Chapter 6 Annual Equivalence Method

Identifying Cash Inflows and Outflows

6.1

$$\begin{aligned} AE(10\%) &= -\$5,000(A/P,10\%,6) \\ &\quad +[\$2,000(P/F,10\%,1)+\cdots \\ &\quad +\$2,500(P/F,10\%,6)](A/P,10\%,6) \\ &= \$1,103.50 \end{aligned}$$

6.2

$$\begin{aligned} AE(12\%) &= \$20,000(A/P,12\%,6)-\$5,000 \\ &\quad -\$3,000(P/G,12\%,5)(P/F,12\%,1)(A/P,12\%,6) \\ &= \$4,303.13 \end{aligned}$$

6.3

$$\begin{aligned} AE(10\%) &= [-\$3,000-\$3,000(P/A,10\%,2) \\ &\quad +\$3,000(P/A,10\%,4)(P/F,10\%,2) \\ &\quad +\$1,000(P/G,10\%,4)(P/F,10\%,2)](A/P,10\%,6) \\ &= \$751.01 \end{aligned}$$

6.4

$$\begin{aligned} AE(13\%) &= [-\$8,000+\$2,000(P/A,13\%,6) \\ &\quad +\$1,000(P/G,13\%,6)-\$4,000(P/F,13\%,2) \\ &\quad -\$2,000(P/F,13\%,4)-\$1,000(P/F,13\%,6)](A/P,13\%,6) \\ &= \$934.92 \end{aligned}$$

6.5

$$\begin{aligned} AE(8\%) &= -\$5,000(A/P,8\%,6)+\$2,000 \\ &\quad -[(\$500+\$1,000(P/A,8\%,2))(P/F,8\%,2) \\ &\quad +\$500(P/F,8\%,5)](A/P,8\%,6) \\ &= \$421.37 \end{aligned}$$

6.6

$$\begin{aligned} AE(10\%)_A &= -\$2,500(A/P,10\%,5)+\$400 \\ &\quad +\$100(A/G,10\%,5) \\ &= -\$78.47 \text{ (Reject)} \\ AE(10\%)_B &= -\$4,500(A/P,10\%,5)+\$500 \\ &\quad +[\$2,500(P/F,10\%,1)+\$1,500(P/F,10\%,2) \\ &\quad +\$500(P/F,10\%,3)](A/P,10\%,5) \\ &= \$338.57 \text{ (Accept)} \\ AE(10\%)_C &= [-\$8,000-\$2,000(P/F,10\%,1) \\ &\quad \cdots+\$2,000(P/F,10\%,5)](A/P,10\%,5) \\ &= \$162.77 \text{ (Accept)} \\ AE(10\%)_D &= [-\$12,000+\$2,000(P/F,10\%,1) \\ &\quad +\cdots+\$4,000(P/F,10\%,5)](A/P,10\%,5) \\ &= \$1,868.31 \text{ (Accept)} \end{aligned}$$

6.7

$$\begin{aligned} AE(12\%) &= [-\$500,000+\$600,000(P/F,12\%,1)+\$400,000(P/F,12\%,2) \\ &\quad +\$300,000(P/F,12\%,3)+\$200,000(P/F,12\%,4)](A/P,12\%,4) \\ &= \$228,894 \end{aligned}$$

6.8

$$\begin{aligned} AE(13\%)_A &= -\$7,500(A/P,13\%,3)+\$15,500(A/F,13\%,3) \\ &= \$1,373.10 \text{ (Accept)} \\ AE(13\%)_B &= -\$4,000(A/P,13\%,3)+\$1,500 \\ &\quad +\$300(A/G,13\%,3) \\ &= \$81.53 \text{ (Accept)} \\ AE(13\%)_C &= -\$5,000(A/P,13\%,3)+\$4,000 \\ &\quad -\$1,000(A/G,13\%,3) \\ &= \$963.62 \text{ (Accept)} \\ AE(13\%)_D &= -\$6,600(A/P,13\%,3)+\$3,800 \\ &= \$1,004.70 \text{ (Accept)} \end{aligned}$$

6.9 Since the project has the same cash flow cycle during the project life, you just can consider the first cycle.

$$
\begin{aligned}
AE(10\%) &= [-\$800 - \$900(P/F,10\%,1) + \$700(P/F,10\%,2) \\
&\quad +\$500(P/F,10\%,3)](A/P,10\%,3) \\
&= \$390.98
\end{aligned}
$$

$\therefore$ Accept the project.

6.10

$$
\begin{aligned}
CE(i) &= \$10,000(P/A,8\%,10) + \frac{\$10,000}{0.06}(P/F,8\%,10) \\
&= \$77,199 + \$67,101 \\
&= \$144,300
\end{aligned}
$$

$\therefore$ The amount of additional funds should be \$44,300.

