Chapter 13 Real-Options Analysis

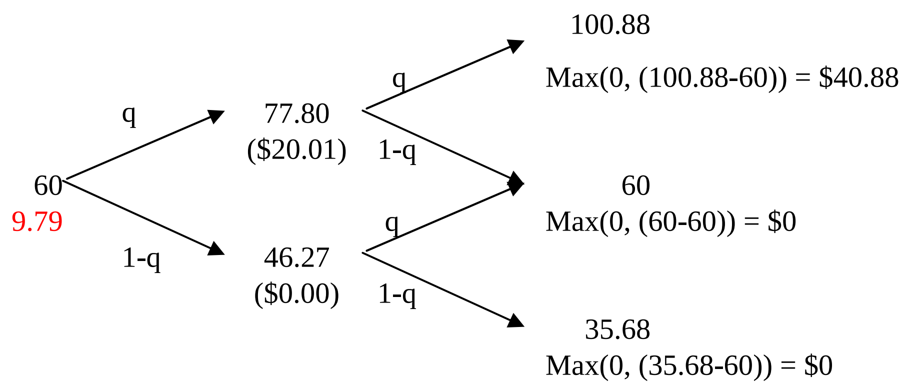
Financial Options

13.1

- $u = e^{\sigma\sqrt{\Delta t}} = e^{0.3\times\sqrt{0.75}} = 1.2967$
- $d = \dfrac{1}{u} = \dfrac{1}{1.2967} = 0.7712$
- Risk neutral probability

$$q = \frac{e^{r\Delta t} - d}{u - d} = \frac{e^{0.05\times 0.75} - 0.7712}{1.2967 - 0.7712} = 0.5081$$

