Chapter 7 Rate-of-Return Analysis

Concept of Rate of Return

Note: Symbol convention---The symbol *i** represents the breakeven interest rate that makes the NPW of the project equal to zero.. The symbol IRR represents the internal rate of return of the investment. For a simple (or pure) investment, IRR = *i**. For a nonsimple investment, generally *i** is not equal to IRR.

7.1

$$\$14{,}500 = \$267(P/A, i, 72)$$
$$i = 0.8148\% \text{ per month}$$
$$r = 0.8148\% \times 12 = 9.7776\%$$
$$i_a = (1 + 0.008148)^{12} - 1 = 10.23\%$$

7.2

$$\$900 = \$30(P/A, 4\%, 8) + F(P/F, 4\%, 8)$$
$$0.7307F = \$698.02$$
$$F = \$955.27$$

7.3

$$\$10{,}649{,}600 = \$1{,}650(F/P, i, 36)$$
$$6{,}454.30 = (1+i)^{36}$$
$$i = 27.6\%$$

7.4

$$\$2.5 = \$1.2(P/A, i, 10)$$
$$i = 46.98\%$$

7.5

$$\$104{,}200{,}000 = \$30{,}000(F/P, i, 54)$$
$$3{,}473.33 = (1+i)^{54}$$
$$i = 16.30\%$$

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Investment Classification and Calculation of *i**

7.6

(a) Simple investment: Project A (Note: Project C is a simple borrowing)

(b) Non simple investment: Project B and Project D

(c)

- Project A:

$$\begin{aligned} PW(i) &= -\$18,000 + \$10,000(P/A,i,3) + \$10,000(P/G,i,3) \\ &= 0 \\ i^* &= 74.23\% \end{aligned}$$

- Project B:

$$\begin{aligned} PW(i) &= -\$20,000 + \$32,000(P/A,i,2) - \$22,000(P/F,i,3) \\ &= 0 \\ i^* &= 111.11\% \end{aligned}$$

- Project C:

$$\begin{aligned} PW(i) &= \$34,578 - \$18,000(P/A,i,3) \\ &= 0 \\ i^* &= 26.08\% \rightarrow \text{Borrowing rate of return} \end{aligned}$$

(d) Project D has no rate of return.

7.7 The equivalent annual cash flow for the first cash flow cycle ($400, $800, $500, $500) will be

$$AE(i) = \$500 + [-\$100(P/F,i,1) + \$300(P,F,i,2)](A/P,i,4)$$

Then, the present worth of the infinite cash flow series is expressed as

$$\begin{aligned} PW(i) &= -\$1,000 + \frac{AE(i)}{i} \\ &= -\$1,000 + \frac{[-\$100(P/F,i,1) + \$300(P,F,i,2)](A/P,i,4)}{i} \\ &= 0 \end{aligned}$$

$$\therefore\ i^* = 54.05\%$$

7.8
(a) Classification of investment projects:

- Simple projects: A,B, and E
- Non simple projects: C and D

(b)

$$-\$100 + \frac{\$60}{1+i} + \frac{\$150}{(1+i)^2} = 0$$

Let $X = \dfrac{1}{(1+i)}$, then,

$$-\$100 + \$60X + \$150X^2 = 0$$
$$X_1 = 0.6406,\ \ X_2 = -1.0406$$

$$\therefore\ i^* = 56.09\%$$

(c) Find i^* by plotting the NPW as a function of interest rate:

Project	i^*
A	56.09%
B	47.94%
C	8.32%
D	178.8%
E	24.21%

7.9
(a) Classification of investment projects:

- Simple projects: A,B, and D
- Non simple projects: C

(b)
- Project A: $i^* = 9.63\%$
- Project B: $i^* = 27.6\%$
- Project C: $i^* = 276.72\%$
- Project D: $i^* = 86.69\%$

(c) Use the PW plot command provided in Cash Flow Analyzer, or you may use the Excel's Chart Wizard

7.10

(a)

$$-\$50,000 + (\$25,000 - \$9,000)(P/A, i, 8) + \$10,000(P/F, i, 8) = 0$$

Solving for i yields $i^* = 28.45\%$

(b) With the geometric expense series

$$-\$50,000 + \$25,000(P/A, i, 8) - \$9,000(P/A_1, 7\%, i, 8)$$
$$+\$10,000(P/F, i, 8) = 0$$

Solving for i^* yields $i^* = 21.47\%$

(c) To maintain $i^* = 28.45\%$

$$PW(i) = -\$50,000 + \$25,000(P/A_1, g, 28.45\%, 8)$$
$$-\$9,000(P/A_1, 7\%, 28.45\%, 8) + \$10,000(P/A, 28.45\%, 8)$$
$$= 0$$

