MACHINE LEARNING - 2 - Assignment

By Prakash Ghosh

1. Build the linear regression model using scikit learn in boston data to predict 'Price' based on other dependent variable.

Here is the code to load the data

In [7]:
```
import numpy as np
import pandas as pd
import scipy.stats as stats
import matplotlib.pyplot as plt
import sklearn
from sklearn.datasets import load_boston
boston = load_boston()
bos = pd.DataFrame(boston.data)
```

- Solution

In [21]:
```
print(" The keys of boston dataset:\t",boston.keys())
print("\n Dataset shape:\t",boston.data.shape)
print("\n Fetures (columns) of Dataset:\t",boston.feature_names)

print('\n First Five Rows:')
bos.head(5)
```

```
 The keys of boston dataset:     dict_keys(['data', 'target', 'feature_names', 'DESCR'])

 Dataset shape:  (506, 13)

 Fetures (columns) of Dataset:   ['CRIM' 'ZN' 'INDUS' 'CHAS' 'NOX' 'RM' 'AGE' 'DIS' 'RAD' 'TAX' 'PTRATIO'
 'B' 'LSTAT']

 First Five Rows:
```

Out[21]:

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0.00632	18.0	2.31	0.0	0.538	6.575	65.2	4.0900	1.0	296.0	15.3	396.90	4.98
1	0.02731	0.0	7.07	0.0	0.469	6.421	78.9	4.9671	2.0	242.0	17.8	396.90	9.14
2	0.02729	0.0	7.07	0.0	0.469	7.185	61.1	4.9671	2.0	242.0	17.8	392.83	4.03
3	0.03237	0.0	2.18	0.0	0.458	6.998	45.8	6.0622	3.0	222.0	18.7	394.63	2.94
4	0.06905	0.0	2.18	0.0	0.458	7.147	54.2	6.0622	3.0	222.0	18.7	396.90	5.33

In [25]:
```
# Description of Boston Dataset
print(boston.DESCR)
```

```
Boston House Prices dataset
===========================

Notes
------
Data Set Characteristics:

    :Number of Instances: 506

    :Number of Attributes: 13 numeric/categorical predictive

    :Median Value (attribute 14) is usually the target

    :Attribute Information (in order):
        - CRIM     per capita crime rate by town
        - ZN       proportion of residential land zoned for lots over 25,000 sq.ft.
        - INDUS    proportion of non-retail business acres per town
        - CHAS     Charles River dummy variable (= 1 if tract bounds river; 0 otherwise)
        - NOX      nitric oxides concentration (parts per 10 million)
        - RM       average number of rooms per dwelling
        - AGE      proportion of owner-occupied units built prior to 1940
        - DIS      weighted distances to five Boston employment centres
        - RAD      index of accessibility to radial highways
        - TAX      full-value property-tax rate per $10,000
        - PTRATIO  pupil-teacher ratio by town
        - B        1000(Bk - 0.63)^2 where Bk is the proportion of blacks by town
        - LSTAT    % lower status of the population
        - MEDV     Median value of owner-occupied homes in $1000's

    :Missing Attribute Values: None

    :Creator: Harrison, D. and Rubinfeld, D.L.

This is a copy of UCI ML housing dataset.
http://archive.ics.uci.edu/ml/datasets/Housing


This dataset was taken from the StatLib library which is maintained at Carnegie Mellon University.

The Boston house-price data of Harrison, D. and Rubinfeld, D.L. 'Hedonic
prices and the demand for clean air', J. Environ. Economics & Management,
```

vol.5, 81-102, 1978. Used in Belsley, Kuh & Welsch, 'Regression diagnostics ...', Wiley, 1980. N.B. Various transformations are used in the table on pages 244-261 of the latter.

The Boston house-price data has been used in many machine learning papers that address regression problems.

