

~~Prashansa  
extreme~~

# 1 Chapter 4

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Prepared by



Simplification of Boolean Function, Badiha Latif Mughal

## Boolean Function

Boolean function is an expression formed with binary variables, the two operators OR and AND, the unary operator NOR, parentheses, and equal sign for a given value of the variable. The function can be either 0 or 1.

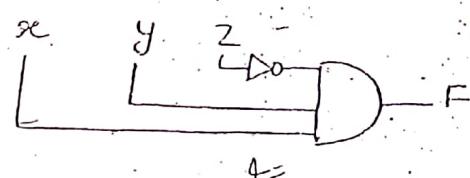
### Examples

$F_1 = xyz'$ . The function is equal to 1 if  $x=1$  and  $y=1$  and  $z'=1$ , otherwise  $F_1=0$ .

### Truth table

x	y	z	$z'$	$F_1 = xyz'$
0	0	0	1	0
0	0	1	0	0
0	1	0	1	0
0	1	1	0	0
1	0	0	1	0
1	0	1	0	0
1	1	0	1	1
1	1	1	0	0

### Gate Implementation

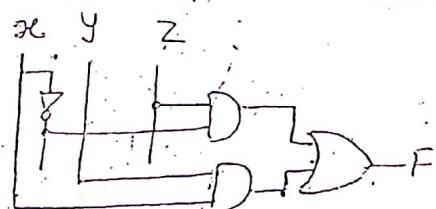


$$\begin{aligned}
 F &= xy + x'z + yz = \cancel{xy} + \cancel{xz} + yz(x + x') \\
 &= \cancel{xy} + x'z + xy2 + x'y2 \\
 &= xy(1+z) + x'z(1+y) \\
 &= xy + x'z
 \end{aligned}$$

### Truth table

x	y	z	$xy$	$xz$	$x'z$	$xy + x'z$
0	0	0	0	1	0	0
0	0	1	0	0	1	1
0	1	0	0	1	0	0
0	1	1	0	1	1	1
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	1	0	0	1
1	1	1	1	0	0	1

### Gate Implementation



### Complement of a Function

The complement of a function  $F$  is  $F'$  or  $\bar{F}$  and is obtained from an interchange of 0's for 1's and 1's for 0's in value of  $F$ . The complement of a function may be determined algebraically through De Morgan's Theorem.

#### Examples

$$F_1 = x'y'z + x'y'z'$$

$$\text{complement of } F_1 = \bar{F}_1$$

$$= (x'y'z + x'y'z')'$$

$$= (x'y'z')' \cdot (x'y'z)'$$

$$= (x+y+z)(x+y+z)$$

$$F_2 = x(y'z' + yz)$$

$$F_2' = [x(y'z' + yz)]'$$

$$= x' + (y'z' + yz)'$$

$$= x' + (y'z')'(yz)'$$

$$= x' + (y+z)(y'+z')$$

min term या 0 तर्म ( $x'$ ) के सभी

max term या 1 तर्म ( $x'$ ) के सभी

#### Minterms and Maxterms:-

			AND Minterm ( $\Sigma$ )	OR Maxterm ( $\Pi$ )	
$x$	$y$	$z$	Term	Designation	Designation
0	0	0	$x'y'z'$	$m_0$	$x+y+z$
0	0	1	$x'y'z$	$m_1$	$x+y+z'$
0	1	0	$x'y'z'$	$m_2$	$x+y'+z$
0	1	1	$x'y z$	$m_3$	$x+y'+z'$
1	0	0	$xey'z'$	$m_4$	$x'+y+z$
1	0	1	$xey'z$	$m_5$	$x'+y+z'$
1	1	0	$xey z'$	$m_6$	$x'+y'+z$
1	1	1	$xey z$	$m_7$	$x'+y'+z'$

$$F = xy + x'z$$

$$= (xy + x')(xz + z)$$

$$= (x+x') (x'+y) (x+z) (y+z)$$

$$= (x'+y) (x+z) (y+z)$$

$$(x'+y) = [(x'+y) + z \underline{z}] = \{ (x'+y) + z \} (x'+y+z) \\ = (x'+y+z) (x'+y+z)$$

$$(x+z) = (x+y+z) (x+y'+z)$$

$$(y+z) = (x+y+z) (x'+y+z)$$

$$\therefore F = (x'+y+z) (x'+y+z') (x+y+z) (x+y'+z) \\ = (x+y+z) (x+y'+z) (x'+y+z) (x'+y+z')$$

$$F = M_0, M_2, M_4, M_5$$

$$F(x,y,z) = \prod (0,2,4,5)$$

3 variables  $\therefore$  total combination  $2^n = 2^3 = 8$

Among 8 (0,2,4,5) outputs are low and rest all are high, hence rest (1,3,6,7) can be presented as sum of products.

$$\therefore F(x,y,z) = \sum (1,3,6,7)$$

### Standard form :-

In standard form, the terms that form the function may contain one, two or any number of literals. There are two types of standard forms.

#### i) Sum of Products (SOP) (Minterm) (लेटर्न)

$$F_1 = y' + x'y + x'y'z'$$

$0 \rightarrow A$   
 $1 \rightarrow A$

#### ii) Product of sums (POS) (Maxterm) (लॉटर्न)

$$F_2 = x(y'+z)(x'+y+z'+w)$$

Non Standard Form: A Boolean function may be expressed in a non standard form.  $F_3 = (AB + CD)(A'B' + C'D)$ .

This function is neither in SOP nor in POS form. It may be changed to a standard form by using distribution law to remove the parenthesis.

$$F_3 = \overbrace{ABA'B' + ABC'D + A'B'C'D + C'DC'D}^{\text{distribution law}}$$

$$= ABC'D + A'B'C'D$$

### Simplification of Boolean function:

#### Simplification of Boolean function / (K-map):

The map method / Karnaugh method / (K-map):  
 → The map method is regarded as a pictorial form of a truth table.

→ The map is a diagram made up of squares. Each represents one minterm. The map presents a visual diagram of all possible ways that a function may be expressed in a standard form.

#### Two variable K-map

→ There are four minterms for two variables; hence it consists of four squares, one for each minterm.

	y	0	1
x	0	$x'y'$ m <sub>0</sub>	$x'y$ m <sub>1</sub>
1	0	$xy'$ m <sub>2</sub>	$xy$ m <sub>3</sub>

#### Three Variable K-map

	y	z	00	01	11	10
x	0	0	000 m <sub>0</sub>	001 m <sub>1</sub>	011 m <sub>3</sub>	010 m <sub>2</sub>
1	0	1	100 m <sub>4</sub>	101 m <sub>5</sub>	111 m <sub>7</sub>	110 m <sub>6</sub>

Four variable K-map

		z			
		00	01		
yz		00	01	11	10
w	00	$m_0$	$m_1$	$m_3$	$m_2$
	01	$m_4$	$m_5$	$m_7$	$m_6$
	11	$m_{12}$	$m_{13}$	$m_{15}$	$m_{14}$
	10	$m_8$	$m_9$	$m_{11}$	$m_{10}$

Five variable K-map

		CDE		AB					
		000	001	011	010	110	111	101	100
A	00	0, 7	1	3	2	6	7	5	4
	01	8	9	11	10	14	15	13	12
	11	24	25	27	26	30	31	29	28
	10	16	17	19	18	22	23	21	20

Example :-  $F(x, y, z) = \Sigma(1, 3, 6)$

yz		00	01	11	10
x	z	1	1	4	1
z	0	-	-	1	1

$$\begin{aligned}
 F &= x'y'z + x'yz + xzy' \\
 &= x'z(y' + y) + x'yz \\
 &= x'z + x'yz
 \end{aligned}$$

Simplifying by K-map

$$F(x, y, z) = x'z + x'yz$$

$$\Rightarrow F(x, y, z) = \Sigma(1, 3, 7)$$

yz		00	01	11	10
x	z	1	1	1	1
z	0	-	-	-	-

$$F(x, y, z) = x'z + yz$$

$$\Rightarrow F(x, y, z) = \Sigma(1, 3, 5, 7)$$

yz		00	01	11	10
x	z	1	1	1	1
z	0	-	-	-	-

$$F(x, y, z) = z$$

Note :- which variable is constant, take that one only.

Example: Simplify the Boolean expression using K.  
 $F(w,x,y,z) = \Sigma(1,3,9,11)$

<del>wx</del>	yz	00	01	11	10
00	.	1	1		
01	.				
11	.				
10	.	1	1		

$$\therefore F(w,x,y,z) = x'y'z'$$

<del>wx</del>	yz	00	01	11	10
00	1				1
01					
11					
10	1				1

$$F(w,x,y,z) = x'y'z'$$

### Product of sums simplification.

Example 1: Simplify the following Boolean function in of sums:

$$F(A,B,C,D) = \Sigma(0,1,2,5,8,9,10)$$

<del>AB</del>	<del>CD</del>	00	01	11	10
00	00	1	1	0	1
01	01	0	1	0	0
11	11	0	0	0	0
10	10	1	0	0	1

$$F' = CD + AB + BD' \quad F = (C'+D')(A'+B')(B'+D)$$

→ take duals.

### Don't Care Condition

Example 2: Simplify the Boolean function

$$F(w,x,y,z) = \Sigma(1,3,7,11,15) \text{ and don't care condition}$$

$$d(w,x,y,z) = \Sigma(0,2,5,7)$$

<del>wx</del>	yz	00	01	11	10
00	X	1	1	X	
01	X	1			
11		1			
10		1			

$$F = w'x' + yz$$

<del>wx</del>	yz	00	01	11	10
00	X	1	1	X	
01	0	X	1	0	
11	0	0	1	0	
10	0	0	1	0	

$$F = z' + wy'$$

- +
- Don't care condition is represented by a symbol 'X'.  
 → It can take either value of 0 or 1 as the need arises.

Example 3: Simplify the Boolean expression

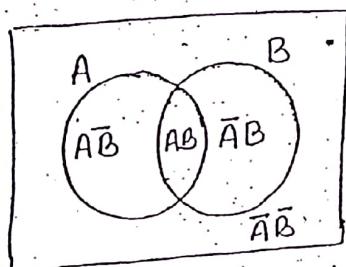
$$f(A, B, C, D, E) = \Sigma(0, 2, 4, 6, 10, 11, 14, 15, 16, 18, 20, 22, 25, 27, 3)$$

		CDE									
		AB		000	001	011	010	110	111	101	100
AB	00	1					1	0	0		1
	01				1		1	1	0		
	11		1	1				1			
	10	1				1	1			1	

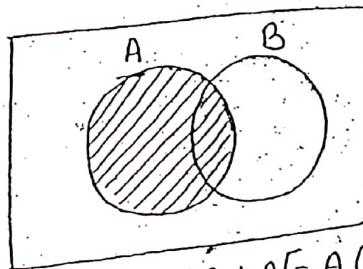
$$f = A'B'D + B'D'E' + ABC'E + B'DE' + BCDE$$

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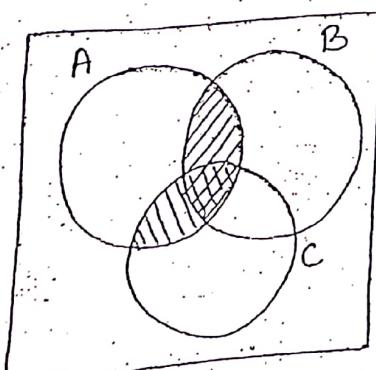
### VENN DIAGRAM



Venn diagram for  
two variables



$$A = A\bar{B} + A[B + \bar{B}]$$



$$A(B+C)$$

$$AB + AC$$

8421

(x) ~~self~~

## Chapter 5

### Combinational Logic

#### # Combinational logic

A combinational ckt consists of logic gates whose outputs at any time are determined directly from the present combination of inputs without regard to previous inputs. Eg. Adders, Subtractors, Multiplexers.

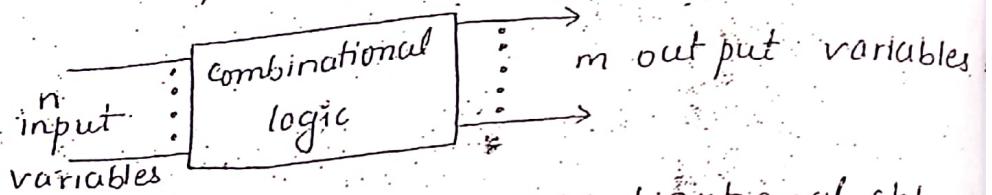


Fig: Block Diagram of Combinational ckt.

#### # Sequential logic

Sequential ckt consists of memory element in addition to logic gates. The outputs of a sequential ckt depend on present inputs and past inputs. Eg. Flip-Flop, Registers, Counters, etc.

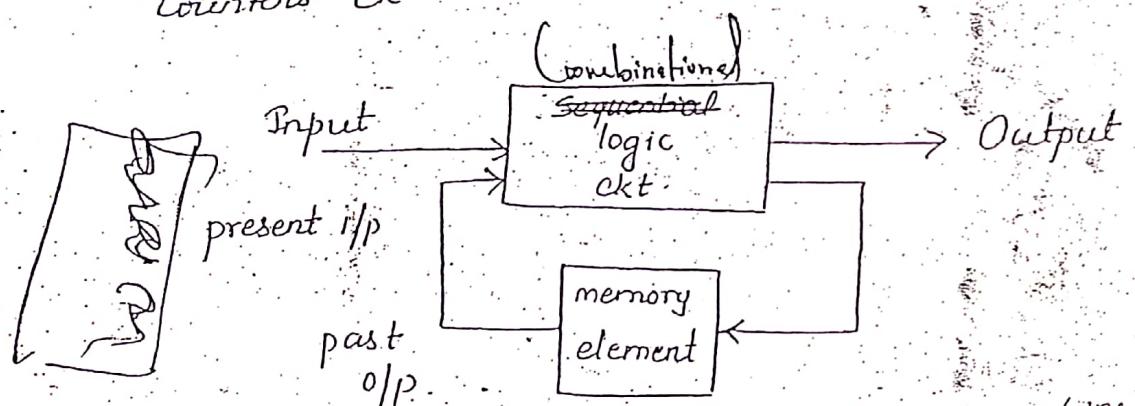


Fig: Block Diagram of a sequential ckt.

#### Design Procedure - combinational ckt

##### steps:

- i) The problem is stated.
- ii) The numbers of available input variables and output variables is determined.
- iii) The input and output variables are assigned letter symbols.

- (iv) The truth table that defines the required relationship between inputs and outputs is determined.
- (v) The simplified Boolean function for each output is obtained.
- (vi) The logic diagrams is drawn.

A practical design method would have to consider such constraints as:

- 1) Minimum numbers of gates.
- 2) Minimum numbers of inputs to gate.
- 3) Minimum propagation time of the signal through the ckt.
- 4) Minimum number of interconnections.
- 5) Limitations of the driving capability of each gate.

### Half Adder

A combinational ckt that performs the addition of two bits is called a half-adder.

#### Step 1

$$0+0 = 00$$

$$0+1 = 01$$

$$1+0 = 01$$

$$1+1 = 10$$

#### Step 2:

We find this ckt needs two binary inputs and two binary outputs.

#### Step 3

We are going to assign symbols x & y to the two inputs and s. (for sum) & c (for carry) to outputs.

#### Step 4

x	y	c	s
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

The  $s$  output represents the least significant bit of the sum.

Step 5

K-map for  $s$

x\y	0	1
0	0	1
1	1	0

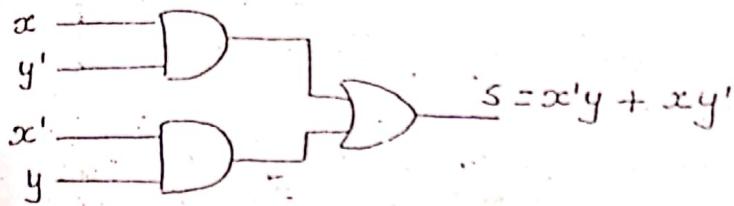
$$\begin{aligned}s &= xy' + x'y \\ &= x \oplus y\end{aligned}$$

K-map for  $c$

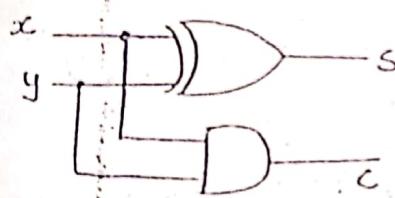
x\y	0	1
0	0	0
1	0	1

$$c = \underline{x}y$$

Step 6



Step 7



Here, logic diagram is further simplified by using XOR gate.

### Full Adder

A full-adder is a combinational circuit that forms the arithmetic sum of three inputs. It consists of 3 inputs and two outputs.

Truth Table

x	y	z	c	s
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

\ k-map for s

	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$$\begin{aligned}
 S &= x'y'z + x'y z' + xy'z' + xyz \\
 &= z(x'y' + xy) + z'(x'y + xy') \\
 &\stackrel{b}{=} z(\bar{x}\bar{y}) + \bar{z}(\bar{x}\oplus y) = z(\pi\oplus y) + z'(\pi\oplus y) \\
 &= x \oplus y \oplus z
 \end{aligned}$$

### k-map for c

ix	yz	00	01	11	10
0	0	0	1	0	
1	0	1	1	1	1

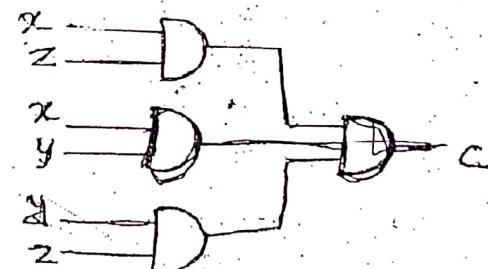
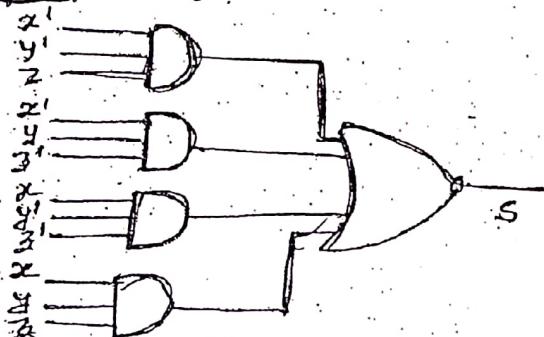
$$c = xz + xy + yz \quad \text{not necessary}$$

$$= xyz + xy'z + xyz' + xy'z' + x'y'z$$

$$= xy + z(x+y) \quad = xyz + xy'z' + xy'z + x'y'z$$

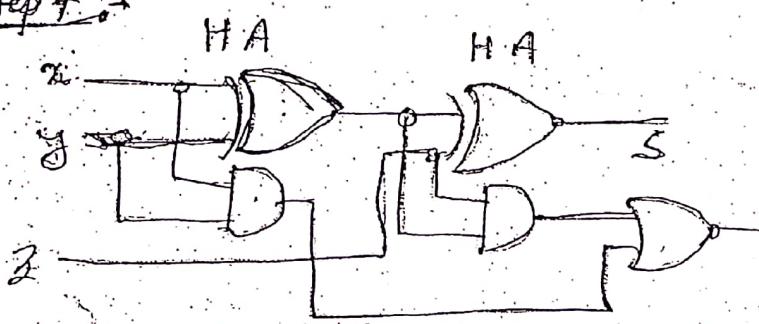
$$\Rightarrow xy + z(x+y) = xy(z+z') + z(xy' + x'y) = xy + z(x+y)$$

$$\text{Step 6} \rightarrow xy + z^d$$



$$= x^2 + y^2$$

### Step 7



## # Half Subtractor

A half-subtractor is a combinational ckt that takes two bits and produces their difference. The half-subtractor needs two inputs and two outputs.

Truth Table

inputs		outputs	
dc	y	B	D
0	0	0	0
0	1	1	1
1	0	0	1
1	1	0	0

$D \rightarrow$  difference

$B \rightarrow$  Borrow

K-map for D

x	y	0	1
0	0	1	
1	1	0	

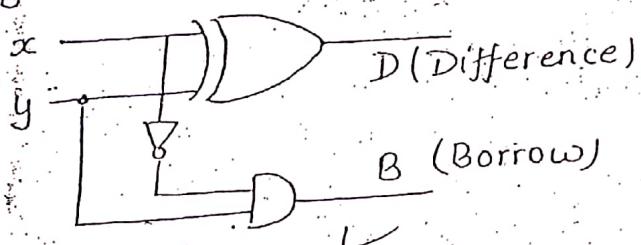
$$\begin{aligned} D &= xy' + x'y \\ &= x \oplus y \end{aligned}$$

K-map for B

x	y	0	1
0	0	1	
1	0	0	

$$B = x'y$$

Logic implementation



## # Full Subtractor

A full subtractor is a combinational ckt that performs subtraction between two bits, taking into account a 1 may have been borrowed by a lower significant stage.

This ckt has three inputs and two outputs.

Step 4

Truth table

13

$$(x-y)-z$$

13

x	y	z	B	D
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	1	0
1	0	0	0	1
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

Step 5

K-map for D

$$D = \Sigma(m_1, m_2, m_4, m_7)$$

x\yz	00	01	11	10
0	1	1	1	1
1	1	1	1	1

$$\therefore D = x'y'z + x'yz' + xy'z' + xyz$$

$$= x'(y'z + yz') + x(y'z' + yz)$$

$$= x'(y \oplus z) + x(y \oplus z')$$

$$= x'(y \oplus z) + x(y \oplus z')$$

$$= x \oplus y \oplus z$$

$y \oplus z$   
 $\Rightarrow (y \oplus z)$

K-map for B

$$B = \Sigma(m_1, m_2, m_3, m_7)$$

x\yz	00	01	11	10
0	1	1	1	1
1	1	1	1	1

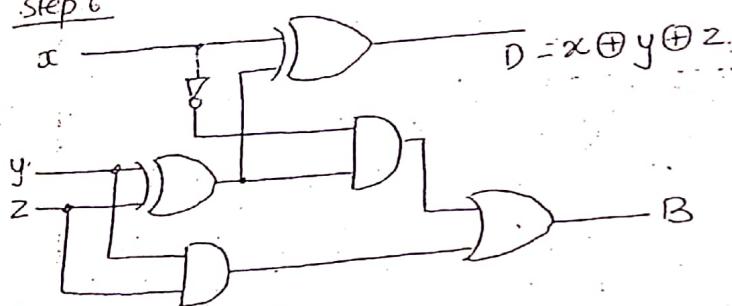
$$B = x'z + x'y + yz$$

$$= x'yz + x'y'z + x'yz + x'y'z' + yz$$

$$= x'yz + x'y'z + x'yz' + yz$$

$$= x'(y'z + yz') + x'yz + yz$$

$$\begin{aligned}
 &= x' \cdot (y \oplus z) + yz(x' + 1) \\
 &= x' \cdot (y \oplus z) + y^2
 \end{aligned}$$

Step 6

$$x - y - z$$

$$= x - \underline{(y + z)}_{HA}$$

$$= x - \underbrace{(w)}_{HS}$$

ENRQD# BCD To Excess-3 Code Conversion:

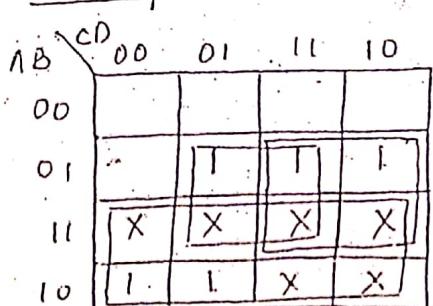
Design a ckt to convert BCD code to excess-3 output.

Step 4

Truth table

	Input BCD				Output Excess 3 cod			
	A	B	C	D	w	x	y	z
0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	1	0	0
2	0	0	1	0	0	1	0	1
3	0	0	1	1	0	1	1	0
4	0	1	0	0	0	1	1	1
5	0	1	0	1	1	0	0	0
6	0	1	1	0	1	0	0	0
7	0	1	1	1	1	0	1	1
8	1	0	0	0	1	0	0	1
9	1	0	0	1	1	1	1	0

$$d = \Sigma(10, 11, 12, 13, 14, 15)$$

K-map for w

$$w = \Sigma(5, 6, 7, 8, 9)$$

$$w = A + BD + BC$$

K-map for  $x$ 

$$x = \Sigma(1, 2, 3, 4, 9)$$

		CD	00	01	11	10
		AB	00	1	1	1
		01	1			
		11	X	X	X	X
		10		1	X	X

$$x = BC'D' + B'D + B'C$$

K-map for  $y$ 

$$y = \Sigma(0, 3, 4, 7, 8)$$

		CD	00	01	11	10	
		AB	00	1		1	
		01	1		1		
		11	X	X	X	X	
		10	1		X	X	

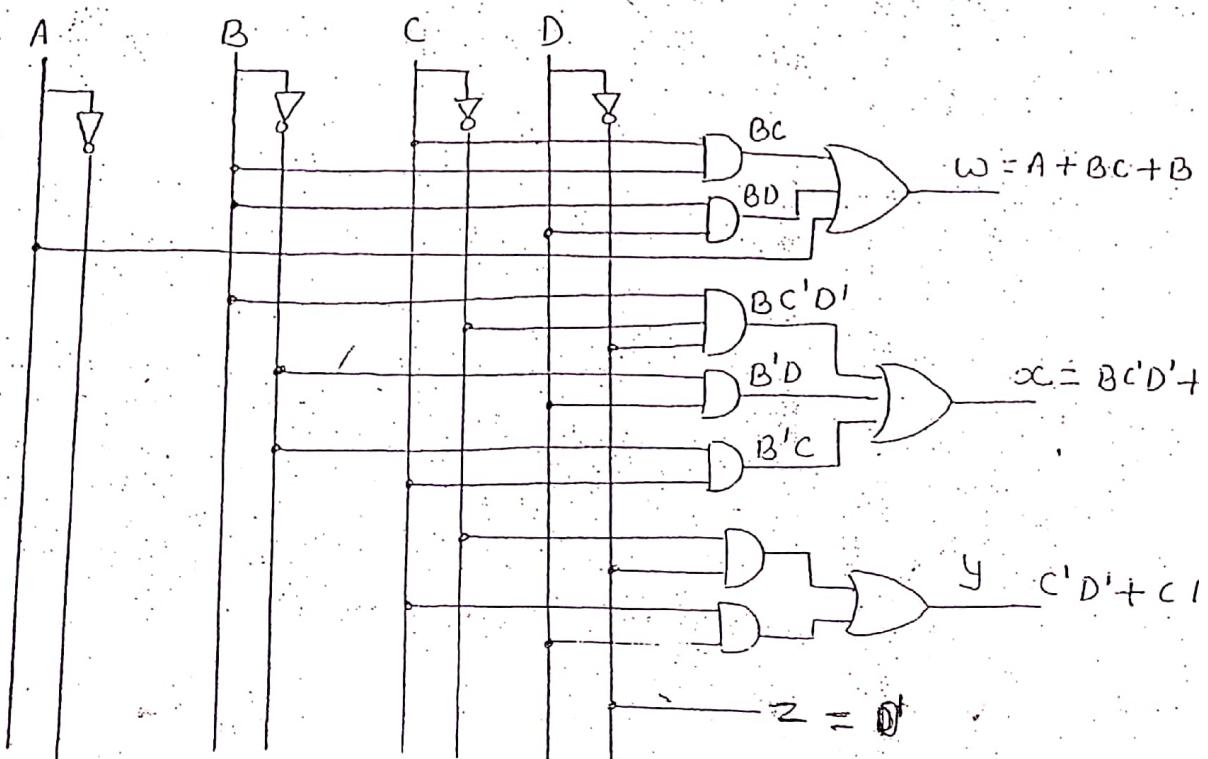
$$y = C'D' + CD$$

K-map for  $z$ 

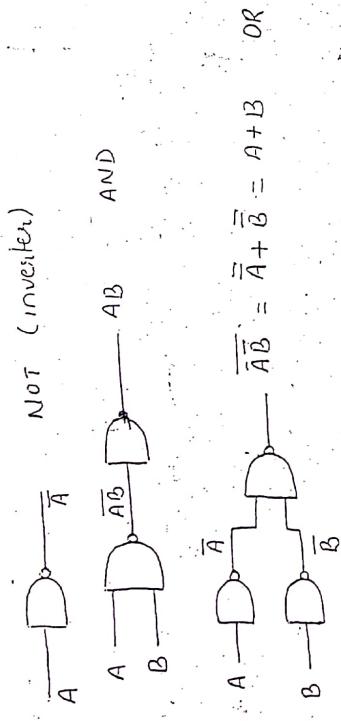
$$z = \Sigma(0, 2, 4, 6, 8)$$

		CD	00	01	11	10	
		AB	00	1			1
		01	1				1
		11	X	X	X	X	X
		10	1		X	X	X

$$\begin{aligned} z &= \bar{A}\bar{D}' + \bar{C}D' \\ &= (\bar{C} + \bar{C})\bar{D}' \\ &= D' \end{aligned}$$

Logic Implementation

## Multilevel NAND Circuit



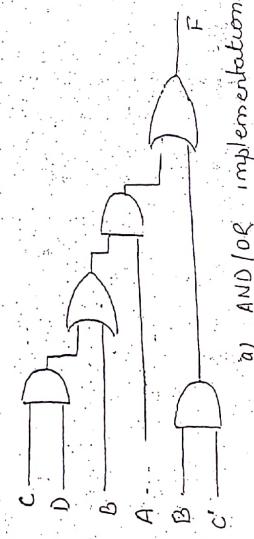
## Boolean Function Implementation

### Steps

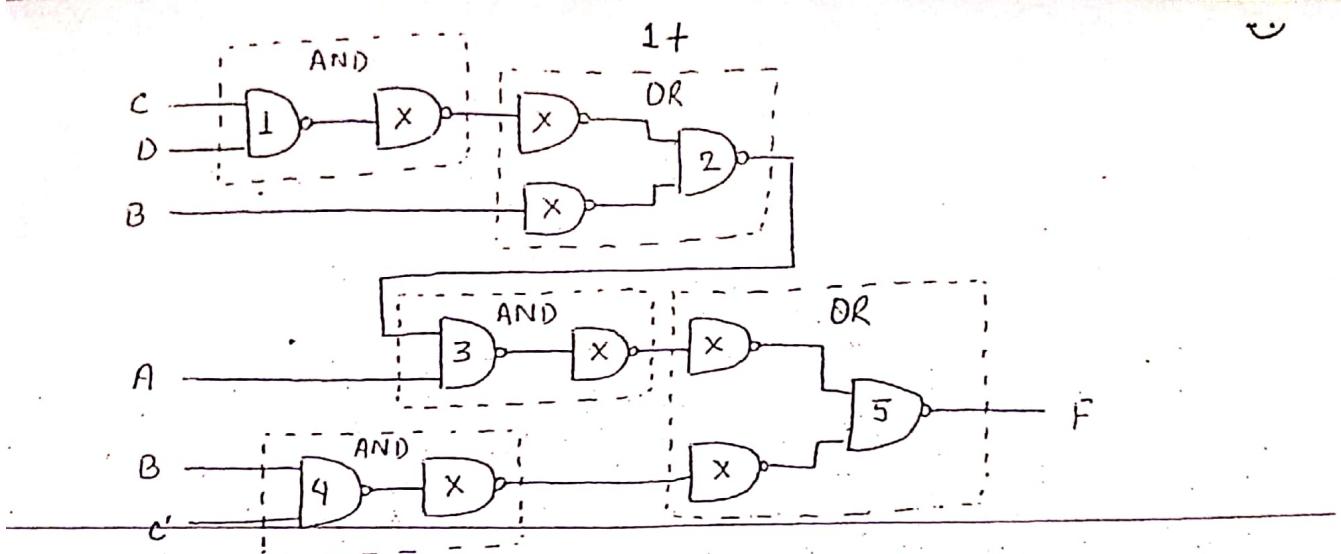
- For the given algebraic expression, draw the logic diagram with AND, OR, and NOT gates. Assume the normal and complement IPs are available.
- Draw a second logic diagram with the equivalent NAND logic, as given in above figures.
- Remove any two cascaded inverters, from the logic diagram since double inversion does not perform a logic function. Remove inverters connected to single external IPs, and the corresponding IP variable. The new logic diagram required NAND gate implementation.

