Chapter 5 Present-Worth Analysis

Identifying Cash Inflows and Outflows

5.1

(a) Cash inflows: (1) savings in labor, \$45,000 per year, (2) salvage value, \$3,000 at year 5.

(b) Cash outflows: (1) capital expenditure = \$30,000 at year 0, (2) operating costs = \$5,000 per year.

(c) Estimating project cash flows:

n	Inflow	Outflow	Net flow
0		\$30,000	-\$30,000
1	\$45,000	5,000	40,000
2	45,000	5,000	40,000
3	45,000	5,000	40,000
4	45,000	5,000	40,000
5	45,000+3,000	5,000	43,000

Payback Period

5.2

(a) Payback period: 0.75 years

(b) Discounted payback period = 0.86 years, assuming continuous payments:

n	Net Cash Flow	Cost of Funds (15%)	Cumulative Cash Flow
0	-\$30,000		-\$30,000
1	+\$40,000	-\$30,000(0.15) = -\$4,500	+\$5,500

5.3

(a) Payback period

Project	A	B	C	D
Payback period	No payback	1.66 years	2.75 years	6.17 years

(b) It maybe viewed as a combination of two separate projects, where the first investment is recovered at the end of year 1 and the investment that made in year 2 and 3 will be recovered at the end of year 6. If you view the total amount of investment to be \$9,500, the payback period could be 6.16years.

(c) Discount payback period

Project	A	B	C	D
Payback period	No payback	1.95 years	3.14 years	6.89 years

NPW Criterion

5.4

(a)

$$PW(10\%)_A = -\$1{,}500 + \$3{,}000(P/F,10\%,3) = \$1{,}412$$

$$PW(10\%)_B = -\$1{,}200 + \$600(P/F,10\%1) + \$800(P/F,10\%,2)$$
$$+\$1{,}500(P/F,10\%,3) = \$1{,}134$$

$$PW(10\%)_C = -\$1{,}600 - \$1{,}800(P/F,10\%1) + \$800(P/F,10\%,2)$$
$$+\$2{,}500(P/F,10\%,3) = -\$697$$

$$PW(10\%)_D = -\$3{,}100 + \$800(P/F,10\%1) + \$1{,}900(P/F,10\%,2)$$
$$+\$2{,}300(P/F,10\%,3) = \$926$$

(b) Not provided. (Can easily obtain the graphs using Excel or other software)

5.5

(a)

$$PW(9\%) = -\$200{,}000 + \$24{,}000(P/A,9\%,35) + \$35{,}000(P/F,9\%,35)$$
$$= \$55{,}317.71 > 0$$

(b)

$$PW(6\%) = -\$200{,}000 + \$24{,}000(P/A,6\%,35) + \$35{,}000(P/F,6\%,35)$$
$$= \$152{,}511.60$$

(c)

$$PW(i) = -\$200{,}000 + \$24{,}000(P/A,i,35) + \$35{,}000(P/F,i,35) = 0$$
$$i = 11.80\%$$

$\therefore$ Therefore, the answer is (d).

5.6 Given: Estimated remaining service life = 25 years, current rental income = \$150,000 per year, O&M costs = \$45,000 for the first year increasing by \$3,000 thereafter, salvage value $= \$50,000$, and $\text{MARR} = 12\%$.

Let A_0 be the maximum investment required to break even.

$$
\begin{aligned}
PW(12\%) = & -A_0 + [\$150,000(F/A,12\%,25) + \$15,000(F/A,12\%,20) \\
& +\$16,500(F/A,12\%,15) + \$18,150(F/A,12\%,10) \\
& +\$19,965(F/A,12\%,5) + \$50,000](P/F,12\%,25) \\
& -\$45,000(P/A,12\%,25) - \$3,000(P/G,12\%,25) \\
& = 0
\end{aligned}
$$

$\therefore$ Solving for $A_0 =$ yields
$A_0 = \$793,113$

5.7

$$
\begin{aligned}
P = & -\$42,000 + [[\$32,400 + [\$33,400 + [\$32,500 + \{\$32,500 \\
& +\$33,000(P/F,12\%,1)\}(P/F,15\%,1)](P/F,13\%,1)] \\
& \times(P/F,11\%,1)]](P/F,10\%,1) \\
& = \$77,417
\end{aligned}
$$

5.8 Units are in thousand

$$
\begin{aligned}
PW(15\%) = & -\$500 - \$3,200(P/F,15\%,1) - \$4,000(P/F,15\%,2) \\
& + (\$4,000 + \$2,000 - \$2,800)(P/A,15\%,8)(P/F,15\%2) \\
& + \$1,200(P/F,15\%,10) \\
& = \$4,847.23
\end{aligned}
$$

5.9

$$
\begin{aligned}
PW(18\%) = & -\$3,000,000 + [\$1,550,000 - \$350,000 - \$150,000](P/A,18\%,10) \\
& + \$200,000(P/F,18\%,10) \\
& = \$1,757,018
\end{aligned}
$$

