Data Mining and Data Warehousing

Chapter 4

Classification and Prediction

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What is classification?



- is a data mining technique used to predict the category of categorical data by building a model based on some predictor variables (to classify data).
- Predictor variable/attribute is called class label attribute (predefined class)



What is classification?



It is a two-step process

- 1. Model Construction (learning step or training phase)
 - build a model to explain the target concept
 - model is represented as classification rules, decision trees, or mathematical formulae.

2. Model Usage

- is used for classifying future or unknown cases

estimate the accuracy of the model



Example

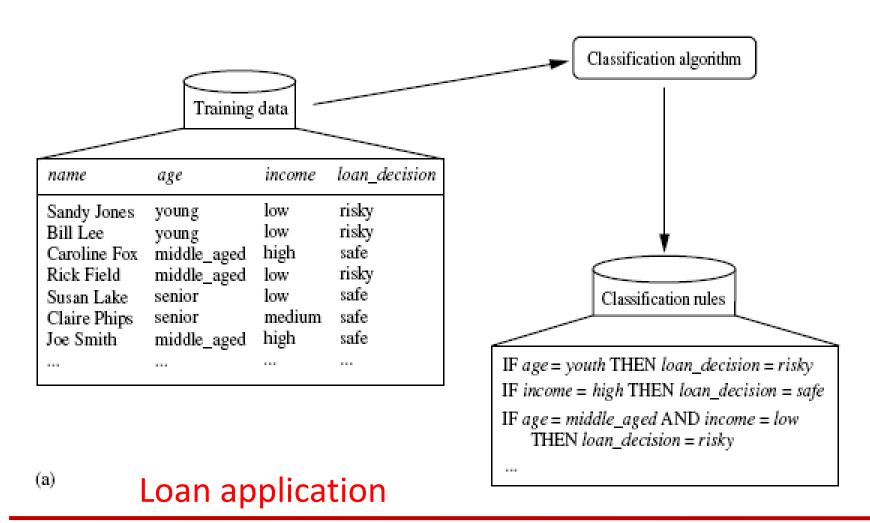


RID	age	income	student	credit_rating	Class: buys_computer
1	youth	high	no	fair	no
2	youth	high	no	excellent	no
3	middle_aged	high	no	fair	yes
4	senior	medium	no	fair	yes
5	senior	low	yes	fair	yes
6	senior	low	yes	excellent	no
7	middle_aged	low	yes	excellent	yes
8	youth	medium	no	fair	no
9	youth	low	yes	fair	yes
10	senior	medium	yes	fair	yes
11	youth	medium	yes	excellent	yes
12	middle_aged	medium	no	excellent	yes
13	middle_aged	high	yes	fair	yes
14	senior	medium	no	excellent	no



Step 1

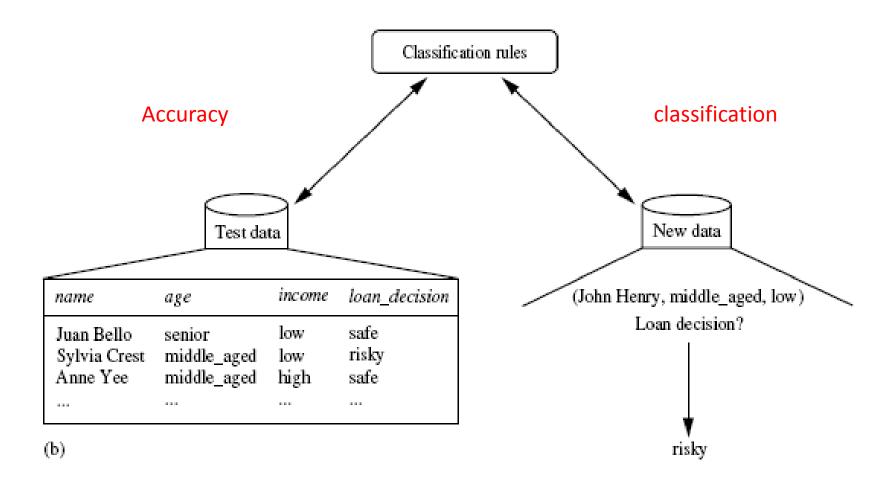






Step 2





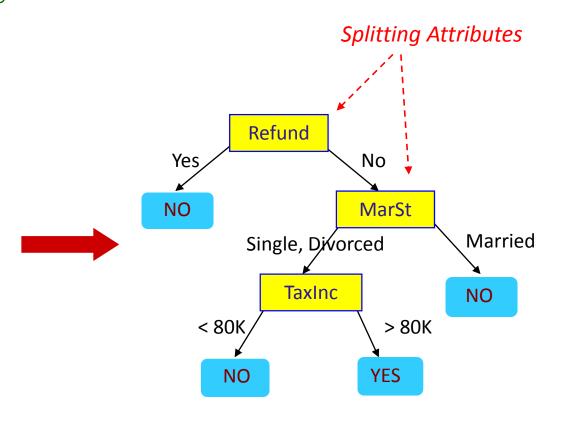


Example of a Decision Tree



categorical continuous

	C	O.	C	O
Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



Training Data

Model: Decision Tree

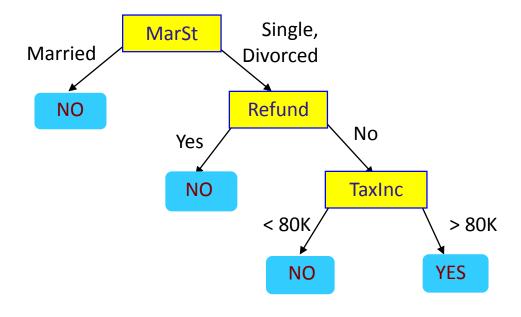


Another Example of Decision Tree



categorical continuous

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



There could be more than one tree that fits the same data!