### Example

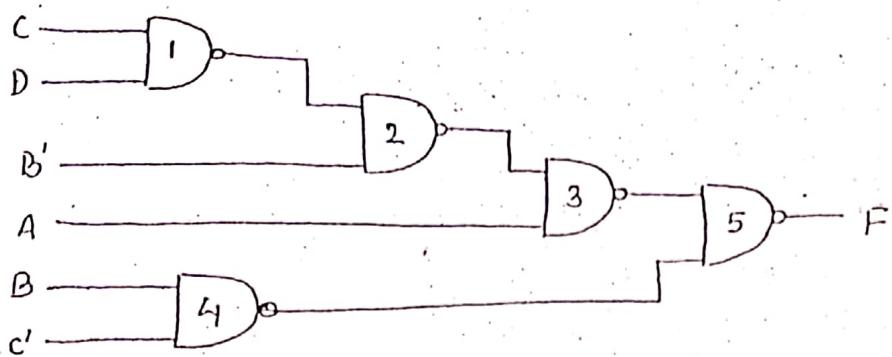
$$F = A(B + CD) + BC'$$



a) AND/OR implementation

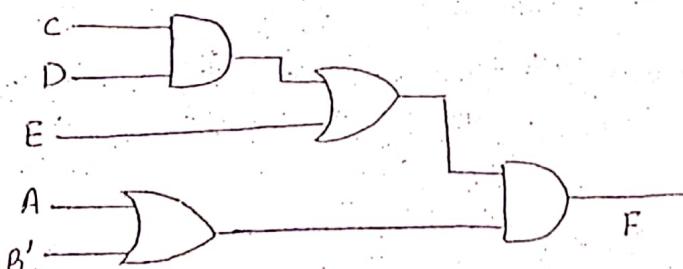


(b) substituting eq. NAND functions form

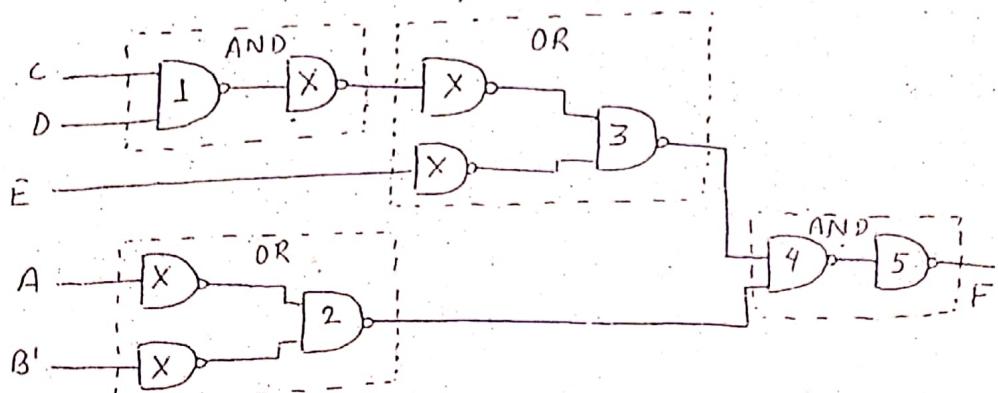


(c) NAND implementation

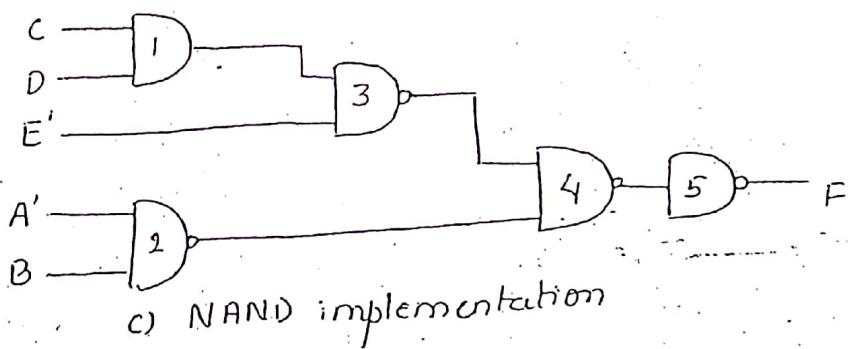
$$F = (A + B') (C D + E)$$



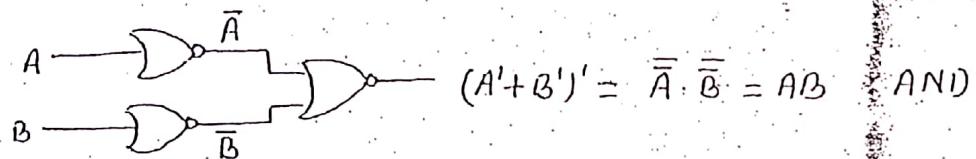
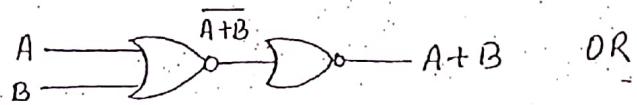
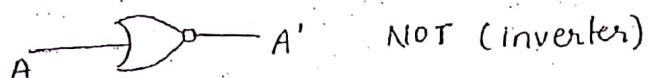
(a) AND/OR implementation



(b) substituting eq. NAND functions



### # Multilevel NOR Circuit

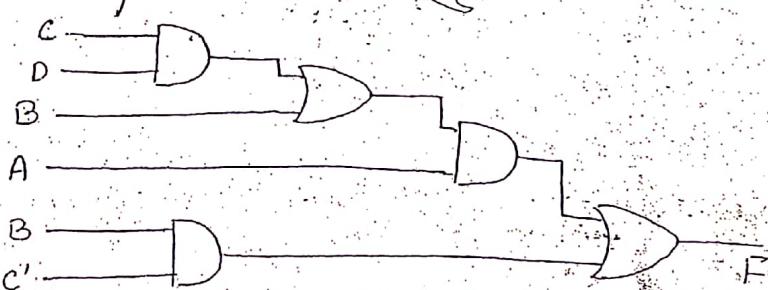


### # Boolean Function Implementation

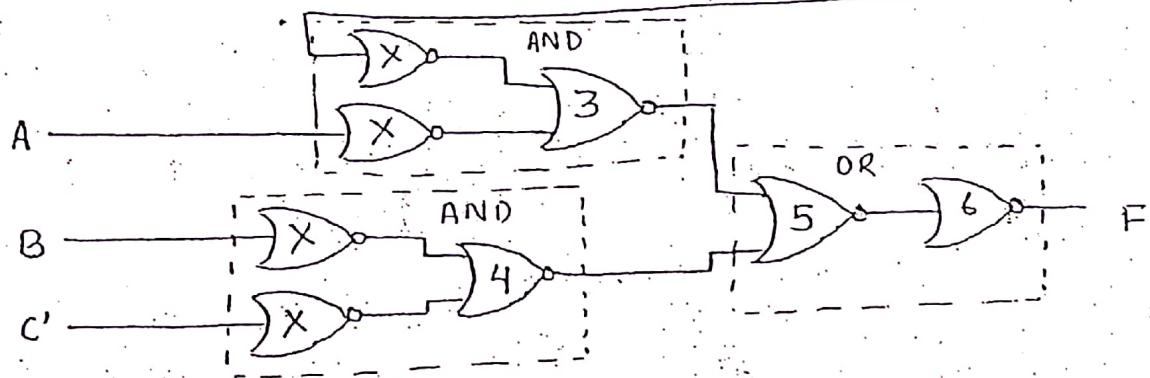
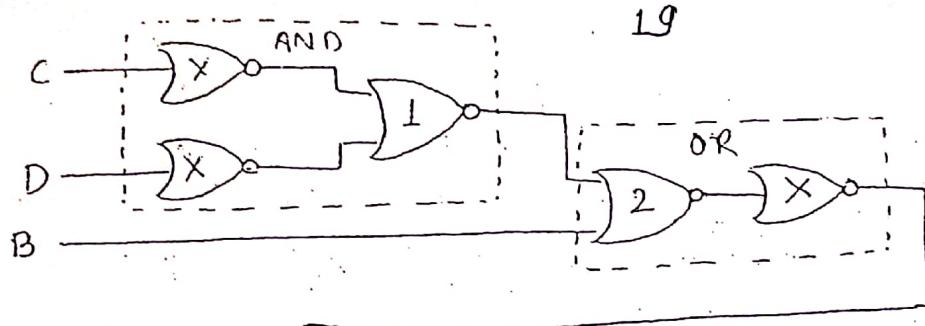
#### Steps

1. Draw the AND-OR logic diagram from the given algebraic expression. Assume that both the normal and the complement P/ps are available.
2. Draw a second logic diagram with e.g. NOR logic, given in Figure, substituted for each AND, OR and NOT gate.
3. Remove pairs of cascaded inverters from the diagram. Remove inverters connected to single external P/ps and complement the corresponding P/p variable.

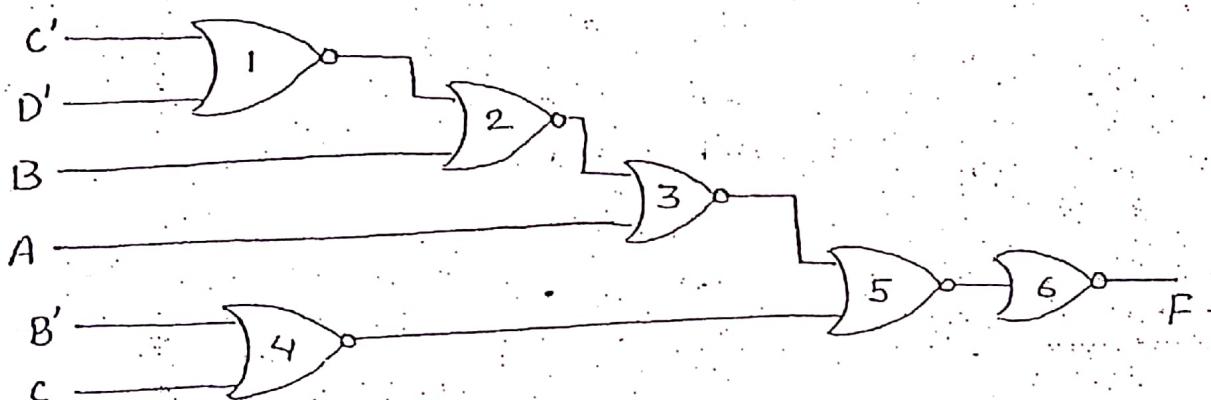
Example  $F = A(B+CD) + BC'$



a) AND/OR implementation



(b) substituting eq. NOR functions



(c) NOR Implementation

### # Analysis Procedure

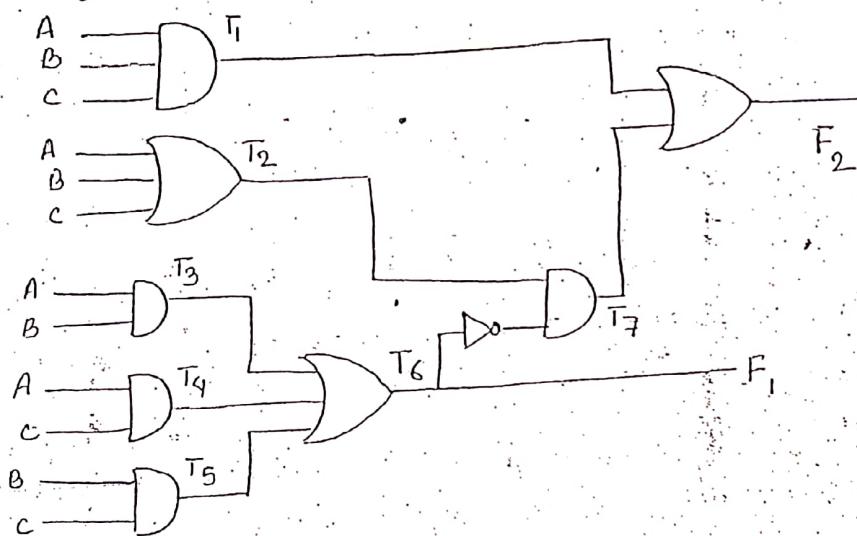
→ Deriving a Boolean function or a truth table from logic ckt diagram is an analysis procedure.

#### Procedure

1. Label with arbitrary symbols for all gate outputs are a function of the input variables. obtain b/c function for each gate.
2. Label with other arbitrary symbols. Those gates which are a function of i/p variables and previously labelled a function of i/p variables and previously labelled
3. obtain o/p function in terms of i/p variables on

#### Example

Analyse the ckt given below



$$T_1 = ABC$$

$$T_2 = (A+B+C)$$

$$T_3 = (AB)$$

$$T_4 = AC$$

$$T_5 = BC$$

$$T_6 = T_3 + T_4 + T_5 = AB + AC + BC$$

$$F_1 = T_6 = AB + AC + BC$$

$$T_7 = T_2 T_6' = (A+B+C)(AB+AC+BC)'$$

$$F_2 = T_1 + T_7 = ABC + (A+B+C) \cdot (AB+AC+BC)'$$

$$= ABC + (A+B+C) (\overline{AB} \cdot \overline{AC} \cdot \overline{BC})$$

$$= ABC + (A+B+C) (A'+B') (A'+C') (B'+C')$$

$$= ABC + (A+B+C)(A'+A'C'+A'B'+B'C')(B'+C')$$

$$= ABC + (A+B+C)(A'B'+A'B'C'+A'B'+B'C'+A'C'+A'C'+A'B'C'+B'C')$$

$$F_2 = ABC + AB'C' + A'BC' + A'B'C$$

Truth table

A	B	C	F <sub>1</sub>	F <sub>2</sub>
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

### Parity generation and checking

#### Parity bit:

A parity bit is an extra bit included with a binary message to make the number of 1's either odd or even so we have two types of parity.

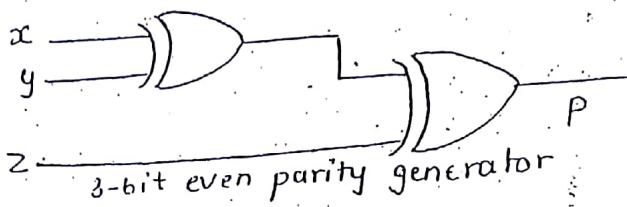
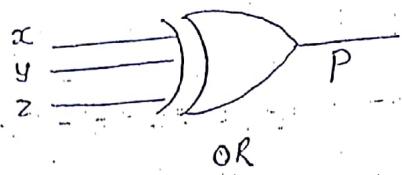
i) odd parity  $\Rightarrow 1$

ii) Even parity  $\Rightarrow 0$

#### Even-parity Generator

Three-bit message			Parity bit
x	y	z	p
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

For even-parity generator,  $\frac{P=1}{P=0}$  when no. of " 1's " is even  
 Hence it is an exclusive-OR gate (XOR Gate)



It can be verified by K-map as well

x \ y	00	01	11	10
0	.	1	1	1
1	1	1	1	

$$\begin{aligned}
 P &= x'y'z + x'y'z' + xy'z' + xyz \\
 &= x'(y'z + yz') + x(y'z' + yz) \\
 &= x'(y \oplus z) + x(y \odot z) \\
 &= x \oplus y \oplus z
 \end{aligned}$$

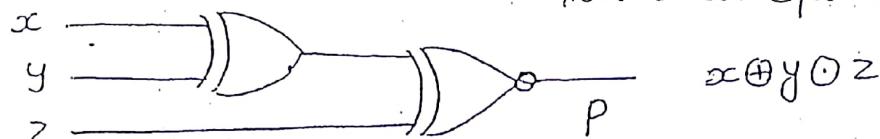
### Odd Parity Generator

Three-bit message			Parity bit
x	y	z	P
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

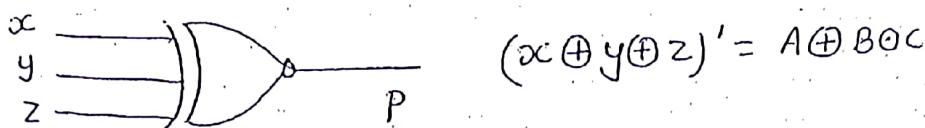
For odd parity generator  $P=1$  when no. of 1's is even  
 $P=0$  when no. of 1's is odd

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Hence odd parity is obtained through complement of either XOR or an equivalence expression.



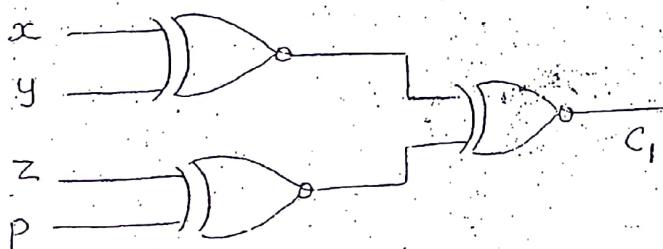
OR



3-bit odd parity generator

### Parity Checker ✓

Four bit received				odd parity checker	even parity checker
$x$	$y$	$z$	$P$	$C_1$	$C_2$
0	0	0	0	1	0
0	0	0	1	0	1
0	0	1	0	0	1
0	0	1	1	1	0
0	1	0	0	0	1
0	1	0	1	1	0
0	1	1	0	1	0
0	1	1	1	0	1
1	0	0	0	0	1
1	0	0	1	1	0
1	0	1	0	1	1
1	0	1	1	0	0
1	1	0	0	1	1
1	1	0	1	0	0
1	1	1	0	0	1
1	1	1	1	1	0



odd parity checker

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K-map for odd parity generator

	$y'z$	$yz$	$y'z'$	$yz'$
$xz$	00	01	11	10
0	1		1	
1		1		1

$$\begin{aligned}
 P &= x'y'z' + x'y'z + xy'z + xyz' \\
 &= x'y'z' + x'y'z + x(y'z + yz') \\
 &= x'(y'z' + yz) + x(yz + y'z') \\
 &= x \oplus (y \odot z) \\
 &= (x \oplus y) \odot z
 \end{aligned}$$

K-map for odd parity checker

	$zp$	$00$	$01$	$11$	$10$
$xy$	00	1		1	
01		1			1
11		1			
10		1			1

$$\begin{aligned}
 P &= x'y'z'p' + x'y'zp + x'y'z'p + x'y'zp' + xyz'p' + xyzp + \\
 &\quad x'y'z'p + x'y'zp' \\
 &= x'y'(z'p' + zp) + x'y(z'p + zp') + xy(z'p' + zp) + xy'(z'p) \\
 &= x'y'(z \oplus p) + x'y(z \oplus p) + xy(z \oplus p) + xy'(z \oplus p) \\
 &= x'(y'(z \oplus p) + y(z \oplus p)) + x(y(z \oplus p) + y'(z \oplus p)) \\
 &= x'(y \odot z \oplus p) + x(y \odot z \oplus p) \\
 &= x \odot y \odot z \oplus p \\
 &= (\bar{x} \odot y) \odot (z \oplus p)
 \end{aligned}$$

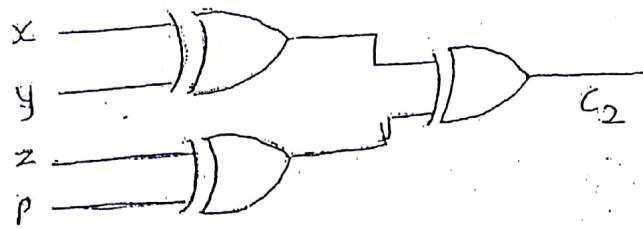
K-map for even parity checker

25

xy \ zp	00	01	11	10
00	1			1
01	1		1	
11		1		1
10	1		1	

$$\begin{aligned} p &= x'y'z'p + x'y'zp' + x'y'z'p' + x'y'zp + x'y'z'p + x'y'zp' \\ &\quad + x'y'z'p' + x'y'zp \\ &= x'y'(z'p + zp') + x'y(z'p' + zp) + x'y(z'p + zp') \\ &\quad + x'y'(z'p' + zp) \\ &= x'y'(z \oplus p) + x'y(z \ominus p) + xy(z \oplus p) + xy'(z \ominus p) \\ &= x'\left[\underset{\bar{a}}{y'}(z \oplus p) + \underset{b}{y}(z \ominus p)\right] + x\left(\underset{\bar{a}}{y}(z \oplus p) + \underset{b}{y'}(z \ominus p)\right) \\ &= x'\left[\underset{\bar{a}}{y} \oplus \underset{b}{y}(z \oplus p)\right] + x\left[\underset{a}{y} \oplus \underset{b}{y'}(z \ominus p)\right] \\ &= x \oplus y \oplus z \oplus p \\ &= \underline{(x \oplus y) \oplus (z \oplus p)} \end{aligned}$$

(x \oplus y) \oplus (z \oplus p)  
even  
(x \oplus y) \oplus (z \oplus p)  
odd



4-bit even parity checker

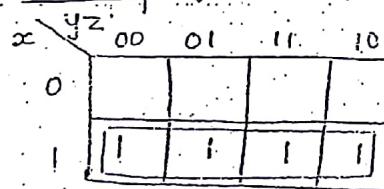
# Some questions

Q) Design a ckt. to convert 3 bit binary to gray

Let  $x, y, z$  be three input variables for binary and  $A, B, C$  be three o/p variables for gray

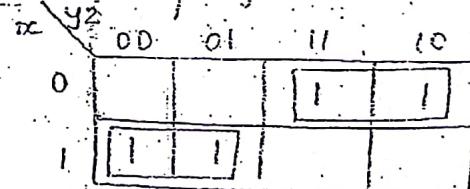
$x$	$y$	$z$	$A$	$B$	$C$
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	1
0	1	1	0	1	0
1	0	0	1	1	0
1	0	1	1	1	1
1	1	0	1	0	1
1	1	1	1	0	0

K-map for A



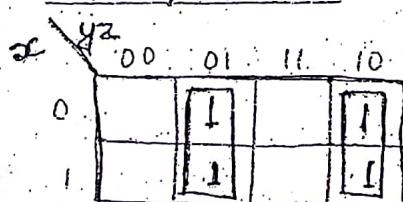
$$A = \bar{x}$$

K-map for B

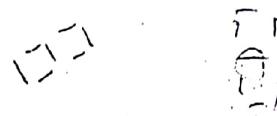
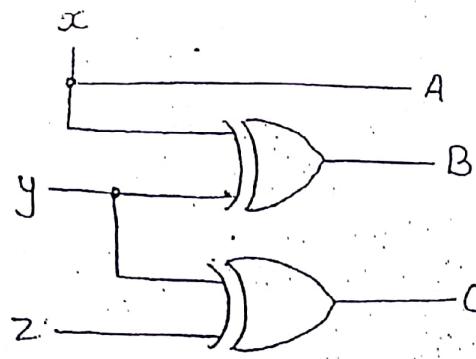


$$B = \bar{x}y + \bar{x}y' = \bar{x} \oplus y$$

K-map for C



$$C = \bar{y}z + yz' = y \oplus z$$



$\begin{array}{c} \overline{x} \\ \overline{y} \\ \overline{z} \end{array}$   
 1  
 1  
 1  
 1  
 0  
 0  
 0  
 0

2) Design the BCD-to-seven-segment decoder ckt.



BCD inputs

w	x	y	z	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	1	0	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	1	0	1	1
0	1	0	1	1	0	1	1	1	1	1
0	1	1	0	1	1	0	1	1	1	1
0	1	1	1	1	1	1	1	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	1	0	1

K-map for a

wx\yz	00	01	11	10
00	1		1	1
01		1	1	1
11	X	X	X	X
10	1	1	X	X

$$\begin{aligned}
 a &= y + w + x'z + x'z' \\
 &= w + y + x \oplus z
 \end{aligned}$$

K-map for b

wx\yz	00	01	11	10
00	1	1	1	1
01	1		1	
11	X	X	X	X
10	1	1	X	X

$$\begin{aligned}
 b &= w + x' + y'z' + yz \\
 &= w + x' + y \oplus z
 \end{aligned}$$

K-map for c

wz\yz	00	01	11	10
00	1	1	1	
01	1	1	1	1
11	X	X	X	X
10	1	1	X	X

$$c = w + z + y + z'$$

K-map for d

wz\yz	00	01	11	10
00	1	-	-	-
01	-	1	-	-
11	X	X	X	X
10	1	1	X	X

$$\begin{aligned}d &= w + yz' + w'z'y' + z'y' \\&= w + yz' + w'x'(z+y) + z'y'\end{aligned}$$

K-map for e

wz\yz	00	01	11	10
00	1	-	-	1
01	-	-	-	1
11	X	X	X	X
10	1	-	X	X

$$\begin{aligned}e &= wz' + yz' + w'x'z' \\&= z'(w + y + w'x')\end{aligned}$$

K-map for f

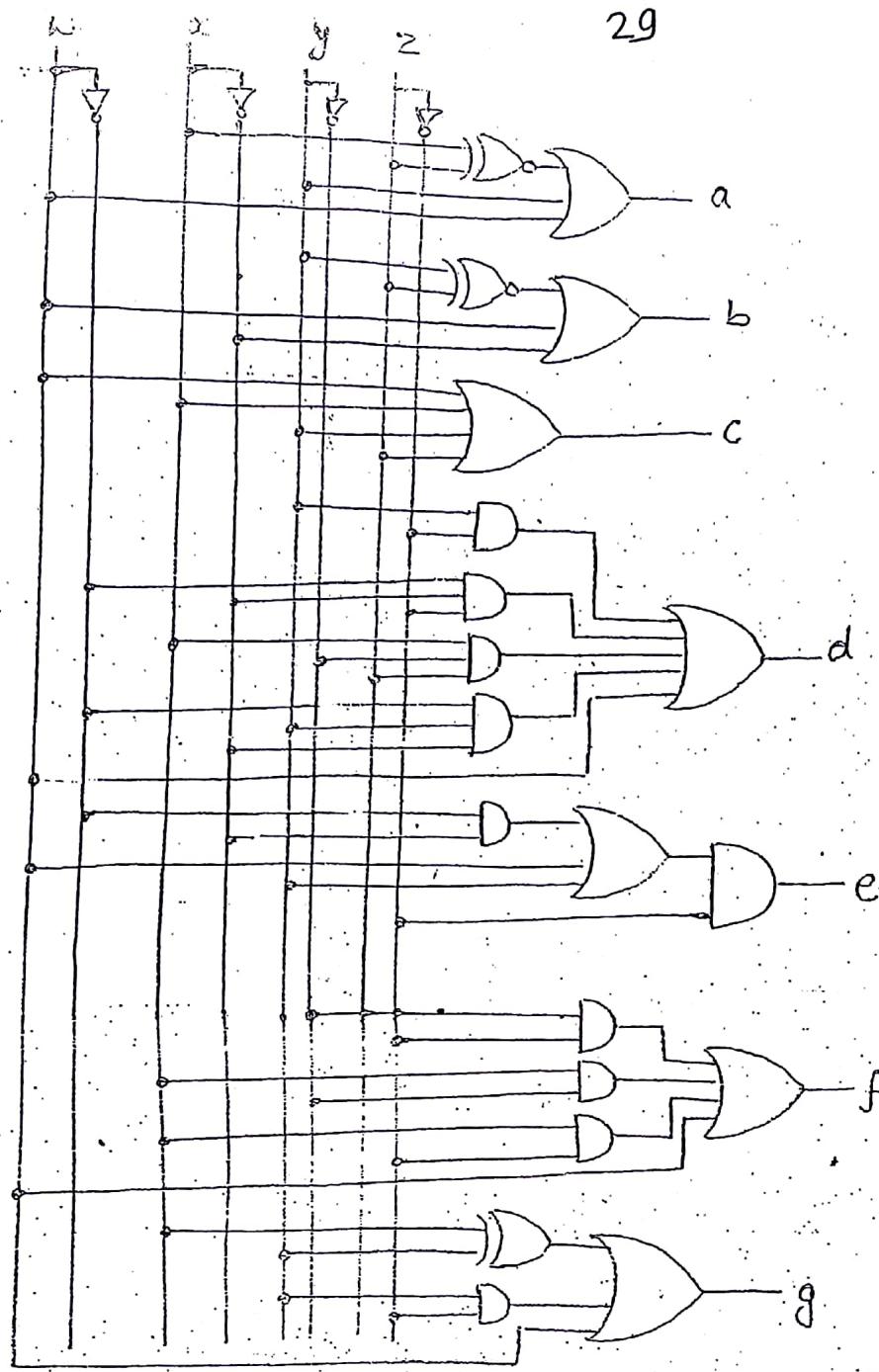
wz\yz	00	01	11	10
00	1	-	-	-
01	1	1	-	1
11	X	X	X	X
10	1	1	X	X

$$f = w + yz' + xy' + xz'$$

K-map for g

wz\yz	00	01	11	10
00	-	-	1	1
01	1	1	-	1
11	X	X	X	X
10	1	1	X	X

$$\begin{aligned}g &= w + yz' + x'y + xy' \\&= w + yz' + x \oplus y\end{aligned}$$



logic implementation

### # Combinational logic with MSI & LSI.

MSI  $\rightarrow$  Medium scale Integration

LSI  $\rightarrow$  Large scale Integration

- The classical procedure assumes that, given two ckt perform the same function, the one that requires f gates is preferable because of its low cost, is not ne true when it is used.
- With integrated ckt, it is not the count of gates that determines the cost but the number & type of ICs employed & the no. of external interconnections needed to implement the given fn.
- Before applying the classical method it is always to investigate the possibility of an alternate method may be more efficient for the particular problem.

### # Binary Parallel Adder:

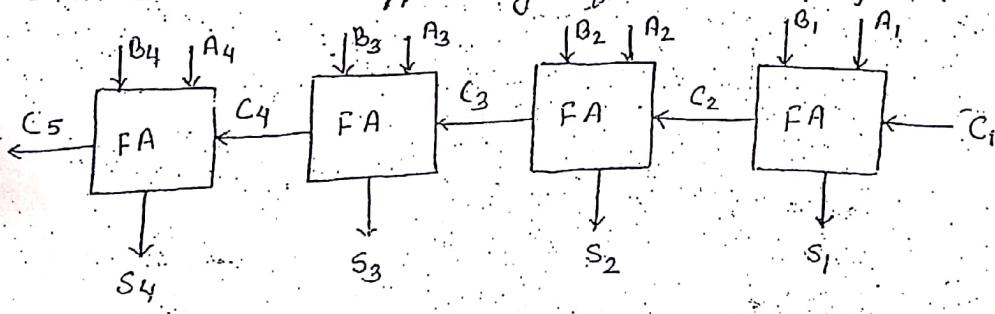
- The full adder is the sum of two bits and a previous carry.
- Two binary numbers of  $n$  bits each can be added by the parallel adder.
- When a pair of bits are added through a full adder the ckt produces a carry that is added to one significant position higher.

	Input carry	1	0	C <sub>i</sub>	<u>full adder</u>
	Augend	1	0	1	
	Addend	0	0	1	
	Sum	1	1	0	
	Output carry	0	0	1	
				C <sub>i+1</sub>	

- The bits are added with full adders, starting from the least significant position, to form sum bit & carry bit.
- The ip carry  $C_i$  in the least significant position must be 0. The value of  $C_{i+1}$  in a given significant position is the o/p carry of the full adder.
- The value is transferred into the ip carry of full adder i.e. one significant position higher to the left.
- Thus the sum of bits are generated from right to left as soon as previous carry bit is generated.
- The sum of bits are generated in two ways.

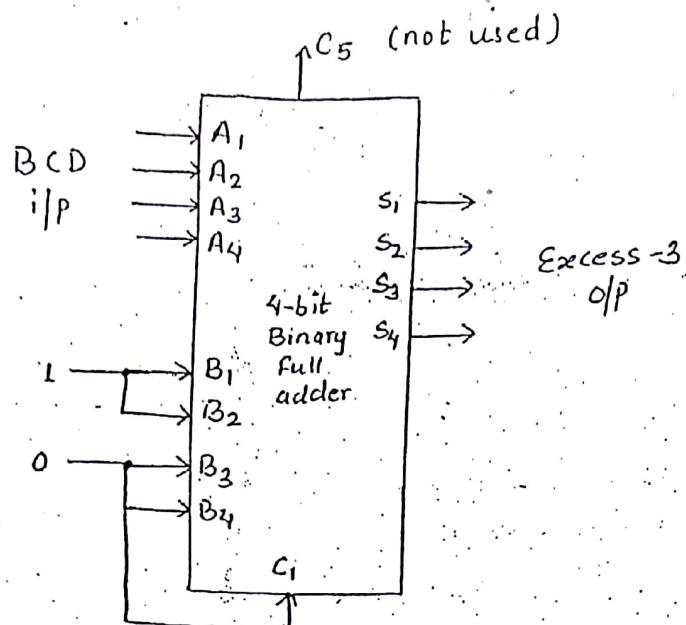
$\left. \begin{array}{l} \rightarrow \text{serial} \\ \rightarrow \text{parallel} \end{array} \right\}$

- The serial addition method uses only one full adder ckt and a storage device to hold the generated output carry. The pair of bits in A & B are transferred serially one at a time, through the single full adder to produce a string of output bits.
- The parallel method uses n full adder cks if all bits of A & B are applied simultaneously. The o/p carry of one full adder is connected to the input carry of next full adder at one significant position higher.
- A binary parallel adder is a digital function that produces the arithmetic sum of two binary numbers in parallel. It consists of full adder connected in cascade. With the output carry from one full adder connected to the ip carry of the next full adder.



4-bit binary Parallel adder

## # Design a BCD to excess-3 code converter

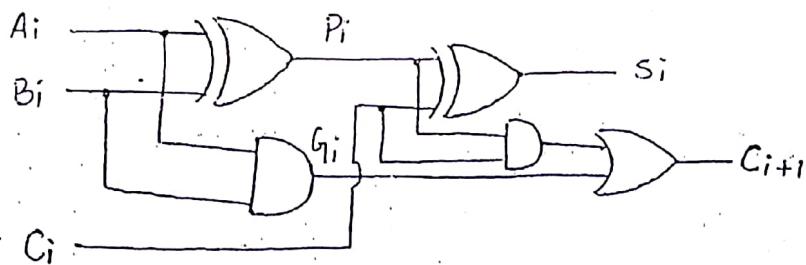


### BCD - to-excess-3 code converter

- The BCD digit is applied to input A. Input B are set to a constant 0011. This is done by applying logic 1 to  $B_1 \& B_2$  & logic 0 to  $B_3 \& B_4$  and also in  $C_1$ .
- The S outputs from the ckt give the excess-3 equivalent code of the input BCD digit.

## # Carry Propagation

- The signal must propagate through the gates before the correct output sum is available in the o/p term.
- The total propagation time is equal to the propagation delay of a typical gate times the no. of gate levels in the ckt.
- The longest propagation delay time in a parallel adder is the time it takes the carry to propagate through the full adders.

Full Adder Circuit

→ If there are four full adders in the parallel adder, the output carry  $C_5$  would have  $2 \times 4 = 8$  gate levels (AND & OR) from  $C_1$  to  $C_5$ .

→ The total propagation time would be the propagation time of one half adder + 8 gate level.

→ There are several techniques to reduce the carry propagation time in a parallel adder. The most widely used technique employs the principle of look-ahead carry.

$$P_i = A_i \oplus B_i$$

$$G_i = A_i B_i$$

The output sum & carry

$$S_i = P_i \oplus C_i$$

$$C_{i+1} = G_i + P_i C_i$$

→ Here  $G_i$  is called a carry generate & it produces an output carry when both  $A_i$  &  $B_i$  are one, regardless of the output carry.

→  $P_i$  is called carry propagate because it is the term associated with the propagation of the carry from  $C_i$  to  $C_{i+1}$ .