Future Worth and Project Balance

5.10

(a)

$$
\begin{aligned}
PW(15\%)_A &= -\$12,500 + \$5,400(P/F,15\%,1) + \$14,400(P/F,15\%,2) \\
& \quad +\$7,200(P/F,15\%,3) = \$7,818 \\
PW(15\%)_B &= -\$11,500 - \$3,000(P/F,15\%,1) + \$21,000(P/F,15\%,2) \\
& \quad +\$13,000(P/F,15\%,3) = \$10,318 \\
PW(15\%)_C &= \$12,500 - \$7,000(P/F,15\%,1) - \$2,000(P/F,15\%,2) \\
& \quad +\$4,000(P/F,15\%,3) = \$7,531 \\
PW(15\%)_A &= -\$13,000 + \$5,500(P/F,15\%,1) + \$5,500(P/F,15\%,2) \\
& \quad +\$8,500(P/F,15\%,3) = \$2,030
\end{aligned}
$$

(b)

$$FW(15\%)_A = \$7{,}818(F/P{,}15\%{,}3) = \$11{,}890$$
$$FW(15\%)_B = \$10{,}318(F/P{,}15\%{,}3) = \$15{,}694$$
$$FW(15\%)_C = \$7{,}531(F/P{,}15\%{,}3) = \$11{,}454$$
$$FW(15\%)_D = \$2{,}030(F/P{,}15\%{,}3) = \$3{,}087$$

$\therefore$ All the projects are acceptable.

5.11

(a) The original cash flows of the project are as follows.

n	A_n	Project Balance
0	-$1,000	-$1,000
1	**($100)**	-$1,100
2	**($520)**	-$800
3	$460	-$500
4	**($600)**	0

(b)

$$PB(i)_3 = -\$800(1+i) + \$460 = -\$500$$

$$\therefore \quad i = 20\%$$

(c) Yes, the project is acceptable.

5.12

$$\begin{aligned} PW(15\%) &= \$1{,}000{,}000(P/F,15\%,1) + \$750{,}000(P/F,15\%,2) \\ &\quad + \$562{,}500(P/F,15\%,3) + \$421{,}875(P/F,15\%,4) \\ &= \$2{,}047{,}734 \end{aligned}$$

5.13

(a) First find the interest rate that is used in calculating the project balances. We can set up the following project balance equations:

$$PB(i)_1 = -\$10{,}000(1+i) + A_1 = -\$11{,}000$$
$$PB(i)_2 = -\$11{,}000(1+i) + A_2 = -\$8{,}200$$
$$PB(i)_3 = -\$8{,}200(1+i) + \$8{,}000 = -\$1{,}840$$
$$PB(i)_4 = -\$1{,}840(1+i) + A_4 = \$3{,}792$$
$$PB(i)_5 = \$3{,}792(1+i) + A_5 = \$7{,}550$$

From $PB(i)_3$, we can solve for i: $i = 20\%$.

Substituting i into other $PB(i)_n$ yields

n	A_n	$PB(i)_n$
0	-$10,000	-$10,000
1	**1,000**	-11,000
2	**5,000**	-8,200
3	8,000	-1,840
4	**6,000**	3,792
5	**3,000**	7,550

(b) Conventional payback period $= 2.5$ years

5.14

(a) From the project balance diagram, note that $PW(24\%)_1 = 0$ for project 1 and $PW(23\%)_2 = 0$ for project 2.

$$PW(24\%)_1 = -\$1{,}000 + \$400(P/F,15\%,1) + \$800(P/F,15\%,2) + X(P/F,15\%,3) = 0$$

$$PW(23\%)_2 = -\$1{,}000 + \$300(P/F,15\%,1) + Y(P/F,15\%,2) + \$800(P/F,15\%,3) = 0$$

$$\therefore\ X = \$299.58, \quad Y = \$493.49$$

(b) Since $PW(24\%)_1 = 0$, $FW(24\%)_1 = 0$.

(c) ⓐ = $593.49, ⓑ = $499.56, ⓒ = 17.91%

5.15

(a) The original cash flows of the project are as follows:

n	A_n	Project Balance
0	-$3,000	-$3,000
1	**($600)**	-$2,700
2	$1,470	-$1,500
3	**($1,650)**	0
4	**(-$300)**	-$300
5	$600	**($270)**

(b)

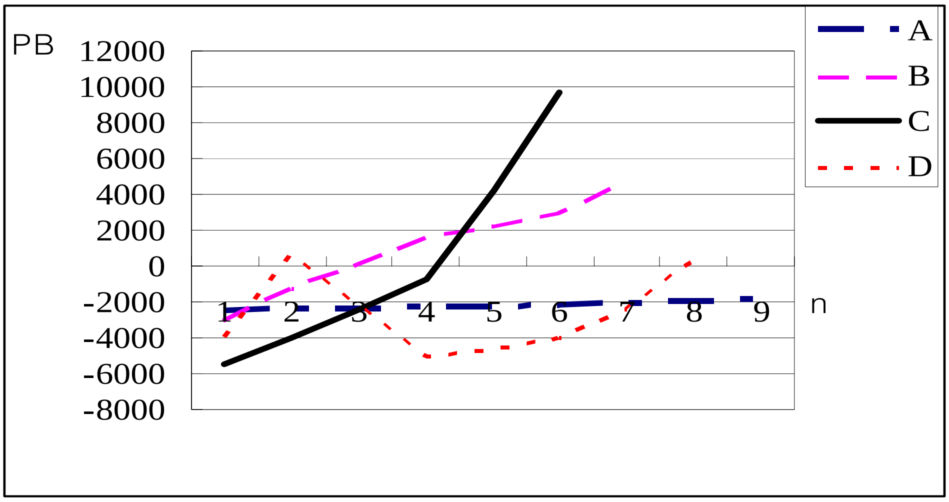
- $PB(i)_2 = -\$2{,}700(1+i) + \$1{,}470 = -\$1{,}500$

$$\therefore\ i = 10\%$$

- $PW(10\%) = \$270(P/F,10\%,5) = \$167.65$

5.16

(a)



(b)

Project	A	B	C	D
PB_2	-$2,395	$70	-$2,455	-$2,340

∴ Project B because it has only a positive balance at the end of year 2.