Decision Tree Classification Task

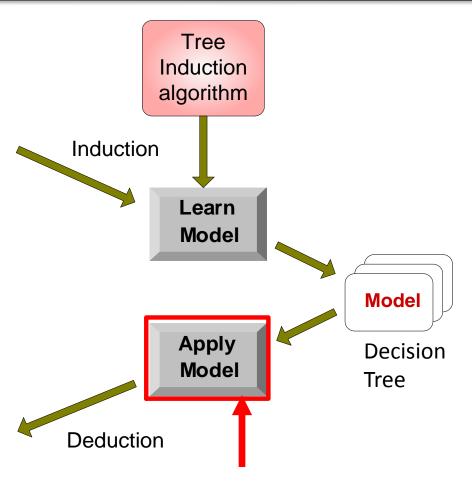


Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	No
2	No	Medium	100K	No
3	No	Small	70K	No
4	Yes	Medium	120K	No
5	No	Large	95K	Yes
6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
9	No	Medium	75K	No
10	No	Small	90K	Yes

Training Set

Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	?
12	Yes	Medium	80K	?
13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

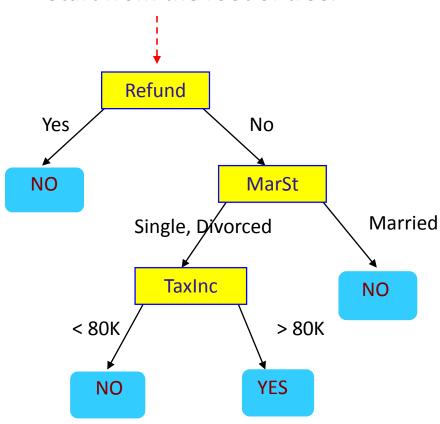
Test Set







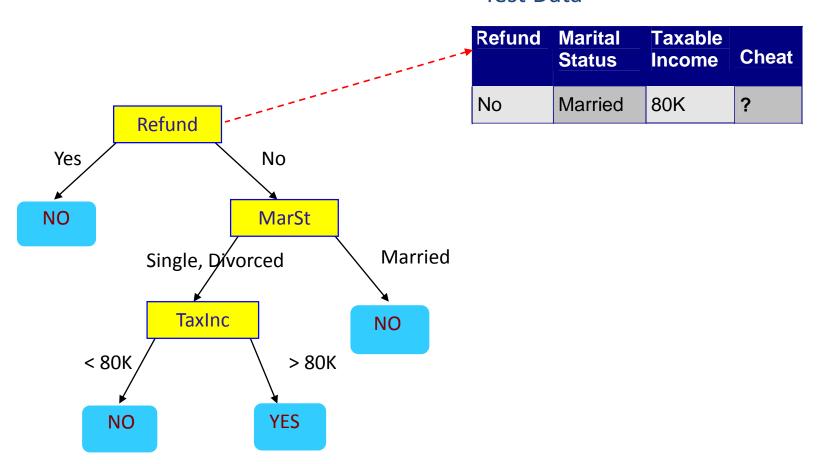
Start from the root of tree.



Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?