Capital (Recovery) Cost / Annual Equivalent Cost

6.11 Given: $I = \$55,000, S = \$6,000, N = 10$ years, $i = 12\%$

(a)

$$
\begin{aligned}
CR(12\%) &= (\$55,000 - \$6,000)(A/P,12\%,10) \\
&\quad +\$6,000(0.12) \\
&= \$9,392
\end{aligned}
$$

(b)

$$
\begin{aligned}
AE(12\%)_{\text{Operating Revenue}} &= \$5,000 + \$2,500(A/G,12\%,10) \\
&= \$13,962
\end{aligned}
$$

(c)

$$
\begin{aligned}
AE(12\%) &= \$13,962 - \$9,392 \\
&= \$4,570
\end{aligned}
$$

$\therefore$ This is a good investment.

6.12

$$
\begin{aligned}
CR(12\%) &= (\$45,000 - \$9,000)(A/P,12\%,4) + (.12)(\$9,000) \\
&= \$11,067 \\
AE(12\%)_{\text{O\&M}} &= \$15,000 + \$2,000(A/G,12\%,5) \\
&= \$18,549 \\
AEC(12\%) &= \$11,067 + \$18,549 \\
&= \$29,616
\end{aligned}
$$

6.13 Given: $I = \$250,000, S = \$40,000, N = 5$ years, $i = 18\%$

$$\begin{aligned} CR(18\%) &= (\$250,000 - \$40,000)(A/P, 18\%, 5) \\ &\quad + \$40,000(0.18) \\ &= \$74,353 \end{aligned}$$

6.14

n	Option 1	Option 2
0	-\$600-\$5,760	
1	0	-\$1,560
2	0	-\$1,680
3	0	-\$1,800
4	+\$100	-\$1,920

$$\begin{aligned} AEC(8\%)_{\text{Option 1}} &= \$6,360(A/P, 8\%, 4) - \$100(A/F, 8\%, 4) \\ &= \$1,897.89 \\ AEC(8\%)_{\text{Option 2}} &= \$1,560 + \$120(A/G, 8\%, 4) \\ &= \$1,728.36 \end{aligned}$$

$\therefore$ Select Option 2.

6.15 Given $i = 6\%$ compounded annually, N =12 years, the effective monthly rate is

$$\begin{aligned} 0.06 &= (1+i)^{12} - 1 \\ i &= 0.487\% \text{ per month} \end{aligned}$$

- Conventional System:

$$AEC(6\%)_{\text{Conventional}} = \$471 + \$576 = \$1,047$$

The equivalent monthly cost = $\$1,047(A/F, 0.487\%, 12) = \$84.94$

- IONETIC System:

$$AEC(6\%)_{\text{IONETICS}} = \$185 + \$1,200(A/P, 6\%, 12) = \$323.13$$

The equivalent monthly cost = $\$328.13(A/F, 0.487\%, 12) = \$26.62$

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6.16
(a)

$$\begin{aligned} AE(13\%) &= -\$4,000(A/P,13\%,4)+\$1,000 \\ &\quad +(X-\$1,000)(P/F,13\%,2)(A/P,13\%,4) \\ &= 0 \\ AE(13\%) &= -\$608.06+0.26328X = 0 \\ X &= \$2,309.55 \end{aligned}$$

(b)

$$\begin{aligned} AE(13\%) &= \$5,500(A/P,15\%,4)-\$1,400 \\ &= \$526.46 > 0 \end{aligned}$$

$\therefore$ Accept project B.

6.17

- Option 1: Purchase-annual installment option:

$$\begin{aligned} A &= \$45,000(A/P,7\%,5) = \$10,975 \\ AEC(10\%)_1 &= \$5,000(A/P,10\%,5)+\$10,975 \\ &= \$12,294 \end{aligned}$$

- Option 2: Cash payment option:

$$\begin{aligned} AEC(10\%)_2 &= \$46,000(A/P,10\%,5) \\ &= \$12,135 \end{aligned}$$

$\therefore$ Option 2 is a better choice**.**

6.18 The total investment consists of the sum of the initial equipment cost and the installation cost, which is $\$135,000$. Let R denote the break-even annual revenue.

$$\begin{aligned} AE(10\%) &= -\$135,000(A/P,10\%,10)-\$30,000 \\ &\quad -\$5,000+\$10,000+R \\ &= 0 \end{aligned}$$

Solving for R yields

$$R = \$46,971$$

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6.19

- Capital cost:

$$\begin{aligned} CR(15\%) &= (\$105{,}000 - \$6{,}000)(A/P, 15\%, 5) + \$6{,}000(0.15) \\ &= \$30{,}432 \end{aligned}$$

- Annual operating costs: $\$45{,}000$

$$AEC(15\%) = \$30{,}432 + \$45{,}000 = \$75{,}432$$

Unit-Cost Profit Calculation

6.20

- Capital recovery cost

$$\begin{aligned} CR(10\%) &= (\$20{,}000 - \$10{,}000)(A/P, 10\%, 2) + \$10{,}000(0.10) \\ &= \$6{,}792 \end{aligned}$$

-

$$\begin{aligned} AE(10\%)_{\text{Revenue}} &= [\frac{\$30{,}000}{1.1} + \frac{\$40{,}000}{1.1^2}](A/P, 10\%, 2) \\ &= \$34{,}762 \\ AE(10\%)_{\text{Net Savings}} &= \$34{,}762 - \$6{,}792 \\ &= \$27{,}970 \end{aligned}$$

