Chapter 3: Interest Rate and Economic Equivalence

Types of Interest

3.1

$$\$10,000 = \$5,000(1+0.08N)$$

- Simple interest: $(1+0.08N) = 2$

$$N = \frac{1}{0.08} = 12.5 \ years$$

$$\$10,000 = \$5,000(1+0.07)^N$$

- Compound interest: $(1+0.07)^N = 2$

$$N = 10.2 \ years$$

3.2

- Simple interest:
  $I = iPN = (0.08)(\$1,000)(5) = \$400$

- Compound interest:
  $I = P[(1+i)^N - 1] = \$1,000(1.4693-1) = \$469$

3.3

- Option 1: Compound interest with 8%:
  $F = \$3,000(1+0.08)^5 = \$3,000(1.4693) = \$4,408$

- Option 2: Simple interest with 9%
  $\$3,000(1+0.09\times 5) = \$3,000(1.45) = \$4,350$

- Option 1 is better

3.4

End of Year	Principal Repayment	Interest payment	Remaining Balance
0			\$10,000
1	\$1,671	\$900	\$8,329
2	\$1,821	\$750	\$6,508
3	\$1,985	\$586	\$4,523
4	\$2,164	\$407	\$2,359
5	\$2,359	\$212	\$0

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Equivalence Concept

3.5

$$P = \$12{,}000(P/F, 5\%, 5) = \$12{,}000(0.7835) = \$9{,}402$$

3.6

$$F = \$20{,}000(F/P, 8\%, 2) = \$20{,}000(1.1664) = \$23{,}328$$

Single Payments (Use of *F/P* or *P/F* Factors)

3.7

(a) $F = \$5{,}000(F/P, 5\%, 8) = \$7{,}388$

(b) $F = \$2{,}250(F/P, 3\%, 12) = \$3{,}208$

(c) $F = \$8{,}000(F/P, 7\%, 31) = \$65{,}161$

(d) $F = \$25{,}000(F/P, 9\%, 7) = \$45{,}700$

3.8

(a) $P = \$5{,}500(P/F, 10\%, 6) = \$3{,}105$

(b) $P = \$8{,}000(P/F, 6\%, 15) = \$3{,}338$

(c) $P = \$30{,}000(P/F, 8\%, 5) = \$20{,}418$

(d) $P = \$15{,}000(P/F, 12\%, 8) = \$3{,}851$

3.9

(a) $P = \$10{,}000(P/F, 13\%, 5) = \$5{,}428$

(b) $F = \$25{,}000(F/P, 13\%, 4) = \$40{,}763$

3.10

$$F = 3P = P(1 + 0.12)^N$$
$$\log 3 = N \log(1.12)$$
$$N = 9.69 \ \textit{years}$$

3.11

$$F = 2P = P(1+0.15)^N$$

- $\log 2 = N \log(1.15)$
  $N = 4.96 \; years$

- Rule of 72: $72/15 = 4.80$ years

Uneven Payment Series

3.12

(a) Single-payment compound amount $(F/P,i,N)$ factors for

n	9%	10%
35	20.4140	28.1024
40	31.4094	45.2593

To find $(F/P,9.5\%,38)$, first, interpolate for $n = 38$

n	9%	10%
38	27.0112	38.3965

Then, interpolate for $i = 9.5\%$

$$(F/P,9.5\%,38) = 32.7039$$

As compared to formula determination

$$(F/P,9.5\%,38) = 31.4584$$

(b) Single-payment compound amount $(P/F,i,N)$ factors for

n	45	50
	0.0313	0.0213

Then, interpolate for $n = 47$

$$(P/F,8\%,47) = 0.0273$$

As compared with the result from formula

$$(P/F,8\%,47) = 0.0269$$

3.13

$$P = \frac{\$32{,}000}{1.08^2} + \frac{\$43{,}000}{1.08^3} + \frac{\$46{,}000}{1.08^4} + \frac{\$28{,}000}{1.08^5} = \$114{,}437$$