Solving for g yields

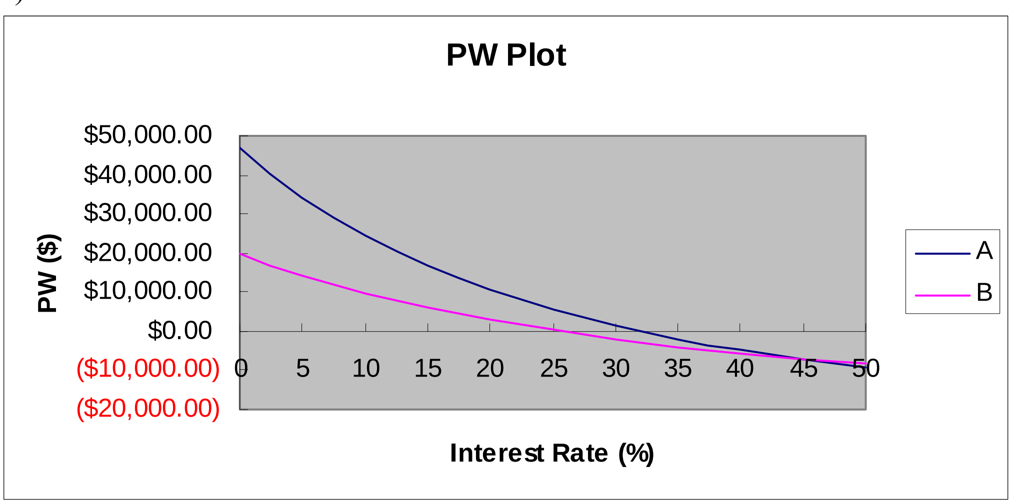
$$g = 4.806\%$$

7.11

(a) Rate of return calculation:

- Project A: $i^* = 32.10\%$
- Project B: $i^* = 25.53\%$

(b)



7.12

(a) Cash flow sign rules:

Projects	Number of Sign Changes	Possible Number of i^*
A	1	0, 1
B	1	0, 1
C	1	0, 1
D	1	0, 1
E	2	0, 1, 2
F	1	0, 1

(b) Use the PW plot command provided in Cash Flow Analyzer, or you may use the Excel's Chart Wizard

(c)

- Project A: $i^* = 228.42\%$
- Project B: $i^* = 500\%$
- Project C: $i^* = 23.27\%$
- Project D: $i^* = 70.99\%$
- Project E: $i^* = 265.41\%$
- Project F: $i^* = 258.91\%$

7.13

(a) IRR = 69.81%

$$PW(i) = -120{,}000 + \frac{94{,}000}{(1+i)} + \frac{144{,}000}{(1+i)^2} + \frac{72{,}000}{(1+i)^3} = 0$$

$$i^* = 69.81\%$$

(b) Use the PW plot command provided in Cash Flow Analyzer, or you may use the Excel's Chart Wizard

(c) Since IRR(69.81%) > MARR (15%), accept the project!

Mixed Investments

7.14

(a)

- Project 1: $i^* = 20\%$
- Project 2: $i^* = 18\%$
- Project 3: $i^*_1 = 32.45\%,\ i^*_2 = -92.45\%$

(b) Investment classification:

- Project 1: simple and pure investment, $IRR = 20\%$
- Project 2: simple and pure investment, $IRR = 18\%$
- Project 3: non simple and mixed investment, $IRR = RIC = 31.07\%$

$$PB(31.07\%,12\%)_0 = -\$1,000$$
$$PB(31.07\%,12\%)_1 = -\$1,000(1+0.3107)+\$1,400 = \$89.30$$
$$PB(31.07\%,12\%)_2 = \$89.30(1+0.3107)-100 = \$0$$

(c) If MARR = 12%, all the projects are acceptable.

7.15

(a)

$$-\$100 + \frac{\$100}{1+i} + \frac{\$24}{(1+i)^2} = 0$$

Let $X = \dfrac{1}{(1+i)}$, then,

$$-\$100 + \$100X + \$24X^2 = 0$$
$$X_1 = 0.8333, \quad X_2 = -5$$
$$\therefore \; i^* = 20\%$$

(b)

- Simple projects: A,B, and D
- Non simple projects: C and E

(c) Apply the cash flow sign rule:

Projects	Number of Sign Changes	Possible Number of i^*	Actual i^*
A	1	0, 1	20%
B	1	0, 1	28.58%
C	2	0, 1, 2	14.63%,210.27%
D	1	0, 1	15.24%
E	2	0, 1, 2	12.63%,41.42%

(d)

- Project B: $IRR_B = 28.59\%$
- Project C: $IRR_C = -15.57\%$

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- Project D: $IRR_D = 15.24\%$
- Project E: $IRR_E = 11.24\%$

Note that, since projects C and E are mixed investments, we need to find the RIC for both C and E by using external interest rate of 10%

(e) Apply the net investment test: Project = pure investment, Project D = pure borrowing

n	Project Balances	
	Project C ($i^* = 14.63\%$)	Project D ($i^* = 12.63\%$)
0	-$5.0	$200
1	$4.3	$325
2	$34.89	-$134
3	0	-$651
4		-$533
5		0