References

- Belsley, Kuh & Welsch, 'Regression diagnostics: Identifying Influential Data and Sources of Collinearity', Wiley, 1980. 244-261.
- Quinlan,R. (1993). Combining Instance-Based and Model-Based Learning. In Proceedings on the Tenth International Conference of Machine Learning, 236-243, University of Massachusetts, Amherst. Morgan Kaufmann.
- many more! (see http://archive.ics.uci.edu/ml/datasets/Housing)

In [26]:
```
#Assign Columns names to Boston DataFrame using feature names
bos.columns=boston.feature_names
bos.head(5)
```

Out[26]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT
0	0.00632	18.0	2.31	0.0	0.538	6.575	65.2	4.0900	1.0	296.0	15.3	396.90	4.98
1	0.02731	0.0	7.07	0.0	0.469	6.421	78.9	4.9671	2.0	242.0	17.8	396.90	9.14
2	0.02729	0.0	7.07	0.0	0.469	7.185	61.1	4.9671	2.0	242.0	17.8	392.83	4.03
3	0.03237	0.0	2.18	0.0	0.458	6.998	45.8	6.0622	3.0	222.0	18.7	394.63	2.94
4	0.06905	0.0	2.18	0.0	0.458	7.147	54.2	6.0622	3.0	222.0	18.7	396.90	5.33

In [29]:
```
# Add target to Boston DataFrame as 'PRICE'
bos['PRICE']=boston.target
bos.head(5)
```

Out[29]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT	PRICE
0	0.00632	18.0	2.31	0.0	0.538	6.575	65.2	4.0900	1.0	296.0	15.3	396.90	4.98	24.0
1	0.02731	0.0	7.07	0.0	0.469	6.421	78.9	4.9671	2.0	242.0	17.8	396.90	9.14	21.6
2	0.02729	0.0	7.07	0.0	0.469	7.185	61.1	4.9671	2.0	242.0	17.8	392.83	4.03	34.7
3	0.03237	0.0	2.18	0.0	0.458	6.998	45.8	6.0622	3.0	222.0	18.7	394.63	2.94	33.4
4	0.06905	0.0	2.18	0.0	0.458	7.147	54.2	6.0622	3.0	222.0	18.7	396.90	5.33	36.2

In [31]:
```
bos.describe()
```

Out[31]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PT
count	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.
mean	3.593761	11.363636	11.136779	0.069170	0.554695	6.284634	68.574901	3.795043	9.549407	408.237154	18.4
std	8.596783	23.322453	6.860353	0.253994	0.115878	0.702617	28.148861	2.105710	8.707259	168.537116	2.16
min	0.006320	0.000000	0.460000	0.000000	0.385000	3.561000	2.900000	1.129600	1.000000	187.000000	12.6
25%	0.082045	0.000000	5.190000	0.000000	0.449000	5.885500	45.025000	2.100175	4.000000	279.000000	17.4
50%	0.256510	0.000000	9.690000	0.000000	0.538000	6.208500	77.500000	3.207450	5.000000	330.000000	19.0
75%	3.647423	12.500000	18.100000	0.000000	0.624000	6.623500	94.075000	5.188425	24.000000	666.000000	20.2
max	88.976200	100.000000	27.740000	1.000000	0.871000	8.780000	100.000000	12.126500	24.000000	711.000000	22.0

- Linear regression using least square method

In [61]:
```
df_feature = bos.iloc[:,:-1]       # Select all features (cloumns) except "PRICE" column
df_price =  bos.iloc[:,-1:]        # Select "PRICE" column
```

In [68]:
```
# import librarie for LinearRegression model
from sklearn.linear_model import LinearRegression as lr
lr_model = lr()

lr_model.fit(df_feature,df_price)                          # Fit the data
lr_model_R_square =lr_model.score(df_feature,df_price)   # Calculate the R square

lr_model_price_pred = lr_model.predict(df_feature)   # Prediction of Price based on features (Training data)

print(" R-square:\t", lr_model_R_square)
print("\n The Estimated Intercept:\t", lr_model.intercept_[0])
print("\n No. of Estimated coefficients :", len(lr_model.coef_[0]), "\n" )
print("\n The Predicted Prices based on features: \n",lr_model_price_pred[0:10])
```

```
 R-square:	 0.7406077428649428

 The Estimated Intercept:	 36.49110328036135

 No. of Estimated coefficients : 13


 The Predicted Prices based on features:
 [[30.00821269]
 [25.0298606 ]
 [30.5702317 ]
 [28.60814055]
 [27.94288232]
 [25.25940048]
 [23.00433994]
 [19.5347558 ]
 [11.51696539]
 [18.91981483]]
```

