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Alert: Some Unsolved Problems I discovered througout the video making journey

1. $\sum_{n=1}^{\infty}(-1)^n \frac{1}{p_n}$ where p_n is nth prime

2. Finding trigamma(1/n) summation formula

Note: All the problems below have a solution video on “Mathalysis World” youtube channel. You can just click on the ‘solution’ that appears at the right of every problem to access solution. Problems that appear early have comparatively lower quality solution video.

$$^{1}\int_0^{\infty} 0.5^{\lfloor x \rfloor} dx = 2 \quad => \textit{solution}$$

$$^{2}\int_0^{\frac{\pi}{4}} \frac{(\sin x + \cos x)}{(9 + 16\sin 2x)} dx = \frac{\ln 3}{20} \quad => \textit{solution}$$

$$^{3}\int \frac{(x+1)}{x(x+\ln x)} dx = \ln(x + \ln x) + c \quad => \textit{solution}$$

$$^{4}\int_0^{1} \frac{\ln(x+1)}{x^2+1} dx = \frac{\pi \ln(2)}{8} \quad => \textit{solution}$$

[1] A surprisingly easy Geometric Integral
[2] King comes to your help
[3] Is MIT integration Bee this easy?
[4] The Mysterious Integral

[5] $$\int_0^1 \left(\sqrt[2022]{1-x^{2020}} - \sqrt[2020]{1-x^{2022}}\right)dx = 0 \quad => solution$$

[6] $$\int_0^1 x^x dx = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^n} \quad => solution$$

[7] $$\int_0^1 x^{-x} dx = \sum_{n=1}^{\infty} \frac{1}{n^n} \quad => solution$$

[8] $$\int_{\pi/6}^{\pi/3} (\sin x^{\cos x^{\sin x}} - \cos x^{\sin x^{\cos x}})dx = 0 \quad => solution$$

[9] $$\int_{-\pi}^{\pi} (\sin x + 2\sin(2x) + 3\sin(3x) + 4\sin(4x) + 5\sin(5x))^2 dx = 55\pi \quad => solution$$

[10] $$\int_0^{\infty} \frac{1}{1+x+x^2+x^3+x^4+x^5} dx = \frac{\pi}{3\sqrt{3}} \quad => solution$$

[11] $$\int tanh^2(x)dx = x - \tanh x \quad => solution$$

[12] $$\int_0^1 \frac{\ln(1+x)}{x} dx = \frac{\pi^2}{12} \quad => solution$$

[13] $$\int_0^1 \prod_{k=0}^{\infty} \left(\frac{1}{1+x^{2^k}}\right) dx = \frac{1}{2} \quad => solution$$

[14] $$\int_{-2}^{2} (x^3 \cos(\frac{x}{2}) + \frac{1}{2})\sqrt{4-x^2} dx = \pi \quad => solution$$

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[5] The Quarrelsome Integral
[6] Sophomore's Dream-i
[7] Sophomore's Dream-ii
[8] The trigonometric towers integral
[9] The Trigonometric BUS Integral
[10] Bro, Are you joking?
[11] How easy is MIT Integration Bee?
[12] Integrating using series in MIT Integration Bee
[13] This is the most easiest difficult question in MIT Integration Bee
[14] Chinese University Wifi Password

[15] $$\int_0^{\pi/2} \frac{\sqrt[3]{\tan(x)}}{(\sin(x)+\cos(x))^2}dx = \frac{2\sqrt{3}\pi}{9} \quad => solution$$

[16] $$\int_0^{1} \frac{x\ln(x)}{x^4+x^2+1}dx = \frac{1}{36}\left(\psi_1(\frac{2}{3}) - \psi_1(\frac{1}{3})\right) \quad => solution$$

[17] $$\int_0^{\pi/2} \ln(\sin(y))dy = -\ln(2)\frac{\pi}{2} \quad => solution$$

[18] $$\int_0^{\infty} \frac{\sin(x)}{x}dx = \frac{\pi}{2} \quad => solution$$

[19] $$\ln(2\sin(x)) = \sum_{n=1}^{\infty} -\frac{\cos(2nx)}{n} \quad => solution$$

[20] $$\ln(2\cos(x)) = \sum_{n=1}^{\infty} (-1)^{n+1}\frac{\cos(2nx)}{n} \quad => solution$$

[21] $$\int_0^{\frac{\pi}{2}} \sqrt{\tan(x)}dx \quad => solution$$

[22] $$\int_0^{\infty} \frac{\sin(x)}{x}dx = \frac{\pi}{2} \quad => solution$$

[23] $$\int_0^{\infty} \frac{\sin(x^n)}{x^n}dx \quad => solution$$

[24] $$\int_0^{\infty} \sin(x^2)dx = \frac{\sqrt{\pi}}{2\sqrt{2}} \quad => solution$$

[25] $$\int_0^{\infty} \cos(x^2)dx = \frac{\sqrt{\pi}}{2\sqrt{2}} \quad => solution$$

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[15] Beta Gamma Function in MIT Integration Bee
[16] Stepping on some hard integrals
[17] proof of why $\int_0^{\pi/2} \ln(\sin(y))dy = -\ln(2)\frac{\pi}{2}$
[18] proof of Dirichlet Integral
[19] Two wonderful Series from Dr. Peyam-i
[20] Two wonderful Series from Dr. Peyam-ii
[21] Best application of Beta Gamma Function
[22] Proof of Dirichlet Integral
[23] Proof of Generalized Dirichlet Integral
[24] Easiest way to prove Fresnel Integrals using Beta Gamma Functions
[25] Easiest way to prove Fresnel Integrals using Beta Gamma Functions

$$^{26}\int_0^\infty \sin(x^n)dx \;\; => solution$$

$$^{27}\int_0^\infty \cos(x^n)dx \;\; => solution$$

$$^{28}\int_0^\infty \sin(x^2)dx = \frac{\sqrt{\pi}}{2\sqrt{2}} \;\; => solution$$

$$^{29} \;\; => solution$$

$$^{30}\int_0^1 \frac{1+x^2}{1+x^4}dx \;\; => solution$$

$$^{31}\int_0^1 \frac{\arctan x}{x}dx \;\; => solution$$

$$^{32}\int_0^{\frac{\pi}{2}} \frac{x}{\sin(x)}dx \;\; => solution$$

$$^{33}\log(-2023) \;\; => solution$$

$$^{34}\int_1^{2023} \log(-x)dx \;\; => solution$$

$$^{35}\int_0^1 x^{x^{x^{x^{x^{x^{x^{\cdot^{\cdot^{\cdot}}}}}}}}} dx = Diverges \;\; => solution$$

[26]Generalized Fresnel Integral-i
[27]Generalized Fresnel Integral-i
[28]Solving Fresnel Integral from Laplace Transform
[29]Solving 20 integrals by Feynman Technique
[30]Another easy integral from MIT Integration Bee
[31]The Catalan Integral
[32]Not the Dirichlet Integral
[33]Shorts: What is log(-2023) ?
[34]Let's go to Complex World
[35]The tower of x integral

$$[36] \int_{-\infty}^{\infty} \Gamma(1+ix)\Gamma(1-ix)dx = \frac{\pi}{2} \;\; => solution$$

$$[37] \int_{0}^{e} W(z)dz \;\; => solution$$

where W(z) is Lambert W function

$$[38] W(z) = \sum_{n=1}^{\infty} \frac{(-n)^{(n-1)}x^n}{n!} \;\; => solution$$

$$[39] \frac{1}{e}^{\frac{1}{e}^{\frac{1}{e}^{\frac{1}{e}^{\frac{1}{e}^{\frac{1}{e}^{\frac{1}{e}^{\frac{1}{e}^{\cdot^{\cdot^{\cdot}}}}}}}}}} = \Omega \;\; => solution$$

$$[40] \int_{0}^{\infty} \frac{x^e}{1+x^{2(e+1)}}dx = \frac{\pi}{2(e+1)} \;\; => solution$$

$$[41] \int_{0}^{\frac{\pi}{4}} \log\left(2\cos\left(x\right)\right)dx \;\; => solution$$