-

$$\begin{aligned} AE(10\%)_{\text{Savings}} &= [\frac{C(6{,}000)}{1.1} + \frac{C(8{,}000)}{1.1^2}](A/P, 10\%, 2) \\ &= \$6{,}952C \\ \$27{,}970 &= 6{,}952C \\ C &= \$4.02 \text{ per hour} \end{aligned}$$

6.21 Given data: Total cost of building: $\$110 \times 16 \times 20 = \$35{,}200$, Salvage value = \$3,520, Annual taxes, insurance, maintenance = \$2,112, Other operating cost = \$1,600, Number of engineer assigned = 3

- Equivalent annual cost of operating the new building:

$$\begin{aligned}AEC(12\%) &= (\$35,200-\$3,520)(A/P,12\%,25)+(0.12)(\$3,520)\\ &\quad +\$2,112+\$1,600\\ &= \$8,173.6\end{aligned}$$

- Required annual increase in productivity per engineer:

$$\$8,173.6/3 = \$2,724.5 \text{ per engineer}$$

6.22

- Equivalent annual cost of owning and operating the vehicle:

$$\begin{aligned}AEC(6\%) &= [\$4,680(P/F,6\%,1)+\$3,624(P/F,6\%,2)\\ &\quad +\$3,421(P/F,6\%,3)](A/P,6\%,3)\\ &= \$3,933\end{aligned}$$

- Annualized cost of owning and operating the vehicle as a function of mileage.

$$\begin{aligned}AEC(6\%) &= [\frac{C(14,500)}{1.06}+\frac{C(13,000)}{1.06^2}+\frac{C(11,500)}{1.06^3}](A/P,6\%,3)\\ &= 13,058C\end{aligned}$$

- So,

$$\begin{aligned}13,058C &= \$3,933\\ C &= \$0.3012 \text{ per mile}\end{aligned}$$

6.23 Let T denote the total operating hours in full load.

- Motor I (Expensive)

  Annual power cost:

$$\frac{150}{0.83}\times(0.746)\times(0.05)\times T = \$6.741T$$

  Equivalent annual cost of operating the motor:

$$\begin{aligned}AEC(6\%)_I &= \$4,500(A/P,6\%,10)+\$675+6.741T\\ &= \$1,286.41+\$6.741T\end{aligned}$$

- Motor II (Less expensive):

  Annual power cost:

$$\frac{150}{0.80} \times (0.746) \times (0.05) \times T = \$6.9938T$$

  Equivalent annual cost of operating the motor:

$$\begin{aligned} AEC(6\%)_{II} &= \$3,600(A/P,6\%,10) + \$540 + 6.9938T \\ &= \$1,026.11 + 6.9938T \end{aligned}$$

- Let $AEC(6\%)_I = AEC(6\%)_{II}$ and solve for T.

$$\$1,286.41 + 6.741T = \$1,029.11 + 6.9938T$$

$$T = 1,017.8 \text{ hours per year}$$

6.24

- Option 1: Purchase units from John Holland

$$\text{Unit cost} = \$25 - (\$35,000)/20,000 = \$23.25$$

- Option 2: Make units in house

$$\begin{aligned} PW(15\%)_{\text{dm}} &= \$63,000(P/A_1,5\%,15\%,5) = \$230,241 \\ PW(15\%)_{\text{dl}} &= \$190,800(P/A_1,6\%,15\%,5) = \$709,491 \\ PW(15\%)_{\text{vo}} &= \$139,050(P/A_1,3\%,15\%,5) = \$490,888 \\ AEC(15\%) &= (\$230,241 + \$709,491 + \$490,888)(A/P,15\%,5) + \$70,000 \\ &= \$496,776 \end{aligned}$$

$$\text{Unit cost} = \$496,776/20,000 = \$24.84$$

$\therefore$ Option 1 is a better choice.

6.25

(a) Determine the unit profit of air sample test by the TEM (in-house).

- Sub-contract Option:

$$\text{Unit profit} = \$400 - \$300 - \$0.50 - \$1,500/1,000 = \$98$$

- TEM Purchase Option:

$$\begin{aligned}AEC(15\%) &= (\$415,000+\$9,500)(A/P,15\%,8)+(\$50,000\\ &\quad +\$6,000+\$18,000+\$20,000)\\ &= \$188,600\end{aligned}$$

$$\begin{aligned}\text{Unit cost} &= \$188,600/1,000 = \$188.60\\ \text{Unit profit} &= \$300-\$188.60 = \$111.40\end{aligned}$$

(b)Let X denote the break-even number of air samples per year.

$$\$400-(\$300+\$0.50+\frac{\$1,500}{X}) = \$300-\frac{\$188,600}{X}$$

Solving for X yields

$$X = 933.17 \approx 934 \text{ air samples per year}$$

6.26

- Option 1: Pay employee \$0.38 per mile (Annual cost: \$8360)
- Option 2: Provide a car to employee:

$$\begin{aligned}CR(10\%) &= (\$25,000-\$8,000)(A/P,10\%,3)+(0.10)(\$8,000)\\ &= \$7,636\\ AEC(10\%)_{\text{operating cost}} &= \$900+(\$0.22)(22,000) = \$5,740\\ AEC(10\%)_{\text{total cost}} &= \$7,636+\$5,740 = \$13,376\\ \text{Operating cost per mile} &= \$13,376/22,000 = \$0.61\end{aligned}$$

$\therefore$ Option 1 is a better choice.