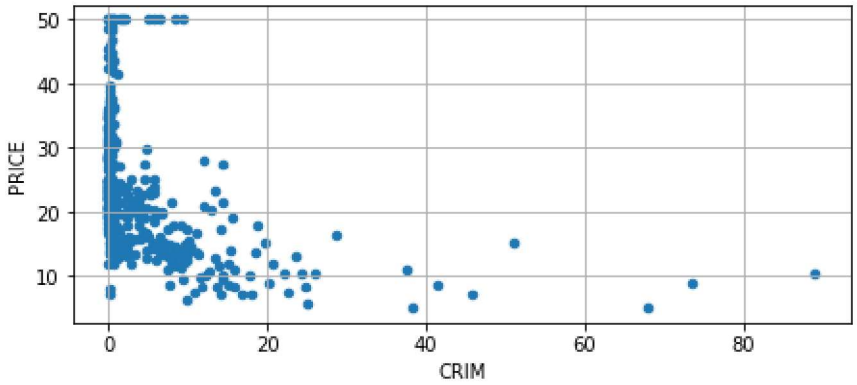
In [71]:
```
# Features and their estimated coefficients
df_lr_coef= pd.DataFrame(list(zip(df_feature.columns, lr_model.coef_[0])), columns=['Features','Estimated_Coefficient
s'])
print("Features and their estimated coefficients")
df_lr_coef
```

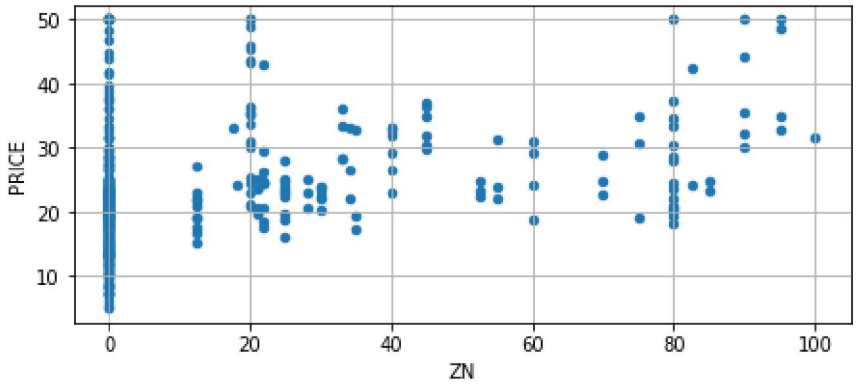
```
Features and their estimated coefficients
```

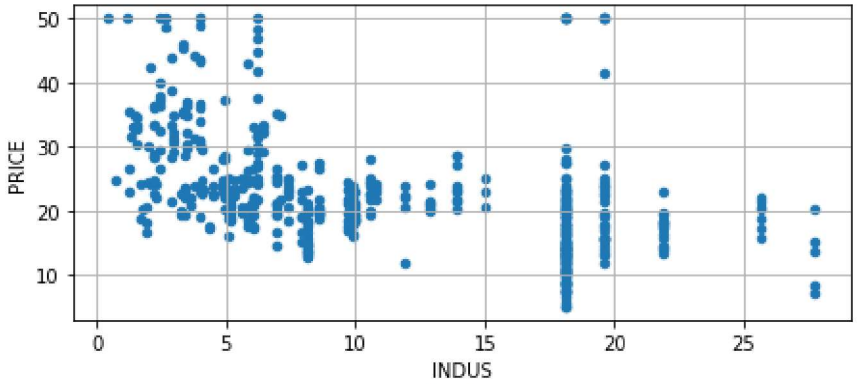
Out[71]:

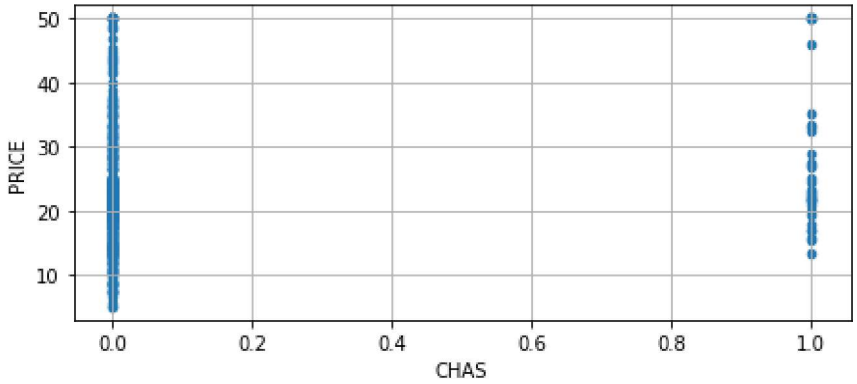
	Features	Estimated_Coefficients
0	CRIM	-0.107171
1	ZN	0.046395
2	INDUS	0.020860
3	CHAS	2.688561
4	NOX	-17.795759
5	RM	3.804752
6	AGE	0.000751
7	DIS	-1.475759
8	RAD	0.305655
9	TAX	-0.012329
10	PTRATIO	-0.953464
11	B	0.009393
12	LSTAT	-0.525467

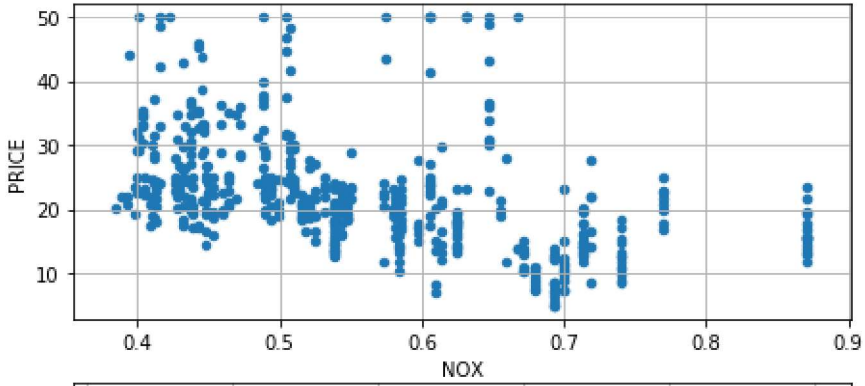
In [72]:
```
# Correlation between Target variable (PRICE) and Fatures
fig, axs = plt.subplots(7, 2 , sharey=False)
bos.plot(kind='scatter', x=bos.columns[0], y='PRICE', ax=axs[0][0] , figsize=(16,25) , grid=True )
bos.plot(kind='scatter', x=bos.columns[1], y='PRICE', ax=axs[0][1], grid=True)
bos.plot(kind='scatter', x=bos.columns[2], y='PRICE', ax=axs[1][0], grid=True)
bos.plot(kind='scatter', x=bos.columns[3], y='PRICE', ax=axs[1][1], grid=True)
bos.plot(kind='scatter', x=bos.columns[4], y='PRICE', ax=axs[2][0], grid=True)
bos.plot(kind='scatter', x=bos.columns[5], y='PRICE', ax=axs[2][1], grid=True)
bos.plot(kind='scatter', x=bos.columns[6], y='PRICE', ax=axs[3][0], grid=True)
bos.plot(kind='scatter', x=bos.columns[7], y='PRICE', ax=axs[3][1], grid=True)
bos.plot(kind='scatter', x=bos.columns[8], y='PRICE', ax=axs[4][0], grid=True)
bos.plot(kind='scatter', x=bos.columns[9], y='PRICE', ax=axs[4][1], grid=True)
bos.plot(kind='scatter', x=bos.columns[10], y='PRICE', ax=axs[5][0], grid=True)
bos.plot(kind='scatter', x=bos.columns[11], y='PRICE', ax=axs[5][1], grid=True)
bos.plot(kind='scatter', x=bos.columns[12], y='PRICE', ax=axs[6][0], grid=True)
plt.show()
```