$$[42] \int_{0}^{1}\int_{0}^{1} \frac{dxdy}{1+x^2y^2} \;\; => solution$$

$$[43] \int_{0}^{3}\int_{x^2}^{9} x^3e^{y^3}dydx \;\; => solution$$

$$[44] \int_{0}^{8}\int_{\sqrt[3]{y}}^{2} \sqrt{x^4+1}dxdy \;\; => solution$$

[36] My take on your ”A satisfying gamma function integral @maths505”
[37] Integrating Lambert W function
[38] Deriving the series of Lambert W function
[39] The Tower of 1/e
[40] Freaking Irrational Integral
[41] Integration using Fourier Series
[42] Most satisfying Double Integral
[43] I can solve the impossible Integral -i
[44] I can solve the impossible Integral - ii

$$^{45} y^{\frac{dy}{dx}} = e^y \quad => solution$$

$$^{46} \int ((1-x)^3+(x-x^2)^3+(x^2-1)^3-3(1-x)(x-x^2)(x^2-1))dx = 0 \quad => solution$$

$$^{47} \int_0^{\frac{\pi}{2}} \frac{\ln(\sec(x))}{\tan(x)} dx = \frac{\zeta(2)}{4} \quad => solution$$

$$^{48} \sum_{n=0}^{\infty} \frac{1}{n!} = e \quad => solution$$

$$^{49} \int_0^1 \int_0^1 \frac{xy\sqrt{x}}{x\sqrt{y}+y\sqrt{x}} dxdy \quad => solution$$

$$^{50} \sum_{n=1}^{\infty} \frac{H_n}{2^n} = ln(4) \quad => solution$$

$$^{51} \int_0^{\infty} \frac{e^{-t}-e^{-tx}}{t} dt = \ln(x) \quad => solution$$

Words of an adolescent $=> solution$

$$^{52} \int_0^{\infty} e^{-t} t^{x-1} dx = \Gamma(x) \quad => solution$$

$$^{53} \int_0^{\infty} t^{m-1}(1-t)^{n-1} dt = \beta(m,n) \quad => solution$$

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[45] Is that even possible?
[46] MIT Integration Bee Qualifier Exam P10
[47] Dear @Maths505, here's my approach
[48] Proving using Beta Gamma Function
You can not have a more difficult proof than this
[49] Using symmetricity in Integrals
[50] A standard technique for such problems: use generating function for harmonic number
[51] This is the best use of Feynman's Method
[52] Origin of Gamma Function
[53] Origin of Beta Function

$$^{54} \quad => solution$$

$$^{55}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1+e^{x\cos(x)}}dx \quad => solution$$

$$^{56}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{(1+e^{x\cos x})(sin^4x+cos^4x)} \quad => solutiondx$$

$$^{57}\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\cdot^{\cdot^{\cdot}}}}}}}}}}}}}} \quad => solution$$

$$^{58}V_n = \frac{\pi^{\frac{n}{2}}}{(\frac{n}{2})!}r^n \quad => solution$$

$$^{59}\sum_{n=2k}^{\infty} V_n = e^{\pi} \quad => solution$$

where V_n is volume of n dimensional Sphere

$$^{60} \quad => solution$$

$$^{61}\int_0^{\frac{\pi}{2}} \tan^{i} x\, dx \quad => solution$$

$$^{62}\int_{-\infty}^{\infty} \frac{\cos(x)}{x^2+1}dx \quad => solution$$

$$^{63}\gamma = \sum_{m=2}^{\infty}(-1)^m\frac{\zeta(m)}{m} \quad => solution$$

[54] Solving the easiest integral using hardest technique i.e. Ramanujan's Master Theorem
[55] You cannot get a more easier integral than this in MIT Integration Bee
[56] The Legend of JEE Mains solved by only 5 percent students
[57] This is the best use of Lambert W function
[58] Deriving the formula for volume of n dimensional sphere
[59] sum of the volumes of all n-dimensional spheres
[60] sum of the volume of even dimensional spheres
[61] A Complex triggy boi
[62] Using Laplace Transform to solve for an absolutely gorgeous result
[63] Short Animation Proof of this absolutely gorgeous result

$$\int_1^\infty \frac{dx}{x\sqrt{x^4-1}} \quad => solution$$ [64]

$$\int_1^2 (x-1)^{\frac{1}{2}}(2-x)^{\frac{1}{2}}dx \quad => solution$$ [65]

$$\lim_{x\to\frac{\pi}{4}} (1+\sin(x)-\cos(x))^{\tan(2x)} = e^{\frac{-1}{\sqrt{2}}} \quad => solution$$ [66]

$$\lim_{x\to 0} \frac{1-x\cot x}{x^2} \quad => solution$$ [67]

$$\int_{-\infty}^\infty \frac{\sin(x)}{x}dx \quad => solution$$ [68]

$$\int_{-\infty}^\infty \frac{1-\cos x}{x^2}dx \quad => solution$$ [69]

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{x} = \frac{1}{a+b+x} \quad => solution$$ [70]

$$\frac{d^{-1}}{dx^{-1}}(x)\frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}}(x)\frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}}(1) \quad => solution$$ [71]

$$\sum_{n=1}^\infty \frac{1}{n^2-x^2} = \frac{\pi\cot(\pi x)}{-2x}+\frac{1}{2x^2} \quad => solution$$ [72]

$$\sum_{n=1}^\infty \frac{\zeta(2n)}{\pi^{2n}} \quad => solution$$ [73]

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[64] MIN Integration Bee 2010 Qualifier Problem 8
[65] MIT Integration Bee 2010 Qualifier Problem 25
[66] An awesome limit problem
[67] How high school student vs University student solve this limit?
[68] A single liner solution using Maz Identity
[69] How Undergrad. Vs Grad solve this integral?
[70] Unseemingly hard Quadratic equation
[71] WTF are these things?)
[72] How come we have cot here?
[73] Stanford Mathematics Tournament

$$^{74}\int_0^\infty \frac{x^{p-1}}{e^x-1}dx = \Gamma(p)\zeta(p) \ \ => solution$$

$$^{75}\int_0^\infty \frac{x^{p-1}}{e^x+1}dx = \Gamma(p)\eta(p) \ \ => solution$$

$$^{76}\eta(s) = (1-\frac{2}{2^s}\zeta(s) \ \ => solution$$

$$^{77}\int_0^\infty \frac{\sin(x)}{x^{\frac{3}{2}}}dx \ \ => solution$$

$$^{78}\Gamma(x) = \int_0^\infty e^{-t}t^{x-1}dt \ \ => solution$$

$$^{79}\Gamma(s) = \lim_{n\to\infty}\frac{n^s}{s}\prod_{k=1}^{n}\frac{k}{s+k} \ \ => solution$$

$$^{80}\frac{1}{\Gamma(x)} = xe^{\gamma x}\prod_{n=1}^{\infty}(1+\frac{x}{n})e^{-\frac{x}{n}} \ \ => solution$$

$$^{81}\psi(x+1) = -\gamma + \sum_{k=1}^{\infty}\frac{1}{k} - \frac{1}{k+x} \ \ => solution$$

$$^{82}\psi(x+1) = -\gamma + H_n \ \ => solution$$

$$^{83}\psi(x+1) = -\gamma + \int_0^\infty \frac{1-x^n}{1-x}dx \ \ => solution$$

$$^{84}\psi(1-n) - \psi(n) = \pi\cot(n\pi) \ \ => solution$$

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[74] $x^{p-1}/e^x - 1$ // Product of Eulers Gamma and Reimann zeta function interms of Bose integral