- Tree valuation



$\therefore$ **European call option value = $9.79**

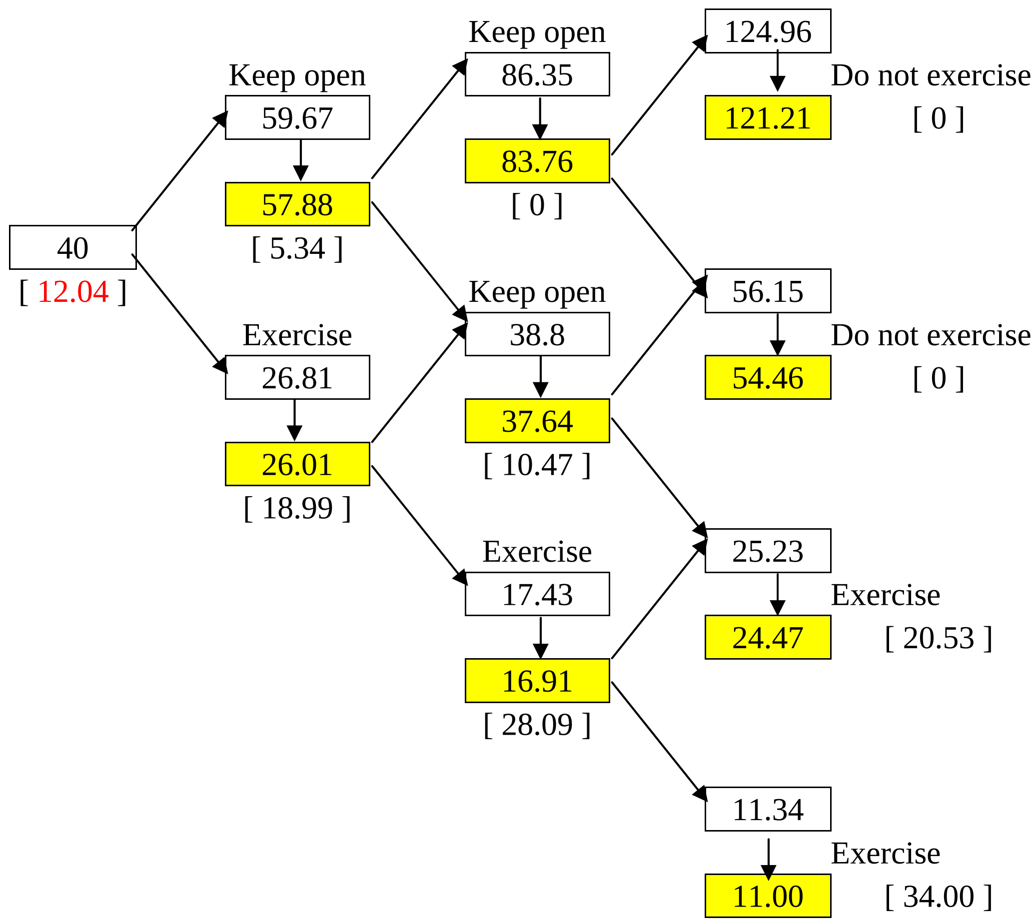
13.2

- $u = e^{\sigma\sqrt{\Delta t}} = e^{0.4\times\sqrt{1}} = 1.4918$
- $d = \dfrac{1}{u} = \dfrac{1}{1.4918} = 0.6703$
- Risk neutral probability

$$q = \frac{e^{r\Delta t} - d}{u - d} = \frac{e^{0.05} - 0.6703}{1.4918 - 0.6703} = 0.4638$$

$$1 - q = 0.5362$$

- Tree valuation:
  Note to instructors: No mention is made about the dividend payment in determining the option premium in the text. In financial option, any dividend payment reduces the value of the option. The figures in yellow represent the adjusted share prices after a 3% dividend payment, i.e., (1 – 0.03)($124.96) = $121.21. If there is no dividend payment, we just use the original share prices in determining the option premium.



∴ **American option value = $12.04**

13.3

Portfolio	Premium	Payoff at stock price $60
A long call with X = $40	$3	$20 - $3 = $17
A short put with X = $45	$4	$4 – 0 = $4
Two short call with X = $35	$5	($5 - $25)×2 = ($40)
Two short stock at $40		($40 - $60)×2 = ($40)
Total		**($59)**

13.4 **Note to the instructors –** The definitions of intrinsic value as well as time value were not given in the text. In financial option, the option premium is viewed as having two types of value. The intrinsic value is the value if you exercise the option immediately. The time value is the value if you wait.

- Intrinsic value = $X - S_0 = \$2$

- Time premium = option premium – intrinsic value = $2

Real-Options Analysis

13.5
- Define the real option parameters for delaying option.

V	I	T	r	σ
$1.9 Million	$2 Million	1 year	0.08	0.4

$$
\begin{aligned}
C_{\text{call}} &= S_0 N(d_1) - Ke^{-r_f T} N(d_2) \\
&= 1.9N(0.2718) - 2e^{-0.08} N(-0.1283) \\
&= \$0.3246\text{M} \\
d_1 &= \frac{\ln(S_0/K) + (r_f + \sigma^2/2)T}{\sigma\sqrt{T}} \\
&= \frac{\ln(1.9/2) + (0.08 + \frac{0.4^2}{2})1}{0.4\sqrt{1}} = 0.2718 \\
d_2 &= d_1 - \sigma\sqrt{T} = 0.2718 - 0.4\sqrt{1} = -0.1283
\end{aligned}
$$

13.6
- Define the real option parameters for license option.

V*	I	T	r	σ
$30 Million	$40 Million	3	0.06	0.2

* V = ($340 - $320) × 1.5M = 30M

$$
\begin{aligned}
C_{\text{call}} &= S_0 N(d_1) - Ke^{-r_f T} N(d_2) \\
&= 30N(-0.0165) - 40e^{-0.18} N(-0.3629) \\
&= \$2.8610\text{M}
\end{aligned}
$$

$$
d_1 = \frac{\ln(S_0 / K) + (r_f + \sigma^2/2)T}{\sigma\sqrt{T}}
$$

$$
= \frac{\ln(30/40) + (0.06 + \frac{0.2^2}{2})3}{0.2\sqrt{3}} = -0.0165
$$

$$
d_2 = d_1 - \sigma\sqrt{T} = -0.0165 - 0.2\sqrt{3} = -0.3629
$$

Switching Options

13.7 A switching option is a special case of a put option.

- Compute the NPW of project B:

$$
\text{NPW}_B = -\$2 + \$1(P/A, 12\%, 10) = \$3.65\text{M}
$$

- Define the real option parameters for the switching option and compute the put option premium by using the Black-Scholes model.

