Chapter 12 Projects Risk and Uncertainty

Sensitivity Analysis

12.1

(a) Project cash flows based on the most-likely estimates: without working capital

	0	1	2	3	4
Income Statement					
Labor Savings		$35,000	$35,000	$35,000	$35,000
Depreciation		21,600	34,560	20,736	6,221
Taxable Income		$13,400	$440	$14,264	$28,779
Income Tax (40%)		5,360	176	5,706	11,512
Net Income		$8,040	$264	$8,558	$17,268
Cash Flow Statement					
Cash From Operation:					
Net Income		8,040	264	8,558	17,268
Depreciation		21,600	34,560	20,736	6,221
Investment&Salvage	(108,000)				30,000
Gains Tax					(2,047)
Net Cash Flow	(108,000)	29,640	34,824	29,294	51,442
PW (10%) =	$4,870				

(b) Project cash flows based on the most-likely estimates: with working capital

	0	1	2	3	4
Income Statement					
Labor Savings		$35,000	$35,000	$35,000	$35,000
Depreciation		21,600	34,560	20,736	6,221
Taxable Income		$13,400	$440	$14,264	$28,779
Income Tax (40%)		5,360	176	5,706	11,512
Net Income		$8,040	$264	$8,558	$17,268
Cash Flow Statement					
Cash From Operation:					
Net Income		8,040	264	8,558	17,268
Depreciation		21,600	34,560	20,736	6,221
Investment&Salvage	(108,000)				30,000
Gains Tax					(2,047)
Working Capital	(5,000)				5,000
Net Cash Flow	(113,000)	29,640	34,824	29,294	56,442
PW (10%) =	$3,285				

$\therefore$ The project is acceptable under either situation.

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(c) Required annual savings (X):

	0	1	2	3	4
Income Statement					
Labor Savings		\$43,370	\$43,370	\$43,370	\$43,370
Depreciation		21,600	34,560	20,736	6,221
Taxable Income		\$21,770	\$8,810	\$22,634	\$37,149
Income Tax (40%)		8,708	3,524	9,053	14,860
Net Income		\$13,062	\$5,286	\$13,580	\$22,289
Cash Flow Statement					
Cash From Operation:					
Net Income		13,062	5,286	13,580	22,289
Depreciation		21,600	34,560	20,736	6,221
Investment & Salvage	(108,000)				30,000
Gains Tax					(2,047)
Net Cash Flow	(108,000)	34,662	39,846	34,316	56,463

PW(18%) = \$0

The required annual savings is \$43,370.

12.2

- Project's IRR if the investment is made now:

$$PW(i) = -\$500,000 + \$200,000(P/A, i, 5) = 0$$

$$\therefore \quad i = 28.65\%$$

- Let X denote the additional after-tax annual cash flow:

$$PW(28.65\%) = -\$500,000 + X(P/A, 28.65\%, 4)(P/F, 28.65\%, 1) = 0$$

$$\therefore \quad X = \$290,240$$

12.3

(a) Economic building height

- $0\% < i < 20\%$: The optimal building height is 5 Floors.
- $20\% < i < 30\%$: The optimal building height is 2 Floors.

Net Cash Flows					
n	2 Floors	3 Floors	4 Floors	5 Floors	
0	($500,000)	($750,000)	($1,250,000)	($2,000,000)	
1	$199,100	$169,200	$149,200	$378,150	
2	$199,100	$169,200	$149,200	$378,150	
3	$199,100	$169,200	$149,200	$378,150	
4	$199,100	$169,200	$149,200	$378,150	
5	$799,100	$1,069,100	$2,149,200	$3,378,150	
Sensitivity Analysis PW(i) as a Function of Interest Rate					
i (%)					Best Floor Plan
5	$832,115	$687,643	$963,010	$1,987,770	5
6	$787,037	$635,190	$873,011	$1,834,680	5
7	$744,141	$585,370	$787,722	$1,689,448	5
8	$703,298	$538,023	$706,879	$1,551,593	5
9	$664,388	$493,002	$630,199	$1,420,666	5
10	$627,298	$450,168	$557,428	$1,296,250	5
11	$591,924	$409,393	$488,330	$1,177,957	5
12	$558,167	$370,556	$422,686	$1,065,427	5
13	$525,937	$333,545	$360,291	$958,321	5
14	$495,148	$298,257	$300,953	$856,326	5
15	$465,720	$264,594	$244,495	$759,148	5
16	$437,580	$232,465	$190,751	$666,513	5
17	$410,657	$201,784	$139,565	$578,166	5
18	$384,885	$172,472	$90,792	$493,867	5
19	$360,205	$144,454	$44,298	$413,393	5
20	$336,557	$117,661	($46)	$336,533	2
21	$313,889	$92,027	($42,357)	$263,091	2
22	$292,150	$67,490	($82,746)	$192,883	2
23	$271,292	$43,993	($121,319)	$125,737	2
24	$251,271	$21,482	($158,173)	$61,490	2
25	$232,044	($95)	($193,399)	($9)	2
26	$213,572	($20,784)	($227,084)	($58,903)	2
27	$195,817	($40,632)	($259,308)	($115,327)	2
28	$178,745	($59,679)	($290,148)	($169,407)	2
29	$162,323	($77,967)	($319,674)	($221,261)	2
30	$146,519	($95,532)	($347,955)	($271,002)	2

(b) Effects of overestimation on resale value:

Resale value	Present Worth as a Function of Number of Floors			
	2	3	4	5
Base	$465,720	$264,594	$244,495	$759,148
10% error	$435,890	$219,898	$145,060	$609,995
Difference	$29,831	$44,696	$99,435	$149,153

12.4 **Note:** In the problem statement, the current book value for the defender is given as \$13,000. This implies that the machine has been depreciated under the alternative MACRS with half-year convention. In other words, the allowed depreciation is based on a 10-year recovery period with straight-line method. With the half-year convention mandated, the book value that should be used in determining the gains tax for the defender (if sold now) is

$$\text{Total depreciation } = \$1,000 + \$2,000 + \$2,000 + \$1,000 = \$6,000$$
$$\text{Book value} = \$20,000 - \$6,000 = \$14,000$$
$$\text{Taxable gain (loss)} = \$6,000 - \$14,000 = (\$8,000)$$
$$\text{Net proceeds from sale} = \$6,000 + \$8,000 \times 0.4 = \$9,200$$

Now the net proceeds from sale can be applied toward purchasing the new machine. Thus, the net investment required for new machine will be only \$2,800. Note that, if you decide to retain the old machine, the current book value will be \$13,000 as you continue to depreciate the asset without any adjustment.

Comments: We will consider the various replacement problems in Chapter 14, where the net proceeds from sale of the old machine is treated as an opportunity cost of retaining the old machine, or simply the new investment required to keep the old machine.

(a) Defender versus Challenger:

Keep the old machine

n	4	5	6	7	8	9	10
Financial Data	0	1	2	3	4	5	6
Depreciation		\$2,000	\$2,000	\$2,000	\$2,000	\$2,000	\$2,000
Book value	\$13,000	\$11,000	\$9,000	\$7,000	\$5,000	\$3,000	\$1,000
Market value							\$1,000
Gain/loss							
Removal cost							1,500
Operating cost		2,000	2,000	2,000	2,000	2,000	2,000
Cash Flow Statement							
Removal							(\$900)
+(.4)*(Depreciation)		800	800	800	800	800	800
Net proceeds from sale							1,000
-(1-0.40)*(Operating cost)		(1,200)	(1,200)	(1,200)	(1,200)	(1,200)	(1,200)
Net Cash Flow	\$0	(\$400)	(\$400)	(\$400)	(\$400)	(\$400)	(\$300)

PW (10%) = (\$1,686)　　AE (10%) = (\$387)

Buy a new machine

Financial Data	*n*	0	1	2	3	4	5	6
Depreciation			$2,400	$3,840	$2,304	$1,382	$1,382	$691
Book value		$12,000	9600	5760	3456	2074	691	0
Market value								2000
Gain/loss								2000
Operating cost			1000	1000	1000	1000	1000	1000
Cash Flow Statement								
Sale of old equipment		9,200						
Investment		(12,000)						
+(.4)*(Depreciation)			960	1,536	922	553	553	276
-(1-0.40)*(Operating cost)			(600)	(600)	(600)	(600)	(600)	(600)
Net proceeds from sale								1,200
Net Cash Flow		($2,800)	$360	$936	$322	($47)	($47)	$876

PW (10%) = ($1,024) AE (10%) = ($235)

∴ Incremental cash flows:

	Net Cash Flow		Incremental Cash flow (new-old)
n	New Machine	Old Machine	
0	$-\$2,800$		$-\$2,800$
1	360	–400	760
2	936	–400	1,336
3	322	–400	732
4	–47	–400	353
5	–47	–400	353
6	876	–300	1,176

$$\text{IRR}_{new-old} = 18.32\%, \quad PW(10\%)_{new-old} = \$669$$

∴ The defender should be replaced now.

(b) Sensitivity analysis: The answer remains unchanged. In fact, it (an increase in O&M) will make the challenger more attractive.($\text{IRR}_{new-old} = 24.26\%$, and $PW(10\%)_{new-old} = \$1,280$)

(c) Break-even trade-in value: Let X denote the minimum trade in value for the old machine. Then, the net proceeds from sale of the old machine will be

$$\begin{aligned}
\text{Total depreciation} &= \$6,000 \\
\text{Book value} &= \$14,000 \\
\text{Salvage value} &= X \\
\text{Taxable gain} &= X - \$14,000 \\
\text{Net proceeds} &= X - (0.40)(X - \$14,000) \\
&= 0.6X + \$5,600
\end{aligned}$$

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To find the break-even trade-in value,

$$PW(10\%)_{old} = -\$400(P/A,10\%,5) - \$300(P/F,10\%,6) = -\$1,686$$
$$\begin{aligned} PW(10\%)_{new} &= -[\$12,000 - (0.6X + \$5,600)] \\ &\quad + \$360(P/F,10\%,1) + \cdots + \$877(P/F,10\%,6) \\ &= 0.6X - \$4,624 \end{aligned}$$

Let $PW(10\%)_{old} = PW(10\%)_{new}$ and solve for X .

$$\therefore \quad X = \$4,897$$

12.5

(a) Total length of a new phone line - 5 miles:

- Option 1-Copper wire:

  5 miles = 5 × 5,280 = 26,400 feet
  First cost = (1,692 + 0.013 × 2,000) × 26,400 = \$731,069 $= \$731,069$
  Annual operating cost = \$731,069(0.184) = \$134,517

$$\therefore \quad \begin{aligned} AEC(15\%)_1 &= \$731,069(A/P,15\%,30) + \$34,517 \\ &= \$111,342 \end{aligned}$$

- Option 2-Fiber optics:

  Cost of ribbon = \$15,000/mile × 5miles = \$75,000
  Cost of terminators = \$30,000 × 6 = \$180,000
  Cost of modulating system $= (\$12,092 + \$21,217)(21)(2)$ $= \$1,398,978$
  Cost of repeater = \$15,000 × 2 = \$30,000
  Total first cost = \$75,000 + \$180,000 + \$1,398,978 + \$30,000 = \$1,683,978
  Annual operating costs $= \$1,398,978(0.125)$ $+\$75,000(0.178)$ $= \$188,222$

$$\therefore \quad \begin{aligned} AEC(15\%)_2 &= \$1,683,978(A/P,15\%,30) + \$188,222 \\ &= \$256,470 \end{aligned}$$

$\therefore$ Select Option 1.