[75] $x^{p-1}/e^x + 1$ // Product of Eulers Gamma and Dirichlet eta function

[76] Relation between Dirichlet Eta and Reimann Zeta Function

[77] MIT Integration Bee: This is the best application of MAZ Identity

[78] Euler Representation of Gamma Function

[79] Gauss Representation of Gamma Function

[80] Weierstrass Representation of Gamma Function

[81] Infinite Sum Representation for Digamma Function

[82] This is the most beautiful equation in mathematics, Deriving from Scratch

[83] Integral Representation for Digamma Function

[84] Reflection formula for Digamma Function

$$^{85}2\psi(2m) = \psi(m) + \psi(m + \frac{1}{2}) + 2\ln(2) \;\; => solution$$

$$^{86}\int_0^\infty (1 - x\sin\left(\frac{1}{x}\right))dx \;\; => solution$$

$$^{87}\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2-0^2}} + \frac{1}{\sqrt{n^2-1^2}} + ... + \frac{1}{\sqrt{n^2-(n-1)^2}}\right) \;\; => solution$$

$$^{88}\lim_{x\to 0}\left(\frac{\sqrt[x]{1+x}}{e}\right)^{\csc x} \;\; => solution$$

$$^{89}\lim_{n\to\infty}\frac{1}{\sqrt{n}}\sum_{k=1}^{n}\frac{1}{\sqrt{n+k}} \;\; => solution$$

$$^{90}\sum_{n=1}^{\infty}\frac{1}{n^2+x^2}\sum_{n=1}^{\infty}\frac{1}{n^2-x^2} \;\; => solution$$

$$^{91}\zeta(s) = \prod_{prime}\frac{1}{1-p^{-s}} \;\; => solution$$

$$^{92}\int_0^\infty f(s)g(s)ds = \int_0^\infty \mathcal{L}\{f\}(t)\;\mathcal{L}^{-1}\{g\}(t)dt \;\; => solution$$

$$^{93}\sum_{n=1}^{\infty} f(n) = \int_0^\infty \frac{\mathcal{L}^{-1}\{f\}(t)}{e^t-1}dt \;\; => solution$$

$$^{94}\sum_{n=2}^{\infty}\frac{4n-3}{n(n^2-1)} = \frac{9}{4} \;\; => solution$$

[85] Duplication formula for Digamma Function

[86] The classic Problem from MIT Integration BEE

[87] Harvard MIT Maths Tournament

[88] A high school limit problem from IIT JEE

[89] Limit involving Reimann Sum

[90] Two important infinite sums

[91] Trivial Proof of Euler's Prime Product Formula // Relation between Reimann Zeta Function and Prime Numbers

[92] Two amazing theorems of MAZ -I

[93] Two amazing theorems of MAZ -II

[94] MAZ theorem helps me solve this infinite sum

95 $$\int_0^{\ln(2)} \frac{e^x - e^{2x} + e^{3x} - e^{4x}}{1 + e^x + e^{2x} + e^{3x}} dx \quad => solution$$

96 $$\int_0^{\infty} \frac{\sin(x)}{e^x - 1} dx = \frac{\pi \coth(\pi) - 1}{2} \quad => solution$$

97 $$\int_0^{\infty} \frac{\sin(ax)}{x^n} dx \quad => solution$$

98 $$\int_1^{\infty} \frac{x^2 - 1}{x^4 \ln(x)} dx \quad => solution$$

99 $$\int_0^{\infty} \frac{\sin^3(x)}{x^2} dx \quad => solution$$

100 $$\lim_{x \to 0} \frac{\sin(x)}{x} = 1 \quad => solution$$

101 $$\sum_{n=2}^{\infty} \frac{(-1)^n \zeta(n)}{2^n} \quad => solution$$

102 $$\int_1^{\infty} \frac{\{x\}}{x^4} dx = \frac{1}{2} - \frac{\zeta(3)}{3} \quad => solution$$

103 $$\int_{\frac{1}{4}}^{\frac{1}{2}} \lfloor \log \lfloor \frac{1}{x} \rfloor \rfloor dx \quad => solution$$

104 $$\int_0^{1} \frac{x^7 - 1}{\log(x)} dx \quad => solution$$

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[95] Monstrous JEE Advanced Integral
[96] MAZ theorem helps me solve this integral
[97] Smashing an improper integral using MAZ Identity
[98] MAZ Identity speed rockets the integral
[99] MAZ Identity speed rockets the integral
[100] 5 Unusual ways to prove this limit
[101] DIGamma Function helps me solve this infinite sum
[102] Integration of Fraction Part for IIT JEE
[103] A tricky GIF Integral from MIT Integration Bee
[104] MIT Integration BEE Problem that needed MAZ Identity

$$^{105}\mathcal{L}\{\ln(x)\} \;\; => solution$$

$$^{106}\int_0^{2022} x^2 - \lfloor x \rfloor \lceil x \rceil dx = \frac{2022}{3} \;\; => solution$$

$$^{107}\int_0^{\frac{1}{2}} \sum_{n=0}^{\infty} {}^{n+3}C_n x^n dx \;\; => solution$$

$$^{108}\int_0^{\infty} \frac{\sin(x)}{\sinh(x)} dx = \frac{\pi}{2}\tanh(\frac{\pi}{2}) \;\; => solution$$

$$^{109}\int_0^{1} \sqrt{1-x^2}dx \int_1^{2} \sqrt{x^2-1}dx \;\; => solution$$

$$^{110}\sum_{n=0}^{\infty} \frac{1}{n+2} - \frac{1}{n+3} \;\; => solution$$

$$^{111}\int_0^{\infty} \frac{f(ax)-f(bx)}{x} dx = (f(\infty)-f(0))\ln(\frac{a}{b}) \;\; => solution$$

$$^{112}\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n+1)!} = \frac{2\pi}{3\sqrt{3}} \;\; => solution$$

$$^{113}\frac{d^i}{dx^i}(x^i) = i! \;\; => solution$$

$$^{114}\lim_{n\to\infty} \left(\frac{n!}{n^n}\right)^{\frac{1}{n}} = \frac{1}{e} \;\; => solution$$

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[105] Laplace Transform of $\ln(x)$

[106] Marriage of floor and ceiling function

[107] A good problem from MIT Integration Bee

[108] An Ridiculously Awesome Integral from Ramanujan's land (India)

[109] Solving Integrals Geometrically

[110] An Introduction to extremely difficult way to do a simple telescoping sum

[111] Frullani's Integral

[112] This ridiculously interesting sum is solved by Beta Function

[113] Imaginary Derivative of imaginary number. wow

[114] A brilliant limit from Stanford Maths Tournament

[115] $$\frac{d^{\pi}}{dx^{\pi}}(x^{\pi}) = \pi! \;\; => solution$$

[116] $$\lim_{n\to\infty} \sqrt[n]{\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{2}{n}\right)\Gamma\left(\frac{3}{n}\right).......\Gamma\left(\frac{n}{n}\right)} \;\; => solution$$

[117] $$\int f(x)^{dx}, \frac{\delta}{\delta x} f(x) \;\; => solution$$

[118] $$\lim_{n\to\infty} \left(\frac{n!}{n^n}\right)^{\frac{1}{n}} = \frac{1}{e} \;\; => solution$$

[119] $$\lim_{n\to\infty} \left(\frac{n!}{n^n}\right)^{\frac{1}{n}} = \frac{1}{e} \;\; => solution$$

[120] $$\infty! \;\; => solution$$

[121] $$\frac{d^{\frac{a}{b}}}{dx^{\frac{a}{b}}} f(x), \int f(x)^{dx}, \frac{\delta}{\delta x} f(x) \;\; => solution$$

[122] $$\int_0^{\frac{\pi}{2}} \sin(x)^{dx} \;\; => solution$$

[123] $$\int_0^1 e^{-x}\ln^2(x)dx = \frac{\pi^2}{6} + \gamma^2 \;\; => solution$$

[124] $$\int_0^1\int_0^1\int_0^1 \tan^{-1}(xyz)dxdydz = -\frac{3\zeta(3)}{32} - \frac{\pi^2}{48} + \frac{\pi}{4} - \frac{\ln(2)}{2} \;\; => solution$$

[115] Differentiation IIT JEE Maths — πth derivative — Application of Derivative

[116] The is the most beautiful problem I ever solved

[117] Product Integral and Product Derivative

[118] Using the powerful stirling approximation for this IIT limit

[119] Proving this IIT Limit using product integral

[120] Infinity factorial, Happy Birthday Bishnu

[121] 500 sub special:: Inventing Math: Fractional Derivative, Product Integral and Product Derivative

[122] Impossible seeming Integrals

[123] Integral with two important constants

[124] Ridiculously Awesome Impossible Integral

125 $$\int_0^1 \int_0^1 \int_0^1 f(xyz)dxdydz = \frac{1}{2}\int_0^1 \ln^2(x)f(x)dx \;\; => solution$$

126 $$\int_0^1 \int_0^1 f(xy)dxdy = -\int_0^1 \ln(x)f(x)dx \;\; => solution$$

127 $$\int_0^1 \int_0^1 \int_0^1 e^{-xyz}dxdydz \;\; => solution$$

128 $$\sum_{k=0}^{10} {}^{10}C_k \; k^2 = 28160 \;\; => solution$$

129

Suggest your favorite integral
in the comment for upcoming Video =>solution

130 $$\lim_{n\to\infty} \frac{n + n^2 + n^3 + ... + n^n}{1^n + 2^n + 3^n + ... + n^n} = 1 - \frac{1}{e} \;\; => solution$$

