield = $6.505\% \times 2 = 13.01\%$ per year
- Effective current yield = 13.433% per year

4.71 Given: Purchase price = \$1,010, par value = \$1,000, coupon rate = 9.5%, bond interest (\$47.50) semiannually, required YTM = 10% per year compounded semiannually, $N = 6$ semiannual periods

$$F + \$47.5(F/A, 5\%, 6) + \$1,010(F/P, 5\%, 6) = 0$$

$$\therefore F = \$1,030.41$$

4.72 Given: Par value = \$1,000, coupon rate = 8%, \$40 bond interest paid semiannually, purchase price = \$920, required YTM = 9% per year compounded semiannually, $N = 8$ semiannual periods

$$\$920 = \$40(P/A, 4.5\%, 8) + F(P/F, 4.5\%, 8)$$

$$\therefore F = \$933.13$$

4.73

- Option 1: Given purchase price = \$513.60, $N = 10$ semiannual periods, par value at maturity = \$1,000

$$\$513.60 = \$1,000(P/F, i, 10)$$

$$i = 6.89\% \text{ semiannually}$$

$$i_a = 14.255\% \text{ per year}$$

- Option 2: Given purchase price = \$1,000, $N = 10$ semiannual periods, \$113 interest paid every 6 months

$$\$1,000 = \$113(P/A, i, 10) + \$1,000(P/F, i, 10)$$

$$i = 11.3\% \text{ semiannually}$$

$$i_a = 23.877\% \text{ per year}$$

\therefore Option 2 has a better yield.

4.74 Given: Par value = \$1,000, coupon rate = 15%, or \$75 interest paid semiannually, purchase price = \$1,298.68, $N = 8$ semiannual periods

$$\$1,298.68 = \$75(P/A, i, 24) + \$1,000(P/F, i, 24)$$

$$i = 5.277\% \text{ semiannually}$$

$$i_a = 10.84\% \text{ per year}$$

4.75 Given: Par value = \$1,000, interest payment = \$75 semiannually or $i = 4.5\%$ semiannually, $N_A = 30$, $N_B = 2$ semiannual periods

$$P_A = \$100(P/A, 4.5\%, 30) + \$1,000(P/F, 4.5\%, 30) = \$1,895.89$$

$$P_B = \$100(P/A, 4.5\%, 2) + \$1,000(P/F, 4.5\%, 2) = \$1,103$$

4.76 Given: Par value = \$1,000, coupon rate = 8.75%, or \$87.5 interest paid annually, $N = 4$ years

(a) Find YTM if the market price is \$1,108:

Contemporary Engineering Economics, Fourth Edition, By Chan S. Park. ISBN 0-13-187628-7.

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$$\begin{aligned} \$1,108 &= \$87.5(P/A, i, 4) + \$1,000(P/F, i, 4) \\ i &= 5.66\% \end{aligned}$$

(b) Find the present value of this bond if $i = 9.5\%$:

$$\begin{aligned} P &= \$87.5(P/A, 9.5\%, 4) + \$1,000(P/F, 9.5\%, 4) \\ &= \$975.97 \end{aligned}$$

\therefore It is good to buy the bond at \$930

4.77 Given: Par value = \$1,000, coupon rate = 12%, or \$60 interest paid every 6 months, $N = 30$ semiannual periods

(a)

$$P = \$60(P/A, 4.5\%, 26) + \$1,000(P/F, 4.5\%, 26) = \$1,227.20$$

(b)

$$P = \$60(P/A, 6.5\%, 26) + \$1,000(P/F, 6.5\%, 26) = \$934.04$$

(c)

Current yield = $\$60 / \$738.58 = 7.657\%$ semiannually. The effective annual current yield = 15.9%

4.78 Given: Par value = \$1,000, coupon rate = 10%, paid as \$50 every 6 months, $N = 20$ semiannual periods,

$$\begin{aligned} P &= \$50(P/A, 3\%, 14) + \$1,000(P/F, 3\%, 14) \\ &= \$1,225.91 \end{aligned}$$

Short Case Studies

ST 4.1

(a)

- Bank A: $i_a = (1 + 0.0155)^{12} - 1 = 20.27\%$

- Bank B: $i_a = (1 + 0.165/12)^{12} - 1 = 17.81\%$

(b) Given $i = 6\% / 365 = 0.01644\%$ per day, the effective interest rate per payment period is $i = (1 + 0.0001644)^{30} - 1 = 0.494\%$ per month.

We also assume that the \$300 remaining balance will be paid off at the end of 24 months. So, the present worth of the annual cost for the credit cards is,

• Bank A:

$$P = \$20 + \$4.65(P / A, 0.494\%, 12) + \$20(P / F, 0.494\%, 12)$$

$$= \$143.85$$

• Bank B:

$$P = \$30 + \$4.13(P / A, 0.494\%, 12) + \$30(P / F, 0.494\%, 12)$$

$$= \$151.55$$

∴ Select bank A

- (c) Assume that Jim makes either the minimum 10% payment or \$20, whichever is larger, every month. It will take 59 months to pay off the loan. The total interest payments are \$480.37.

