Estimating time-variation in matching efficiency and match elasticity for the US labor market

Prajyna Barua

Economics, School of Economic, Political and Policy Sciences

University of Texas at Dallas

ppb190000@utdallas.edu

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Abstract

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The paper estimates time-varying matching efficiency and match elasticity within a Cobb-Douglas matching function for the U.S. non-farm sector, using a state-space model. It evaluates two specifications: (i) no constant returns to scale (CRS) with time variation (my baseline model) and (ii) CRS with time variation. I compare these two models with the standard model without time variability, and the results show significant time variation in both parameters, even under stochastic volatility. Counterfactual vacancy estimates reveal that the non-CRS baseline model fits the data best. This model suggests a gradual decline in matching efficiency, indicating growing labor market inefficiencies. Matching efficiency and match elasticity with respect to unemployment are found to be procyclical. The procyclicality in efficiency can be attributed to reduced sectoral reallocation, as seen in CPS data, driven by structural economic shifts. Procyclical match elasticity points to the nonlinear effect of labor market tightness on the job-finding rate. The paper also finds a rise in the efficient unemployment rate after COVID-19, as estimated by the baseline model.

Introduction

Unemployment is one of the important indicators of slack in the labor market. It is countercyclical, and according to Shimer (2012), around 90% of the fluctuations in the unemployment rate since 1987 can be explained by movements in the probability of finding a job. Matching efficiency is one of the factors that affect job-finding probability; however, it is not observed. The matching efficiency represents the productivity of the process through which jobs are matched to job seekers (Hall and Schulhofer-Wohl (2018)). Therefore, it is important to estimate the matching efficiency; as highlighted by Barnichon and Figura (2011), a decline in matching efficiency would mean that fewer job matches are formed each period when holding unemployment and vacancies fixed. This would shift the Beveridge curve outward, suggesting a higher level of unemployment for any given level of vacancy which changes the trade-off between the unemployment rate and the vacancy rate at any point in time assuming the labor market is at a steady state [1]. Changes in matching efficiency also affect the natural rate of unemployment (Friedman (1968)[2]), an important input in policy consideration. Hence, it is important to understand how the efficiency has changed over time and to explore the possible explanations for such a change.

In this paper, I first estimate the matching efficiency and match elasticity using a Cobb-Douglas matching function and introduce time variation in these coefficients to understand how they evolved. A matching function [3] provides a mechanism through which jobs are matched to job-seekers to form hires and match elasticity is the responsiveness of the vacancies to hires. I primarily compare the two models, namely: no-CRS with time variation (baseline model-model1) and constant returns to scale (CRS) with time variation (model2). I also estimate the conventional matching function, which is CRS with no time variation using restricted OLS. With the introduction of time variation, I find a decline in matching efficiency over time. I also find that matching efficiency is procyclical in nature, and recovery was slow after the Great Recession, but recovery was rapid after COVID-19. Therefore I find

[1] A Steady state is a state where the rate of growth of the unemployment rate equals zero. Several studies have found that for the US, the actual unemployment rate is close to the steady state.

[2] Friedman (1968) argued that improvements in employment exchanges, availability of information about job vacancies, and labor supply, and so on would lower the natural rate of unemployment. All these factors affect matching efficiency.

[3] The underlying matching function assumes that the Beveridge Curve holds; Pissarides (2009); Elsby et al. (2015); Michaillat and Saez (2021).

decline in matching efficiency as one of the reason for an outward shift in the Beveridge Curve. Using the individual-level CPS [4] data, I find that there was a simultaneous decline in the reallocation of individuals across industries in response to structural change in the economy during both the COVID-19 pandemic and the Great Recession. The similarities in the movement in reallocation of the matching efficiency over time offer a possible explanation for the decline in the matching efficiency.

The matching elasticity also exhibits evidence of time variation and indicates that hires have become more responsive to vacancies. The increase in elasticity also suggests non-linearity of the job finding rate with respect to the labor market tightness, where the effect of a decrease in labor market tightness on job-finding rate increases during recession than during expansion. I also calculate the elasticity of the Beveridge curve through a steady-state analysis and find that vacancies have become less responsive to changes in unemployment over time. The analysis also shows that the functional form of the matching function matters, as the estimate of the unemployment gap differs. I estimate the efficient rate of unemployment using Michaillat and Saez (2021) (MS) methodology for all competing models. The analysis clearly shows different implications for different functional forms, suggesting that allowing for time variation without CRS not only provides a different estimate for the unemployment gap but also a different conclusion for the overall dynamics of the labor market.

I contribute to the literature by estimating matching efficiency through continuous time variation, and I attempt to provide a possible explanation of the mechanism affecting aggregate matching efficiency. Section one provides a brief review of the literature and motivation for exploring functional forms. Section two explains the data sources and some labor market variables. Section three explains the empirical framework and the estimation strategy. The results are presented and discussed in section

[4]The Current Population Survey (CPS) is a monthly survey of U.S. households conducted jointly by the U.S. Census Bureau and the Bureau of Labor Statistics (BLS). The IPUMS-CPS project provides a harmonized and integrated set of microdata drawn from the CPS, covering the period from 1962 onward. To support long-term comparative research, IPUMS-CPS standardizes variable coding across years and includes comprehensive documentation on comparability issues. The database retains all substantive variables from the original CPS samples. For more details, see: Sarah Flood, Miriam King, Renae Rodgers, Steven Ruggles, J. Robert Warren, Daniel Backman, Annie Chen, Grace Cooper, Stephanie Richards, Megan Schouweiler, and Michael Westberry. IPUMS CPS: Version 12.0 [dataset]. Minneapolis, MN: IPUMS, 2024. `https://doi.org/10.18128/D030.V12.0`

four, followed by the conclusion.

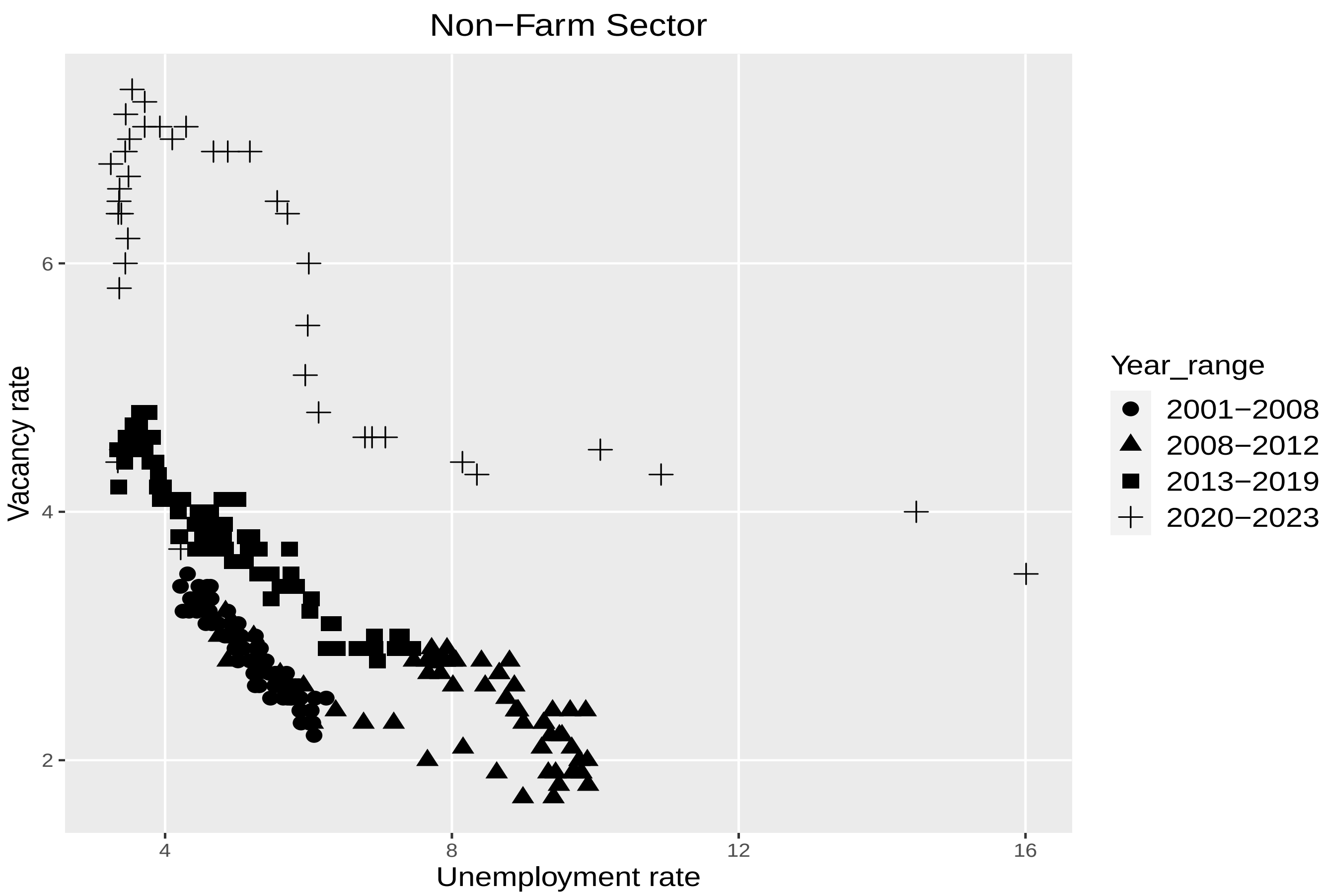
1 Related literature

To motivate the analysis, figure 1 plots the Beveridge curve (BC): that is, the vacancy rates against unemployment rates for the aggregate non-farm sector.[5] The Beveridge curve describes a negative theoretical relation between the vacancy rate and the unemployment rate such that the flows into and out of unemployment are matched so that the net change in unemployment is zero. At the steady state, the slope of the Beveridge curve depicts the trade-off between unemployment and vacancy rates. If inflows are greater than outflows, then the curve shifts to the right. The figure suggests both an outward shift and a change in the slope of the empirical Beveridge curve. These stylized facts parallel earlier analysis by Benati and Lubik (2014), were they find that the Beveridge curve changes shape and location after each recession. The change in shape suggests changes in slope, and the location change suggests an outward or inward shift.

Barlevy et al. (2024) points out several reasons for inflows to be different from outflows and finds that for each recession it was different. For instance, they found that during the Great Recession, the decrease in unemployment outflows led to an outward shift in the Beveridge curve. One of the reasons for such a decline was a decrease in matching efficiency, suggesting a mismatch between employers and skills of the unemployed, in addition to a decrease in recruitment intensity and increase in share of long-term unemployment. During COVID-19, both outflows and inflows changed. In the net, the curve shifted outward because of a decline in match efficiency due to an increase in quit rates and a change in labor demand by employers.

Therefore, to understand the impact of matching efficiency in reducing the outflow from unemployment, this unobserved variable needs to be estimated. It is standard in the literature to assume a time-invariant match efficiency, which is not in line with the recent empirical evidence that points to-

[5]BLS computes vacancy rate/job openings rate by dividing the number of job openings by the sum of the number of people employed and the number of job openings, and the unemployment rate is reported as the number of unemployed as a percentage of the total labor force (civilian).



Source: Calculated from JOLTS and BLS

Figure 1: Empirical Beveridge curve for the Non-Farm Sector

wards a time-varying pattern in the Beveridge Curve. Therefore, the paper uses a Cobb-Douglas match function and introduces time variation in match efficiency and match elasticity. Since the Global Financial Crisis, several papers have introduced time variation in several ways in these models. Lubik (2013) calibrated the values of separation rates and estimated matching efficiency and match elasticity using nonlinear least squares. Blanchard et al. (2022) extract time-varying match efficiency from the Cobb-Douglas match function by fixing match elasticity and finding declining matching efficiency during COVID. Hall and Schulhofer-Wohl (2018) estimated time-varying match efficiency using residuals after regressing job-finding rate on labor market tightness after accounting for labor market heterogeneity. They found that overall matching efficiency declined smoothly from 2001 through 2013. Lubik et al. (2016) modeled the time variation in matching efficiency as endogenous switching regimes with high and low match efficiency regimes. The endogenous threshold is determined by the level of output and labor market performance, and variations in the efficiency of the match indicate a structural shift in the labor market. This is unlike the Markov-switching model where regime change is exogenous.

Michaillat and Saez (2021) estimated structural breaks in Beveridge curve elasticity and then estimated match elasticity for 1951-2019 under the Diamond Mortensen Pissarides (DMP) model [6], suggesting time variation. Hence, the literature provides evidence for time variation. Bernstein et al. (2021) found that fixed match elasticity does not describe the data well. They found that by using the DRW (Den Haan, Rogerson, and Wright) matching function[7] they get time variation in match elasticity which suggests a non-linear relation between job-finding rate and productivity, suggesting that functional form of the matching function affects the outcome of these relations. This paper explores the time variation in these latent variables together by estimating them using state-space modeling and assuming continuous time variation.

In addition, there is also precedence for using non-CRS matching functions in the literature. Blanchard and Diamond (1989) found mildly increasing returns to scale for the Cobb-Douglas matching

[6]Under DMP model, Beveridge curve elasticity and match elasticity are directly related using the steady state equation of the Beveridge Curve, Michaillat and Saez (2021).