6.27

- Capital costs:

$$\begin{aligned}CR(7\%) &= (\$25,000-\$2,000)(A/P,7\%,12)+(0.07)(\$2,000)\\ &= \$3,036\end{aligned}$$

- Annual battery replacement cost:

$$\begin{aligned}AEC(7\%) &= [\$3,000(P/F,7\%,3)+(P/F,7\%,6)\\ &\quad +(P/F,7\%,9)](A/P,7\%,12)\\ &= \$765.41\end{aligned}$$

- Annual recharging cost:

$$AEC(7\%) = (\$0.015)(20,000) = \$300$$

- Total annual costs:

$$\begin{aligned} AEC(7\%) &= \$3,036 + \$765.41 + \$300 + \$700 \\ &= \$4,801.41 \end{aligned}$$

- Costs per mile:

$$\text{cost/mile} = \$4,801.41 / 20,000 = \$0.2401$$

6.28

- Minimum operating hours:

$$\begin{aligned} AEC(9\%) &= (\$30,000 - \$2,000)(A/P, 9\%, 15) \\ &\quad + (0.09)(\$2,000) + \$500 \\ &= \$4,153.65 \end{aligned}$$

Let T denote the annual operating hours. Then the total kilowatt-hours generated would be $40T$. Since the value of the energy generated is considered to be $\$0.08$ per kilowatt-hour, the annual energy cost is

$$\$0.08 \times 40T = \$4,153.65$$

Solving for T yields

$$T = 1,298 \text{ hours}$$

- Annual worth of the generator at full load operation:

$$AE(9\%) = (\$0.08)(100,000) - \$4,153.65 = \$3,846.35$$

- Discounted payback period at full load of operation:

n	Investment	Revenue	Maintenance cost	Net Cash flow
0	-\$30,000			-\$30,000
1		\$8,000	-\$500	7,500
⋮	⋮	⋮	⋮	⋮
15	+\$2,000	8,000	-500	9,500

$$\$30,000 = \$7,500(P/A, 9\%, n)$$

Solving for n yields

$$n = 5.185 \text{ years}$$

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6.29

- Capital recovery cost:

$$
\begin{aligned}
CR(6\%) &= (\$150,000 - \$3,000)(A/P, 6\%, 12) + (0.06)(\$3,000) \\
&= \$17,714
\end{aligned}
$$

- Annual operating costs:

$$
\begin{aligned}
AEC(6\%)_{\text{O\&M}} &= \$50,000 + \$10,000 + \$3,000 \\
&= \$63,000
\end{aligned}
$$

- Total annual system costs:

$$AEC(6\%) = \$17,714 + \$63,000 = \$80,714$$

- Number of rides required per year:

$$\text{Number of rides} = \$80,714/(\$0.10) = 807,140 \text{ rides}$$

6.30 Given: Investment cost $=\$7$ million, plant capacity $= 200,000$ 1bs/hour, plant operating hours $= 3,600$ hours per year, O&M cost $= \$4$ million per year, useful life $= 15$ years, salvage value $=$ \$700,000, and MARR = 15%.

(a)

$$
\begin{aligned}
PW(15\%) &= -\$7,000,000 + (R - \$4,000,000)(P/A, 15\%, 6) \\
&= 3.7844R - \$22,137,900 \\
&= 0
\end{aligned}
$$

Solving for R yields

$$R = \$5,849,700 \text{ per year}$$

(b) Minimum processing fee per 1b (after-tax):

$$\frac{\$5,849,700}{(200,000)(3,600)} = \$0.0081 \text{ per 1b}$$

Comments: The minimum processing fee per 1b should be higher before-tax basis.

6.31 Given: Investment $=\$3$ million, plant capacity $=0.4$ acre, useful plant life $=20$ years, salvage value = negligible, O&M cost $=\$250,000$ per year, MARR $=10\%$ compounded annually (or effective monthly rate of 0.7974%)

$$\begin{aligned} AEC(10\%) &= \$3,000,000(A/P,10\%,20)+\$250,000 \\ &= \$602,379 \end{aligned}$$

Monthly water bill for each household:

$$\frac{\$602,379(A/F,0.7974\%,12)}{295}=\$162.83$$

6.32

- Annual total operating hours:

  $(0.70)(8,760)=6,132$ hours per year

- The amount of electricity generated annually:

  $50,000\times 6,132=306,600,000$ kilowatt-hours

- Equivalent annual cost:

$$\begin{aligned} AEC(14\%) &= \$85,000,000(A/P,14\%,25)+\$6,000,000 \\ &= \$18,367,364 \end{aligned}$$