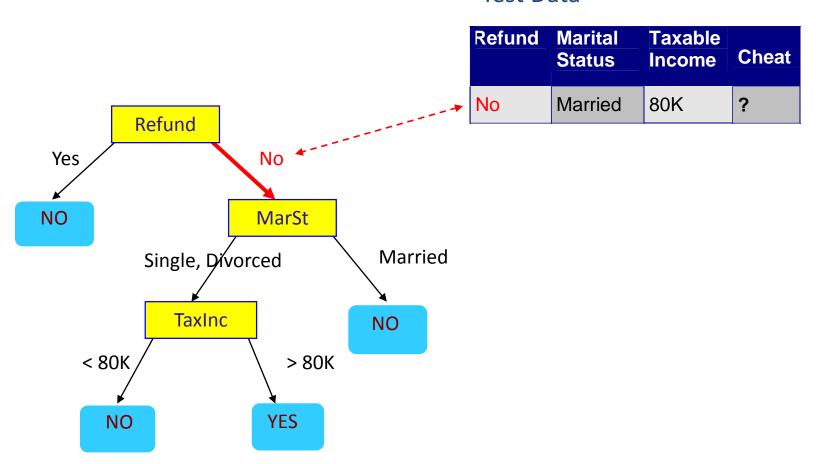






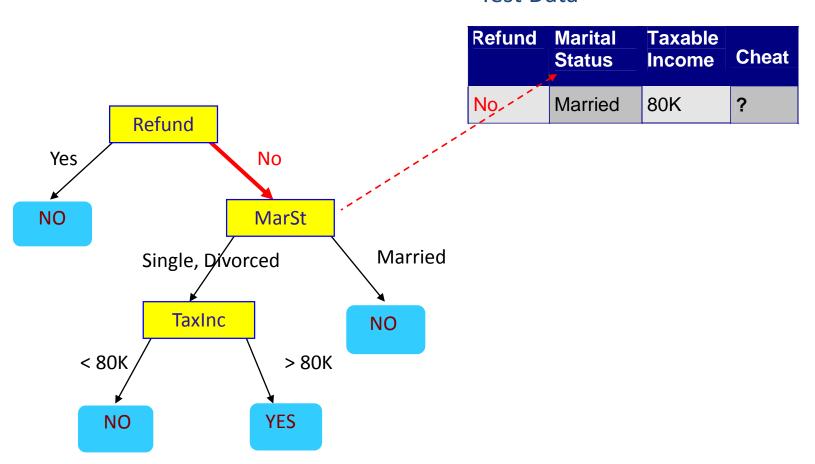








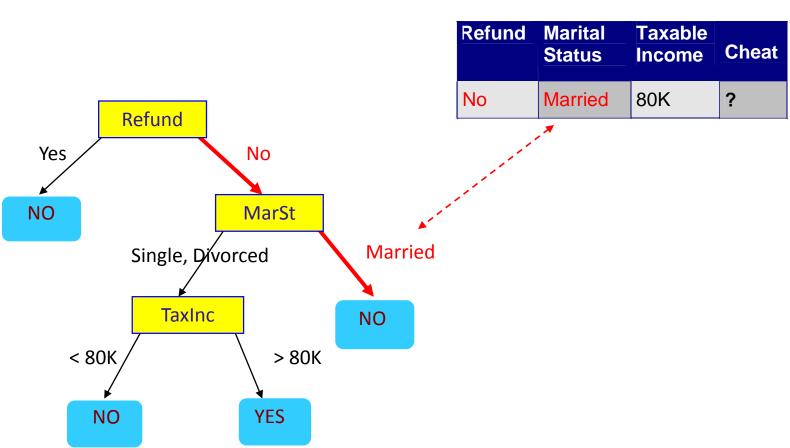






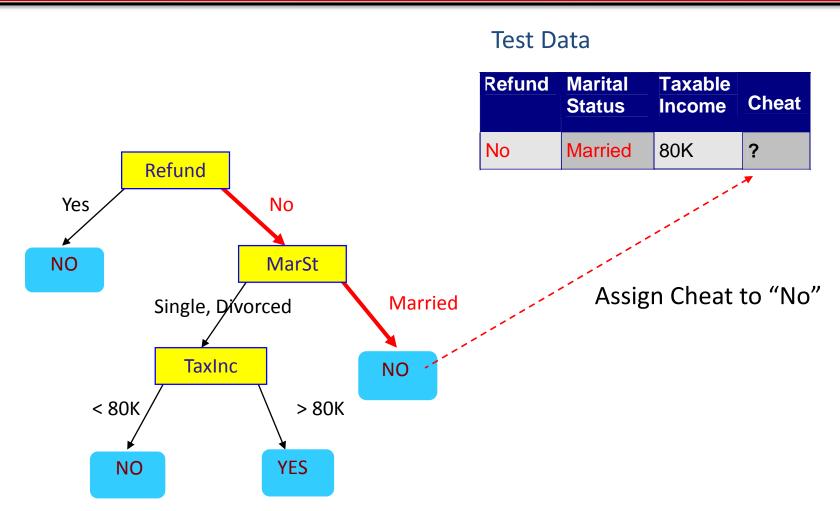








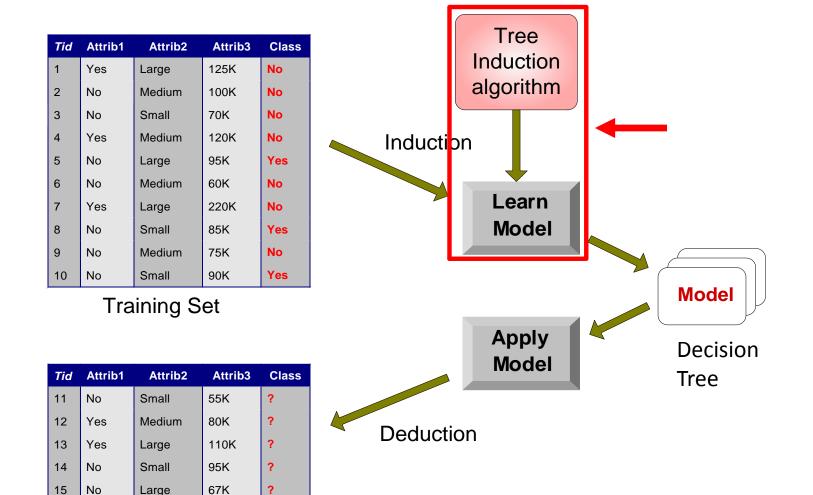






Decision Tree Classification Task





Test Set



What is Prediction?



- Models continuous-valued functions, i.e. predicts unknown or missing values (numeric)
- Lost terminology of "class label attribute", instead we use "predicted attribute"
- Viewed as a mapping or function y = f(X)
- Example: predict the amount (in dollars) that would be safe for the bank to loan an application



Supervised & Unsupervised Learning



- Supervised Learning (Classification)
 - Supervision: The training data are accompanied by labels indicating the class of the observations
 - New data is classified based on the training set
- Unsupervised Learning (Clustering)
 - The class labels of training data is unknown
 - Given a set of measurements, observations, etc. with the aim of establishing the existence of classes or clusters in the data



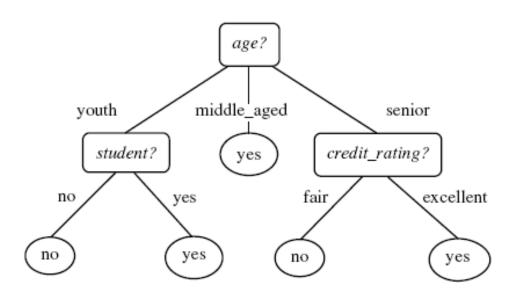
Classification by decision tree induction



What is decision tree?

- flow chart like tree structure
- internal node denotes a test on an attribute
- each branch represents an outcome of the test
- each leaf node holds a class label

Decision tree for the concept buys computer





Why decision tree?



- Construction does not require any domain knowledge
- Can handle high dimensional data
- Learning step is simple and fast
- In general have good accuracy
- People are able to understand decision tree models after a brief explanation



Attribute selection measures



Table 6.1 Class-labeled training tuples from the *AllElectronics* customer database.