(b) Sensitivity Analysis

- Total length of phone lines -10 miles:

$$AEC(15\%)_1 = \$491,718$$
$$AEC(15\%)_2 = \$471,749$$

$\therefore$ Option 2 is the better choice.

- Total length of the phone lines - 25 miles:

$$AEC(15\%)_1 = \$1,229,293$$
$$AEC(15\%)_2 = \$552,920$$

$\therefore$ Option 2 is the better choice.

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12.6

(a) With infinite planning horizon: We assume that both machines will be available in the future with the same cost. (Select Model A)

Model A										
Financial Data										
	n	0	1	2	3	4	5-7	8		
Depreciation			$857	$1,469	$1,049	$749	$536	$268		
Book value		$6,000	$5,143	$3,673	$2,624	$1,874	$1,339	$0		
Market value								$500		
Gains/Losses								$500		
O&M			$700	$700	$700	$700	$700	$700		
Cash Flow Statement										
Investment		($6,000)								
+(.30)*(Depreciation)			$257	$441	$315	$225	$161	$80		
-(1-0.30)*(O&M)			($490)	($490)	($490)	($490)	($490)	($490)		
Net proceeds from sale								$350		
Net Cash Flow		($6,000)	($233)	($49)	($175)	($265)	($329)	($60)		
PW (10%) =	($7,152)	AEC(10%) =	$1,341							

Model B										
Financial Data										
	n	0	1	2	3	4	5-7	8	9	10
Depreciation			$1,215	$2,082	$1,487	$1,062	$759	$379		
Book value		$8,500	$7,285	$5,204	$3,717	$2,655	$1,896	($0)	($0)	($0)
Market value										$1,000
Gains/Losses										$1,000
O&M			$520	$520	$520	$520	$520	$520	$520	$520
Cash Flow Statement										
Investment		($8,500)								
+(.30)*(Depreciation)			$364	$624	$446	$318	$228	$114	$0	$0
-(1-0.30)*(O&M)			($364)	($364)	($364)	($364)	($364)	($364)	($364)	($364)
Net proceeds from sale										$700
Net Cash Flow		($8,500)	$0	$260	$82	($46)	($136)	($250)	($364)	$336
PW (10%) =	($8,627)	AEC(10%) =	$1,404							

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(b) Break-even annual O&M costs for machine A: Let X denotes a before-tax annual operating cost for model.

$$\begin{aligned} PW(10\%)_A &= -\$6,000 + (\$257 - 0.7X)(P/F,10\%,1) + \cdots \\ &\quad + (\$458 - 0.7X)(P/F,10\%,8) \\ &= -\$4,526 - 3.734X \\ AEC(10\%)_A &= \$849 + 0.7X \end{aligned}$$

Let $AEC(10\%)_A = AEC(10\%)_B$, and solve for *X*.

$$\$849 + 0.7X = \$1,404$$

$$\therefore \quad X = \boxed{\$793} \text{ per year}$$

(c) With a shorter service life:

n	Net Cash Flow	
	Model A	Model B
0	-$6,000	-$8,500
1	-233	0
2	-49	260
3	-175	82
4	-265	-46
5	2,172	2,883
PW(10%)	-$5,216	-$6,464

$\therefore$ Model A is still preferred over Model B.

12.7 Assuming that all old looms were fully depreciated

(a)

- Project cash flows: Alternative 1

Alternative 1										
Financial Data										
	n	0	1	2	3	4	5	5	7	8
Depreciation			$306,669	$525,564	$375,342	$268,040	$191,641	$191,426	$191,641	$95,713
Book value		$2,146,036	1,839,367	1,313,803	938,462	670,422	478,781	287,354	95,713	0
Market value										169,000
Gain/Loss										169,000
Annual sales			7,915,748	7,915,748	7,915,748	7,915,748	7,915,748	7,915,748	7,915,748	7,915,748
Annual labor cost			261,040	261,040	261,040	261,040	261,040	261,040	261,040	261,040
Annual O&M cost			1,092,000	1,092,000	1,092,000	1,092,000	1,092,000	1,092,000	1,092,000	1,092,000
Cash Flow Statement										
Investment		($2,108,836)								
+(0.40)*Dn			122,667	210,226	150,137	107,216	76,656	76,571	76,656	38,285
+(0.60)*Sales			4,749,449	4,749,449	4,749,449	4,749,449	4,749,449	4,749,449	4,749,449	4,749,449
-(0.60)*Labor			(156,624)	(156,624)	(156,624)	(156,624)	(156,624)	(156,624)	(156,624)	(156,624)
-(0.60)*O&M			(655,200)	(655,200)	(655,200)	(655,200)	(655,200)	(655,200)	(655,200)	(655,200)
Net proceeds from sale										101,400
Net Cash Flow		($2,108,836)	$4,060,292	$4,147,850	$4,087,761	$4,044,841	$4,014,281	$4,014,195	$4,014,281	$4,077,310
		PW (18%) =	$14,471,800		AE (18%) =	$3,549,127				

Note: Cost basis for the new looms = $ 2,119,170 + $ 26,866 = $ 2,146,036

Net investment required = Cost basis - Net proceeds from sale of the old looms

= $ 2,146,036 - $ 62,000 (1-0.40) = $ 2,108,836

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- Project cash flows: Alternative 2

Alternative 2

Financial Data								
n	0	1	2	3	4	5	5	7
Depreciation		$160,083	$274,347	$195,930	$139,918	$100,038	$99,926	$100,038
Book value	$1,120,242	960,159	685,812	489,882	349,964	249,926	150,000	49,963
Market value								
Gain/Loss								
Annual sales		7,455,084	7,455,084	7,455,084	7,455,084	7,455,084	7,455,084	7,455,084
Annual labor cost		422,080	422,080	422,080	422,080	422,080	422,080	422,080
Annual O&M cost		1,560,000	1,560,000	1,560,000	1,560,000	1,560,000	1,560,000	1,560,000
Cash Flow Statement								
Investment	($1,083,042)							
+(0.40)*Dn		64,033	109,739	78,372	55,967	40,015	39,970	40,015
+(0.60)*Sales		4,473,050	4,473,050	4,473,050	4,473,050	4,473,050	4,473,050	4,473,050
-(0.60)*Labor		(253,248)	(253,248)	(253,248)	(253,248)	(253,248)	(253,248)	(253,248)
-(0.60)*O&M		(936,000)	(936,000)	(936,000)	(936,000)	(936,000)	(936,000)	(936,000)
Net proceeds from sale								
Net Cash Flow	($1,083,042)	$3,347,835	$3,393,541	$3,362,175	$3,339,770	$3,323,817	$3,323,773	$3,323,817

PW (18%) = $12,575,319 AE (18%) = $3,084,026

Note: Cost basis for the new looms = $ 1,071,240 + $ 49,002 = $ 1,120,242
Net investment required = Cost basis - Net proceeds from sale of the old looms
= $ 1,120,242 - $ 62,000 (1-0.40) = $ 1,083,042

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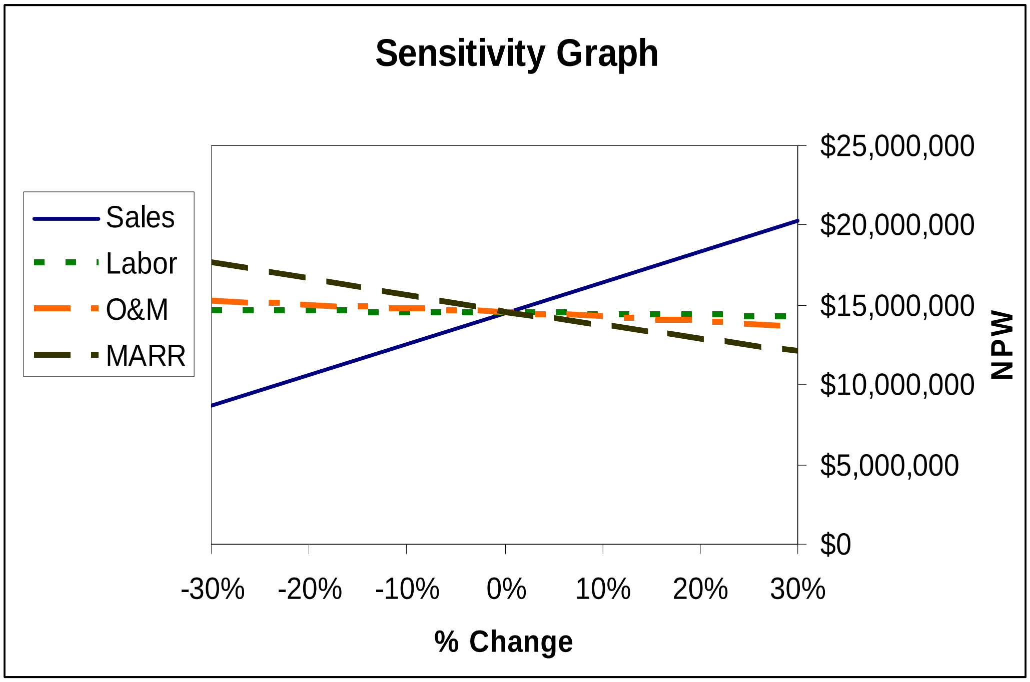
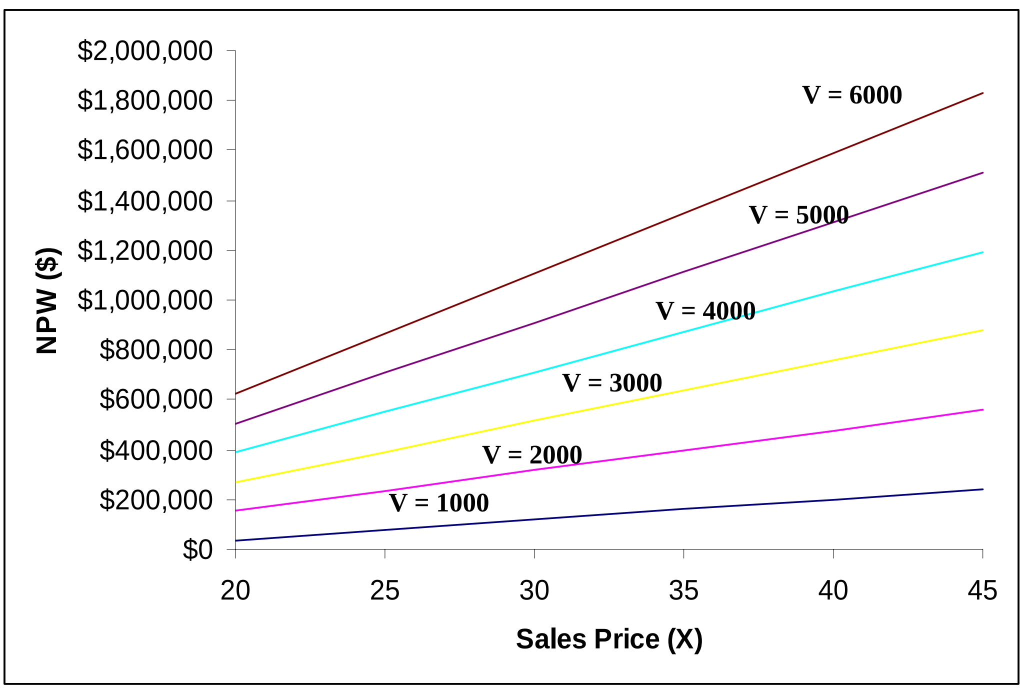
(b) Sensitivity graph