3.14

$$F = \$1{,}500(F/P,6\%,15) + \$1{,}800(F/P,6\%,13) + \$2{,}000(F/P,6\%,11) = \$11{,}231$$

3.15

$$P = \$3,000,000 + \$2,400,000(P/F,8\%,1) + \cdots$$
$$+\$3,000,000(P/F,8\%,10)$$
$$= \$20,734,618$$

Or,

$$P = \$3,000,000 + \$2,400,000(P/A,8\%,5)$$
$$+\$3,000,000(P/A,8\%,5)(P/F,8\%,5)$$
$$= \$20,734,618$$

3.16

$$P = \$7,500(P/F,6\%,2) + \$6,000(P/F,6\%,5) + \$5,000(P/F,6\%,7) = \$14,484$$

Equal Payment Series

3.17

(a) With deposits made at the end of each year
$F = \$1,000(F/A,7\%,10) = \$13,816$

(b) With deposits made at the beginning of each year
$F = \$1,000(F/A,7\%,10)(1.07) = \$14,783$

3.18

(a) $F = \$3,000(F/A,7\%,5) = \$16,713$

(b) $F = \$4,000(F/A,8.25\%,12) = \$77,043$

(c) $F = \$5,000(F/A,9.4\%,20) = \$267,575$

(d) $F = \$6,000(F/A,10.75\%,12) = \$134,236$

3.19

(a) $A = \$22,000(A/F,6\%,13) = \$1,166$

(b) $A = \$45,000(A/F,7\%,8) = \$4,388$

(c) $A = \$35,000(A/F,8\%,25) = \$479.5$

(d) $A = \$18{,}000(A/F, 14\%, 8) = \$1{,}361$

3.20

$$\$30{,}000 = \$1{,}500(F/A,\ 7\%,\ N)$$
$$(F/A,\ 7\%,\ N) = 20$$
$$N = 12.94 \approx 13 years$$

3.21

$$\$15{,}000 = A(F/A,\ 11\%,\ 5)$$
$$A = \$2{,}408.56$$

3.22

(a) $A = \$10{,}000(A/P, 5\%, 5) = \$2{,}310$

(b) $A = \$5{,}500(A/P, 9.7\%, 4) = \$1{,}723.70$

(c) $A = \$8{,}500(A/P, 2.5\%, 3) = \$2{,}975.85$

(d) $A = \$30{,}000(A/P, 8.5\%, 20) = \$3{,}171$

3.23

- Equal annual payment:
  $A = \$25{,}000(A/P, 16\%, 3) = \$11{,}132.5$

- Interest payment for the second year:

End of Year	Principal Repayment	Interest payment	Remaining Balance
0			\$25,000
1	\$7,132.5	\$4,000	\$17,867.5
2	\$8,273.7	\$2,858.8	\$9,593.8
3	\$9,593.8	\$1,535	-

3.24

(a) $P = \$800(P/A, 5.8\%, 12) = \$6{,}781.2$

(b) $P = \$2{,}500(P/A, 8.5\%, 10) = \$16{,}403.25$

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(c) $P = \$900(P/A, 7.25\%, 5) = \$3{,}665.61$

(d) $P = \$5{,}500(P/A, 8.75\%, 8) = \$30{,}726.3$

3.25

(a) The capital recovery factor $(A/P, i, N)$ for

n	6%	7%
35	0.0690	0.0772
40	0.0665	0.0750

To find $(A/P, 6.25\%, 38)$, first, interpolate for $n = 38$

n	6%	7%
38	0.0675	0.0759

Then, interpolate for $i = 6.25\%$; $(F/P, 6.25\%, 38) = 0.0696$

As compared with the result from formula

$$(F/P, 6.25\%, 38) = 0.0694$$

(b) The equal payment series present-worth factor $(P/A, i, 85)$ for

i	9%	10%
	11.1038	9.9970

Then, interpolate for $i = 9.25\%$

$$(P/A, 9.25\%, 85) = 10.8271$$

As compared with the result from formula

$$(P/A, 9.25\%, 85) = 10.8049$$

Linear Gradient Series

3.26

$$\begin{aligned} F &= F_1 + F_2 \\ &= \$5{,}000(F/A, 8\%, 5) + \$2{,}000(F/G, 8\%, 5) \\ &= \$5{,}000(F/A, 8\%, 5) + \$2{,}000(A/G, 8\%, 5)(F/A, 8\%, 5) \\ &= \$50{,}988.35 \end{aligned}$$