Period	Beg. Bal	Interest	Payment	End. Bal.
0				\$ 1,500.00
1	\$ 1,500.00	\$ 20.63	\$ 76.03	\$ 1,444.59
2	\$ 1,444.59	\$ 19.86	\$ 73.22	\$ 1,391.23
3	\$ 1,391.23	\$ 19.13	\$ 70.52	\$ 1,339.85
4	\$ 1,339.85	\$ 18.42	\$ 67.91	\$ 1,290.35
12	\$ 991.49	\$ 13.63	\$ 50.26	\$ 954.87
13	\$ 954.87	\$ 13.13	\$ 48.40	\$ 919.60
14	\$ 919.60	\$ 12.64	\$ 46.61	\$ 885.63
15	\$ 885.63	\$ 12.18	\$ 44.89	\$ 852.92
16	\$ 852.92	\$ 11.73	\$ 43.23	\$ 821.42
17	\$ 821.42	\$ 11.29	\$ 41.64	\$ 791.07
18	\$ 791.07	\$ 10.88	\$ 40.10	\$ 761.85
19	\$ 761.85	\$ 10.48	\$ 38.62	\$ 733.71
20	\$ 733.71	\$ 10.09	\$ 37.19	\$ 706.61
21	\$ 706.61	\$ 9.72	\$ 35.82	\$ 680.51
38	\$ 372.27	\$ 5.12	\$ 20.00	\$ 357.39
39	\$ 357.39	\$ 4.91	\$ 20.00	\$ 342.30
40	\$ 342.30	\$ 4.71	\$ 20.00	\$ 327.01
41	\$ 327.01	\$ 4.50	\$ 20.00	\$ 311.50
53	\$ 126.23	\$ 1.74	\$ 20.00	\$ 107.96
54	\$ 107.96	\$ 1.48	\$ 20.00	\$ 89.45
55	\$ 89.45	\$ 1.23	\$ 20.00	\$ 70.68
56	\$ 70.68	\$ 0.97	\$ 20.00	\$ 51.65
57	\$ 51.65	\$ 0.71	\$ 20.00	\$ 32.36
58	\$ 32.36	\$ 0.44	\$ 20.00	\$ 12.81
59	\$ 12.81	\$ 0.18	\$ 12.98	\$ 0.00
		\$ 480.37	\$ 1,980.37	

ST 4.2 To explain how Trust Company came up with the monthly payment scheme, let's assume that you borrow \$10,000 and repay the loan over 24 months at 13.4% interest compounded monthly. Note that the bank loans up to 80% of the sticker price. If you are borrowing \$10,000, the sticker price would be \$12,500. The assumed residual value will be 50% of the sticker price, which is \$6,250.

(a) Monthly payment:

$$\begin{aligned} A &= \$10,000(A/P, 13.4\% / 12, 24) - \$6,250(A/F, 13.4\% / 12, 24) \\ &= \$249 \end{aligned}$$

(b) Equivalent cost of owning or leasing the automobile:

- Alternative Auto Loan:

$$\begin{aligned} P &= \$211(P/A, 8\% / 12, 36) + \$6,250(P/F, 8\% / 12, 36) \\ &= \$11,653.73 \end{aligned}$$

- Conventional Loan:

$$\begin{aligned} P &= \$339(P/A, 8\% / 12, 36) \\ &= \$10,818.10 \end{aligned}$$

∴ It appears that the conventional loan is a better choice.

ST 4.3 We need to determine the annual college expenses at a 4-year state school when the newborn goes to college at the age of 18. If costs continued to rise at the annual rate of at least 7%, four-year schooling would cost \$36,560. Assuming that the first year's college expense (\$X) would be paid at the beginning of age 18 (or at the end of age 17), we can establish the following equivalence relationship.

$$\begin{aligned} \$36,560 &= X(1 + 1.07 + 1.07^2 + 1.07^3) \\ X &= \$8,234 \end{aligned}$$

Then, the annual expenses during the subsequent years would be \$8,810, \$9,427, and \$10,087, respectively. To meet these future collage expenses, the state-run program must earn

$$\begin{aligned}
 \$6,756(F/P, i, 17) &= \$8,234 + \$8,810(P/F, i, 1) \\
 &\quad + \$9,427(P/F, i, 2) + \$10,087(P/F, i, 3) \\
 i &= 9.55\%
 \end{aligned}$$

Therefore, it would be a good program to join if you cannot invest your money elsewhere at a rate greater than 9.55%.