12.8 Sensitivity graph



12.9

- AEC(10%)s for 200 shift:

$$\begin{aligned} AEC(10\%)_{Electric} &= \$38,058(A/P,10\%,7) = \$7,817 \\ AEC(10\%)_{LPG} &= \$69,345(A/P,10\%,7) = \$14,244 \\ AEC(10\%)_{Gasoline} &= \$54,971(A/P,10\%,7) = \$11,280 \\ AEC(10\%)_{Diesel} &= \$49,994(A/P,10\%,7) = \$10,269 \end{aligned}$$

	0	1	2	3	4	5	6	7
Electric Power								
O&M		($2,025)	($2,025)	($2,025)	($2,025)	($2,025)	($2,025)	($2,025)
Initial cost	($29,739)							
Salvage								$3,000
Net cash flow	($29,739)	($2,025)	($2,025)	($2,025)	($2,025)	($2,025)	($2,025)	$975
PW(10%) =	($38,058)							
LPG								
O&M		($10,100)	($10,100)	($10,100)	($10,100)	($10,100)	($10,100)	($10,100)
Initial cost	($21,200)							
Salvage								$2,000
Net cash flow	($21,200)	($10,100)	($10,100)	($10,100)	($10,100)	($10,100)	($10,100)	($8,100)
PW(10%) =	($69,345)							
Gasoline								
O&M		($7,372)	($7,372)	($7,372)	($7,372)	($7,372)	($7,372)	($7,372)
Initial cost	($20,107)							
Salvage								$2,000
Net cash flow	($20,107)	($7,372)	($7,372)	($7,372)	($7,372)	($7,372)	($7,372)	($5,372)
PW(10%) =	($54,971)							
Diesel Fuel								
O&M		($5,928)	($5,928)	($5,928)	($5,928)	($5,928)	($5,928)	($5,928)
Initial cost	($22,263)							
Salvage								$2,200
Net cash flow	($22,263)	($5,928)	($5,928)	($5,928)	($5,928)	($5,928)	($5,928)	($3,728)
PW(10%) =	($49,994)							

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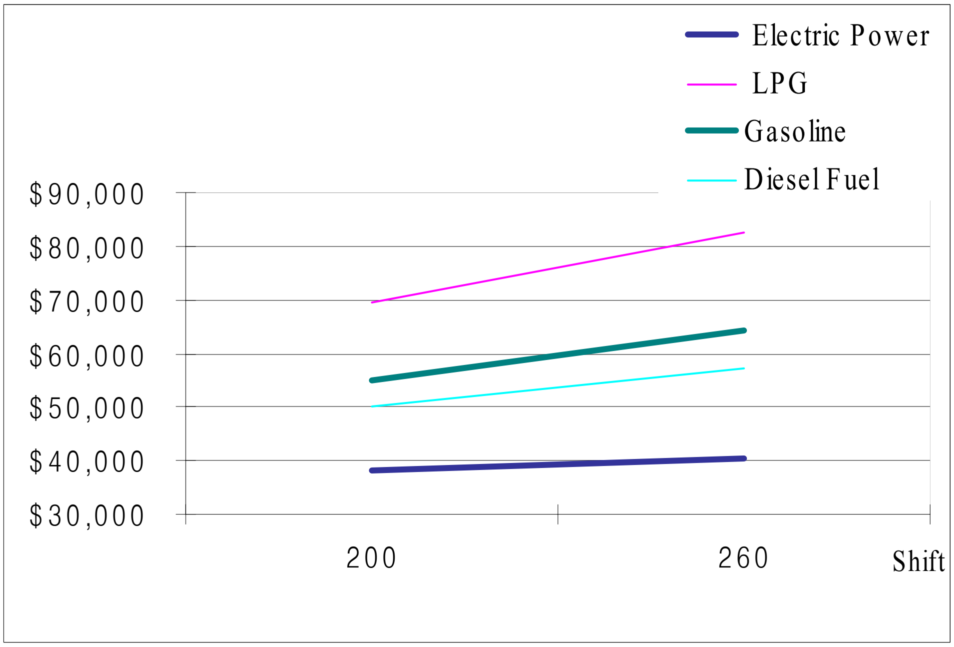
- AEC(10%)s for 260 shift:

$$AEC(10\%)_{Electric} = \$8,275$$
$$AEC(10\%)_{LPG} = \$16,974$$
$$AEC(10\%)_{Gasoline} = \$13,203$$
$$AEC(10\%)_{Diesel} = \$11,748$$

- Sensitivity graph: PW cost as a function of number of shifts



Break-Even Analysis

12.10

- PW of net investment:

$$P_0 = -\$2,200,000 - \$600,000 - \$400,000 = -\$3,200,000$$

- PW of after-tax revenue:

$$P_1 = -\$4,000(365)X(1-0.31)(P/A,10\%,25)$$
$$= \$9,144,210X$$

- PW of after-tax operating costs:

$$P_2 = -(\$230,000 + \$170,000X)(1-0.31)(P/A,10\%,25)$$
$$= -\$1,440,526 - 1,064,737X$$

- PW of tax credit (shield) on depreciation:

n	Depreciation		Combined
	Building	Furniture	Tax savings
1	\$54,060	\$57,143	\$111,203(0.31) = \$34,473
2	56,410	97,959	154,369(0.31) = 52,018
3	56,410	69,971	126,381(0.31) = 39,178
4	56,410	49,979	106,389(0.31) = 32,981
5	56,410	35,699	92,109(0.31) = 28,554
6	56,410	35,699	92,109(0.31) = 28,554
7	56,410	35,699	92,109(0.31) = 28,554
8	56,410	17,850	74,260(0.31) = 23,021
9-24	56,410	0	56,410(0.31) = 17,487
25	54,060	0	54,060(0.31) = 16,759

$$P_3 = \$34,473(P/F,10\%,1) + \$52,018(P/F,10\%,2)$$
$$+\cdots+ \$16,759(P/F,10\%,25)$$
$$= \$250,902$$

- PW of net proceeds from sale:

Property (asset)	Cost basis	Salvage value	Book value	Gains (losses)	Gains Taxes
Furniture	\$400,000	\$0	\$0	\$0	\$0
Building	2,200,000	0	794,450	(794,450)	(246,280)
Land	600,000	2,031,813	600,000	1,431,813	443,862

$$\text{Net proceeds from sale} = \$2,031,813 + \$246,280 - \$443,862$$
$$= \$1,834,231$$
$$P_4 = \$1,834,231(P/F,10\%,25)$$
$$= \$169,292$$

$$PW(10\%) = P_0 + P_1 + P_2 + P_3 + P_4$$
$$= -\$4,220,332 + 8,079,473X$$
$$= 0$$

$$X = \boxed{52.24\%}$$

12.11 Useful life of the old bulb:

$$14,600/(19 \times 365) = 2.1 \text{ years}$$

For computational simplicity, let's assume the useful life of 2 years for the old bulb. Then, the new bulb will last 4 years. Let X denote the price for the new light bulb. With an analysis period of 4 years, we can compute the equivalent present worth cost for each option as follows:

$$PW(15\%)_{\text{old}} = (1-0.40)[\$45.90 + \$45.90(P/F,15\%,2)]$$
$$= \$65.23$$
$$PW(15\%)_{\text{new}} = (1-0.40)(X + \$16)$$

The break-even price for the new bulb will be

$$0.6X + 9.6 = \$65.23$$

$$X = \boxed{\$92.72}$$

$\therefore$ Since the new light bulb costs only \$60, it is a good bargain.

12.12

- PW of net investment:

$$P_0 = -\$250,000$$

- PW of after-tax rental revenue:

$$P_1 = X(1-0.30)(P/A,15\%,20)$$
$$= \$4.3815X$$

- PW of after-tax operation costs:

$$P_2 = -(1-0.30)\$12,000(P/A,15\%,20)$$
$$= -\$52,578$$

- PW of tax credit (shield) on depreciation: (In this problem, we assume that the purchasing cost of $250,000 does not include any land value. Therefore, the entire purchasing cost will be the cost basis for depreciation purpose.)