3.27

$$\begin{aligned} F &= \$3{,}000(F/A, 7\%, 5) - \$500(F/G, 7\%, 5) \\ &= \$3{,}000(F/A, 7\%, 5) - \$500(P/G, 7\%, 5)(F/P, 7\%, 5) \\ &= \$11{,}889.47 \end{aligned}$$

3.28

$$P = \$100 + [\$100(F/A, 9\%, 7) + \$50(F/A, 9\%, 6) + \$50(F/A, 9\%, 4)$$
$$+ \$50(F/A, 9\%, 2)](P/F, 9\%, 7)$$
$$= \$991.26$$

3.29

$$A = \$15{,}000 - \$1{,}000(A/G, 8\%, 12)$$
$$= \$10{,}404.3$$

3.30

(a) $P = \$6{,}000{,}000(P/A_1, -10\%, 12\%, 7) = \$21{,}372{,}076$

(b) Note that the oil price increases at the annual rate of 5% while the oil production decreases at the annual rate of 10%. Therefore, the annual revenue can be expressed as follows:

$$A_n = \$60(1+0.05)^{n-1}1000{,}000(1-0.1)^{n-1}$$
$$= \$6{,}000{,}000(0.945)^{n-1}$$
$$= \$6{,}000{,}000(1-0.055)^{n-1}$$

This revenue series is equivalent to a decreasing geometric gradient series with g = -5.5%.
So, $P = \$6{,}000{,}000(P/A_1, -5.5\%, 12\%, 7) = \$23{,}847{,}896$

(c) Computing the present worth of the remaining series (A_4, A_5, A_6, A_7) at the end of period 3 gives

$$P = \$5{,}063{,}460(P/A_1, -5.5\%, 12\%, 7) = \$14{,}269{,}652$$

3.31

$$P = \sum_{n=1}^{20} A_n (1+i)^{-n}$$
$$= \sum_{n=1}^{20} (2{,}000{,}000) n (1.06)^{n-1} (1.06)^{-n}$$
$$= (2{,}000{,}000 / 1.06) \sum_{n=1}^{20} n\left(\frac{1.06}{1.06}\right)^n$$
$$= \$396{,}226{,}415$$

Note: if $i \neq g$,

$$\sum_{n=1}^{N} nx^n = \frac{x[1-(N+1)x^N + Nx^{N+1}}{(1-x)^2}$$

When $g = 6\%$ and $i = 8\%$,

$$x = \frac{1+g}{1+i} = 0.9815$$

$$P = \frac{\$2,000,000}{1.06}\left[\frac{0.9815[1-(21)(0.6881)+20(0.6756)]}{0.0003}\right]$$

$$= \$334,935,843$$

3.32

(a) The withdrawal series would be

Period	Withdrawal
11	\$5,000
12	\$5,000(1.08)
13	\$5,000(1.08)(1.08)
14	\$5,000(1.08)(1.08)(1.08)
15	\$5,000(1.08)(1.08)(1.08)(1.08)

$$P_{10} = \$5,000(P/A_1, 8\%, 9\%, 5) = \$22,518.78$$

Assuming that each deposit is made at the end of each year, then;

$$\$22,518.78 = A(F/A,\ 9\%,\ 10) = 15.1929A$$
$$A = \$1482.19$$

(b) $P_{10} = \$5,000(P/A_1, 8\%, 6\%, 5) = \$24,491.85$

$$\$24,491.85 = A(F/A, 6\%,\ 10) = 13.1808A$$
$$A = \$1858.15$$

Various Interest Factor Relationships

3.33

(a) $(P/F, 8\%, 67) = (P/F, 8\%, 50)(P/F, 8\%, 17) = (0.0213)(0.2703) = 0.0058$

$(P/F, 8\%, 67) = (1+0.08)^{67} = 0.0058$

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(b) $(A/P,i,N)=\dfrac{i}{1-(P/F,i,N)}$