V	I	T	r	σ
\$4 Million	\$3.65 Million	5	0.06	0.5

$$
\begin{aligned}
C_{\text{put}} &= 3.65e^{-3.0} N(0.2088) - 4N(-0.9093) \\
&= \$0.85\text{M}
\end{aligned}
$$

$$
d_1 = \frac{\ln(4/3.65) + (0.06 + \frac{0.5^2}{2})5}{0.5\sqrt{5}} = 0.9093
$$

$$
d_2 = 0.9093 - 1.118 = -0.2088
$$

Contemporary Engineering Economics, Fourth Edition, by Chan S. Park. ISBN 0-13-187628-7.

- Determine the combined option value

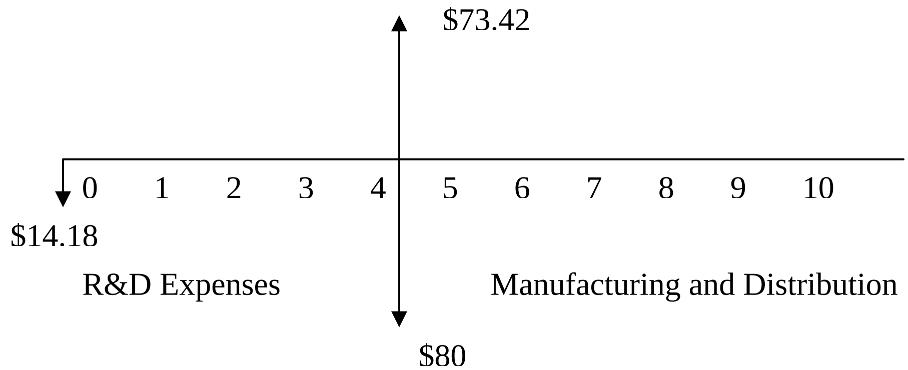
Combined Option Value = Value of project A + Option to switch to project B

$$\$4 + \$0.85 = \$4.85\text{M}$$

R&D Options

13.8

- Assuming MARR = 12%, the cash flow diagram transforms to:



- Define the real option parameters for R&D option.

V	I	T	r	σ
$46.66 Million	$80 Million	4	0.06	0.5

$$\begin{aligned} C_{\text{call}} &= S_0 N(d_1) - Ke^{-r_f T} N(d_2) \\ &= 46.66 N(0.2009) - 80e^{-0.08} N(-0.7991) \\ &= \$13.70\text{M} \end{aligned}$$

$$d_1 = \frac{\ln(46.66/80) + (0.06 + \frac{0.5^2}{2})4}{0.5\sqrt{4}} = 0.2009$$

$$d_2 = 0.2009 - 0.5\sqrt{4} = -0.7991$$

- Therefore, the total value is:

Combined option premium = Cost for R&D + Option value

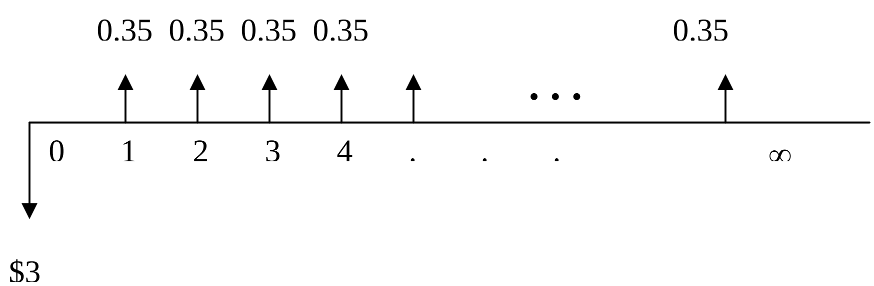
$$= -\$14.18 + \$13.7 = -\$0.48M$$

The maximum amount the firm should spend on R&D for this project is equal to \$13.7M.