7.16

(a)

- Project 1: $i^* = 612.695\%$
- Project 2: $i^*_1 = 14.64\%,\ i^*_2 = 210.28\%$
- Project 3: $i^* = 100\%$

(b) Apply the net investment test:

- Project 1:

$$PB(612.695\%)_0 = -\$1,600$$
$$PB(612.695\%)_1 = -\$1,600(1+6.12695) + \$10,000 = -\$1,403.2$$
$$PB(612.695\%)_2 = -\$1,403.2(1+6.12695) + \$10,000 = 0$$

(-,-,0), a pure investment

- Project 2:

$$PB(14.64\%)_0 = -\$5,000$$
$$PB(14.64\%)_1 = -\$5,000(1+0.1464) + \$10,000 = \$4,268$$
$$PB(14.64\%)_2 = \$4,268(1+0.1464) + \$30,000 = \$34,892.80$$
$$PB(14.64\%)_3 = \$34,892.8(1+0.1464) - \$40,000 = 0$$

(-,+,0), a mixed investment

- Project 3:

$$PB(100\%)_0 = -\$1,000$$
$$PB(100\%)_1 = -\$1,000(1+1) + \$4,000 = \$2,000$$
$$PB(100\%)_2 = \$2,000(1+1) - \$4,000 = 0$$

(-,+,0), a mixed investment

(c)
- Project 1: $IRR_1 = 612.695\%,\ PW(12\%) = \$15,300$
- Project 2: $IRR_2 = RIC = -2.04\%,\ PW(12\%) = -\$623 < 0$
- Project 3: $IRR_3 = RIC = -57.14\%, PW(12\%) = -\$617 < 0$

(d) Only project 1 is acceptable.

7.17
(a) Simple investments: A and B (simple as well as pure)

(b) Mixed investments: C

(c) Project A: $IRR_A = 23.24\%$, Project B: $IRR_B = 21.11\%$, Project C: $IRR_C = 12.24\%$ at an external rate of 12%

(d) All three projects are acceptable.

7.18
(a) There are two sign changes in cash flow, indicating multiple i^*s.

$$i^*_1 = 20\%,\ i^*_2 = 40\%$$

Apply the net investment test:

$$PB(20\%)_0 = -\$100$$
$$PB(20\%)_1 = -\$100(1.2) + \$260 = \$140$$
$$PB(20\%)_2 = \$140(1.2) - \$168 = 0$$

$\therefore$ (-,+,0), a mixed investment

(b) At an external interest rate of 12%, RIC = IRR = 10% < 12%.

(c) The project is not acceptable.

7.19

(a) $i^*_1 = 15.99\%$, $i^*_2 = 0\%$

(b) Since all projects pass the net investment test, all projects are pure investments.

(c) All projects are acceptable at MARR = 10%.

7.20

(a) Project A: $i^*_1 = 10\%$, $i^*_2 = 100\%$, Project B: $i^*_1 = 350.33\%$, $i^*_2 = -80.83\%$

(b) Pure investment: C, mixed investments: A,B, D and E

(c) Project A: $IRR_A = 13.57\%$, Project B: $IRR_B = 342.16\%$, Project C: $IRR_C = 18\%$, Project D: $IRR_D = 31.07\%$, Project E: $IRR_E = 19.66\%$

(d) All projects are acceptable at MARR = 12%.

7.21

(a) Use the PW plot command provided in Cash Flow Analyzer.
From the plot, we get $i^*_1 = 14.64\%$, $i^*_2 = 210.27\%$

(b) Apply the net investment test:

$$PB(14.64\%)_0 = -\$5,000$$
$$PB(14.64\%)_1 = -\$5,000(1+0.1464)+\$10,000 = \$4,268$$
$$PB(14.64\%)_2 = \$4,268(1+0.1464)+\$30,000 = \$34,892.80$$
$$PB(14.64\%)_3 = \$34,892.8(1+0.1464)-\$40,000 = 0$$

(-,+,+,0): The project is a mixed investment.

(c) Since RIC = IRR = 33.92% > 18%, the project is acceptable.

7.22

(a) IRR for the incremental investment:

n	Net Cash Flow		
	Project A	Project B	B-A
0	-\$300	-\$800	-\$500
1	0	\$1,150	\$1,150
2	\$690	\$40	-\$650

$$i^*_{B-A} = 0\% \text{ or } 30\%$$

Since this is a mixed incremental investment, we need to find the RIC using an external interest rate of 15%.

$$RIC_{\text{B-A}} = IRR_{\text{B-A}} = 16.95\% > 15\%$$

$\therefore$ Project B is preferred

(b) Use the PW plot command provided in Cash Flow Analyzer.

7.23

(a) Apply the net investment test using $i^* = 10\%$

$$PB(10\%)_0 = -\$100,000$$
$$PB(10\%)_1 = -\$100,000(1.1) + \$310,000 = \$200,000$$
$$PB(10\%)_2 = \$200,000(1.1) - \$220,000 = 0$$

$\therefore$ (-,+,0), a mixed investment

(b) RIC = IRR = 6.29% <8%. So the project is not acceptable.