$$(P/F,8\%,42)=(P/F,8\%,40)(P/F,8\%,2)=0.0394$$

$$(A/P,8\%,42)=\frac{0.08}{1-0.0394}=0.0833$$

$$(A/P,8\%,42)=\frac{0.08(1.08)^{42}}{(1.08)^{42}-1}=0.0833$$

(c) $(P/A,i,N)=\dfrac{1-(P/F,i,N)}{i}=\dfrac{1-(P/F,8\%,100)(P/F,8\%,35)}{0.08}=12.4996$

$$(A/P,8\%,135)=\frac{(1.08)^{135}-1}{0.08(1.08)^{135}}=12.4996$$

3.34

(a)

$$(F/P,i,N)=i(F/A,i,N)+1$$

$$\begin{aligned}(1+i)^N &= i\frac{(1+i)^N-1}{i}+1\\ &=(1+i)^N-1+1\\ &=(1+i)^N\end{aligned}$$

(b)

$$(P/F,i,N)=1-(P/A,i,N)i$$

$$\begin{aligned}(1+i)^{-N} &=1-i\frac{(1+i)^N-1}{i(1+i)^N}\\ &=\frac{(1+i)^N}{(1+i)^N}-\frac{(1+i)^N-1}{(1+i)^N}\\ &=(1+i)^{-N}\end{aligned}$$

(c)

$$(A/F,i,N)=(A/P,i,N)-i$$

$$\begin{aligned}\frac{i}{(1+i)^N-1} &=\frac{i(1+i)^N}{(1+i)^N-1}-i=\frac{i(1+i)^N}{(1+i)^N-1}-\frac{i[(1+i)^N-1]}{(1+i)^N-1}\\ &=\frac{i}{(1+i)^N-1}\end{aligned}$$

(d)

$$(A/P,i,N) = \frac{i}{[1-(P/F,i,N)]}$$

$$\frac{i(1+i)^N}{(1+i)^N - 1} = \frac{i}{\dfrac{(1+i)^N}{(1+i)^N} - \dfrac{1}{(1+i)^N}}$$

$$= \frac{i(1+i)^N}{(1+i)^N - 1}$$

(e) (f) & (g) Divide the numerator and denominator by $(1+i)^N$ and take the limit $N \to \infty$.

Equivalence Calculations

3.35

$$P = [\$100(F/A,12\%,9) + \$50(F/A,12\%,7) + \$50(F/A,12\%,5)](P/F,12\%,10)$$
$$= \$740.49$$

3.36

$$P(1.08) + \$200 = \$200(P/F,8\%,1) + \$120(P/F,8\%,2) + \$120(P/F,8\%,3)$$
$$+ \$300(P/F,8\%,4)$$
$$P = \$373.92$$

3.37 Selecting the base period at n = 0, we find

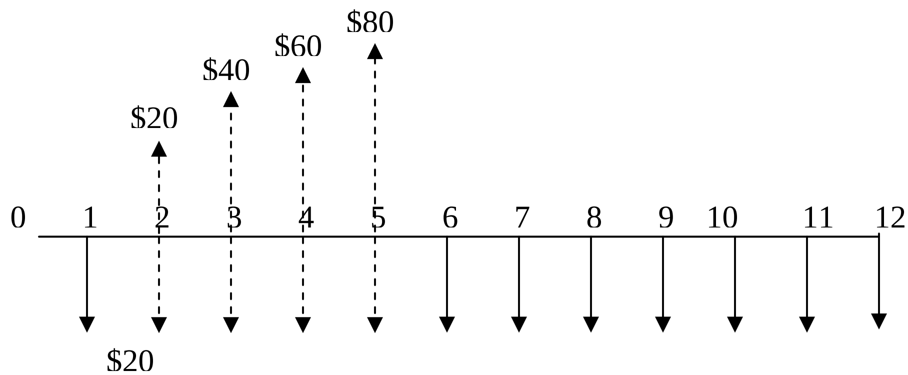
$$\$100(P/A,13\%,5) + \$20(P/A,13\%,3)(P/F,13\%,2) = A(P/A,13\%,5)$$
$$\$351.72 + \$36.98 = (3.5172)A$$
$$A = \$110.51$$