Depreciation		Combined
n	Building	Tax savings
1	$6,143	$6,143(0.30) = $1,843
2-19	6,410	6,410(0.30) = 1,923
20	6,143	6,143(0.30) = 1,843

$$\begin{aligned} P_3 &= \$1,843(P/F,15\%,1) + \$1,923(P/A,15\%,19)(P/F,15\%,1) \\ &\quad + \$1,843(P/F,15\%,20) \\ &= \$11,962 \end{aligned}$$

- PW of net proceeds from sale:

Total depreciation $= \$127,666$
Book value $= \$250,000 - \$127,666 = \$122,334$
Salvage value $= \$250,000(1.05)^{20} = \$663,324$
Taxable gain $= \$663,324 - \$122,334 = \$540,990$
Gains tax $= \$540,990(0.30) = \$162,297$
Net proceeds from sale $= \$663,324 - \$162,297 = \$501,027$

$$\begin{aligned} P_4 &= \$501,027(P/F,15\%,20) \\ &= \$30,613 \end{aligned}$$

- The break-even rental:

$$\begin{aligned} PW(10\%) &= P_0 + P_1 + P_2 + P_3 + P_4 \\ &= -\$260,003 + 4.3815X \\ &= 0 \end{aligned}$$

$$\therefore \quad X = \boxed{\$59,341}$$

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12.13 Let X denotes the additional annual revenue (above \$16,000) for model A that is required to breakeven.

- Generalized cash flow for model A:

Cash flow elements	0	1	2	3	4	5	6
Investment	(\$80,000)						
Net proceeds							12,000
$+0.6R_n$		\$9,600 $+0.6X$	\$9,600 $+0.6X$	\$9,600 $+0.6X$	\$9,600 $+0.6X$	\$9,600 $+0.6X$	\$9,600 $+0.6X$
$+0.4D_n$		\$6,400	\$10,240	\$6,144	\$3,686	\$3,686	\$1,843
-(0.6)O&M		(\$13,200)	(\$13,200)	(\$13,200)	(\$13,200)	(\$13,200)	(\$13,200)
Net cash flow	(\$80,000)	\$2,800 $+0.6X$	\$6,640 $+0.6X$	\$2,544 $+0.6X$	\$86 $+0.6X$	\$86 $+0.6X$	\$10,234 $+0.6X$

(Columns 0–6: End of Year)

$$PW(20\%)_A = -\$68,077 + 1.9953X$$

- Generalized cash flow for model B:

Cash flow elements	0	1	2	3	4	5	6
Investment	(\$52,000)						
Net proceeds							9000
$+0.6R_n$		\$0	\$0	\$0	\$0	\$0	\$0
$+0.4D_n$		\$4,160	\$6,656	\$3,994	\$2,396	\$2,396	\$1,198
-(0.6)O&M		(\$10,200)	(\$10,200)	(\$10,200)	(\$10,200)	(\$10,200)	(\$10,200)
Net cash flow	(\$52,000)	(\$6,040)	(\$3,544)	(\$6,206)	(\$7,804)	(\$7,804)	(\$2)

(Columns 0–6: End of Year)

$$PW(20\%)_B = -\$69,985$$

- By letting $PW(20\%)_A = PW(20\%)_B$

$$-\$68,077 + 1.9953X = -\$69,985$$
$$X = -\$957$$

$\therefore$ The required break-even annual revenue for model A is then

$$\$16,000 + X = \$15,043$$

12.14 Let X denote the number of copies to breakeven.

- A/T annual revenue $= (0.6)[\$0.05 + (\$0.25 - \$0.05)]X = 0.15X$

- A/T O&M cost $= -(0.60)[\$300,000(12) + \$0.10X] = \$2,160,000 + 0.06X$

$$\begin{aligned}\text{Depreciation tax credit} &= (0.40)[\$85,714(P/F,13\%,1)+\cdots \\ &\quad +\$26,775(P/F,13\%,8)](A/P,13\%,10) \\ &= \$29,285\end{aligned}$$

$$\begin{aligned}CR(13\%) &= (\$600,000-\$60,000)(A/P,13\%,10)+(0.15)\$60,000 \\ &= \$107,316\end{aligned}$$

$$\begin{aligned}AE(13\%) &= 0.15X - \$2,160,000 - 0.06X + \$29,285 - \$107,316 \\ &= 0.09X - \$2,238,031 = 0\end{aligned}$$

$\therefore\ X = \boxed{24,867,011}$ copies per year or 82,890 copies per day

Probabilistic Analysis

12.15

$$\begin{aligned}PW(12\%)_{\text{light}} &= -\$8,000,000+\$1,300,000(P/A,12\%,3) \\ &= -\$4,800,000 \\ PW(12\%)_{\text{moderate}} &= -\$8,000,000+\$2,500,000(P/A,12\%,4) \\ &= -\$406,627 \\ PW(12\%)_{\text{high}} &= -\$8,000,000+\$4,000,000(P/A,12\%,4) \\ &= \$4,149,000 \\ E[PW(12\%)] &= -\$4,800,000(0.20)-\$406,627(0.40) \\ &\quad +\$4,149,000(0.40) \\ &= \boxed{\$536,947}\end{aligned}$$

$\therefore$ Since $E[PW]$ is positive, it is good to invest.

12.16

(a) The PW distribution for project 1:

Event (x,y)	Joint Probability	$PW(10\%)$
($20,$10)	0.24	$8,000
($20,$20)	0.36	$16,000
($40,$10)	0.16	$32,000
($40,$20)	0.24	$64,000

(b) The mean and variance of the NPW for Project 1:

$$\begin{aligned} E[PW(10\%)]_1 &= \$8,000(0.24) + \$16,000(0.36) + \$32,000(0.16) \\ &\quad + \$64,000(0.24) \\ &= \$28,160 \\ Var[PW(10\%)]_1 &= (8,000 - 28,160)^2(0.24) + (16,000 - 28,160)^2(0.36) \\ &\quad + (32,000 - 28,160)^2(0.16) + (64,000 - 28,160)^2(0.24) \\ &= 461,414,400 \end{aligned}$$

(c) The mean and variance of the NPW for Project 2:

$$\begin{aligned} E[PW(10\%)]_2 &= \$8,000(0.24) + \$16,000(0.20) + \$32,000(0.36) \\ &\quad + \$64,000(0.20) \\ &= \$32,000 \\ Var[PW(10\%)]_2 &= (8,000 - 32,000)^2(0.24) + (16,000 - 32,000)^2(0.20) \\ &\quad + (32,000 - 32,000)^2(0.36) + (64,000 - 32,000)^2(0.20) \\ &= 394,240,000 \end{aligned}$$

(d) Project 2 is preferred over project 1 because its mean is greater than that of project 1 but its variance is smaller than that of project 1.

12.17

(a) Expected value criterion: Assume that the opportunity cost rate is 7.5%.

- Option 1:

$$\begin{aligned} E[R]_1 &= \$2,450(0.25) + \$2,000(0.45) + \$1,675(0.30) \\ &\quad - \$150(F/P, 7.5\%, 1) \\ &= \$1,854 \end{aligned}$$

- Option 2:

$$E[R]_2 = \$25,000(0.075) = \$1,875$$

$\therefore$ Option 2 is the better choice based on the principle of the expected value maximization.

(c) Here we are looking for the value of perfect information. If we know for sure that the bond yield would be \$2,450, then our strategy would be to purchase the bond. In the absence of this perfect information, Option 2 was the better strategy. In terms of regret not selecting Option 1 is

$$\$2,450(P/F, 7.5\%, 1) - \$1,875 = \$414$$

If we know for sure that the bond yield would be either $2,000 or $1,675, clearly we do not invest in bond, so there will be no regret. Since there is only a 25% chance that the bond yield will be $2,450, we should not solicit professional advice at any expense higher than (0.25)($414) = $104.

12.18 Let X denote the annual revenue in constant dollars and Y the general inflation during the first year. Then Z is defined as $(1+Y)$.

(a) NPW as functions of X and Z:

Cash elements	End of Period		
	0	1	2
Investment	-$9,000		
Salvage value			$4,000Z^2$
Gains tax			$-(4,000Z^2$ -5,200)(0.40)
$(0.4)\,D_n$		1,200	800
$(0.6)\,R_n$		0.6ZX	$0.6Z^2X$
working capital	-2,000	2,000(1-Z)	2,000Z
Net cash flow	-$11,000	3,200-2,000Z +0.6ZX	$2{,}400Z^2$+3,000 $+0.6Z^2$X+2,000Z

Note that the market interest rate is a random variable as the general inflation rate becomes a random variable. There are nine joint events for X and Y. For a joint event where $x = 10,000$ and $y = 0.05$ (or $z = 1.05$), we first calculate the market interest rate and then evaluate the PW function at this market interest rate.

$$\begin{aligned} i &= i' + \bar{f} + i'\bar{f} \\ &= 0.10 + 0.05 + (0.1)(0.05) \\ &= 15.5\% \end{aligned}$$

$$\begin{aligned} PW(15.5\%) = &-\$11,000 + [3,200 - 2,000(1.05) \\ &+0.6(10,000)(1.05)](P/F, 15.5\%, 1) + [2,400(1.05)^2 \\ &+3,000 + 0.6(10,000)(1.05)^2 + 2,000(1.05)](P/F, 15.5\%, 2) \\ = &\$6,172 \end{aligned}$$

You repeat the process for the remaining joint events.

(b) Mean and variance calculation:

Event No.	X	Z	A0	A1	A2	PW(15.5%)	
1	$ 10,000	1.03	($11,000)	$7,320	$13,972	$5,811	
2	$ 10,000	1.05	($11,000)	$7,400	$14,361	$6,172	
3	$ 10,000	1.07	($11,000)	$7,480	$14,757	$6,538	
4	$ 20,000	1.03	($11,000)	$13,500	$20,337	$15,933	
5	$ 20,000	1.05	($11,000)	$13,700	$20,976	$16,585	
6	$ 20,000	1.07	($11,000)	$13,900	$21,627	$17,246	
7	$ 30,000	1.03	($11,000)	$19,680	$26,702	$26,055	
8	$ 30,000	1.05	($11,000)	$20,000	$27,591	$26,999	
9	$ 30,000	1.07	($11,000)	$20,320	$28,496	$27,954	

Event No.	PW(15.5%)	P(x)	P(z)	P(x,z)	Weighted PW(15.5%)	Deviation	Weighted Deviation
1	$5,811	0.3	0.35	0.105	$610	153,014,104	16,066,481
2	$6,172	0.3	0.5	0.15	$926	38,094,719	5,714,208
3	$6,538	0.3	0.25	0.075	$490	42,749,655	3,206,224
4	$15,933	0.4	0.35	0.14	$2,231	253,864,423	35,541,019
5	$16,585	0.4	0.5	0.2	$3,317	275,072,678	55,014,536
6	$17,246	0.4	0.25	0.1	$1,725	297,429,425	29,742,942
7	$26,055	0.3	0.35	0.105	$2,736	67,881,135	71,282,519
8	$26,999	0.3	0.5	0.15	$4,050	728,921,068	109,338,160
9	$27,954	0.3	0.25	0.075	$2,097	781,424,027	58,606,802
				E[PW]=	$18,181	Var[PW] =	384,512,892

Comparing Risky Projects

12.19

(a)

$$E[PW]_1 = (\$2,000)(0.20) + (\$3,000)(0.60) + (\$3,500)(0.20) - \$1,000$$
$$= \$1,900$$
$$E[PW]_2 = (\$1,000)(0.30) + (\$2,500)(0.40) + (\$4,500)(0.30) - \$800$$
$$= \$1,850$$

$\therefore$ Project 1 is preferred over Project 2.