- Cost per kilowatt-hour:

  $\$18,367,364/306,600,000=\$0.06$ per kilowatt-hour

6.33 Let X denote the average number of round-trip passengers per year.

- Capital costs:

$$\begin{aligned} CR(15\%) &= (\$12,000,000-\$2,000,000)(A/P,15\%,15) \\ &\quad +(0.15)(\$2,000,000) \\ &= \$2,010,171 \end{aligned}$$

- Annual crew costs: \$225,000

- Annual fuel costs for round trips:

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$$(\$1.10)(3,280)(2)(3)(52) = \$1,125,696$$

- Annual landing fees:

$$(\$250)(3)(52)(2) = \$78,000$$

- Annual maintenance, insurance, and catering costs:

$$\$237,500 + \$166,000 + \$75X = \$403,500 + \$75X$$

- Total equivalent annual costs:

$$\begin{aligned} AEC(15\%) &= \$2,010,171 + \$225,000 + \$1,125,696 \\ &\quad + \$78,000 + \$403,500 + \$75X \\ &= \$3,400X \end{aligned}$$

Solving for X yields

$$X = 1,156 \text{ Passengers per year}$$

Or

$$1,156/(52)(3) = 7.41 \approx 8 \text{ passengers per round trip}$$

Comparing Mutually Exclusive Alternatives by the AE Method

6.34

(a)

$$\begin{aligned} AE(15\%)_A &= -\$12,000(A/P,15\%,4) + [\$9,120 - \$2,280(A/G,15\%,4)] \\ &= \$1,892.95 \\ AE(15\%)_B &= -\$12,000(A/P,15\%,4) + \$6,350 \\ &= \$2,146.82 \end{aligned}$$

(b) Process A: $\$1,892.95/2,000 = \$0.9465/\text{hour}$
Process B: $\$2,146.82/2,000 = \$1.0734/\text{hour}$

(c) Process B is a better choice.

6.35

- Capital recovery cost for both motors:

$$CR(13\%)_{\text{CV}} = \$13,000(A/P,13\%,20) = \$1,851$$
$$CR(13\%)_{\text{PE}} = \$15,600(A/P,13\%,20) = \$2,221$$

- Annual operating cost for both motors:

$$\text{CV}: \frac{18.650 \text{ kW}}{0.895} \times \frac{\$0.07}{\text{kWh}} \times \frac{3{,}120 \text{ hrs}}{\text{year}} = \$4,551$$
$$\text{PE}: \frac{18.650 \text{ kW}}{0.93} \times \frac{\$0.07}{\text{kWh}} \times \frac{3{,}120 \text{ hrs}}{\text{year}} = \$4,380$$

(a) Savings per kWh:

$$AEC(13\%)_{\text{CV}} = \$1,851 + \$4,551 = \$6,402$$
$$AEC(13\%)_{\text{PE}} = \$2,221 + \$4,380 = \$6,601$$

Comments: At 3,120 annual operating hours, it will cost the company an additional \$370, but energy savings are only \$171, which results in a \$199 loss from each motor. The total output power is 58,188kWh per year (25 HP × 0.746 kW/HP × 3120 hrs/year). Therefore, the savings (losses) per operating hour from switching from conventional motor to the PE is

$$\frac{(\$199/\text{yr})}{58{,}188 \text{ kWh/yr}} = (\$0.034)/\text{kWh}$$

(b)

$$\$1,851 + 1.4587T = \$2,221 + 1.4038T$$
$$0.0549T = 370$$
$$T = 6,737 \text{ hours}$$

6.36

- New lighting system cost:

$$AEC(12\%) = \$50,000(A/P,12\%,20) + (\$8,000 + \$3,000)$$
$$= \$17,694$$

- Old lighting system cost:

$$AEC(12\%) = \$20,000$$

Annual savings from installing the new lighting system $= \$2,306$

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6.37 Given: $i = 6\%$ interest compounded monthly, the effective annual interest $= (1.005)^{12} - 1 = 6.17\%$ per year, effective semiannual interest $= (1.005)^{6} - 1 = 3.04\%$ per semiannual

- Option 1: Buying a bond

$$\begin{aligned} AE(3.04\%)_1 &= -\$2,000(A/P, 3.04\%, 6) + \$100 + \$2,000(A/F, 3.04\%, 6) \\ &= \$39.20 \text{ per semiannual} \\ AE(6.17\%) &= \$39.20(F/A, 3.04\%, 2) \\ &= \$79.59 \text{ per year} \end{aligned}$$

- Option 2: Buying and holding a growth stock for 3 years

$$\begin{aligned} AE(6.17\%)_2 &= -\$2,000(A/P, 6.17\%, 3) + \$2,735.26(A/F, 6.17\%, 3) \\ &= \$107.17 \end{aligned}$$

- Option 3: Receiving \$150 interest per year for 3 years

$$\begin{aligned} AE(6.17\%)_3 &= -\$2,000(A/P, 6.17\%, 3) + \$150 + \$2,000(A/F, 6.17\%, 3) \\ &= \$126.60 \end{aligned}$$

$\therefore$ Making the personal loan is the best option.