Now, Boolean function for the carry output of each stage for 4 bit adder,

$$C_2 = G_1 + P_1 C_1$$

$$C_3 = G_2 + P_2 C_2 = G_2 + P_2 (G_1 + P_1 C_1) = G_2 + G_1 P_2 + P_1 P_2 C_1$$

$$C_4 = G_3 + P_3 C_3 = G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 C_1$$

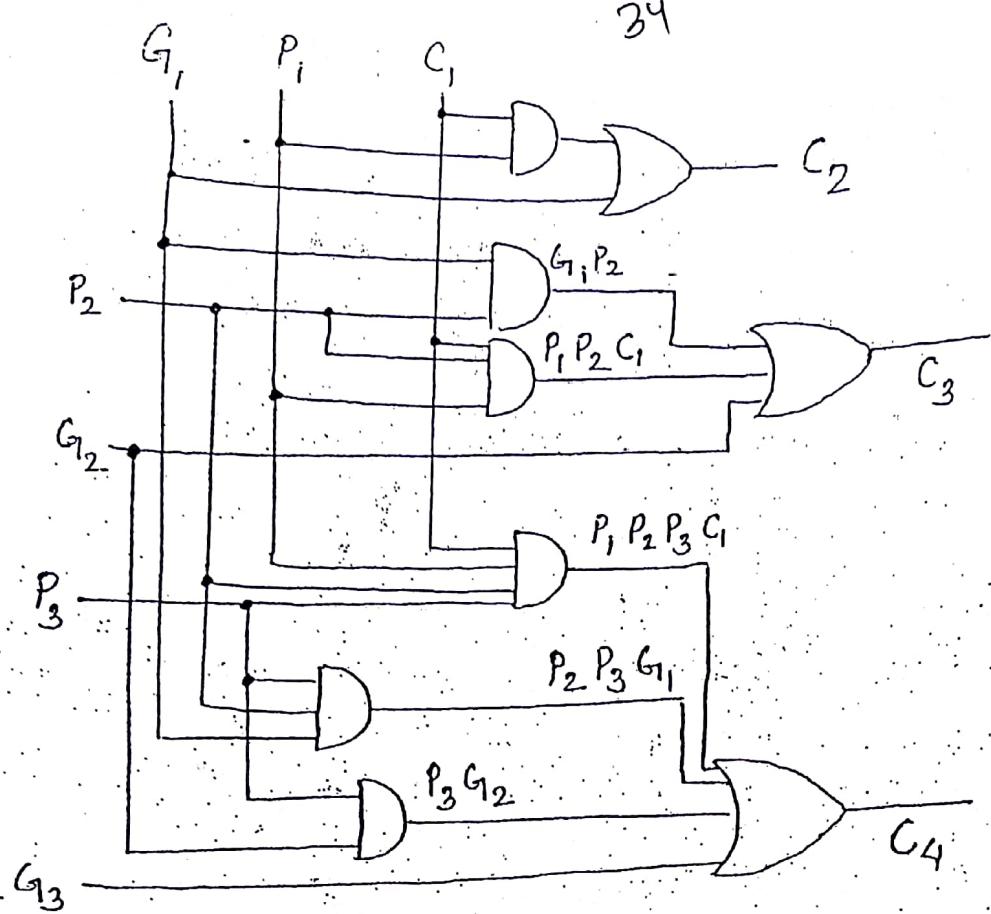
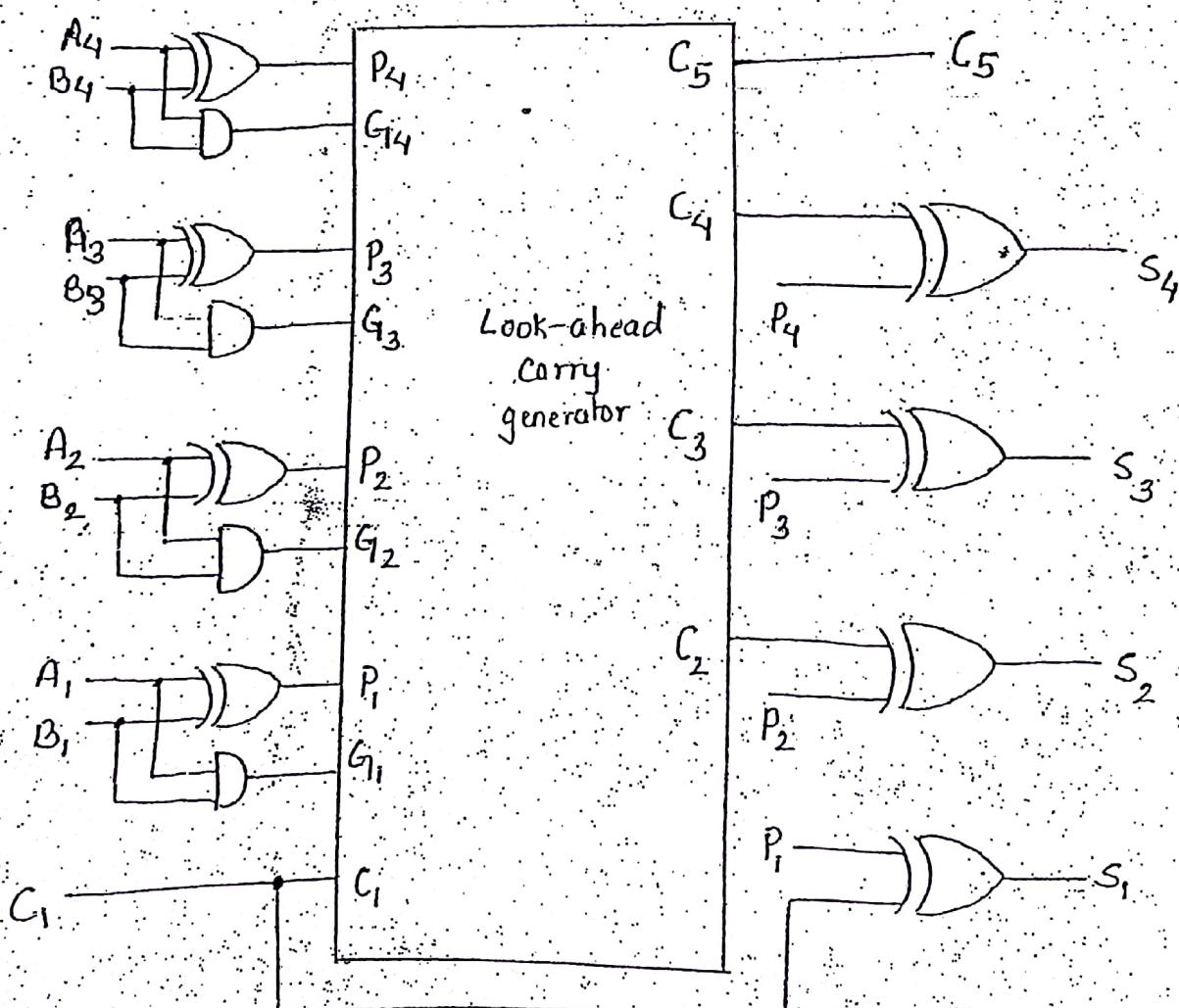


Fig: logic diagram of a look-ahead carry generator



## # Binary Subtractor:

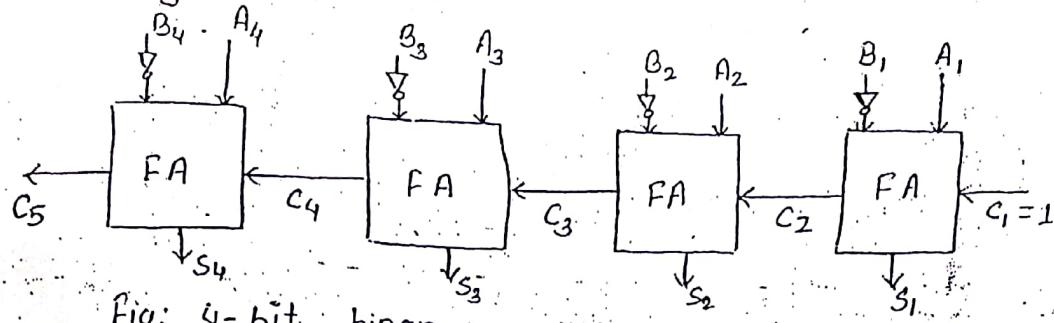


Fig: 4-bit binary parallel subtractor

$$\begin{array}{r}
 A_4 \quad A_3 \quad A_2 \quad A_1 \\
 + \overline{B}_4 \quad \overline{B}_3 \quad \overline{B}_2 \quad \overline{B}_1 \\
 \hline
 S_4 \quad S_3 \quad S_2 \quad S_1
 \end{array}
 \quad ! \leftarrow \text{Carry in } [C_1]$$

→ The circuit consists of a parallel adder with inverters placed at each data input of B & the initial input carry  $C_1$  is set to 1. For subtracting  $(A-B)$ .

→ The inverter ckt produces 1's complement of B & carry in  $C_1=1$  is added to the 1's complement, so the effect is equivalent to adding A & 2's complement of B. The final carry is ignored.

## # Binary Adder - Subtractor

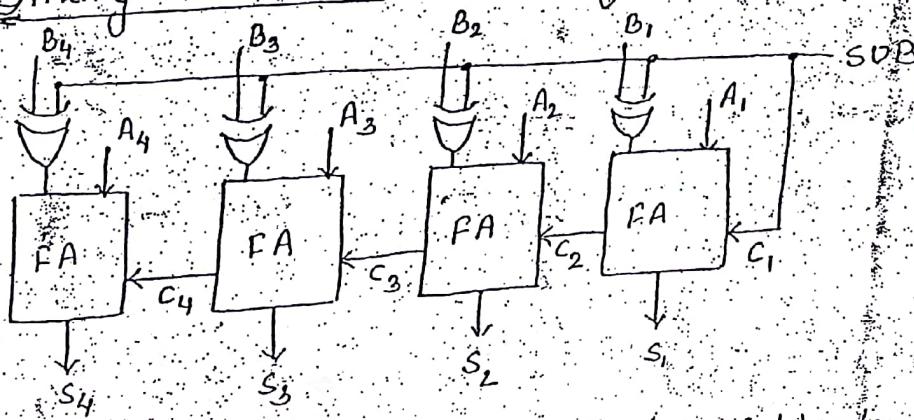


Fig: 4-bit binary adder-subtractor

→ The addition & subtraction can be combined into one circuit with one common adder with an exclusive-OR gate at each input of B to the full adder. The SUB input controls the operation when

$SUB = 0$ , the ckt perform as binary adder.

As  $SUB = 0$

$$B \oplus 0 = B'0 + B'0 = B$$

with initial carry input  $C_1 = 0$

when  $SUB = 1$ , the ckt perform as binary subtractor.

$$\text{As } SUB = 1, B \oplus 1 = B'1 + B'1 = B'$$

with initial carry input  $C_1 = 1$

i.e. 2's complement of B is added to A.

## # BCD Adder

→ Two decimal digits in BCD are applied to a 4-bit binary adder. The adder will perform the sum & produces output which may range from 0 to 19.

sum cannot be greater than  $1+9+9 = 19$

↓ possible carry from a previous

→ The binary sum are listed below in the table and are labelled by symbols  $K$ ,  $Z_8$ ,  $Z_4$ ,  $Z_2$  &  $Z_1$ . Here  $K$  is the carry and the subscripts under the letter Z represents the weight 8, 4, 2, & 1 that can be assigned to four bits in the BCD code.

### Derivation of a BCD adder

Binary Sum					BCD Sum					Decimal
<u><math>K</math></u>	<u><math>Z_8</math></u>	<u><math>Z_4</math></u>	<u><math>Z_2</math></u>	<u><math>Z_1</math></u>	<u><math>C</math></u>	<u><math>S_8</math></u>	<u><math>S_4</math></u>	<u><math>S_2</math></u>	<u><math>S_1</math></u>	
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	1	1
0	0	0	1	0	0	0	0	1	0	2
0	0	0	1	1	0	0	0	1	1	3
0	0	1	0	0	0	0	1	0	0	4
0	0	1	0	1	0	0	1	0	1	5
0	0	1	0	0	0	0	1	1	0	6
0	0	1	1	0	0	0	1	1	1	7
0	1	1	1	1	0	0	1	1	1	8

K	$Z_8$	$Z_4$	$Z_2$	$Z_1$	C	$S_8$	$S_4$	$S_2$	$S_1$	Decimal
0	1	0	0	0	0	1	0	0	0	8
0	1	0	0	1	0	1	0	0	1	9
0	1	0	1	0	1	0	0	0	0	10
0	1	0	1	1	1	0	0	0	1	11
0	1	1	0	0	1	0	0	1	0	12
0	1	1	0	1	1	0	0	1	1	13
0	1	1	1	0	1	0	1	0	0	14
0	1	1	1	1	1	0	1	0	1	15
1	0	0	0	0	1	0	1	1	0	16
1	0	0	0	1	1	0	1	1	1	17
1	0	0	1	0	1	1	0	0	0	18
1	0	0	1	1	1	1	0	0	1	19

- first 5 columns list the binary sum & the second 5 columns list the BCD sum.
- The sum of binary and BCD are same till 1001 (9).
- When binary sum is greater than 1001 (9), BCD invalid codes are generated.
- The addition of binary 0110 (6) converts the invalid into valid BCD & also produces an output carry.
- It is obvious from the table that correction is needed when binary sum has output carry i.e.  $C=1$  and whenever  $Z_8 = 1$  and either  $Z_4$  or  $Z_2$  or both are 1.
- Thus we can derive output carry as

$$C = K + Z_8 Z_4 + Z_8 Z_2$$

When,  $C=1$ , it is necessary to add  $\underline{0110 (6)}$  to binary sum.

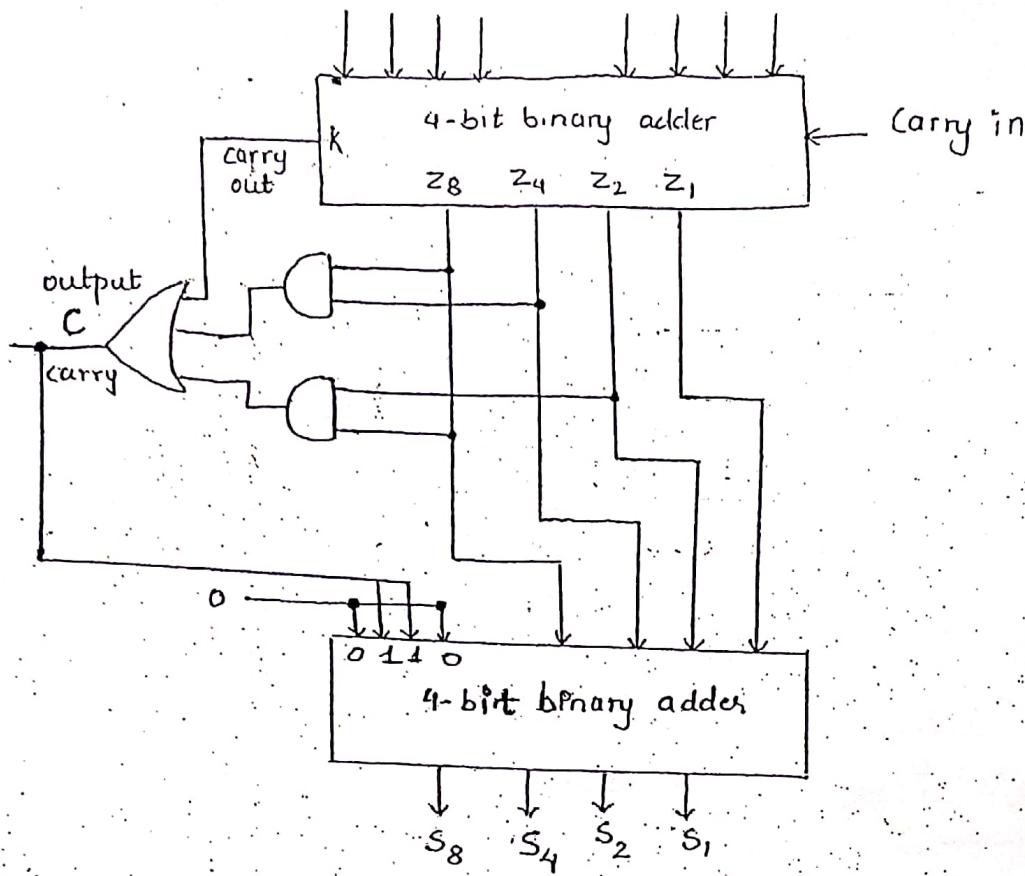


Fig: Block diagram of a BCD adder

# Magnitude Comparator

→ A magnitude comparator is a combinational circuit which compares two numbers  $A \& B$  and determines their relative magnitude.

→ The outcome of the comparison is specified by three binary variables that indicate whether  $A > B$ ,  $A = B$  or  $A < B$ .

→ Consider two numbers  $A \& B$  with four digits each write the coefficient of the numbers with descending significance position.

$$A = A_3 A_2 A_1 A_0$$

$$B = B_3 B_2 B_1 B_0$$

For  $A=B$ 

→ The two numbers are equal if all pairs of significant digits are equal i.e. if  $A_3 = B_3$  &  $A_2 = B_2$  &  $A_1 = B_1$  &  $A_0 = B_0$ .

→ When the numbers are binary, the digits are either 0 or 1.

→ If both the inputs are same the output must be high i.e. the equivalence X-NOR function.

$$x_i = A_i' B_i + A_i B_i \quad i=0, 1, 2, 3 \quad \boxed{A \oplus B} \quad A_i' B_i + 1$$

→ If all the significant digits are equal then

$$(A=B) = x_3 \cdot x_2 \cdot x_1 \cdot x_0$$

$$m_i = A_i' B_i + \bar{A}_i' \bar{B}_i$$

$$A = B = m_3 m_2 m_1 m_0$$

$$A > B = A_3' B_3 + m_3 A_2' B_2 + m_3 m_2$$

$$A > B = A_3' B_3 + m_3 A_2' B_2 + m_3 m_2 m_0$$

For  $A > B$ 

→ If  $A$  is greater than or less than  $B$ , we inspect the relative magnitudes of pairs of significant digits starting from the most significant position.

If the two digits are equal, we compare the next lower significant pair of digits. This comparison continues until a pair of unequal digits is reached.

→ If the corresponding digit of  $A = 1$  & that of  $B = 0$ . Then  $A > B$ .

$$(A > B) = A_3' B_3 + x_3 A_2' B_2 + x_3 x_2 A_1' B_1 + x_3 x_2 x_1 A_0' B_0$$

For  $A < B$ 

→ If the corresponding digit of  $A$  is 0 & that of  $B$  is 1, we have  $A < B$ .

$$(A < B) = A_3' B_3 + x_3 A_2' B_2 + x_3 x_2 A_1' B_1 + x_3 x_2 x_1 A_0' B_0$$

(TTL type 7485 is a 4-bit magnitude comparator)

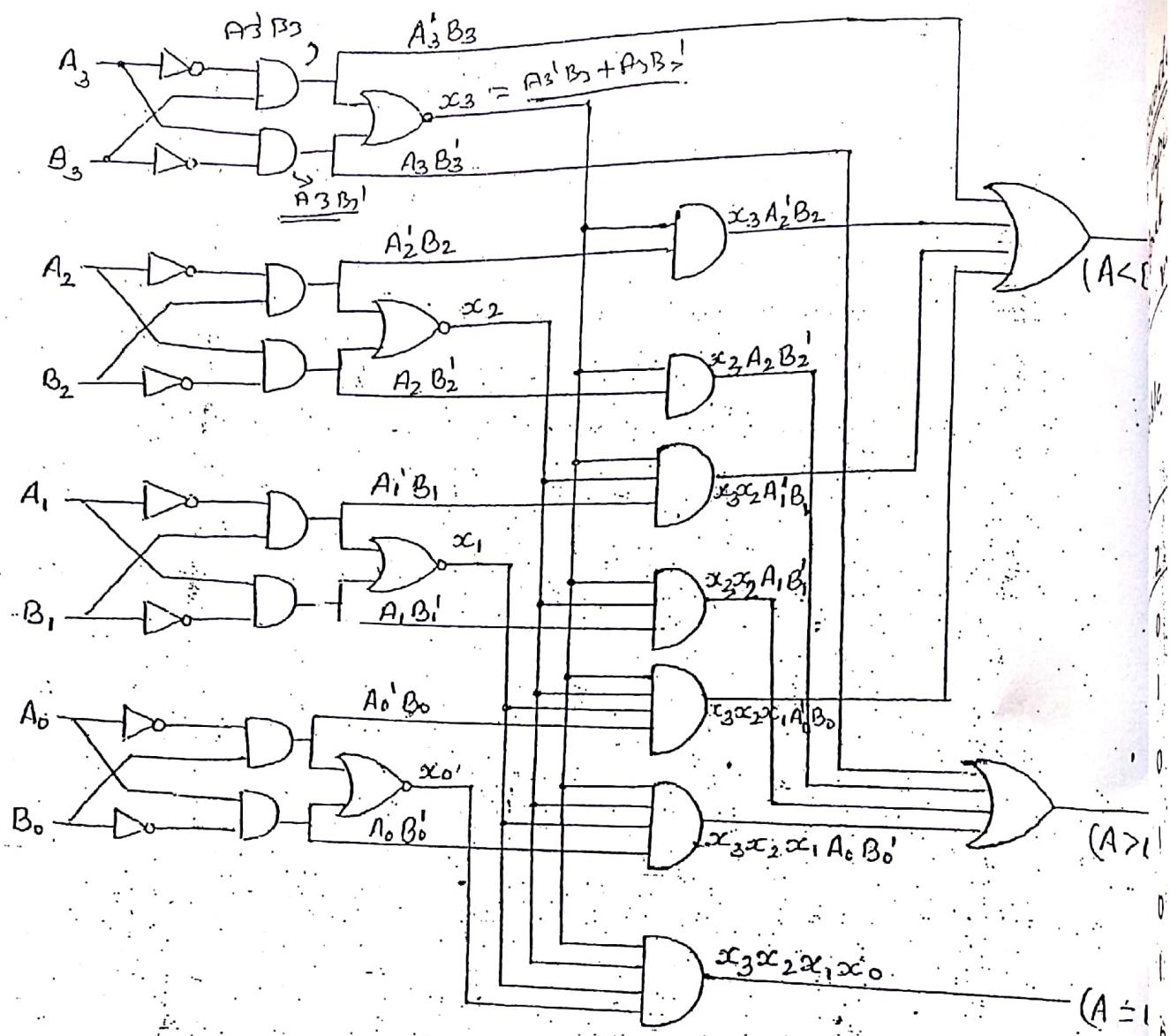


Fig: 4-bit magnitude comparator

## # Decoder:

→ A decoder is a combinational circuit that converts binary information from  $n$  input lines to a maximum of  $2^n$  unique output lines.

→ The basic function of a decoder is to detect the presence of a specified combination of bits (code) on its inputs and to indicate the presence of that code by a specified output level.

→ If the  $n$  bit information from  $n$  input lines has unused or don't care combinations, the decoder output will have less than  $2^n$  outputs.

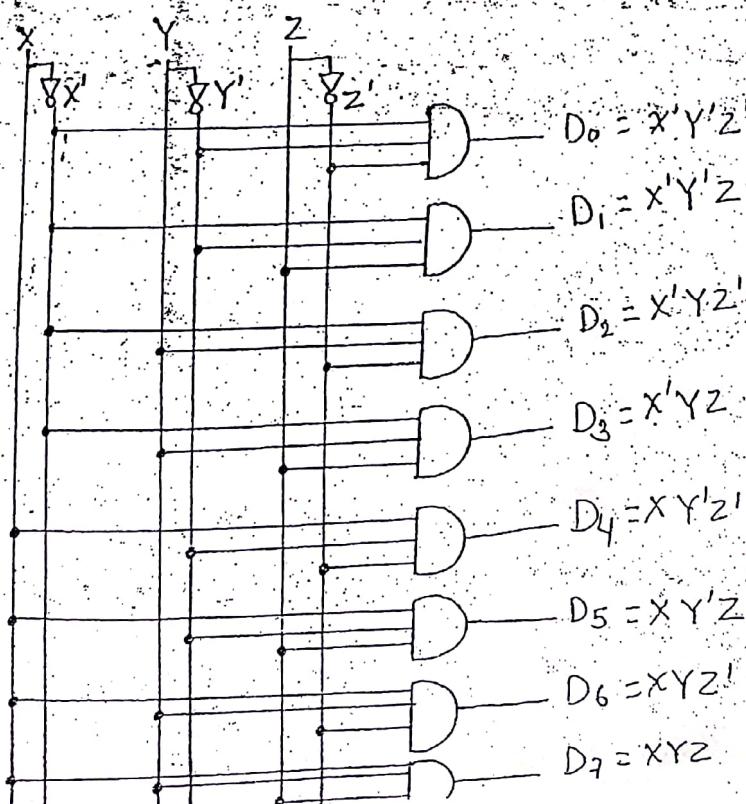
$\rightarrow n$  are the input lines and  $m$  are the output lines  
The in decoder.  
 $m \leq 2^n$

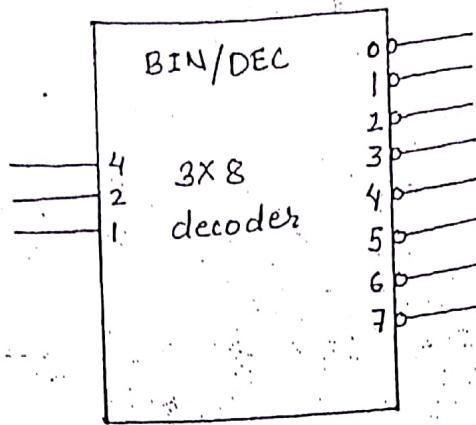
### 3-bit decoder

$\rightarrow$  The 3 input lines are decoded into eight outputs,  
each output representing one of the minterms of the  
3-input variables. Also, called 3 to 8 line decoder.

Truth table of 3 to 8 line decoder

Inputs			outputs							
X	Y	Z	D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	D <sub>6</sub>	D <sub>7</sub>
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1





- A logic symbol for 3 to 8 lines decoders with active low outputs is shown above.
- The BIN/DEC label indicates that a binary input may be the corresponding decimal output. The input label 4, 2, & 1 represents the binary input positional weight respectively.

~~Design a BCD to Decimal decoder~~

- It has 4 inputs & 10 outputs, one for each decimal digit.
- 4-10 line BCD to decimal decoder.
- Since the ckt has 10 outputs 10 K-maps are required to simplify each one output function.
- 6 don't care cond<sup>n</sup> at input can be used to simplify each of the output function.
- Instead of drawing 10-Kmap, all output variables D<sub>0</sub> to D<sub>9</sub> is drawn in 1 K-map to simplify each output separately with don't care conditions.

w\z	y\z	00	01	11	10
00	D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	
01	D <sub>4</sub>	D <sub>5</sub>	D <sub>7</sub>	D <sub>6</sub>	
11	X	X	X	X	
10	D <sub>8</sub>	D <sub>9</sub>	X	X	

$$\text{so, } D_0 = w'x'y'z$$

$$D_1 = w'x'y'z$$

$$D_2 = x'yz'$$

$$D_3 = x'yz$$

$$D_4 = x'y'z'$$

$$D_5 = xy'z$$

$$D_6 = xcyz'$$

$$D_7 = xyz$$

$$D_8 = wz'$$

$$D_9 = wz$$

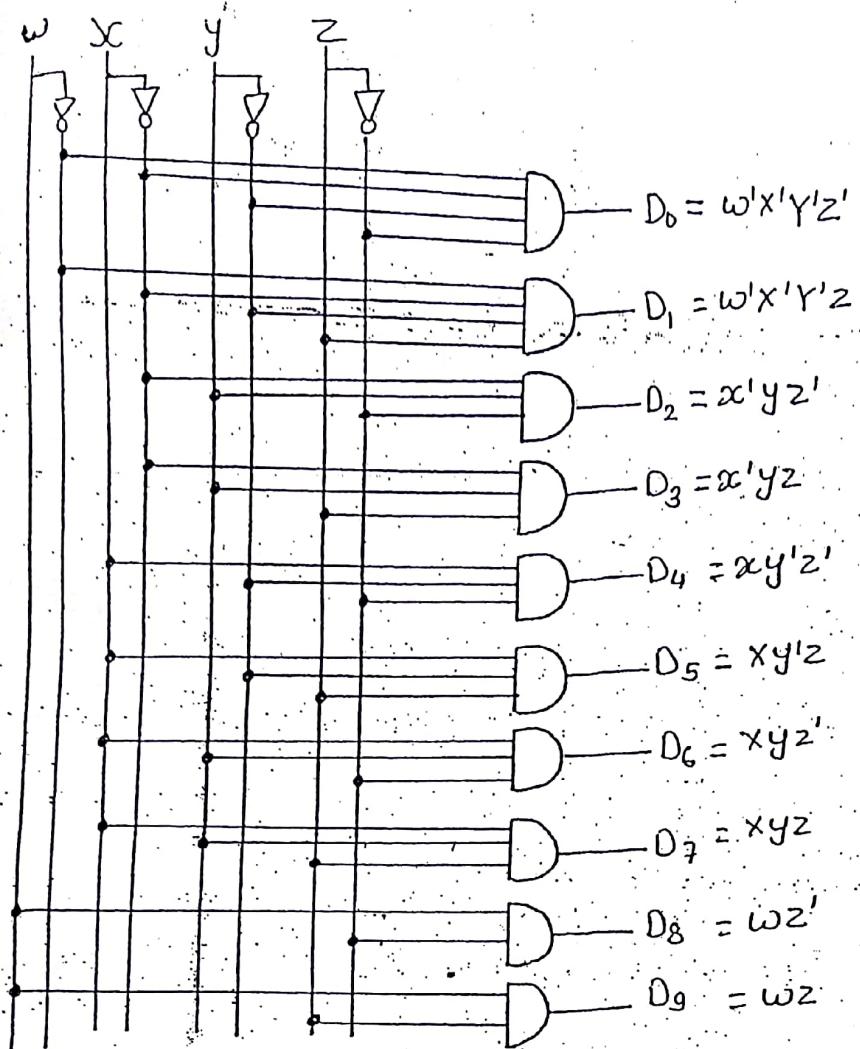


Fig: BCD to decimal decoder

Example question

Implement a full adder circuit with a decoder and two OR gates. (S) C

→ A decoder provides the  $2^n$  minterms of  $n$  input variables. Since any Boolean function can be expressed in sum of minterms canonical form, one can use a decoder to generate the minterms and a external OR gate to form the sum.

$$S(x_1, y_1, z) = \sum(1, 2, 4, 7)$$

$$C(x_1, y_1, z) = \sum(3, 5, 6, 7)$$

Since there are three inputs and a total of eight minterms, we need 3 to 8 line decoder.

x	y	z	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

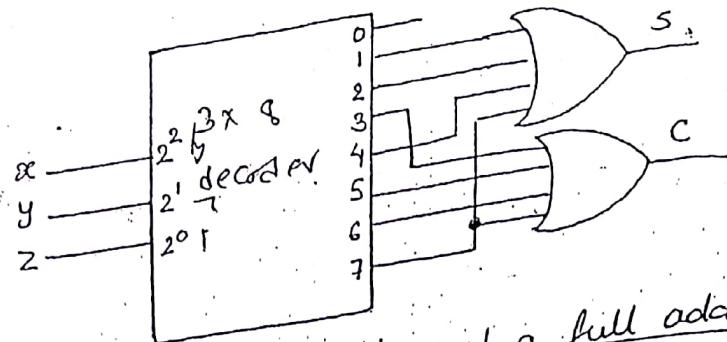


Fig: Implementation of a full adder with a decoder

### # Encoder

- An encoder produces reverse operation from that decoder.
- It has  $2^n$  (or less) input lines &  $n$  output lines.
- The output line generates the binary code for the  $2^n$  input lines.
- The process of converting from familiar symbols or numbers to a coded format is called encoding.

### # Octal to Binary encoder

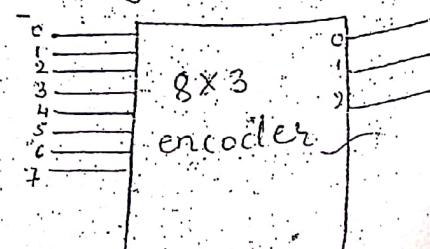
Inputs								Outputs		
D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	D <sub>6</sub>	D <sub>7</sub>	X	Y	Z
1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	1	1
0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	1	0	0	1	0	1
0	0	0	0	0	0	1	0	1	1	0
0	0	0	0	0	0	0	1	1	1	1

Fig: Truth table of octal to binary encoder

$$X = D_4 + D_5 + D_6 + D_7$$

$$Y = D_2 + D_3 + D_6 + D_7$$

$$Z = D_1 + D_3 + D_5 + D_7$$



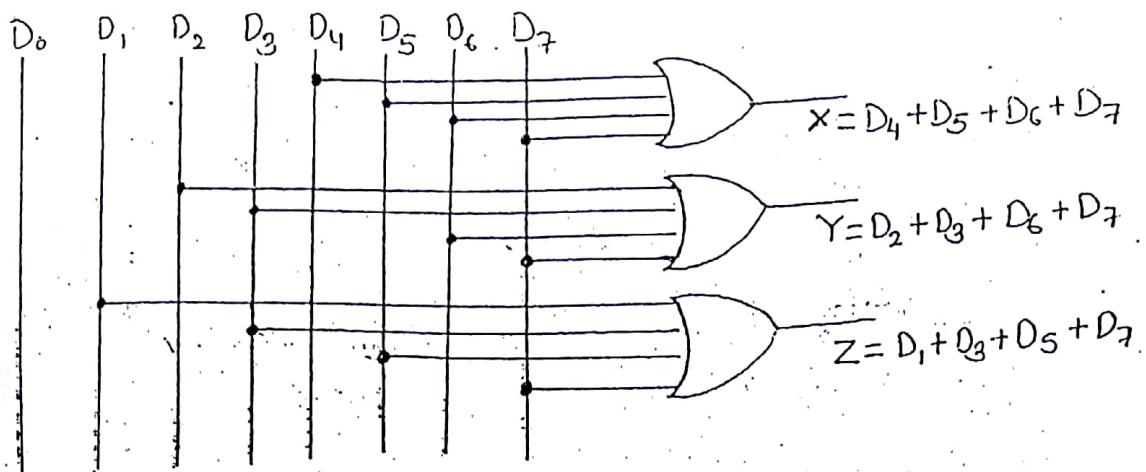


Fig: logic implementation of octal to Binary encoder

Here, D<sub>0</sub> is not connected to any OR gate, the binary output must be all 0 in this case. An all 0's output is also obtained when all inputs are 0.

→ Priority Encoder: These encoder establish an input priority to ensure that the highest-priority input line is encoded. eg. If both D<sub>2</sub> & D<sub>5</sub> are logic 1 simultaneously, the output will be 101 because D<sub>5</sub> has a highest priority over D<sub>2</sub>.

# Multiplexer (many input select 2<sup>n</sup> output)  
(Many to one)

→ A digital multiplexer is a combinational circuit that selects binary information from one of many input lines and directs it into a single output line.

→ The selection of a particular input line is controlled by a set of selection lines.

→ Normally, there are 2<sup>n</sup> input lines and n selection lines whose bit combinations determine which input is selected.

→ A multiplexer is also called a data selector since it selects one of many inputs & gives the binary information to the output line.

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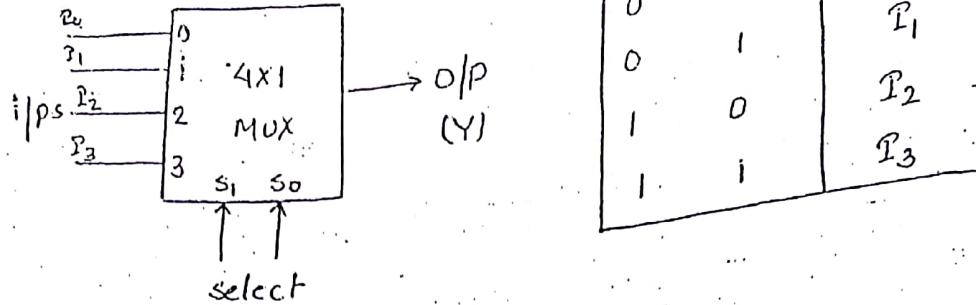


Fig: Block diagram

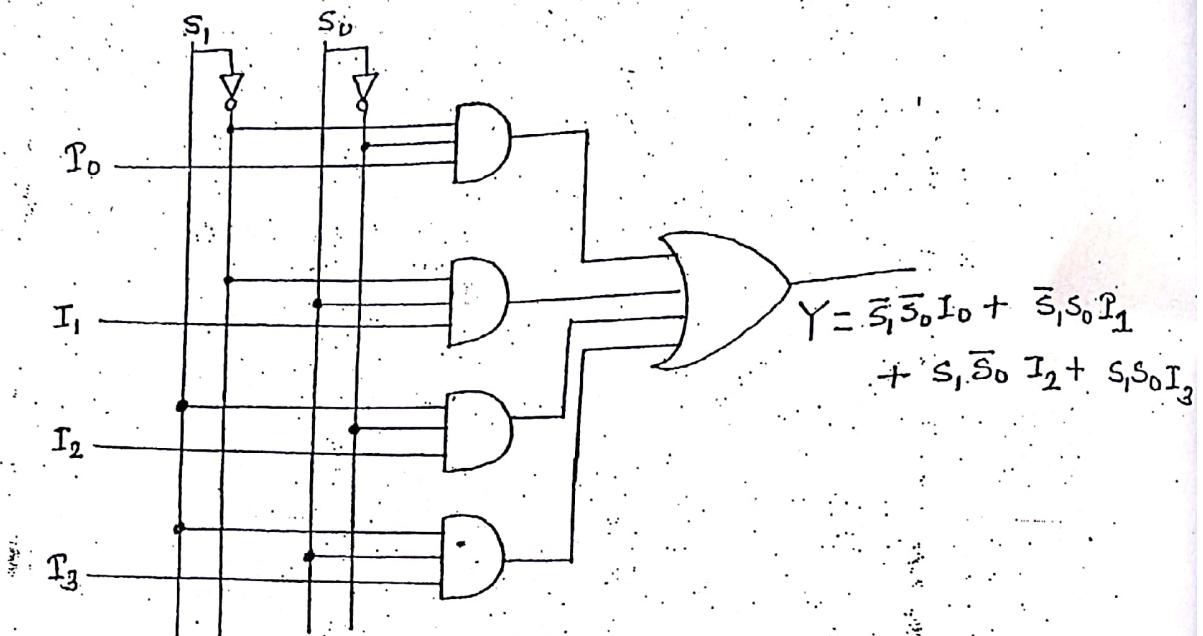


Fig: 4x1 multiplexer

→ It is used for connecting two or more sources to a single destination among computer units & it is useful for constructing a common bus system. It can be used to implement any boolean function.