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(b)

$$
\begin{aligned}
Var[PW]_1 &= (2,000-1,900)^2(0.20)+(3,000-1,900)^2(0.60) \\
&\quad +(3,500-1,900)^2(0.20) \\
&= 1,240,000 \\
Var[PW]_2 &= (1,000-1,850)^2(0.30)+(2,500-1,850)^2(0.40) \\
&\quad +(4,500-1,850)^2(0.30) \\
&= 2,492,500
\end{aligned}
$$

$\therefore$ Project 1 is still preferred, as we have $Var_1 < Var_2$ and $E_1 > E_2$.

12.20

(a) Mean and variance calculations:

$$
\begin{aligned}
E[PW]_1 &= (\$100,000)(0.20)+(\$50,000)(0.40)+(0)(0.40) \\
&= \$40,000 \\
E[PW]_2 &= (\$40,000)(0.30)+(\$10,000)(0.40)+(-\$10,000)(0.30) \\
&= \$13,000 \\
Var[PW]_1 &= (100,000-40,000)^2(0.20)+(50,000-40,000)^2(0.40) \\
&\quad +(0-40,000)^2(0.40) \\
&= 1,400,000,000 \\
Var[PW]_2 &= (40,000-13,000)^2(0.30)+(10,000-13,000)^2(0.40) \\
&\quad +(-10,000-13,000)^2(0.30) \\
&= 381,000,000
\end{aligned}
$$

It is not a clear case, because $E_1 > E_2$ but also $Var_1 > Var_2$ .If she makes decision solely based on the principle of maximization of expected value, she may prefer contract A.

(b) Assuming that both contracts are statistically independent form each other,

Joint event $(PW_A > PW_B)$	Joint Probability
($100,000,$40,000)	(0.20)(0.30) = 0.06
($100,000,$10,000)	(0.20)(0.40) = 0.08
($100,000,-$10,000)	(0.20)(0.30) = 0.06
($50,000,$40,000)	(0.40)(0.30) = 0.12
($50,000,$10,000)	(0.40)(0.40) = 0.16
($50,000,-$10,000)	(0.40)(0.30) = 0.12
($0,-$10,000)	(0.40)(0.30) = 0.12
	$\Sigma = 0.72$

12.21

(a)

- Machine A:

$$
\begin{aligned}
CR(10\%)_A &= (\$60,000-\$22,000)(A/P,10\%,6)+(0.10)(\$22,000) \\
&= \$10,924 \\
E[AE(10\%)]_A &= (\$5,000)(0.20)+(\$8,000)(0.30) \\
&\quad +(\$10,000)(0.30)+(\$12,000)(0.20)+\$10,924 \\
&= \$19,725 \\
Var[AE(10\%)]_A &= (15,924-19,725)^2(0.20)+(18,924-19,725)^2(0.30) \\
&\quad +(20,924-19,725)^2(0.30)+(22,924-19,725)^2(0.20) \\
&= 5,560,000
\end{aligned}
$$

- Machine B:

$$
\begin{aligned}
CR(10\%)_B &= \$35,000(A/P,10\%,4) \\
&= \$11,042 \\
E[AE(10\%)]_B &= (\$8,000)(0.10)+(\$10,000)(0.30) \\
&\quad +(\$12,000)(0.40)+(\$14,000)(0.20)+\$11,042 \\
&= \$22,442 \\
Var[AE(10\%)]_B &= (19,042-22,442)^2(0.10)+(21,042-22,442)^2(0.30) \\
&\quad +(23,042-22,442)^2(0.40)+(25,042-22,442)^2(0.20) \\
&= 3,240,000
\end{aligned}
$$

(b) Prob$[AE(10\%)_A > AE(10\%)_B]$:

Joint event $(\text{O\&M}_A, \text{O\&M}_B)$	$(AE_A > AE_B)$	Joint Probability
($10,000, $8,000)	($20,924, $19,042)	(0.30)(0.10) = 0.03
($12,000, $8,000)	($22,924, $19,042)	(0.20)(0.10) = 0.02
($12,000, $10,000)	($22,924, $21,042)	(0.20)(0.30) = 0.06
		$\Sigma = 0.11$

12.22

(a) Mean and variance calculation (Note: For a random variable Y, which can be expressed as a linear function of another random variable X (say, $Y = aX$, where a is a constant) the variance of Y can be calculated as a function of variance of X, $Var[Y] = a^2 Var[X]$.

$$E[PW]_A = -\$5,000 + \$4,000(P/A,15\%,2)$$
$$= \$1,502.84$$
$$E[PW]_B = -\$10,000 + \$6,000(P/F,15\%,1) + \$8,000(P/F,15\%,2)$$
$$= \$1,266.54$$
$$V[PW]_A = 1,000^2 + (P/F,15\%,1)^2 1,000^2 + (P/F,15\%,2)^2 1,500^2$$
$$= 3,042,588$$
$$V[PW]_B = 2,000^2 + (P/F,15\%,1)^2 1,500^2 + (P/F,15\%,2)^2 2,000^2$$
$$= 7,988,336$$

(b) Comparing risky projects

	Project A	Project B
$E[PW]$	$1,503	$1,267
$V[PW]$	3,042,588	7,988,336

$\therefore$ Project A is preferred.

Decision Tree Analysis

12.23

(a) Let's define the symbols:

P: Party is taking place
NP: No party is planned
TP: Tipster says "P"
TNP: Tipster says "NP"

Then,

$$P(\text{NP} \cap \text{TP})P(\text{TP}) = P(\text{NP})P(\text{TP}|\text{NP})$$
$$= (0.4)(0.2) = 0.08$$

(b)

- Optimal decision without sample information:

$$EMV = (0.6)(100) + (0.4)(-50) = 40 \text{ points}$$

$\therefore$ Raid the dormitories.

- Joint probabilities:

$$P(\mathrm{P} \cap \mathrm{TP}) = P(\mathrm{P})P(\mathrm{TP}\,/\,\mathrm{P}) = (0.60)(0.40) = 0.24$$
$$P(\mathrm{P} \cap \mathrm{TNP}) = P(\mathrm{P})P(\mathrm{TNP}\,/\,\mathrm{P}) = (0.60)(0.60) = 0.36$$
$$P(\mathrm{NP} \cap \mathrm{TP}) = P(\mathrm{NP})P(\mathrm{TP}\,/\,\mathrm{NP}) = (0.40)(0.20) = 0.08$$
$$P(\mathrm{NP} \cap \mathrm{TNP}) = P(\mathrm{NP})P(\mathrm{TNP}\,/\,\mathrm{NP}) = (0.40)(0.80) = 0.32$$

- Marginal probabilities:

$$\begin{aligned} P(\mathrm{TP}) &= P(\mathrm{P} \cap \mathrm{TP}) + P(\mathrm{NP} \cap \mathrm{TP}) \\ &= 0.24 + 0.08 = 0.32 \\ P(\mathrm{TNP}) &= P(\mathrm{P} \cap \mathrm{TNP}) + P(\mathrm{NP} \cap \mathrm{TNP}) \\ &= 0.36 + 0.32 = 0.68 \end{aligned}$$

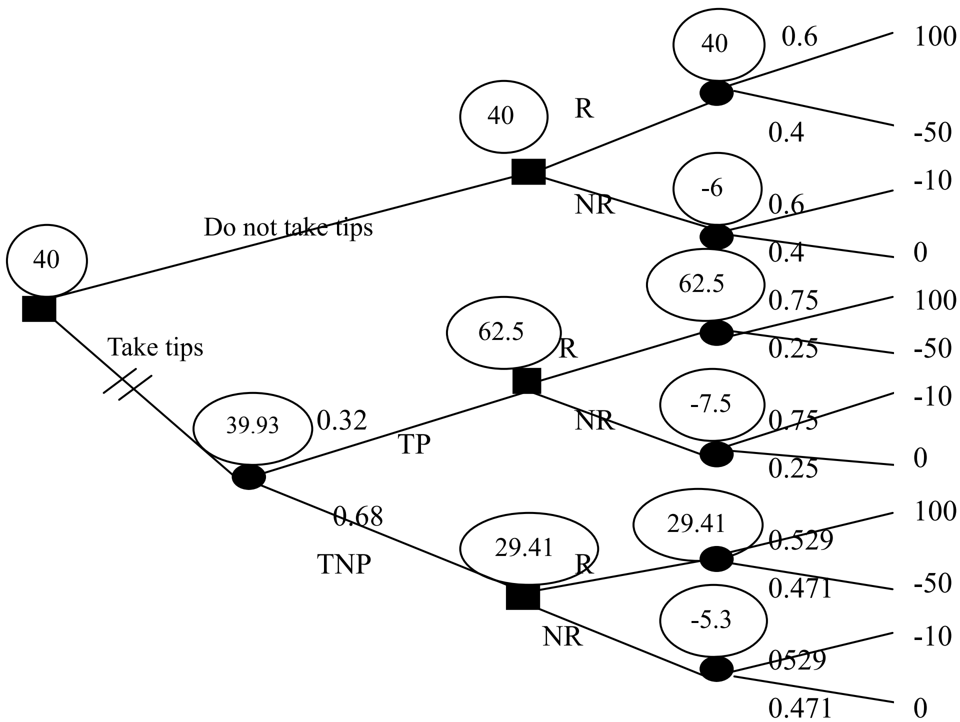
State of Nature		Tipster says		Marginal Probability
		P	NP	
Actual	P	0.24	0.36	0.6
	NP	0.08	0.32	0.4
Marginal Probability		0.32	0.68	1

- Revised probabilities after receiving the tips:

$$P(\mathrm{P/TP}) = \frac{P(\mathrm{P} \cap \mathrm{TP})}{P(\mathrm{TP})} = \frac{0.24}{0.32} = 0.75$$
$$P(\mathrm{NP/TP}) = \frac{P(\mathrm{NP} \cap \mathrm{TP})}{P(\mathrm{TP})} = \frac{0.08}{0.32} = 0.25$$
$$P(\mathrm{P/TNP}) = \frac{P(\mathrm{P} \cap \mathrm{TNP})}{P(\mathrm{TNP})} = \frac{0.36}{0.68} = 0.5294$$
$$P(\mathrm{NP/TNP}) = \frac{P(\mathrm{NP} \cap \mathrm{TNP})}{P(\mathrm{TNP})} = \frac{0.32}{0.68} = 0.4706$$

- Optimal decision after receiving the tips: The tipster's information has no value, even though it costs nothing. Do not reply on the tips.