[7]The DRW matching function, proposed by Den Haan, Rogerson, and Wright (Den Haan et al., 2000) is an alternate form of matching function where the job-finding rate is allowed to vary with match elasticity, unlike the Cobb-Douglas specification where the relation is constant.

function for aggregate economy using IV estimation[8]. They find returns to scale ranging between 1.03-1.07. For the manufacturing sector, they find increasing returns to scale (1.35). Based on their study, increasing returns to scale can happen when markets are active, which may lead to easier matches with or without intensive search. They found the coefficient of vacancy to be greater than the unemployment coefficient. Warren (1996) found increasing returns to scale for the manufacturing sector. Petrongolo and Pissarides (2001) found that generally constant returns to scale fit the data well. To further explore this issue, the paper, therefore, compares models with CRS and non-CRS including time variation.

I find that the baseline model fits the data well, supporting time variation and mildly increasing returns to scale. There is a decline in match efficiency and an increase in match elasticity over time. Several other papers have also suggested a decline in matching efficiency (m_t) as a reason for the shift in BC during the Great Recession. Lubik (2013) found that a shift in the Beveridge curve during the Great Recession is consistent with a decrease in matching efficiency, an increase in separation rate, and a reduction in match elasticity. He suggested that the decline in matching efficiency was because of mismatch. Bleakley and Fuhrer (1997) provided three reasons for an outward shift- an increase in the degree of labor reallocation, growth in the labor force, and a decrease in matching efficiency[9]. The Beveridge curve shifted outward again after COVID-19, and Blanchard et al. (2022) explained this outward shift as a result of stronger economic activity, lower matching efficiency, and higher reallocation.

The reallocation argument becomes important as there is evidence that an economy may go through structural change during recessions. In general, business cycles may affect industries asymmetrically. Hence, some industries may fail to recover fully and therefore lose jobs permanently. The labor and capital may, therefore, reallocate to other industries. In case the reallocation of labor is not fast enough, this can lead to mismatch and thus affect the BC through an outward shift. Briggs (2022) explained the outward shift as a result of the decline in job search intensity of unemployed workers depicted by a decrease in the fraction who actively submit job applications. Another factor that can affect the location of the BC is changes in the separation rate which was ruled out by Lubik (2013) as data suggested

[8] As instrumental variables (IVs), they used lags of unemployment, vacancies, and industrial production.

[9] As summarized by Hobijn and Sahin (2013) there are several reasons for decline in matching efficiency which can be due to occupational, industrial and skill mismatch between labor supply and labor demand, increase in houselock leading to geographical mismatch or extension of unemployment benefits during recessions.

stable separation rates[10] (ρ_t) except during crisis. Benati and Lubik (2014) suggested a change in the slope of the curve after each business cycle. One of the factors affecting slope is match elasticity (α_t), which was found to fall during the Great Recession. Lange and Papageorgiou (2020) estimated match function non-parametrically and found support for procyclical match elasticity that fluctuated between 0.15-0.3.

Both match elasticity and matching efficiency are used in the calculation of the slope and elasticity of the steady-state BC. Hence, I estimate the slope and elasticity of the Beveridge curve using the time-varying latent variables and find falling elasticities of vacancies with respect to unemployment, which may suggest declining responsiveness of vacancies with respect to unemployment over time. I then use these parameters to estimate efficient unemployment rates and the unemployment gap as per the methodology suggested in Michaillat and Saez (2021). As will be seen in the later section, estimating the unemployment gap with no time variation in these variables provides a different conclusion of the state of the labor market. The baseline model suggests that the unemployment gap closely follows the business cycle, where it increases during recessions and declines during expansions, thus following the general intuition about the labor market. However, the conclusions are not that intuitive when I draw them from model 2, which suggests that the labor market was mostly inefficiently slack until 2015, after which it became inefficiently tight.

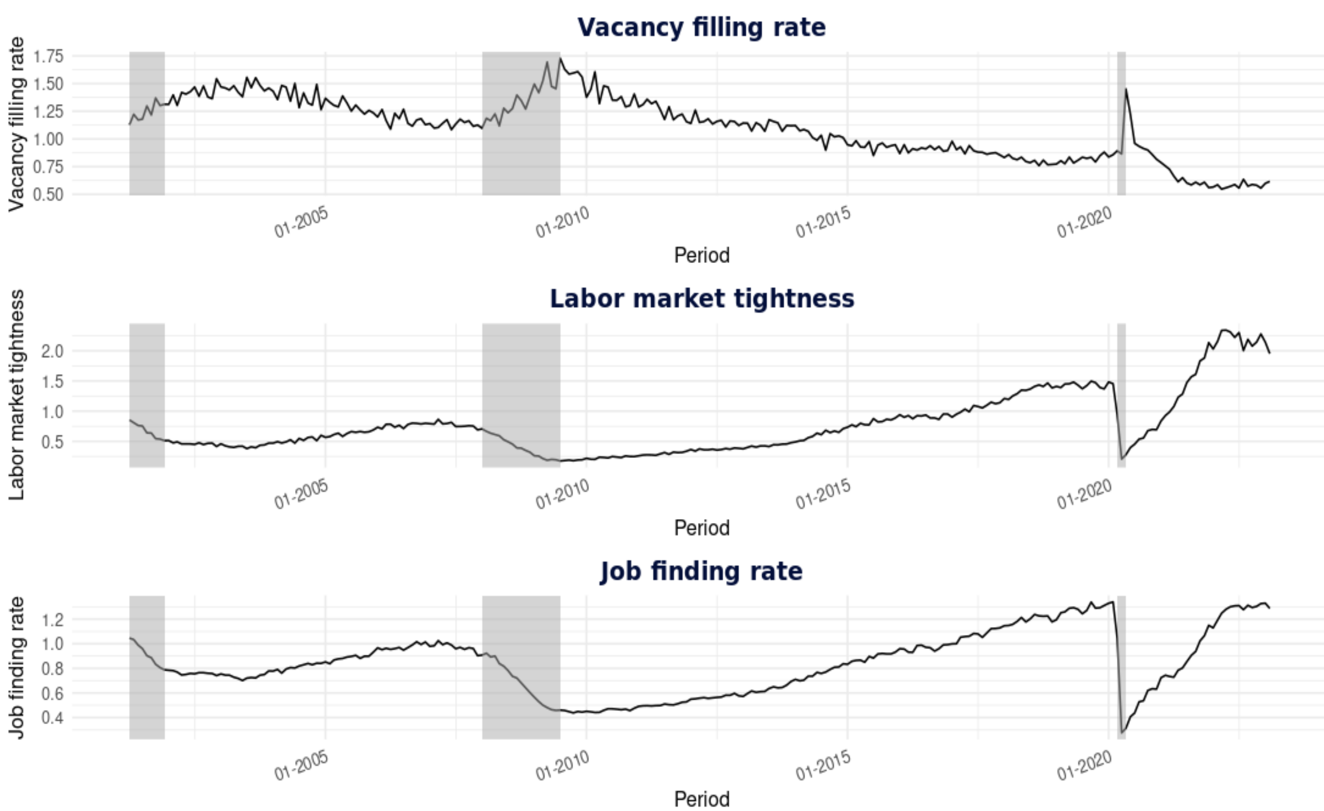
2 Data

The paper uses survey data primarily from the Job Opening and Labor Turnover Survey (JOLTS) and the Bureau of Labor Statistics (BLS) for the number of unemployed people per month. Data from JOLTS begins from December 2000 and therefore, the period under study is from December 2000 to March 2023. Variables considered for analysis includes Hires (H_t), Vacancies (V_t), Unemployed (U_t), separations (S_t). All variables are seasonally adjusted and are in levels unless otherwise stated.

Hires are gross hires, which include new hires, retired employees, part-time employees, and per-

[10] The separation rate is computed by dividing the number of workers separated from their jobs by the number of people employed.

manent and short-term employees. The hires are reported during the month. Vacancies are the number of job openings that are open the last business day of the reference month and meet all the three criteria, namely- (i) a specific position exists and work is available for that position, (ii) the job could start within 30 days, and (iii) employer is actively looking for workers outside the firm. Vacancies exclude positions for internal transfers and promotions. Separations include people who are separated from payroll for the entire reference month, which includes quits (either voluntary or retirement), layoffs, and discharges.



Source: Calculated from JOLTS and BLS database

Figure 2: Labor Market Ratios

I provide several labor market indicators in figure 2 to provide some understanding and stylized facts for the aggregate non-farm sector. The shaded areas represent NBER recession periods. Labor market tightness is defined as the ratio of the number of vacancies and unemployed ($\theta_t = \frac{V_t}{U_t}$) is procyclical and, on average, shows an upward trend after each recession. This is intuitive as during recessions, unemployment increases and vacancies decline, resulting in lower labor market tightness. The job-finding rate ($f_t = \frac{H_t}{U_{t-1}}$) is also unsurprisingly procyclical and moves in tandem with labor market tightness. This implies that it becomes difficult for people to find a job during the crisis and easier during the boom. In contrast, it becomes easier for firms to fill a vacancy during the recession and difficult for firms to fill it during the boom. The vacancy-filling rate is given by ($q_t = \frac{H_t}{V_{t-1}}$). These ratios are rates and not probabilities and, therefore, can be greater than 1. For instance, the job-finding rate f_t can be greater than 1 since the gross hires at period t can be from unemployed at t-1, from people out of the labor force, and from people who experience job-to-job transfers without facing unemployment. This would mean that hires can be greater than unemployed. The JOLTS database does not provide monthly data on job-to-job transfers or re-hires. Because of this, the job-finding probabilities are calculated from the Integrated Public-Use Microdata Series (IPUMS)-Current Population Survey (CPS), which tells us the probability of finding a job each month by only considering those individuals who transitioned from unemployment to employment that month. It can be seen from figure 15 in appendix V that the seasonally adjusted job-finding probabilities broadly follow the trend of job-finding rates.

3 Empirical framework to estimate the time-varying parameters

3.1 Empirical framework

The starting point for this analysis is the estimation of a Cobb-Douglas matching function, Equation 1. This is the most restricted model, which assumes CRS and no time variation in the parameters of interest, namely matching efficiency (m) and match elasticity (α). The matching function suggests that total hires (H_t) in any given period are an increasing function of the number of unemployed (U_{t-1}), the number of job vacancies from the previous period, (V_{t-1}) and match elasticity and matching efficiency. I assume no contemporaneous relationship between hires and vacancies and unemployment. The JOLTS data reports job openings at the month's end, while hires are reported during the month by the surveyed establishments. For the unemployment data, CPS conducts interviews around the middle of the month. Hence, it is a reasonable approximation to assume that hires at time (t) come from vacancies and unemployed at time $(t-1)$. This is both driven by the data and the literature. Empirically, on average, it takes more than a month to find a job across industries. Table1 provides an approximation of the average [11] number of months it takes to find a job in each industry and it generally exceeds one month [12]. In the literature, a similar specification was used for example in Blanchard and Diamond (1989).

$$H_t = mU_{t-1}^{1-\alpha}V_{t-1}^{\alpha} \tag{1}$$

Industry	Average Job finding rate ($\frac{H}{U}$)	Average number of months to find a job
Education and Health	0.81	1.23
Leisure and Hospitality	0.91	1.1
Professional and Business Services	1.12	0.89
Transportation and Utilities	0.63	1.58
Financial activities	0.72	1.39
Information	0.62	1.62
Mining and Quarrying	0.96	1.05
Construction	0.55	1.82
Government Workers	0.58	1.73
Wholesale and retail trade	0.75	1.33
Manufacturing	0.49	2.04

Table 1: Average number of months to find a job at Industry level

After introducing time variation in the matching efficiency and match elasticity and not assuming constant returns to scale in Equation 1, I have my baseline model, Equation 2. Including time variation in these parameters suggests that both matching efficiency and match elasticity do not have a constant

[11] Average is calculated from December 2000-March 2023.