6.38

- Equivalent annual cost:

$$\begin{aligned} AEC(13\%)_A &= (\$1,200,000 - \$60,000)(A/P, 13\%, 20) \\ &\quad + (0.13)(\$60,000) + \$50,000 + \$40,000 \\ &= \$260,083 \\ AEC(13\%)_B &= (\$750,000 - \$30,000)(A/P, 13\%, 10) \\ &\quad + (0.13)(\$30,000) + \$80,000 + \$30,000 \\ &= \$216,395 \end{aligned}$$

- Processing cost per ton:

$$\begin{aligned} C_A &= \$260,083/(20)(365) = \$35.63 \text{ per ton} \\ C_B &= \$216,395/(20)(365) = \$29.64 \text{ per ton} \end{aligned}$$

$\therefore$ Incinerator B is a better choice.

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6.39

(a)

$$AE(15\%)_A = [-\$2,500 + \$1,000(P/F,15\%,1) + \cdots + \$400(P/F,15\%,4)](A/P,15\%,4) = \$216.06$$

$$AE(15\%)_B = [-\$4,000 + \$100(P/F,15\%,1)](A/P,15\%,4) + \$1,500 = \$129.40$$

$\therefore$ Project A is a better choice.

(b)

$$AE(15\%)_B = \$129.40$$

$$AE(15\%)_C = [-\$5,000 - \$200(P/A,15\%,2)](A/P,15\%,4) + \$2,000 = \$134.79$$

$\therefore$ Project C is a better choice.

Life-Cycle Cost Analysis

6.40 Assumption: jet fuel cost = \$2.10/gallon

- System A : Equivalent annual fuel cost: A_1 = (\$2.10/gal)(40,000gals/1,000 hours)(2,000 hours) $= \$168,000$ (assuming an end of-year convention)

$$AEC(10\%)_{\text{fuel}} = [\$168,000(P/A_1,6\%,10\%,3)](A/P,10\%,3) = \$177,623$$

$$AEC(10\%)_A = (\$100,000 - \$10,000)(A/P,10\%,3) + (0.10)(\$10,000) + \$177,623 = \$\$214,813$$

- System B : Equivalent annual fuel cost: A_1 = (\$2.10/gal)(32,000gals/1,000 hours)(2,000 hours = \$134,400

$$AEC(10\%)_{\text{fuel}} = [\$134,400(P/A_1,6\%,10\%,3)](A/P,10\%,3) = \$142,099$$

$$AEC(10\%)_B = (\$200,000 - \$20,000)(A/P,10\%,3) + (0.10)(\$20,000) + \$142,099 = \$216,480$$

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- Cost of owning and operating:

$$\text{System } A = \$214,813/2,000 = \$107.41 \text{ per hour}$$
$$\text{System } B = \$216,480/2,000 = \$108.24 \text{ per hour}$$

$\therefore$ System A is a better choice.

6.41 Since the required service period is 12 years and the future replacement cost for each truck remains unchanged, we can easily determine the equivalent annual cost over a 12-year period by simply finding the annual equivalent cost of the first replacement cycle for each truck.

- Truck A: Four replacements are required

$$\begin{aligned} AEC(12\%)_A &= (\$15,000 - \$5,000)(A/P, 12\%, 3) \\ &\quad +(0.12)(\$5,000) + \$3,000 \\ &= \$7,763.50 \end{aligned}$$

- Truck B: Three replacements are required

$$\begin{aligned} AEC(12\%)_B &= (\$20,000 - \$8,000)(A/P, 12\%, 4) \\ &\quad +(0.12)(\$8,000) + \$2,000 \\ &= \$6,910.80 \end{aligned}$$

$\therefore$ Truck B is a more economical choice.

6.42

(a) Number of decision alternatives (required service period = 5 years):

Alternative	Description
A1	Buy Machine A and use it for 4 years. Then lease the machine for one year.
A2	Buy Machine B and use it for 5 years.
A3	Lease a machine for 5 years.

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A4	Buy Machine A and use it for 4 years. Then buy another Machine A and use it for just one year.
A5	Buy Machine A and use it for 4 years. Then buy Machine B and use it for one year.

Both A4 and A5 are feasible but may not be practical alternatives. To consider these alternatives, we need to know the salvage values of the machines after one-year use.

(b) With lease, the O&M costs will be paid by the leasing company:

- For A1:

$$\begin{aligned} PW(10\%)_1 &= -\$6,500 + (\$600 - \$100)(P/F,10\%,4) \\ &\quad -\$800(P/A,10\%,4) - \$200(P/F,10\%,3) \\ &\quad -\$100(P/F,10\%,2) - (\$3,000 + \$100)(P/F,10\%,5) \\ &= -\$10,976 \\ AEC(10\%)_1 &= \$10,976(A/P,10\%,5) \\ &= \$2,895.44 \end{aligned}$$

- For A2:

$$\begin{aligned} PW(10\%)_2 &= -\$8,500 + \$1,000(P/F,10\%,5) \\ &\quad -\$520(P/A,10\%,5) - \$280(P/F,10\%,4) \\ &= -\$10,042 \\ AEC(10\%)_2 &= \$10,042(A/P,10\%,5) \\ &= \$2,649 \end{aligned}$$

- For A3:

$$\begin{aligned} AEC(10\%)_3 &= [\$3,000 + \$3,000(P/A,10\%,4)](A/P,10\%,5) \\ &= \$3,300 \end{aligned}$$

$\therefore$ A2 is a better choice.