*** Decision Tree**



(c) EVPI = 60 - 40 = 20

Comments: Note that if a party is planned, "raid" and earn 100 points. If no party is planned, do not raid and earn no point. The expected profit with perfect information is

$$\text{EPPI} = (0.6)(100) + (0.4)(0) = 60 \text{ points}$$

12.24

(a) Given:

Tax rate		40%
Depreciation	Year 1	D_1 = 0.024573($500,000)
	Year 2 - 14	$D_2 - D_{14}$ =0.025641($500,000)
	Year 15	D_{15} = 0.024573($500,000)
	Total	$191,240
Book Value		$308,760
Salvage Value		$100,000
Net proceeds from sale		$183,504

- Case 1: With high demand,

$$\begin{aligned} PW(15\%) = &-\$500{,}000 + 0.6(\$1{,}000{,}000)(P/A,15\%,15) \\ &+ 0.4(\$12{,}287)(P/F,15\%,1) \\ &+ 0.4(\$12{,}821)(P/A,15\%,13)(P/F,15\%,1) \\ &+ 0.4(\$12{,}287)(P/F,15\%,15) + \$183{,}504(P/F,15\%15) \\ = &\$3{,}060{,}145 \end{aligned}$$

- Case 2: With medium demand, $PW(15\%) = \$1{,}305{,}934$

- Case 3: With lower demand, $PW(15\%) = -\$728{,}950$

$$\begin{aligned} \text{EMV}_{\text{open}} &= (0.3)(\$3{,}060{,}145) + (0.4)(\$1{,}305{,}934) + (0.3)(-\$728{,}950) \\ &= \$1{,}221{,}732 \end{aligned}$$

$$\text{EMV}_{\text{do not open}} = 0$$

$\therefore$ Open the store.

- EVPI = EPPI – EMV = $218,685

$$\begin{aligned} \text{EPPI} &= (0.3)(\$3{,}060{,}145) + (0.4)(\$1{,}305{,}934) + (0.3)(0) \\ &= \$1{,}440{,}417 \end{aligned}$$

(b) Investment decision with sample information. Let's define the symbols.

H = High demand
M = Medium demand
L = Low demand
SH = Survey predicts a "H" demand.
SM = Survey predicts an "M" demand.
SL = Survey predicts a "L" demand.

- Joint / marginal probabilities:

$$
\begin{aligned}
P(\mathrm{H} \cap \mathrm{SH}) &= P(\mathrm{H})P(\mathrm{SH/H}) = (0.30)(0.70) = 0.21 \\
P(\mathrm{H} \cap \mathrm{SM}) &= P(\mathrm{H})P(\mathrm{SM/H}) = (0.30)(0.25) = 0.075 \\
P(\mathrm{H} \cap \mathrm{SL}) &= P(\mathrm{H})P(\mathrm{SL/H}) = (0.30)(0.05) = 0.015 \\
&\vdots \\
P(\mathrm{L} \cap \mathrm{SL}) &= P(\mathrm{L})P(\mathrm{SL/L}) = (0.30)(0.75) = 0.225
\end{aligned}
$$

- Marginal probabilities:

$$
\begin{aligned}
P(\mathrm{SH}) &= P(\mathrm{H} \cap \mathrm{SH}) + P(\mathrm{M} \cap \mathrm{SH}) + P(\mathrm{L} \cap \mathrm{SH}) \\
&= 0.24 + 0.08 + 0.15 = 0.305 \\
P(\mathrm{SM}) &= 0.075 + 0.24 + 0.06 = 0.375 \\
P(\mathrm{SL}) &= 0.015 + 0.08 + 0.215 = 0.320
\end{aligned}
$$

State of Nature		Survey says			Marginal Probability
		High	Medium	Low	
	High	0.21	0.075	0 015	0.3
Actual	Medium	0.08	0.24	0.08	0.4
	Low	0.015	0.06	0.225	0.3
Marginal Probability		0.305	0.375	0.320	1

- Revised probabilities after seeing the survey results:

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$$P(\text{H/SH}) = \frac{P(\text{H} \cap \text{SH})}{P(\text{SH})} = \frac{0.21}{0.305} = 0.6885$$

$$P(\text{M/SH}) = \frac{P(\text{M} \cap \text{SH})}{P(\text{SH})} = \frac{0.08}{0.305} = 0.2623$$

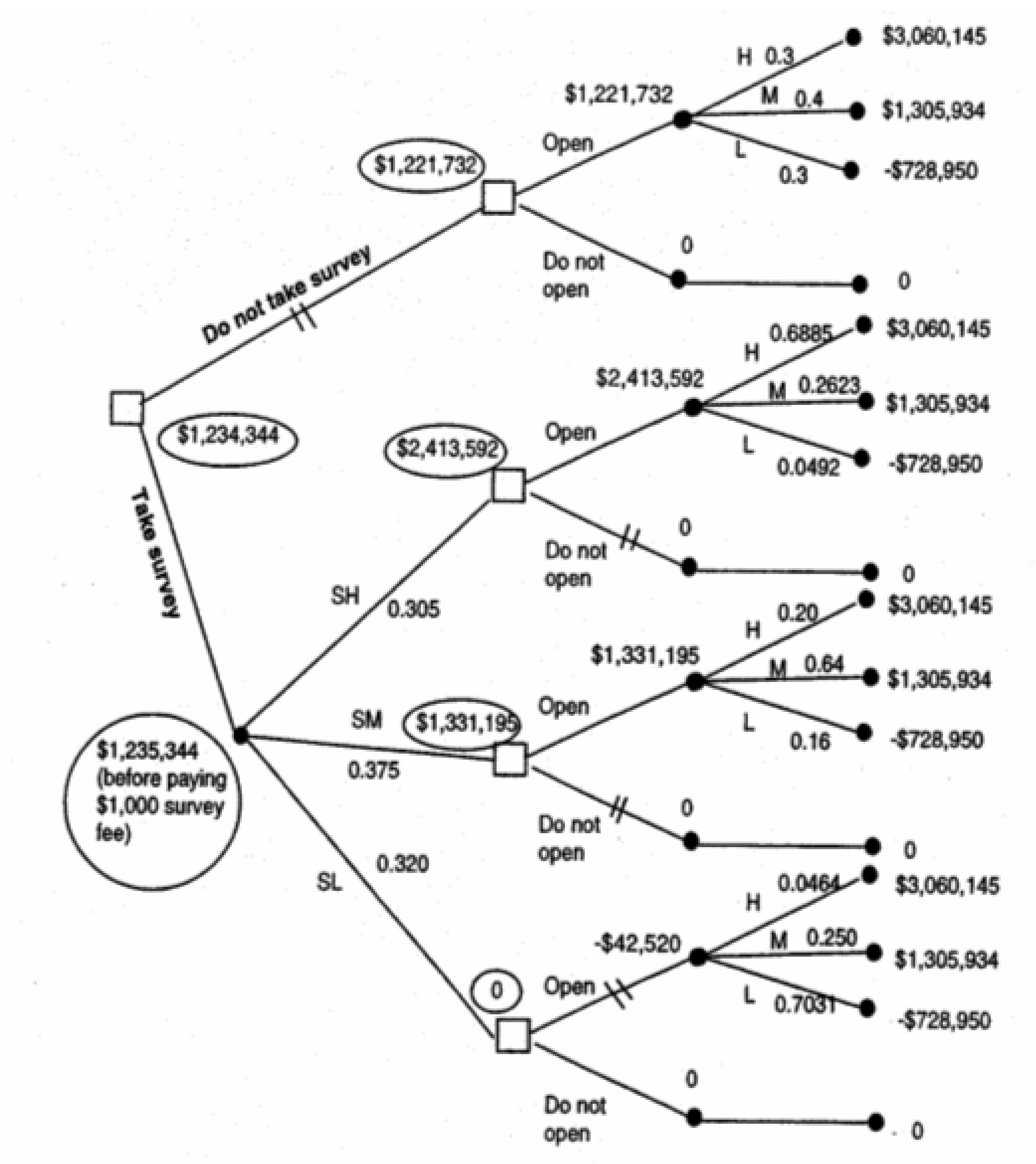
$$\vdots$$

$$P(\text{L/SL}) = \frac{P(\text{L} \cap \text{SL})}{P(\text{SL})} = \frac{0.225}{0.320} = 0.7031$$

- Optimal decision: Take a survey. With either “High” or “Low” result from the survey, open the store. Otherwise, do not open the store.

- The expected monetary value after taking the survey is \$1,234,344.

*** Decision Tree (Problem 12.24)**



Short Case Studies

ST 12.1

Discussion: In Virginia, one investment group came tantalizingly close to cornering the market on all possible combinations of six numbers from 1 to 44. State lottery officials say that the group bought 5 million of the possible 7 million tickets (precisely 7,059,052). Each ticket cost $1 each. The lottery had a more than $27 million jackpot.

$$C(44,6) = \frac{44!}{6!(44-6)!} = 7,059,052$$

• Economic Logic: If the jackpot is big enough, provided nobody else buys a winning ticket, it makes economic sense to buy one lottery ticket for every possible combination of numbers and be sure to win. A group in Australia apparently tried to do this in the February 15 (1992) Virginia Lottery drawing.

• The Cost: Since 7,059,052 combinations of numbers are possible[3] and each ticket costs $1, it would cost $7,059,052 to cover every combination. The total remains the same regardless of the size of the jackpot.

• The Risk: The first prize jackpot is paid out in 20 equal yearly installments, so the actual payoff on all prizes is $2,261,565 the first year and $1,350,368 per year for the next 19 years. If more than one first prize-winning ticket is sold, the prize is shared so that the maximum payoff depends on an ordinary player not buying a winning ticket. Since Virginia began its lottery in January 1990, 120 of the 170 drawings have not yielded a first-prize winner.