7.24 (c)

IRR Analysis

7.25 The present worth of the project cash flow is

$$PW(i) = -\$10M + \$1.8M(P/A, i, 8) + \$1M(P/F, i, 8) = 0$$

$\therefore$ Since $IRR = 10.18\%$ > MARR, accept the project

7.26 The present worth of the project cash flow is

$$PW(i) = -\$5,000 + \$4,840(P/F, i, 2) + \$1,331(P/F, i, 3) = 0$$

$\therefore$ Since $IRR = 10\%$ = MARR, the project breaks even.

7.27

(a) Since IRR = 10% and PW(10%) = 0, we have,

$$PW(10\%) = -\$2,000 + \$800(P/F,10\%,1) + \$900(P/F,10\%,2) + X(P/F,10\%,3) = 0$$

$$\therefore\ X = \$704$$

(b) Since IRR > 8%, the project is acceptable.

7.28

- Let X be the annual rent per apartment unit. Then net cash flow table is:

N	Capital Investment	Revenue	Maintenance	Manager	Net Cash Flow
0	-12,500,000				-12,500,000
1		50 X	-250,000	-80,000	50X - 330,000
2		50 X	-300,000	-80,000	50X - 380,000
3		50 X	-350,000	-80,000	50X - 430,000
4		50 X	-400,000	-80,000	50X - 480,000
5	14,000,000	50 X	-450,000	-80,000	50X - 13,470,000

$$\begin{aligned} PW(15\%) &= -12,500,000 + (50X - 330,000)(P/A,15\%,5) \\ &\quad -50,000(P/G,15\%,5) + 12,940,000(P/F,15\%,5) \\ &= 0 \\ X^* &= \$41,373 \text{ or } \$3,448 \text{ per month} \end{aligned}$$

7.29

- Let X be the annual savings in labor, then.

$$PW(10\%) = -\$25,000 + (X - \$3,000)(P/A,10\%,6) + \$5,000(P/F,10\%,6) = 0$$

- $X = \$8,092.15$

7.30

- Net cash flow table:

n	Land	Building	Equipment	Revenue	Expenses	Net Cash Flow
0	($1.50)	($3)				($4.50)
1			($4)			($4.00)
2				$3.50	($1.40)	$2.10
3				$3.68	($1.47)	$2.21
4				$3.86	($1.54)	$2.32
5				$4.05	($1.62)	$2.43
6				$4.25	($1.70)	$2.55
7				$4.47	($1.79)	$2.68
8				$4.69	($1.88)	$2.81
9				$4.92	($1.97)	$2.95
10				$5.17	($2.07)	$3.10
11				$5.43	($2.17)	$3.26
12				$5.43	($2.17)	$3.26
13				$5.43	($2.17)	$3.26
14	$2	$1.40	$0.50	$5.43	($2.17)	$7.16

- Rate of return calculation:

$$\begin{aligned} PW(i) &= -\$4.5 - \$4(P/F,i,1) + \$2.1(P/F,i,2) + \cdots \\ &\quad + \$7.16(P/F,i,14) = 0 \end{aligned}$$

$$\therefore\ i^* = 24.85\%$$

- Since this is a simple investment, IRR = 24.85%. At MARR = 15%, the project is economically attractive.

7.31
(a)

$$\begin{aligned} PW(i) &= -\$20 - \$8(P/F,i,1) + \$17(P/F,i,2) + \$19(P/F,i,3) \\ &\quad + \$18(P/F,i,4) + \$10(P/F,i,5) + \$3(P/F,i,6) \\ &= 0 \end{aligned}$$

This is a simple investment. Therefore, $IRR = i^* = 60.52\%$.

$\therefore$ Since IRR > MARR (18%), the product is worth marketing.

(b) IRR = 67.03%

(c) IRR = 48.07%

Comparing Alternatives

7.32

(a) Project A: $\text{IRR} = 11.71\%$
Project B: $\text{IRR} = 19.15\%$

(b) Only project B is acceptable

(c) Since project A is not acceptable, select project B.

7.33 Option 1:Buy a certificate,
Option 2: Purchase a bond, and assume that $\text{MARR} = 9\%$

	Net Cash Flow		
n	Option 1	Option 2	Option 1 – Option 2
0	-$10,000	-$10,000	0
1	0	1,000	-1,000
2	0	1,000	-1,000
3	0	1,000	-1,000
4	0	1,000	-1,000
5	16,105	11,000	5,105

The rate of return on incremental investment is

$$i^*_{1-2} = 10\% > 9\%$$

$\therefore$ Option 1 is a better choice.

7.34 Determine the cash flow on incremental investment:

	Net Cash Flow		
n	Project A	Project B	B - A
0	-$2,000	-$3,000	-$1,000
1	1,400	2,400	1,000
2	1,640	2,000	360

$$i^*_{B-A} = 28.11\% > 15\%$$

$\therefore$ Select project B.