[12] Inverse of the Job-finding rate.

effect on the job-finding rate but vary because of the business cycle fluctuations. This baseline model is compared with models assuming constant returns to scale with and without time variation.

$$H_t = m_t U_{t-1}^{\beta_t} V_{t-1}^{\alpha_t} \tag{2}$$

The parameters (efficiency and elasticities) are affected by labor market heterogeneity and endogeneity in search intensity. The presence of endogeneity implies that workers are generally more inclined to job search during better economic conditions as it is easier to find jobs in the tight labor market, and this can affect the productivity of the matching process (m_t). Acknowledging this, I initially assume no heterogeneity and endogeneity in the labor market. I rewrite Equation 2 as Equation 3 in terms of job-finding rate and labor market tightness by dividing it by U_{t-1} and taking logarithmic transformations on both sides.

$$ln\left(\frac{H_t}{U_{t-1}}\right) = ln\left(m_t\right) + \delta_t ln\left(U_{t-1}\right) + \alpha_t ln\left(\frac{V_{t-1}}{U_{t-1}}\right) + \epsilon_t \tag{3}$$

where $\delta_t = \alpha_t + \beta_t - 1$

Under constant returns to scale β_t becomes $1 - \alpha_t$ which implies $\delta_t = 0$. Therefore, under constant returns to scale the job finding becomes the function of only labor market tightness. For estimation purposes, I rewrite equation 3 both in terms of non-CRS and CRS and get Equation 4 (baseline model-Model 1), and Equation 5 (model 2), respectively. I also estimate Equation 5 but without any time variation in $\tilde{m}$ and α either through restricted OLS or the state space model with constant parameters.

$$\tilde{f}_t = \tilde{m}_t + \delta_t U_{t-1} + \alpha_t \tilde{\theta}_{t-1} + \epsilon_t \tag{4}$$

$$\tilde{f}_t = \tilde{m}_t + \alpha_t \tilde{\theta}_{t-1} + \epsilon_t \tag{5}$$

Models 1 and 2 are estimated by specifying state-space equations. The data is seasonally adjusted on a monthly frequency. The model consists of observation equations, which link the observed variables to the latent states, and state equations, which describe the evolution of unobserved variables over time. Let t denote time, $t = 1, 2, ..., T$. The state and the observation equations are given below. This specification does not account for stochastic volatility. However, the assumption is relaxed in subsequent estimation.[13] Model 2 is represented below.

$$\begin{bmatrix} \tilde{f}_t \\ \tilde{\theta}_{t-1} \end{bmatrix} = \begin{bmatrix} \tilde{m}_t \\ a \end{bmatrix} + \begin{bmatrix} \alpha_t & 0 \\ 0 & c \end{bmatrix} . \begin{bmatrix} \tilde{\theta}_{t-1} \\ \tilde{\theta}_{t-2} \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ \epsilon_{\tilde{\theta}_{t-1}} \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} \tilde{m}_t \\ \alpha_t \end{bmatrix} = \begin{bmatrix} \mu \\ 0 \end{bmatrix} + \begin{bmatrix} \rho & 0 \\ 0 & 1 \end{bmatrix} . \begin{bmatrix} \tilde{m}_{t-1} \\ \alpha_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{\tilde{m}t} \\ \epsilon_{\alpha t} \end{bmatrix} \tag{7}$$

$$\epsilon_i \sim \mathcal{N}(0,\, \sigma_i^2) \qquad where,\ i \in \{\tilde{\theta}, \tilde{m}, \alpha, \epsilon\} \tag{8}$$

The observation equations and state equations are given by Equation 6 and Equation 7, respectively. The residuals are assumed to be normally distributed and given by Equation 8. $\tilde{m}_t$ is matching efficiency in log levels and is modeled as an AR(1) process similar to Lubik (2013). α_t is match elasticity and I assume a random walk process following the literature. $\tilde{\theta}_t$ is the logarithm of labor market tightness, which is modeled as an AR(1) process after analyzing the acfs and pacfs. $\tilde{f}_t$ is the logarithmic value of observed job finding rate. Allowing for stochastic volatility, model 2 is rewritten below.

$$\begin{bmatrix} \tilde{f}_t \\ \tilde{\theta}_{t-1} \end{bmatrix} = \begin{bmatrix} \tilde{m}_t \\ a \end{bmatrix} + \begin{bmatrix} \alpha_t & 0 \\ 0 & c \end{bmatrix} . \begin{bmatrix} \tilde{\theta}_{t-1} \\ \tilde{\theta}_{t-2} \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ \epsilon_{\tilde{\theta}_{t-1}} \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} \tilde{m}_t \\ \alpha_t \\ h_t \end{bmatrix} = \begin{bmatrix} \mu \\ 0 \\ (1-\phi)\mu_h \end{bmatrix} + \begin{bmatrix} \rho & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \phi \end{bmatrix} . \begin{bmatrix} \tilde{m}_{t-1} \\ \alpha_{t-1} \\ h_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{\tilde{m}t} \\ \epsilon_{\alpha t} \\ \epsilon_{\eta t} \end{bmatrix} \tag{10}$$

$$\epsilon_t \sim \mathcal{N}(0,\, e^{h_t}) \qquad where\ e\ is\ an\ exponential\ function \tag{11}$$

[13] The results after accounting for stochastic volatility do not vary significantly from the results for the benchmark model and are presented in the Appendix.

$$\epsilon_i \sim \mathcal{N}(0,\, \sigma_i^2) \qquad\qquad where,\ i \in \{\tilde{\theta}, \tilde{m}, \alpha, \eta\} \tag{12}$$

Here, ϵ_t has a heteroscedastic variance represented by Equation 11, and the errors of all the other variables are assumed to be normally distributed and given by Equation 12. The observation equations and state equations are given by Equation 9 and Equation 10, respectively. The stochastic volatility of the error term is modeled through h_t. ϕ is assumed to be less than one to ensure the stationarity of the variable.

Bayesian estimation is applied using a Gibbs sampler, which iteratively samples for each parameter using conditional distribution, given their current values. For each estimation, between 250,000 and 300,000 iterations were performed across four independent Markov Chain Monte Carlo (MCMC) chains, each initialized with distinct starting values, following single-move filtering updates. The initial values were taken as the previous estimates of the parameters of interest from the literature. The first 10,000 iterations of each chain were discarded as burn-in to eliminate dependence on initial values. This approach ensures robust posterior inference and thorough exploration of the parameter space. Both m_t and α_t are expected to be positive and constrained between 0 and 1. However, no explicit constraints were imposed during the sampling. Instead, the posterior distributions of these parameters were inferred directly from the data, and the estimates remained within reasonable bounds throughout the iterations. A similar procedure was applied to the estimation of Model one, both with and without stochastic volatility. The detailed model structure for model one is presented in Appendix I, while information on the prior distributions is provided in Appendix II for both models. The results presented in the next section are based on the estimations with loose priors, where lower precision reflects weaker prior information. Additional details about the link between my empirical model an the canonical DMP model are in Appendix III.

3.2 Framework to estimate unemployment gap

The approach used here can also be used to extract an estimated unemployment gap, which is both a variable that is empirically interesting and relevant for policy on its own, and a variable that can be used to evaluate the performance of the different models considered here. Introducing time-variation in matching efficiency and match elasticity can affect the slope of the Beveridge curve, which is calculated in a steady state. The curve theoretically depicts a negative relationship between the unemployment rate and vacancy rates. In the steady state, the inflow into unemployment equals the outflow from unemployment, such that the net change in unemployment is zero. The parameters estimated from the matching function can be used to estimate the trade-off between unemployment and vacancies, which is the slope of the Beveridge curve, under the assumption that the empirical Beveridge curve functions very close to the Beveridge curve in the steady state. According to Michaillat and Saez (2021), the deviation from the unemployment rate of the Beveridge curve decreases about 50% within one month and about 90% within one quarter; therefore, it is accurate to assume that $\Delta u = 0$ and, therefore, the labor market is always on the Beveridge curve. The motivation to calculate the slope is to estimate the efficient rate of unemployment and understand the effect of these different models on the state of the labor market.

Under the steady state, equation 13 describes the law of motion for unemployment where s_{t-1} is the separation rate from the previous period, f_t is the job-finding rate, and $\Delta u = 0$. It states that unemployment today depends on the people moving from employment to unemployment plus those who were already unemployed (or inflow into unemployment) minus those who got out of unemployment to employment in the previous period. The Beveridgean unemployment rate can be calculated from the differential equation 13, and Michaillat and Saez (2021) found that this unemployment rate decays at the rate s+f.

$$u_t = s_{t-1}(1 - u_{t-1}) - f_t u_{t-1} + u_{t-1} \tag{13}$$

The unemployment and vacancy rates are expressed as a percentage of the labor force (L). By rewriting Equation 2 in terms of the labor force ratio and normalizing by the unemployment rate, I derive Equation 14. Substituting Equation 14 into Equation 13 yields Equation 15, which, in the steady state, simplifies to Equation 16—the equation for the Beveridge curve.

This formulation represents a generalized Beveridge curve equation without imposing constant returns to scale (CRS). The slope of the Beveridge curve in the steady state is derived from Equation 16, leading to Equation 17. Under the assumption that the labor market operates on the Beveridge curve, I estimate the relevant variables to compute the Beveridge curve elasticity. The elasticity of the vacancy rate with respect to unemployment, which characterizes the Beveridge curve, is given by Equation 18.

$$f_t = m_t u_{t-1}^{\delta_t} \theta_{t-1}^{\alpha_t} \tag{14}$$

$$v_{t-1} = \left(\frac{(s_{t-1}(1-u_{t-1}) - \Delta u_t)}{m_t . u^{\beta_t}}\right)^{\frac{1}{\alpha_t}} \tag{15}$$

$$v = \left(\frac{s(1-u)}{m.u^{\alpha}}\right)^{\frac{1}{\beta}} \tag{16}$$

$$\frac{dv}{du} = -\frac{1}{\alpha}\left(\frac{s}{m(u)^{\beta}}\right)^{\frac{1}{\alpha}}(1-u)^{(\frac{1}{\alpha})-1}.(1+\beta\frac{1-u}{u}) \tag{17}$$

$$\eta = \frac{dv/v}{du/u} = \frac{dv}{du}.\frac{u}{v} \tag{18}$$

Michaillat and Saez (2021), estimate the efficient rate of unemployment (u^*) and the unemployment gap (un_{gap}) using the slope of the Beveridge curve. They define u^* as a combination of unemployment and vacancy on the iso-welfare curve that maximizes social welfare, subject to the Beveridge curve constraint. In other words, at u^*, the marginal cost of creating a vacancy equals the cost of additional unemployment, Ahn and Crane (2020). According to Michaillat and Saez (2021), as the Beveridge curve gets steeper, the elasticity of the Beveridge curve increases; therefore, to maintain the same level of welfare, the Beveridge curve must shift to the right, increasing the efficient unemployment rate. However, this assumes that κ (the recruiting cost) and ζ (the social value of non-work) are constant.

$$u* = \left(\frac{\kappa\epsilon}{1-\zeta}.\frac{v}{u^{-\epsilon}}\right)^{\frac{1}{1+\epsilon}} \tag{19}$$

$$un_{gap} = u - u* \tag{20}$$

$$un_{gap} < 0 \tag{21}$$

According to Michaillat and Saez (2021), in a decentralized economy, this problem is solved using the Beveridge curve, wage equation, and job creation condition within the Diamond-Mortensen-Pissarides (DMP) model. They used Equation 19, where they assumed κ (recruiting cost) = 0.92; ζ (social value of non-work) = 0.26. I will also take their chosen values to estimate the unemployment gap. The unemployment gap can be calculated from Equation 20. Michaillat and Saez (2021) states the labor market is efficient when $u = u^*$. A positive unemployment gap indicates that the labor market is inefficiently slack. While a negative unemployment gap indicates that the labor market is inefficiently tight.

4 Model Estimates and Discussion of Findings

4.1.1 Matching efficiency estimates

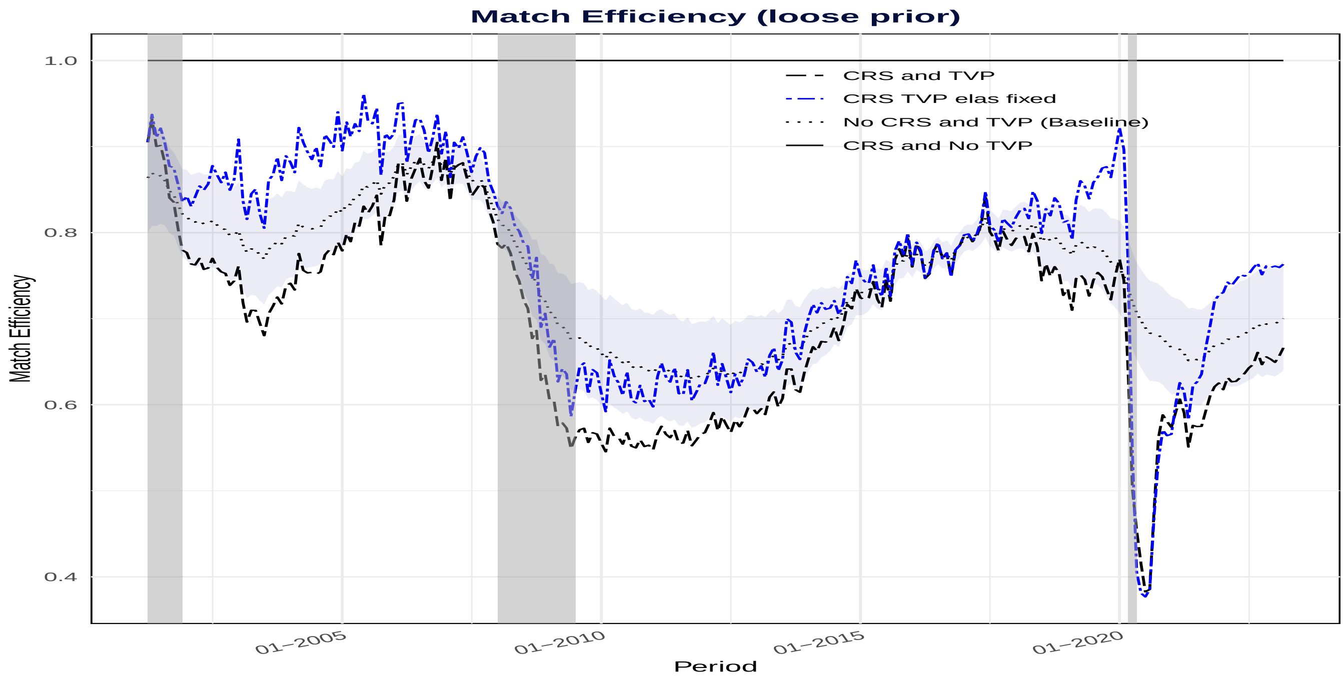
Matching efficiency exhibits pro-cyclicality, with a sharp decline during each recession (Figure 3). The figure compares the three models along with an additional model that assumes constant returns to scale (CRS) and time-varying efficiency while keeping the match elasticity fixed (in blue). Except for the efficiency estimated from the restricted model 1, where the parameters are held fixed, all other models suggest time-variability. The matching efficiency declines during recessions, indicating it is harder to match the job to job seekers during recessions than during expansions. The estimates from the alternative models provide similar estimates during expansions but differ in their impact during recessions. For instance during COVID-19, model 2 suggests a sharp decline in matching efficiency, while the baseline model suggests a gradual decline.