Abandonment Options

13.9

- Standard NPV approach



$$\text{NPW}_0 = -\$3 + \frac{\$0.35}{0.12} = -\$0.08\text{M}$$

- Abandon Option value through the binomial tree
  - Option parameters

V	I	T	r	σ
\$2.92 Million	\$2.2 Million	5	0.06	0.5

  - Option valuations

Time	0	1	2	3	4	5
	2.92	4.81	7.94	13.09	21.58	35.57
		1.77	2.92	4.81	7.94	13.09
			1.07	1.77	2.92	4.81
Monetary Value				0.65	1.07	1.77
					0.40	0.65
						0.24

Contemporary Engineering Economics, Fourth Edition, by Chan S. Park. ISBN 0-13-187628-7.

	0.50	**0.23**	**0.06**	**0.00**	**0.00**	**0.00**
		0.77	**0.42**	**0.12**	**0.00**	**0.00**
Option value			**1.13**	**0.69**	**0.23**	**0.00**
				1.55	**1.13**	**0.43**
					1.80	**1.55**
						1.96

* Early abandonment decisions could occur in the shaded periods.

$\therefore$ Combined option value = -0.08 + 0.50 = 0.42M

Scale-Down Options

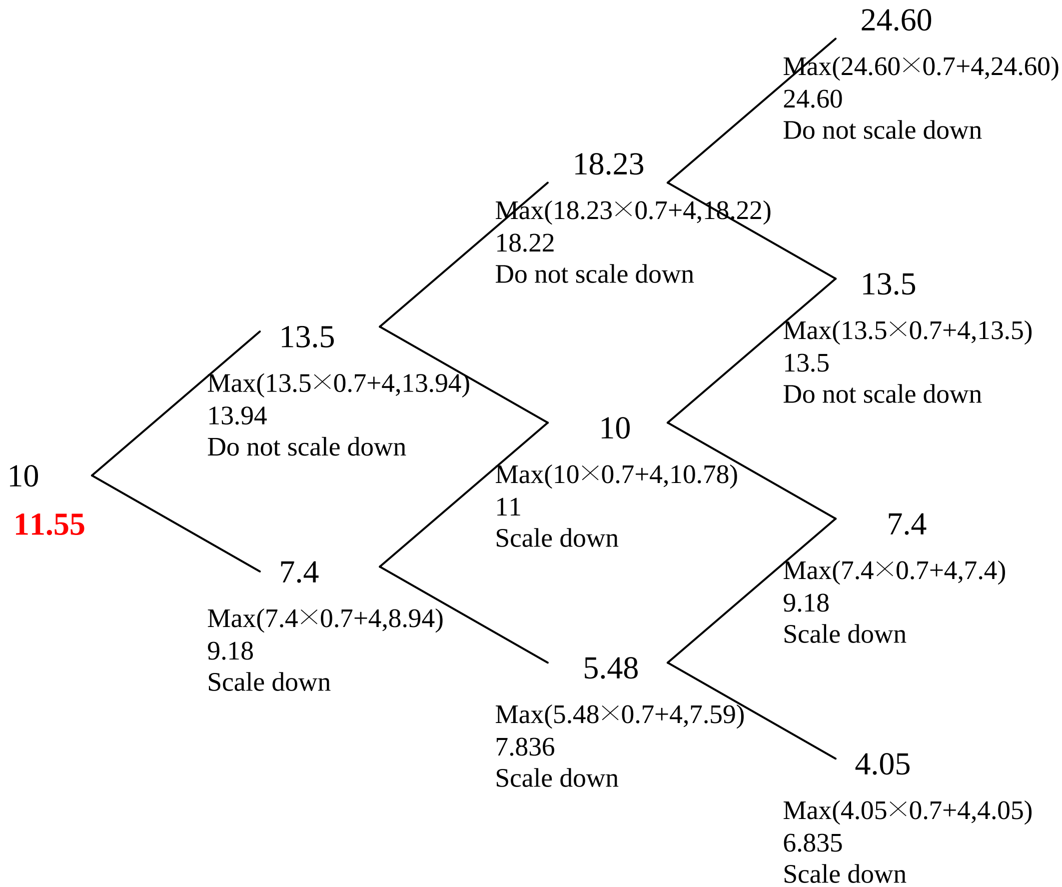