RID	age	income	student	credit_rating	Class: buys_computer
1	youth	high	no	fair	no
2	youth	high	no	excellent	no
3	middle_aged	high	no	fair	yes
4	senior	medium	no	fair	yes
5	senior	low	yes	fair	yes
6	senior	low	yes	excellent	no
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8	youth	medium	no	fair	no
9	youth	low	yes	fair	yes
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11	youth	medium	yes	excellent	yes
12	middle_aged	medium	no	excellent	yes
13	middle_aged	high	yes	fair	yes
14	senior	medium	no	excellent	no

Information Gain, gain ratio and gini index



Attribute selection measures



Information Gain

the expected information needed to classify a tuple in *D* : (entropy of *D*)

$$Info(D) = -\sum_{i=1}^{m} p_i \log_2(p_i),$$

$$> Pi = /C$$

 \triangleright A having ν distinct values, $\{a_1, a_2, ..., a_{\nu}\}$

$$\blacktriangleright$$
 $D_1,\,D_2,\,...,\,D_v$
$$\mathit{Info}_A(D) = \sum_{j=1}^v \frac{|D_j|}{|D|} \times \mathit{Info}(D_j).$$



Attribute selection measures



$$Gain(A) = Info(D) - Info_A(D)$$
. (Choose maximum value)

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11	youth	medium	yes	excellent	yes
12	middle_aged	medium	no	excellent	yes
13	middle_aged	high	yes	fair	yes
14	senior	medium	no	excellent	no



Example A is discrete-valued



$$Info(D) = -\frac{9}{14}\log_2\left(\frac{9}{14}\right) - \frac{5}{14}\log_2\left(\frac{5}{14}\right) = 0.940 \text{ bits.}$$

$$\begin{split} Info_{age}(D) &= \frac{5}{14} \times (-\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5}) \\ &+ \frac{4}{14} \times (-\frac{4}{4} \log_2 \frac{4}{4} - \frac{0}{4} \log_2 \frac{0}{4}) \\ &+ \frac{5}{14} \times (-\frac{3}{5} \log_2 \frac{3}{5} - \frac{2}{5} \log_2 \frac{2}{5}) \\ &= 0.694 \text{ bits.} \end{split}$$

$$Gain(age) = Info(D) - Info_{age}(D) = 0.940 - 0.694 = 0.246 \text{ bits.}$$

Gain(income)=0.029, Gain(student)=0.151, Gain(credit_rating)=0.048







age?

youth

middle_aged senior

income	student	credit_rating	class
high high medium low medium	no no no yes yes	fair excellent fair fair excellent	no no no yes yes

income	student	credit_rating	class
medium low low medium medium	no yes yes yes no	fair fair excellent fair excellent	yes yes no yes no

income	student	credit_rating	class	
high low medium high	no yes no yes	fair excellent excellent fair	yes yes yes yes	



Tree Pruning



- When a decision tree is built, many of the branches will reflect anomalies in the training data due to noise or outliers. Tree pruning methods address this problem of *overfitting* the data.
- There are two common approaches for tree pruning
 - Pre-pruning (early stopping rule):
 - The tree is "pruned" by halting its construction early
 - Or stop algorithm before it becomes a fully grown tree
 - Most popular test: chi-squared test



Tree Pruning



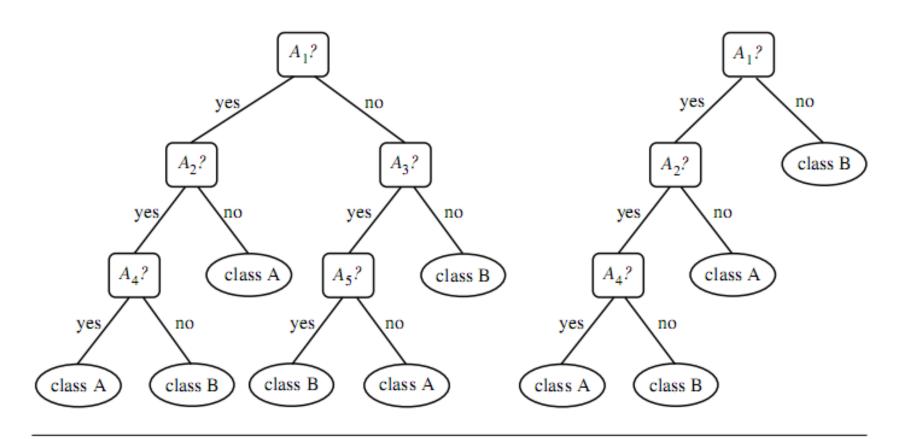
Post-pruning

- removes sub-trees from a "fully grown" tree: get a sequence of progressively pruned trees
- Possible strategies: error estimation, significance testing, MDL pruning
- preferred in practice



Example..





An unpruned decision tree and a pruned version of it.