The confidence interval is for the baseline model. Between the two models (models one and two), the matching efficiency estimates are similar during expansions but differ significantly during recessions and recoveries. Hence, the main difference between the baseline and CRS time-varying models occurs during recessions, which is driven not by time variation but by assumptions on returns to scale. The decline in matching efficiency is much steeper for the CRS model than the baseline model. This decline is similar to the steep decline suggested by Blanchard et al. (2022) but for a much shorter period.

4.1.2 Potential mechanism

Evidence of the procyclical nature of matching efficiency can be also found in the literature. Barlevy et al. (2024) suggests a decline in matching efficiency was one of the reasons for the shift in the Beveridge curve. Such a decline affects the outflow from unemployment and thus increases unemployment at any level of vacancy. They suggested several reasons behind such decline- the extension of unemployment insurance benefits during recessions, increase in hiring standards by the firm, increase in a mismatch between location and qualification of the unemployed, and the scarring effect of recession on the unemployed. Barnichon and Figura (2011) also found a decrease in matching efficiency using two measurable factors- the composition of the unemployment pool and the dispersion in labor market conditions. I use a similar framework to explain the movements in matching efficiency.

According to the composition effect, the average job-finding probability will decline more than what the matching function would imply if a group with a lower-than-average job-finding probability gets overrepresented in the unemployment pool. Shimer (2012) also suggests that the cyclicality in the unemployment rate since 2006 was mainly driven by cyclicality in job-finding probability (which controls the outflow out of unemployment) rather than cyclicality in job separation rates. In other words, unemployment during a recession increases mainly because it is hard to find a job and not because more people are joining the workforce. I find the evidence in support of this argument using the micro-level data that the unemployment rate among people without a degree is much higher during the recession than with a degree, Figure 4. The difference between unemployment rates increases much more during a recession. Generally, people without a college degree (associate or above) have a lower probability of finding a job than people with a degree. In addition, there was also an increase in long-term unem-



Source: Own Estimates

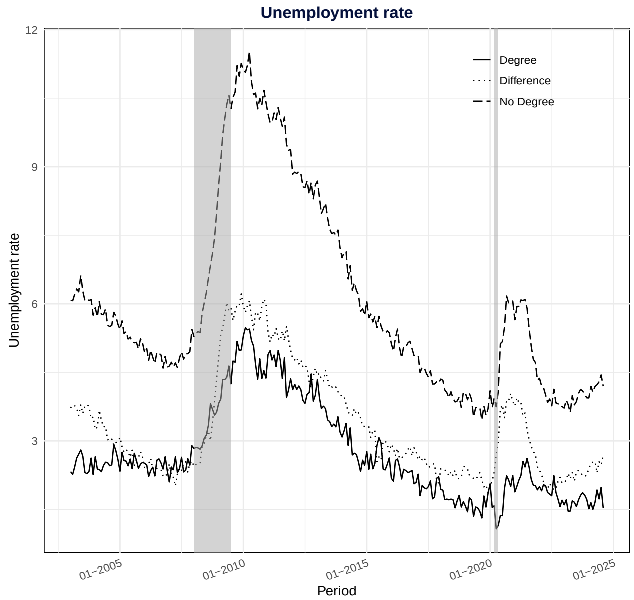
Figure 3: Matching efficiency

ployment [14] during the Great Recession, further increasing the people with low job-finding probability in the unemployment pool. Both of the pieces of evidence indirectly support the composition effect on the unemployment pool that Barnichon and Figura (2011) suggested.

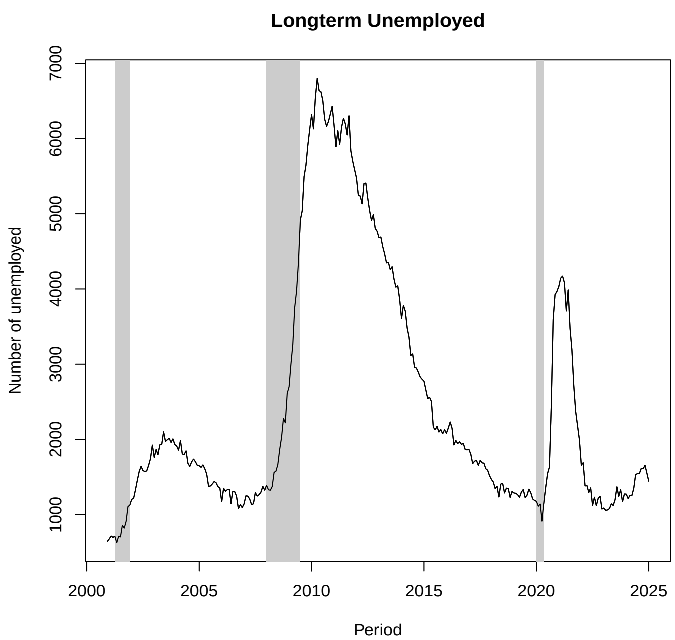
Looking at the job-finding probabilities (Figure 5) it becomes clear that the probabilities reduce significantly during recessions, reducing the outflow of unemployment as people with lower job-finding probabilities [15] become over-represented in the unemployment pool. The probability represents the transition from unemployment to employment each month and is calculated from individual-level data. The estimation process of job-finding probabilities is detailed in Appendix IV. The figure also suggests that the two recessions were different with respect to the recoveries in the job-finding probabilities. While the job-finding probability recovered quickly after COVID-19, this was not the case during the Great Recession, where it took around 7-8 years for those probabilities to return to the pre-recession levels.

[14] Defined as people unemployed for 27 weeks or longer.

[15] The unemployment rate of people without a degree (either associate or bachelors) and people with long term unemployment.



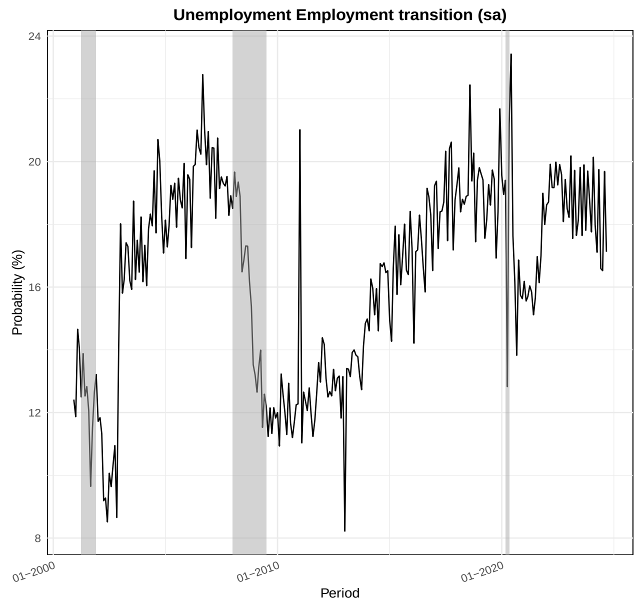
(a)



(b)

Source: a) Author's estimates from CPS data; b) FRED database

Figure 4: Unemployment rate as per degree and Number of Long-term unemployed



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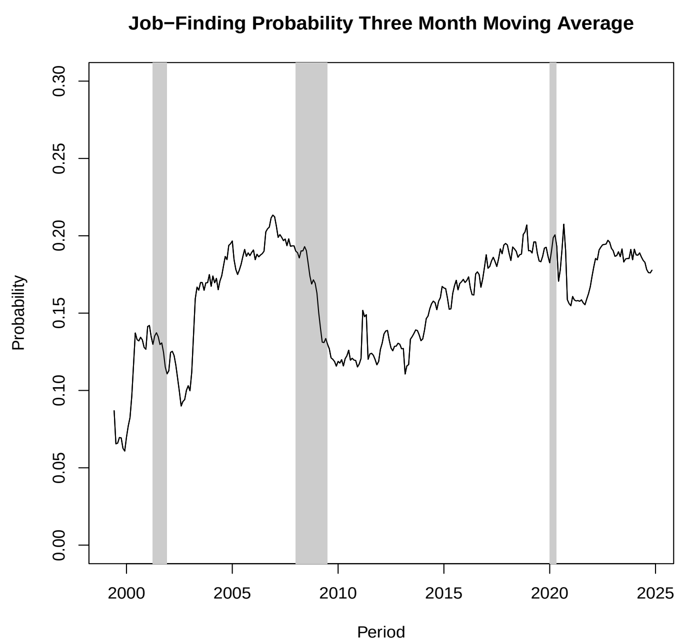


(b)

Source: Author's calculation using CPS data

Figure 5: Job finding probability for non-farm sector

A possible explanation for the slow recovery of matching efficiency following the Great Recession can be the reallocation of labor across industries, which led to a slow recovery of the job-finding probabilities, a dispersion argument. This argument is particularly relevant given the structural transformation of the economy, which shifted from manufacturing and mining toward service-based sectors. As pointed by Pissarides (2000), this structural shift in demand can effect the flows into unemployment. This structural change is measured in terms of industry-level employment share, as defined by Equation 22. Industries are classified according to the 2-digit NAICS classification. To quantify structural change, I employ the Norm of Absolute Values (NAV) index, which captures shifts in employment shares across sectors, Dietrich (2012). The NAV index is computed as the absolute difference in employment share for sector i between periods t and s. The index ranges from 0 to 1, where 0 implies no structural change and 1 otherwise. To standardize the measure, the absolute differences are summed and then divided by two. The data used is seasonally adjusted. Figure 6 illustrates that significant structural change occurred during both recessions. To better isolate the effects of the Great Recession, the right side of Figure 6 excludes the COVID-19 recession, allowing for a clearer analysis.

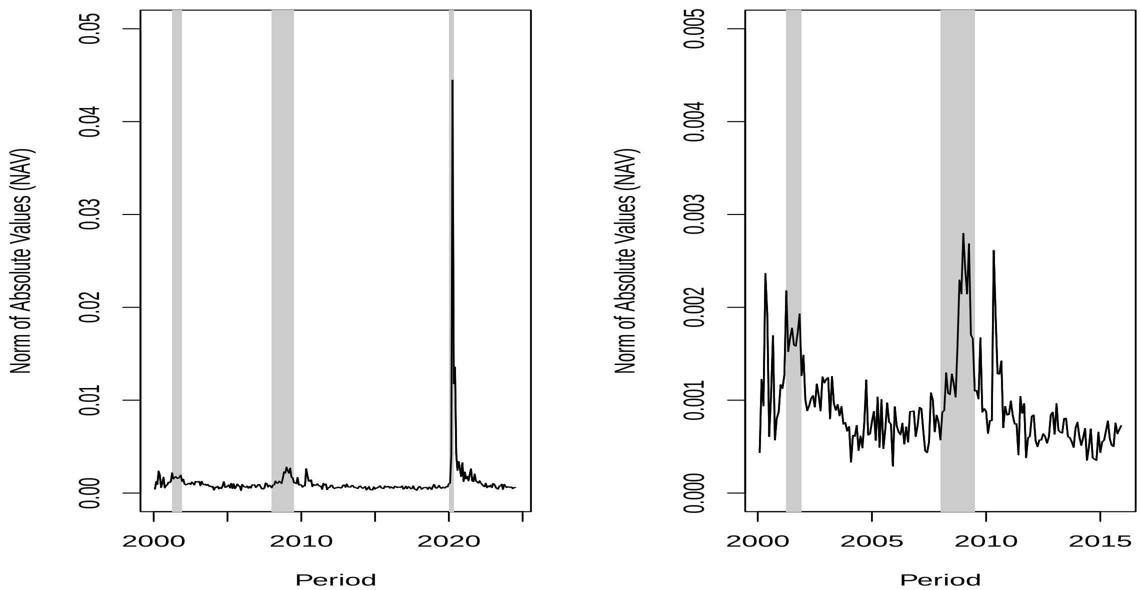
$$NAV_{i,t} = 0.5.\Sigma_{i=1}^{n}|x_{it} - x_{is}| \tag{22}$$

According to Barnichon and Figura (2011), dispersion in labor market conditions can include changes in the location or nature of the job, however this may have a negative effect on matching efficiency as people may take a bit longer to reallocate to other industries. I define reallocation as the change in the industry by the worker. I define average reallocation as reallocation (r_t) between industries weighted by their employment share. Further details regarding estimation are in the next sub-section. Here, X_{it} is employment in industry i, X_t is total employment in the non-farm sector, and a_{it} is the percentage of people changing their industry.

$$r_t = \Sigma_{i=1}^{13} \frac{X_{it}}{X_t} a_{it} \tag{23}$$

4.1.3 Calculation of monthly average reallocation

Here, reallocation is defined as an individual changing industries from one month to the next, and therefore, it signifies a shift in industry from one month to another. The individual-level data are extracted



Source: Author's calculation from the BLS Database

Figure 6: Index for changes in the sectoral composition in employment for the US non-farm sector

from the CPS data at a monthly frequency and span the period from 2003 to 2024. An individual may change industries multiple times within the 4-8-4 cycle, till the household is represented in the panel; hence, (s)he may go back to the same industry after changing industries before, and therefore, in that sense, it includes reallocation as well. I have dropped from the sample those individuals who were currently out of the labor force or do not have any industrial classification, and an individual is flagged as reallocated/shifted industry if the person (either employed or unemployed) at time t changes industry at time t+1, given that the person is employed at time t+1.