Boolean function implementation of Multiplexer

$$F(A_1, B_1, C) = \Sigma(1, 3, 5, 6)$$

→ If we have a Boolean function of  $n+1$  variables, we take  $n$  of these variables and connect them to the selection lines of a multiplexer. The remaining single variable of a function is used for the inputs of the multiplexer.

→ If A is this single variable, the inputs of the multiplexer are chosen to be either A or A' or 1 or 0. By using these four values for the inputs and by connecting the other variables to the selection lines, one can implement any Boolean function with a MUX.

Minterm	A	B	C	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

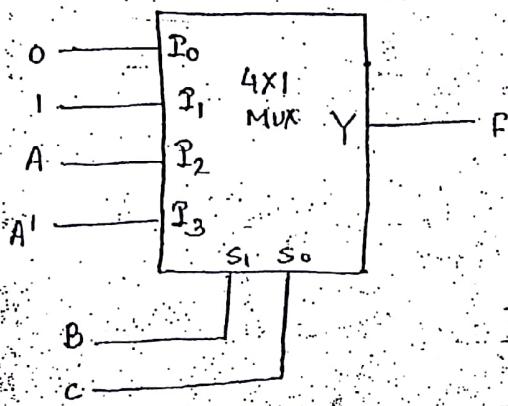
a) Truth table

	I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>	I <sub>3</sub>
A'	0	1	2	3
A	4	5	6	7
	0	1	A	A'

A = 1 for 4, 5, 6 & 7

A = 0 for 0, 1, 2 & 3

b) Implementation table



c) MUX implementation

For implementation table

→ If the two minterms in a column are not circled, apply 0 to the corresponding multiplexer input.

→ If the two minterms are circled, apply 1 to the corresponding multiplexer input.

→ If the bottom minterm is circled and the top is not circled, apply A to the corresponding multiplexer i/p.

→ If the top minterm is circled and the bottom is not circled, apply A' to the corresponding multiplexer i/p.

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~~Implement the Boolean function with a MUX~~

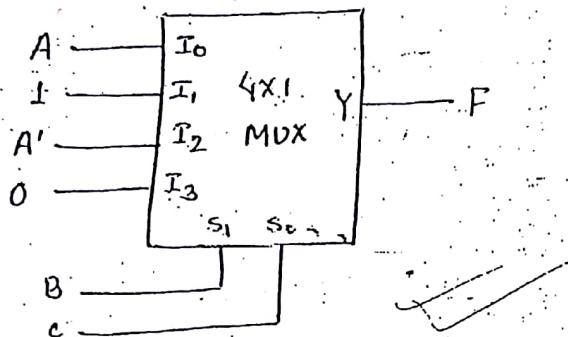
$$f(A, B, C) = \sum (1, 2, 4, 5)$$

minterm	A	B	C	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	1
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	0

	I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>	I <sub>3</sub>
A'	0	1	2	3
A	4	5	6	7
A	1	A'	0	0

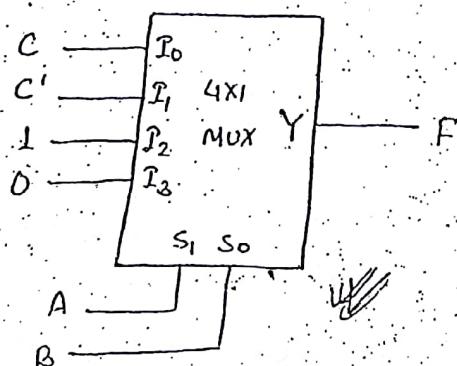
b) Implementation table

a) Truth table



If we take A, B as selection inputs then,

	I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>	I <sub>3</sub>
C'	0	2	4	6
C	1	3	5	7
C	C'	1	0	



Multiplexer implementation

Boolean function  $F(W, X, Y, Z) = \sum(1, 2, 5, 7, 11, 15)$   
Implement using 8 to 1 MUX.

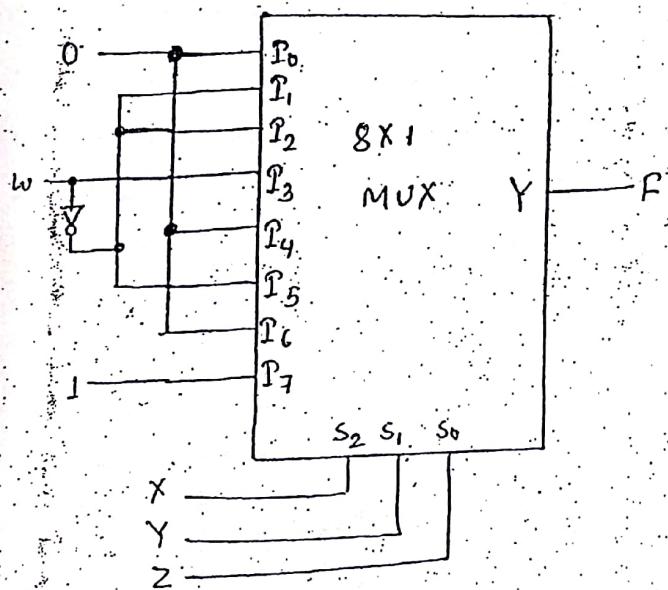
soln:

minterm	w	x	y	z	F
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	1
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	1

w'	P <sub>0</sub>	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	P <sub>6</sub>	P <sub>7</sub>
w	1	0	1	2	3	4	5	6
w	8	9	10	11	12	13	14	15
w'	0	w'	w'	w	0	w'	0	1

b) Implementation table

a) Truth table



Multiplexer implementation

- # Demultiplexer
- A demultiplexer is a circuit that receives information on a single line & transmits this information on 2^n possible output lines.
  - The selection of a specific output line is controlled by the bit values of n selection lines.

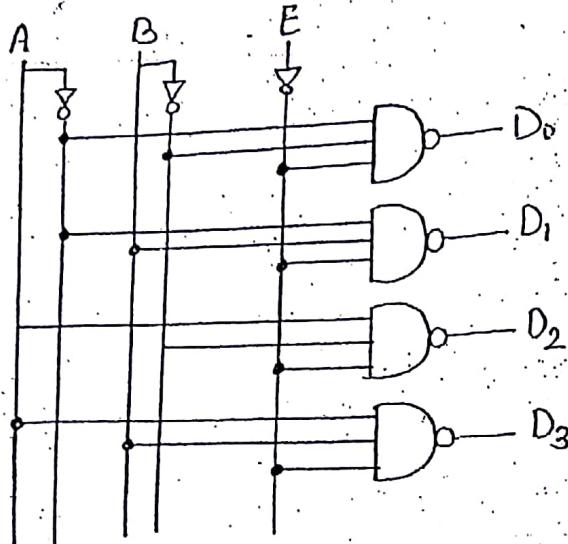
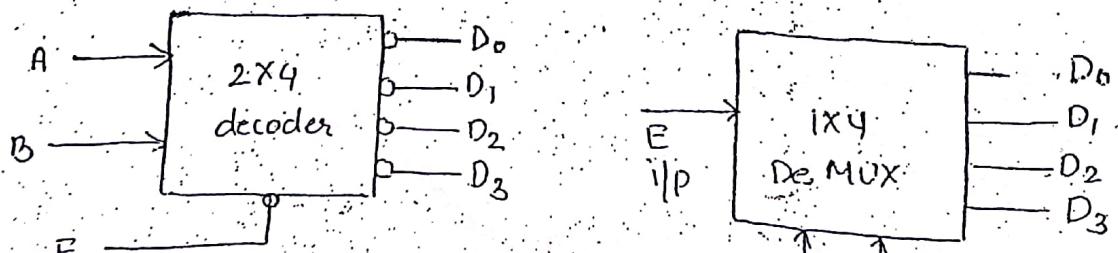


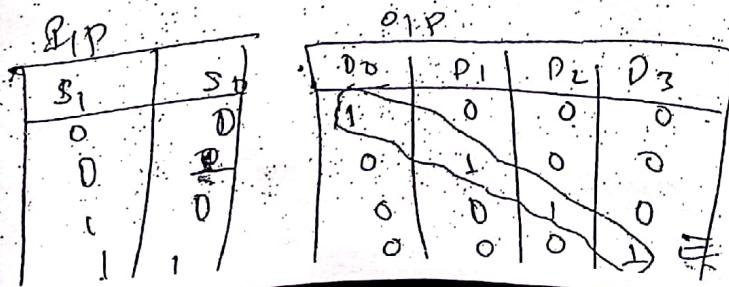
Fig: Logic Diagram of a 2 to 4 line decoder with enable (E) input.

E	A	B	D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>
-1	X	X	1	1	1	1
0	0	0	0	1	1	1
0	0	1	1	0	1	1
0	1	0	1	1	0	1
0	1	1	1	1	1	0

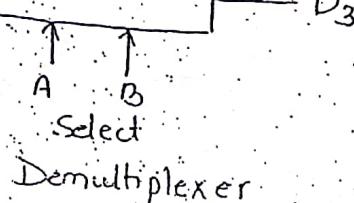
Pruth table



Decoder with enable



\* It is also called distributor  
\* It is similar to demux perform one to many output. Mux N to device but demux is device.



Demultiplexer

→ A decoder can function as demultiplexer if the E(enable) line is taken as a data input & lines A & B are taken as the selection lines.

→ The single input variable E has a path to all four outputs, but the input is directed to only one of the o/p lines at a time, which is selected by selection line.  
for eg.

→ If selection, AB = 00, o/p D<sub>0</sub> will have the same data as the i/p value E has.

→ If AB = 10, o/p D<sub>2</sub> has the same data as the i/p value E has.

→ A decoder with an enable i/p is referred to as a decoder / demultiplexer. It is the enable input that makes the ckt a demultiplexer.

→ Decoder/demultiplexer ckt can be connected to form a larger decoder ckt.

→ two 3x8 decoder with enable inputs connected to form a 4x16 decoder

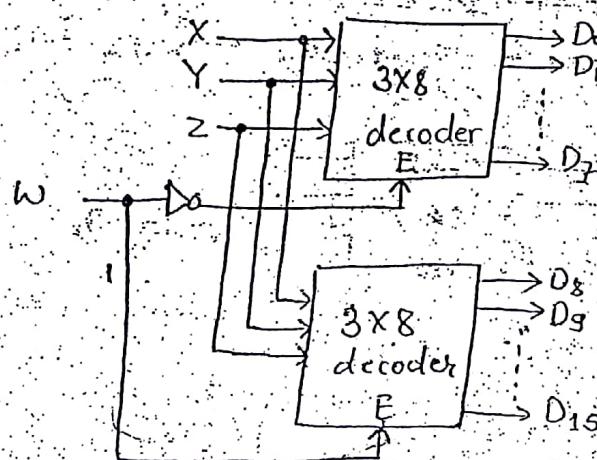


Fig: A 4x16 decoder constructed with two 3x8 decoders

→ When w=0, the top decoder is enabled and the bottom is disabled.

→ when w=1, the bottom one is enabled if the top is disabled.

Hence, the enable lines are a convenient feature to two or more IC package for the purpose of expand digital function into a similar function with more outputs.

## # Programmable Logic Device (PLD)

- A programmable logic device (PLD) is an integrated circuit with internal logic gates that are connected by electronic fuses.
- Programming the device involves the blowing of fuses along the paths that must be disconnected so as to achieve a particular configuration.
- The initial state of a PLD has all the fuses intact.
- Programming the device involves blowing of internal fuses to achieve desired logic function.

## \* Simple Programmable Logic Devices (SPLDs)

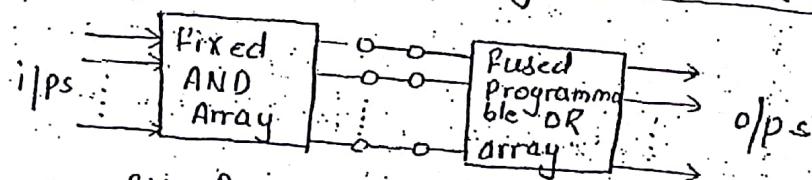


Fig: Programmable Read Only Memory (PROM)

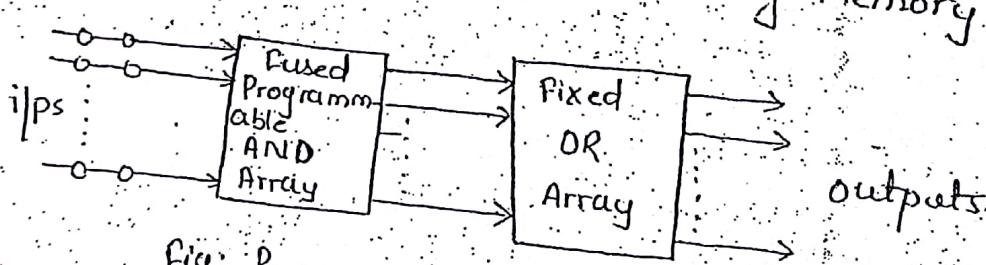


Fig: Programmable Array Logic (PAL)

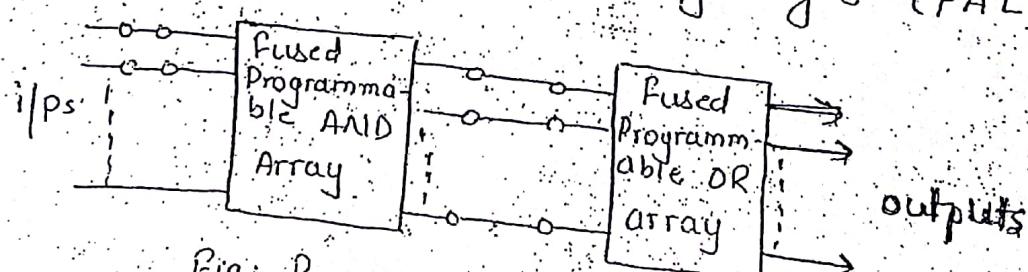


Fig: Programmable Logic Array (PLA)

## Read Only Memory (ROM)

- It consists of n inputs lines & m output lines.
- Each bit combination of the i/p variable is called an address.
- Each bit combination that comes out of the o/p lines is called a word.
- An address is essentially a binary numbers that denotes one of the minterms of n variables.
- The numbers of distinct address possible with n i/p variables as  $2^n$ .
- An o/p word can be selected by a unique address and since there are  $2^n$  distinct addresses in a ROM.
- There are  $2^n$  distinct words which are said to store in the unit.
- A ROM is characterised by the number of word  $2^n$  and the number of bits per word m.
- Internally, the ROM is a combinational logic ckt with AND gates connected as a decoder and a number of OR gates equal to the number of output in the unit.

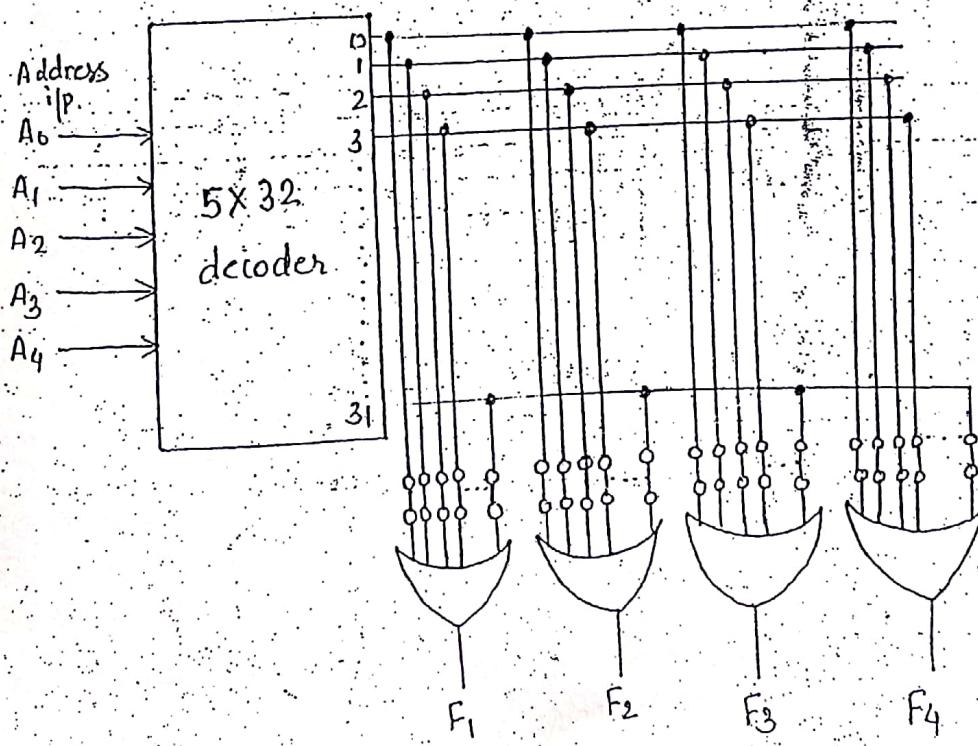


fig: logic Construction of a  $32 \times 4$  ROM  
128 bit

## Combinational logic Implementation

Example:

1) Implement  $F_1(A_1, A_0) = \Sigma(1, 2, 3)$

$F_2(A_1, A_0) = \Sigma(0, 2)$  with a ROM

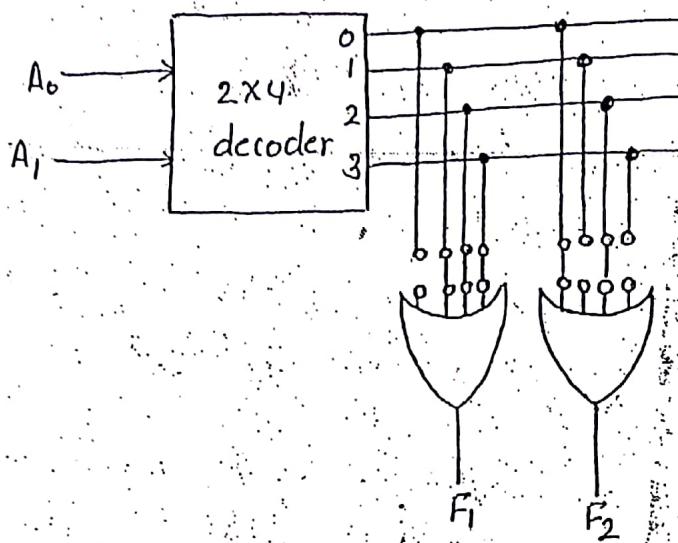


Fig: Combinational-ckt implementation with a  $4 \times 1$  ROM with AND-OR gates.

2) Implement

$$F_1(A_1, B, C) = \Sigma(0, 1, 5, 7)$$

$$F_2(A_1, B, C) = \Sigma(2, 3, 4)$$

$$F_3(A_1, B, C) = \Sigma(0, 1, 6)$$

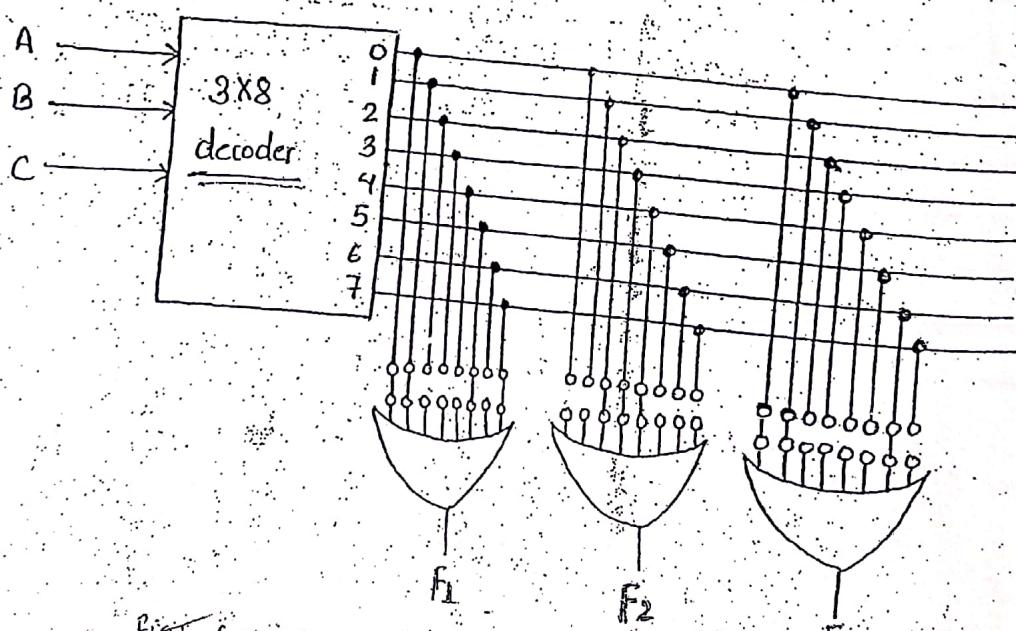


Fig: Combinational-ckt implementation with a  $8 \times 3$  ROM with AND-OR gates.

## # Programmable Logic Array (PLA)

- A PLA is similar to a ROM in concept but the PLA does not provide full decoding of the variables and does not generate all the minterms as in the ROM.
- In PLA, the decoder is replaced by a group of AND gates, each of which can be programmed to generate a product term of the input variables.
- The AND and OR gates inside the PLA are initially fabricated with fuses among them.
- The specific Boolean functions are implemented in sum of products form by blowing appropriate fuses and leaving the desired connection.
- The size of the PLA is specified by the number of inputs, the number of product terms and the number of outputs.
- A typical PLA has 16 inputs, 48 product terms & 8 outputs. Then the no. of programmed fuses is  

$$[2n \times K + Kxm + m]$$
  

$$= [2 \times 16 \times 48 + 48 \times 8 + 8]$$

where as that of ROM is  $2^n \times m$ .

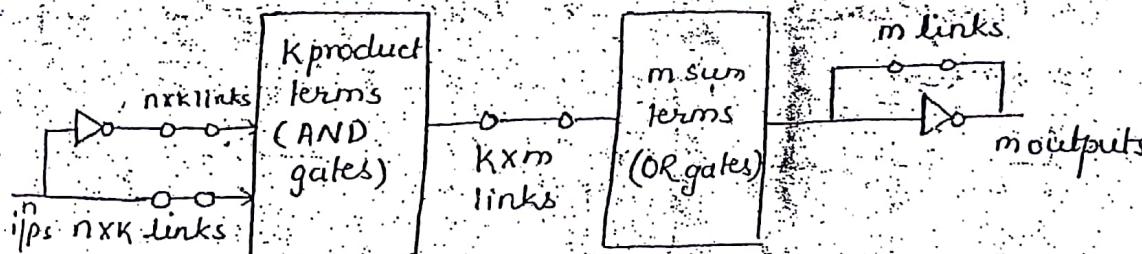


Fig: PLA block diagram

3  
2

Example Implement

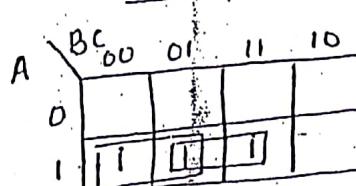
$$F_1 = AB' + AC$$

$$F_2 = AC + BC$$

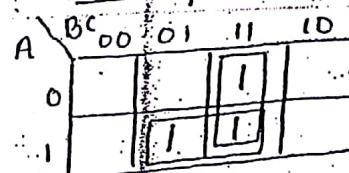
A	B	C	$F_1$	$F_2$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	1	1
1	1	0	0	0
1	1	1	1	1

Truth table

$$F_1 = \Sigma(4, 5, 7), F_2 = \Sigma(3, 5, 7)$$

K-map for  $F_1$ 

$$F_1 = AB' + AC$$

K-map for  $F_2$ 

$$F_2 = BC + AC$$

Product terms	Inputs			Outputs	
	A	B	C	$F_1$	$F_2$
$AB'$ (1)	1	0	-	1	-
$AC$ (2)	1	-	1	-	1
$BC$ (3)	-	1	1	-	-
				T	T/C

~~T~~ implies output dictates that the link across the output inverters remains in place.

~~C~~ (complement) - specifies that the corresponding link be broken.

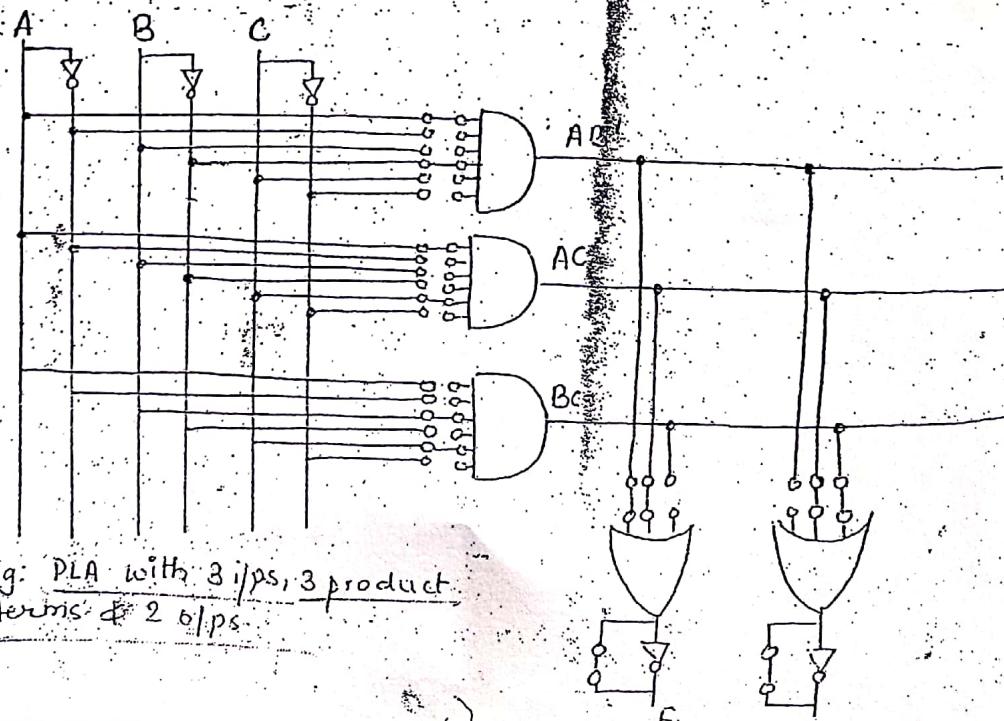


Fig: PLA with 3 inputs, 3 product terms & 2 outputs

## # Programmable Logic Array (PLA)

- A PLA is similar to a ROM in concept but the PLA does not provide full decoding of the variables and does not generate all the minterms as in the ROM.
- In PLA, the decoder is replaced by a group of AND gates, each of which can be programmed to generate a product term of the input variables.
- The AND and OR gates inside the PLA are initially fabricated with fuses among them.
- The specific Boolean functions are implemented in sum of products form by blowing appropriate fuses and leaving the desired connection.
- The size of the PLA is specified by the number of inputs, the number of product terms and the number of outputs.
- A typical PLA has 16 inputs, 48 product terms & 8 outputs. Then the no. of programmed fuses is

$$[2n \times K + Kxm + m]$$

$$= [2 \times 16 \times 48 + 48 \times 8 + 8]$$

where as that of ROM is  $2^n \times m$ .

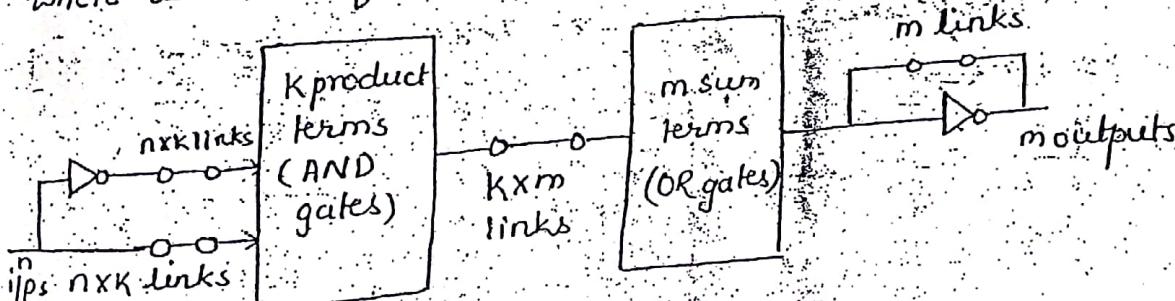


Fig: PLA block diagram

Example Implement

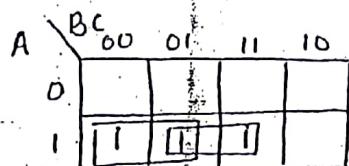
$$F_1 = AB' + AC$$

$$F_2 = AC + BC$$

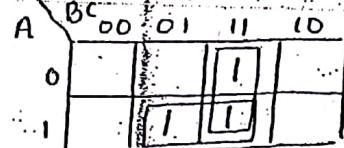
A	B	C	$F_1$	$F_2$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	1	1
1	1	0	0	0
1	1	1	1	1

Truth table

$$F_1 = \Sigma(4, 5, 7), F_2 = \Sigma(3, 5, 7)$$

K-map for  $F_1$ 

$$F_1 = AB' + AC$$

K-map for  $F_2$ 

$$F_2 = BC + AC$$

Product terms	Inputs			Outputs	
	A	B	C	$F_1$	$F_2$
$AB'$ (1)	1	0	-	1	-
$AC$ (2)	1	-	1	-	1
$BC$ (3)	-	1	1	-	-
				T	TIC

T implies output dictates that the link across the o/p inverters remains in place.

C (complement) - specifies that the corresponding link be broken.

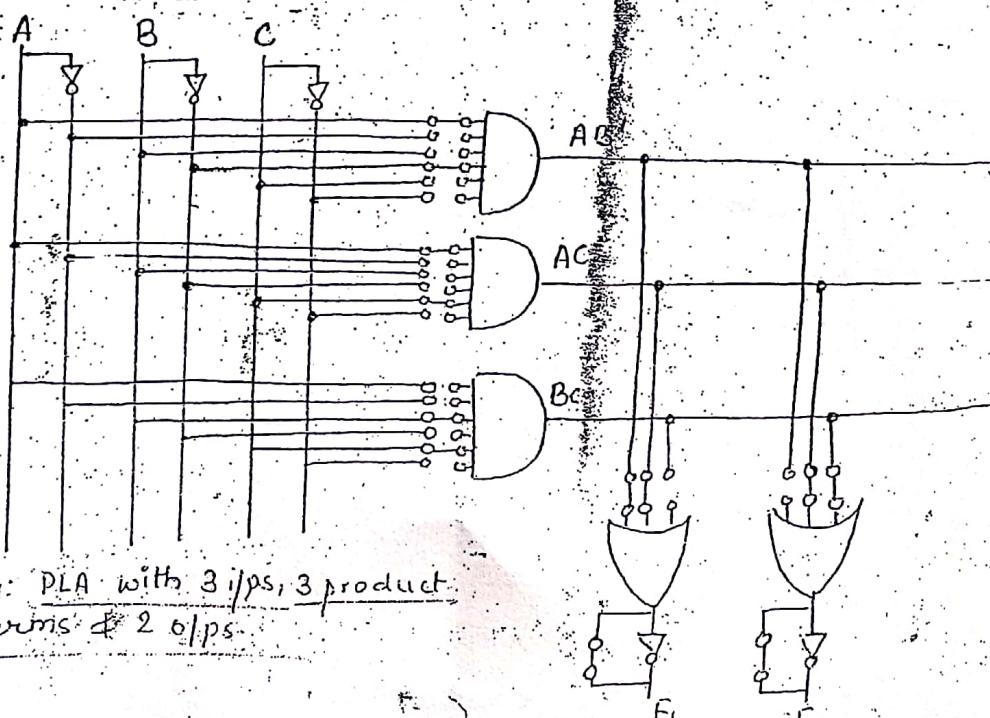


Fig: PLA with 3 ips, 3 product terms & 2 o/p's

A combinational circuit is defined by the functions

$$F_1(A, B, C) = \Sigma(3, 5, 6, 7)$$

$$F_2(A, B, C) = \Sigma(0, 2, 4, 7)$$

Implement the circuit with a PLA having three inputs, four product terms and two outputs.

Soln-

K-map for  $F_1$

		BC		00		01		11		10	
		A	B	0	1	0	1	0	1	0	1
0				1	1	1	1	1	1	1	1
1				1	1	1	1	1	1	1	1

$$F_1 = BC + AC + AB$$

K-map for  $F_2$

		BC		00		01		11		10	
		A	B	0	1	0	1	0	1	0	1
0				1	1	1	1	1	1	1	1
1				1	1	1	1	1	1	1	1

$$F_2 = A'C' + B'C' + ABC$$

→ Here total no. of product terms are 6 but we should use only four. Therefore let us see from pos soln.