Solution

In the Virginia game, 7,059,052 combinations of numbers are possible. The following table summarizes the winning odds for various prizes for a one ticket-only purchase.

Number of Prizes	Prize Category	Winning Odds
1	First prize	0.0000001416
228	Second prizes	0.0000323
10,552	Third prizes	0.00149
168,073	Fourth prizes	0.02381

So a person who buys one ticket has odds of 1 in slightly more than 7 million. Holding more tickets increases the odds of winning, so that 1,000 tickets have odds of 1 in 7,000 and 1 million tickets have odds of 1 in 7. Since each ticket costs $1, it would receive at least a share in the jackpot and many of the second, third, and fourth place prizes. Together these combined prizes (second through fourth) were worth $911,197 payable in one lump sum. Suppose that the Australia group bought all the tickets (7,059,052). We may consider two separate cases.

- Case 1: If none of the prizes was shared, the rate of return on this lottery investment, with prizes paid at the end of each year would be

$$\begin{aligned} PW(i) &= -\$7,059,052 + \$911,197(P/F, i, 1) \\ &\quad +\$1,350,368(P/A, i, 20) \\ &= 0 \\ i^* &= 20.94\% \end{aligned}$$

The first-prize payoff over 20 years is equivalent to putting the same $7,059,052 in a more conventional investment that pays a guaranteed 20.94% return before taxes for 20 years, a rate available only from speculative investments with fairly high risk. (If the prizes are paid at the beginning of each year, the rate of return would be 27.88 %.)

- Case 2: If the first prize is shared with one other ticket, the rate of return on this lottery investment would be 8.87%. (With the prizes paid at the beginning of each year, the rate of return would be 10.48%.) Certainly, if the first prize is shared by more than one, the rate of return would be far less than 8.87%.

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ST 12.2 Since the amount of annual labor savings is the same for both alternatives, this labor savings factor is not considered in the following analysis.

(a) After-tax cash flows:

n	After-Tax Cash Flows Lectra System	Tex System
0	-$136,150	-$195,500
1	117,927	149,075
2	124,462	158,459
3	117,491	148,449
4	113,308	142,443
5	113,308	142,443
6	122,171	146,939
PW(12%)	$350,189	$415,383
AE(12%)	$85,175	$101,032

Based on the most-likely estimates, the Tex system is the better choice.

(b) Let X and Y denote the annual material savings for the Lectra system and Tex system, respectively.

n	After-Tax Cash Flows Lectra System	Tex System
0	-$136,150	-$195,500
1	0.6X-20,073	0.6Y-15,325
2	0.6X-13,538	0.6Y-5,941
3	0.6X-20,508	0.6Y-15,951
4	0.6X-24,691	0.6Y-21,956
5	0.6X-24,691	0.6Y-21,956
6	0.6X-15,828	0.6Y-17,461

- Lectra System:

$$
\begin{aligned}
AE(12\%)_{\text{Lectra}} &= -\$52,824 + 0.6X \\
E[X] &= \$224,000 \\
Var[X] &= 899,000,000 \\
E[AE(12\%)]_{\text{Lectra}} &= -\$52,824 + 0.6E[X] \\
&= \$81,576 \\
Var[AE(12\%)]_{\text{Lectra}} &= (0.6)^2 Var[X] \\
&= 323,640,000
\end{aligned}
$$

- Tex System:

$$
\begin{aligned}
AE(12\%)_{\text{Tex}} &= -\$63,368 + 0.6Y \\
E[Y] &= \$259,400 \\
Var[Y] &= 1,718,440,000 \\
E[AE(12\%)]_{\text{Tex}} &= -\$63,368 + 0.6E[Y] \\
&= \$92,272 \\
Var[AE(12\%)]_{\text{Tex}} &= (0.6)^2 Var[Y] \\
&= 618,638,400
\end{aligned}
$$

(c) Probabilistic analysis:

Variable	Savings	Probability	AE(12%)
	\$150,000	0.25	\$37,176
X	230,000	0.40	85,176
	270,000	0.35	109,176
	200,000	0.30	56,632
Y	274,000	0.50	101,032
	312,000	0.20	123,832

Joint Event		Joint Probability
(x, y)	$AE_{\text{Lectra}} > AE_{\text{Tex}}$	
(230,000 ∩ 200,000)	\$85,176 > \$56,632	(0.40)(0.30) = 0.120
(270,000 ∩ 200,000)	\$190,176 > \$56,632	(0.35)(0.30) = 0.105
(270,000 ∩ 274,000)	\$109,176 > \$101,032	(0.35)(0.50) = 0.175
		$\sum = 0.400$

ST 12.3

(a) Project cash flows:

	0	1	2	3	4	5	6	7	8-19	20
Income Statement										
Revenue:										
Steam Sales		$1,550,520	$1,550,520	$1,550,520	$1,550,520	$1,550,520	$1,550,520	$1,550,520	$1,550,520	$1,550,520
Tipping Fee		976,114	895,723	800,275	687,153	553,301	395,161	208,585	0	0
Expenses:										
O&M		832,000	832,000	832,000	832,000	832,000	832,000	832,000	832,000	832,000
Depreciation										
Interest(11.5%)		805,000	805,000	805,000	805,000	805,000	805,000	805,000	805,000	805,000
Taxable Income		$889,634	$809,243	$713,795	$600,673	$466,821	$308,681	$122,105	($86,480)	($86,480)
Income Tax (0%)		0	0	0	0	0	0	0	0	0
Net Income		$889,634	$809,243	$713,795	$600,673	$466,821	$308,681	$122,105	($86,480)	($86,480)
Cash Flow Statement										
Cash From Operation:										
Net Income		889,634	809,243	713,795	600,673	466,821	308,681	122,105	(86,480)	(86,480)
Depreciation		0	0	0	0	0	0	0	0	0
Investment&Salvage	(6,688,800)									300,000
Gains Tax										
Loan Repayment	6,688,800									(7,000,000)
Net Cash Flow	0	889,634	809,243	713,795	600,673	466,821	308,681	122,105	(86,480)	(6,786,480)
PW (10%) =	$1,639,723									

Note: As a municipal government, the City of Opelika pays no income taxes.

(b) Let X denote the steam charge per pound. Then,

$$\text{Annual steam charge} = 1,061,962(0.001X)(365) = 387,616X$$

n	Revenue	Expenses
1	$387,616X	-$660,886
2	$387,616X	-$741,277
3	$387,616X	-$836,725
4	$387,616X	-$949,847
5	$387,616X	-$1,083,699
6	$387,616X	-$1,241,830
7	$387,616X	-$1,428,415
8-19	$387,616X	-$1,637,000
20	$387,616X	-$8,337,000

$$\begin{aligned} AE(10\%) &= \$387,616X - [\$660,886(P/F,10\%,1) + \cdots \\ &\quad + \$8,337,000(P/F,10\%,20)](A/P,10\%,20) \\ &= \$387,616X - \$1,357,918 \\ &= 0 \end{aligned}$$

$$X = \boxed{\$3.503} \text{ per lb}$$

or $3,503 per thousand lbs

(c) Sensitivity graph (not given)

ST 12.4

(a) Project cash flows based on most-likely estimates:

	0	1	2	3	4	5	5	7	8
Income Statement									
Revenue:									
Bill savings		$3,000,000	$3,000,000	$3,000,000	$3,000,000	$3,000,000	$3,000,000	$3,000,000	$3,000,000
Mile Savings		1,250,000	1,250,000	1,250,000	1,250,000	1,250,000	1,250,000	1,250,000	1,250,000
Expenses:									
Depreciation		2,000,000	3,200,000	1,920,000	1,152,000	1,152,000	576,000		
Taxable Income		2,250,000	1,050,000	2,330,000	3,098,000	3,098,000	3,674,000	4,250,000	4,250,000
Income Tax (38%)		855,000	399,000	885,400	1,177,240	1,177,240	1,396,120	1,615,000	1,615,000
Net Income		$1,395,000	$651,000	$1,444,600	$1,920,760	$1,920,760	$2,277,880	$2,635,000	$2,635,000
Cash Flow Statement									
Cash From Operation:									
Net Income		1,395,000	651,000	1,444,600	1,920,760	1,920,760	2,277,880	2,635,000	2,635,000
Depreciation		2,000,000	3,200,000	1,920,000	1,152,000	1,152,000	576,000	0	0
Investment&Salvage	(10,000,000)								
Net Cash Flow	(10,000,000)	3,395,000	3,851,000	3,364,600	3,072,760	3,072,760	2,853,880	2,635,000	2,635,000
PW (18%) =	$3,204,044								

(b) Sensitivity analysis:

Percentage deviation	Savings In T.B	PW(18%)	Savings In D.M	PW(18%)
-30%	$2,100,000	$928,762	$875,000	$2,256,010
-20%	2,400,000	1,687,189	1,000,000	2,572,021
-10%	2,700,000	2,445,616	1,125,000	2,888,032
0 (base)	3,000,000	3,204,044	1,250,000	3,204,044
+10%	3,300,000	3,962,471	1,375,000	3,520,055
+20%	3,600,000	4,720,898	1,500,000	3,836,066
+30%	3,900,000	5,479,325	1,625,000	4,152,078

ST 12.5

(a) Incremental project cash flows (FMS – CMT)

No. of part types		3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000	3,000
No. of pieces per year		544,000	544,000	544,000	544,000	544,000	544,000	544,000	544,000	544,000	544,000
Year	0	1	2	3	4	5	5	7	8	9	10
Income Statement											
Revenue (Savings):											
Labor		$462,400	$462,400	$462,400	$462,400	$462,400	$462,400	$462,400	$462,400	$462,400	$462,400
Material		233,920	233,920	233,920	233,920	233,920	233,920	233,920	233,920	233,920	233,920
Overhead		1,200,000	1,200,000	1,200,000	1,200,000	1,200,000	1,200,000	1,200,000	1,200,000	1,200,000	1,200,000
Tooling		170,000	170,000	170,000	170,000	170,000	170,000	170,000	170,000	170,000	170,000
Inventory		109,500	109,500	109,500	109,500	109,500	109,500	109,500	109,500	109,500	109,500
Expenses:											
Depreciation		928,850	1,591,850	1,136,850	811,850	580,450	579,800	580,450	289,900		
Taxable Income		1,246,970	583,970	1,038,970	1,363,970	1,595,370	1,596,020	1,595,370	1,885,920	2,175,820	2,175,820
Income Tax (40%)		498,788	233,588	415,588	545,588	638,148	638,408	638,148	754,368	870,328	870,328
Net Income		$748,182	$350,382	$623,382	$818,382	$957,222	$957,612	$957,222	$1,131,552	$1,305,492	$1,305,492
Cash Flow Statement											
Cash From Operation:											
Net Income		748,182	350,382	623,382	818,382	957,222	957,612	957,222	1,131,552	1,305,492	1,305,492
Depreciation		928,850	1,591,850	1,136,850	811,850	580,450	579,800	580,450	289,900		
Investment&Salvage	(6,500,000)										500,000
Gains Tax (40%)											(200,000)
Net Cash Flow	$ (6,500,000)	$ 1,677,032	$ 1,942,232	$ 1,760,232	$ 1,630,232	$ 1,537,672	$ 1,537,412	$ 1,537,672	$ 1,421,452	$ 1,305,492	$ 1,605,492