Rule Based Classification



- has learned model as a set of IF-THEN rules
- We need to
 - How to generate the model
 - Examine how to use model to classify data
- IF-THEN rule is an expression of the form
 IF condition THEN conclusion

Ex:

R1:IF age=youth and student=yes THEN buys_computer=yes
Or

R1: (age=youth)Λ(student=yes)=>(buys_computer=yes)



Using IF-THEN rules for classification (1/7)



- IF part is called rule antecedent or precondition, it can consist of one or more attributes test
- THEN part is called rule consequent, it consist a class prediction
- A rule R can be assessed by its coverage and accuracy
 - Given a tuple X from a data D
 - Let n_{cover}: # of tuples covered by R
 - n_{correct}: # of tuples correctly classify by R
 - |D|: # of tuples in D

$$coverage(R) = \frac{n_{covers}}{|D|}$$



$$accuracy(R) = \frac{n_{correct}}{n_{covers}}.$$



Using IF-THEN rules for classification (2/7)



Ex: of assessing R

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12	middle_aged	medium	no	excellent	yes
13	middle_aged	high	yes	fair	yes
14	senior	medium	no	excellent	no



Using IF-THEN rules for classification (3/7)



R: IF age=youth AND student=yes THEN buys_computer=yes

$$n_{cover} = 2$$

$$n_{correct} = 2$$

$$cov erage(R) = \frac{2}{14} = 14.28\%$$

$$accuracy(R) = \frac{2}{2} = 100\%$$



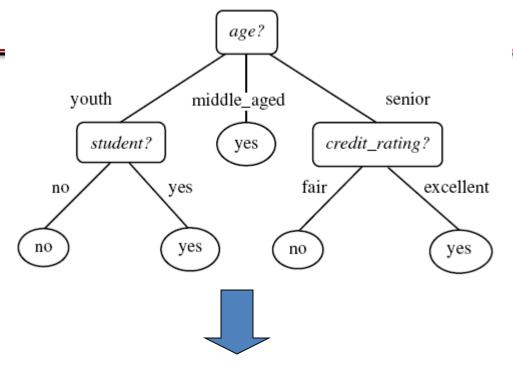
Rule Extraction from a Decision Tree



- One rule is created for each path from the root to a leaf node
- Each splitting criterion is logically AND to form the rule antecedent (IF part)
- Leaf node holds the class prediction for rule consequent (THEN part)
- Logical OR is implied between each of the extracted rules







*R*1: IF age = youth AND student = no

R2: IF age = youth AND student = yes

 $R3: IF age = middle_aged$

R4: IF age = senior AND $credit_rating = excellent$ THEN $buys_computer = yes$

R5: IF age = senior AND $credit_rating = fair$ THEN $buys_computer = no$

THEN $buys_computer = no$

THEN $buys_computer = yes$

THEN $buys_computer = yes$



Bayesian classification



- are statistical classifiers
- based on Baye's theorem
- Simple Bayesian classifier called naïve Bayesian classifier
- the effect of an attribute value on a given class is independent of the values of the other attributes: this assumption is called class conditional independence



Baye's theorem



- Is name after Thomas Bayes, nonconformist English clergyman who did early work in probability and decision theory during the 18th century
- Let X: a data tuple, evidence, measure on n attributes.

H: some hypothesis such as that $X \in class C$

⇒We want to calculate

P(H|X): prob that hypothesis H holds given evidence X or prob that $X \in C$



Baye's theorem



 P(H|X): is a posterior prob or posteriori prob condition on X

Ex: X=(age=35, income=\$40,000)

H: hypothesis that our customer will buy computer

=> P(H|X): prob that customer X will buy computer giving that we know their age and income



Baye's theorem



Baye's theorem

$$P(H \mid X) = \frac{P(X \mid H)P(H)}{P(X)}$$



Naïve Bayesian Classification



- Works as follow
- From data set D
 - Associated class label
 - n dimensional att vector X=(x1,x2,x3,...,xn), depiction n measurement made on the tuple from n atts. A1, A2, A3,..., An
- 2. Suppose we have m classes c1, c2, ...,cm Giving tuple X, classifier will predict X belong to highest posterior probability, condition on X.

$$X \in Ci \text{ iif } P(Ci|X) > P(Cj|X) \text{ for } 1 \le j \le m, j \in A$$

Ci, for which P(Ci|X) is maximized is called maximum posterior hypothesis; $P(C_i \mid X) = \frac{P(X \mid C_i)P(Ci)}{P(X)}$



Naïve Bayesian Classification



- 3. P(X) is constant for all classes
- \Rightarrow Maximize $P(X|C_i)P(C_i)$

If class prior prob are not know, commonly assumed that $P(C_1)=P(C_2)=...=P(C_m)$

=>maximize P(X|C_i)

Else maximize P(X | Ci)P(Ci)

$$P(Ci) = \frac{|C_{i,D}|}{|D|}$$



Naïve Bayesian Classification



4. Calculate P(X|Ci) is extremely expensive Naïve assumes class conditional independence is made.

=>

$$P(X \mid Ci) = \prod_{k=1}^{n} (x_k \mid Ci)$$

= $P(x_1 \mid Ci).P(x_2 \mid Ci)...P(x_n \mid Ci)$

Where x_k is the value of att. A_k for X If A is category

$$=> P(x_k \mid C_i) = \frac{\#of_tuple_of_class_Ci_inD_have_value_Xk}{\mid C_{i,D}\mid}$$



Example



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- A={age, income, student, credit_rating}
- Class label buy_computer = Yes | No
- C1: buy_computer = Yes
- C2: buy_computer = No
- X=(age=youth, income=medium, student=y, creditrating=fair)
- We need to maximize P(X | Ci)P(Ci)





$$P(buys_computer = yes) = 9/14 = 0.643$$

$$P(buys_computer = no) = 5/14 = 0.357$$

To compute $PX|C_i$), for i = 1, 2, we compute the following conditional probabilities:

$$P(age = youth \mid buys_computer = yes)$$
 = $2/9 = 0.222$

$$P(age = youth \mid buys_computer = no)$$
 = 3/5 = 0.600

$$P(income = medium \mid buys_computer = yes) = 4/9 = 0.444$$

$$P(income = medium \mid buys_computer = no) = 2/5 = 0.400$$

$$P(student = yes \mid buys_computer = yes) = 6/9 = 0.667$$

$$P(student = yes \mid buys_computer = no)$$
 = $1/5 = 0.200$

$$P(credit_rating = fair \mid buys_computer = yes) = 6/9 = 0.667$$

$$P(credit_rating = fair \mid buys_computer = no) = 2/5 = 0.400$$



Using the above probabilities, we obtain

$$P(X|buys_computer = yes) = P(age = youth \mid buys_computer = yes) \times \\ P(income = medium \mid buys_computer = yes) \times \\ P(student = yes \mid buys_computer = yes) \times \\ P(credit_rating = fair \mid buys_computer = yes) \\ = 0.222 \times 0.444 \times 0.667 \times 0.667 = 0.044.$$