		BC		00		01		11		10	
		A	B	0	1	0	1	0	1	0	1
0				0	0	0	0	0	0	0	0
1				0	0	0	0	0	0	0	0

$$F'_1 = A'B'C' + A'C' + B'C'$$

		BC		00		01		11		10	
		A	B	0	1	0	1	0	1	0	1
0				0	0	0	0	0	0	0	0
1				0	0	0	0	0	0	0	0

$$F'_2 = A'C + B'C + ABC'$$

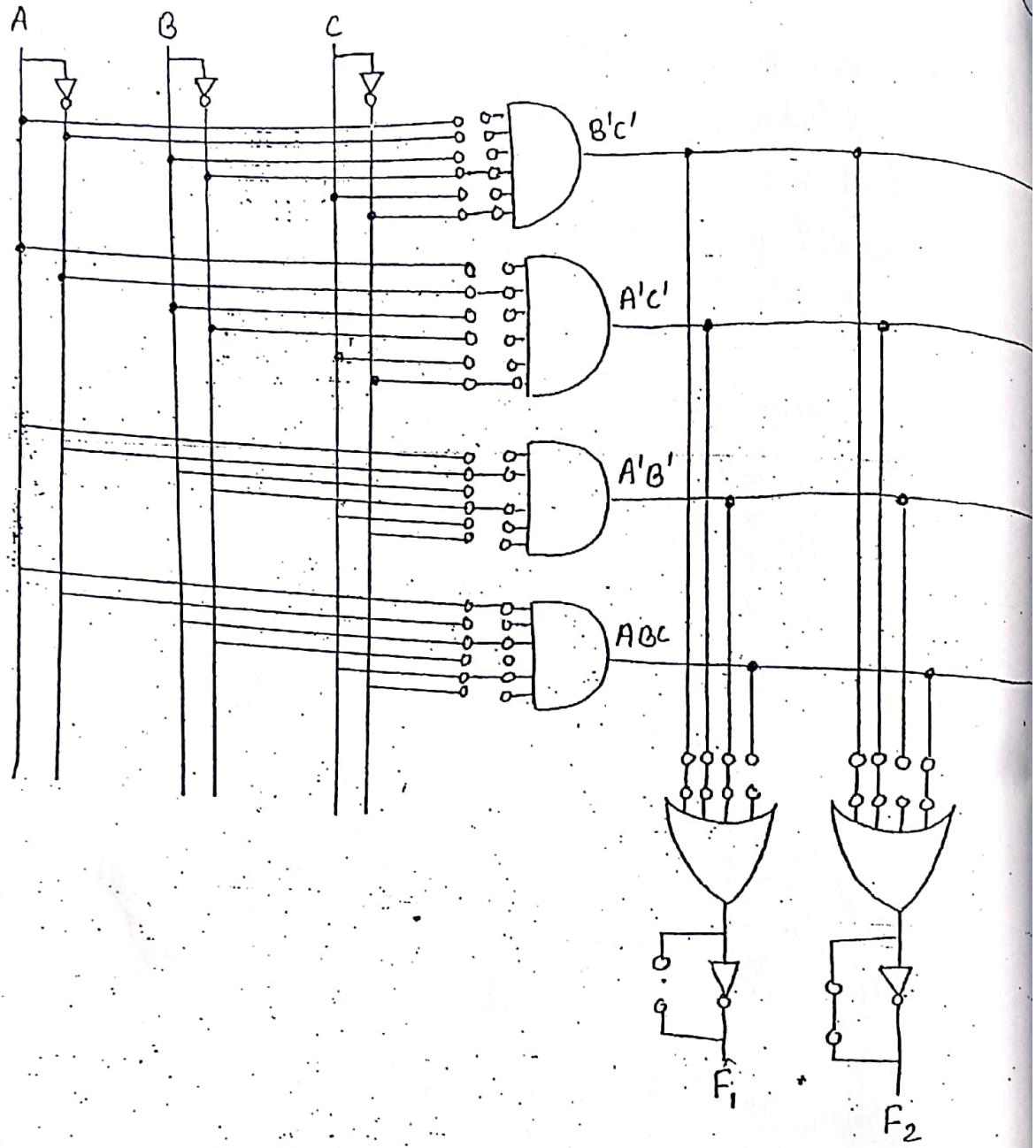
→ For four product terms we can use  $F'_1$  &  $F'_2$

PLA program table

Product terms	Inputs			Outputs			
	A	B	C	$F_1$	$F_2$		
1 $B'C'$	-	0	0	1	1		
2 $A'C'$	0	-	0	1	0		
3 $A'B'$	0	0	-	0	1		
4 $ABC$	1	1	1	0	0	T	TIC

$$F_1 = (B'C' + A'C' + A'B')$$

$$F_2 = B'C' + A'C' + ABC$$



## Chapter - 7

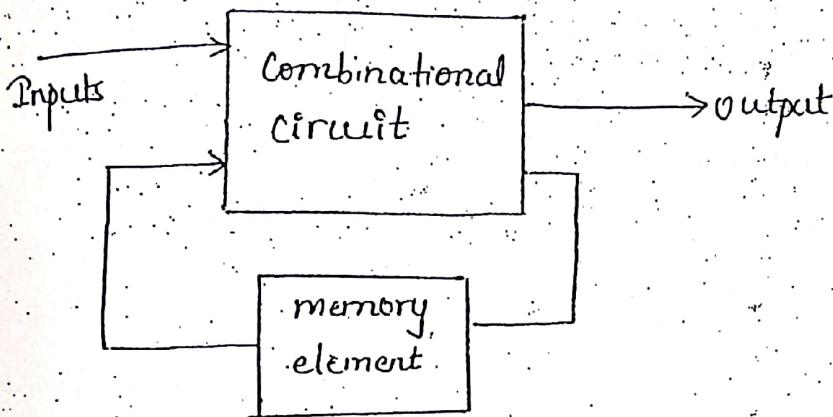
### Sequential logic

Prepared By:  
Buddha Laxmi Mahayani

①

#### Sequential logic

→ A sequential logic circuit consists of a combinational circuit to which memory elements are connected to form a feedback path.



[in chapter 5 photo copi  
it is written that  
sequential ckt isn't  
the box but that  
is wrong. It must  
be combinational  
ckt.]

Fig: Block diagram of a sequential circuit

→ The outputs of a sequential circuit depends on present inputs and past or previous state. Eg. flipflop, registers, counters.

#### Types of sequential logic :-

##### i) Synchronous:

→ In this system signals that affect the memory elements only at discrete instants of time.

→ Synchronous is achieved by a time device called a master-clock generator which generates a periodic train of clock pulses.

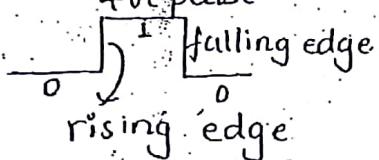
→ Synchronous sequential circuits that use clock pulses in the input of memory elements are called clocked sequential circuit.

## 2) Asynchronous:

→ The behaviour of an asynchronous sequential logic system depends upon the order in which its input signals change and can be affected at any instant of time.

### Clock

→ A clock is a control signal that periodically makes a transition from a 0 to 1 & then back to 0 again. We usually denote the clock by the symbol CLK or CP.



### Flipflop

→ Flipflops are binary cells, capable of storing one bit information.

→ A flipflop circuit has two outputs: one for the normal value & one for the complement value of the bit stored in it.

→ Binary information can enter in a flipflop in a variety of ways, a fact which gives rise to different types of flipflops.

### Basic flip-flop circuit

#### SR flipflop or RS flipflop

R (Reset)

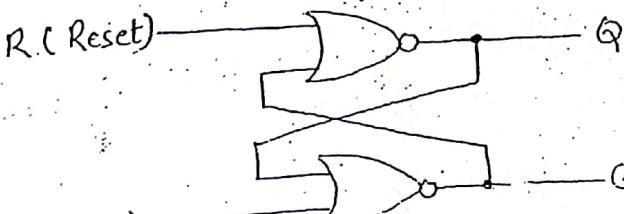


Fig: Active high i/p SR latch

→ An active high i/p SR (Set-Reset) latch is formed with crossed complement

#### Note

\* if  $S = 1 \rightarrow$  Set  
i.e.  $Q = 1, \bar{Q} = 0$

\* if  $R = 1 \rightarrow$  Reset  
i.e.  $Q = 0, \bar{Q} = 1$

\* if  $S = 0 \& R = 0$  No change

(2)

Inputs		Output		Comment
S	R	Q	$\bar{Q}$	
0	0	NC	NC	No change
0	1	0	1	latch reset
1	0	1	0	latch set
1	1	0	0	invalid condition

Truth table for active-high input SR latch

### # $\overline{S}\overline{R}$ flip flop

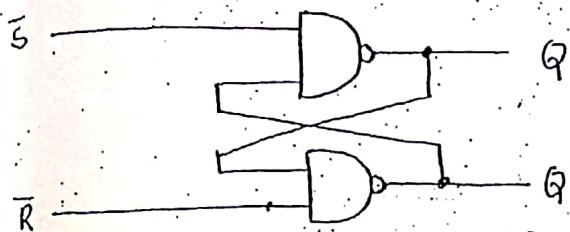


Fig: Active low input  $\overline{S}\overline{R}$  latch

→ An active-low input S-R latch is formed with cross-coupled NAND gates.

Input	Output	Comment
S, $\overline{R}$	Q, $\bar{Q}$	
0, 0	1, 1	invalid condition
0, 1	1, 0	set
1, 0	0, 1	Reset
1, 1	NC, NC	No change

if  $S=0 \& \overline{R}=0$  invalid  
 $\overline{S}=1 \& \overline{R}=1$  NC  
 $\overline{S}=1$  Reset  
 $\overline{R}=1$  Set

Truth table for active-low input  $\overline{S}\overline{R}$  latch

### # Clocked SR flip flop

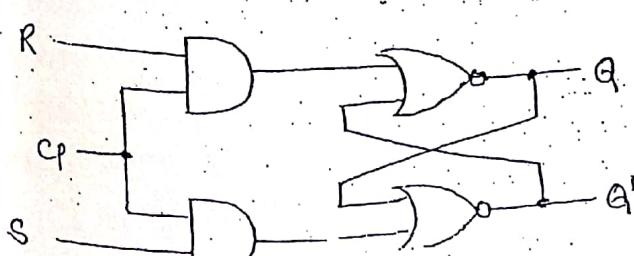
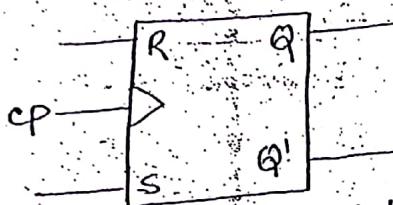


Fig: logic Diagram

Inputs			outputs		Comment
S	R	CP	Q	$Q'$	
0	0	X	NC	NC	No change
0	1	↑	0	1	Reset
1	0	↑	1	0	Set
1	1	↑	?	?	Invalid

Truth table



Graphical Symbol

Q(t)		SR	00	01	11	10
0	0			X	1	
1	1			X	1	

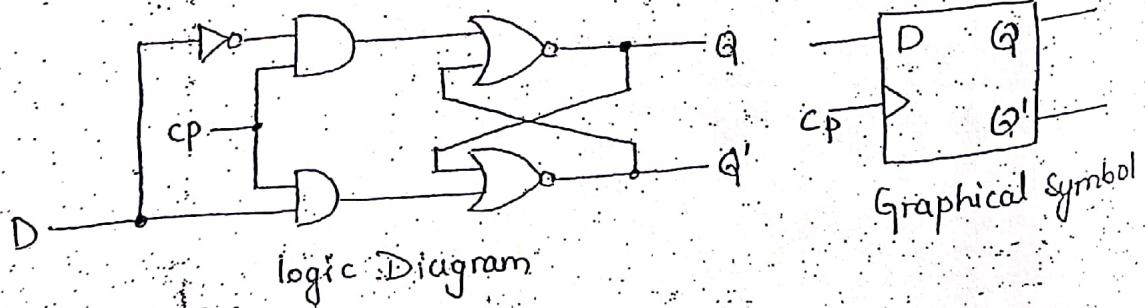
$$Q(t+1) = S + QR'$$

Characteristic Equation

Q(t)	S	R	Q(t+1)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	invalid X
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0
1	1	1	invalid X

Characteristic table

~~SR~~ flip flop : (Ability to transfer 'Data')  
 $\Rightarrow$  It is an RS flip flop with an inverter in the R input



Q(t)	D	Q(t+1)
0	0	0
0	1	1
1	0	0
1	1	1

Characteristic table

Q(t)	D
0	1
1	1

$$Q(t+1) = D$$

Characteristic Equation

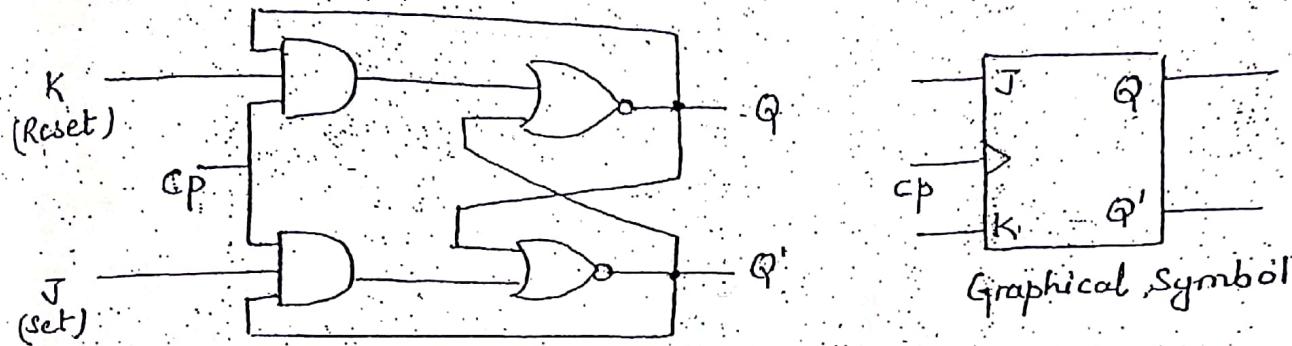
(3)

## JK flip flop

→ It is a refinement of RS flip flop. The indeterminate (invalid) state of RS flip flop is defined in JK flip flop.

→ Input J & K behaves like S & R i.e. Set & Reset respectively.

J → set      K → reset



logic diagram

Q	S	R	Q(t+1)	comment
Q	J	K	Q(t+1)	
0	0	0	0	No change
0	0	1	0	Reset
0	1	0	1	Set
0	1	1	1	Complement or toggle
1	0	0	1	No change
1	0	1	0	Reset
1	1	0	1	Set
1	1	1	0	Toggle

characteristic table

JK	00	01	11	10
Q				
0			1	1
1	1			1

characteristic Equation

$$Q(t+1) = Q'J + QK'$$

when  $J=0, K=0$  flip flop retains its previous state.

when  $J=0, K=1$  flip flop goes to reset state  $Q(t+1)=0$

when  $J=1, K=0$  flip flop goes to set state  $Q(t+1)=1$

when  $J=1, K=1$  flip flop complements its previous state.

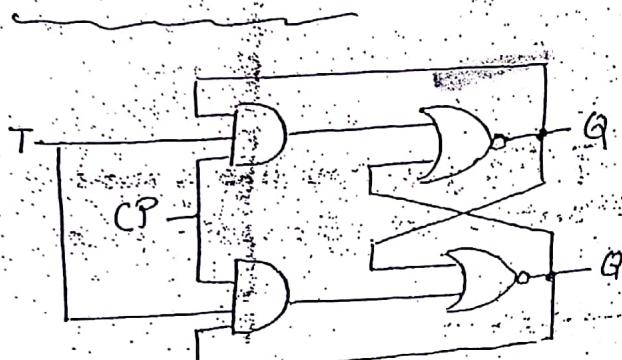
$$Q(t+1) = Q'$$

→ The feedback is taken from  $Q'$  to J so that the flip flop goes to set ...

→ similarly the feedback is taken from  $Q$  to  $K$  so that the flip flop goes to reset only when  $G=1$ .

### T - flip flop

→ T is the same input version of JK flip flop. The name T comes from the ability to toggle or complement the state when  $T=1$ .



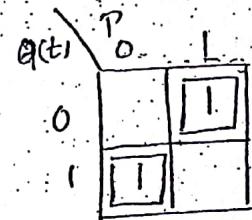
logic diagram.



Graphic symbol

$Q(t)$	T	$Q(t+1)$	Comment
0	0	0	No change
0	1	1	Toggle
1	0	1	No change
1	1	0	Toggle

characteristic table



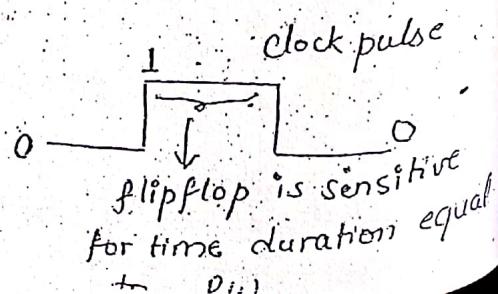
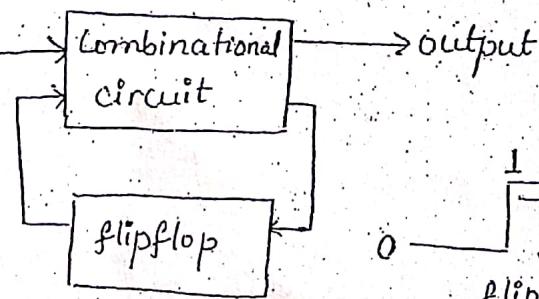
$$Q(t+1) = Q'T + Q'T'$$

Characteristic Eqn

### Triggering of flip flops

→ Clocked flip flops are triggered by pulses.

→ A pulse starts from an initial value of 0 goes momentarily to 1 & after a short time returns to its initial value.

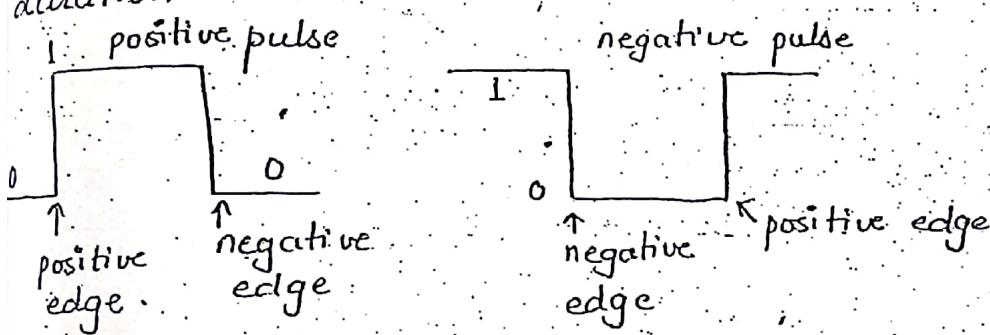


Note: The path can produce instability if the outputs of flipflops are changing while the outputs of the combinational circuit that goes to flip-flop inputs are being sampled by the clock pulse.

→ This timing problem can be prevented if the outputs of the flip flop do not start changing until the pulse input has returned to 0.

### Solution:

This problem can be solved if the flipflops are made sensitive to the pulse transition rather than the pulse duration.



→ A clock pulse may be either positive or negative.

→ The pulse goes through two signal transitions from 0 to 1 and the return from 1 to 0.

→ The positive transition is defined as positive edge & the negative transition is defined as the -ve edge. This definition applies also to negative pulse.

### Master-slave flipflop

(Pulse-triggered master-slave)

→ Data are entered into the flipflop at the leading edge of the clock pulse but the output does not reflect the input state until the trailing edge.

→ The pulse triggered master-slave flip-flop does not allow data to change while the clock pulse is active.

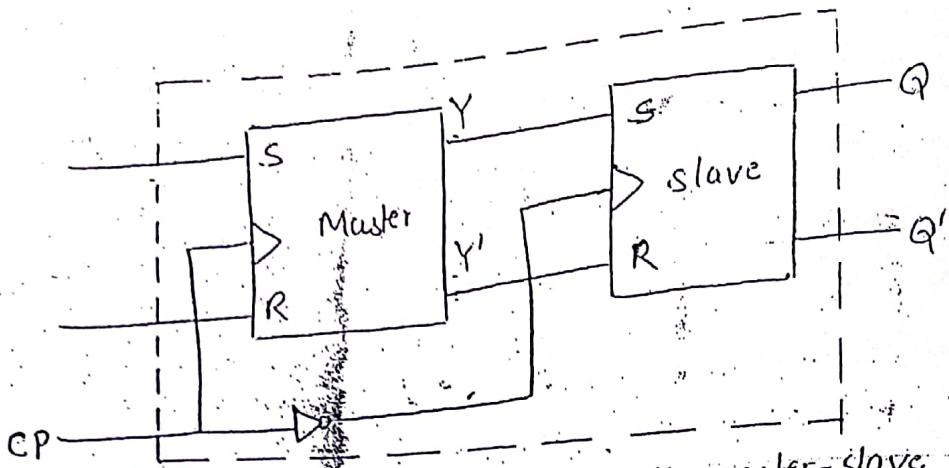


Fig. logic diagram of master-slave flip-flop

→ A master-slave flip-flop is constructed from two separate flip-flops, one circuit serves as a master and the another circuit as slave. The overall circuit is referred to as a Master-slave flipflop.

When  $CP = 0$

- The o/p of the inverter is 1, the slave flip-flop is enable &  $Q = Y$ , while  $Q'$  is  $Y'$ .
- the master is disabled because  $CP = 0$ .

When  $CP = 1$

- The information at the external R & S inputs is transmitted to the master flipflop. The slave flip-flop however is isolated as long as the pulse is at its 1 level.

→ when the pulse returns to 0, the master flip-flop is isolated. The slave flip-flop then goes to the same state as the master flipflop.

→ It shows that the state changes in all flip-flops coincide with the negative edge transition of the pulse. If an additional inverter is added bet'n the CP terminal & the input of the master flipflop, Master-slave flip-flop will be sensitive to the positive edge of the clock pulse. It is called positive edge triggering.

(5)

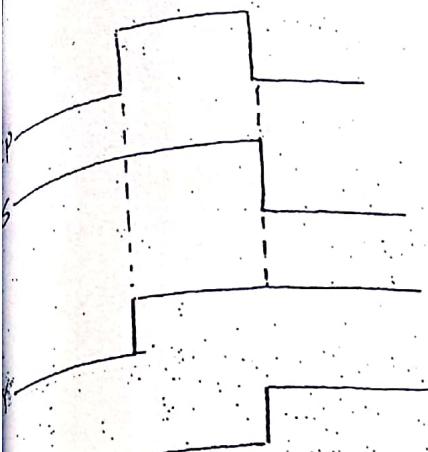
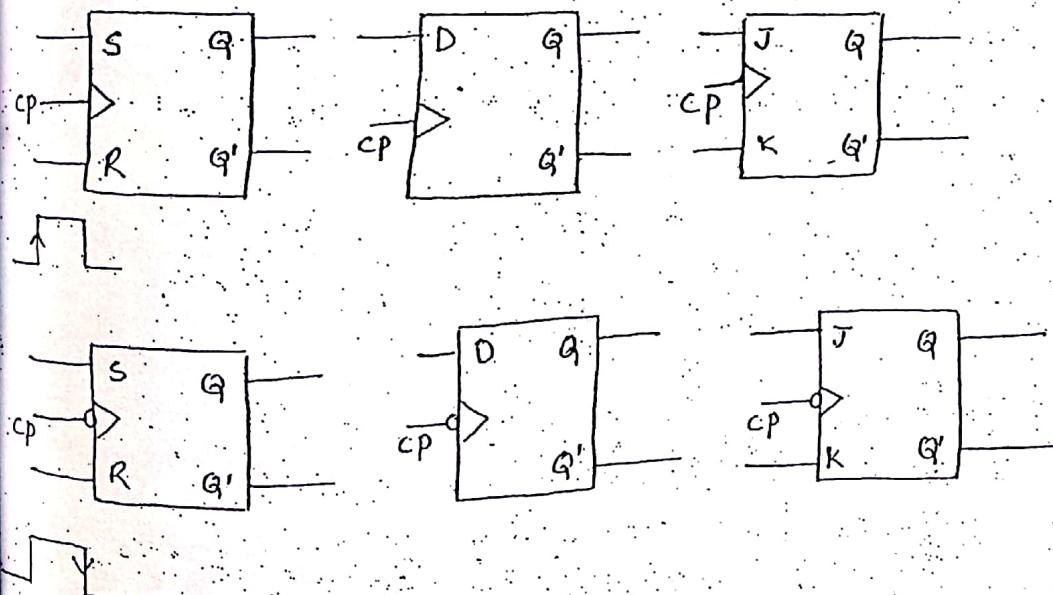


Fig: Timing relationships in a master-slave flip-flop

### Edge triggered flip-flop

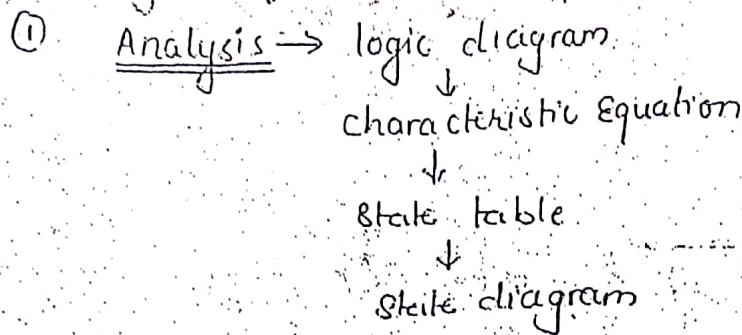
→ An edge triggered flip-flop changes states either at the positive edge (rising edge) or at the negative edge (falling edge) of the clock pulse & it is sensitive to its inputs only at the clock (cp) input. This triangle is called the dynamic input indicator "▷".



Truth table for a positive edge-triggered SR flip-flop

Input	Output	Comment			
S 0	R 0	CP X	Q NC	Q' NC	No change
0	1	↑	0	1	Reset
1	0	↑	1	0	Set
1	1	↑	?	?	Invalid condition

## III Analysis of clocked Sequential Circuits



(II) Design → Verbal statement

↓  
 state diagram

↓  
 state table

↓  
 characteristic equation

↓  
 logic diagram

→ The analysis of sequential circuit consists of obtaining a table & a state diagram for the time sequence of input, output & internal states. It is also possible to write Boolean expression that describe the behaviour of sequential circuit called characteristic eq<sup>n</sup> or state eq<sup>n</sup>.

State table :-

The time sequence of inputs, outputs & flip flops states are listed in state table.

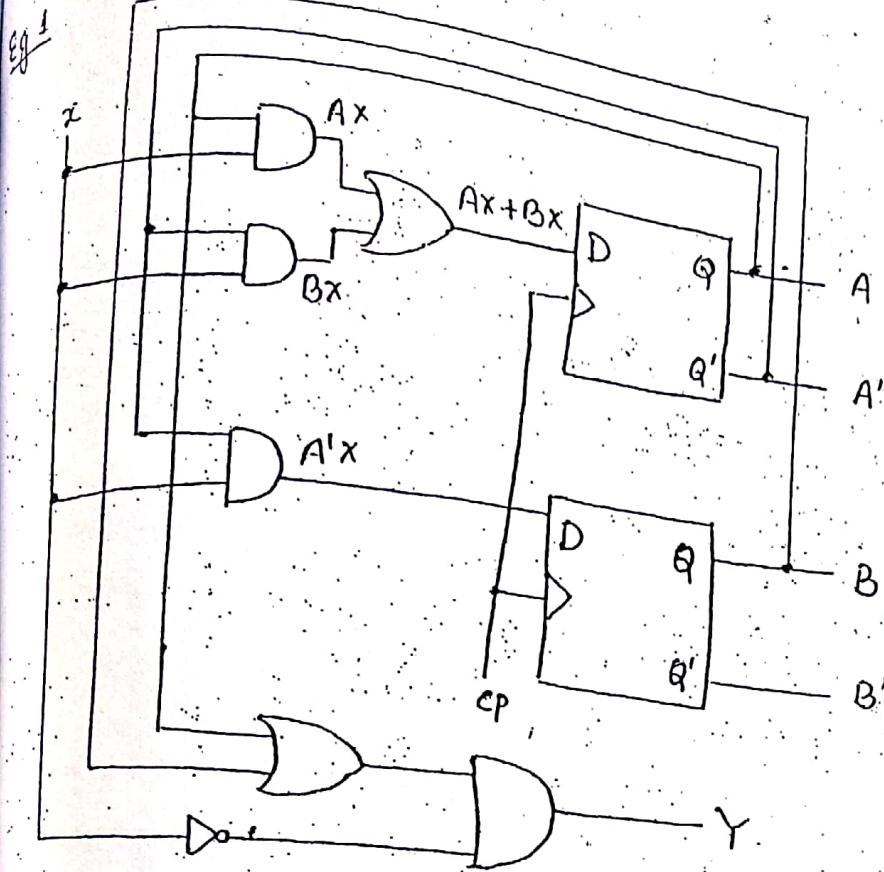
State diagram :-

→ The information available in the state table may be represented graphically in a state diagram.

→ If there are 'n' number of flip flop then there will be  $2^n$  number of states.

State Eq<sup>n</sup> or characteristic Eq<sup>n</sup>

→ A state equation is an algebraic expression that specifies the conditions for a flip flop state transition.



Characteristic eqn

$$D_A(t+1) = A(t)x(t) + B(t)x'(t)$$

$$D_B(t+1) = A'(t)x'(t)$$

$$\text{Output } Y(t+1) = [A(t) + B(t)]x'(t)$$

$$\Rightarrow D_A = Ax + Bx'$$

$$D_B = A'x'$$

$$Y = (A + B)x'$$

State table: 2 flipflops so  $2^2 = 4$  states

Present state		Next state		Output	
A	B	$x=0$	$x=1$	$x=0$	$x=1$
0	0	A = B	A' B	Y	Y
0	1	0 0	0 1	0	0
1	0	0 0	1 1	1	0
1	1	0 0	1 0	1	0

state diagram

Mealy Model :- output are the function of the present state & inputs.

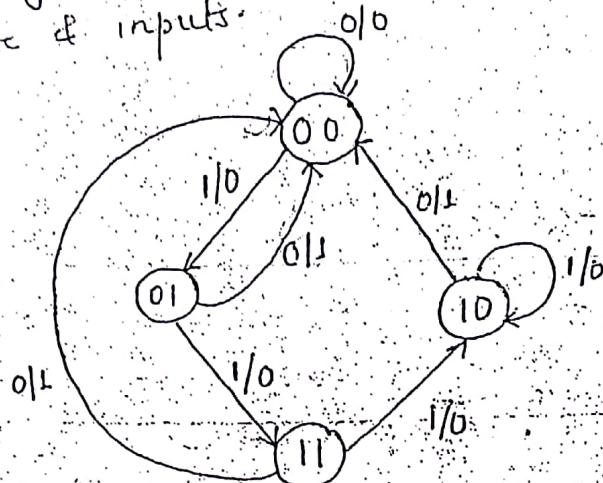
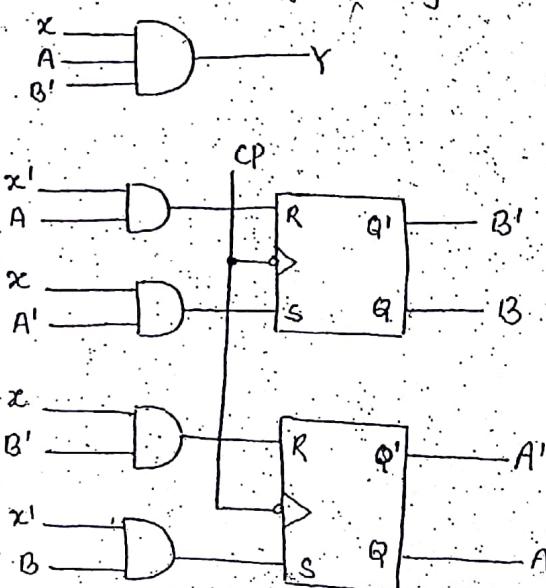


Fig: Mealy Model State diagram

In Moore Model :- Outputs are the function of the present state only [nothing is written along the direction line]

Eg 2:- Analysis the following clocked sequential logic



State Equations

Characteristic eqn of SR flip flop is  
 $Q(t+1) = S + QR'$

Here,

$$R = Ax$$

$$S = xA'$$
 for B

$$R = \bar{x}B'$$

$$S = \bar{x}B$$
 for A

73

(73)

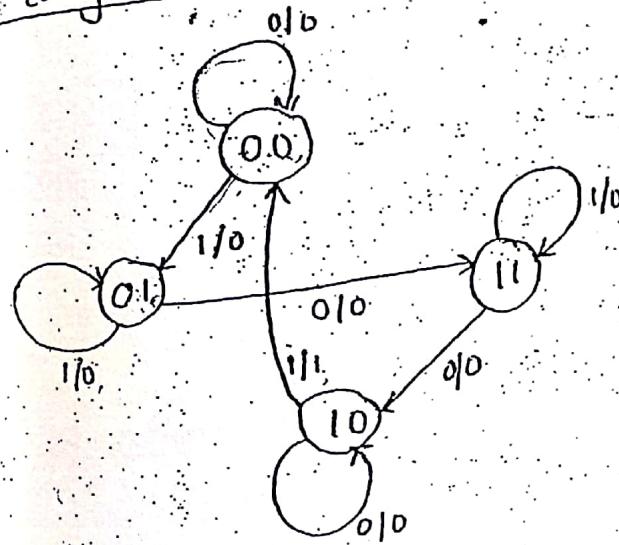
$$\begin{aligned}
 A(t+1) &= Bx' + (B'x)^T A = AB + A'n' + Bn' \\
 B(t+1) &= A'x + (A'x')^T B = A'x + A'B + Bx
 \end{aligned}
 \quad \left. \begin{array}{l} \text{These can also} \\ \text{be identified} \\ \text{by state table} \\ \text{using K-map.} \end{array} \right\}$$

$Y = xAB'$

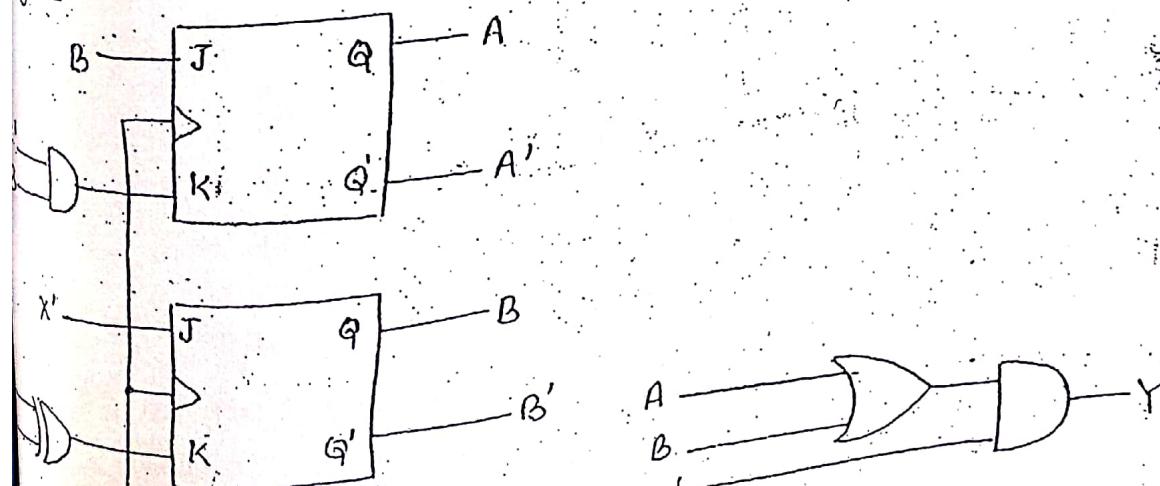
state table  
2 flipflops so  $2^2 = 4$  states