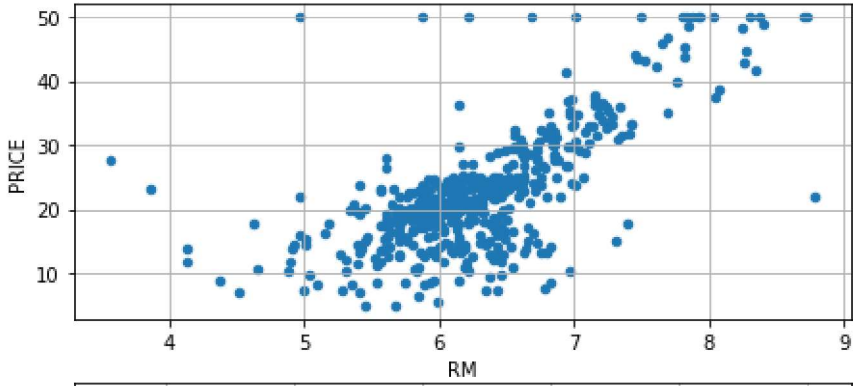

PRICE
CRIM

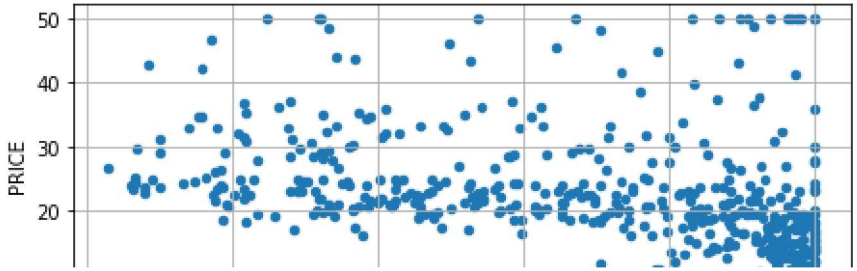

PRICE
ZN

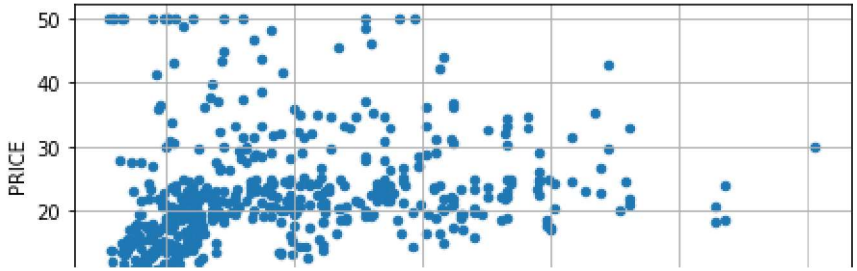

PRICE
INDUS


PRICE
CHAS

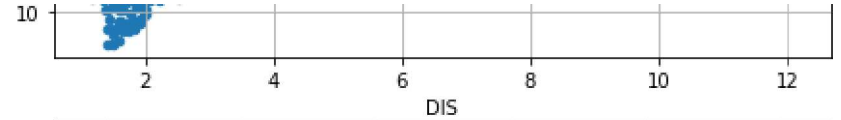

PRICE
NOX

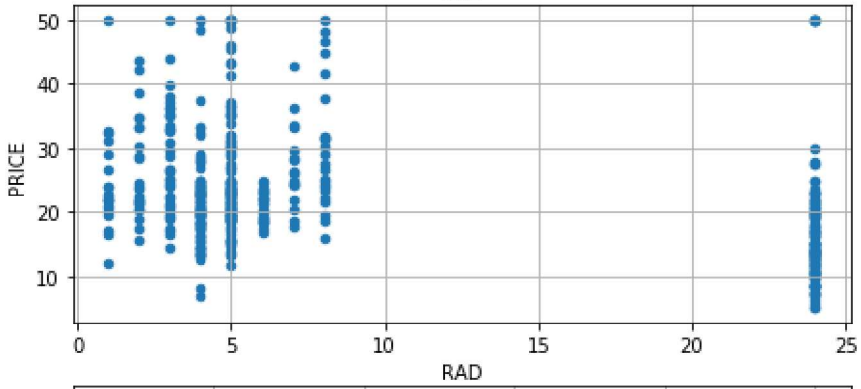

PRICE
RM

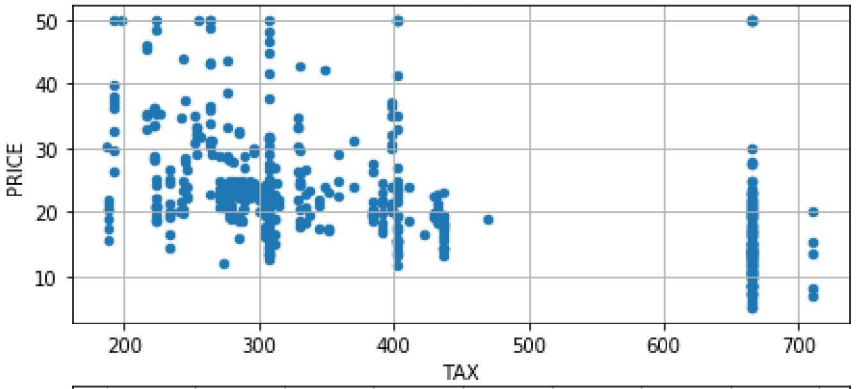

PRICE

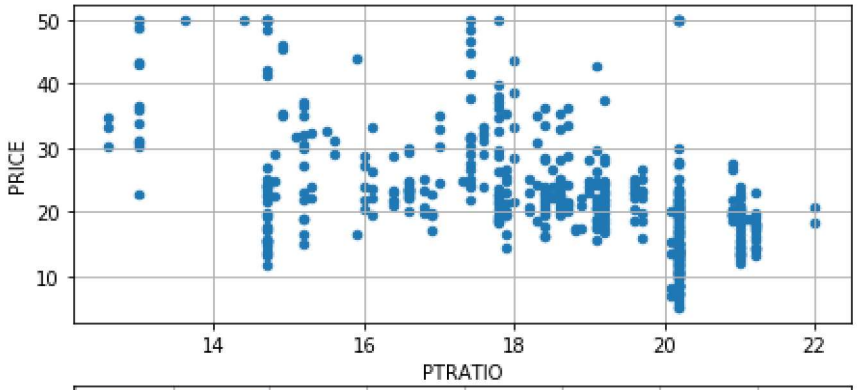

PRICE