7.35

(a) IRR on the incremental investment:

	Net Cash Flow		
n	Project A1	Project A2	A2 – A1
0	-10,000	-12,000	-\$2,000
1	5,000	6,100	1,100
2	5,000	6,100	1,100
3	5,000	6,100	1,100

$$i^*_{A2-A1} = 29.92\%$$

(b) Since it is a simple incremental investment, $\text{IRR}_{\text{A2-A1}} = 29.92\% > 10\%$. Therefore, select project A2.

7.36

(a) IRR on the incremental investment:

n	Net Cash Flow		
	A1	A2	A2 – A1
0	-\$15,000	-\$20,000	-\$5,000
1	7,500	8,000	500
2	7,500	15,000	7,500
3	7,500	5,000	-2,500

Since the incremental cash flow series portrays a nonsimple investment, we need to find the RIC at 10%, which is $RIC_{A2-A1} = 7.36\% < 10\%$. So, select A1.

(b) We can verify the same result by applying the NPW criterion.

$$\begin{aligned} PW(10\%)_{A1} &= -15,000 + \$7,500(P/A, 10\%, 3) \\ &= \$3,651 \\ PW(10\%)_{A2} &= -20,000 + \$8,000(P/F, 10\%, 1) + \$15,000(P/F, 10\%2) \\ &\quad + \$5,000(P/F, 10\%, 3) \\ &= \$3,426 \end{aligned}$$

$\therefore$ Select project A1.

7.37 Incremental cash flows (Model A – Model B):

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n	A – B
0	-\$2,376
1	0
2	0
3	0
4	2,500

$$\text{IRR}_{A-B} = 1.28\%$$

$\therefore$ If MARR < 1.28%, Model A is a preferred.

7.38 IRR for Model A: 6.01%, IRR for Model B: 7.25%

$\therefore$ The best decision is "do-nothing."

7.39

(a) The least common multiple project lives = 6 years → Analysis period 6 years

n	Net Cash Flow		
	Project A	Project B	B – A
0	-\$100	-\$200	-100
1	60	120	60
2	50	150-200	-100
3	50-100	120	170
4	60	150-200	-110
5	50	120	70
6	50	150	100

Since the incremental cash flow series indicates a nonsimple investment, but it is a pure incremental investment.

$$IRR_{B-A} = 15.98\% > 15\%$$

$\therefore$ So, select project B.

(b) Incremental analysis between C and D:

n	Net Cash Flow Project C	 Project D	 C – D
0	-$4,000	-$2,000	-$2,000
1	2,410	1,400	1,010
2	2,930	1,720	1,210

$\therefore$ $\text{IRR}_{C-D} = 7.03\%$, Project D is preferred

(c) Incremental analysis between E and F:

n	Net Cash Flow Project E	 Project F	 F –E
0	-$2,000	-$3,000	-$1,000
1	3,700	2,500	-1,200
2	1,640	1,500	-140

$\therefore$ No IRR, Project E dominates project F. Select E.

7.40 Let A0 = current practice, $A1 =$ just-in-time system, $A2 =$ stock-less supply system.

- Comparison between A0 and A1:

n	A0	A1	A1 – A0
0	0	-$2,500,000	-$2,500,000
1-8	-5,000,000	-2,900,000	2,100,000

$$i^*_{A1-A0} = \text{IRR}_{A1-A0} = 83.34\% > 10\%$$

$\therefore$ A1 is a better choice.

- Comparison between A1 and A2:

n	A2	A1	A2 – A1
0	-$5,000,000	-$2,500,000	-$2,500,000
1-8	-1,400,000	-2,900,000	1,500,000

$$i^*_{A2-A1} = \text{IRR}_{A2-A1} = 58.49\% > 10\%$$

$\therefore$ A2 is a better choice. That means that the stockless supply system is the final choice.

7.41

(a)

- Project A vs. Project B

n	Net Cash Flow Project A	Project B	B – A
0	-$1,000	-$1,000	0
1	900	600	-$300
2	500	500	0
3	100	500	400
4	50	100	50

$\therefore\ i^*_{B-A} = \text{IRR}_{B-A} = 21.27\% > 12\%$, select B.

- Project B vs. Project C

n	Net Cash Flow Project B	Project C	C – B
0	-$1,000	-$2,000	-$1,000
1	600	900	300
2	500	900	400
3	500	900	400
4	100	900	800

$\therefore\ i^*_{C-B} = \text{IRR}_{C-B} = 26.32\% > 12\%$, select C.

(b)

$$\$1,000 = \$300(P/A, i, 4)$$
$$i = 7.71\%$$

(c) Since BRR (borrowing rate of return) is less than MARR, project D is acceptable.