13.10

- Scale down option parameters

V	I	T	r	σ
$10 Million	$4 Million	3	0.06	0.3

- Decision tree for a scale-down option through one-year time increment.

- $u = e^{\sigma\sqrt{\Delta t}} = e^{0.3\times\sqrt{1}} = 1.35$
- $d = \frac{1}{u} = \frac{1}{1.35} = 0.74$
- $q = \frac{e^{r\Delta t} - d}{u - d} = \frac{e^{0.06\times 1} - 0.74}{1.35 - 0.74} = 0.53,\ 1 - q = 0.47$



- From the result of the tree we can get the combined option value as follows:

Combined option value = NPW + Option value = \$11.55

$\therefore$ Option value = \$11.55 - \$10 = \$1.55M

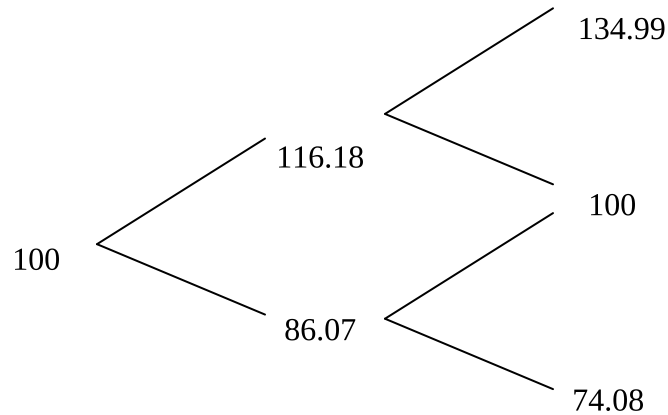
Expansion-Contraction Options

13.11

(a) Binomial lattice tree

- $u = e^{\sigma\sqrt{\Delta t}} = e^{0.15\times\sqrt{1}} = 1.1618$
- $d = \dfrac{1}{u} = \dfrac{1}{1.1618} = 0.8007$

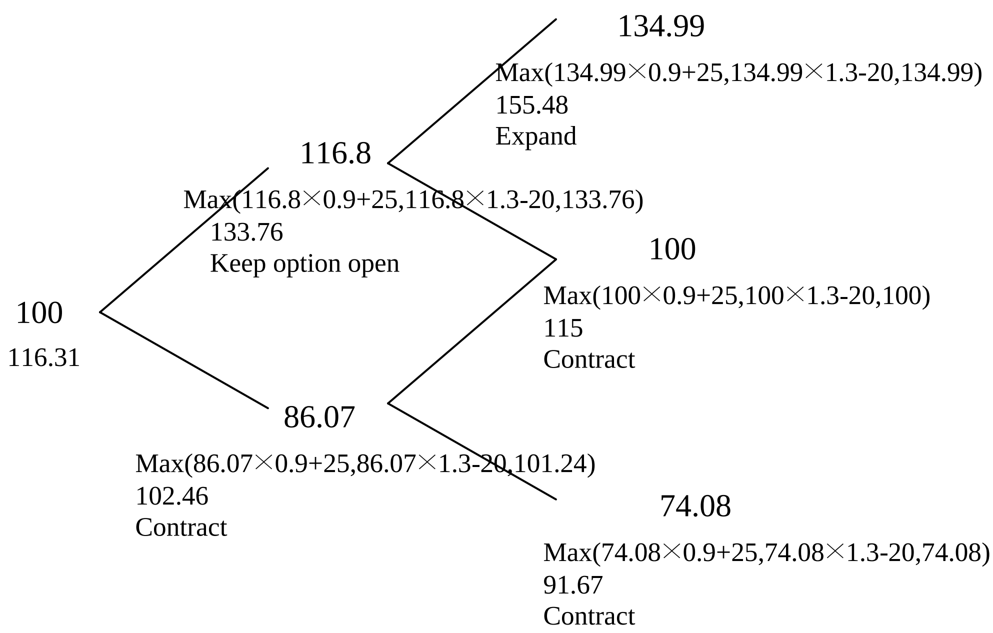
• Tree with a one-year incremental period