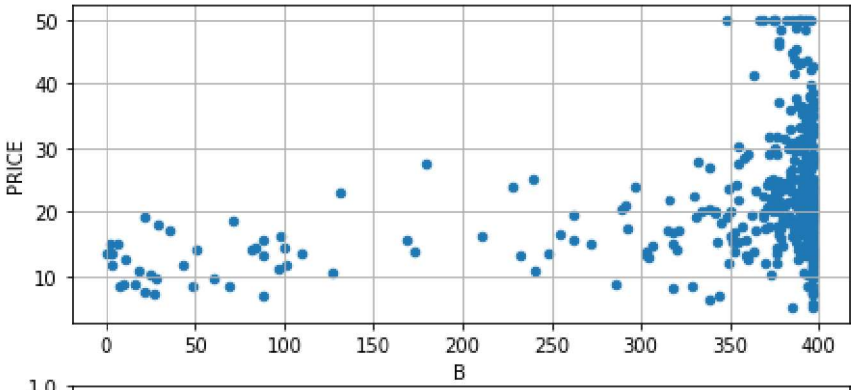

AGE

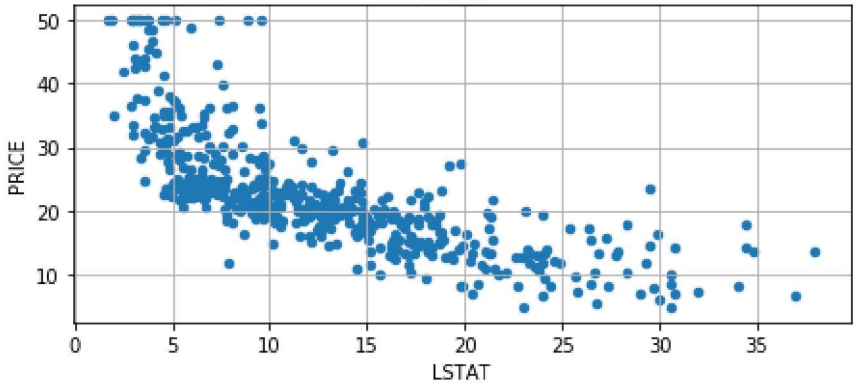

DIS

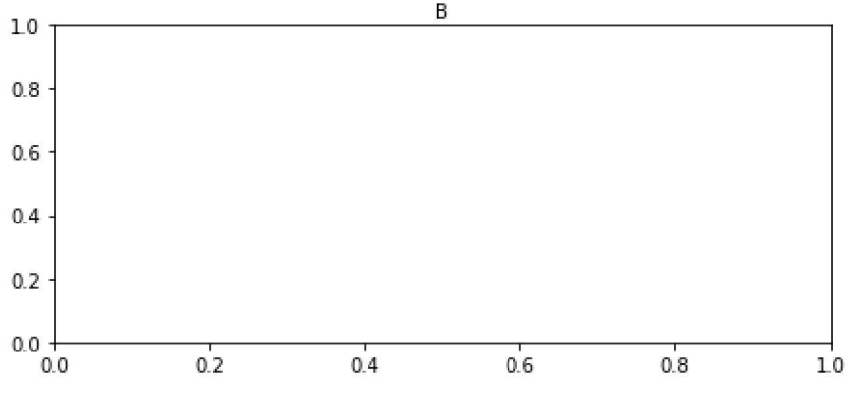

PRICE
RAD


PRICE
TAX


PRICE
PTRATIO


PRICE
B


PRICE
LSTAT



In [78]:
```
# Calulation of mean square error
mean_square_error = np.mean((bos['PRICE'] - lr_model.predict(df_feature).flatten())**2)
print("Mean Squared Error:\t\t",mean_square_error)
# Residual sum of squares
residual_square_error = np.sum((bos.PRICE - lr_model.predict(df_feature).flatten()) ** 2)
print("Residual Sum of Squares:\t", residual_square_error)
```

```
Mean Squared Error:             21.897779217687496
Residual Sum of Squares:        11080.276284149873
```

In [81]:
```
# import regression libraries
import statsmodels.api as sm
from statsmodels.formula.api import ols
```