(d)

n	Net Cash Flow Project C	Project E	C – E
0	-$2,000	-$1,200	-$800
1	900	400	500
2	900	400	500
3	900	400	500
4	900	400	500

$\therefore\ i^*_{C-E} = \text{IRR}_{C-E} = 50.23\% > 12\%$, select C.

7.42

(a)

$$i_1^* = 85.08\%, i_2^* = 48.11\%, \text{ and } i_3^* = 44.31\%$$

(b)

• Project 1 versus Project 2:

n	Project 1	Project 2	2 – 1
0	-$1,000	-$5,000	-$4,000
1	500	7,500	7,000
2	2,500	600	-1,900

This is a nonsimple incremental investment. So, we need to compute RIC at 15%.

$$RIC_{2-1} = 33.69\% > 15\%$$

$\therefore$ Select Project 2.

• Project 2 versus Project 3:

n	Project 2	Project 3	2 – 3
0	-$5,000	-$2,000	-$3,000
1	7,500	1,500	6,000
2	600	2,000	-1,400

This is another nonsimple incremental investment, so we need to calculate RIC on the incremental investment.

$$RIC_{2-3} = 59.42\% > 15\%$$

$\therefore$ Again, select Project 2.

7.43

(a) $IRR_B = 25.99\%$

(b) $PW(15\%)_A = -\$10,000 + \$5,500(P/A,15\%,3) = \$2,558$

(c) Incremental analysis:

Net Cash Flow			
n	Project A	Project B	B – A
0	-$10,000	-$20,000	-$10,000
1	5,500	0	-5,500
2	5,500	0	-5,500
3	5,500	40,000	34,500

$\therefore$ Since $\text{IRR}_{B-A} = 24.24\% > 15\%,$ select project B.

7.44 Select Model C. Note that all three projects would be acceptable individually, as each project's IRR exceeds the MARR. The incremental IRR of Model (C – B) is 40%, indicating that Model C is preferred over Model B at MARR = 12%. Similarly, the incremental IRR of Model (C – A) is 15% which exceeds the MARR. Therefore, Model C is again preferred.

7.45 All projects would be acceptable because individual ROR exceed the MARR. Based on the incremental analysis, we observe the following relationships:

$$\text{IRR}_{A2-A1} = 10\% < 15\% \text{ (Select A1)}$$
$$\text{IRR}_{A3-A1} = 18\% > 15\% \text{ (Select A3)}$$
$$\text{IRR}_{A3-A2} = 23\% > 15\% \text{ (Select A3)}$$

$\therefore$ Therefore, A3 is the best alternative.

7.46 From the incremental rate of return table, we can deduce the following relationships:

$$\text{IRR}_{A2-A1} = 9\% < 15\% \text{ (Select A1)}$$
$$\text{IRR}_{A3-A2} = 42.8\% > 15\% \text{ (Select A3)}$$
$$\text{IRR}_{A4-A3} = 0\% < 15\% \text{ (Select A3)}$$
$$\text{IRR}_{A5-A4} = 20.2\% > 15\% \text{ (Select A5)}$$
$$\text{IRR}_{A6-A5} = 36.3\% > 15\% \text{ (Select A6)}$$

It is necessary to determine the preference relationship among A1, A3, and A6.

$$\text{IRR}_{A3-A1} = 16.66\% > 15\% \text{ (Select A3)}$$
$$\text{IRR}_{A6-A3} = 20.18\% > 15\% \text{ (Select A6)}$$
$$\text{IRR}_{A6-A1} = 18.24\% > 15\% \text{ (Select A6)}$$

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$\therefore$ A6 is the best alternative.

7.47 For each power saw model, we need to determine the incremental cash flows over the "by-hand" operation that will result over a 20-year service life.

Power Saw			
Category	Model A	Model B	Model C
Investment cost	$4,000	$6,000	$7,000
Salvage value	$400	$600	$700
Annual labor savings	$1,296	$1,725	$1,944
Annual power cost	$400	$420	$480
Net annual savings	$896	$1,305	$1,464

Net Cash Flow			
n	Model A	Model B	Model C
0	-$4,000	-$6,000	-$7,000
1	896	1,305	1,464
2	896	1,305	1,464
⋮	⋮	⋮	⋮
20	400+896	600+1,305	700+1,464
IRR	22.03%	21.35%	20.46%
PW(10%)	$3,688	$5,199	$5,568

- Model A versus Model B:

$$\begin{aligned} PW(i)_{B-A} &= -\$2,000 + \$409(P/A, i, 20) + \$200(P/F, i, 20) \\ &= 0 \\ \text{IRR}_{B-A} &= 19.97\% > 10\% \end{aligned}$$

$\therefore$ **Select Model B.**

- Model B versus Model C:

$$\begin{aligned} PW(i)_{C-B} &= -\$1,000 + \$159(P/A, i, 20) + \$100(P/F, i, 20) \\ &= 0 \\ \text{IRR}_{C-B} &= 15.03\% > 10\% \end{aligned}$$

$\therefore$ Select Model C

Unequal Service Life

7.48 With the least common multiple of 6 project years,

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	Net Cash Flow		
n	Project A	Project B	B – A
0	-$100	-$200	-$100
1	60	120	60
2	50	150-200	-100
3	50-100	120	170
4	60	150-200	-110
5	50	120	70
6	50	150	100

Since the incremental cash flow series is a nonsimple investment, we may abandon the IRR analysis, and use the PW decision rule.

$$\begin{aligned} PW(15\%)_{B-A} &= -\$100 + \$60(P/F, 15\%, 1) \\ &\quad + \cdots + \$100(P/F, 15\%, 6) \\ &= \$3.48 \end{aligned}$$

Since $PW(15\%)_{B-A} > 0$, or $PW(15\%)_B > PW(15\%)_A$, select project B.