5.17

(a)

$$
\begin{aligned}
FW(12\%)_A &= -\$1{,}800(F/P,12\%,5) - \$500(F/P,12\%,4) \\
&\quad + \cdots - \$700 \\
&= \$700.18 \\
FW(12\%)_B &= -\$5{,}200(F/P,12\%,5) + \$2{,}500(F/P,12\%,4) \\
&\quad + \cdots + \$3{,}000 \\
&= \$5{,}141.91 \\
FW(12\%)_C &= -\$3{,}800(F/P,12\%,5) + \$4{,}000(F/P,12\%,4) \\
&\quad + \cdots + \$12{,}000 \\
&= \$18{,}160.7 \\
FW(12\%)_D &= -\$4{,}000(F/P,12\%,5) + \$500(F/P,12\%,4) \\
&\quad + \cdots + \$1{,}250 \\
&= \$6{,}040.45 \\
FW(12\%)_E &= -\$6{,}500(F/P,12\%,5) + \$1{,}000(F/P,12\%,4) \\
&\quad + \$3{,}600(P/F,12\%,3) + \$2{,}400(F/P,12\%,2) \\
&= -\$1{,}813.4
\end{aligned}
$$

(b) Accept all projects except E

5.18

(a) You may plot the future worth for each project as a function of interest rate, using Excel software.

(b)

n	A	B	C	D	E
0	-$1,800.00	-$5,200.00	-$3,800.00	-$4,000.00	-$6,500.00
1	-$2,516.00	-$3,324.00	-$4,256.00	-$3,980.00	-$6,280.00
2	-$1,917.92	-$7,722.88	-$4,766.72	-$2,457.60	-$3,433.60
3	-$848.07	-$3,649.63	-$1,338.73	$247.49	-$1,445.63
4	$1,250.16	$1,912.42	$5,500.63	$4,277.19	-$1,619.11
5	$700.18	$5,141.91	$18,160.70	$6,040.45	-$1,813.40

(c) Note that $PB(12\%)_N = FW(12\%)$

5.19 From the project balance table shown below, it will take about 7.57 years.

n	Cash flow	Balance
0	-$20,000.00	-$20,000.00
1	$5,000.00	-$18,000.00
2	$5,000.00	-$15,700.00
3	$5,000.00	-$13,055.00
4	$5,000.00	-$10,013.25
5	$5,000.00	-$6,515.24
6	$5,000.00	-$2,492.52
7	$5,000.00	$2,133.60
8	$5,000.00	$7,453.64
9	$5,000.00	$13,571.68
10	$5,000.00	$20,607.44

5.20

- Equivalent investment made or required at $n = 0$:

$$PW(20\%)_{\text{investment}} = \$10\text{M}(F/A, 20\%, 5) + \$30\text{M} = \$104.416\text{M}$$

- Anticipated future benefits with $104.414M investment:

$$PW(20\%)_{\text{revenue}} = \$100\text{M}(P/A, 20\%, 10) = \$419.25\text{M}$$

$\therefore$ Potential asking price = $104.416M + $419.25M = $523.67M

Comments: The absolute minimum asking price is $104.42, because that is how much you have invested in the project. However, if you go with this absolute minimum, you are giving up the future benefits that the R&D project would generate. Certainly, you have no way of knowing that Merck would pay this price, but at least we where you stand at the time of negotiation.

5.21

(a)

$$PW(10\%)_A = \$404.40$$
$$PW(10\%)_B = \$191.59$$
$$PW(10\%)_C = \$0.53$$

(b)

$$FW(10\%)_A = \$404.40(F/P,10\%,6) = \$716.42$$
$$FW(10\%)_B = \$191.59(F/P,10\%,5) = \$308.56$$
$$FW(10\%)_C = \$0.53(F/P,10\%,3) = \$0.7$$

(c)

$$\begin{aligned}FW(i)_A &= -\$400(F/P,10\%,3)(F/P,15\%,3)\\ &\quad +\$150(F/P,10\%,2)(F/P,15\%,3)\\ &\quad \vdots\\ &\quad +\$300\\ &= \$745.08\end{aligned}$$

$$\begin{aligned}FW(i)_B &= -\$300(F/P,10\%,3)(F/P,15\%,3)\\ &\quad +\$140(F/P,10\%,2)(F/P,15\%,3)\\ &\quad \vdots\\ &\quad +\$110(F/P,15\%,2)\\ &= \$369.46\end{aligned}$$

$$\begin{aligned}FW(10\%)_C &= \$100(F/P,10\%,3)(F/P,15\%,3)\\ &\quad -\$40(F/P,10\%,2)(F/P,15\%,3)\\ &\quad -\$40(F/P,10\%,1)(F/P,15\%,3)\\ &\quad -\$40(F/P,15\%,3)\\ &= \$1.06\end{aligned}$$

5.22

(a) Since the project's terminal project balance is equivalent to its future worth, we can easily find the equivalent present worth for each project by

$$\begin{aligned}PW(10\%)_A &= \$105(P/F,10\%,5)\\ &= \$65.20\\ PW(10\%)_C &= -\$1,000(P/F,20\%,5)\\ &= -\$401.80\end{aligned}$$