131 $$\lim_{n\to\infty} \pi(n)\left(\sqrt[n]{n} - 1\right) = 1 \;\; => solution$$

132 $$\lim_{n\to\infty} \frac{\lfloor e^{\frac{1}{n}} \rfloor + \lfloor e^{\frac{2}{n}} \rfloor + \lfloor e^{\frac{3}{n}} \rfloor + ... + \lfloor e^{\frac{n}{n}} \rfloor}{n} \;\; => solution$$

133 $$\int_0^\infty \lfloor x \rfloor e^{1-\lfloor x \rfloor} dx \;\; => solution$$

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125 Ridiculously Awesome Impossible Integral
126 Ridiculously Awesome Impossible Integral
127 Ridiculously Awesome Integral
128 A sum from World International Mathematics Olympiad Final 2019
129 Suggest
130 The nightmare limit Problem
131 Limit involving Prime counting Function
132 JEE Advanced Limit (Model Question)
133 Easy Integral by Himanshu

[134] $$F(n)=\int_0^{\infty}\frac{x^{2n-2}}{(x^4-x^2+1)^n}dx, F(5)=?\ \ => solution$$

[135] $$\lim_{n\to\infty}\frac{(2n)!.(2n+1)!}{(n!\,.2^n)^4}\ \ => solution$$

[136] $$\int_0^1\int_0^1\frac{\sqrt{x}+\sqrt{y}}{\sqrt{\sqrt{xy}}(1-xy)}dxdy\ \ => solution$$

[137] $$\int_1^{\int_1^{\int_1^{\int_1^{\int_1^{\cdot^{\infty}}xdx}xdx}xdx}xdx}xdx}xdx=2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cdot_{\cdot\infty}}}}}\ \ => solution$$

[138] $$\sum_{n=0}^{2023}\frac{1}{5^n+\sqrt{5^{2023}}}\ \ => solution$$

[139] $$\int_0^1\frac{x-1}{\ln(x)}dx=\ln(2),\int_0^{\infty}\frac{\sin(x)}{x}=\frac{\pi}{2},\int_0^1\frac{\sin(\ln(x))}{\ln(x)}=\frac{\pi}{4},$$

$$\int_0^{\infty}\frac{e^{-x^2}\sin(x^2)}{x^2}dx=\sqrt{\pi\sqrt{2}}\sin(\frac{\pi}{8}),\int_0^{\infty}e^{-x^2}\cos(5x)dx=\frac{\sqrt{\pi}}{2}e^{-\frac{25}{4}}\ \ => solution$$

[140] $$\int W(x)dx\ \ => solution$$

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[134] Hard Integral by Himanshu

[135] This is the best use of stirling's approximation

[136] This is the best use of digamma function

[137] Surprise!!!

[138] Sum involving King's Rule

[139] Destroying five harsh integrals using Feynman's Technique

[140] Integral of Lambert W function

$$^{141}\sum_{n=0}^{\infty}\frac{1}{(4n)!} \;\; => solution$$

$$min = f(x) = \left(\frac{0.05}{2}e^{\frac{0.05}{2}(2\times17+0.05\times20^2-2x)}\times erfc\left(\frac{17+0.05\times20^2-x}{\sqrt{2}\times20}\right)\right)\times\frac{3030}{0.0153}$$

$$^{142}\int_0^{\infty} e^{-x^2}dx \;\; => solution$$

$$^{143}\int \sin(\sin(x))$$

$$^{144}A difficult integral problem made easy, shorts$$

$$^{145}\frac{d}{dx}W(x) \;\; => solution$$

$$^{146}\int \sqrt{\sin(x)}dx = -2E(\frac{\pi}{4}-\frac{x}{2}\,|\,2)+c \;\; => solution$$

$$^{147}\int_0^{\frac{\pi}{2}} \ln(\sin(x))dx \; \int_0^{\frac{\pi}{2}} \ln(\cos(x))dx \; \int_0^{\frac{\pi}{2}} \ln(\tan(x))dx \;\; => solution$$

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[141] Can you solve this sum?
[142] 6 proofs of Gaussian Integral
[143] Horse shoe Integral
[144] MIT would not want to listen this hack about MIT Integration BEE
[145] Differentiation of Lambert W function
[146] Elliptic Integral of the second kind
[147] A nice family of Integrals

[148] $\int_0^{\frac{\pi}{4}} \ln(\sin(x))dx \int_0^{\frac{\pi}{4}} \ln(\cos(x))dx \int_0^{\frac{\pi}{4}} \ln(\tan(x))dx \;\; => solution$

[149] $\int_0^{\frac{\pi}{4}} \ln(1+\tan(x)) \int_0^{\frac{\pi}{4}} \ln(1-\tan(x)) \;\; => solution$

[150] $\int_0^{\infty} \frac{\ln^2(x)}{1-x^2}dx \int_0^{1} \frac{\ln^2(x)}{1-x^2}dx \int_0^{1} \frac{\ln^2(x)}{1+x^2}dx \int_0^{\infty} \frac{\ln^2(x)}{1+x^2}dx \;\; => solution$

[151]

1. Proof of Lhopital's rule in both algebraic and visual way
2. Fundamental theorem of calculus in a visual way
3. Derivative of sin(x) and cos(x) in visual way
3. Calculation of Escape Velocity by Newton in 17th Century
4. Introduction to Epsilon-Delta Definition
5. Zeno's paradox in limits
6. What does it mean to be undefined at a point but have limiting value at a point
7. Different notation for differentiation of Newton and Leibniz
8. Rigorous proof of Euler's Identity from level 0 9. Applications of Differential equation: NEwton's Law of Cooling 10. When to swap the sum and integrals 11. Why does the nth root test work?

[152]

1. Cut then multiply method // Demontration in a visual way that (n/x)$^x maximizesate^x i.e.e^(n/e).//Proofusingfirstderivativethatitinfactis$
2. The reciprocal of the Basel sum answers the question: What is the probability that two numbers selected at random are relatively prime?

[148] A nice family of Integrals
[149] One of them is easy and other is hard
[150] A happy get-together of integrals
[151] Some cool concepts to explain
[152] Some concepts to use MANIM animation

Excusion in Number Theory Page: 29-35 Citation: Ogilvy, C. S.; Anderson, J. T. (1988). Excursions in Number Theory. Dover Publications. pp. 29–35. ISBN 0-486-25778-9.

3.

[153]

1. Fractional root of a Matrix, exponential of a matrix, logarithm of a matrix

2. Contour Integration 3. Usage of Epsilon-Delta Definition and when does it fail ? 4. Deriving Gamma'(1)= - gamma and Gamma"(1) = gamma$^2 + zeta(2) and digamma(1/2) = -gamma + 2ln(2) 5. Finding the value of Reimann Zeta of 2 from Level 0 through Digamma Function$

$$^{154}\int \sqrt{\tan(x)}dx \int \sqrt{\cot(x)}dx \;\; => solution$$

$$^{155}\sin(z) = 2 \;\; => solution$$

$$^{156}\int_0^\infty \frac{x\cos(x)}{e^x - 1}dx \;\; => solution$$

$$^{157}\int_0^\pi xf(\sin(x))dx = \frac{\pi}{2}\int_0^\pi f(\sin(x))dx \;\; => solution$$

$$^{158}\int f^{-1}(x)dx = xf^{-1}(x) - F(f^{-1}(x)) + c \;\; => solution$$

[153]Some videos on my Checklist
[154]A story of two brothers
[155]This has a solution!!!
[156]A Ridiculously Awesome integral
[157]A nice Lemma for my nice viewers
[158]Proof and Usage of Inverse Integration Technique

$$^{159}\int_a^b f(x)dx = \int_a^b f(a+b-x)dx \quad => solution$$

$$^{160}\frac{d}{dx}(f^{-1}(x)) = \frac{1}{f'(f^{-1}(x))} \quad => solution$$

$$^{161}\int_a^b f(x)dx + \int_{f(a)}^{f(b)} f^{-1}(x)dx = bf(b) - af(a) \quad => solution$$

$$^{162}(f(x)g(x))^n = \sum_{r=0}^{n} {}^nC_r f^r(x) g^{n-r}(x) \quad => solution$$

$$^{163}\frac{d}{dy}\left(\int_a^b f(x,y)dx\right) = \int_a^b \frac{\partial}{\partial y}(f(x,y))dx \quad => solution$$