In [86]:
```
# statistical analysis of the linear regression model
bos_ols = ols(formula='PRICE ~ CRIM + ZN + INDUS + CHAS + NOX + RM + AGE + DIS + RAD + TAX + PTRATIO + B + LSTAT + PRI
CE ', \
              data=bos).fit()
            # Pass the feature to create the formula for Ordinary least squares
print(' Statistical Analysis:', bos_ols.summary())
```

```
 Statistical Analysis:                            OLS Regression Results
==============================================================================
Dep. Variable:                  PRICE   R-squared:                       1.000
Model:                            OLS   Adj. R-squared:                  1.000
Method:                 Least Squares   F-statistic:                 6.603e+30
Date:                Sat, 27 Oct 2018   Prob (F-statistic):               0.00
Time:                        18:06:52   Log-Likelihood:                 15214.
No. Observations:                 506   AIC:                        -3.040e+04
Df Residuals:                     491   BIC:                        -3.033e+04
Df Model:                          14
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept  -2.132e-14   2.43e-14     -0.877      0.381    -6.9e-14    2.64e-14
CRIM       -8.039e-16    1.5e-16     -5.367      0.000    -1.1e-15    -5.1e-16
ZN         -1.388e-17   6.29e-17     -0.221      0.825   -1.37e-16     1.1e-16
INDUS       4.718e-16   2.79e-16      1.694      0.091   -7.55e-17    1.02e-15
CHAS        8.882e-16   3.94e-15      0.225      0.822   -6.86e-15    8.63e-15
NOX         1.354e-14   1.77e-14      0.766      0.444   -2.12e-14    4.83e-14
RM           1.11e-15   2.05e-15      0.542      0.588   -2.91e-15    5.13e-15
AGE         3.747e-16   5.98e-17      6.262      0.000    2.57e-16    4.92e-16
DIS        -9.992e-16   9.53e-16     -1.049      0.295   -2.87e-15    8.72e-16
RAD         4.094e-16   3.07e-16      1.334      0.183   -1.94e-16    1.01e-15
TAX        -6.072e-17   1.72e-17     -3.526      0.000   -9.46e-17   -2.69e-17
PTRATIO     2.914e-16   6.24e-16      0.467      0.641   -9.34e-16    1.52e-15
B          -3.816e-17   1.23e-17     -3.101      0.002   -6.23e-17    -1.4e-17
LSTAT      -1.013e-15   2.53e-16     -3.997      0.000   -1.51e-15   -5.15e-16
PRICE          1.0000   2.04e-16    4.9e+15      0.000       1.000       1.000
==============================================================================
Omnibus:                       26.847   Durbin-Watson:                   0.509
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               70.889
Skew:                          -0.189   Prob(JB):                     4.04e-16
Kurtosis:                       4.794   Cond. No.                     1.59e+04
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.59e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
```

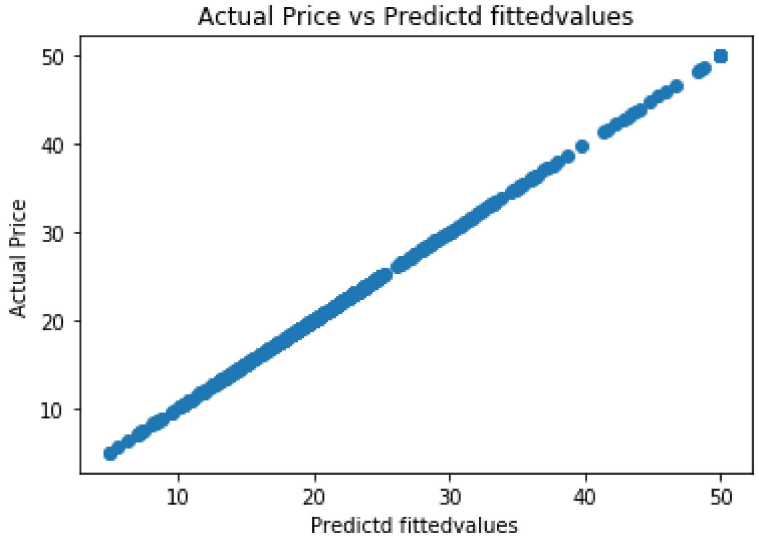
In [93]:
```
# Scatter plot between Actual Price Data and Predictd Price
print("Actual Price vs Predictd Price")
fig = plt.figure()
plt.title('Actual Price vs Predictd Price')
plt.scatter(bos['PRICE'],lr_model.predict(df_feature) ,  linewidths=2)
plt.xlabel("Actual Price")
plt.ylabel("Predictd Price")
```

```
Actual Price vs Predictd Price
```

Out[93]: Text(0,0.5,'Predictd Price')

In [94]:
```
# Scatter plot between Actual Price Data and Predictd fittedvalues
plt.scatter(bos_ols.fittedvalues, bos.PRICE)
plt.xlabel("Predictd fittedvalues")
plt.ylabel("Actual Price")
plt.title("Actual Price vs Predictd fittedvalues")
```

Out[94]: Text(0.5,1,'Actual Price vs Predictd fittedvalues')



In [96]:
```
print("R-sqaure value:\t\t\t",bos_ols.rsquared )
print("Adjusted R-sqaure value:\t",bos_ols.rsquared_adj)
```

```
R-sqaure value:                 1.0
Adjusted R-sqaure value:        1.0
```