The algorithm identifies the individual using a unique individual identifier provided by CPS (CPSIDV). The industry classification for the individual is taken at time t+1. Therefore, the algorithm flags reallocation/shift to other industries instead of reallocation from. For the unemployed at time (t), the CPS classifies them into industry according to their last industry worked within the past 12 months. Once individuals who moved to other industries are flagged, I aggregated individuals at the level of year, month, and industry using weights provided by CPS. For any time t, the percentage of reallocation represents the number of people who reallocated to industry k as a ratio of the total number of people employed at t. The ratio was seasonally adjusted using ARIMA X-13 seats. Average monthly reallocation Figure 7 was calculated using equation 23.

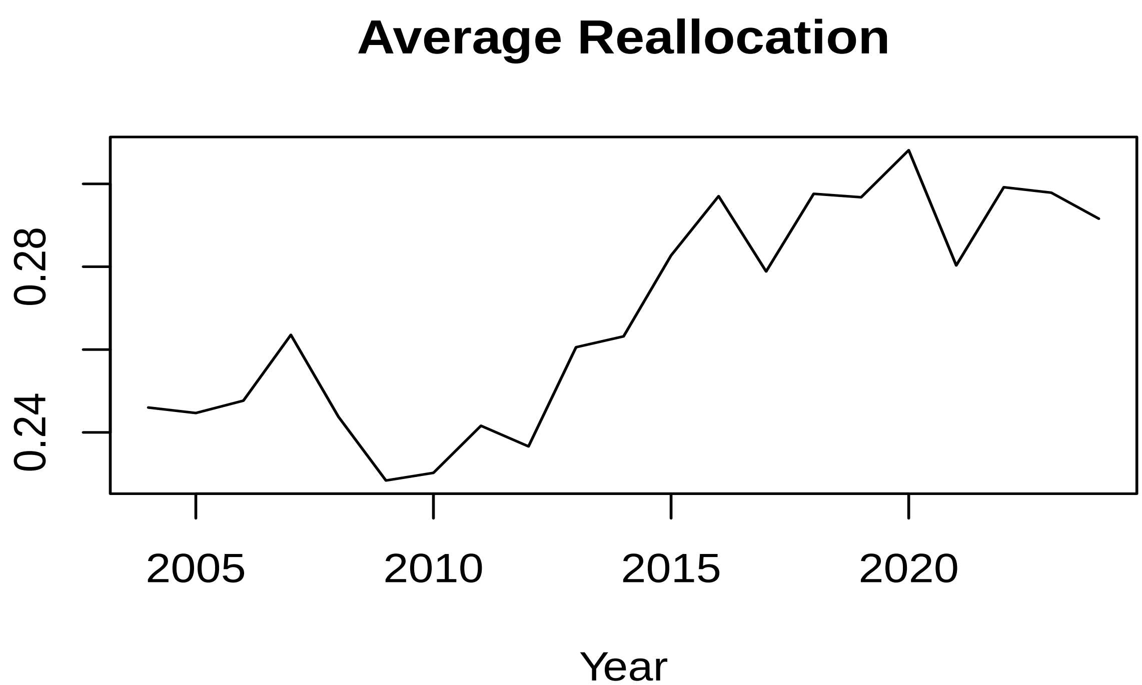


Figure 7: Average reallocation

Source: Own calculation

To evaluate the strength of the composition effect and dispersion effect on matching efficiency, I use a simple linear regression where my dependent variable represents past matching efficiency, the difference in unemployment rates between degree and non degree holders, long term unemployment rate, and the average reallocation for the non-farm sector. As previously stated, average reallocation captures the dispersion effect while the long-term unemployment rate and the difference in unemployment rates between degree and non degree holders represents the composition effects. Since matching efficiency was modeled as an AR(1), it's lagged values are also taken as a dependent variable. Newey-West standard errors are used to account for autocorrelation. Several specifications were tried before selecting the current model specification, such as, is the relationship between dependent variables and independent variable is only contemporaneous or is there a lagged relationship as well, and how the model behaves in pre-COVID times. I find that lagged coefficients of reallocation variable and difference in unemployment rate and long term unemployment rate are insignificant and therefore suggest that there is a contemporaneous link between these variables. Also, for the pre-COVID sample, there is a problem of multicollinearity between the long-term unemployment rate and lagged matching efficiency. Therefore, the long run unemployment rate was dropped from the pre-COVID model.

The regression results Table 2 show that both average reallocation across industries and the difference in unemployment rates between degree and non-degree holders are negatively associated with matching efficiency, although the estimates are generally not statistically significant for the full sample (2003-24). The negative relation suggests that higher worker movement between industries or greater disparities in joblessness across education groups may reduce the efficiency with which unemployed workers are matched to vacancies, but the evidence is weak. In the pre-COVID sample, however, both variables become marginally significant, with average reallocation and the unemployment gap exerting a stronger negative effect on matching efficiency. These findings imply that structural shifts in the labor market and heterogeneity in worker characteristics can undermine the smooth functioning of the matching process for the pre-COVID sample, as people take time to match. Significant negative coefficients on regression dummies which indicates that matching efficiency is procyclical and affected by business cycle shocks.

Table 2

	Dependent variable:		
	Matching efficiency		
	Full Sample	Pre-COVID	Pre-COVID
Lagged matching efficiency	0.998^{***}	0.956^{***}	0.976^{***}
	(0.009)	(0.019)	(0.011)
Average reallocation	-0.164	-0.200	-0.231^{*}
	(0.145)	(0.158)	(0.156)
Difference in unemployment rate	-0.092	-0.105	-0.170^{*}
	(0.070)	(0.092)	(0.077)
Longterm unemployment rate	0.009	-0.025	
	(0.010)	(0.020)	
Covid dummy	-0.013^{***}		
	(0.004)		
GR dummy	-0.009^{***}	-0.011^{***}	-0.009^{***}
	(0.002)	(0.002)	(0.002)
Constant	0.006	0.049^{**}	0.030^{*}
	(0.010)	(0.020)	(0.013)
Observations	241	202	202
R^2	0.994	0.994	0.993
Adjusted R^2	0.993	0.993	0.993
Residual Std. Error	0.006 (df = 234)	0.006 (df = 196)	0.006 (df = 197)
F Statistic	$6,071.198^{***}$ (df = 6; 234)	$6,008.622^{***}$ (df = 5; 196)	$7,486.367^{***}$ (df = 4; 197)
Note:			$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

4.2 Match elasticities

Figure 8 plots the estimates of the elasticity over time. As illustrated in the figure, the impact of vacancies on hires has increased over time. Hence, after allowing for time variation, the job-finding rate has become increasingly more responsive to labor market tightness. The estimate from the model that imposes constant elasticity is around 0.75, which is higher than other elasticities estimated in the literature (probably due to the inclusion of the COVID period). However, the average [16] match elasticity under time variation for the CRS and non-CRS model is 0.46 and 0.48, respectively. As per Bernstein et al. (2021), there are other estimates of match elasticities in table 3 which are closer to my estimates. The counter-cyclicality of the match elasticity implies that hires are more sensitive to vacancies during recessions and during expansions are more sensitive to the number of unemployed. This indicates non-linearities in the labor market dynamics suggested in the literature (Lubik (2013), Bernstein et al. (2021)).

Imposing CRS would mean that the elasticity of the job-finding rate to unemployment will decline if match elasticity increases. However, without the CRS restriction, this need not be the case. Even then, the unemployment elasticity has declined (Figure 9), suggesting that hires are more impacted by the increase in vacancies than by the same increase in unemployment. Bernstein et al. (2021) uses the search and matching model and links job finding rate (f_t) and labor productivity (a_t) under two matching functions- Cobb Douglas matching function with constant matching elasticity with respect to unemployed searching and DRW match function with time-varying match elasticity with respect to unemployed searching.

They find a procyclical match elasticity under the DRW match function, which suggests that given the concavity of the job-finding rate with respect to productivity, a lower match elasticity during the recession would mean a steeper slope of f_t. Therefore, a negative labor productivity shock during a recession will lead to a larger decline in the job-finding rate than the same negative shock during an expansion since the slope will be flatter. With constant match elasticity with respect to unemployed searching under the Cobb-Douglas matching function, the slope of the job-finding rate is constant and

[16] Average is the average of mean match elasticities.

negative. This would imply that a small negative shock to productivity will cause the same decline in the job-finding rate during both expansions and recessions.

Intuitively, time variability makes sense. For example, a supply chain disruption causing a decline in labor productivity will lead to a larger decline in the job-finding rate during recessions than during expansions. A similar argument was made by Lubik (2021), who examined time variation in the correlation coefficient between the residuals of unemployment and vacancies. Their findings suggest the correlation is higher during recessions and lower during expansions. As a result, a positive shock to vacancies is likely to cause a larger decline in the unemployment rate during recessions compared to expansions.

The procyclicality of the elasticity with respect to the unemployment rate, as shown in Figure 9, suggests that this phenomenon may arise under a Cobb-Douglas matching function with time variation. However, the paper does not develop a theoretical search and matching model to formally derive the equilibrium conditions linking the job-finding rate to labor productivity when matching elasticity varies over time. This topic is left for further research.

Authors	Methods	Sample	Match elasticity estimates
Michaillat and Saez (2021)	OLS with Break points	1951-2019	0.51-0.61
Sahin et al. (2014)	GMM IV	2001-12	0.24-0.66
Rogerson and Shimmer (2011)	OLS, multiplicative noise	2001-09	0.42

Table 3: Match elasticity estimates in the literature

I find that on average, the aggregate labor market for the non-farm sector experiences mildly increasing returns to scale for the baseline model, Figure 10. With a 68% credible interval, the returns to scale vary between 1.02 to 0.99 and are very stable over the period except during crisis. The returns to scale increased on average during the Great Recession and before and during COVID-19, leading to