$$^{164}For f(x,y) = 0\, \frac{dy}{dx} = -\frac{f_x}{f_y} \quad => solution$$

$$^{165}\int_{-a}^{a} odd(x)dx = 0 \int_{-a}^{a} even(x)dx = 2\int_0^a even(x)dx \quad => Solution$$

$$^{166}\int_a^b f(x)dx = -\int_b^a f(x)dx \quad => Solution$$

[159] Proof (algebraic and geometric) and usage of King's Rule
[160] Proof and usage of inverse derivative technique
[161] Proof(algebraic and geometric) and usage of definite inverse integration technique
[162] Verification and usage of Leibniz Rule
[163] Proof and application of Feynman's Technique
[164] proof and application of complete differentiation using partial differentiation
[165] proof (algebraic and geometric) and usage of odd/ even function
[166] Proof and usage of reflection formula

$$^{167}\int f(x)g(x)dx = f\left(\int g\right) - f'\left(\int\int g\right) + f''\left(\int\int\int g\right) - f'''\left(\int\int\int\int g\right) + ... \ => Solut$$

$$\int_{-\infty}^{\infty} sech^n(t)dt = \frac{\sqrt{\pi}\,\Gamma(\frac{n}{2})}{\Gamma(\frac{n+1}{2})} \ => Solution$$

$$^{168}1^z = 3 \ => Solution$$

$$^{169}\int_0^{\frac{\pi}{2}} \frac{x}{\tan(x)}dx \ => Solution$$

$$^{170}\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)} \ => Solution$$

$$^{171}\Gamma(z) = \lim_{n\to\infty} \frac{n^z}{z}\prod_{k=1}^{n}\frac{k}{z+k} ||| \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+z)\Gamma(y-z)} = \prod_{k=0}^{\infty}\left[\left(1+\frac{z}{x+k}\right)\left(1-\frac{z}{y+k}\right)\right] \ => Solution$$

$$^{172}\int_0^{\infty} \frac{e^{-t}\cosh(a\sqrt{t})}{\sqrt{t}}dt \ => Solution$$

$$^{173}\Gamma(n+\frac{1}{2}) = \frac{(2n)!}{4^n\,n!}\sqrt{\pi} \ => solution$$

$$^{174}\Gamma(x)\Gamma(1-x) = \frac{\pi}{\sin(\pi x)} \ => Solution$$

$$^{175}\left(-\frac{1}{2}\right)! = \sqrt{\pi} \ => Solution$$

[167] Proof and Usage of DI(Differentiation Integration) Method
[168] Everything is possible in the realm of complex numbers
[169] Feynman's Technique is never obvious
[170] Proof of beta-gamma function using Laplace Tranform and convolution Integral
[171] A simple problem involving Gauss Representation of Gamma Function
[172] This is the best use of Legendre's Duplication Formula
[173] A common sense proof of Legendre's Duplication formula
[174] Proving the Euler's Reflection using Sine Product Formula
[175] Finding (-1/2)! without gaussian integral

[176] $$\frac{2.2}{1.3}.\frac{4.4}{3.5}.\frac{6.6}{5.7}.\frac{8.8}{7.9}.... = \frac{\pi}{2} \quad => \textit{Solution}$$

[177] $$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + ... = \frac{\pi^2}{6} \quad => \textit{Solution}$$

[178] $$\frac{\sin(\pi x)}{\pi x} = \prod_{n=1}^{\infty} \left(1 - \frac{x^2}{n^2}\right) \quad => \textit{Solution}$$

[179] $$\int_0^{\frac{\pi}{2}} \ln\left(\sqrt{\sin(x)} + \sqrt{\cos(x)}\right) dx$$

[180] $$\psi\left(\frac{1}{2}\right) = -\gamma - 2\ln(2) \quad => \textit{Solution}$$

[181] $$\int_0^{\infty}\int_0^{\infty} \frac{\tan^{-1}(x^2)\tan^{-1}(y^4)}{x^2y^3} dxdy \quad => \textit{Solution}$$

[182] $$\int_0^1\int_0^1\int_0^1 \ln\left(\frac{1}{1+xyz} + \frac{1}{1-xyz}\right) dxdydz \quad => \textit{Solution}$$

[183] $$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(\frac{1}{1+x^2+y^2}\right)^n dxdy, n\epsilon N, n > 1 \quad => \textit{Solution}$$

[176] Proving Wallis Product using Sine Product Formula
[177] Finding Reimann zeta function of 2 using Sine product formula
[178] Proving the Sine Product Formula using Digamma Function
[179] A symmetric Integral
[180] Finding the value of digamma(1/2)
[181] A simple problem for practice
[182] A bonus assignment problem from my mentor
[183] This is the best use of polar coordinates

[184] $$\int_0^1 \frac{z^n}{(1-z)^{\frac{1}{2}}}dz = 2.\frac{(2n)!!}{(2n+1)!!} \quad => \mathit{Solution}$$

[185] $$\psi\left(\frac{1}{2}\right) = -\gamma - 2\ln(2) \quad => \mathit{Solution}$$

[186] $$\arcsin(x) = \sum_{n=0}^{\infty} \frac{1}{4^n}\binom{2n}{n}\frac{x^{2n+1}}{2n+1} \quad => \mathit{Solution}$$

$$\arccos(x) = \frac{\pi}{2} - \sum_{n=0}^{\infty} \frac{1}{4^n}\binom{2n}{n}\frac{x^{2n+1}}{2n+1} \quad => \mathit{Solution}$$

$$\arctan(x) = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1} \quad => \mathit{Solution}$$

[187] $$\int_0^{\infty} \frac{x^{n-1}}{1+x}dx = \Gamma(1-n)\Gamma(n) = \frac{\pi}{\sin(n\pi)} \quad => \mathit{Solution}$$

[188] $$\int_0^{\infty} \frac{x^{n-1}}{1-x}dx = \psi(1-n) - \psi(n) = \frac{\pi}{\tan(n\pi)} \quad => \mathit{Solution}$$

[189] => Taylor series in multivariable

[190] $$\lim_{k\to\infty} \frac{\sqrt{k}}{e^k}e^{\int_0^{\infty}\lfloor ke^{-x}\rfloor dx} \quad => \mathit{Solution}$$

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[184] When can double factorial be helpful?

[185] Finding digamma (1/2) without using Legendre's Duplication Formula

[186] Taylor expansion of $\arcsin(x), \arccos(x) and \arctan(x)$

[187] A sad story of two brothers

[188] A sad story of two brothers-II

[189] There's something important to look at

[190] Can you see the factorial in the problem? If yes, this problem is for you

$$^{191}x^{x^3}=729||2^x+x=5||2^x=32x||^2x=16||3^x=x^9||3^x+3^y+3^z=333 \quad => Solution$$

$$^{192}\Gamma(1+x)=1+\frac{(-\gamma)}{1!}x+\frac{(\gamma^2+\zeta(2))}{2!}x^2+O(x^3) \quad => Solution$$

$$^{193}\int_0^\infty \frac{tan^{-1}(x^2)}{1+x^2}+\frac{1}{2}\int_0^\infty \frac{tan^{-1}(4x^2)}{1+4x^2}+\frac{1}{3}\int_0^\infty \frac{tan^{-1}(9x^2)}{1+9x^2}+..=\frac{\pi^4}{48} \quad => Solution$$

$$^{194}\int_0^1 \left(\frac{1}{\log(x)}+\frac{1}{1-x}\right)dx=\gamma \quad => Solution$$

$$^{195}\int_0^\infty \frac{\log(x)}{e^x}dx \quad => Solution$$

$$^{196}\zeta(2)=\frac{\pi^2}{6}\,\zeta(4)=\frac{\pi^4}{90}\,\frac{\sin(\pi x)}{\pi x}=\prod_{n=1}^\infty\left(1-\frac{x^2}{n^2}\right) \quad => Solution$$

$$^{197}\sum_{n=2}^\infty(\zeta(n)-1)=1\;\sum_{n=1}^\infty(\zeta(2n)-1)=\frac{3}{4}\;\sum_{n=1}^\infty(\zeta(2n+1)-1)=\frac{1}{4} \quad => Solution$$

$$^{198}\sum_{n=2}^\infty\frac{\zeta(n)-1}{n}=1-\gamma$$

[191] Five fake olympiad problems on youtube and a nice problem
[192] Series Expansion for Gamma Function
[193] A nice Problem from Romanian Mathematical Magazine
[194] A beautiful integral for the Euler-Mascheroni Constant
[195] For the love of e
[196] How euler found zeta(2) and zeta(4) from Sine product formula?
[197] Some fun manipulations on zeta function
[198] Something that involves infinite series of digamma and euler-mascheroni constant