PW (15%) = $1,756,225

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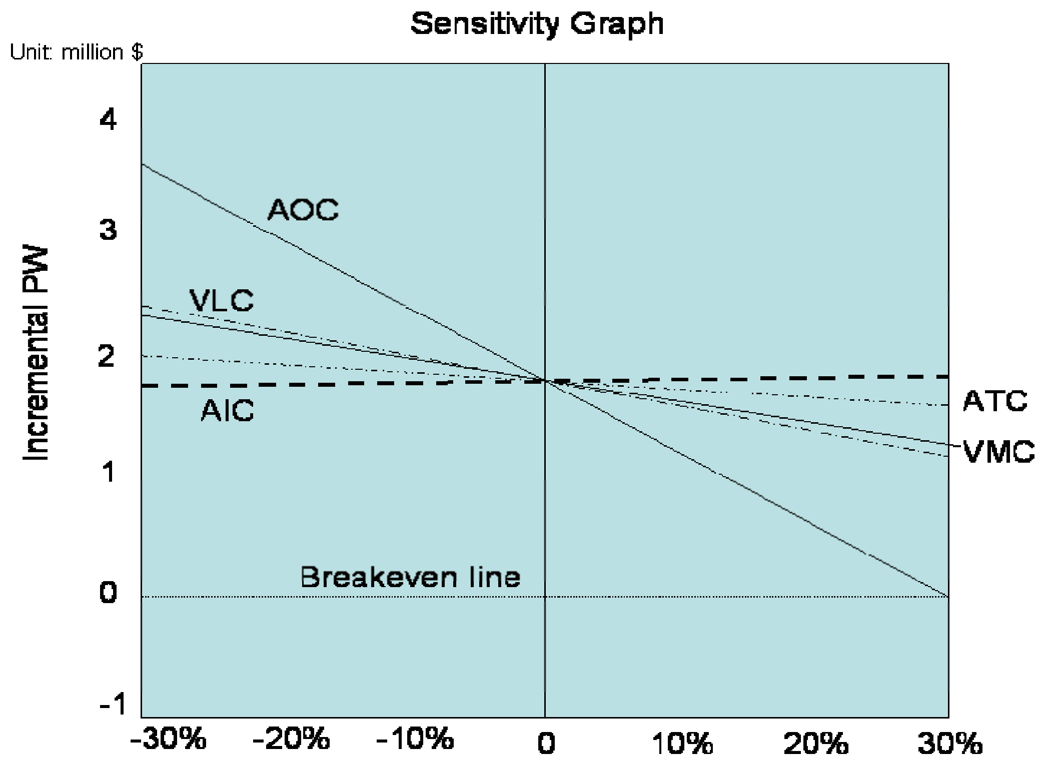
(b) & (c) Sensitivity analysis:

AOC = average overhead cost
VLC = variable labor cost
AIC = average inventory cost
ATC = average tooling cost
VMC = variable material cost



(c) Best and worst scenarios:

- Best case: Material cost = \$1.00 per part, annual inventory cost = \$25,000

$$PW(15\%)_{FMS-CMT} = \$1,939,384$$

- Worst case: Material cost = \$1.40 per part, annual inventory cost = \$100,000

$$PW(15\%)_{FMS-CMT} = \$1,058,289$$

(d) Mean and variance:

- $E[PW(15\%)_{FMS-CMT}]$: \$1,595,123
- $Var[PW(15\%)_{FMS-CMT}]$: 46,073,274,329

Event No.	(x,y)	P(x)	P(y)	P(x,y)	NPV(15%)	WT NPW
1	(\$1, \$25,000)	0.25	0.1	0.025	\$ 1,939,611	\$ 48,490
2	(\$1, \$31,000)	0.25	0.3	0.075	\$ 1,921,544	\$ 144,116
3	(\$1, \$50,000)	0.25	0.2	0.05	\$ 1,864,330	\$ 93,217
4	(\$1, \$80,000)	0.25	0.2	0.05	\$ 1,773,992	\$ 88,700
5	(\$1, \$100,000)	0.25	0.2	0.05	\$ 1,713,767	\$ 85,688
6	(\$1.1, \$25,000)	0.3	0.1	0.03	\$ 1,775,799	\$ 53,274
7	(\$1.1, \$30,000)	0.3	0.3	0.09	\$ 1,757,731	\$ 158,196
8	(\$1.1, \$50,000)	0.3	0.2	0.06	\$ 1,700,517	\$ 102,031
9	(\$1.1, \$80,000)	0.3	0.2	0.06	\$ 1,610,179	\$ 96,611
10	(\$1.1, \$100,000)	0.3	0.2	0.06	\$ 1,549,954	\$ 92,997
11	(\$1.2, \$25,000)	0.2	0.1	0.02	\$ 1,611,986	\$ 32,240
12	(\$1.2, \$31,000)	0.2	0.3	0.06	\$ 1,593,918	\$ 95,635
13	(\$1.2, \$50,000)	0.2	0.2	0.04	\$ 1,536,705	\$ 61,468
14	(\$1.2, \$80,000)	0.2	0.2	0.04	\$ 1,446,367	\$ 57,855
15	(\$1.2, \$100,000)	0.2	0.2	0.04	\$ 1,386,141	\$ 55,446
16	(\$1.3, \$25,000)	0.2	0.1	0.02	\$ 1,448,173	\$ 28,963
17	(\$1.3, \$31,000)	0.2	0.3	0.06	\$ 1,430,106	\$ 85,806
18	(\$1.3, \$50,000)	0.2	0.2	0.04	\$ 1,372,892	\$ 54,916
19	(\$1.3, \$80,000)	0.2	0.2	0.04	\$ 1,282,554	\$ 51,302
20	(\$1.3, \$100,000)	0.2	0.2	0.04	\$ 1,222,329	\$ 48,893
21	(\$1.4, \$25,000)	0.05	0.1	0.005	\$ 1,284,361	\$ 6,422
22	(\$1.4, \$31,000)	0.05	0.3	0.015	\$ 1,266,293	\$ 18,994
23	(\$1.4, \$50,000)	0.05	0.2	0.01	\$ 1,209,079	\$ 12,091
24	(\$1.4, \$80,000)	0.05	0.2	0.01	\$ 1,118,741	\$ 11,187
25	(\$1.4, \$100,000)	0.05	0.2	0.01	\$ 1,058,516	\$ 10,585
					E[PW(15%)] = \$	1,595,123

- $Var[PW(15\%)_{FMS-CMT}]$:

Event No.	(x,y)	P(x)	P(y)	P(x,y)	NPW	(NPW-u)^2	WT (NPW-m)^2
1	($1, $25,000)	0.25	0.1	0.025	1,939,611	118,671,982,144	2,966,799,554
2	($1, $31,000)	0.25	0.3	0.075	1,921,544	106,550,669,241	7,991,300,193
3	($1, $50,000)	0.25	0.2	0.05	1,864,330	72,472,408,849	3,623,620,442
4	($1, $80,000)	0.25	0.2	0.05	1,773,992	31,994,119,161	1,599,705,958
5	($1, $100,000)	0.25	0.2	0.05	1,713,767	14,076,398,736	703,819,937
6	($1.1, $25,000)	0.3	0.1	0.03	1,775,799	32,643,816,976	979,314,509
7	($1.1, $30,000)	0.3	0.3	0.09	1,757,731	26,441,361,664	2,379,722,550
8	($1.1, $50,000)	0.3	0.2	0.06	1,700,517	11,107,895,236	666,473,714
9	($1.1, $80,000)	0.3	0.2	0.06	1,610,179	226,683,136	13,600,988
10	($1.1, $100,000)	0.3	0.2	0.06	1,549,954	2,040,238,561	122,414,314
11	($1.2, $25,000)	0.2	0.1	0.02	1,611,986	284,360,769	5,687,215
12	($1.2, $31,000)	0.2	0.3	0.06	1,593,918	1,452,025	87,122
13	($1.2, $50,000)	0.2	0.2	0.04	1,536,705	3,412,662,724	136,506,509
14	($1.2, $80,000)	0.2	0.2	0.04	1,446,367	22,128,347,536	885,133,901
15	($1.2, $100,000)	0.2	0.2	0.04	1,386,141	43,673,476,324	1,746,939,053
16	($1.3, $25,000)	0.2	0.1	0.02	1,448,173	21,594,302,500	431,886,050
17	($1.3, $31,000)	0.2	0.3	0.06	1,430,106	27,230,610,289	1,633,836,617
18	($1.3, $50,000)	0.2	0.2	0.04	1,372,892	49,386,617,361	1,975,464,694
19	($1.3, $80,000)	0.2	0.2	0.04	1,282,554	97,699,379,761	3,907,975,190
20	($1.3, $100,000)	0.2	0.2	0.04	1,222,329	138,975,366,436	5,559,014,657
21	($1.4, $25,000)	0.05	0.1	0.005	1,284,361	96,573,020,644	482,865,103
22	($1.4, $31,000)	0.05	0.3	0.015	1,266,293	108,129,168,900	1,621,937,534
23	($1.4, $50,000)	0.05	0.2	0.01	1,209,079	149,029,969,936	1,490,299,699
24	($1.4, $80,000)	0.05	0.2	0.01	1,118,741	226,939,809,924	2,269,398,099
25	($1.4, $100,000)	0.05	0.2	0.01	1,058,516	287,947,072,449	2,879,470,724
						Var[PW(15%)] =	46,073,274,329

(e) The FMS option dominates the CMT option over the relevant range of decision variables.