Similarly,

$$P(X|buys_computer = no) = 0.600 \times 0.400 \times 0.200 \times 0.400 = 0.019.$$

To find the class, C_i , that maximizes $P(X|C_i)P(C_i)$, we compute

$$P(X|buys_computer = yes)P(buys_computer = yes) = 0.044 \times 0.643 = 0.028$$

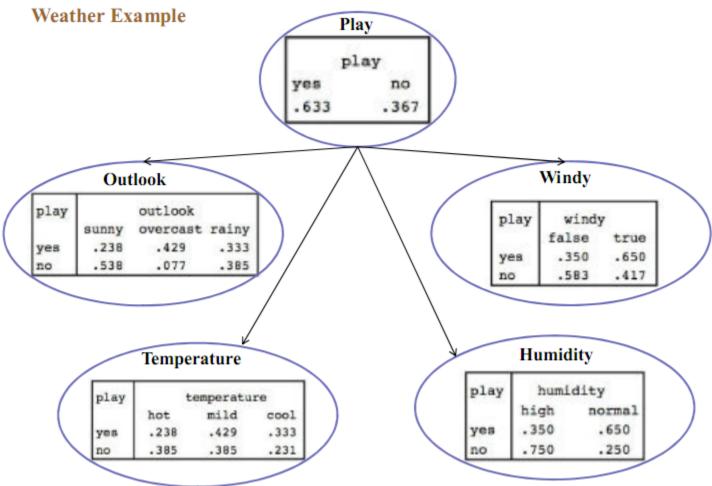
$$P(X|buys_computer = no)P(buys_computer = no) = 0.019 \times 0.357 = 0.007$$

Therefore, the naïve Bayesian classifier predicts $buys_computer = yes$ for tuple X.











Example



Suppose given $X=\{\text{outlook=rainy, temperature=cool, humidity=high, and windy=true}\}$, need to find out which play value (yes or no) should be assigned to X

```
Compare p(\text{play=no}|X) and p(\text{play=yes}|X)

p(\text{play=no}|X) = p(\text{play=no and }X)/p(X), p(\text{play=yes}|X) = p(\text{play=yes and }X)/p(X)

p(\text{play=no and }X) = p(\text{play=no}) \times p(\text{outlook=rainy}|\text{play=no}) \times p(\text{temperature=cool}|\text{play=no}) \times p(\text{humidity=high}|\text{play=no}) \times p(\text{windy=true}|\text{play=no}) = 0.367 \times 0.385 \times 0.231 \times 0.750 \times 0.417 = 0.010

p(\text{play=yes and }X) = p(\text{play=yes}) \times p(\text{outlook=rainy}|\text{play=yes}) \times p(\text{temperature=cool}|\text{play=yes}) \times p(\text{humidity=high}|\text{play=yes}) \times p(\text{windy=true}|\text{play=yes}) = 0.633 \times 0.333 \times 0.333 \times 0.350 \times 0.650 = 0.016

p(\text{play=yes and }X) > p(\text{play=no and }X), so assign X to play value of "yes"
```

Advantages and Disadvantages Naïve Bayesian Classification

Advantages:

- Easy to implement
- Obtain good results in most of the cases

Disadvantages

- Assumption of class conditional independence usually doesn't hold
- Dependencies among these cannot be modeled by this classifier

How to deal with these Dependencies?

Bayesian Belief Networks



Bayesian networks



- A simple, graphical notation for conditional independence assertions and hence for compact specification of full joint distributions
- Syntax:
 - a set of nodes, one per variable

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- a directed, acyclic graph (link ≈ "directly influences")
- a conditional distribution for each node given its parents:

 $P(X_i \mid Parents(X_i))$

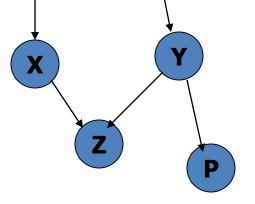
• In the simplest case, conditional distribution represented as a **conditional probability table** (CPT) giving the distribution over X_i for each combination of parent values



Bayesian Networks



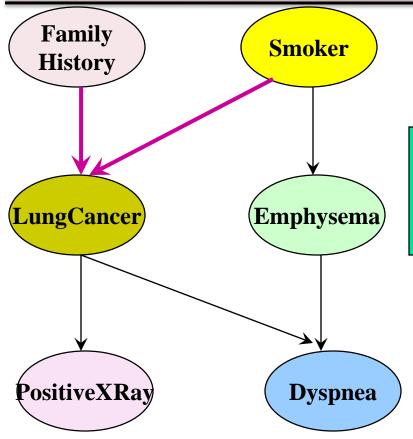
- Bayesian belief network allows a subset of the variables conditionally independent
- A graphical model of causal relationships
 - Represents dependency among the variables
 - Gives a specification of joint probability distribution



- ■Nodes: random variables
- □Links: dependency
- $\square X, Y$ are the parents of Z, and Y is the
- parent of P
- ■No dependency between Z and P
- □ Has no loops or cycles