Present state B x	Next state $x=0$ $x=1$				Output	
	A	B	$A'$	$B'$	$x=0$	$x=1$
0	0	0	0	1	0	0
1	1	1	0	1	0	0
0	1	0	0	0	0	1
1	0	1	1	0	0	0

state diagram



Q.31



For JK flip flop

$$Q(t+1) = \bar{J}Q' + K'Q$$

$$A(t+1) = BA' + (x'B)'A$$

$$= A'B + Ax + AB'$$

$$B(t+1) = x'B' + (x \oplus A)'B$$

$$= x'B' + (x \otimes A)B$$

$$= x'B' + xAB + x'A'B$$

for A

$$J = B$$

$$K = x'B$$

for B

$$J = x$$

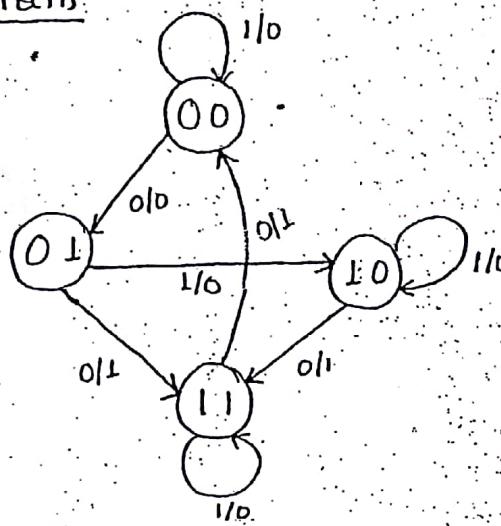
$$K = x \oplus A$$

State table

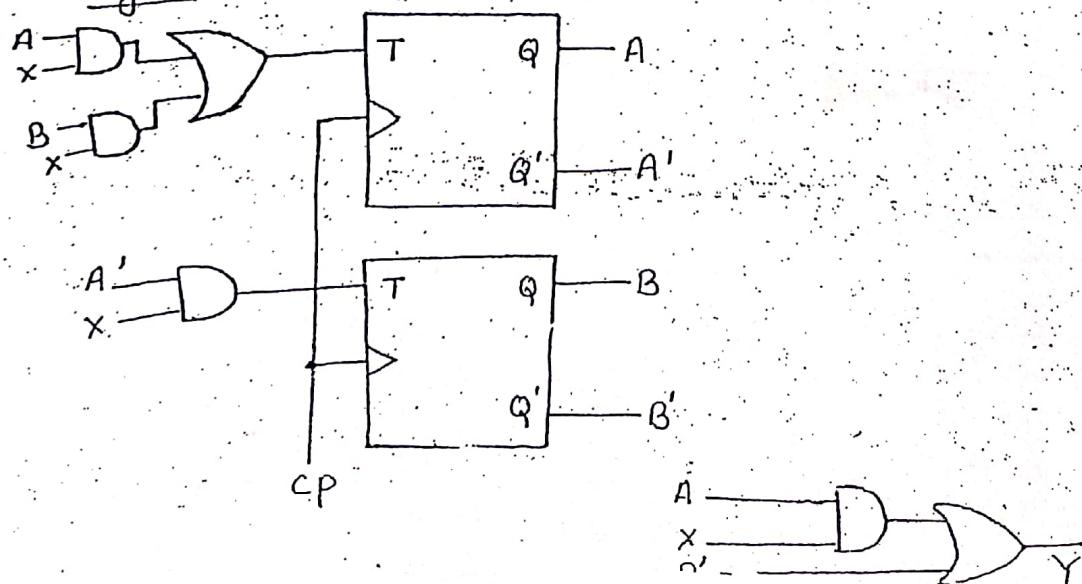
$$Y = (A+B)x'$$

Present state		Next state				Output	
A	B	x = 0	x = 1	A	B	A	B
0	0	0	1	0	0	0	0
0	1	1	1	1	0	1	0
1	0	1	1	1	0	1	0
1	1	0	0	1	1	1	0

State diagram



Eg: 4



for A

$$T = AX + BX = x(A+B)$$

(F5)

$$Y = AX + B'$$

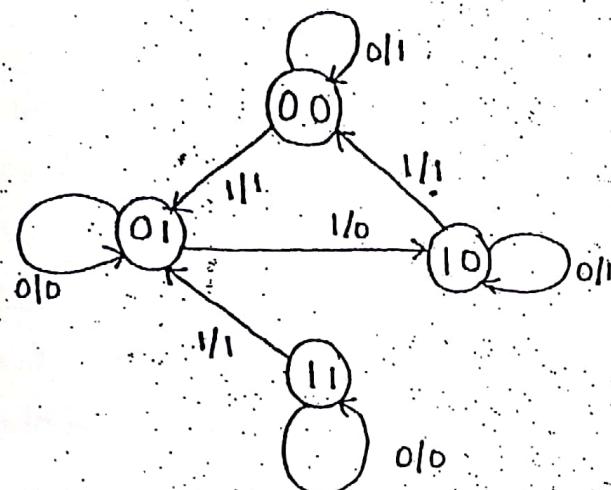
for B

$$T = A'x$$

state table

		Present State		Next State		Output	
A	B	$x=0$	$x=1$	$A$	$B$	$x=0$	$x=1$
0	0	0 0	0 1	0 0	0 1	0	1
0	1	0 1	1 1	0 1	1 0	1	1
1	0	1 0	1 0	1 0	0 0	0	0
1	1	1 1	0 0	1 1	0 1	1	1

State diagram



State equation

For T flip flop

$$T(t+1) = TQ' + T'Q$$

Here for A flip flop

$$A(t+1) = TA' + TA$$

$$= (AX + BX)A' + (AX + BX)'A$$

$$= A'BX + \bar{A}\bar{X} \cdot B \bar{X} A$$

$$= A'BX + (\bar{A} + \bar{X})(\bar{B} + \bar{X})A = A'BX + A\bar{X}\bar{B} + A\bar{X}$$

$$= A'BX + A\bar{B}'x' + A\bar{x}'$$

for B flip flop

$$B(t+1) = TB' + T'B = A'x B' + (A'x)'B$$

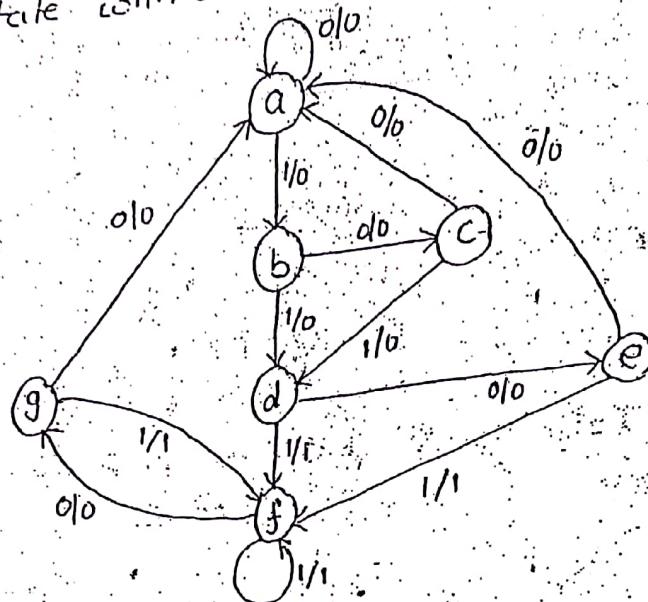
$$= A'B'x + (A + x')B$$

$$= A'B'x + AB + BX'$$

From state table.

### State Reduction & Assignment

Two states are said to be equal if they give the same output and goes to the same next state upon the application of same input. Hence we can replace one state with another.



state diagram

In the above state diagram, minimize the number of states. Consider the input sequence 0101001010 starting from the initial state a.

State: a a b c d e f f g f g  
 Input: 0 1 0 1 0 1 1 0 1 0 → (Keep it as your wish)  
 Output: a 0 0 0 1 1 0 1 0 → you can keep any number

initial state = a

input applied = 0

output = 0

so next state = a

2<sup>nd</sup>

initial state = a

input = 1

output = 0

so next state = b & so on

#### Step 1

Find out the two present states that go to the same next state & hence the same output for the both states of input.

State table

Present state	Next state		Output	
	$x=0$	$x=1$	$x=0$	$x=1$
a	a	b	0	0
b	c	d	0	0
c	a	d	0	0
d	e	f/d	0	1
e	a	f/d	0	1
f (cancelled)	g/e	f	0	1
g (cancelled)	a	f	0	1

→ Here stage e & g both go to a & f & have same o/p 0 & 1 for the input  $x=0$  &  $x=1$  respectively.

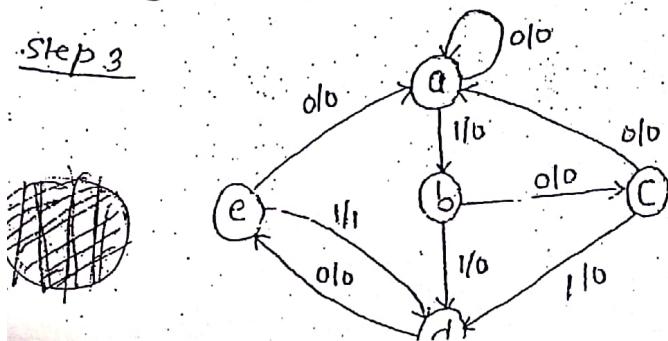
→ These states g & e are equivalent, so one can be cancelled.

Step 2:

The row with state g at present state is crossed out & state g at all other place is replaced by e each time it occurs, in the next state column.

Reduced state table

Present state	Next state		Output	
	$x=0$	$x=1$	$x=0$	$x=1$
a	a	b	0	0
b	c	d	0	0
c	a	d	0	0
d	e	d	0	1
e	a	d	0	1

Step 3

Step 4

State:

Input: a a b c d e d d e d e

Output: 0 1 0 1 0 1 1 0 1 0

Output: 0 0 0 0 0 1 1 0 1 0

Conclusion: For 7 states  $\rightarrow$  3 flipflops are required

7 states are assigned binary values & the last state can be taken for don't care condition.

$\rightarrow$  Reduced to 5 states  $\rightarrow$  also required 3 flipflop

5 states are only assigned binary values & the last 3 states can be taken for the don't care condition.

$\rightarrow$  In this case as the reduced state state from 7 to 5 state does not reduce the number of flipflop but the circuit will be more simple than before.

### State Assignment:

State assignment procedure is the method of assigning binary values to state in such a way to reduce the

cost of the combinational circuit that drives the

flip flop.

State	Assignment 1	Assignment 2	Assignment 3
a	000	000	000
b	001	010	100
c	010	011	010
d	011	101	101
e	100	111	011

Three possible binary state assignment

Present state	Next state		output	
	$x=0$	$x=1$	$x=0$	$x=1$
$a = 000$	000	001	0	0
$b = 001$	010	011	0	0
$c = 010$	010	011	0	0
$d = 011$	100	011	0	1
$e = 100$	000	011	0	1

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## # Flip-flop Excitation table

Characteristic

S	R	$Q(t+1)$
0	0	$Q(t)$
0	1	0
1	0	1
1	1	?

Excitation

$Q(t)$	$Q(t+1)$	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

SR flipflop

JK characteristic

J	K	$Q(t+1)$
0	0	$Q(t)$
0	1	0
1	0	1
1	1	$Q'(t)$

Excitation

$Q(t)$	$Q(t+1)$	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

D characteristic

D	$Q(t+1)$
0	0
1	1

Excitation

$Q(t)$	$Q(t+1)$	D
0	0	0
0	1	1
1	0	0
1	1	1

Same

T characteristic

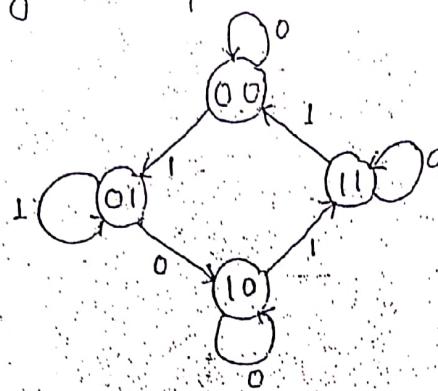
T	$Q(t+1)$
0	$Q(t)$
1	$Q(t)$

Excitation

$Q(t)$	$Q(t+1)$	T
0	0	0
0	1	1
1	0	1
1	1	0

Imp

# Design a sequential ckt using JK flip flop.



Characteristic table for JK

J	K	$Q(t+1)$
		$Q(t)$
0	0	$Q(t)$
0	1	0
1	0	1
1	1	$Q(t)$

Excitation

$Q(t)$	$Q(t+1)$	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

Present state			Input	Next state		flip flop inputs			
A	B	$\alpha$		A	B	$J_A$	$K_A$	$J_B$	$K_B$
0	0	0		0	0	0	X	0	X
0	0	1		0	1	0	X	1	X
0	1	0		1	0	1	X	X	1
0	1	1		0	1	0	X	X	0
1	0	0		1	0	X	0	0	X
1	0	1		1	1	X	0	1	X
1	1	0		1	1	X	0	X	0
1	1	1		0	0	X	1	X	1

K-map for  $J_A$

A	$Bx'$			
	00	01	11	10
0				1
1	X	X	X	X

$$J_A = Bx'$$

for  $K_A$

A	$Bx'$			
	00	01	11	10
0	X	X	X	X
1			1	

$$K_A = Bx'$$

for  $J_B$

A	$Bx'$			
	00	01	11	10
0				1
1	1	X	X	X

$$J_B = X$$

for  $K_B$

A	$Bx'$			
	00	01	11	10
0	X	X		1
1	X	X	1	

$$K_B = A'x' + Ax = A \oplus x$$

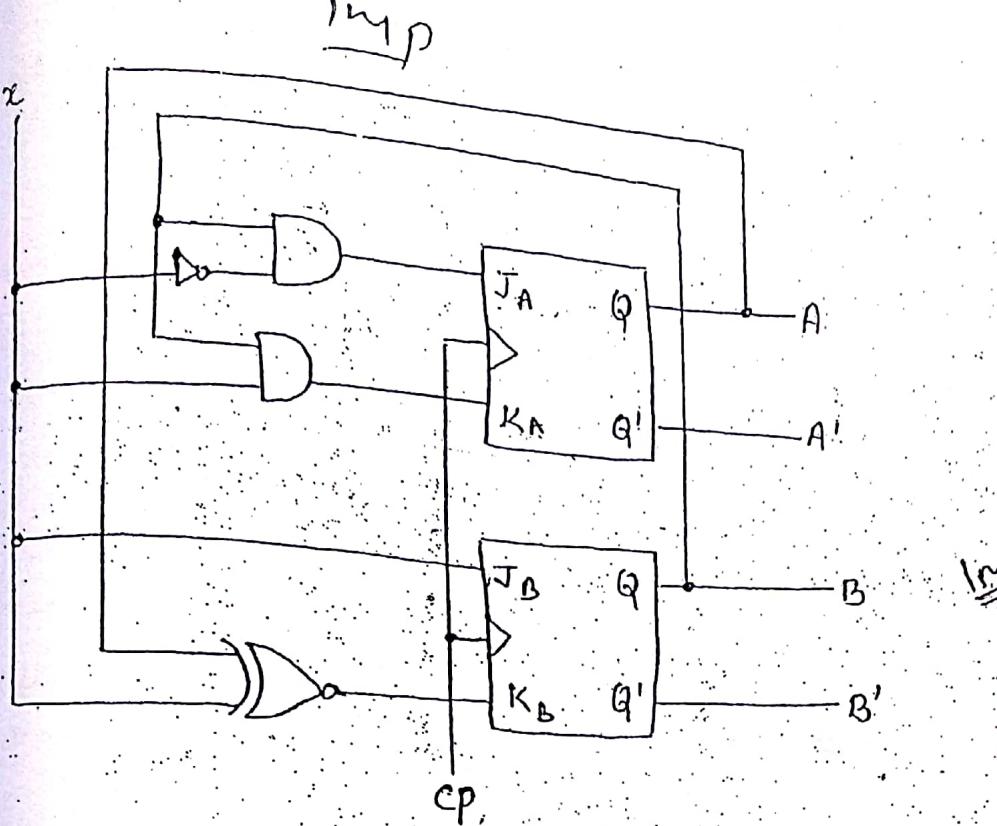


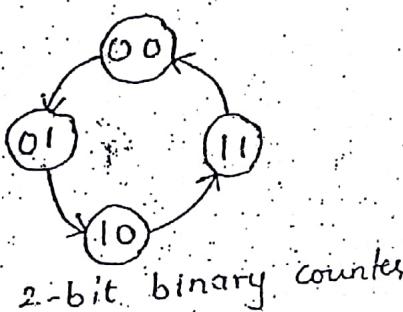
Fig: logic diagram of sequential ckt.

### Design of Counter

Counter is a circuit which does not have any input and it counts repeatedly the respective sequence of states upon the application of pulse.

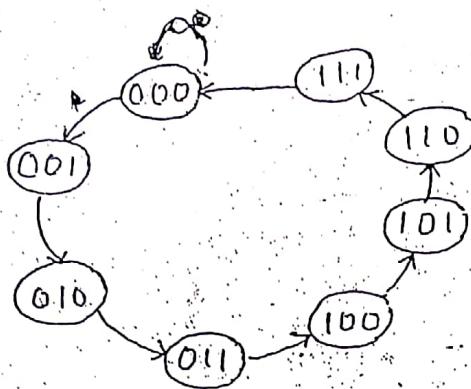
In a counter, the sequence of states may follow a binary count or any other sequence of states.

A counter that follows binary sequence is called binary counter. A  $n$  bit counter requires  $n$  flip flops and can count maxm of 0 to  $2^n - 1$ .



2-bit binary counter

Design a 3 bit binary counter using JK flip flop.



Characteristic for JK

J	K	$Q(t+1)$
0	0	$Q(t)$
0	1	'0
1	0	1
1	1	$Q'(t)$

Excitation for JK

$Q(t)$	$Q(t+1)$	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

Excitation table

Present state			Next state			flipflop inputs					
A	B	C	A	B	C	$J_A$	$K_A$	$J_B$	$K_B$	$J_C$	$K_C$
0	0	0	0	0	1	0	X	0	X	1	X
0	0	1	0	1	0	0	X	1	X	1	1
0	1	0	0	1	1	0	X	X	0	1	X
0	1	1	1	0	0	1	X	X	1	X	1
1	0	0	1	0	1	X	0	0	X	1	1
1	0	1	1	1	0	X	0	1	X	1	X
1	1	0	1	1	1	X	0	X	0	1	X
1	1	1	0	0	0	X	1	X	1	X	1

K-map for  $J_A$ 

A	BC		
	00	01	11
0			1
1	X	X	X

$$J_A = BC$$

for  $J_B$ 

A	BC		
	00	01	11
0			X
1	X	X	X

$$J_B = C$$

for  $J_C$ 

A	BC		
	00	01	11
0			1
1	X	X	X

$$J_C = 1$$

for  $K_A$ 

A	BC		
	00	01	11
0	X	X	X
1			1

$$K_A = BC$$

for  $K_B$ 

A	BC		
	00	01	11
0	X	X	1
1	X	X	1

$$K_B = C$$

for  $K_C$ 

A	BC		
	00	01	11
0	X	1	1
1	X	1	1

$$K_C = 1$$

Chapter 8Register, Counter & Memory Unit

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Prepared by

Budelha Laxmi Mahar

sequential circuit are classified into three categories

Registers  
Counters  
Memory Unit

Registers :-

- A register is a group of binary storage cells suitable for holding binary information.
- It is a group of flip flops & gates.
- An  $n$  bit register has a group of  $n$  flip flops & is capable of storing any binary information counting  $n$  bits.

Counters :-

- A counter is essentially a register that goes through a predetermined sequence of states upon the application of input pulses.

Memory Unit :-

- It a collection of storage cells together with associated circuits needed to transfer information in and out of storage.

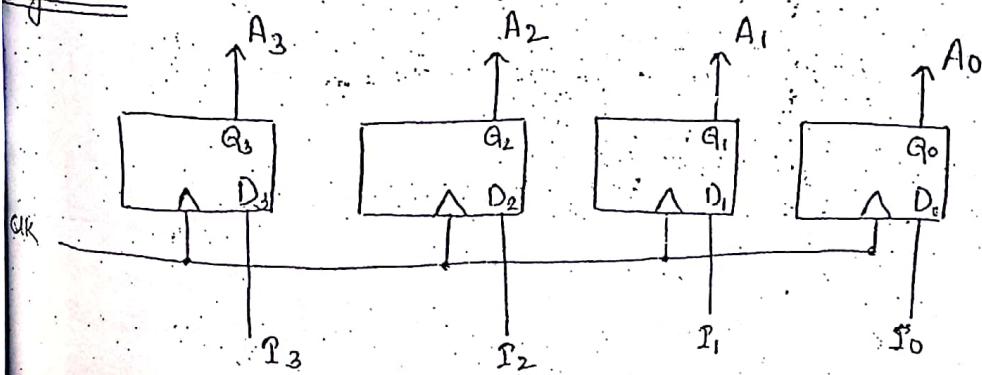
Registers

Fig: 4 bit register

→ A group of flipflops sensitive to pulse duration is usually called as Gated Latch.

→ A group of flipflops sensitive to pulse transition is called Register.

→ Gated latch is used for temporary storage.

### # Register with Parallel load

→ The transfer of new information into a register is referred to as Loading the register.

→ If all bits of the register are loaded simultaneously with a single clock pulse we say that the loading is done in parallel.

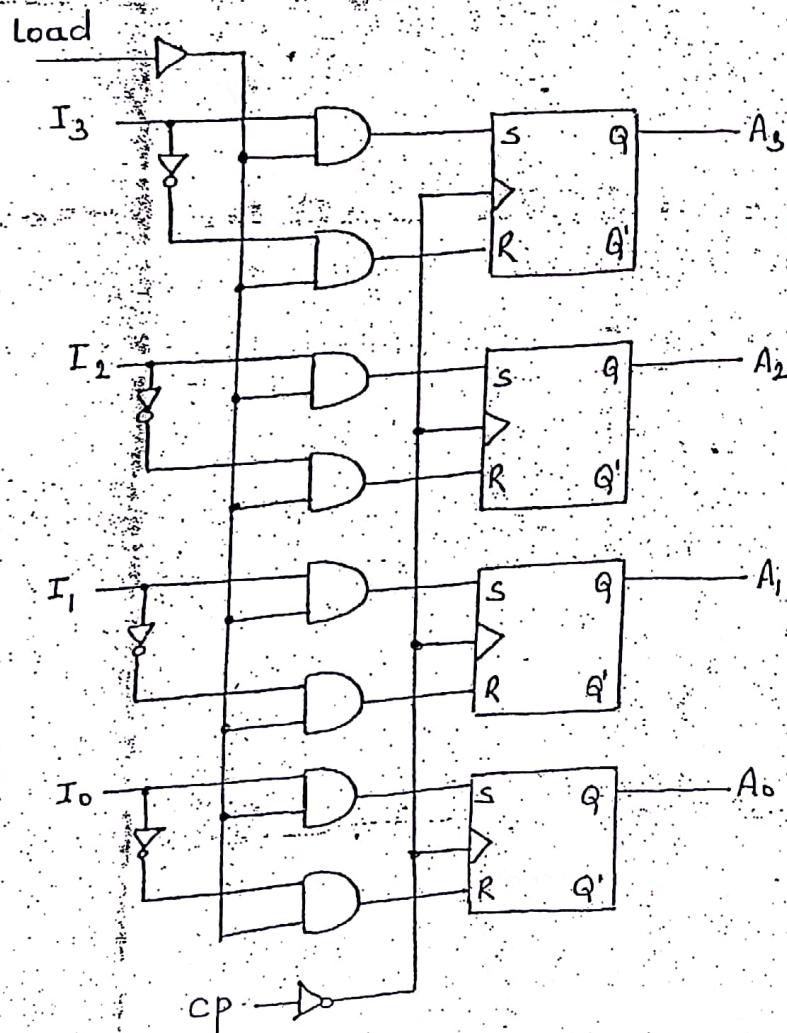


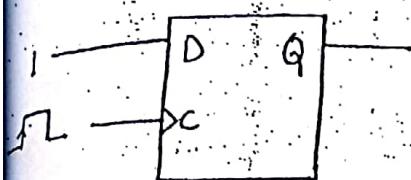
Fig: 4-bit register with parallel load.

when load = 1,  $I_0, I_1, I_2, I_3$  is loaded with falling edge of the clock pulse. Clear is maintained at high level for the synchronous operation.

### Shift Register

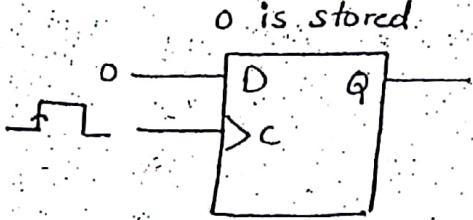
A register is a digital circuit with two basic functions, Data storage & data movement.

1 is stored



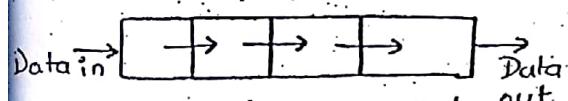
When a 1 is on D, Q becomes a 1 at the triggering edge of clock or remains 1 if already in a SET state.

0 is stored



when a 0 is on D, Q becomes a 0 at the triggering edge of clock or remains a 0 if already in the RESET state.

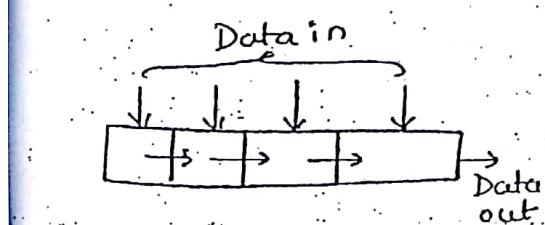
→ The shifting capability of a register permits the movement of data from stage to stage within the register or into or out of the register upon application of clock pulses.



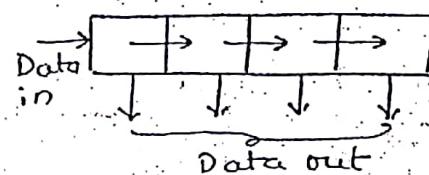
a) serial in / shift right / serial out



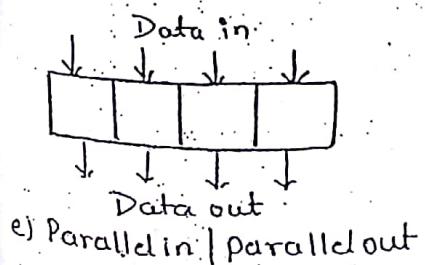
b) serial in / shift left / serial out



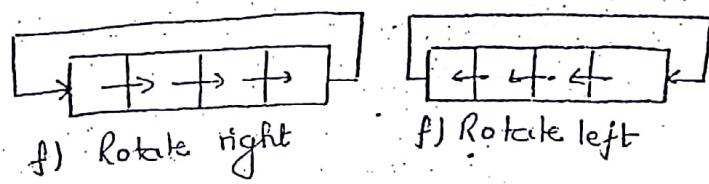
c) parallel in / serial out



d) serial in / parallel out



e) parallel in / parallel out



f) Rotate right

g) Rotate left

Pig: Basic data movement in shift registers

### i) Serial IN / Serial Out Shift Register

→ A register capable of shifting its binary information either to the right or to the left is called a shift register.

→ Serial IN / serial OUT shift register accepts data serially that is one bit at a time on a single line. It produces the stored information on its output also in a serial form.

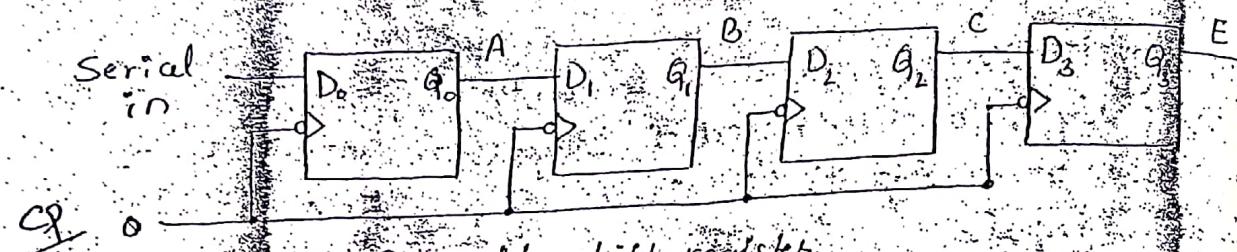


Fig: The right shift register

→ The serial input determines what goes into the left-most flipflop during the shift. The serial output is taken from the output of the right-most flipflop prior to the application of a pulse.

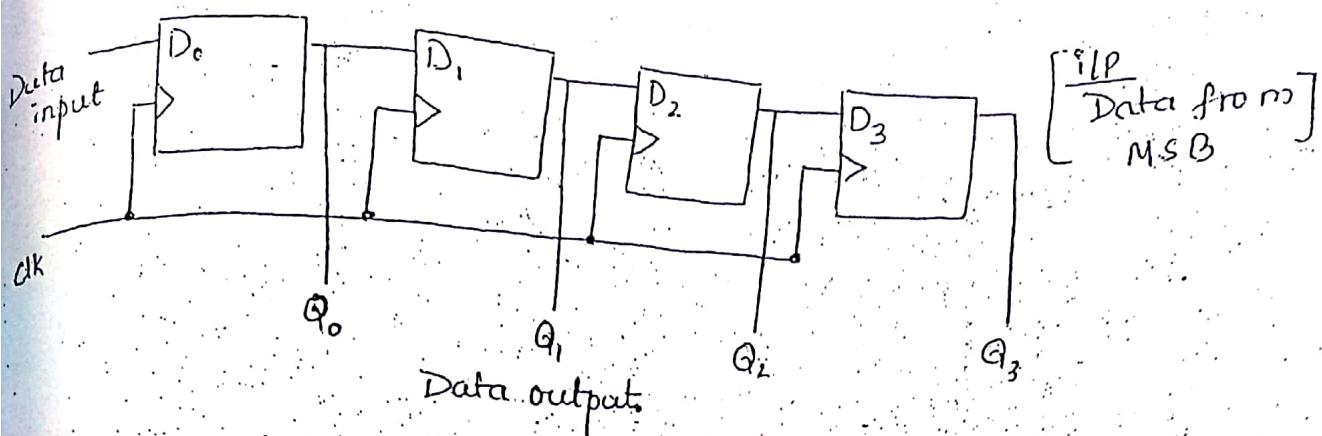
→ Let us assume the input sequence is 1010. Then the shifting sequence table follows as below.

	A	B	C	E	(Data from MSB)
Initial content	0	0	0	0	
clk	1	0	0	0	
clk	0	1	0	0	
clk	0	0	1	0	
clk	1	0	0	1	

right shifting the data

### ii) Serial IN / Parallel OUT Shift Register

→ Data bits are entered serially (right-most bit first) into this type of register in the same manner. The difference is the way in which the data bits are taken out of the register; in the parallel out register, the output of each stage is available.



### Parallel In/ Serial Out shift Register :-

→ For a register with parallel data inputs, the bits are entered simultaneously into their respective stages on parallel lines rather than on a bit by bit basis on one line as with serial data inputs.