(b)

$$PB(10\%)_0 = A_0 = -\$1,000$$
$$PB(10\%)_1 = PB(10\%)_0(1+0.10) + A_1 = -\$1,000$$
$$PB(10\%)_2 = PB(10\%)_1(1+0.10) + A_2 = -\$900$$
$$PB(10\%)_3 = PB(10\%)_2(1+0.10) + A_3 = -\$690$$
$$PB(10\%)_4 = PB(10\%)_3(1+0.10) + A_4 = -\$359$$
$$PB(10\%)_5 = PB(10\%)_4(1+0.10) + A_5 = \$105$$

From the project balance equations above, we derive

$A_0 = \$1,000, A_1 = \$100, A_2 = \$200, A_3 = \$300, A_4 = \$400,$ and $A_5 = \$500$.

(c)

$$FW(20\%) = PB(20\%)_5 = -\$1,000$$

(d)

$$\begin{aligned} PW(i)_B &= FW(i)_B(P/F, i, N) \\ &= \$198/(1+i)^5 \\ &= \$79.57 \end{aligned}$$

$\therefore$ Solving for i yields $i = 20\%$

5.23

(a)

$$PW(0\%)_A = 0$$
$$PW(18\%)_B = \$575(P/F, 18\%, 5) = \$251.34$$
$$PW(12\%)_C = 0$$

(b) Assume that $A_2 = \$500$.

$$PW(12\%)_0 = -\$1,000$$
$$PW(12\%)_1 = -\$1,000(1.12) + A_1 = -\$530$$
$$PW(12\%)_2 = -\$530(1.12) + \$500 = X$$

Solving for *X* yields *X* = -\$93.60.

(c) The net cash flows for each project are as follows:

Net Cash Flow			
n	A	B	C

0	-\$1,000	-\$1,000	-\$1,000
1	\$200	\$500	\$590
2	\$200	\$500	\$500
3	\$200	\$300	-\$106
4	\$200	\$300	\$147
5	\$200	\$300	\$100

(d)

$$FW(0\%)_A = 0$$
$$FW(18\%)_B = \$575$$
$$FW(12\%)_C = 0$$

Capitalized Equivalent Worth

5.24

(a)

$$PW(13\%) = \$50{,}000(P/A,13\%,5) + \$70{,}000(P/A,13\%,5)(P/F,13\%,5) + (\$90{,}000/0.13)(P/F,13\%,10) = \$513{,}435.45$$

(b)

$$PW(13\%) = \frac{A}{i}$$
$$A = \$513{,}435.45(0.13) = \$66{,}746.61$$

5.25

- Find the equivalent annual series for the first cycle:

$$A = \$100 - [\$60(F/A,14\%,3) + \$20](A/F,14\%,5) = \$65.75$$

- Capitalized equivalent amount:

$$CE = \frac{\$65.75}{0.14} = \$469.64$$

5.26 Given: r = 6% compounded monthly, maintenance cost = \$30,000 per year

$$i_a = (1 + \frac{0.06}{12})^{12} - 1 = 6.17\%$$

$$\therefore \quad CE(6.17\%) = \frac{\$30{,}000}{0.0617} = \$486{,}223.66$$

5.27 Given: Construction cost = \$5,000,000, renovation cost = \$1,000,000 every 15 years, annual $O\&M$ costs = \$100,000 and $i = 5\%$ per year

(a)

$$P_1 = \$5,000,000$$

$$P_2 = \frac{\$1,000,000(A/F,5\%,15)}{0.05}$$

$$= \$926,846$$

$$P_3 = \$100,000/0.05$$

$$= \$2,000,000$$

$$CE(5\%) = P_1 + P_2 + P_3$$

$$= \$7,926,846$$

(b)

$$P_1 = \$5,000,000$$

$$P_2 = \frac{\$1,000,000(A/F,5\%,20)}{0.05}$$

$$= \$604,852$$

$$P_3 = \$100,000/0.05$$

$$= \$2,000,000$$

$$CE(5\%) = P_1 + P_2 + P_3$$

$$= \$7,604,852$$

(c)

- A 10-year cycle with 10% of interest:

$$P_1 = \$5,000,000$$

$$P_2 = \frac{\$1,000,000(A/F,10\%,15)}{0.10}$$

$$= \$314,738$$

$$P_3 = \$100,000/0.10$$

$$= \$1,000,000$$

$$CE(10\%) = \$6,314,738$$

- A 20-year cycle with 10% of interest:

$$P_1 = \$5,000,000$$

$$P_2 = \frac{\$1,000,000(A/F,10\%,20)}{0.10}$$

$$= \$174,596$$

$$P_3 = \$100,000/0.10$$

$$= \$1,000,000$$

$$CE(10\%) = \$6,174,596$$

As interest rate increases, CE value decreases.