Comments: Even though the incremental flow is a nonsimple, it has a unique rate of return. As shown in Problem 7.39, this incremental cash flow series will pass the net investment test, indicating that the incremental cash flow is a pure investment.

$$\text{IRR}_{B-A} = 15.98\% > 15\%$$

7.49

(a) Since there is not much information given regarding the future replacement options and the required service period, we may assume that the required service period is indefinite and both projects can be repeated at the same cost in the future.

(b) The analysis period may be chosen as the least common multiple of project lives, which is 3 years.

n	A2 – A1
0	-$5,000
1	0
2	0
3	15,000

$$\text{IRR}_{A2-A1} = 44.195\%$$

$\therefore$ The MARR must be less than 44.195% for Project A1 to be preferred.

Short Case Studies

ST 7.1

(a) Analysis period of 40 years:

- Without the "mothballing" cost:

$$PW(i) = -\$1,500,000 + \$138,000(P/A_1, 0.05\%, i, 40)$$
$$= 0$$
$$i^* = 8.94\%$$

- With the "mothballing" cost of \$0.75 billion:

$$PW(i) = -\$1,500,000 + \$138,000(P/A_1, 0.05\%, i, 40)$$
$$= -\$750,000(P/F, i, 40)$$
$$i^* = 8.77\%$$

For a 40-year analysis period, the drop of IRR with the mothballing cost is only 1.9%, which is relatively insignificant.

(b) Analysis period of 25 years (unit: thousand \$):

- Without the "mothballing" cost:

$$PW(i) = -\$1,500,000 + (\$207,000 - \$69,000)(P/A_1, 0.05\%, i, 25)$$
$$= 0$$
$$i^* = 8.94\%$$

- With the "mothballing" cost of \$0.75 billion:

$$PW(i) = -\$1,500,000 + (\$207,000 - \$69,000)(P/A_1, 0.05\%, i, 25)$$
$$-\$750,000(P/F, i, 25)$$
$$= 0$$
$$i^* = 6.80\%$$

For a 25-year analysis period, the drop of IRR with the mothballing cost is about 13.27%, which is relatively significant.

ST 7.2

(a) Assumptions required:

- There is no information regarding the expected cash flows from the current operation if B&E Cooling decides to defer the introduction of the absorption technology for 3 years. Therefore, we need to make an explicit assumption of the expected cash flows for the first three years if B&E Cooling decides to defer the decision. Assume that the annual cash flow during this period would be *X*.
- Another assumption we have to make is about the analysis period. Assuming that the firm will be in business for an indefinite period, we also need to make an explicit assumption regarding the future cooling technology. Since there is no information about the future cooling technology options, we may assume that the best cooling technology will be the absorption technology that will be introduced 3 years from now. Therefore, if B&E Cooling decides to select Option1, we could assume that, at the end of 8 years, Option 2 (the best cooling technology at that time) will be adopted for an indefinite period.

(b) Investment decision:

- Present worth analysis: First, we will determine the equivalent present worth for each option:

$$\begin{aligned} PW(i)_1 &= -\$6 + \$9(P/A,i,8) + \$1(P/F,i,8) \\ &\quad + \frac{[-\$5 + \$4(P/A,i,8) + \$2(P/F,i,8)](A/P,i,8)}{i} \\ &\quad \times (P/F,i,8) \\ PW(i)_2 &= X(P/A,i,3) \\ &\quad + \frac{[-\$5 + \$4(P/A,i,8) + \$2(P/F,i,8)](A/P,i,8)}{i} \\ &\quad \times (P/F,i,3) \end{aligned}$$

Now we can determine the value *X* that makes the two options economically equivalent at an interest rate of 15%. In other words, if we evaluate the two present worth functions at $i = 15\%$, we have

$$PW(15\%)_1 = \$41.31$$
$$PW(15\%)_2 = 2.2832X + \$13.28$$

Letting $PW(15\%)_1 = PW(15\%)_2$ and solving for *X* gives

$$X = \$12.28$$

As long as the current operation continues to generate annual net revenue of

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\$12.28 millions for 3 years, Option 2 is a better choice.