3.38 Selecting the base period at n =0, we find

$$P_1 = \$200 + \$100(P/A,6\%,5) + \$50(P/F,6\%,1) + \$50(P/F,6\%,4) + \$100(P/F,6\%,5)$$
$$= \$782.75$$
$$P_2 = X(P/A,6\%,5) = \$782.75$$
$$X = \$185.82$$

3.39

$$P = \$20(P/G,10\%,5) - \$20(P/A,10\%,12)$$
$$= \$0.96$$



3.40 Establish economic equivalent at $n = 8$:

$$C(F/A,8\%,8) - C(F/A,8\%,2)(F/P,8\%,3) = \$6{,}000(P/A,8\%,2)$$
$$10.6366C - (2.08)(1.2597)C = \$6{,}000(1.7833)$$
$$8.0164C = \$10{,}699.80$$
$$C = \$1{,}334.73$$

3.41 The original cash flow series is

n	A_n	n	A_n
0	0	6	\$900
1	\$800	7	\$920
2	\$820	8	\$300
3	\$840	9	\$300
4	\$860	10	\$300 − \$500
5	\$880		

3.42 Establishing equivalence at n = 8, we find

$$\$300(F/A,10\%,8) + \$200(F/A,10\%,3) = 2C(F/P,10\%,8) + C(F/A,10\%,7)$$
$$\$4{,}092.77 = 2C(2.1436) + C(9.4872)$$
$$C = \$297.13$$

3.43 Establishing equivalence at $n = 5$

$$\begin{aligned}\$200(F/A,8\%,5) - \$50(F/P,8\%,1) \\ &= X(F/A,8\%,5) - (\$200 + X)[(F/P,8\%,2) + (F/P,8\%,1)] \\ \$1,119.32 &= X(5.8666) - (\$200 + X)(2.2464) \\ X &= \$185.09\end{aligned}$$

3.44 Computing equivalence at $n = 5$

$$X = \$3,000(F/A,9\%,5) + \$3,000(P/A,9\%,5) = \$29,623.2$$

3.45 (2), (4), and (6)

3.46 (2), (4), and (5)

3.47

$$A_1 = (\$50 + \$50(A/G,10\%,5) - [\$50 + \$50(P/F,10\%,1)](A/P,10\%,5) = \$115.32$$
$$A_2 = A + A(A/P,10\%,5) = 1.2638A$$
$$A = \$91.25$$

3.48 (a)

3.49 (b)

3.50 (b)

$$\begin{aligned}\$25,000 + \$30,000(P/F,10\%,6) \\ &= C(P/A,10\%,12) + \$1,000(P/A,10\%,6)(P/F,10\%,6)] \\ \$41,935 &= 6.8137C + \$2,458.60 \\ C &= \$5,793\end{aligned}$$

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Solving for an Unknown Interest Rate of Unknown Interest Periods

3.51

$$2P = P(1+i)^5$$
$$\log 2 = 5 \quad \log \ (1+i)$$
$$i = 14.87\%$$

3.52 Establishing equivalence at $n = 0$

$$\$2{,}000(P/A, i, 6) = \$2{,}500(P/A_1, -25\%, i, 6)$$

Solving for I with Excel, we obtain $i = 92.35\%$

3.53

$$\$35{,}000 = \$10{,}000(F/P, i, 5) = \$10{,}000(1+i)^5$$
$$i = 28.47\%$$

3.54

$$\$1{,}000{,}000 = \$2{,}000(F/A, 6\%, N)$$
$$500 = \frac{(1+0.06)^N - 1}{0.06}$$
$$31 = (1+0.06)^N$$
$$\log 30 = N \log 1.06$$
$$N = 58.37 \text{ years}$$

Short Case Studies

ST 3.1 Assuming that they are paid at the beginning of each year

(a)

$$\$15.96 + \$15.96(P/A, 6\%, 3) = \$58.62$$

It is better to take the offer because of lower cost to renew.