Bayesian Belief Network: An Example



(FH, S) $(FH, \sim S)$ $(\sim FH, S)$ $(\sim FH, \sim S)$	(FH, S)	$(FH, \sim S)$	(~FH, S)	(~FH,	~ S
---	---------	----------------	----------	-------	------------

LC	0.8	0.5	0.7	0.1
~LC	0.2	0.5	0.3	0.9

The **conditional probability table (CPT)** for the variable LungCancer:

Shows the conditional probability for each possible combination of its parents

Bayesian Belief Networks

$$P(z1,...,zn) = \prod_{i=1}^{n} P(z_i | Parents(Z_i))$$



Example

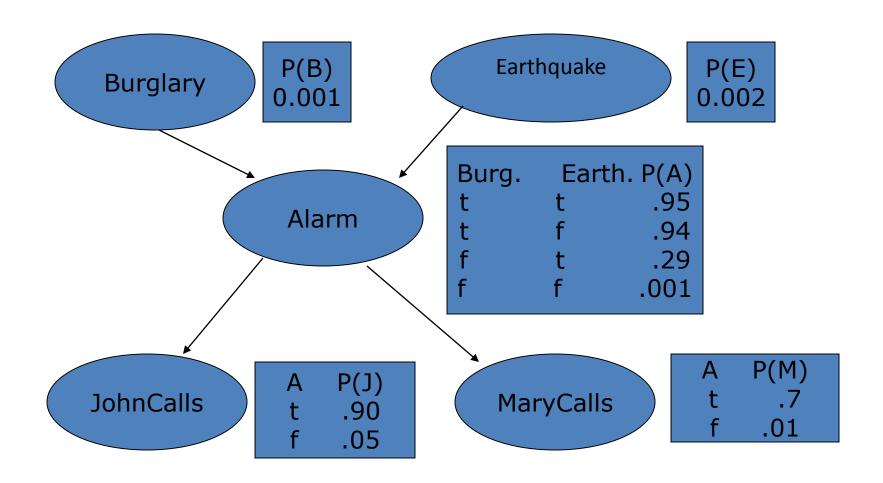


- I'm at work, neighbor John calls to say my alarm is ringing, but neighbor Mary doesn't call. Sometimes it's set off by minor earthquakes. Is there a burglar?
- Variables: Burglary, Earthquake, Alarm, JohnCalls, MaryCalls
- Network topology reflects "causal" knowledge:
 - A burglar can set the alarm off
 - An earthquake can set the alarm off
 - The alarm can cause Mary to call
 - The alarm can cause John to call



Belief Networks









Case Based Reasoning



Faced this situation before?



- Oops the car stopped.
 - What could have gone wrong?
- Aah.. Last time it happened, there was no petrol.
 - Is there petrol?
 - Yes.
 - Oh but wait I remember the tyre was punctured
- This is the normal thought process of a human when faced with a problem which is similar to a problem he/she had faced before.



How do we solve problems?



- By knowing the steps to apply
 - from symptoms to a plausible diagnosis
- How does an expert solve problems?
 - uses same "book learning" as a novice
 - but quickly selects the right knowledge to apply
- Heuristic knowledge ("rules of thumb")
 - "I don't know why this works but it does and so I'll use it again!"



So what?



- Reuse the solution experience when faced with a similar problem.
- This is Case Based Reasoning (CBR)!
 - memory-based problem-solving
 - re-using past experiences
- Experts often find it easier to relate stories about past cases than to formulate rules



What's CBR?



- To solve a new problem by remembering a previous similar situation and by reusing information and knowledge of that situation
- Ex: Medicine
 - doctor remembers previous patients especially for rare combinations of symptoms
- Ex: Law
 - case histories are consulted
- Ex: Management
 - decisions are often based on past rulings
- Ex: Financial
 - performance is predicted by past results



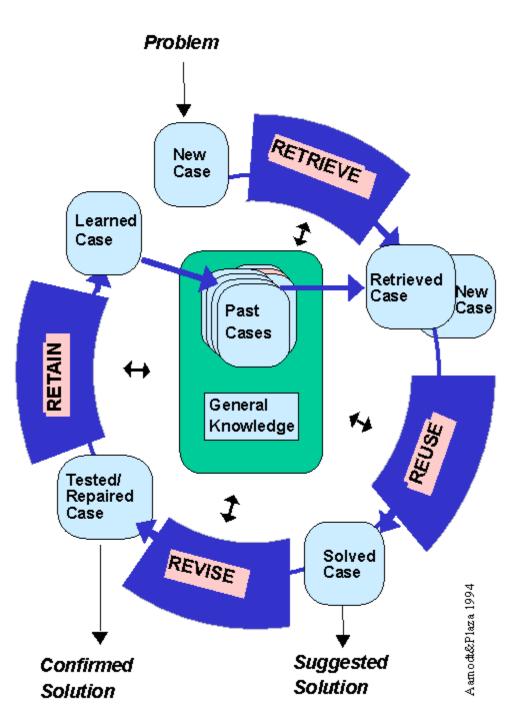
Definitions of CBR



- Case-based reasoning is [...] reasoning by remembering - Leake, 1996
- A case-based reasoner solves new problems by adapting solutions that were used to solve old problems - Riesbeck & Schank, 1989
- Case-based reasoning is a recent approach to problem solving and learning [...] - Aamodt & Plaza, 1994