(b) Option valuation

• Risk neutral probability

$$q = \frac{e^{r\Delta t} - d}{u - d} = \frac{e^{0.05\times 1} - 0.8007}{1.1618 - 0.8007} = 0.6328,\ 1 - q = 0.3672$$



∴ Option value = $116.31 - $100 = $16.31M

Compound Options

13.12

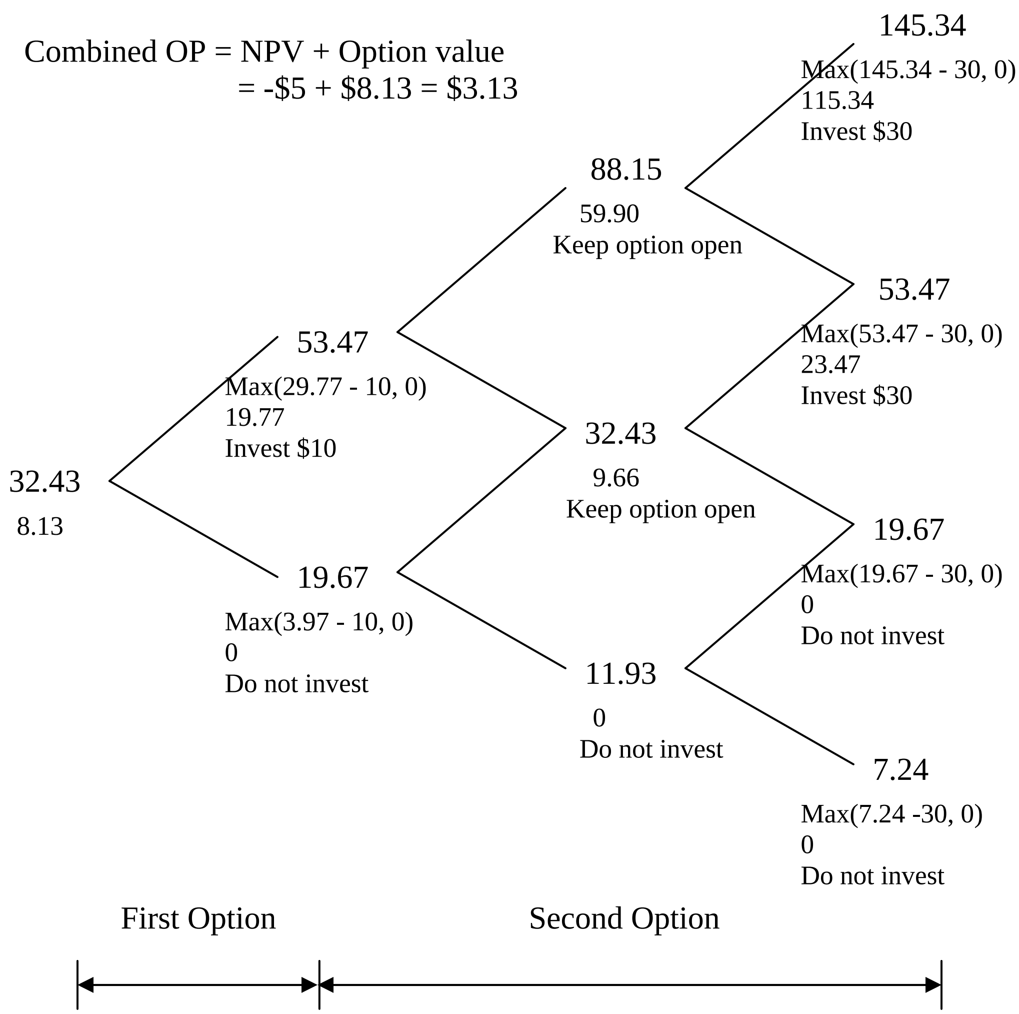
- Compound option parameters

V_0	I_1	I_2	T_1	T_2	r	σ
\$32.43	\$10	\$30	1	3	0.06	0.5

- Decision tree for a scale-down option with a one-year time increment.

- $u = e^{\sigma\sqrt{\Delta t}} = e^{0.5\times\sqrt{1}} = 1.6487$