[199] $$\ln(x)=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n}||Li_s(z)=\sum_{n=1}^{\infty}\frac{z^n}{n^s}||\zeta(s)=\sum_{n=0}^{\infty}\frac{1}{n^s}||Li_2(z)=Di{-}Logarithm\ \ => Solution$$

[200] $$Li_s(z)=\sum_{n=1}^{\infty}\frac{z^n}{n^s}||Li_s(z)=\frac{1}{\Gamma(s)}\int_0^{\infty}\frac{t^{s-1}}{\frac{e^t}{z}-1}dt\ \ => Solution$$

[201] $$z\frac{\partial(Li_s(z)}{\partial z}=Li_{s-1}(z)||\int_0^z\frac{Li_s(z)}{z}dz=Li_{s+1}(z)\ \ => Solution$$

[202] $$Li_s(z)+Li_s(-z)=2^{1-s}Li_s(z^2)\ \ => Solution$$

[203] $$\int\frac{tanh^{-1}(x)}{x}dx=\frac{Li_2(x)-Li_2(-x)}{2}||\int_0^1 Li_2(\sqrt{x})dx=\zeta(2)-\frac{3}{4}\ \ => Solution$$

[204] $$\int_0^1\frac{1-e^{-x}}{x}dx-\int_1^{\infty}\frac{e^{-x}}{x}dx=\gamma\ \ => Solution$$

[205] $$\frac{Li_s(x)-Li_s(-x)}{2}=\chi_s(x)\ \ => Solution$$

[206] $$\int_0^2\frac{\ln(1+x)}{x^2-x+1}dx\ \ => Solution$$

[199] A sweet introduction to polylogarithm
[200] Sum and Integral Representation of Polylogarithm
[201] Properties of Polylogarithm under Differentiation and Integration
[202] Reflection Property of Polylogarithm and some more ideas
[203] Two integrals involving Di-logarithm function
[204] Gauss loves the definition of e
[205] An introduction to Legendre's Chi function
[206] The u-substitution in this problem is unbelievable . Credit: @nicogehren6556

[207] $$\lim_{n\to\infty}\left(\frac{\zeta(2)}{\Gamma(n-2)}+\frac{\zeta(3)}{\Gamma(n-3)}+.......+\frac{\zeta(n-1)}{\Gamma(1)}\right) \; => Solution$$

[208] $$\int_0^\infty \frac{\ln(x)}{e^x}dx=-\gamma \; => Solution$$

[209] $$\psi(z)=\int_0^1\left(\frac{-1}{\log(t)}-\frac{t^{z-1}}{1-t}\right)dt \; => Solution$$

[210] $$\psi(z)=\int_0^\infty\left(\frac{e^{-t}}{t}-\frac{e^{-zt}}{1-e^{-t}}\right)dt \; => Solution$$

[211] $$\psi(z)=\int_0^\infty\left(\frac{e^{-t}}{t}-\frac{(1+t)^{-z}}{t}\right)dt \; => Solution$$

[212] $$\int_0^1 \frac{(1-x^a)(1-x^b)(1-x^c)}{(1-x)(-\log(x))}dx=\log\left[\frac{\Gamma(a+b+1)\Gamma(b+c+1)\Gamma(c+a+1)}{\Gamma(a+1)\Gamma(b+1)\Gamma(c+1)\Gamma(a+b+c+1)}\right] \; => Solution$$

[213] $$\int_0^\infty\left(e^{-bx}-\frac{1}{1+ax}\right)\frac{dx}{x} \; => Solution$$

[207] Limit Problem by Cornel Loan Valean on American Mathematical Monthly

[208] Solving this integral for euler mascheroni constant without digamma function Suggestional Credit: @sigmapoint8333

[209] second representation for digamma function credit: Advanced Integration Techniques by Zaid Alyafeai

[210] third representation for digamma function credit: Advanced Integration Techniques by Zaid Alyafeai

[211] fourth representation of digamma function credit: Advanced Integration Techniques by Zaid Alyafeai

[212] Big but easy integral Credit:Advanced Integration Techniques by Zaid Alyafeai

[213] This integral invokes the fourth integral representation for Di-gamma function Credit:Advanced Integration Techniques by Zaid Alyafeai

[214] $$\int_{-\frac{\pi}{4}}^{0} \prod_{n=0}^{\infty} (1+\tan^{2^n}(x))dx = \frac{\ln(2)}{4} + \frac{\pi}{8} \ \ => Solution$$

[215] $$\int_0^\infty e^{-x}dx || \int_0^\infty e^{-x^2}dx || \int_0^\infty e^{-x^3}dx || \int_0^\infty e^{-x^4}dx || \int_0^\infty e^{-x^5}dx || \int_0^\infty e^{-x^6}dx || \int_0^\infty e^{-x^7}dx ||$$
$$\int_0^\infty e^{-x^8}dx || \int_0^\infty e^{-x^9}dx || \int_0^\infty e^{-x^{10}}dx \ \ => Solution$$

[216] $$\lim_{n\to\infty} n! = \sqrt{2\pi n}\left(\frac{n}{e}\right)^n \ \ => Solution$$

[217] $$\lim_{n\to\infty} \Gamma(\frac{1}{n}) = n - \gamma \ \ => Solution$$

// To be proved (1 proved)

[218] $$\sum_{n=1}^{\infty} \left(\frac{H_n}{n}\right)^2$$

[219] $$\sum_{n=1}^{\infty} \left(\frac{H_n}{n}\right)^3$$

[220] $$\int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2} || \int_0^\infty e^{-ax^2}dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

[214] An easy problem from Maths Stack Exchange Credit: Guillermo Garc
[215] Let's solve all of these
[216] Proof of Stirling's approximation
[217] Proof of the approximation for $\Gamma(\frac{1}{n})$
[218] Proof of the quadratic series of Au-Yeung
[219] Proof of the quadratic series of Au-Yeung
[220] Evaluating the variant of the Integral

$$\int_0^\infty e^{-(\ln(x))^2}dx = \sqrt[4]{e}\sqrt{\pi}||\int_0^\infty e^{-(W(x))^2}dx = e^{\frac{1}{4}}\left(\frac{3\sqrt{\pi}}{4}+\frac{e^{-\frac{1}{4}}}{2}+\frac{3\sqrt{\pi}}{4}erf(\frac{-1}{2})\right) = 3.0953$$

$$\int_0^\infty e^{-(H_x)^2}dx =$$

$$\int_0^\infty e^{-\Gamma(x)^2}dx = 0.717||\int_0^\infty e^{-\psi(x)^2}dx||\int_0^\infty e^{-Li_2(x)^2}dx$$

$$\int_0^\infty e^{-erf(x)^2}dx = DNC||\int_0^\infty e^{erfc(x)^2}dx = DNC||\int_0^\infty e^{-erfi(x)^2}dx = 0.728473$$

$$\int_1^\infty e^{-\zeta(x)^2}dx = DNC||\int_0^\infty e^{-\eta(x)^2}dx||\int_0^\infty e^{-\mathcal{L}(x)^2}dx = DNC$$

$$\int_0^\infty e^{-(\arcsin(x))^2}dx = DNC||\int_0^\infty e^{-(\arccos(x))^2}dx = DNC||||\int_0^\infty e^{-(\arctan(x))^2}dx = DNC$$

$$\int_0^1 e^{-(\arcsin(x))^2}dx = \frac{\sqrt{\pi}e^{-\frac{1}{4}}}{4}\left(erfc(\frac{i}{2})+erfc(\frac{-i}{2})+ierfi(\frac{1}{2}-\frac{i\pi}{2})-ierfi(\frac{1}{2}+\frac{i\pi}{2})-2\right)$$

$$\int_0^1 e^{-(\arccos(x))^2}dx = \frac{\sqrt{\pi}}{4e^{\frac{1}{4}}}\left(2erfi(\frac{1}{2})-erfi(\frac{1}{2}-\frac{i\pi}{2})-erfi(\frac{1}{2}+\frac{i\pi}{2})\right)$$

$$\int_0^1 e^{-(\arctan(x))^2}dx = DNC$$

$$\int_0^\infty e^{-\sin^2(x)}\int_0^\infty e^{-\cos^2(x)}\int_0^\infty e^{-\tan^2(x)}\int_0^\infty e^{-\csc^2(x)}\int_0^\infty e^{-\sec^2(x)}\int_0^\infty e^{-\cot^2(x)}$$