5.28 Given: Cost to design and build $=\$650,000$, rework cost $=\$100,000$ every 10 years, a new type of gear $=\$50,000$ at the end of 5th year, annual operating costs $=\$30,000$ for the first 15 years and $\$35,000$ thereafter

$$\begin{aligned}CE(8\%) &= \$650,000+\frac{\$100,000(A/F,8\%,10)}{0.08}\\ &\quad +\$50,000(P/F,8\%,5)\\ &\quad +\$30,000(P/A,8\%,15)\\ &\quad +\frac{\$35,000}{0.08}(P/F,8\%,15)\\ &= \$1,165,019\end{aligned}$$

Mutually Exclusive Alternatives

5.29

$$\begin{aligned}PW(12\%)_A &= -\$800-\$1,500(P/F,12\%,1)\\ &\quad +\cdots+\$660(P/F,12\%,10)\\ &= \$988.91\\ PW(12\%)_B &= -\$2,635-\$565(P/F,12\%,1)\\ &\quad +\cdots+\$840(P/F,12\%,10)\\ &= \$1,696.01\end{aligned}$$

$\therefore$ Select project B.

5.30

(a)

$$\begin{aligned}PW(15\%)_A &= -\$1,500-\$1,350(P/F,15\%,1)+\cdots\\ &\quad +\$100(P/F,15\%,4)\\ &= \$467.52\\ PW(15\%)_B &= -\$1,500+\$1,000(P/F,15\%,1)+\cdots\\ &\quad +\$150(P/F,15\%,4)\\ &= \$586.26\end{aligned}$$

Select project B.

(b)

$$\begin{aligned}FW(15\%)_D &= \$1,500(F/P,15\%,4)-\$450(F/A,15\%,4)\\ &= \$376.49\\ FW(15\%)_E &= -\$1,800(F/P,15\%,4)+\$600(F/A,15\%,4)\\ &= -\$152.18\end{aligned}$$

Select project D.

Contemporary Engineering Economics, Fourth Edition, By Chan S. Park. ISBN 0-13-187628-7.

(c)

$$\begin{aligned} PW(15\%)_C &= -\$3,000 + \$1,000(P/F,15\%,1) \\ &\quad + X(P/F,15\%,2) \\ &\quad + \$1,500(P/F,15\%,3) + X(P/F,15\%,4) \\ &= 1.3279X - \$1,144.16 \end{aligned}$$

To be acceptable, it must satisfy the following condition:

$$\begin{aligned} PW(15\%)_C &> 0 \\ 1.3279X - \$1,144.16 &> 0 \\ X &> \$861.63 \end{aligned}$$

(d)

$$\begin{aligned} PW(18\%)_D &= \$1,500 - \$450(P/A,18\%,4) \\ &= \$289.47 > 0 \end{aligned}$$

Yes, project D is acceptable.

(e) If MARR < 10.40%, project E is better. Otherwise, project D is better.

5.31

(a)

$$\begin{aligned} PW(12\%)_A &= -\$14,500 + \$12,610(P/F,12\%,1) + \$12,930(P/F,12\%,2) \\ &\quad + \$12,300(P/F,15\%,3) = \$15,821.54 \\ PW(12\%)_B &= -\$12,900 + \$11,210(P/F,12\%,1) + \$11,720(P/F,12\%,2) \\ &\quad + \$11,500(P/F,12\%,3) = \$14,637.51 \end{aligned}$$

$\therefore$ Select project A.

(b)

$$\begin{aligned} FW(12\%)_A &= \$15,821.54(F/P,12\%,3) = \$22,228.13 \\ FW(12\%)_B &= \$14,637.51(F/P,12\%,3) = \$20,564.65 \end{aligned}$$

$\therefore$ Select project A.

5.32

(a)

$$\begin{aligned} PW(15\%)_A &= -\$6,000 + \$800(P/F,15\%,1) + \$14,000(P/F,15\%,2) \\ &= \$5,281.66 \\ PW(15\%)_B &= -\$8,000 + \$11,500(P/F,15\%,1) + \$400(P/F,15\%,2) \\ &= \$2,302.46 \end{aligned}$$

$\therefore$ Select project A.

(b) Project A dominates project B at any interest rate between 0% and 50%.

5.33

- Model A:

$$CE(12\%)_A = \$60,000 + \frac{\$25,000(A/F,12\%,5)}{0.12} = \$92,791.67$$

- Model B:

$$CE(12\%)_B = \$150,000 + \frac{\$180,000(A/F,12\%,50)}{0.12} = \$150,625.5$$

$\therefore$ Since CE(12%) values above represent cost, project A is preferred.

5.34

- Standard Lease Option:

$$\begin{aligned} PW(0.5\%)_{\text{SL}} &= -\$5,500 - \$1,150(P/A, 0.5\%, 24) \\ &\quad +\$1,000(P/F, 0.5\%, 24) \\ &= -\$30,560.10 \end{aligned}$$

- Single Up-Front Option:

$$\begin{aligned} PW(0.5\%)_{\text{SU}} &= -\$31,500 + \$1,000(P/F, 0.5\%, 24) \\ &= -\$30,612.82 \end{aligned}$$

$\therefore$ Select the standard lease option as you will save \$52.72 in present worth.

5.35

- Machine A:

$$\begin{aligned} PW(13\%) &= -\$75,200 - (\$6,800 + \$2,400)(P/A, 13\%, 6) \\ &\quad +\$21,000(P/F, 13\%, 6) \\ &= -\$101,891 \end{aligned}$$

- Machine B:

$$\begin{aligned} PW(13\%) &= -\$44,000 - \$11,500(P/A, 13\%, 6) \\ &= -\$89,971 \end{aligned}$$

$\therefore$ Machine B is a better choice.