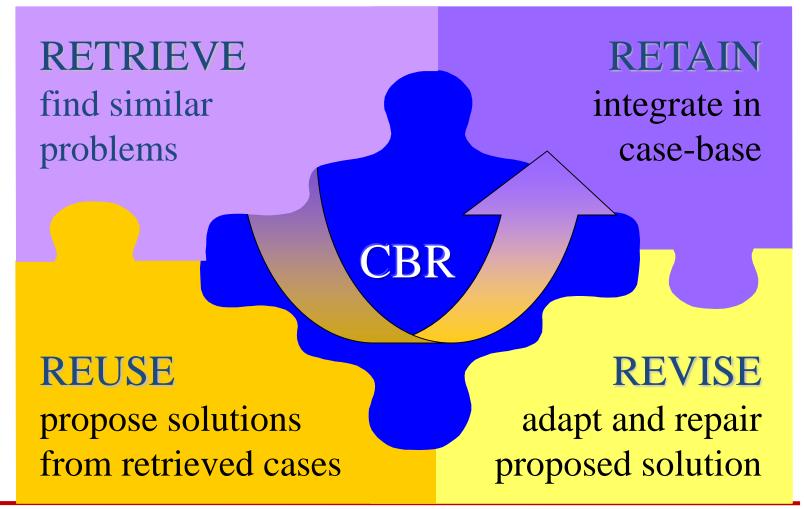


CBR Cycle



R⁴ Cycle







CBR System Components



- Case-base
 - database of previous cases (experience)
- Retrieval of relevant cases
 - index for cases in library
 - matching most similar case(s)
 - retrieving the solution(s) from these case(s)
- Adaptation of solution
 - alter the retrieved solution(s) to reflect differences between new case and retrieved case(s)



CBR Assumption(s)



- The main assumption is that:
 - Similar problems have similar solutions:
 - e.g., an aspirin can be taken for any mild pain
- Two other assumptions:
 - The world is a regular place: what holds true today will probably hold true tomorrow
 - (e.g., if you have a headache, you take aspirin, because it has always helped)
 - Situations repeat: if they do not, there is no point in remembering them
 - (e.g., it helps to remember how you found a parking space near that restaurant)





Problem (Symptoms) Problem: Front light doesn't work Car. VW Golf II, 1.6 L Year: 1993 Battery voltage: 13,6 V State of lights: 0 K State of light switch: 0 K Solution Diagnosis: Front light fuse defect Repair: Replace front light fuse

Technical Diagnosis of Car Faults

	Problem (Symptoms)		
	Problem: Front light doesn't work		
C	Car: Audi A6		
Α	 Year: 1995 		
s	Battery voltage : 12,9 V		
E	State of lights: surface damaged		
	State of light switch: O K		
2	Solution		
	Diagnosis: Bulb defect		
	 Repair: Replace front light 		



What Is Prediction?



- (Numerical) prediction is similar to classification
 - construct a model
 - use model to predict continuous or ordered value for a given input
- Prediction is different from classification
 - Classification refers to predict categorical class label
 - Prediction models continuous-valued functions
- Major method for prediction: regression
 - model the relationship between one or more independent or predictor variables and a dependent or response variable
- Regression analysis
 - Linear and multiple regression
 - Non-linear regression
 - Other regression methods: generalized linear model, Poisson regression, log-linear models, regression trees



Linear Regression



<u>Linear regression</u>: involves a response variable y and a single predictor variable x

$$y = W_0 + W_1 x$$

where w_0 (y-intercept) and w_1 (slope) are regression coefficients

Method of least squares: estimates the best-fitting straight line

$$w_{1} = \frac{\sum_{i=1}^{|D|} (x_{i} - \overline{x})(y_{i} - \overline{y})}{\sum_{i=1}^{|D|} (x_{i} - \overline{x})^{2}} \qquad w_{0} = \overline{y} - w_{1}\overline{x}$$

- Multiple linear regression: involves more than one predictor variable
 - Training data is of the form $(\mathbf{X_1}, \mathbf{y_1}), (\mathbf{X_2}, \mathbf{y_2}), ..., (\mathbf{X_{|D|}}, \mathbf{y_{|D|}})$
 - Ex. For 2-D data, we may have: $y = w_0 + w_1 x_1 + w_2 x_2$
 - Solvable by extension of least square method or using SAS, S-Plus
 - Many nonlinear functions can be transformed into the above



Linear Regression



A regression model is comprised of a dependent, or response, variable and an independent, or predictor, variable.

Dependent Variable = Independent Variable(s)



Prediction Relationship

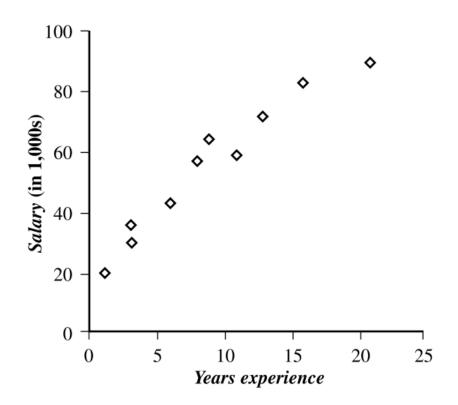






Salary data.

x years experience	y salary (in \$1000s)
3	30
8	57
9	64
13	72
3	36
6	43
11	59
21	90
1	20
16	83



Plot of data: Although the points do not fall on a straight line, the overall pattern suggests a linear relationship between x(years experience) and y (salary)







Given the above data, we compute $\bar{x} = 9.1$ and $\bar{y} = 55.4$. Substituting these values into Equations (6.50) and (6.51), we get

$$w_1 = \frac{(3-9.1)(30-55.4) + (8-9.1)(57-55.4) + \dots + (16-9.1)(83-55.4)}{(3-9.1)^2 + (8-9.1)^2 + \dots + (16-9.1)^2} = 3.5$$

$$w_0 = 55.4 - (3.5)(9.1) = 23.6$$

Thus, the equation of the least squares line is estimated by y = 23.6 + 3.5x. Using this equation, we can predict that the salary of a college graduate with, say, 10 years of experience is \$58,600.