6.43

- Option 1:

$$
\begin{aligned}
AEC(18\%)_1 &= \$200,000(180)(A/P,18\%,20) \\
&\quad -\$(0.08)(200,000)(180)(A/F,18\%,20) \\
&\quad +(\$0.005+0.215)(180,000,000) \\
&= \$46,305,878 \\
\text{cost/lb} &= \$46,305,878/180,000,000 \\
&= \$0.2573 \text{ per 1b}
\end{aligned}
$$

- Option 2:

$$
\begin{aligned}
AEC(18\%)_2 &= (\$0.05+\$0.215)(180,000,000) \\
&= \$47,700,000 \\
\text{cost/lb} &= \$47,700,000/180,000,000 \\
&= \$0.2650 \text{ per 1b}
\end{aligned}
$$

$\therefore$ Option 1 is a better choice.

6.44 Given: Required service period = indefinite, analysis period = indefinite

Plan A: Incremental investment strategy:

- Capital investment :

$$
\begin{aligned}
AEC(10\%)_1 &= [\$400,000+\$400,000(P/F,10\%,15)](A/P,10\%,\infty) \\
&= \$49,576
\end{aligned}
$$

- Supporting equipment:

$$
\begin{aligned}
AEC(10\%)_2 &= [(\$75,000+\$75,000/3.1772)(P/F,10\%,30)] \\
&\quad \times(A/P,10\%,\infty) \\
&= \$565
\end{aligned}
$$

Note that the effective interest rate for 15-year period is

$$(1+0.1)^{15}-1=3.1772$$

- Operating cost:

$$
\begin{aligned}
AEC(10\%)_3 &= [\$31,000(P/A,10\%,15) \\
&\quad +\$62,000(P/A,10\%,5)(P/F,10\%,15)] \\
&\quad +[\frac{\$63,000}{0.10}+\$1,000(P/G,10\%,\infty)] \\
&\quad \times(P/F,10\%,20)](A/P,10\%,\infty) \\
&= \$40,056
\end{aligned}
$$

Note that $(P/G,i,\infty)=1/i^2$ or $(P/G,10\%,\infty)=100$

- Total equivalent annual worth:

$$AEC(10\%)_A = \$49,576+\$565+\$40,056 = \boxed{\$90,197}$$

Plan B: One time investment strategy:

- Capital investment:

$$
\begin{aligned}
AEC(10\%)_1 &= \$550,000(A/P,10\%,\infty) \\
&= \$55,000
\end{aligned}
$$

- Supporting equipment:

$$
\begin{aligned}
AEC(10\%)_2 &= \frac{\$150,000}{16.4494}(A/P,10\%,\infty) \\
&= \$912
\end{aligned}
$$

Note that the effective interest rate for 30-year period is

$$(1+0.1)^{30}-1=16.4494$$

- Operating cost:

$$
\begin{aligned}
AEC(10\%)_3 &= [\$35,000(P/A,10\%,15) \\
&\quad +\$55,000(P/A,10\%,\infty)(P/F,10\%,15)] \\
&\quad \times(A/P,10\%,\infty) \\
&= \$39,788
\end{aligned}
$$

- Total equivalent annual worth:

$$AEC(10\%)_B = \$55,000 + \$912 + \$39,788 = \boxed{\$95,700}$$

$\therefore$ Plan A is a better choice.

Minimum Cost Analysis

6.45

(a)

- Energy loss in kilowatt-hour:

$$\frac{6.516}{A}(24 \times 365)(\$0.0825) = \frac{\$4,709.11}{A}$$

- Material weight in pounds:

$$(200)(12)(555)\frac{A}{12^3} = (770.83)A$$

- Total material costs:

$$(770.83)A(\$6) = \$4,625A$$

- Capital recovery cost:

$$\begin{aligned} CR(11\%) &= [\$4,625A - \$1 \times 770.83A](A/P, 11\%, 25) + \$1 \times 770.83A \times 0.11 \\ &= 542.44A \end{aligned}$$

- Total equivalent annual cost:

$$AEC(11\%) = 542.44A + \frac{4,709.11}{A}$$

- Optimal cross-sectional area:

$$\frac{dAEC(11\%)}{dA} = 542.44 - \frac{4,709.11}{A^2} = 0$$

$$A = 2.9464 \text{ inches}^2$$

(b) Minimum annual equivalent total cost:

$$AEC(11\%) = 542.44(2.9464) + \frac{4,709.11}{2.9464} = \$3,196.51$$

(c) Graph is not shown

Short Case Studies

ST 6.1 This case problem appears to be a trivial decision problem as one alternative (laser blanking method) dominates the other (conventional method). The problem of this nature (from an engineer's point of view) involves more strategic planning issues than comparing the accounting data. We will first calculate the unit cost under each production method. Since all operating costs are already given in dollars per part, we need to convert the capital expenditure into the required capital recovery cost per unit.