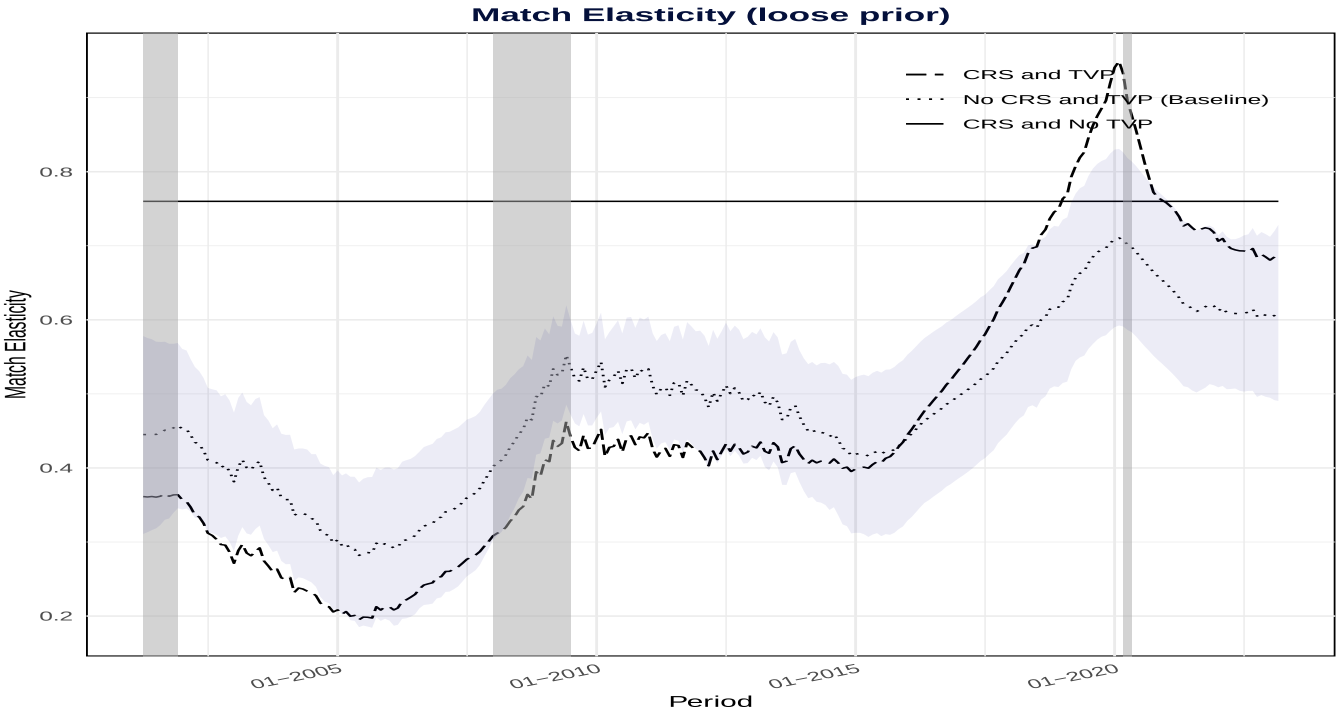


Figure 8: Match Elasticity

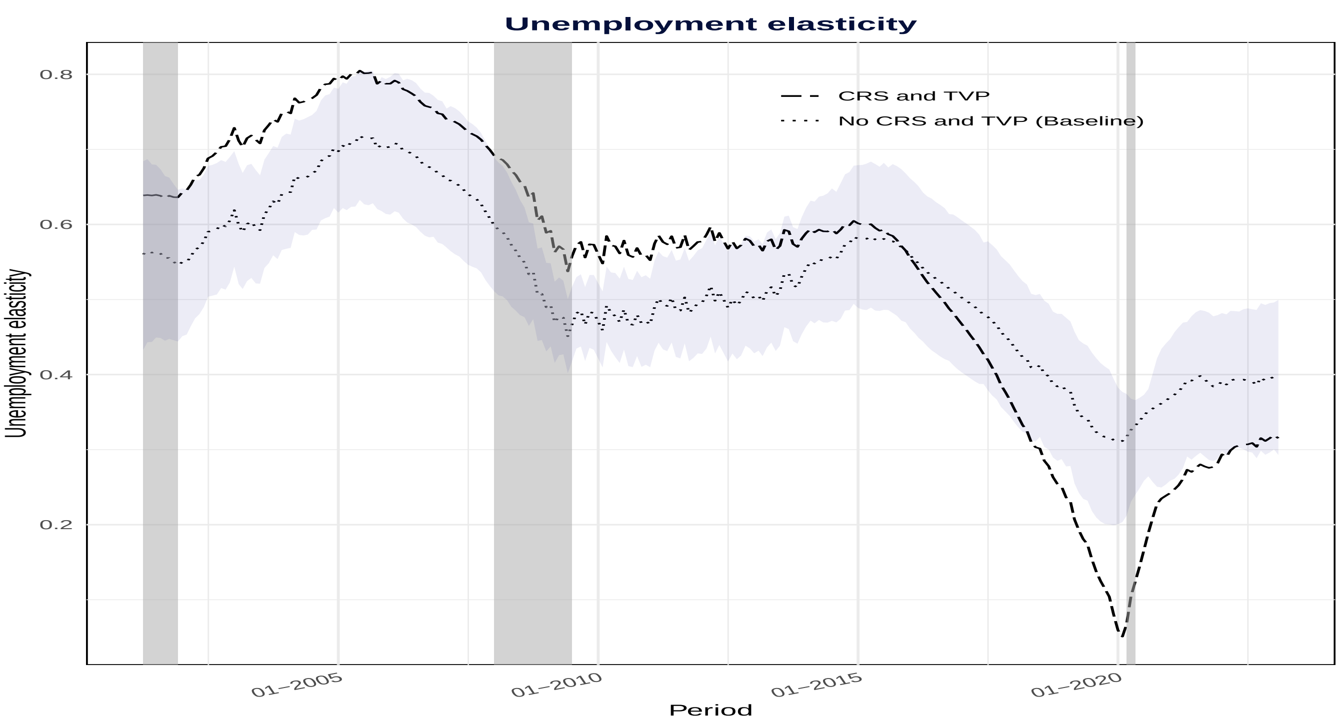


Figure 9: Unemployment Elasticity

mildly increasing returns to scale. According to Blanchard and Diamond (1989) increasing returns to scale can happen when markets are active, which may lead to easier matches with or without intensive search.

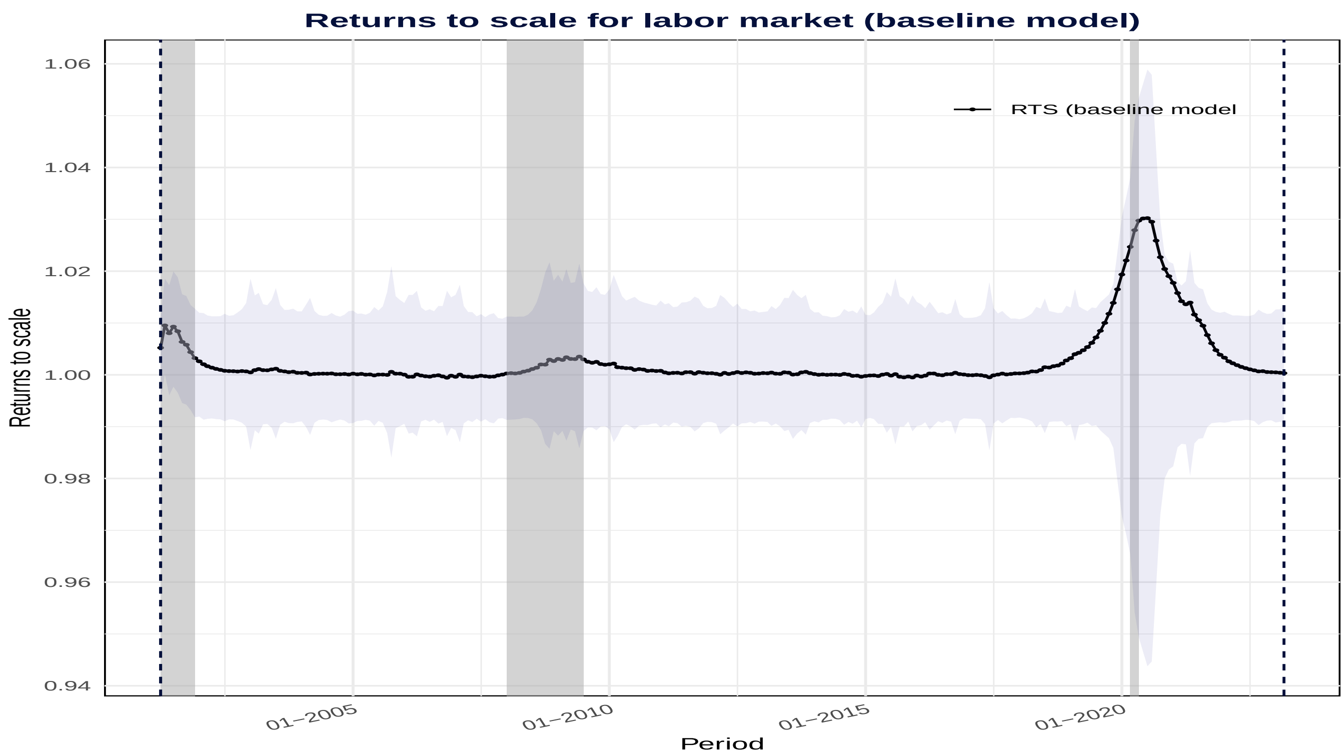


Figure 10: Returns to scale

4.3 Model selection

Table 4 presents the results on model performance as per the Deviance Information Criteria (DIC), where the model with lower DIC is considered better in terms of balancing fit and complexity. The results suggests that even with greater model complexity under non-CRS assumption, it supports the data better. To see how the models perform with respect to actual data, I follow the intuition from the methodology suggested in Ahn and Crane (2020). They estimated the counterfactual vacancies from the log-linearized law of motion for unemployment. Using the equation, they obtained its first-order Taylor approximation at given values of the unemployment rate, separation rate, average matching efficiency, and fixed match elasticity to understand the effect on log vacancies due to changes in separation rate, matching efficiency, and out-of-steady state dynamics. Such decomposition allowed them to understand the individual effects of each factor on the vacancies, given actual unemployment.

Model	Mean Deviance	Penalty	Penalized Deviance
Model 1	-1792	255.3	-1537
Model 2	-1599	187.7	-1412

Table 4: Deviance Information Criteria (DIC)

I use a similar principle to understand the effects of these different models on estimated vacancies given actual unemployment. The idea was to understand how the model performs with respect to actual data on vacancies. Therefore, I estimate the counterfactual vacancies from the models' respective matching functions using observed unemployment, hires, and parameter estimates for all three models. I then compare those counterfactual vacancies with observed vacancies to understand how much over- or underestimation each model suggests. While both time-varying models overestimate the vacancies, the fixed-parameter model underestimates the vacancies from actual vacancies. However, the over-estimation of vacancies under the non-CRS model is lower than in the CRS model. The difference between CRS and the baseline model are most pronounced during the COVID period.

Next, I calculate the root mean square error for all the models and compare it with the baseline model. The result suggests that the baseline model fits the data well, especially after including COVID-19, table 5. I also estimate the baseline model with stochastic volatility and found that the mean estimates for both matching efficiency and match elasticity were within the confidence interval of the baseline model without stochastic volatility, suggesting no significant differences (Figure 14).

Ratio of Model RMSEs to Baseline RMSEs	CRS and No TVP	CRS and TVP
Whole period (2001-2023)	1.9	2.2
Before COVID (2001- 2019)	1.1	1.2

Table 5: Model Comparison (RMSE)

4.4 The Beveridge curve slope and elasticity

Using the baseline model and equations 17 and 18, I calculate the slope of the Beveridge curve and the elasticity. This is based on the assumption that the empirical Beveridge curve is close to the steady-state Beveridge curve. The slope of the BC has shown a declining trend from 2000 onward because

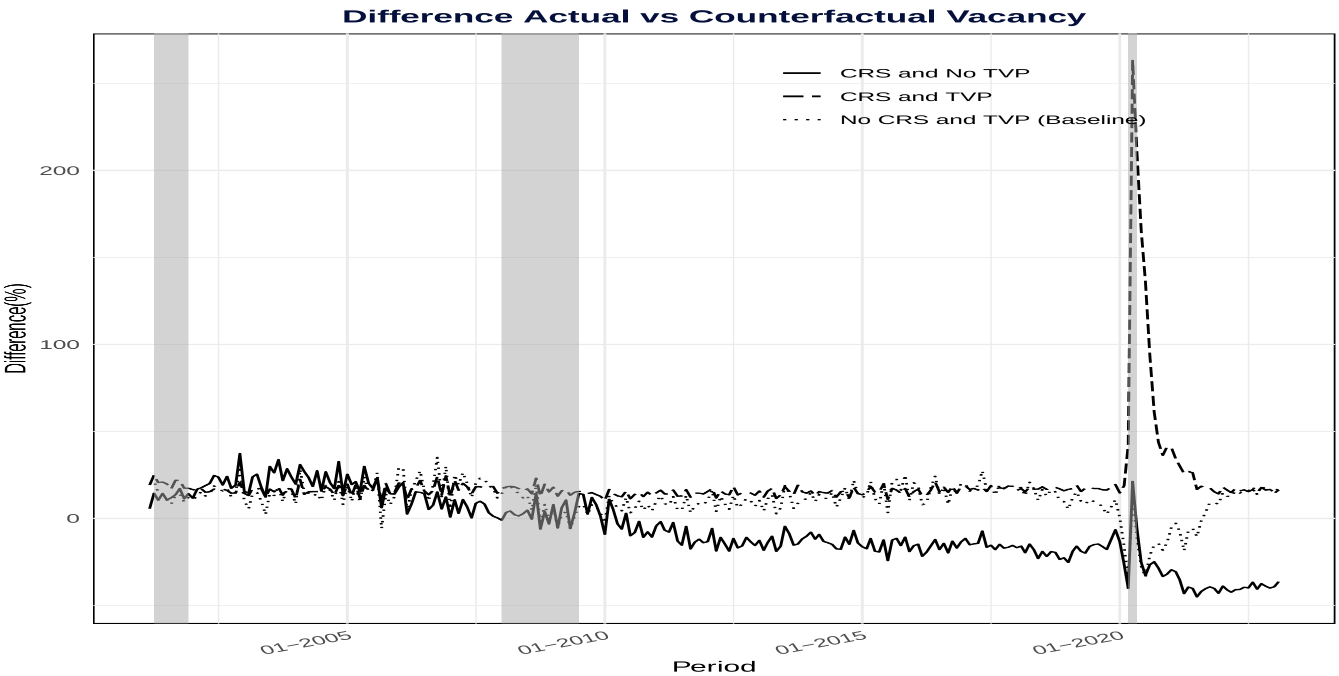
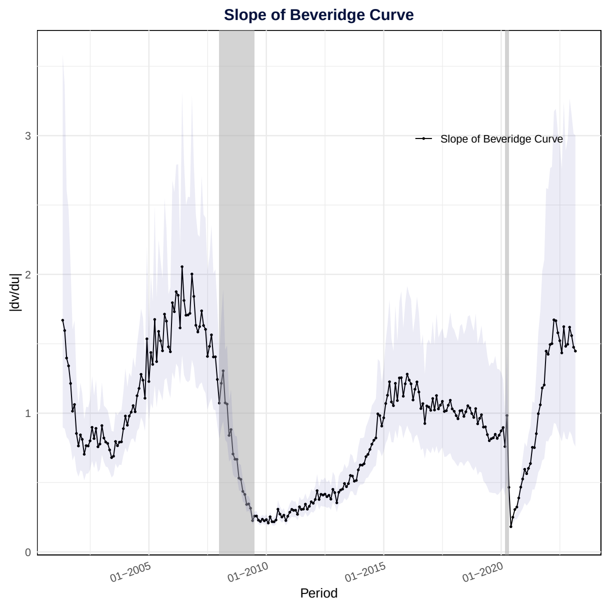


Figure 11: Counterfactual Vacancy

of an increase in match elasticity. However, after COVID-19, the slope has increased. Similarly, the elasticity declined from 2000 onward, but after COVID-19, the elasticity of BC increased. Figure 12b shows the BC's mean elasticity, which ranges from 0.5 to 3, excluding the COVID period. The Beveridge curve elasticity was between 0.84 and 1.02 from 1951 to 2019 as per Michaillat and Saez (2021) estimates. The average elasticity for the period is 0.90 (excluding 2020 and onward), which is again close to the estimate of 0.91 for 1951 to 2019 in Michaillat and Saez (2021). They suggested that the Beveridge curve elasticity underwent five structural breaks during this period and showed a decline from 2000 to 2019. I also find that the elasticity is declining from 2000 through 2023, Figure 12b. Declining elasticity for the baseline model suggests that vacancies are becoming less responsive to changes in unemployment and this may suggest an increase in labor market inefficiencies over time.