$$\int_0^{\frac{\pi}{2}} e^{-\sin^2(x)} = \frac{\pi}{2\sqrt{e}}I_0(\frac{1}{2})\int_0^{\frac{\pi}{2}} e^{-\cos^2(x)} = \frac{\pi}{2\sqrt{e}}I_0(\frac{1}{2})\int_0^{\frac{\pi}{2}} e^{-\tan^2(x)} = \frac{e\pi}{2}erfc(1)$$

$$\int_0^{\frac{\pi}{2}} e^{-\csc^2(x)} = \frac{\pi}{2} erfc(1) \int_0^{\frac{\pi}{2}} e^{-\sec^2(x)} = \frac{\pi}{2} erfc(1) \int_0^{\frac{\pi}{2}} e^{-\cot^2(x)} = \frac{e\pi}{2} erfc(1)$$

$$\int_0^{\infty} e^{-(arcsinh(x))^2} dx = DNC || \int_0^{\infty} e^{-(arccosh(x))^2} dx = DNC |||| \int_0^{\infty} e^{-(arctanh(x))^2} dx = DNC$$

$$\int_0^{1} e^{-(arcsinh(x))^2} dx = \frac{\sqrt{\pi}}{2} e^{\frac{1}{4}}$$

$$\int_0^{1} e^{-(arccosh(x))^2} dx = \frac{\sqrt{\pi}}{4} e^{\frac{1}{4}} \left[erf(\frac{1}{2} - \frac{i\pi}{2}) + erf(\frac{1}{2} + \frac{i\pi}{2}) \right]$$

$$\int_0^{1} e^{-(arctanh(x))^2} dx = DNC$$

$$\int_0^{\infty} e^{-\sinh^2(x)} \int_0^{\infty} e^{-\cosh^2(x)} \int_0^{\infty} e^{-\tanh^2(x)} \int_0^{\infty} e^{-csch^2(x)} \int_0^{\infty} e^{-sech^2(x)} \int_0^{\infty} e^{-coth^2(x)}$$

$$\int_0^{1} e^{-\sinh^2(x)} \int_0^{1} e^{-\cosh^2(x)} \int_0^{1} e^{-\tanh^2(x)} = \int_0^{1} e^{-csch^2(x)} \int_0^{1} e^{-sech^2(x)} \int_0^{1} e^{-coth^2(x)}$$

[221] $$\int_0^{\infty} e^{-x^2} x^n dx = \frac{1}{2} \Gamma \left(\frac{n+1}{2} \right) \quad => Solution$$

$$\int_0^{\infty} e^{-x^2} \cos(ax) dx = \frac{\sqrt{\pi}}{2} e^{\frac{-a^2}{4}} \quad => Solution$$

$$\int_0^{\infty} e^{-x^2} \sin(ax) dx = \frac{\sqrt{\pi}}{2} e^{\frac{-a^2}{4}} erfi(\frac{a}{2}) \quad => Solution$$

$$\int_0^{\infty} e^{-x^2} \ln(x) dx = -\frac{\sqrt{\pi}}{4} (\gamma + \ln(4)) \quad => Solution$$

[221] Few Gauss-like Integrals

$$\int_0^\infty e^{-x^2}\cosh(ax)dx = \frac{\sqrt{\pi}}{2}e^{\frac{a^2}{4}} \quad => Solution$$

$$\int_0^\infty e^{-x^2}\sinh(ax)dx = \frac{\sqrt{\pi}}{2}e^{\frac{a^2}{4}}erf(\frac{a}{2}) \quad => Solution$$

$$\int_0^\infty e^{-x^2}erf(ax)dx = \frac{\arctan(a)}{\sqrt{\pi}} \quad => Solution$$

$$\int_0^\infty e^{-x^2}erfc(ax)dx = \frac{\arctan(\frac{1}{a})}{\sqrt{\pi}} \quad => Solution$$

[222] $$\sum_{n=1}^{\infty}\frac{H_n}{n^q} = \frac{(q+2)\zeta(q+1)}{2} - \frac{1}{2}\sum_{k=1}^{q-2}\zeta(k+1)\zeta(q-k)$$

[223] $$\int_0^\infty x^2\frac{\sin(x)}{\sinh(x)}dx \quad => Solution$$

[224] $$Li_2\left(\frac{x}{1-y}\right)+Li_2\left(\frac{y}{1-x}\right)-Li_2\left(\frac{xy}{(1-x)(1-y)}\right) = Li_2(x)+Li_2(y)+\ln(1-x)\ln(1-y)$$

[225] $$\sum_{k=1}^{n}a_kb_k = b_{n+1}A_n - \sum_{k=1}^{n}(b_{k+1}-b_k)A_k$$

$$\text{where } A_x = \sum_{i=1}^{x}a_i$$

[222] Proof of the Classical Euler Sum
[223] Integral by @sigmapoint8333
[224] Abel's Identity for dilogarithm
[225] Abel Summation

// To be proved

$$^{226}\int_0^\infty e^{-ax}\left(\frac{1}{x}-\coth(x)\right)dx \;\; => Solution$$

$$^{227}\frac{t}{e^t-1}=\sum_{k=0}^{\infty}\frac{B_k}{k!}t^k$$

$$B_0=1, B_1=\frac{-1}{2}, B_2=\frac{1}{6}, B_3=0, B_4=\frac{-1}{30}, B_5=0, B_6=\frac{1}{42}, B_7=0, B_8=\frac{-1}{30}, B_9=0, B_{10}=\frac{5}{66}$$

$$B_{11}=0, B_{12}=\frac{-691}{2730}, B_{13}=0, B_{14}=\frac{7}{6}, B_{15}=0 \;\; => Solution$$

[228] Prove that odd Bernoulli numbers are zero.

$$B_k : k \geq 3;\ \text{k is odd} = 0\ Credit : Arizona, planetmath.org \;\; => Solution$$

$$^{229}B_0=1; \sum_{k=0}^{n}\binom{n+1}{k}B_k=0 \text{ for } n>0 \;\; => Solution$$

$$^{230}1+2+....+n=\frac{n(n+1)}{2}$$

$$1^2+2^2+.......+n^2=\frac{n(n+1)(2n+1)}{6}$$

$$1^3+2^3+.......+n^3=\frac{n^2(n+1)^2}{4}$$

$$1^4+2^4+.......+n^4=\frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

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[226] This involves the definition of digamma function Credit:Advanced Integration Techniques by Zaid Alyafeai

[227] How do you find the Bernoulli numbers?

[228] Come. let me show you the beauty of mathematics

[229] Proof of this interesting result

[230] the most fascinating use of Bernoulli numbers

$$1^5 + 2^5 + ... + n^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$$

$$1^6 + 2^6 + ... + n^6 = \frac{n(n+1)(2n+1)(3n^4+6n^3-3n+1)}{42}$$

$$1^k + 2^k + ... + n^k = ? \ => Solution$$

[231] $$\sum_{m=1}^{\infty}\sum_{n=1}^{m} x^m \overline{H_n} \ => Solution$$

[232] $$Li_2(z) \ => Solution$$

[233] $$Li_2(z) + Li_2(-z) = \frac{1}{2}Li_2(z^2) \ => Solution$$

[234] $$Li_2(z) + Li_2(1-z) = \zeta(2) - \ln(z)\ln(1-z) \ => Solution$$

[235] $$Li_2(-z) + Li_2(\frac{z}{1+z}) = -\frac{1}{2}\log^2(z+1) \ => Solution$$

[236] $$Li_2(z) + Li_2(\frac{1}{z}) = -\zeta(2) - \frac{1}{2}\log^2(-z) \ => Solution$$

[237] $$\int_0^1 \frac{\log(1-x)\log(x)}{x}dx \ => Solution$$

[238] $$\int_0^1 \frac{x\sqrt{x}\ln(x)}{x^2-x+1}dx||\psi'(x) \ => Solution$$

[231] Interchanging sum is really beneficial in such cases Credit: An Introduction To The Harmonic Series And Logarithmic Integrals For High School Students Up To Researchers by Ali Shadhar Olaikhan

[232] Some proofs related to di-logarithm function (also known as Spencer's function) Credit: Maths Stack Exchange , Felix Marin , N3buchadnezzar, Raymond Manzoni