5.36

(a)

- Required HP to produce 10 HP:

  -Motor A: $X_1 = 10/0.85 = 11.765$ HP
  -Motor B: $X_2 = 10/0.90 = 11.111$ HP

- Annual energy cost:

  -Motor A: $11.765(0.7457)(1,500)(0.07) = \$921.18$
  -Motor B: $11.111(0.7457)(1,500)(0.07) = \$869.97$

- Equivalent cost:

$$\begin{aligned} PW(8\%)_A &= -\$800 - \$921.18(P/A, 8\%, 15) \\ &\quad +\$50(P/F, 8\%, 15) \\ &= -\$8,669 \\ PW(8\%)_B &= -\$1,200 - \$869.97(P/A, 8\%, 15) \\ &\quad +\$100(P/F, 8\%, 15) \\ &= -\$8,614 \end{aligned}$$

$\therefore$ Motor B is preferred.

(b) With 2,000 operating hours:

$$\begin{aligned} PW(8\%)_A &= -\$800 - \$1,535.26(P/A, 8\%, 15) \\ &\quad +\$50(P/F, 8\%, 15) \\ &= -\$13,925 \\ PW(8\%)_B &= -\$1,200 - \$1,449.97(P/A, 8\%, 15) \\ &\quad +\$100(P/F, 8\%, 15) \\ &= -\$13,579 \end{aligned}$$

$\therefore$ Motor B is still preferred.

5.37 Given: Required service period = infinite, analysis period = least common multiple service periods (6 years)

- Model A:

$$\begin{aligned} PW(12\%)_{\text{cycle}} &= -\$20,000 + \$17,500(P/F, 12\%, 1) + \$17,000(P/F, 12\%, 2) \\ &\quad +\$15,000(P/F, 12\%, 3) \\ &= \$19,854.00 \end{aligned}$$

$$\begin{aligned} PW(12\%)_{\text{total}} &= \$19,854[1+(P/F,12\%,3)] \\ &= \$33,985.69 \end{aligned}$$

- Model B:

$$\begin{aligned} PW(12\%)_{\text{cycle}} &= -\$25,000 + \$25,500(P/F,12\%,1) + \$18,000(P/F,12\%,2) \\ &= \$12,117.35 \end{aligned}$$

$$\begin{aligned} PW(12\%)_{\text{total}} &= \$12,117.35[1+(P/F,12\%,2)+(P/F,12\%,4)] \\ &= \$29,478.02 \end{aligned}$$

$\therefore$ Model A is preferred.

5.38

(a) Without knowing the future replacement opportunities, we may assume that both alternatives will be available in the future with the identical investments and expenses. We further assume that the required service period will be indefinite.

(b) With the common service period of 24 years,

- Project A1:

$$\begin{aligned} PW(10\%)_{\text{cycle}} &= -\$900 - \$400(P/A,10\%,3) \\ &\quad +\$200(P/F,10\%,3) \\ &= -\$1,744.48 \\ PW(10\%)_{\text{total}} &= -\$1,744.48[1+(P/A,33.10\%,7)] \\ &= -\$6,302.63 \end{aligned}$$

Note that the effective interest rate for a 3-year cycle is

$$(1.10)^3 - 1 = 33.10\%$$

- Project A2:

$$\begin{aligned} PW(10\%)_{\text{cycle}} &= -\$1,800 - \$300(P/A,10\%,8) \\ &\quad +\$500(P/F,10\%,8) \\ &= -\$3,167.22 \\ PW(10\%)_{\text{total}} &= -\$3,167.22[1+(P/F,10\%,8) \\ &\quad +(P/F,10\%,16)] \\ &= -\$5,334.03 \end{aligned}$$

$\therefore$ Project A2 is preferred.

(c)

$$PW(10\%)_{A1} = -\$1{,}744.48$$

$$PW(10\%)_{A2} = -\$1{,}800 - \$300(P/A,10\%,3) + S(P/F,10\%,3)$$
$$= -\$2{,}546.06 + 0.7513S$$

Let $PW(10\%)_{A1} = PW(10\%)_{A2}$ and solve for *S*.

$$S = \$1{,}067$$

5.39

(a) Assuming a common service period of 15 years

- Project B1:

$$PW(12\%)_{\text{cycle}} = -\$18{,}000 - \$2{,}000(P/A,12\%,5) + \$2{,}000(P/F,12\%,5)$$
$$= -\$24{,}075$$
$$PW(12\%)_{\text{total}} = -\$24{,}075[1+(P/A,76.23\%,2)]$$
$$= -\$45{,}487$$

Note: $(1.12)^5 - 1 = 76.23\%$

- Project B2:

$$PW(12\%)_{cycle} = -\$15{,}000 - \$2{,}100(P/A,12\%,3) + \$1{,}000(P/F,12\%,3)$$
$$= -\$19{,}332$$
$$PW(12\%)_{total} = -\$19{,}332[1+(P/A,40.49\%,4)]$$
$$= -\$54{,}826$$

Note: $(1.12)^3 - 1 = 40.49\%$

$\therefore$ Select project B1.