4.5 Estimates of the unemployment gap

According to Michaillat and Saez (2021), the efficient rate of unemployment is at the intersection of the BC and the iso-welfare curve. Therefore, an increase in the slope of BC will disturb the equilibrium, as the initial intersection between the iso-welfare curve and the BC will no longer be efficient. For the economy to be on the same iso-welfare curve, the BC must shift to the right of the initial equilibrium



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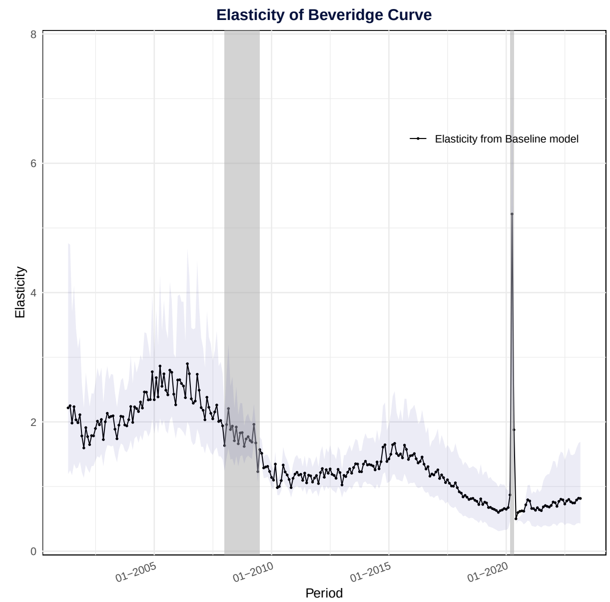

(b)

Figure 12: Beveridge curve slope and elasticity for the Baseline model

point, which would suggest an increase in the efficient rate of unemployment. Now the data suggests that the slope of BC has declined from 2000 through 2019 and briefly increased from 2020 to 2023, figure 12a. This suggests a leftward shift of the BC to be on the same iso-welfare curve and hence a decline in the efficient rate of unemployment from 2000- 2019 and then a right shift of BC, which suggests a slight increase in the efficient rate of unemployment. This analysis is also supported by the data which suggests for the baseline model a decrease in the efficient rate of unemployment from 2015-2019 to below 5% and then an increase to around 5% post-COVID for the baseline model, Figure 13(a).

The efficient unemployment rate for the CRS time-varying model fluctuates between 1.5% to 5.6% and for the non-time-varying model between 5% to 10%, excluding the COVID period and onward. This assumes κ (recruiting cost) = 0.92; ζ (social value of non-work) = 0.26, from MS (2021). From here, it is evident that a slight difference in parameter estimates from different models changes the estimates for the efficient unemployment rate. The non-cyclical rate of unemployment (NROU) by CBO [17] is between 4.1-5.3% for the same period and suggests a declining efficient unemployment rate since 1979. Looking at the unemployment gap, the CRS non-time-varying model suggests that the la-

[17] U.S. Congressional Budget Office.

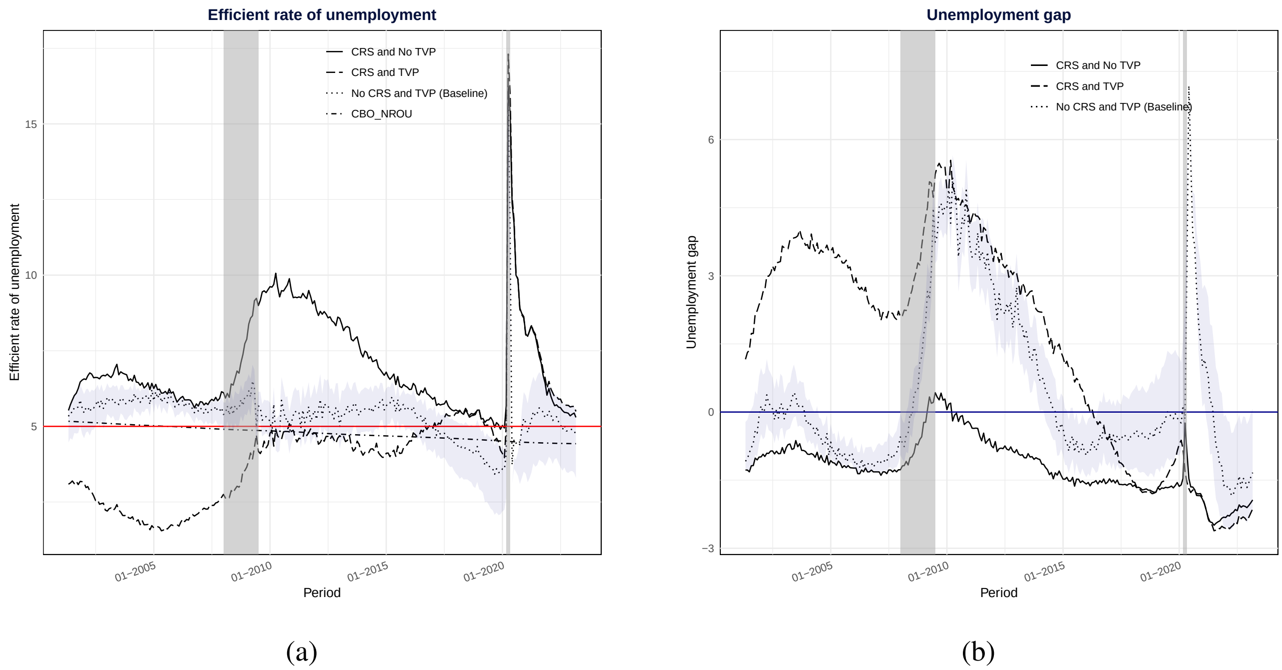

Figure 13: Efficient unemployment rate and unemployment gap

bor market was inefficiently tight, while the CRS time-varying model indicates that it was inefficiently slack for the most part. The baseline model suggests that the labor market was inefficiently tight and inefficiently slack. This follows the general intuition about the labor market where the unemployment gap was negative when the labor market was tight and positive otherwise. Hence, the choice of the functional form can significantly change our estimates of the unemployment gap, and our inference about the state of the labor market. Bernstein et al. (2021) similarly suggested a strong link between the choice of matching function and the non-linearities [18] in labor market dynamics and therefore functional form of the match function matters. My findings here show that not accounting for time-variation in the labor market can lead to estimates of the unemployment gap that are counterintuitive in magnitude and in sign, whereas the estimated gap from the model with time-variation are countercyclical and comparable in magnitude to other commonly used estimates such as the CBO.

[18] Non-linearities is defined in terms of transmission of productivity to job-finding rate which is a function of match elasticity. A constant match elasticity generates a linear relationship between job finding rate and productivity while a varying elasticity generates a non-linear relationship.

Conclusion

The dynamics in the US labor market had changed even before the pandemic. The shift in the Beveridge curve space suggests the need for the estimation of time-varying parameters which are widely taken to be constant in literature. I find that time variation exists in both the parameters and indicates that they are affected by business cycle fluctuations. Overall, the productivity of the process to match jobs to job seekers has declined for the non-farm sector of the aggregate labor market. This could be due to the composition effect on the unemployment pool and the dispersion effect because of structural change in the economy. The impact of vacancies on hires has increased over time as a 1% increase in vacancies has a larger impact on hires than a 1% increase in unemployed, irrespective of the model. The policy implication will be that for a higher number of matches, the availability of jobs is more important. Also, over time vacancies have become less responsive to the unemployed, depicted by Beveridge Curve elasticity for the baseline model and is due to the introduction of time variability. The same model suggests that the efficient rate of unemployment to be between 4-6%, which is closer to the non-cyclical unemployment rate of CBO (4.1-5.3% for the same period). The choice of model changes the conclusion on the labor market. As per the baseline model, the labor market has been working above and below the efficiency as per the business cycle.

This paper contributes to the literature by estimating matching efficiency through continuous time variation and attempts to provide a possible explanation of the mechanism affecting aggregate matching efficiency using microlevel data. The paper also suggests the increasing importance of hires, which has potential policy implications. Further, since all these parameters are used in the estimation of u^*, they reveal important information about the state of the labor market, which has important policy implications.

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Appendix I- State Space Model

Assuming no stochastic volatility

Model 1- Estimates no-CRS with time variation. The model estimates three observation equations Equation 24 and, three state equations for latent variables Equation 25.

$$\begin{bmatrix} \tilde{f}_t \\ \tilde{\theta}_{t-1} \\ \tilde{\theta}_{t-2} \\ \tilde{u}_{t-1} \end{bmatrix} = \begin{bmatrix} \tilde{m}_t \\ a \\ 0 \\ \rho \end{bmatrix} + \begin{bmatrix} \delta_t & \alpha_t & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & j \end{bmatrix} \cdot \begin{bmatrix} \tilde{u}_{t-1} \\ \tilde{\theta}_{t-1} \\ \tilde{\theta}_{t-2} \\ \tilde{u}_{t-2} \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ \epsilon_{\tilde{\theta}_{t-1}} \\ 0 \\ \epsilon_{\tilde{u}_{t-1}} \end{bmatrix} \tag{24}$$

$$\begin{bmatrix} \tilde{m}_t \\ \alpha_t \\ \delta_t \end{bmatrix} = \begin{bmatrix} \mu \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \rho & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \tilde{m}_{t-1} \\ \alpha_{t-1} \\ \delta_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{\tilde{m}_t} \\ \epsilon_{\alpha_t} \\ \epsilon_{\delta_t} \end{bmatrix} \tag{25}$$

The residuals are assumed to be normally distributed and given by Equation 26.

$$\epsilon_i \sim \mathcal{N}(0,\, \sigma_i^2) \qquad\qquad where,\ i \in \{\tilde{\theta}, \alpha, \tilde{u}, \delta, \tilde{m}, \epsilon\} \tag{26}$$

- $\delta_t = \alpha_t + \beta_t - 1$
- β_t is the coefficient for unemployment and is assumed to follow a random walk.
- Unemployment is assumed to follow an AR(1) process with drift after analyzing the acfs and pacfs.