(b)

$$\$57.12 = \$15.96 + \$15.96(P/A, i, 3)$$
$$i = 7.96\%$$

ST 3.2 The equivalent future worth of the prize payment series at the end of Year 20 (or beginning of Year 21) is

$$F_1 = \$1,952,381(F/A, 6\%, 20)$$
$$= \$71,819,490$$

The equivalent future worth of the lottery receipts is

$$F_2 = (\$36,100,000 - \$1,952,381)(F/P, 6\%, 20)$$
$$= \$109,516,040$$

The resulting surplus at the end of Year 20 is

$$F_2 - F_1 = \$109,516,040 - \$71,819,490$$
$$= \$37,696,550$$

ST 3.3

(a) Compute the equivalent present worth (in 2006) for each option at $i = 6\%$.

$$P_{\text{Deferred}} = \$2,000,000 + \$566,000(P/F, 6\%, 1) + \$920,000(P/F, 6\%, 2) + \cdots$$
$$+\$1,260,000(P/F, 6\%, 11)$$
$$= \$8,574,490$$

$$P_{\text{Non-Deferred}} = \$2,000,000 + \$900,000(P/F, 6\%, 1) + \$1,000,000(P/F, 6\%, 2) + \cdots$$
$$+\$1,950,000(P/F, 6\%, 5)$$
$$= \$7,431,560$$

$\therefore$ At $i = 6\%$, the deferred plan is a better choice.

(b) Using either Excel or Cash Flow Analyzer, both plans would be economically equivalent at $i = 15.72\%$

ST 3.4 The maximum amount to invest in the prevention program is

$$P = \$14,000(P/A, 12\%, 5) = \$50,467$$

ST 3.5 Using the geometric gradient series present worth factor, we can establish the equivalence between the loan amount \$120,000 and the balloon payment series as

1) $$\$120,000 = A_1(P/A_1,10\%,9\%,5) = 4.6721A_1$$
$$A_1 = \$25,684.38$$

2) Payment series

n	Payment
1	\$25,684.38
2	\$28,252.82
3	\$31,078.10
4	\$34,185.91
5	\$37,604.50

ST 3.6

1) Compute the required annual net cash profit to pay off the investment and interest.

$$\$70,000,000 = A(P/A,10\%,5) = 3.7908A$$
$$A = \$18,465,759$$

2) Decide the number of shoes, X

$$\$18,465,759 = X(\$100)$$
$$X = 184,657$$

ST 3.7

$$\$1,000(P/F,9.4\%,5) + \$500(F/A,9.4\%,5) = \$4,583.36$$
$$\$4,583.36(F/P,9.4\%,60) = \$1,005,132$$

The main question is whether or not the U.S. government will be able to invest the social security deposits at 9.4% interest over 60 years.

ST 3.8

(a)

$$\begin{aligned} P_{\text{Contract}} &= \$3,875,000 + \$3,125,000(P/F,6\%,1) \\ &\quad + \$5,525,000(P/F,6\%,2) + \ldots \\ &\quad + \$8,875,000(P/F,6\%,7) \\ &= \$39,547,242 \end{aligned}$$

(b)

$$\begin{aligned} P_{\text{Bonus}} &= \$1,375,000 + \$1,375,000(P/A,6\%,7) \\ &= \$9,050,775 > \$8,000,000 \end{aligned}$$

Stay with the original deferred plan.

ST 3.9

Basically we are establishing an economic equivalence between two payment options. Selecting n = 0 as the base period, we can calculate the equivalent present worth for each option as follows:

$$\text{Option 1: } P = \$140,000$$
$$\text{Option 2: } P = \$32,639(P/A, i\%, 9)$$

Or,

$$\begin{aligned} \$140,000 &= \$32,639(P/A, i\%, 9) \\ i &= 18.10\% \end{aligned}$$

If Mrs. Setchfield can invest her money at a rate higher than 18.10%, it is better to go with Option 1. However, it may be difficult for her to find an investment opportunity that provides a return exceeding 18%.