- Conventional method:

$$\begin{aligned} CR(16\%) &= \$106,480(A/P,16\%,10) \\ &= \$22,031 \text{ per year} \end{aligned}$$

$$\text{Unit capital recovery cost} = \frac{\$22,031}{3,000}$$

$$= \$7.34 \text{ per year}$$

- Laser blanking method:

$$\begin{aligned} CR(16\%) &= \$83,000(A/P,16\%,10) \\ &= \$17,173 \text{ per year} \end{aligned}$$

$$\text{Unit capital recovery cost} = \frac{\$17,173}{3,000}$$

$$= \$5.72 \text{ per part}$$

Blanking Method		
Description	Conventional	Laser
Steel cost/part	$14.98	$8.19
Transportation cost/part	$0.67	$0.42
Blanking cost/part	$0.50	$0.40
Capital cost/part	$7.34	$5.72
Total unit cost	$23.49	$14.73

It appears that the window frame production by the laser blanking technique would save about $8.76 for each part produced. If Ford decides to make the window frame in house, the part cost would range between $14.73 and $23.49, depending upon the blanking method adopted. If Ford relies on an outside supplier, the subcontracting work should be in this cost range. If Ford were producing the window frames by the conventional method, the die investment had already been made. In this case, one of the important issues is to address if it is worth switching to the laser blanking this time or later. If Ford decides to go with the laser blanking, it will take 6 months to reach the required production volume. What option should Ford exercise to meet the production need during this start-up period?

ST 6.2 Not given

ST 6.3 Given: annual energy requirement $= 145,000,000,000$ BTUs, 1-metric ton $= 2,204.6$ lbs (an approximation figure of 2,000 lbs was mentioned in the case problem), net proceeds from demolishing the old boiler unit $= \$1,000$

(a) Annual fuel costs for each alternative:

- Alternative 1:

$$\text{Weight of dry coal} = \frac{145,000,000,000 \text{ BTUs}}{(0.75)(14,300)}$$
$$= 13,519,814 \text{ lbs}$$
$$= \frac{13,519,814}{2,204.6}$$
$$= 6,132.45 \text{ tons}$$

$$\text{Annual fuel cost} = 6,132.45 \times \$55.5$$
$$= \$340,351$$

- Alternative 2:

$$\text{Gas cost} = \$9.5\left(\frac{145,000,000,000(0.94)}{(0.78)(1,000,000)}\right)$$
$$= \$1,660,064$$

$$\text{Oil cost} = \$1.35\left(\frac{145,000,000,000(0.06)}{(0.81)(139,400)}\right)$$
$$= \$104,016$$

$$\text{Annual fuel cost} = \$1,660,064 + \$104,016$$
$$= \$1,764,080$$

(b) Unit cost per steam pound:

- Alternative 1: Assuming a zero salvage value of the investment

$$
\begin{aligned}
AEC(10\%) &= (\$1,770,300 + \$100,000 \\
&\quad -\$1,000)(A/P, 10\%, 20) \\
&\quad +\$340,351 \\
&= \$644,571
\end{aligned}
$$

$$\text{Unit cost} = \frac{\$644,571}{145{,}000{,}000} = \$0.0044 \text{ per steam lb}$$

- Alternative 2:

$$
\begin{aligned}
AEC(10\%) &= (\$889,200 - \$1,000)(A/P, 10\%, 20) \\
&\quad +\$1,764,080 \\
&= \$1,868,408
\end{aligned}
$$

$$\text{Unit cost} = \frac{\$1,868,408}{145{,}000{,}000} = \$0.0129 \text{ per steam lb}$$

(c) Select alternative 1.

ST 6.4 Assuming that the cost of your drainage pipe has experienced a 4% annual inflation rate, I could estimate the cost of the pipe 20 years ago as follow:

$$\$4,208(P/F, 4\%, 20) = \$4,208(1.04)^{-20} = \$1,920.48$$

If the pipe had a 50-year service life with a zero salvage value when it was placed in service 20 years ago, the annual capital recovery cost to the owner would be as follows: (I assumed the owner's interest rate would be 5% per year. In other words, the owner could invest his $1,920.48 at 5% annual interest, if he did not purchase the pipe.)

$$
\begin{aligned}
CR(5\%) &= \$1,920,48(A/P, 5\%, 50) \\
&= \$105.20 \text{ per year}
\end{aligned}
$$

You can view this number as the annual amount he would expect to recover from his investment considering the cost of money. With only a 20-year's usage, he still has 30 more years to go. So, the unrecovered investment at the current point is

$$\$105.20(P/A, 5\%, 30) = \$1,617.15.$$

The owner could claim this number, but the city's interest rate could be different from the owner's, so there is some room for negotiation.

Assumed interest rate	Claim cost

0%	$1,152.24
3%	$1,462.98
4%	$1,545.89
5%	$1,617.15

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