(b)

- Project B1 with two replacement cycles:

$$PW(12\%) = -\$24{,}075 - \$24{,}075(P/F,12\%,5)$$
$$= -\$37{,}736$$

- Project B2 with four replacement cycles where the 4th replacement ends at the end of the first operating year:

$$PW(12\%) = -\$19{,}332[1+(P/F,12\%,3)+(P/F,12\%,6)]$$
$$-[\$15{,}000 - (\$2{,}100 - \$6{,}000)(P/F,12\%,1)](P/F,12\%,9)$$
$$= -\$47{,}040$$

$\therefore$ Project B1 is still a better choice

5.40 Since only Model B is repeated in the future, we may have the following sequence of replacement cycles:

- Option 1: Purchase Model A now and repeat Model A forever.
- Option 2: Purchase Model B now and replace it at the end of year 2 by Model A. Then repeat Model A forever.

$$\begin{aligned}
PW(15\%)_{\text{Model A}} &= -\$6,000 + \$3,500(P/A,15\%,3) \\
&= \$1,991.29 \\
A &= \$1,991.29(A/P,15\%,3) \\
&= \$872.14 \\
PW(15\%)_{\text{Model B}} &= -\$15,000 + \$10,000(P/A,15\%,2) \\
&= \$1,257.09 \\
A &= \$1,257.09(A/P,15\%,2) \\
&= \$773.26
\end{aligned}$$

(a)

- Option 1:

$$\begin{aligned}
PW(15\%)_{AAA\cdots} &= \frac{\$872.14}{0.15} \\
&= \$5,814.27
\end{aligned}$$

- Option 2:

$$\begin{aligned}
PW(15\%)_{BAA\cdots} &= \$1,257.09 + \frac{\$872.14}{0.15}(P/F,15\%,2) \\
&= \$5,653.51
\end{aligned}$$

$\therefore$ Option 1 is a better choice.

(b) Let S be the salvage value of Model A at the end of year 2.

$$-\$6,000 + \$3,500(P/A,15\%,2) + S(P/F,15\%,2) = \$1,257.09$$

Solving for S yields

$$S = \$2,072.50$$

5.41

- Since either tower will have zero salvage value after 20 years, we may select the analysis period of 35 years:

$$\begin{aligned}
PW(11\%)_{\text{Bid A}} &= -\$137,000 - \$2,000(P/A,11\%,35) \\
&= -\$154,710
\end{aligned}$$

$$PW(11\%)_{\text{Bid B}} = -\$128,000 - \$3,300(P/A,11\%,35)$$
$$= -\$157,222$$

$\therefore$ Bid A is a better choice.

- If you assume an infinite analysis period, the present worth of each bid will be:

$$PW(11\%)_{\text{Bid A}} = \frac{[-\$137,000 - \$2,000(P/A,11\%,40)](A/P,11\%,40)}{0.11}$$
$$= -\$157,296$$
$$PW(11\%)_{\text{Bid B}} = \frac{[-\$128,000 - \$3,300(P/A,11\%,35)](A/P,11\%,35)}{0.11}$$
$$= -\$161,367$$

$\therefore$ Bid A is still preferred.

5.42

(a)

$$PW(15\%)_{A1} = -\$15,000 + \$9,500(P/F,15\%,1)$$
$$+\$12,500(P/F,15\%,2) + \$7,500(P/F,15\%,3)$$
$$= \$7,644.04$$

(b)

$$PW(15\%)_{A2} = -\$25,000 + X(P/F,15\%,2)(P/F,15\%,1)$$
$$= \$9,300$$
$$X = \$24,263$$

(c) Note that the net future worth of the project is equivalent to its terminal project balance.

$$PB(15\%)_3 = \$7,644.04(F/P,15\%,3)$$
$$= \$11,625.63$$

(d) Select A2.

5.43

(a) Project balances as a function of time are as follows:

Project Balances		
n	A	D
0	-\$2,500	-\$5,000
1	-2,100	-6,000
2	-1,660	-7,100

3	-1,176	-3,810
4	-694	-1,191
5	-163	1,690
6	421	3,859
7	763	7,245
8	1,139	

All figures above are rounded to nearest dollars.

(b) Knowing the relationship $FW(i) = PB(i)_N$,

$$FW(10\%)_A = \$1{,}139$$
$$FW(10\%)_D = \$7{,}245$$

(c) Assuming a required service period of 8 years

$$\begin{aligned} PW(10\%)_B &= -\$7{,}000 - \$1{,}500(P/A,10\%,8) \\ &\quad -\$1{,}000(P/F,10\%,1) - \$500(P/F,10\%,2) \\ &\quad -\$1{,}500(P/F,10\%,7) - \$1{,}500(P/F,10\%,8) \\ &= -\$17{,}794 \\ PW(10\%)_C &= -\$5{,}000 - \$2{,}000(P/A,10\%,7) \\ &\quad -\$3{,}000(P/F,10\%,8) \\ &= -\$16{,}136 \end{aligned}$$

$\therefore$ Select Project C.

5.44

- Option 1: Non-deferred plan

$$\begin{aligned} PW(12\%)_1 &= -\$200{,}000 - \$21{,}000(P/A,12\%,8) \\ &= -\$304{,}320 \end{aligned}$$

- Option 2: Deferred plan

$$\begin{aligned} PW(12\%)_2 &= -\$100{,}000(P/F,12\%,2) - \$6{,}000(P/A,12\%,3)(P/F,12\%,2) \\ &\quad -\$160{,}000(P/F,12\%,5) - \$15{,}000(P/A,12\%,3)(P/F,12\%,5) \\ &\quad -\$140{,}000(P/F,12\%,8) \\ &= -\$258{,}982 \end{aligned}$$

$\therefore$ Option 2 is a better choice.