- $d = \dfrac{1}{u} = \dfrac{1}{1.6487} = 0.6065$
- $q = \dfrac{e^{r\Delta t} - d}{u - d} = \dfrac{e^{0.06\times 1} - 0.6065}{1.6487 - 0.6065} = 0.4369$

Combined OP = NPV + Option value
= -\$5 + \$8.13 = \$3.13



Short Case Studies

ST 13.1

(a) American option value

- Option parameters

S	K	T	Δt	r	σ
\$150	\$100	5	1 year	0.05	0.3

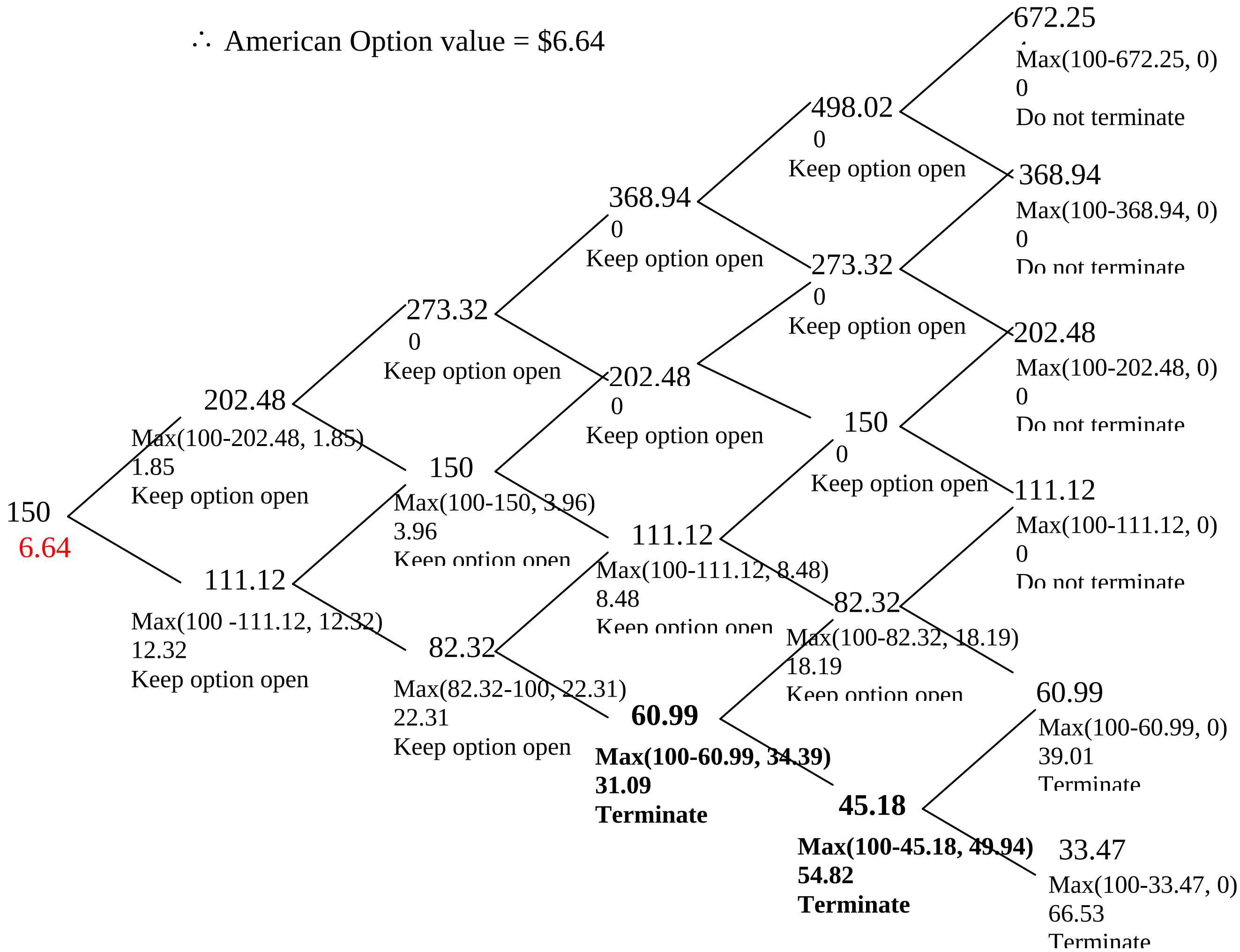
- $u = e^{\sigma\sqrt{\Delta t}} = e^{0.3\times\sqrt{1}} = 1.3499$