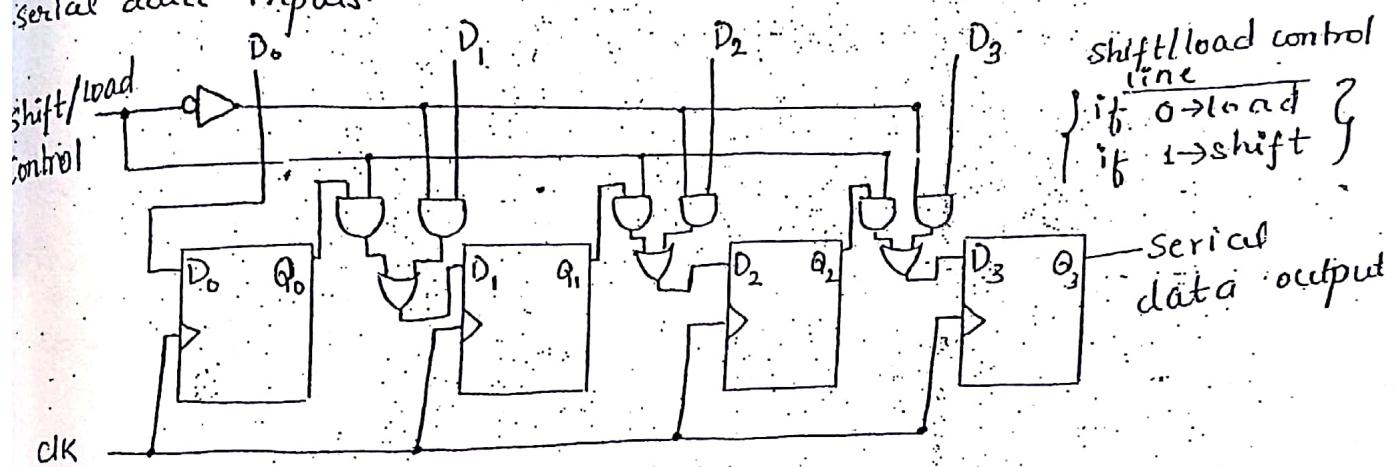


Fig: logic diagram

### Parallel In/ Parallel Out shift Register

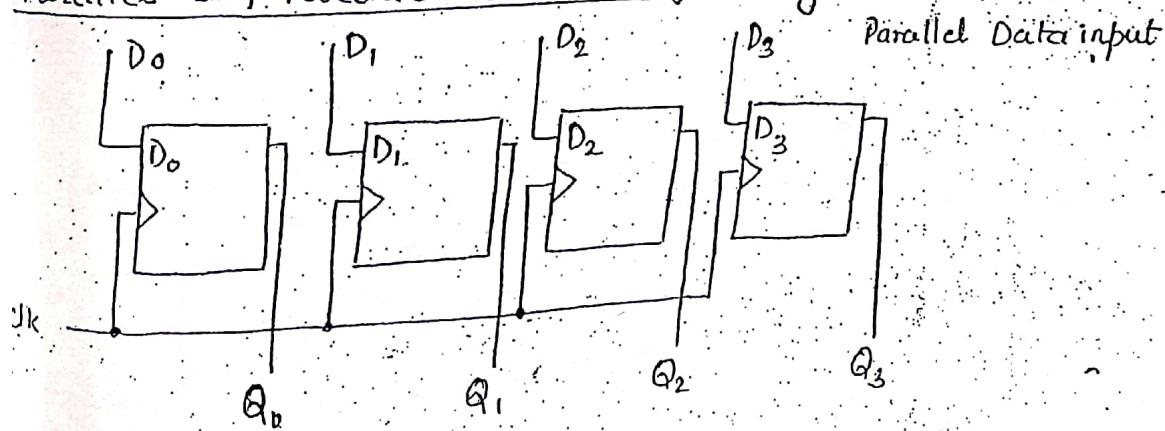
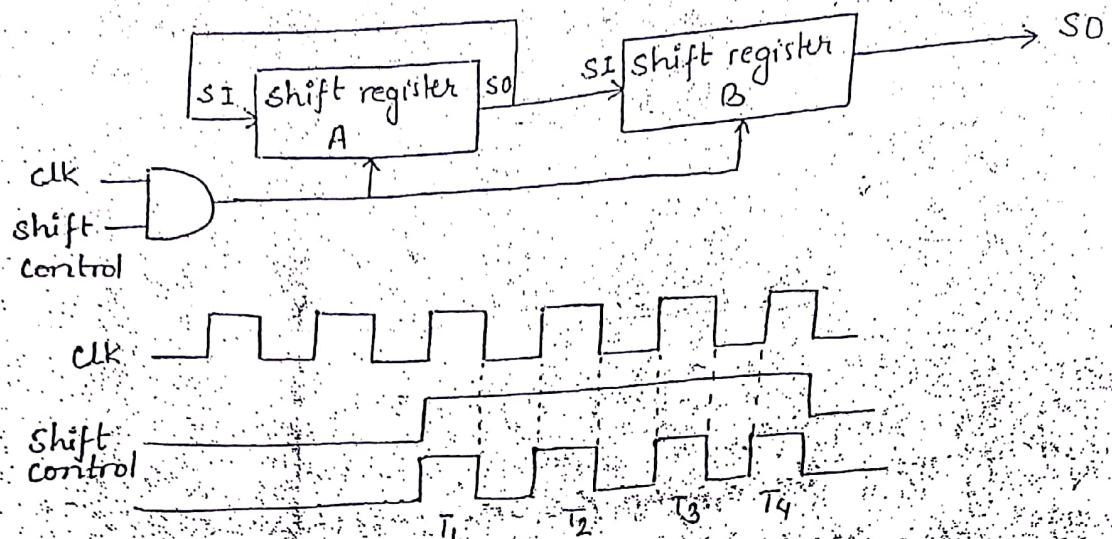
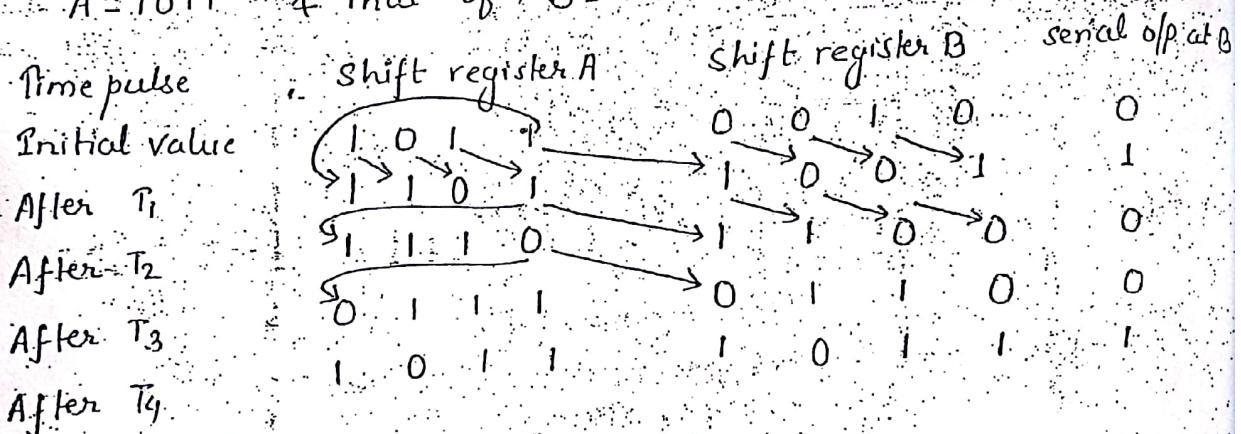


Fig: logic diagram

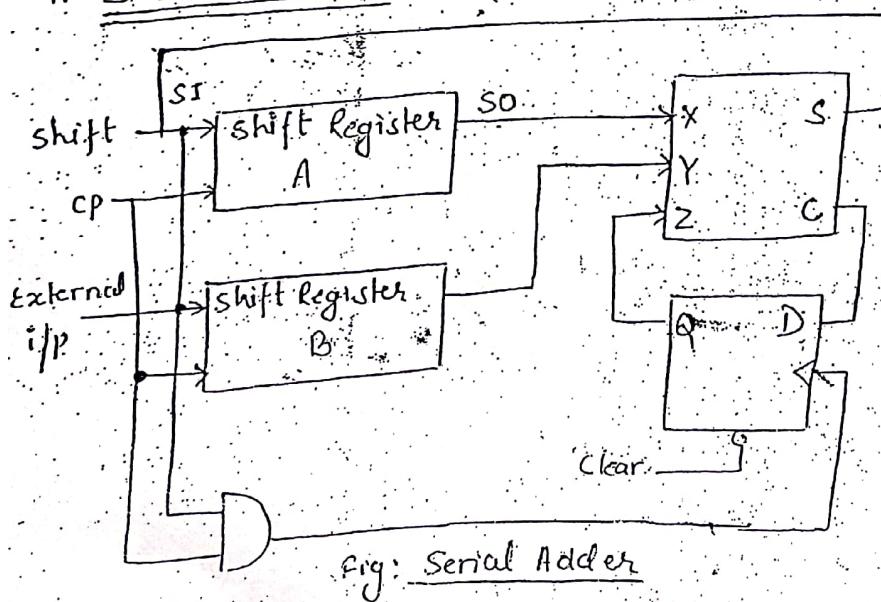
## # Serial Transfer



Let us assume that the initial content of shift register  
 $A = 1011$  & that of  $B = 0010$



## # Serial Adder (Assignment, refer book)



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Assignment

Design a serial adder using sequential logic procedure by using JK flipflop. (Refer book)

Bidirectional shift Register with parallel load. (Refer Book)

Counter

- Synchronous: All flipflops are triggered by the same clock pulse.
- Asynchronous: The o/p of the one flipflop is used to trigger the another flipflop.

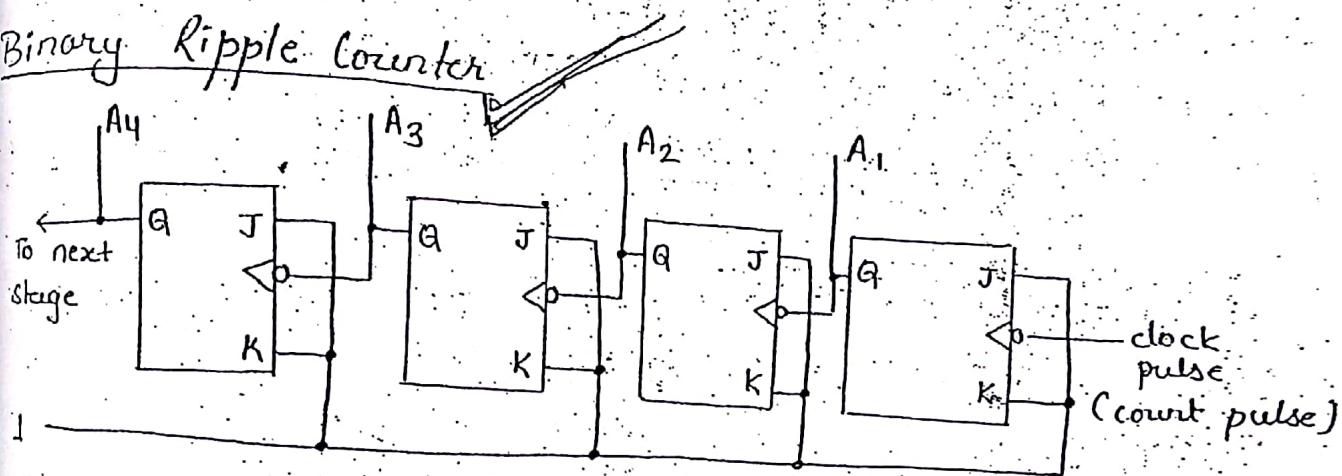
Binary Ripple Counter

Fig: 4-bit binary ripple counter

Whenever the first flipflop get the count pulse it complements the previous state i.e. it is always in the complementing.

Count SequenceConditions for complementing flipflops

$A_4$	$A_3$	$A_2$	$A_1$	Conditions for complementing flipflops
0	0	0	0	complement $A_1$
0	0	0	1	complement $A_1$ , $A_1$ will go from 1 to 0 & complement $A_2$
0	0	1	0	complement $A_1$
0	1	1	1	complement $A_1$ , $A_1$ will go from 1 to 0 & complement $A_2$
0	1	1	0	complement $A_1$ , $A_1$ will go from 1 to 0 & complement $A_2$
0	1	0	1	complement $A_1$ , $A_1$ will go from 1 to 0 & complement $A_2$
0	1	0	0	complement $A_1$ , $A_1$ will go from 1 to 0 & complement $A_2$
1	0	0	0	complement $A_1$

- Binary ripple counter consists of a series connection of complementing flipflops, with the output of each flipflop connected to the CP input of the next higher-order flipflop.
- The flipflop holding least significant bit receives the incoming count pulse.
- The LSB  $A_1$  must be complemented with each count pulse.
- Every time  $A_1$  goes from 1 to 0, it complements  $A_2$ . Every time  $A_2$  goes from 1 to 0, it complements  $A_3$ , and so on.
- Ripple counters are sometime called asynchronous counters.
- A binary counter with a reverse count is called a binary down-counter. The above circuit functions as a binary down counter if the output are taken from the complement terminals  $Q'$  of all flipflops.

## ii) BCD Ripple Counter

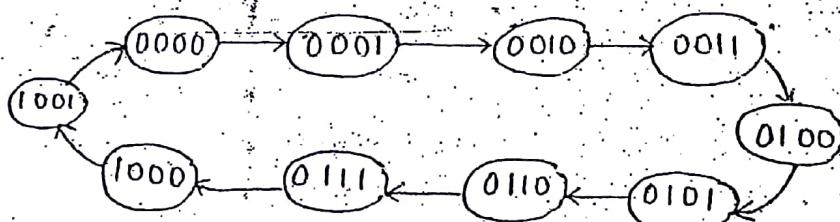


fig: state diagram of a decimal BCD counter.

- Counters can also be designed to have a number of states in their sequence that is less than the maximum of  $2^n$  states. Counter with ten states in their sequence are called decade counter.
- To obtain a truncated sequence, it is necessary to force the counter to recycle before going through all of its possible states.
- For example: the BCD decade counter must recycle back to the 0000 state after the 1001 state. NAND gate is used to decade the next count (1010) and the output of the NAND gate is applied to the clear (CLR) inputs of the flipflops.

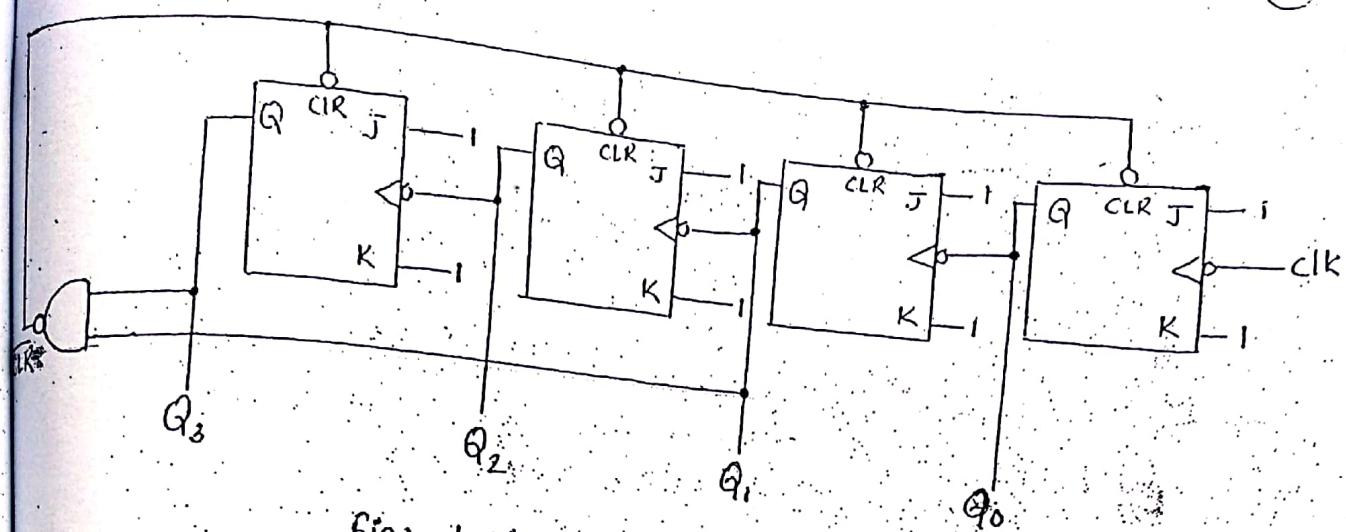


Fig: logic Implementation of asynchronous decade counter (BCD Ripple counter)

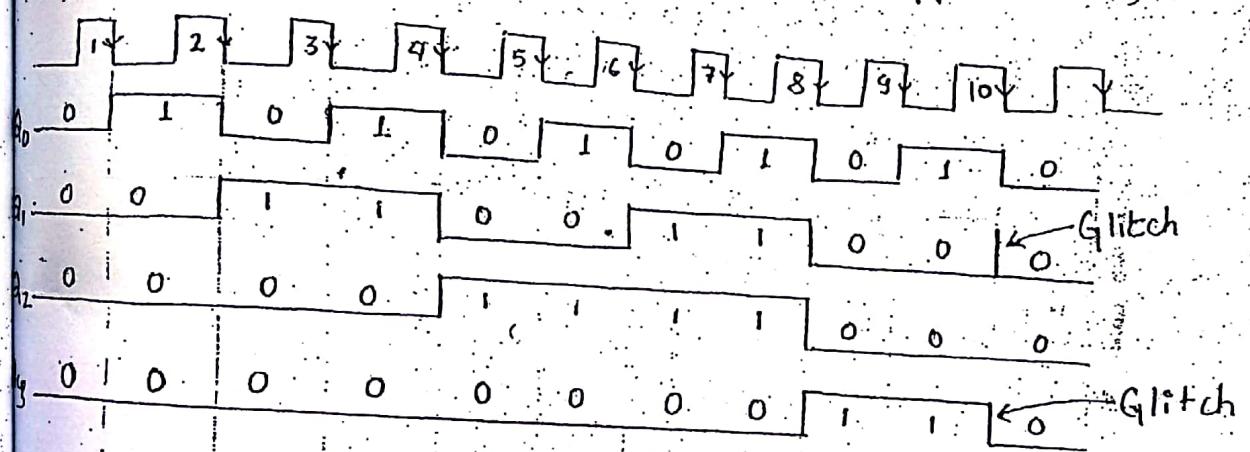
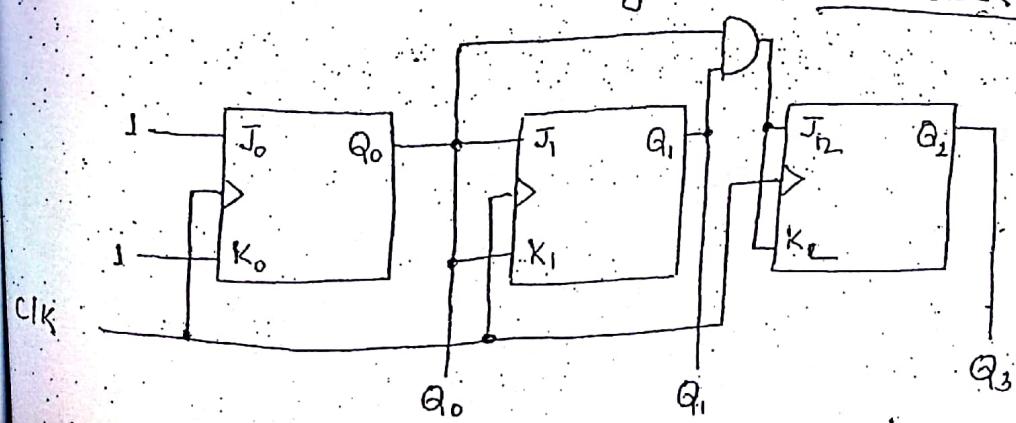


Fig: Timing diagram of a synchronous BCD ripple counter

### Synchronous Counter

→ Synchronous means that all the flipflop in the counter are clocked at the same time by a common clock pulse.



3-bit synchronous binary counter

### Timing diagram

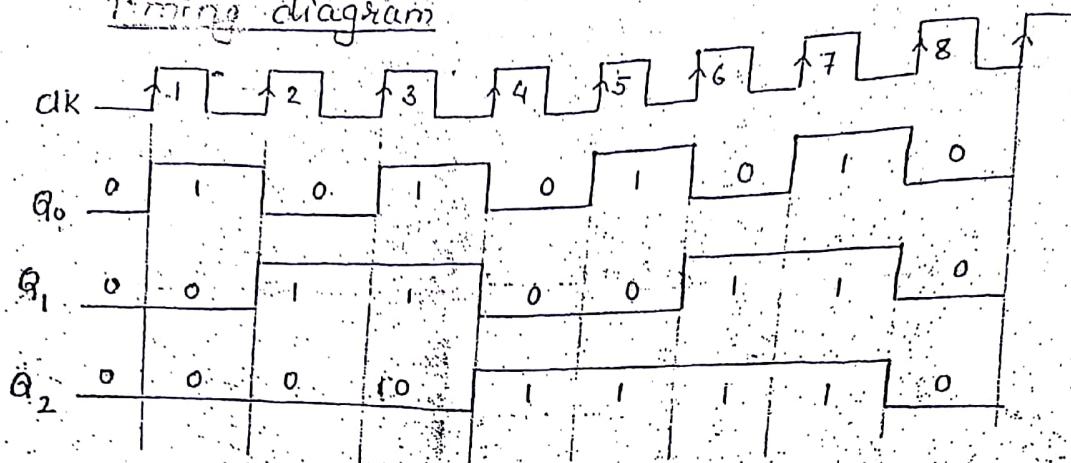


Fig: Timing diagram of 3 bit synchronous binary counter

#### \* Note

Synchronous counter can be designed by following the designing procedure as in the previous chapter.

Design of synchronous counter include following steps:

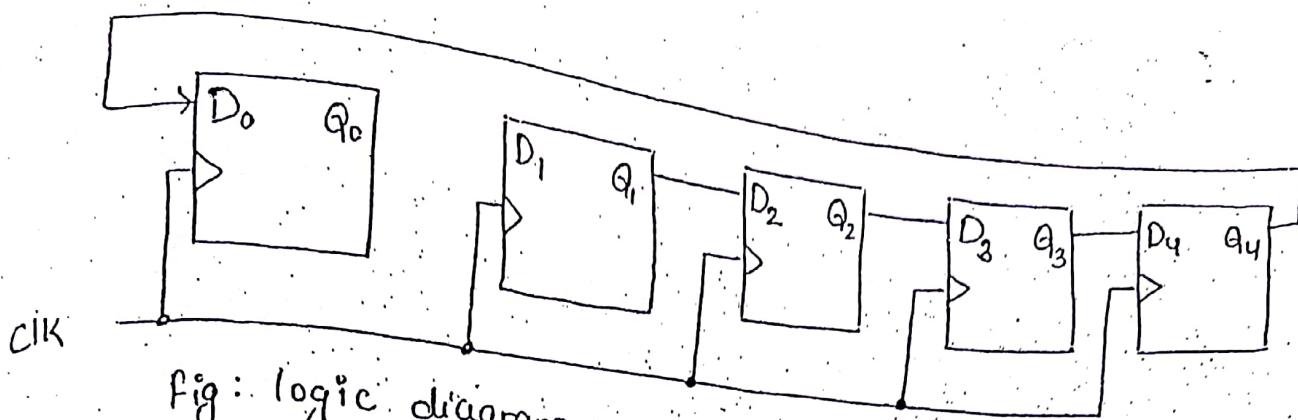
- 1) State diagram
- 2) Excitation table of the flipflop being used
- 3) K-map & simplified Boolean expression
- 4) Logic implementation.

### Ring Counter

→ A ring counter is a type of counter composed of circular shift register.

→ The output of last shift register is feed back to the input of first register.

→ In ring counter circulation is done by shifting with only one flip flop being at a time & all other are cleared.



clk

Fig: logic diagram of 5 bit ring counter

clk	$Q_0$	$Q_1$	$Q_2$	$Q_3$	$Q_4$
0	1	0	0	0	0
1	0	1	0	0	0
2	0	0	1	0	0
3	0	0	0	1	0
4	0	0	0	0	1
5	1	0	0	0	0
6	0	1	0	0	0

Fig: Sequence table

Johnson Counter :-  $D \rightarrow$ 

→ In a Johnson counter the complement of the output of the last flip-flop is connected back to the D input of the first flip flop.

→ This feedback arrangement produces a unique sequence of states so called twisted ring counter.

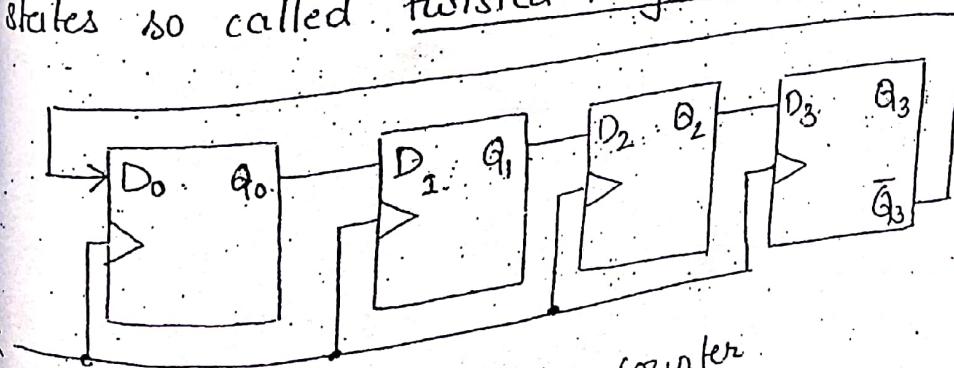


Fig: 4-bit Johnson counter

Clock pulse	$Q_0$	$Q_1$	$Q_2$	$Q_3$
0	0	0	0	0
1	1	0	0	0
2	1	1	0	0
3	1	1	1	0
4	1	1	1	1
5	0	1	1	1
6	0	0	1	1
7	0	0	0	1

Fig: Sequence table of 4-bit Johnson counter

→ The 4-bit sequence has a total of 8 states or bit patterns. Similarly, the 5-bit sequence has a total of 10 states.

### Assignment

- # Design a 4-bit binary synchronous up-down counter.
- # Design a 4-bit binary counter with parallel load.

## The Memory Unit

The registers in a digital computers may be classified as either operational or storage type.

### Operational register

An operational register is capable of storing binary information in its flip-flops and in addition has combinational gate capable of data processing tasks.

### Storage register

→ A storage register is used solely for temporary storage of binary information. This information can not be altered when transferred in and out of the register.

→ A memory unit stores binary information in groups called words, each word being stored in a memory register.

### Memory Unit

→ A memory unit is a collection of storage registers together with the associated circuit needed to transfer information in & out of the registers.

→ The storage registers in a memory unit are called memory registers.

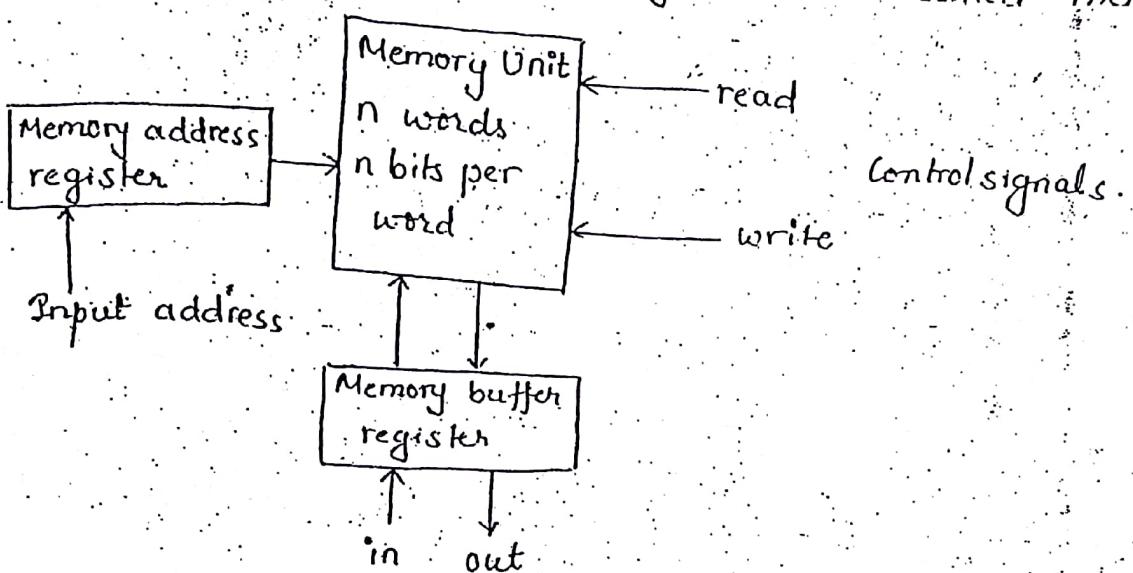


Fig: Block diagram of a memory unit showing communication with environment.

→ The communication between a memory unit and its environment is achieved through two control signals & two external registers.

### Memory address Register (MAR):

→ It specifies the memory word selected.

#### Read control signal

→ A read signal specifies a transfer out function.

#### Write control signal

→ A write signal specifies transfer in function.

### Memory Buffer Register (MBR):

→ The information transfer to and from registers in memory & the external environment is communicated through the commun

## Integrated - Circuit Memory

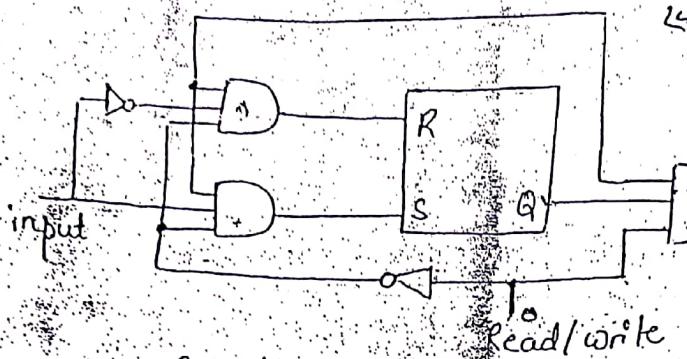


Fig: logic diagram

R.A.M  
J.B.J.

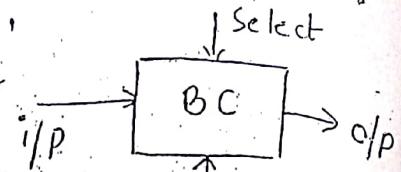
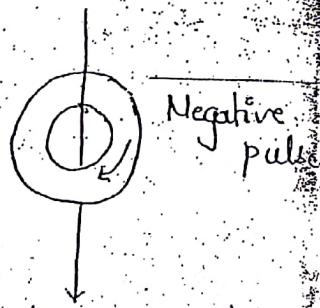


Fig: Block diagram of Memory cell

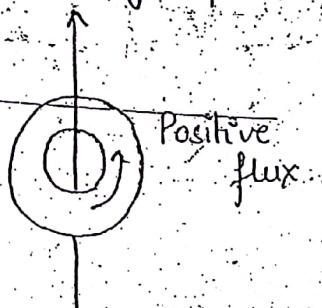
- The binary cell has three inputs & one output.
- The select input enables the cell for reading or writing.
- The read / write input determines the cell operation when it is selected.
- A 1 in read / write input forms a path from the flip flop to the output terminal. The information in the input terminal is transferred into the flip flop when the read / write control is 0.

## Magnetic - core memory

- A magnetic core memory uses magnetic core to store binary information.
- A magnetic core employs three physical quantities, current, magnetic flux & voltage for its operation.
- The signal that excites the core is a current pulse in a wire passing through the core.
- The binary information stored is represented by the direction of magnetic flux within the core.
- The output binary information is extracted from a wire linking the core in the form of a voltage pulse.



a) Store 0



b) Store 1

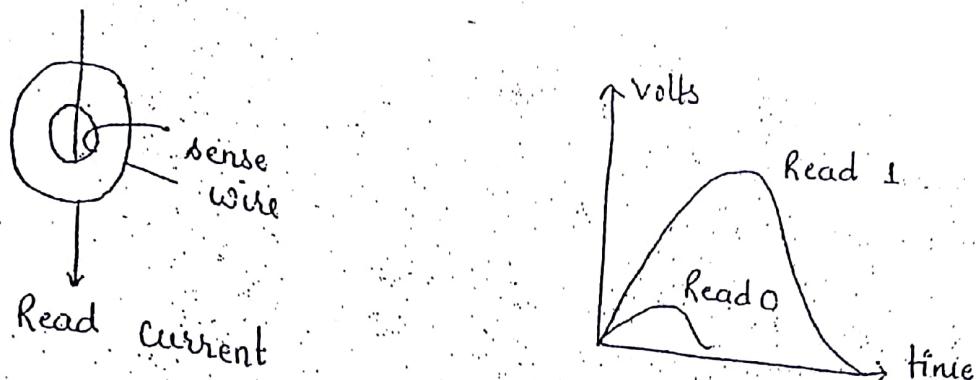


Fig: Reading a bit from a magnetic core.

- Read out is accomplished by applying a current in the negative direction. If the core is in the 1 state, the current reverses the direction of magnetization & the resulting change of the flux produces a voltage pulse in the sense wire.
- If the core is already in the 0 state, the negative current leaves the core magnetized in the same direction causing a very slight disturbance of magnetic flux which results in a very small output voltage in the sense wire.
- Note that this is a destructive read out, since the read current always returns the core to the 0 state. The previously stored value is lost.

### # RAM (Random Access Memory):-

- A memory unit is a collection of storage cell together with associated circuit needed to transfer information in and out of the device.
- Memory cells can be accessed for information transfer to or from any desired random location & hence the name Random access memory.
- A memory unit stores binary information in group of bits called words. A word in memory is an ability of bits that moves in and out of storage as a unit.
- The capacity of a memory unit is usually stated as the total number of bits it can store.

→ The communication between a memory & its environment is achieved through data input & output lines, address selection lines, & control lines that specify the direction of transfer.

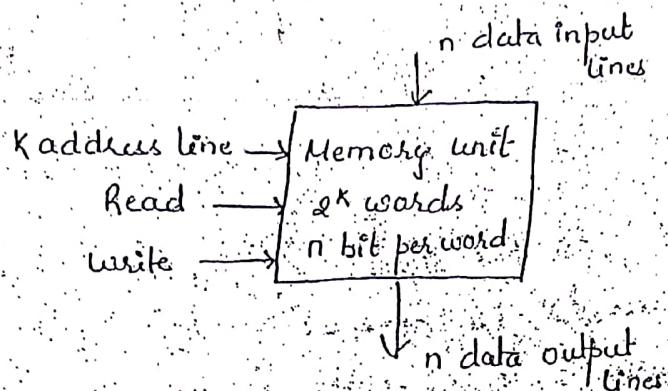


fig:- Block diagram of a memory unit

The memory unit is specified by the numbers of words it contains & the number of bits in each word.

Memory address		
Binary	Decimal	
00000 00000	0	00011111 00111110
00000 00001	1	00101010 01010111
00000 00010	2	
11111 11101	1021	
11111 11110	1022	
11111 11111	1023	00000111 00110011

Fig: Content of a 1024 X 16 memory

Control Inputs to Memory Chip

Memory Enable	Read / write	Memory operation
0	X	None
1	0	Write to selected word
1	1	Read from selected word

Access time:

- The access time of a memory is the time required to select a word and either read or write it.
- In RAM, the access time is always the same regardless of the particular location of the word.
- In a sequential access memory, the time it makes to access

a word depends on the reading-head position of the word with respect to the types of memories & therefore the access time is variable.

## Types of Memories

- Integrated-circuit RAM units are available in two possible operating modes.

  - i) static (use flip-flop)
  - ii) dynamic (MOS transistors)

→ Memory units that lose the stored information when the power is turned off are said to be volatile.

→ Integrated-circuit RAMs, both static & dynamic are of this category since the binary cells need external power to maintain the stored information.

→ In contrast, nonvolatile memory, such as magnetic disk, retains its stored information after removal of power. This is because the data stored on magnetic compounds is manifested by the direction of magnetization, which is retained after power is turn off. Another non volatile memory is the read only memory (ROM).

Hazard S. 2012  
2017 2009

- Hazards are unwanted switching transients that may appear at the output of a circuit because different paths exhibit different propagation delays.
  - Hazards occur in combinational circuits where they may cause a temporary false output value. When this condition occurs in asynchronous sequential circuits it may result in a transition of a wrong stable state.
  - A hazard is a condition where a single variable change produces a momentary output change when no output change should occur.

### Types of hazard

1) Static 1 - hazard



2) Static 0 - hazard



3) Dynamic hazard

Dynamic hazard causes the output to change three or more times it should change from 1 to 0 or vice versa.

### combinational CKT

output variable at any instant of time are depend only on the present input variables.

- i) It does not have memory.
- ii) It does not have feed back
- iii) They are fast
- iv) Easy to Design

Building blocks are basic gate

### sequential CKT

- ① The output variable at any instant are depend on the present & past of input variable.
- ② It has memory
- ③ It has a feed back
- ④ They slow
- ⑤ comparatively hard to design
- ⑥ Building block basic gate & (combinational)

### Synchronous

They are external clock pulse for transition from one to next state

slower than Asynchronous sequential CKT

More reliable.

Required more hardware so more cost

common clock pulse may be used.

### Asynchronous sequential CKT

- ② They do not need external clock pulse for transition from one to next.
- ③ faster than synchronous CKT
- ④ less Reliable.
- ⑤ require less Hard ware so cheap.
- ⑥ common clock pulse can be used.

a word depends on the position of the word with respect to the reading-head position & therefore, the access time is variable.

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# Arithmetic Logic Unit (ALU)

(10)

prepared by

Buddha Laxmi Ma

## # Arithmetic logic Unit

- An arithmetic logic Unit is a multioperation, combinational-logic digital function.
- It can perform a set of basic arithmetic operations and a set of logic operations.
- The ALU has a number of selection lines to select a particular operation in the unit.
- The selection lines are decoded within the ALU so that  $K$  selection variables can specify upto  $2^K$  distinct operations.

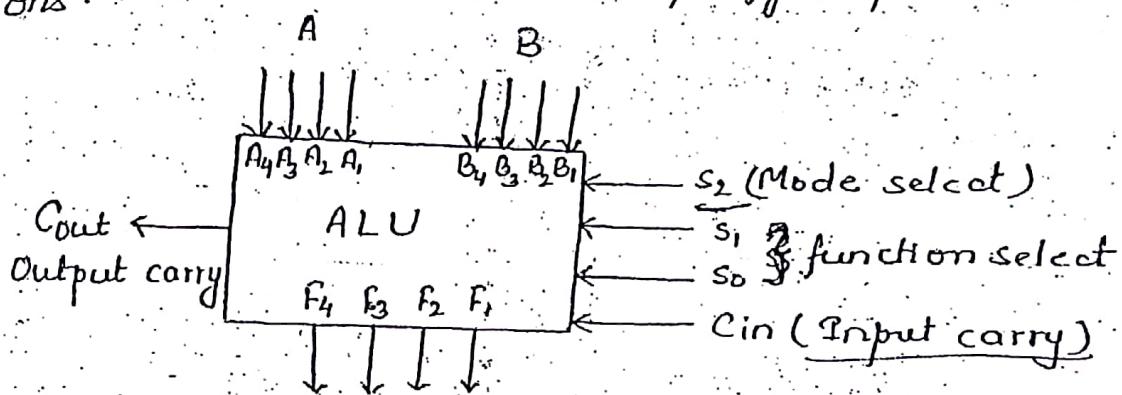


Fig: Block diagram of a 4-bit ALU

- The four data inputs from A are combined with the four inputs from B to generate an operation at the F output.
- The mode-select input  $s_2$  distinguishes between arithmetic & logic operations. The two function-select inputs  $s_1$  &  $s_0$  specify the particular arithmetic or logic operation to be generated.
- With three selection variables, it is possible to specify four arithmetic operations & four logic operations.
- The i/p & o/p carry has meaning only during arithmetic operation.