5.45

- Alternative A: Once-for-all expansion

$$\begin{aligned} PW(15\%)_A &= -\$30M - \$0.40M(P/A,15\%,25) \\ &\quad +\$0.85M(P/F,15\%,25) \\ &= -\$32,559,839 \end{aligned}$$

- Alternative B: Incremental expansion

$$\begin{aligned} PW(15\%)_B &= -\$10M - \$18M(P/F,15\%,10) \\ &\quad -\$12M(P/F,15\%,15) + \$1.5M(P/F,15\%,25) \\ &\quad -\$0.25M(P/A,15\%,25) \\ &\quad -\$0.10M(P/A,15\%,15)(P/F,15\%,10) \\ &\quad -\$0.10M(P/A,15\%,10)(P/F,15\%,15) \\ &= -\$17,700,745 \end{aligned}$$

$\therefore$ Select alternative B.

5.46

- Option 1: Tank/tower installation

$$PW(12\%)_1 = -\$164,000$$

- Option 2: Tank/hill installation with the pumping equipment replaced at the end of 20 years at the same cost

$$\begin{aligned} PW(12\%)_2 &= -(\$120,000 + \$12,000) \\ &\quad -(\$12,000 - \$1,000)(P/F,12\%,20) \\ &\quad +\$1,000(P/F,12\%,40) - \$1,000(P/A,12\%,40) \\ &= -\$141,373 \end{aligned}$$

$\therefore$ Option 2 is a better choice.

Short Case Studies

ST 5.1

- Option 1: Process device A lasts only four years. You have a required service period of 6 years. If you take this option, you must consider how you will satisfy the rest of the required service period at the end of the project life. One option would subcontract the remaining work for the duration of the remaining required service period. If you select this subcontracting option along with the device A, the equivalent net present worth would be

$$\begin{aligned} PW(12\%)_1 &= -\$100,000 - \$60,000(P/A,12\%,4) \\ &\quad +\$10,000(P/F,12\%,4) \\ &\quad -\$100,000(P/A,12\%,2)(P/F,12\%,4) \\ &= -\$383,292 \end{aligned}$$

- Option 2: This option creates no problem because its service life coincides with the required service period.

$$\begin{aligned} PW(12\%)_2 &= -\$150,000 - \$50,000(P/A,12\%,6) \\ &\quad +\$30,000(P/F,12\%,6) \\ &= -\$340,371 \end{aligned}$$

- Option 3: With the assumption that the subcontracting option would be available over the next 6 years at the same cost, the equivalent present worth would be

$$\begin{aligned} PW(12\%)_3 &= -\$100,000(P/A,12\%,6) \\ &= -\$411,141 \end{aligned}$$

With the restricted assumptions above, option 2 appears to be the best alternative.

Notes to Instructors: This problem is deceptively simple. However, it can make the problem interesting with the following embellishments.

- If the required service period is changed from 6 years to 5 years, what would be the best course of action?
- If there are price differentials in the subcontracting option (say, \$55,000 a year for a 6-year contract, \$60,000 for a 5-year contract, \$70,000 a year for a 4-year contract and \$75,000 a year for any contract lasting less than 4 years), what would be the best option?
- If both processes A and B would be available in the subsequent years, but the required investment and salvage value would be increasing at the annual rate of 10%, what would be the best course of action?
- If both device A and B will be available in the subsequent years, but the required investment and salvage value(as well as the O & M costs) would be decreasing at the annual rate of 10%, what would be the best course of action?

ST 5.2 **Note to Instructors**: This case problem requires several pieces of information. (1) No minimum attractive rate return figure is given for Tampa Electrics. (2) What would be a typical number of accidents in line construction work? (3) How does a typical electric utility handle the nesting problems? If there is some cleaning cost, how much and how often?

- First, we may calculate the equivalent present value (cost) for each option without considering the accident costs and nesting problems.

Design Options				
Factors	Option 1 Cross Arm	Option 2 Triangular	Option 3 Horizontal Line	Option 4 Stand Off
Investment:				
Construction cost	$495,243	$396,813	$402,016	$398,000
Accident cost				
Annual cost:				
Flashover repair	$6,000	$3,000	$3,000	$3,000
Cleaning nest				
Annual savings:				
Inventory	0	$4,521	$4,521	$4,521

Assuming that Tampa Electrics' required rate of return would be 12%. The equivalent present value cost for each option is as follows:

$$
\begin{aligned}
PW(12\%)_1 &= -\$495,243 - \$6,000(P/A, 12\%, 20) \\
&= -\$540,060 \\
PW(12\%)_2 &= -\$396,813 - \$3,000(P/A, 12\%, 20) \\
&\quad +\$4,521(P/A, 12\%, 20) \\
&= -\$385,452 \\
PW(12\%)_3 &= -\$402,016 - \$3,000(P/A, 12\%, 20) \\
&\quad +\$4,521(P/A, 12\%, 20) \\
&= -\$398,654 \\
PW(12\%)_4 &= -\$398,000 - \$3,000(P/A, 12\%, 20) \\
&\quad +\$4,521(P/A, 12\%, 20) \\
&= -\$386,639
\end{aligned}
$$

It appears that the triangular design configuration be the most economical.

- If we consider the potential accident costs ($65,000 per accident) during line construction work, it is likely to change the outcome. If we expect only a couple of accidents, option 2 still appears to be the best. However, if you expect more than three accidents, the conventional cross-arm design appears to be more economical. If the nest cleaning cost were factored into the analysis, the accident cost would be reduced to the extent of the annual cleaning cost, indicating the preference of the triangular design.