- $d = \dfrac{1}{u} = \dfrac{1}{1.3499} = 0.7408$

- Risk neutral probability

$$q = \frac{e^{r\Delta t} - d}{u - d} = \frac{e^{0.05\times 1} - 0.7408}{1.3499 - 0.7408} = 0.5097\,,\ 1 - q = 0.4903$$

∴ American Option value = \$6.64



(b) European option value is \$6.09 by B-S equation

ST 13.2

(a) Since \$4 M is lower than the option price, it is a good investment for Merck.

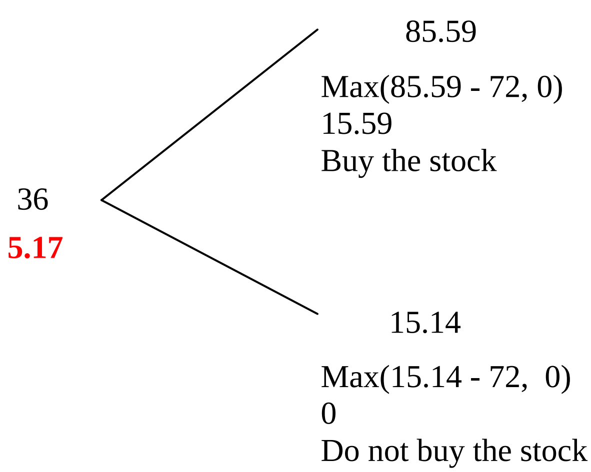
- To give a range for the option value, first use one period lattice.

V	K	T	Δt	r	σ
\$36M (\$30×1.2M)	\$72M (\$60×1.2M)	3	3 years	0.06	0.5

- $u = e^{\sigma\sqrt{\Delta t}} = e^{0.5\times\sqrt{3}} = 2.3774$
- $d = \dfrac{1}{u} = \dfrac{1}{2.3774} = 0.4206$
- Risk neutral probability

$$q = \frac{e^{r\Delta t} - d}{u - d} = \frac{e^{0.06\times 3} - 0.4206}{2.3774 - 0.4206} = 0.3969\,,\ 1 - q = 0.6031$$

- One period lattice and option price: \$**5.17M**



- Through the B-S model, the option price should be **\$6.54M.**