The input carry in the least significant position of an ALU is quite often used as a fourth selection variable that can double the number of arithmetic operations.

### Design of Arithmetic Circuit

→ The basic component of the arithmetic section of ALU is a parallel adder. By controlling the data i/p to the parallel adder, it is possible to obtain different types of arithmetic operations.

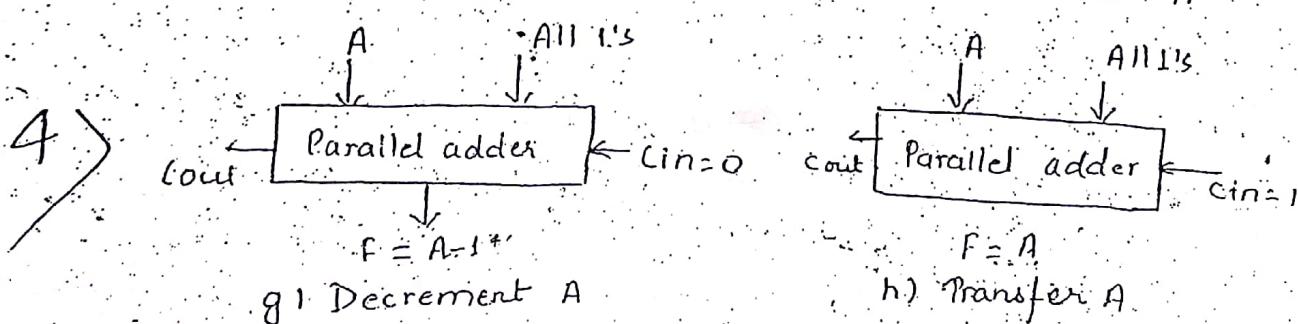
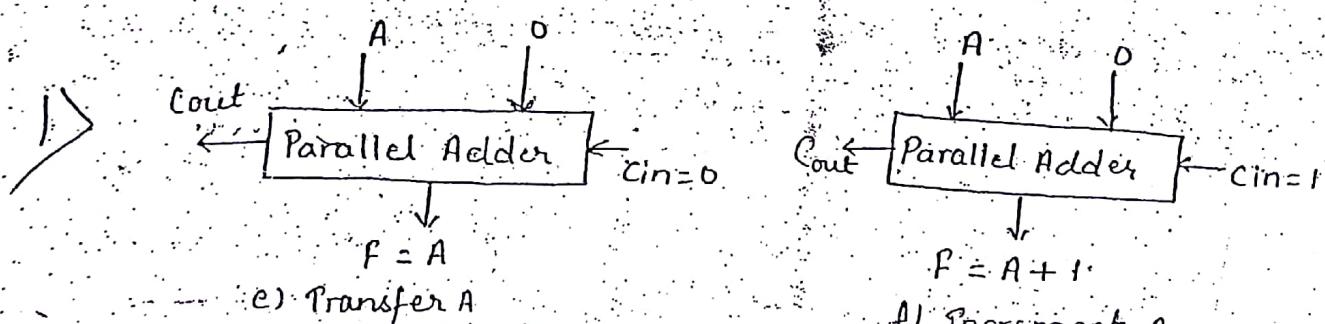
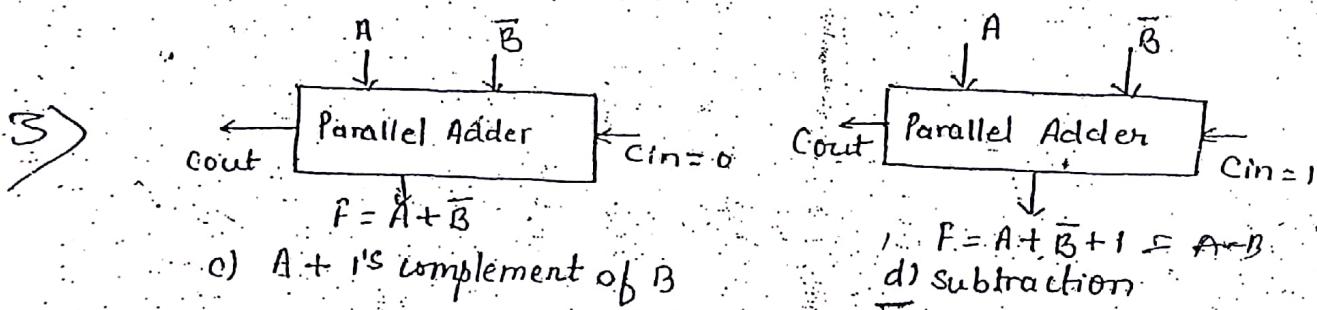
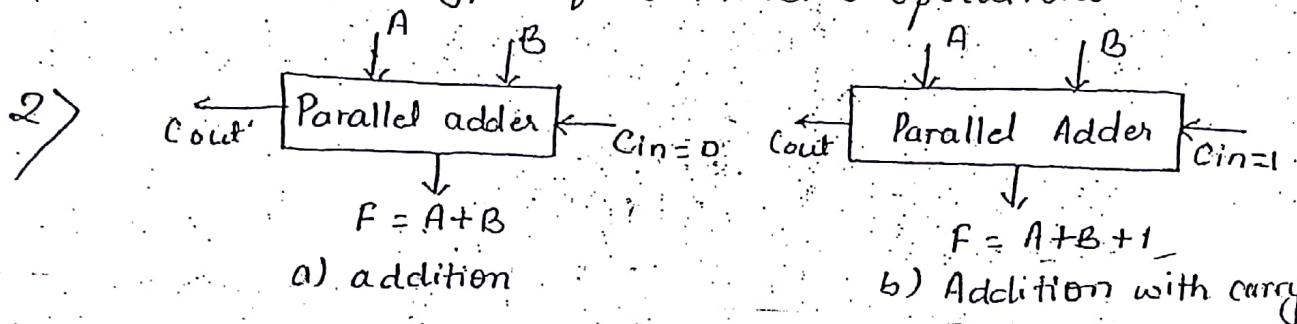


Fig: Operations obtained by controlling one set of i/p's to a parallel adder.

### Case (8) Decrement of A

- Consider a parallel adder with  $n$  full-adder circuits.
- When the output carry is 1, represents the number  $2^n$  because in binary consists of a 1 followed by  $n$  0's.
- Subtracting 1 from  $2^n$ , we obtain  $2^n - 1$ , which in binary is a number of  $n$  1's.
- Adding  $2^n - 1$  to A, we obtain  $F = A + 2^n - 1 = 2^n + A - 1$ . If the output carry  $2^n$  is removed, we obtain  $F = A - 1$ .

### Example

$$A = 0000\ 1001 = (9)_{10}$$

$$2^n = 10000\ 0000 = (256)_{10} \quad (2^8)$$

$$2^n - 1 = 1111\ 1111 = (255)_{10}$$

$$A + 2^n - 1 = 0000\ 1000 = (256 + 8)_{10}$$

Removing the output carry  $2^n = 256$ , we obtain  $8 = 9 - 1$ .

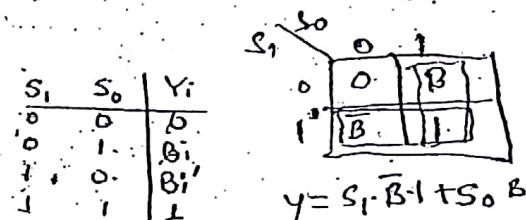
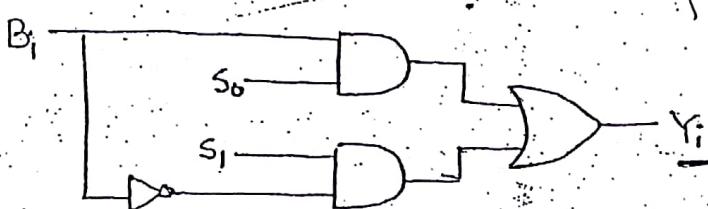


Fig: Prue / complement, one/zero circuit

2010, 2011, 2011, 2003,

Function table for the arithmetic circuit given above

Function select			Y equals o/p equals	Function
$S_1$	$S_0$	Cin		
0	0	0	0	$F = A$ Transfer A
0	0	1	0	$F = A + 1$ Increment A
0	1	0	B	$F = A + B$ Add B to A
0	1	1	B	$F = A + B + 1$ Add B to A plus 1
1	0	0	$\bar{B}$	$F = A + \bar{B}$ Add 1's complement of B to A
1	0	1	$\bar{B}$	$F = A + \bar{B} + 1$ Add 2's complement of B to A
1	1	0	All 1's	$F = A - 1$ Decrement A
1	1	1	All 1's	$F = A$ Transfer A

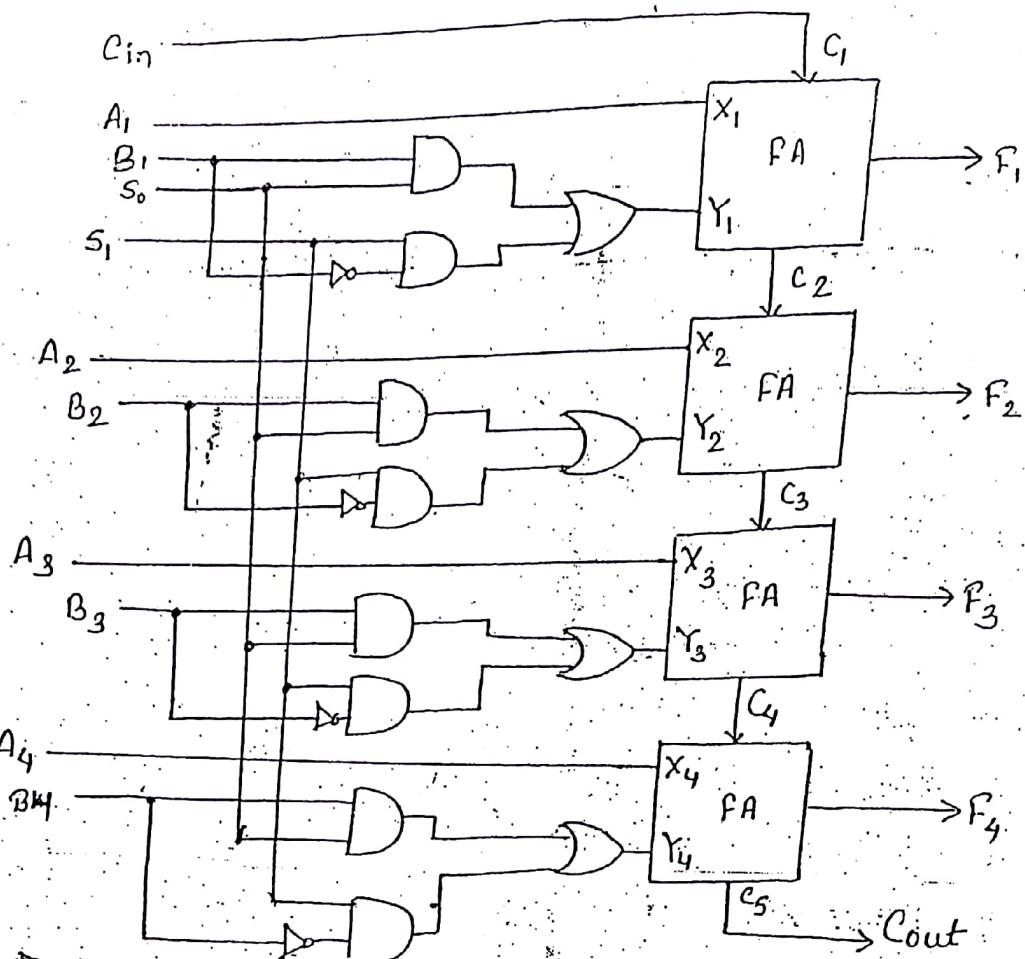


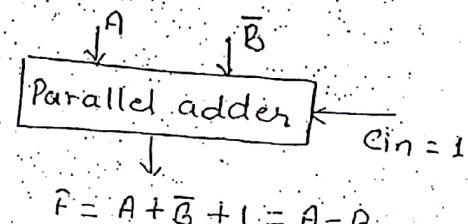
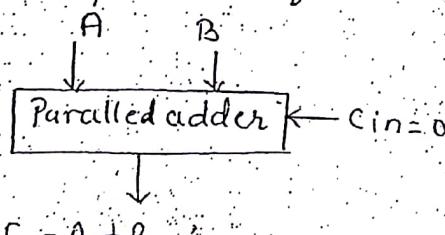
Fig: logic diagram of arithmetic circuit (4-bit)

$$X_i = A_i$$

$$Y_i = \underline{B_i} \underline{S_0} + \underline{B_i} \underline{S_1}, \quad i = 1, 2, \dots, n$$

### Example

\* Design an adder / subtractor circuit with one selection variable  $S$  and two inputs  $A$  and  $B$ . When  $S=0$  the circuit performs  $A+B$ . When  $S=1$  the circuit performs  $A-B$  by taking the 2's complement of  $B$ .



a) function specification

## # Design of logic circuit

- The logic microoperations manipulate the bits of the operands separately and treat each bit as a binary variable
- Consider different four logic operations AND, OR, NOT & XOR, for these operations we need two selection variables.
- Let us design a logic circuit

$S_1$	$S_0$	Output	Operation
0	0	$F_i = A_i + B_i$	OR
0	1	$F_i = A_i \oplus B_i$	XOR
1	0	$F_i = A_i \cdot B_i$	AND
1	1	$F_i = A_i'$	NOT

Fig: Function table

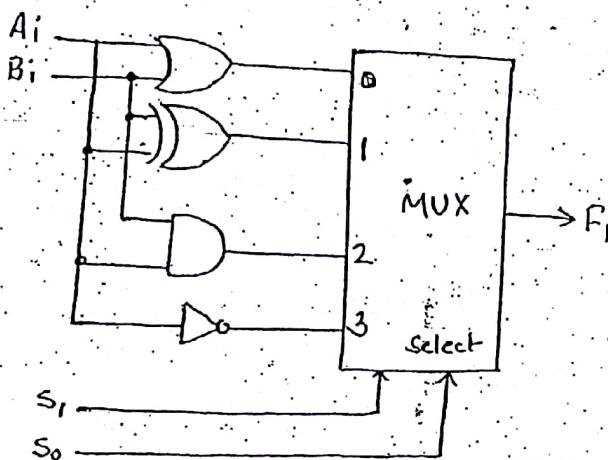
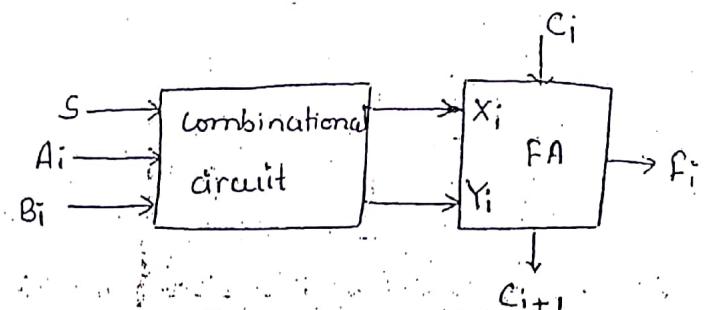


Fig: logic diagram

- The simplest and most straightforward way to design a logic circuit is shown in above figure.
- The diagram shows one typical stage designated by subscript  $i$ . The ckt must be repeated  $n$  times for an  $n$ -bit logic circuit.
- The four gates generated the four logic operations OR, XOR, AND, and NOT.
- The two selection variables in the multiplexer select one of the gates for the output.
- The function table lists the output logic generated as a function of the two selection variables.

S	$X_i$	$Y_i$	$Cin$
0	$A_i$	$B_i$	0
1	$A_i$	$B'_i$	1



b) specifying combinational circuit

S	$A_i$	$B_i$	$X_i$	$Y_i$
0	0	0	0	0
0	0	1	0	1
0	1	0	1	0
0	1	1	1	1
1	0	0	0	1
1	0	1	0	0
1	1	0	1	1
1	1	1	1	0

for  $X_i$

S	$A_iB_i$	00	01	11	10
0	00	0	0	1	1
1	01	0	0	1	1

$$X_i = A_i$$

for  $Y_i$

S	$A_iB_i$	00	01	11	10
0	00	0	1	1	0
1	01	1	0	0	1

$$Y_i = S'B_i + SB_i \\ = S \oplus B_i$$

$$Cin = S$$

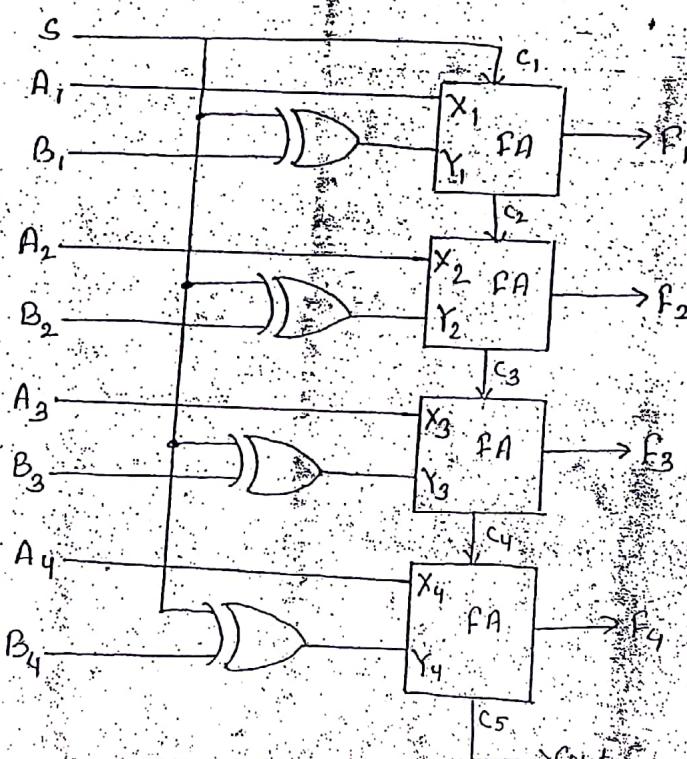


Fig: 4-bit adder/ subtractor circuit

- The logic circuit can be combined with the arithmetic circuit to produce one arithmetic logic unit.
- Selection variables  $s_1$  &  $s_0$  can be made common to both and  $s_2$  is used to differentiate b/w the two. When  $s_2=0$ , the arithmetic o/p is selected, but when  $s_2=1$  the logic o/p is selected.

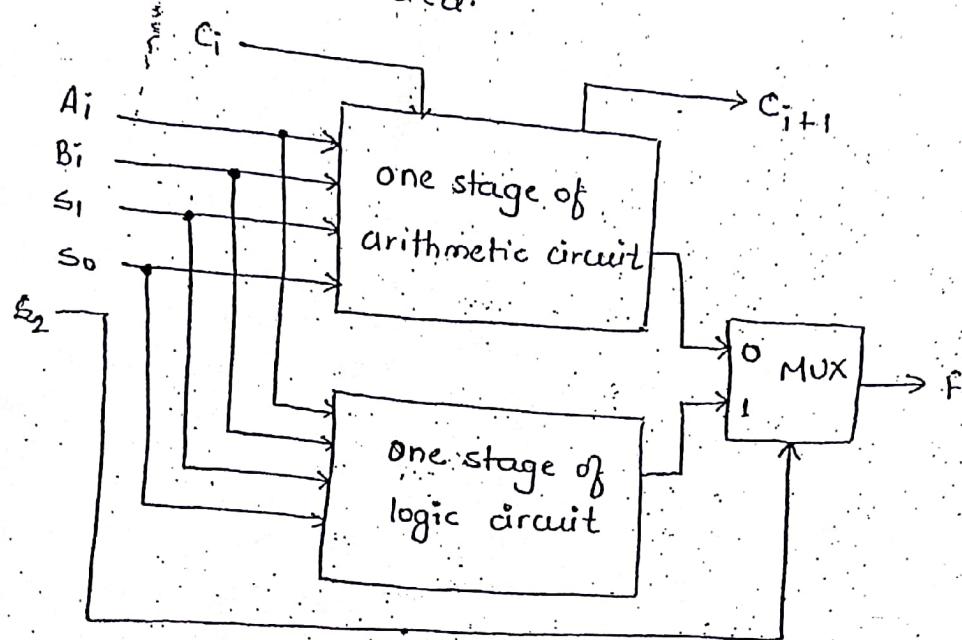


Fig: Combining ALU

#### Note

\* The more efficient way to design ALU is if we investigate the possibility of generating logic operations in an already available arithmetic circuit.

→ The full adder output is

$$F_i = X_i \oplus Y_i \oplus C_i$$

When  $s_2=1$  the logic operation is selected,  $C_i=0$

$$F_i = X_i \oplus Y_i \quad [\because x \oplus 0 = x]$$

Thus, with input carry to each stage equal to 0, the full adder circuit generates XOR operation.

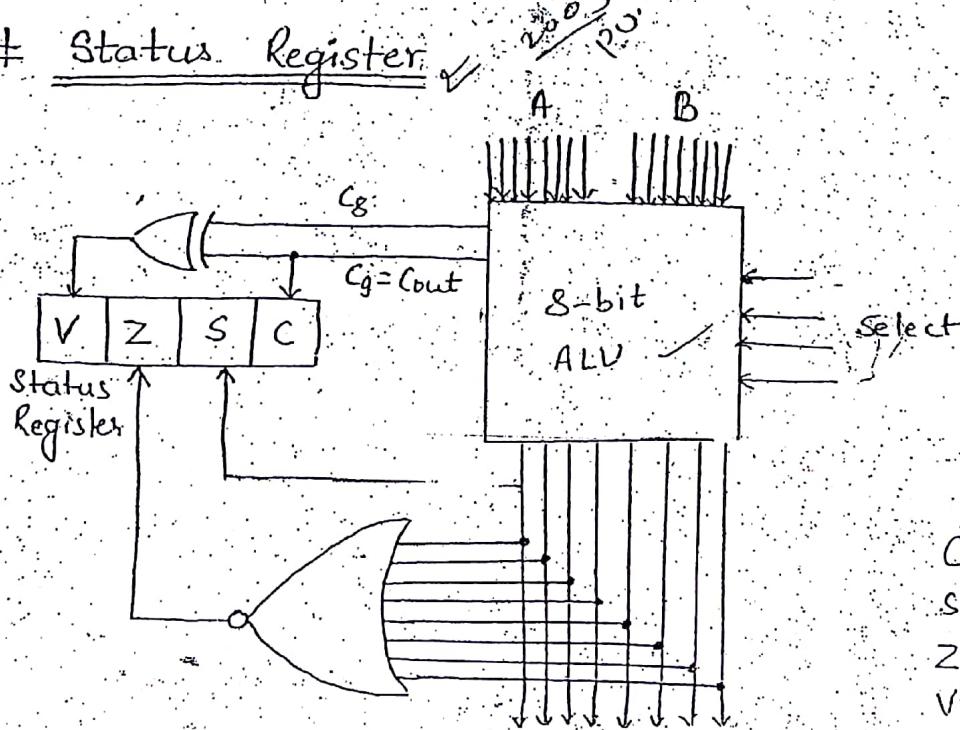
$s_1$	$s_0$	$X_i$	$Y_i$	$C_i$	$F_i = X_i \oplus Y_i$	Operation	Required Operation
1	0	$A_i$	0	0	$F_i = A_i$	Transfer	OR
1	0	$A_i$	$B_i$	0	$F_i = A_i \oplus B_i$	XOR	XOR
1	1	$A_i$	$B_i$	0	$F_i = A_i \ominus B_i$	Equivalence	AND
1	1	$A_i$	1	0	$F_i = A_i'$	NOT	NOT

Output  $F_i$  is then equal to  $X_i \oplus Y_i$  and produces the  $X_0$  and NOT operations. When  $S_2 S_1 S_0 = 100$ , each  $A_i$  is ORed with  $B_i$  to provide OR operation. When  $S_2 S_1 S_0 = 110$ , each  $A_i$  is ORed with  $B_i'$  to provide the AND operation.

Selection				Output	Function
$S_2$	$S_1$	$S_0$	Cin		
0	0	0	0	$F = A$	Transfer A
0	0	0	1	$F = A + 1$	Increment A
0	0	1	0	$F = A + B$	Addition
0	0	1	1	$F = A + B + 1$	Add with carry
0	1	0	0	$F = A + \bar{B} (A - B - 1)$	Subtract with borrow
0	1	0	1	$F = A - B (A + \bar{B} + 1)$	Subtraction
0	1	1	0	$F = A - 1$	Decrement A
0	1	1	1	$F = \bar{A}$	Transfer A
1	0	0	X	$F = A \vee B$	OR
1	0	1	X	$F = A \oplus B$	XOR
1	1	0	X	$F = A \wedge B$	AND
1	1	1	X	$F = \bar{A}$	Complement A

Function table for the ALU

## # Status Register



C - Carry  
S - Sign  
Z - Zero  
V - overflow

Fig: Setting bits in a status register

- Status register is an additional register with ALU which gives the status of the final output of ALU.
- Status bits of status register are used for further analysis or Flag bits.
- Status-bit conditions are sometime called Condition-codes.

→ Figure shows the block diagram of an 8-bit ALU with 4-bit status register.

→ The four status bits are symbolized by C, S, Z & V. The bits are set or cleared as a result of an operation performed in the ALU.

1. Bit C is set if the output carry of the ALU is 1. It is cleared if the output carry is 0.
2. Bit S is set if the highest-order bit of the result in the output of the ALU (the sign bit) is 1. It is cleared if the highest-order bit is 0.
3. Bit Z is set if the output of the ALU contains all 0's and cleared otherwise. Z=1 if the result is zero, and Z=0 if the result is nonzero.
4. Bit V is set if the exclusive-OR of carries  $C_8 \oplus C_9$  is 1, and cleared otherwise. This is the condition for overflow when the numbers are in sign-2's-complement representation. For the 8-bit ALU, V is set if the result is greater than 127 or less than -128.

### Application

A = 10121100 when x is the bit to be checked. The AND operation of A with

B = 00010000 produces result

- i) if  $x=0$ , then Z status bit is set.
- ii) if  $x=1$ , then Z bit is cleared since the result is not zero.

9.01/2021

## Design of shifter :-

→ The shift unit attached to a processor transfers the output of the ALU onto the output bus. The shifter may transfer the information directly without a shift or it may shift the information to the right or left.

→ Provision is sometimes made for no transfer from the ALU to the output bus. The shifter provides the shift operations commonly not available in a

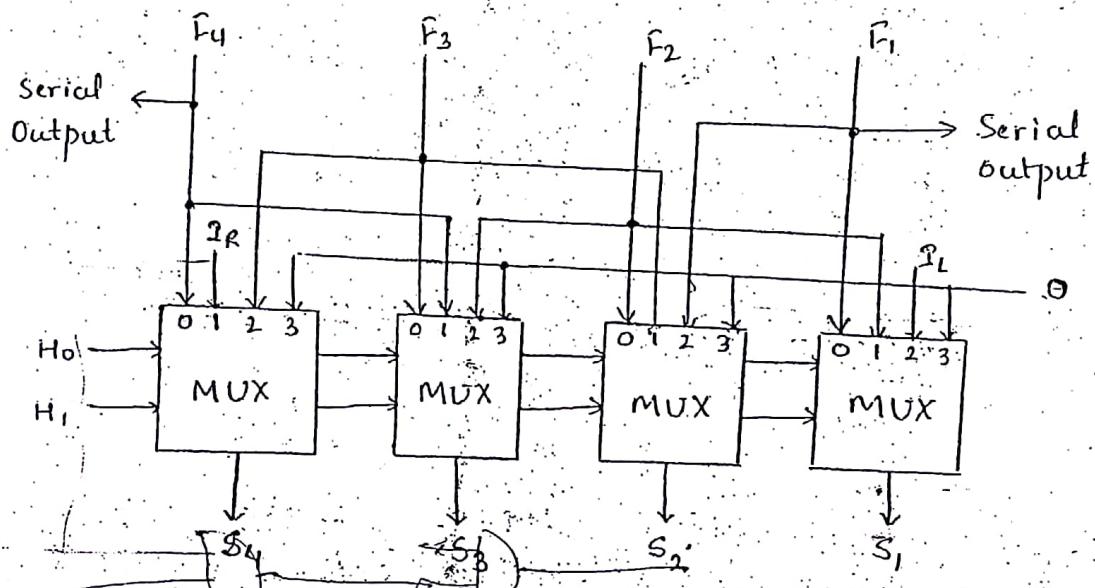


Fig: 4-bit combinational-logic shifter

$H_1 \backslash H_0$	operation	function
0 0	$S \leftarrow F$	Transfer F to S (no shift)
0 1	$S \leftarrow \text{shr } F$	shift-right F into S
1 0	$S \leftarrow \text{shl } F$	shift-left F into S
1 1	$S \leftarrow 0$	Transfer 0's into S

Fig: Function table for shifter

The selection variables in a processor unit control the micro-operations executed within the processor during any given clock pulse. The selection variable control the buses, the ALU, the shifter and the destination register.

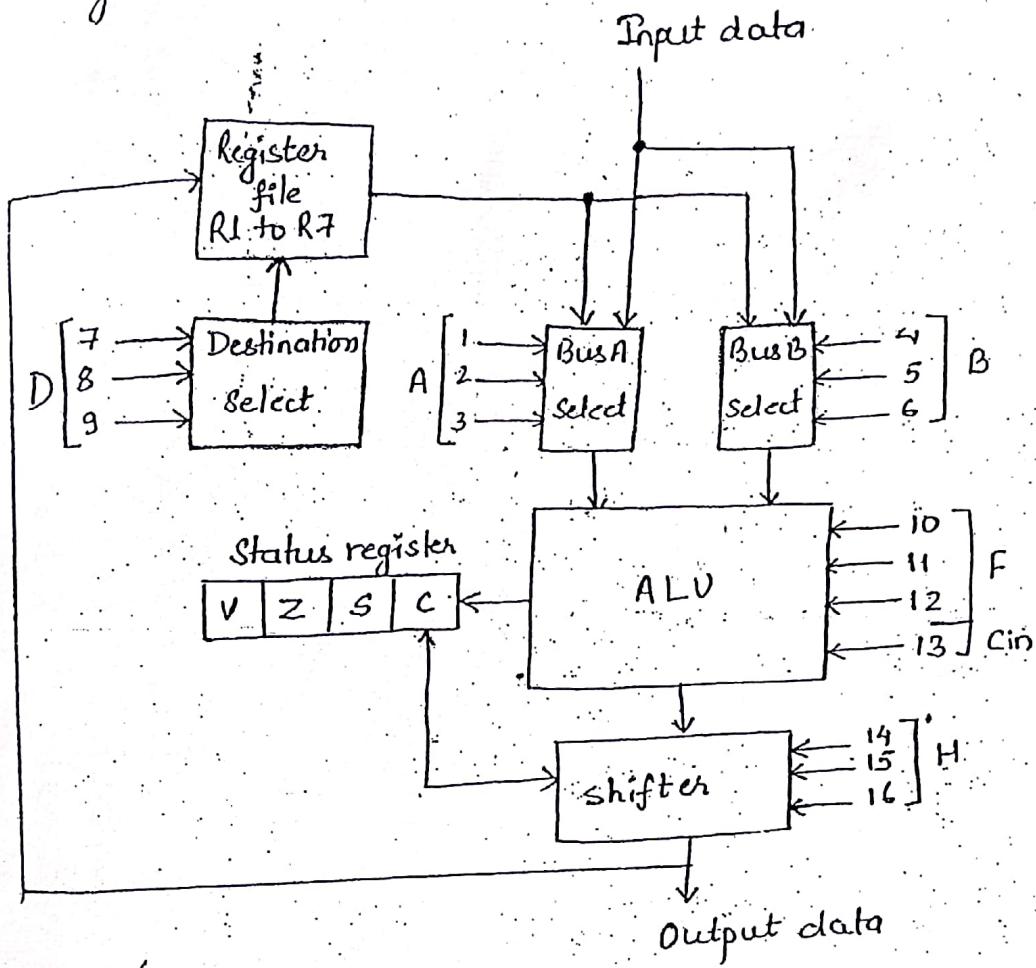


Fig: Block diagram

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	B	D	F	Cin	H										

Control word

- Processor unit consists of seven registers R<sub>1</sub> through R<sub>7</sub> and status register. The output of the seven registers go through two multiplexers to select the i/p to the ALU.
- Input data from an external source are also selected by the same multiplexers.
- The o/p of ALU goes through a shifter and then to a set of external o/p terminal. The o/p from the shifter can be transferred to any one of the registers or to an external destination.
- There are 16 selection variables in the unit and their function is

specified by a control word. The 16 bit control word, when applied to the selection variables in the processor, specifies a given micro operation.

→ All fields, except Cin, have a code of three bits. The three bits of A select a source register for the input to left side of the ALU. The B field is the same, but it selects the source information for the right input of the ALU. D field selects the destination register. The field, together with the bin in Cin select a function for the ALU.

→ The H field select the type of shift in the shift unit.

→ The H field select the type of shift in the shift unit.

→ A control word of 16 bits is needed to specify a micro-operation for the processor unit. The most efficient way to generate control words with so many bits is to store them in a memory unit which functions as a control memory where all control words are stored.

→ The sequence of control words is then read from the control memory, one word at a time, to initiate the desired sequence of micro-operations. This type of control operation is called micro programming.

## # Design of Accumulator:

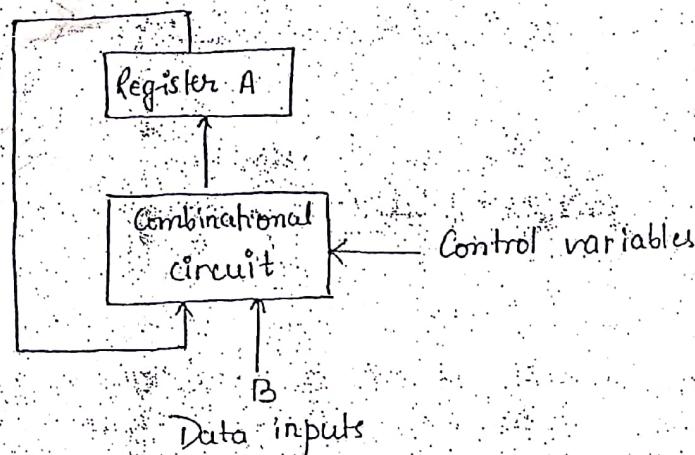


Fig: Block diagram of accumulator

→ Some processor units distinguish one register from all others and call it an accumulator register.

→ The accumulator register is essentially a bidirectional shift register with parallel load which is connected to an ALU