ST 4.4

(a) $A = \$60,000(A/P, 13\% / 12, 360) = \663.70

(b)

$$\begin{aligned}
 \$60,000 &= \$522.95(P/A, i, 12) \\
 &\quad + \$548.21(P/A, i, 12)(P/F, i, 12) \\
 &\quad + \$574.62(P/A, i, 12)(P/F, i, 24) \\
 &\quad + \$602.23(P/A, i, 12)(P/F, i, 36) \\
 &\quad + \$631.09(P/A, i, 12)(P/F, i, 48) \\
 &\quad + \$661.24(P/A, i, 300)(P/F, i, 60)
 \end{aligned}$$

Solving for i by trial and error yields

$$i = 1.0028\%$$

$$i_a = 12.72\%$$

Comments: With Excel, you may enter the loan payment series and use the **IRR** (range, guess) function to find the effective interest rate. Assuming that the loan amount (-\$60,000) is entered in cell A1 and the following loan repayment series in cells A2 through A361, the effective interest rate is found with a guessed value of 11.8/12%:

$$=IRR(A1:A361, 0.9833\%) = 0.010028$$

(c) Compute the mortgage balance at the end of 5 years:

- Conventional mortgage:

$$B_{60} = \$663.70(P/A, 13\% / 12, 300) = \$58,848.90$$

- FHA mortgage (not including the mortgage insurance):

$$B_{60} = \$635.28(P/A, 11.5\% / 12, 300) = \$62,498.71$$

(d) Compute the total interest payment for each option:

- Conventional mortgage(using either Excel or Loan Analysis Program from the book's website—<http://www.prenhall.com/park>):

$$I = \$178,932.34$$

- FHA mortgage:

$$I = \$163,583.79$$

- (e) Compute the equivalent present worth cost for each option at $i = 6\% / 12 = 0.5\%$ per month:

- Conventional mortgage:

$$P = \$663.70(P / A, 0.5\%, 360) = \$110,699.59$$

- FHA mortgage including mortgage insurance:

$$\begin{aligned} P &= \$522.95(P / A, 0.5\%, 12) \\ &\quad + \$548.21(P / A, 0.5\%, 12)(P / F, 0.5\%, 12) \\ &\quad + \$574.62(P / A, 0.5\%, 12)(P / F, 0.5\%, 24) \\ &\quad + \$602.23(P / A, 0.5\%, 12)(P / F, 0.5\%, 36) \\ &\quad + \$631.09(P / A, 0.5\%, 12)(P / F, 0.5\%, 48) \\ &\quad + \$661.24(P / A, 0.5\%, 300)(P / F, 0.5\%, 60) \\ &= \$105,703.95 \end{aligned}$$

\therefore The FHA option is more desirable (least cost).

ST 4.5

Let A_i the monthly payment for i^{th} year and B_j the balance of the loan at the end of j^{th} month. Then,

$$A_1 = \$95,000(A / P, 8.125\% / 12, 360) = \$705.37$$

$$B_{12} = \$705.37(P / A, 8.125\% / 12, 348) = \$94,225.87$$

$$A_2 = \$94,225.87(A / P, 10.125\% / 12, 348) = \$840.17$$

$$B_{24} = \$840.17(P / A, 10.125\% / 12, 336) = \$93,658.39$$

$$A_3 = \$93,658.39(A / P, 12.125\% / 12, 336) = \$979.77$$

$$B_{36} = \$979.77(P / A, 12.125\% / 12, 324) = \$93,234.21$$

$$A_{4-30} = \$93,407.58(A / P, 13.125\% / 12, 324) = \$1,050.71$$

(a) The monthly payments over the life of the loan are

$$A_1 = \$95,000(A / P, 8.125\% / 12, 360) = \$705.37$$

$$A_2 = \$94,225.87(A / P, 10.125\% / 12, 348) = \$840.17$$

$$A_3 = \$93,658.39(A / P, 12.125\% / 12, 336) = \$979.77$$

$$A_{4-30} = \$93,407.58(A / P, 13.125\% / 12, 324) = \$1,050.71$$

(b)

$$(\$705.37 \times 12) + (\$840.17 \times 12) + (\$979.77 \times 12) + (\$1,050.71 \times 324) - \$95,000 \\ = \$257,187.28$$

(c)

$$\begin{aligned} \$95,000 &= \$705.37(P / A, i, 12) + \$840.17(P / A, i, 12)(P / F, i, 12) \\ &\quad + \$979.77(P / A, i, 12)(P / F, i, 24) \\ &\quad + \$1,050.71(P / A, i, 324)(P / F, i, 36) \end{aligned}$$

$$i = 1.0058\% \text{ per month}$$

$$\text{APR}(r) = 1.0058\% \times 12 = 12.0696\%$$

$$i_a = (1 + 0.010058)^{12} - 1 = 12.77\% \text{ per year}$$