Incorporating stochastic volatility

Model 1- Estimates no-CRS with time variation. The model estimates three observation equations Equation 27 and, three state equations for latent variables Equation 28.

$$\begin{bmatrix} \tilde{f}_t \\ \tilde{\theta}_{t-1} \\ \tilde{\theta}_{t-2} \\ \tilde{u}_{t-1} \end{bmatrix} = \begin{bmatrix} \tilde{m}_t \\ a \\ 0 \\ \rho \end{bmatrix} + \begin{bmatrix} \delta_t & \alpha_t & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & j \end{bmatrix} . \begin{bmatrix} \tilde{u}_{t-1} \\ \tilde{\theta}_{t-1} \\ \tilde{\theta}_{t-2} \\ \tilde{u}_{t-2} \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ \epsilon_{\tilde{\theta}_{t-1}} \\ 0 \\ \epsilon_{\tilde{u}_{t-1}} \end{bmatrix} \tag{27}$$

$$\begin{bmatrix} \tilde{m}_t \\ \alpha_t \\ \delta_t \\ h_t \end{bmatrix} = \begin{bmatrix} \mu \\ 0 \\ 0 \\ (1-\phi)\mu_h \end{bmatrix} + \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \phi \end{bmatrix} . \begin{bmatrix} \tilde{m}_{t-1} \\ \alpha_{t-1} \\ \delta_{t-1} \\ h_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{\tilde{m}_t} \\ \epsilon_{\alpha_t} \\ \epsilon_{\delta_t} \\ \epsilon_{\eta t} \end{bmatrix} \tag{28}$$

Where, ϵ_t has a heteroscedastic variance represented by Equation 11. The residuals ϵ_t, θ_t, $\tilde{m}_t$, α_t, $\tilde{u}_t$, η and, δ_t are assumed to be normally distributed and given by Equation 29.

$$\epsilon_i \sim \mathcal{N}(0,\, \sigma_i^2) \qquad\qquad where,\ i \in \{\tilde{\theta}, \alpha, \tilde{u}, \delta, \tilde{m}, \eta\} \tag{29}$$

Appendix II- Priors

Variables	Distribution	Mean	Std Dev.
$\tilde{m}[1]$	Normal	0.7	1
$\alpha[1]$	Normal	0.5	1
$\theta[1]$	Normal	1	1
z[1]	Normal	1	1
a	Normal	0	1
$\delta[1]^*$	Normal	0.5	1
p^*	Normal	0	1
j^*	unif(-1,1)	0	0.58
c	unif(-1,1)	0	0.58
μ	beta(1,1)	0.5	0.3
ρ	beta(1,1)	0.5	0.3
σ_y	gamma**	6.21@	1
σ_x	gamma**	2.54@	1
σ_m	gamma**	7	10
σ_α	gamma**	7	10
σ_z^*	gamma**	10.88@	1
σ_δ^*	gamma**	7	10

m[1], α [1], x[1], z[1], $\delta[1]$ are the initial values of these variables. M denotes matching efficiency, α denotes match elasticity, θ denotes labor market tightness, z denotes the variable used for unemployment and d is the elasticity with respect to unemployment. Variables with an asterisk (*) are additional variables for the non-CRS time-varying model. For gamma distributions, ** represents scale and shape parameters instead of mean and variance. Variables having @ denotes that their shape parameter equals the variable's variance. Rjags takes precision in the models instead of variance where precision is 1/variance. Posterior means for these variables are time-varying. Burn-in =10,000 and chains run =4. Priors for m and b are different for each chain. Each chain had 250,000 draws. Point estimates on Rhat are either equal to or less than 1.02 suggesting convergence.

Table 7: Details on Priors

Appendix III- Theoretical Motivation

The theoretical motivation for time-varying parameters comes from Benati and Lubik (2014) and Bernstein et.al. (2021) which takes one of the matching functions as a Cobb-Douglas matching function and constant returns to scale Equation 30. However, in this paper, I assume no contemporaneous relation between hires and unemployment and vacancies. I also assume a standard matching function with CRS in this section since it is only to motivate time variation in these parameters. Here hires, vacancies and unemployment are as proportion the of labor force. Since the period is monthly, I assume that the labor force at t is approximately equal to that at t-1. This is a safe assumption since the labor force is affected by several structural factors that take time to evolve and therefore usually do not change suddenly between two months.

$$h_t = m.u_{t-1}^{1-\alpha}.v_{t-1}^{\alpha} \tag{30}$$

$$f_t = \frac{h_t}{u_{t-1}} = m\theta_{t-1}^{\alpha} \tag{31}$$

The job finding rate and vacancy filling rate are given by Equation 31 and Equation 32. It says that the at job-finding rate at period t is a function of labor market tightness (θ) at t-1. Similarly, Equation 32 states that the vacancy filling rate at t is dependent on labor market tightness at t-1.

$$q_t = \frac{h_t}{v_{t-1}} = m\theta_t^{\alpha-1} \tag{32}$$

Their model assumes linear production function Equation 33 where a_t is marginal labor productivity which is assumed to follow AR(1) process. Here output y_t is represented as a proportion of labor force L_t. Consumer faces a constant relative risk aversion utility function and time is monthly Equation 34.

$$y_t = a_t.n_t \tag{33}$$

$$U(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} \tag{34}$$

The law of motion of employment and unemployment is given by Equation 35 and Equation 38, respectively where separation rate (s) is assumed to be constant. The law of motion is written as a proportion of the labor force.

$$n_t = (1-s)n_{t-1} + mu_{t-1}^{1-\alpha}v_{t-1}^{\alpha} \tag{35}$$

$$1 - n_t = 1 - (1-s)(1-u_{t-1}) + f_t.u_{t-1} \tag{36}$$

$$u_t = 1 - (1-u_{t-1}) + s(1-u_{t-1}) + f_t.u_{t-1} \tag{37}$$

$$u_t = s(1-u_{t-1}) + u_{t-1}(1-f_t) \tag{38}$$

The firm chooses the optimal number of vacancies V_t and employment level N_t by maximizing the intertemporal profit function Equation 39, subject to the law of motion for employment Equation 35. The firm faces a fixed cost for vacancy creation κ where $\kappa > 0$ and a time-varying wage rate w_t. The value function below adds up the representative firm's present and future profits using a discount factor β such that $\beta \in (0,1)$. The value function (F_t) Equation 40 is maximized for employment at t subject to the law of motion of employment.

$$E\sum_{t=0}^{\infty} \beta^t [a_t n_t - w_t n_t - \kappa v_{t-1}] \tag{39}$$

$$F_t = a_t n_t - w_t n_t - \kappa v_{t-1} + \beta E_t[F_{t+1}] \tag{40}$$

The first-order equation is given by Equation 44 where the left-hand side (LHS) shows the real marginal cost of vacancy creation and the right-hand side (RHS) shows the real marginal benefit from creating a vacancy for the representative firm. Taking the LHS and substituting Equation 32, I get Equation 45, which clearly shows that as matching efficiency increases, ceteris paribus, the real marginal cost of vacancy creation for the representative firm decreases, hence motivating time-variation in this parameter. Similarly, as hires become more responsive to vacancies, the real marginal cost of vacancy creation decreases, ceteris paribus.

$$\frac{\partial F_t}{\partial n_t} = a_t - w_t - \frac{\kappa}{q_t} + \beta E_t[\frac{\partial F_{t+1}}{\partial n_{t+1}}.\frac{\partial n_{t+1}}{\partial n_t}] \tag{41}$$

where

$$\frac{\partial F_{t+1}}{\partial n_{t+1}} = a_{t+1} - w_{t+1} - \frac{\kappa}{q_{t+1}} \tag{42}$$

and

$$\frac{\partial n_{t+1}}{\partial n_t} = (1 - s) \tag{43}$$

Hence we get the first-order condition for profit maximization of the representative firm with respect to employment.

$$\frac{\kappa}{q_t} = a_t - w_t + \beta(1 - s)E_t[a_{t+1} - w_{t+1} - \frac{\kappa}{q_{t+1}}] \tag{44}$$

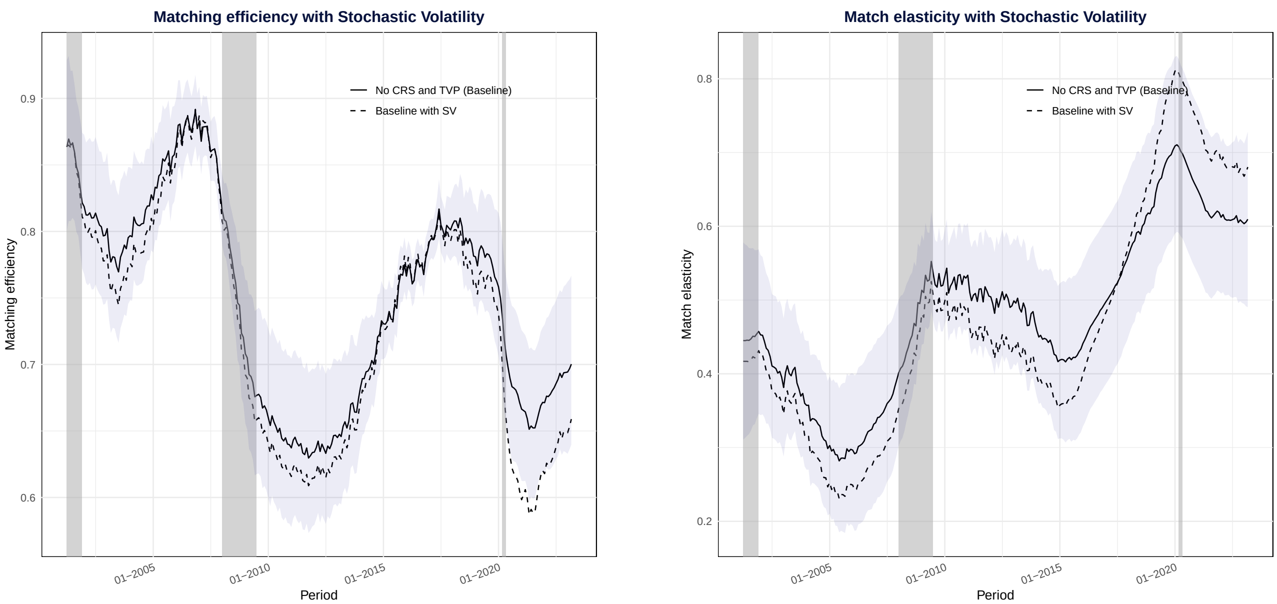
$$\frac{\kappa}{q_t} = \frac{\kappa.\theta_{t-1}^{1-\alpha}}{m} \tag{45}$$

Appendix IV- Labor Market Summaries and Variables Calculated Using CPS data

The aggregate job-finding probabilities in the non-farm sector signify the transition from unemployment to employment for the month. Monthly data is used, where I traced the individuals who changed their labor market status from unemployment to employment and therefore were hired for that month. Since CPS rotates a household in a 4-8-4 cycle, where a household is interviewed for 4 consecutive months, leaves the sample for 8 months, and then returns to the sample for the same 4 months of the following year. Hence individual who is unemployed in the last month of the first cycle and getting employed in the first month of the 2nd cycle is also included. This was done to increase the sample size. Since I define job finding rate as $\frac{H_t}{U_{t-1}}$, I define the job finding probabilities as people transitioning from unemployment to employment in time t divided by total unemployment at time t-1. The probabilities are seasonally adjusted using the X-13ARIMA SEATS package.

Average reallocation: I used the database at an individual level and used both annual and monthly data to identify the individuals who switched between industries. CPS conducts longitudinal annual surveys in March of every year. Hence, I matched the individuals from the annual database with the individuals from the monthly database using the unique individual identifier given by CPS (CPISDV). For instance, the annual database for March 2004 was matched with the monthly database for March 2003 and the industries for the same individual were compared. The variable of interest in the annual database was primarily identifying the "industry code of primary job last year" and the variable of interest in the monthly database was identifying the "industry code of primary job last week". This essentially asks the question did the individual switch industries after a year? Since CPS rotates a household as a 4-8-4 cycle, a_{it} captures the percentage of people who switched or reallocated to a different industry after a year. CPS uses census code to classify industries and the industries were taken at a 4-digit classification. I reclassified industries to a 2-digit NAICS classification using the appropriate classification list provided by the U.S. Census Bureau. To project appropriately for the population, ASECWT weights were used.

Appendix V- Additional Figures



(a) (b)

Figure 14: Matching efficiency and match elasticity after accounting for stochastic volatility, comparison with the baseline model

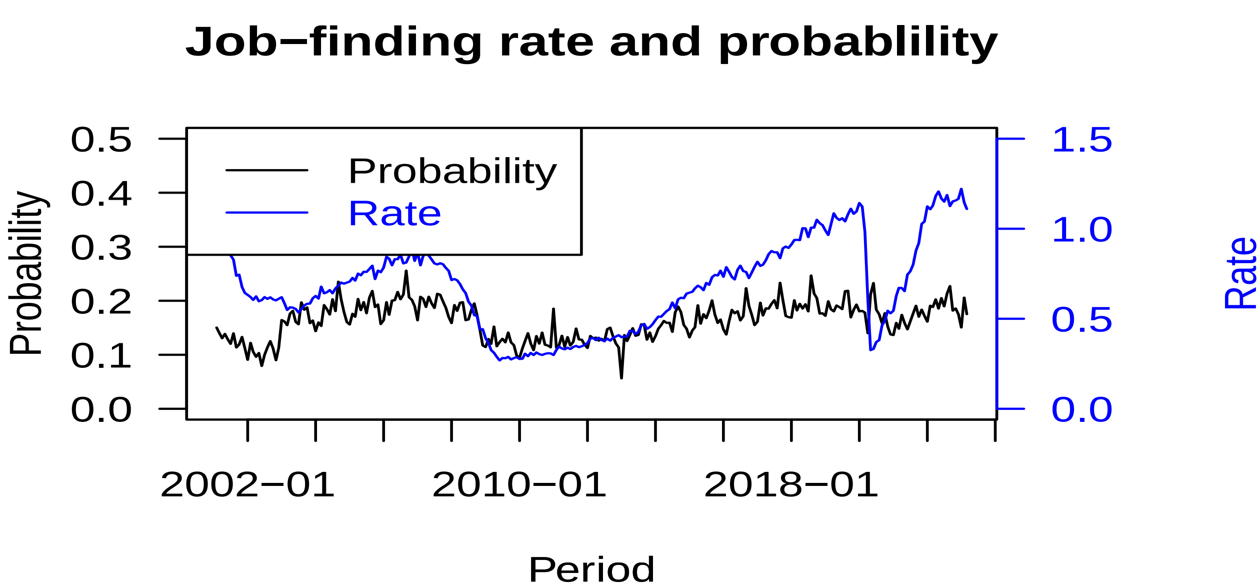


Figure 15: Job-finding rate and Unemployment-Employment transition probabilities