- Rate of return analysis: The present worth analysis above indicates that, if $X = \$12.28$, the break-even rate of return on incremental investment is $i^*_{1-2} = 15\%$

Therefore, the ultimate choice will depend on the level of annual revenues generated during the first 3 years when the advanced cooling technology is deferred. Clearly, if

$X < \$12.28$, then $i^*_{1-2} > 15\%$, Option 1 is preferred.

ST 7.3

n	Current Pump(A)	Larger Pump(B)	B-A	
0	\$0	-\$1,600,000	-\$1,600,000	
1	\$10,000,000	\$20,000,000	\$10,000,000	
2	\$10,000,000	\$0	-\$10,000,000	
		IRR =	25%	
			400%	

The incremental cash flows result in multiple rates of return (25% and 400%), so we may abandon the rate of return analysis. Using the PW analysis,

$$PW(20\%) = -\$1.6M + \$10M(P/F, 20\%, 1) - \$10M(P/F, 20\%, 2)$$
$$= -\$0.21 < 0$$

Reject the larger pump.

Return on Invested Capital: Using the procedure outlined in Section 7.3.4,

$$PB(i, 20\%)_0 = -\$1,600,000$$
$$PB(i, 20\%)_1 = -\$1,600,000(1+i) + \$10,000,000$$
$$= 8,400,000 - 1,600,000i$$

If $i < 525\%$, then $PB(i, 20\%)_1 > 0$.

$$PB(i, 20\%)_2 = (8,400,000 - 1,600,000i)(1+0.20) - \$10,000,000$$
$$= 80,000 - 1,920,000i$$
$$= 0$$
$$i = 4.17\% < 20\%$$

If $i > 525\%$, then $PB(i, 20\%)_1 < 0$, no RIC exists. So, the RIC on the incremental cash flows should be 4.17%, which indicates "Select a smaller pump."

ST 7.4

(a) Whenever you need to compare a set of mutually exclusive projects based on the rate of return criterion, you should perform an incremental analysis. In our example, the incremental cash flows would look like the following:

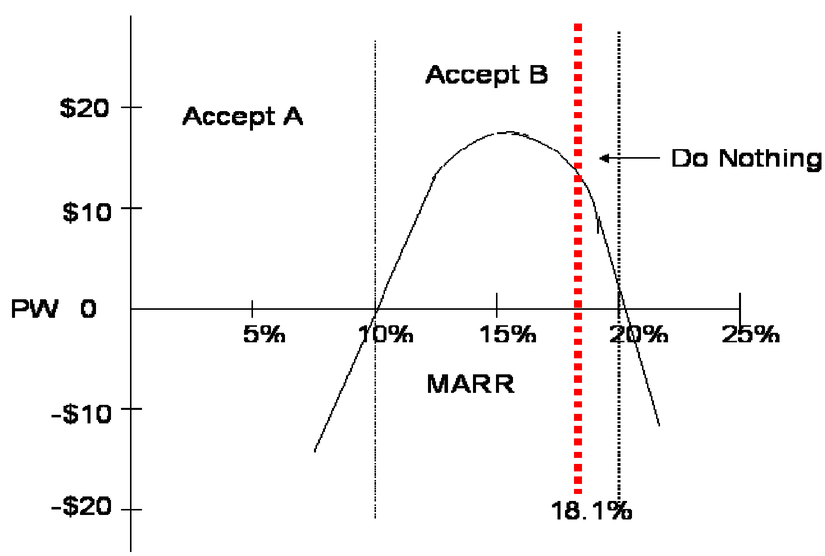
n	B - A
0	-$10,000
1	+23,000
2	-13,200

This is a nonsimple investment with two rates of return.

$$i^*_{B-A} = 10\% \text{ or } 20\%$$

We could abandon the IRR analysis and use either the PW analysis to rank the projects.

(b) If we plot the present worth as a function of interest rate, we will observe the following:



- If MARR < 10% select project A.
- If 10% ≤ MARR ≤ 18.1%, select project B.
- If MARR > 18.1%, do nothing.

Return on Invested Capital: Using the procedure outlined in Section 7.3.4, the true rate of return can be found as a function of MARR.

Let $i = \text{IRR}$ and assume $i < 1.3$

$$\begin{aligned}
PB(i, \text{MARR})_0 &= -\$10,000 \\
PB(i, \text{MARR})_1 &= -\$10,000(1+i) + \$23,000 \\
&= \$13,000 - 10,000i \\
PB(i, \text{MARR})_2 &= (\$13,000 - 10,000i)(1 + \text{MARR}) \\
&\quad -\$13,200 \\
&= 0
\end{aligned}$$

(Note that, if, $i > 1.3,$ there will be no feasible solution.) Rearranging the terms in $PB(i, \text{MARR})_2$ gives an expression of IRR as a function of MARR.

$$\text{IRR} = 1.3 - \frac{1.32}{1 + MARR}$$

For example, at $\text{MARR} = 15\%$

$$\text{IRR}_{B-A} = 15.3\% > 15\%$$

$\therefore$ Project B (the higher cost investment) would be justified.

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