[233] Double Identity

[234] Euler's Reflection Formula

[235] Landen's Identity

[236] Inversion Formula

[237] A cute integral for Apery's constant Credit:Advanced Integration Techniques by Zaid Alyafeai

[238] Let's invoke the tri-gamma function Credit: mathematical reflections, awesomemath.org

[239] $$\int_a^b\int_a^b\left(\frac{x}{x^4+y^2}+\frac{y}{y^4+x^2}\right)dxdy\leq \ln^2\left(\frac{b}{a}\right)\ \ =>Solution$$

[240] $$\int_0^\infty x^2\frac{\sin(x)}{\sinh(x)}dx=\frac{\pi^3}{4}\tanh(\frac{\pi}{2})\text{sech}^2(\frac{\pi}{2})\ \ =>Solution$$

[241] $$\int_0^1 \arcsin(x)\ln(x)dx=2-\frac{\pi}{2}-\ln(2)\ \ =>Solution$$

[242] $$\frac{d}{d(x)}\left(\beta(x,k)\right)=\beta(x,k)(\psi(x)-\psi(x+k))\ \ =>Solution$$

[243] How to write matrix A as PDP^{-1} where D is Diagonal Matrix and P some other matrix

$$\begin{bmatrix}2&2\\2&2\end{bmatrix}=\begin{bmatrix}1&1\\-1&1\end{bmatrix}\begin{bmatrix}0&0\\0&4\end{bmatrix}\begin{bmatrix}1&1\\-1&1\end{bmatrix}^{-1}\ \ =>Solution$$

[244] If D be a digonal matrix $\begin{bmatrix}a&0\\0&b\end{bmatrix}$, and matrix $A=PDP^{-1}$ for some matrix P , then

$$i)A^n=PD^nP^{-1}=P\begin{bmatrix}a^n&0\\0&b^n\end{bmatrix}P^{-1}$$

$$ii)e^A=Pe^DP^{-1}=P\begin{bmatrix}e^a&0\\0&e^b\end{bmatrix}P^{-1}$$

$$iii)\ln(A)=P\ln(D)P^{-1}=P\begin{bmatrix}\ln(a)&0\\0&\ln(b)\end{bmatrix}P^{-1}\ \ =>Solution$$

[239] An easy inequality from Romanian Mathematical Magazine Credit: Daniel Sitaru

[240] An astonishing integral as a tribute to quad-gamma function Problem Credit: @sigma8333 Solution Credit: Ankush Kumar Parcha

[241] You will find this integral so cool that you will suffer from cold

[242] Did you know about this stuff

[243] Diagonalising a matrix

[244] Some property of Matrix Algebra

$$^{245}\sqrt{\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}} \quad => Solution$$

$$^{246}e^{\begin{bmatrix} 0 & -\pi \\ \pi & 0 \end{bmatrix}} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad => Solution$$

$$^{247}\ln \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \frac{\pi}{2} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad => Solution$$

$$^{248}W(\begin{bmatrix} 0 & \pi \\ -\pi & o \end{bmatrix}) \quad => Solution$$

$$^{249}\sqrt{\begin{bmatrix} 1 & 3 & -3 \\ 0 & 4 & 5 \\ 0 & 0 & 9 \end{bmatrix} \quad => Solution}$$

$$^{250}\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}^{35} \quad => Solution$$

$$^{251}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}^{\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}} \quad => Solution$$

[245] Square root of a Matrix

[246] Exponential of a matrix and some cool insights —— Is this just a Miracle? Did this happen by chance

[247] Logarithm of a Matrix —— Is this just a coincidence??

[248] Lambert W of a matrix —— This is the most crazy thing on earth

[249] Berkeley Qualifying Exam Question, University of California

[250] Let's do this under few second

[251] Matrix raised to a Matrix

[252] $$\sqrt[\begin{bmatrix}2 & -1\\ -3 & 2\end{bmatrix}]{\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}} || \begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}^{\frac{1}{\begin{bmatrix}2 & -1\\ -3 & 2\end{bmatrix}}} \quad => Solution$$

[253] $$\zeta(2)=\frac{\pi^2}{6}||\zeta(4)=\frac{\pi^4}{90}||\zeta(6)=\frac{\pi^6}{945}||\zeta(8)=\frac{\pi^8}{9450}||\zeta(10)=\frac{\pi^{10}}{93555}$$

$$\zeta(12)||\zeta(14)||\zeta(16)||\zeta(18)||\zeta(20)||\zeta(22)||\zeta(24)||\zeta(26)||\zeta(28)||\zeta(30) \quad => Solution$$

[254] $$\zeta(s,a)=\frac{\psi_{s-1}(a)}{(-1)^s(s-1)!} \quad => Solution$$

[255] $$\int_0^x \frac{\ln^2(1-t)}{t}dt, 0<x<1 \quad => Solution$$

[256] $$\int_0^1\int_0^1\int_0^1 \frac{x^2y^2z^2\ln(xyz)}{1-x^2y^2z^2}dxdydz=-\frac{\pi^4}{32}+3 \quad => Solution$$

[257] $$\int_0^\infty \frac{x}{e^x-1}dx||\int_0^a \frac{x}{e^x-1}dx \quad => Solution$$

[258] $${}_2F_1(a,b;c;z)=\sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(c)_n}\frac{z^n}{n!} \quad => Solution$$

[252] Matrixth root of a Matrix
[253] Finding all of these zeta values
[254] Relation between Hurwitz zeta and poly-gamma function
[255] Let's get adapted with Di-logarithm
[256] Monstrous but easy integral
[257] Everyone can solve the first one. Can you solve the second one?
[258] A short introduction to hyper geometric function

where (k)$_n = k(k+1)..(k+n-1)$

[259]Logarithm $\ln(1+z) =_2 F_1(1,1;2;-z)z$ => *Solution*
Power Function $(1-z)^{-a} =_2 F_1(a,1;1;z)$ => *Solution*
Arcsin Function $\arcsin(x) =_2 F_1(\frac{1}{2},\frac{1}{2};\frac{3}{2};z^2)z$ => *Solution*
Geometric Series $(1-z)^{-1} =_2 F_1(1,1;1;z)$ => *Solution*
Exponential Function $e^z =_2 F_1(-,-;-;z)$ => *Solution*
Sine Function $\sin(z) =_2 F_1(-,-;\frac{3}{2};\frac{-z^2}{4})z$ => *Solution*
Cosine Function $\cos(z) =_2 F_1(-,-;\frac{1}{2};\frac{-z^2}{4})$ => *Solution*

[260] $\int_0^1 x^{-\frac{1}{2}}(1-x)^{-\frac{1}{4}}dx || \int x^{-\frac{1}{2}}(1-x)^{-\frac{1}{4}}dx$ => *Solution*

[261] $\beta(c-b,b)_2F_1(a,b;c;z) = \int_0^1 t^{b-1}(1-t)^{c-b-1}(1-tz)^{-a}dt$ => *Solution*

[262] ${}_2F_1(a,b;c;z) = (1-z)^{-a}\ {}_2F_1(a,c-b;c;\frac{z}{z-1}) = (1-z)^{-b}\ {}_2F_1(c-a,b;c;\frac{z}{z-1})$ => *Solution*

[263] ${}_2F_1(a,b;c;z) = (1-z)^{c-a-b}\ {}_2F_1(c-a,c-b;c;z)$ => *Solution*

[264] ${}_2F_1(a,b;c;1) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}$ => *Solution*

[265] ${}_2F_1(a,b;1+a-b;-1) = \frac{\Gamma(1+a-b)\Gamma(1+\frac{a}{2})}{\Gamma(1+\frac{a}{2}-b)\Gamma(1+a)}$ => *Solution*

[259]Representation of some famous functions using hyoer-geometric functions
[260]You can solve the first integral. But can you solve the second integral? An application of Hyper-Geometric Function
[261]Proving the Integral Representation for Hyper-Geometric function
[262]Proof of the Pfaff tranformation of Hyper-geometric function
[263]Proof of the Euler Transformation of Hyper-geometric function
[264]Some Special Values of Hyper-geometric function at 1
[265]Some special values of Hyper-geometric function at -1

$$^{266}\frac{\sqrt{\pi}}{2}erf(x)=\int_0^x e^{-t^2}dt||\frac{\sqrt{\pi}}{2}erfc(x)=\int_x^\infty e^{-t^2}dt||\frac{\sqrt{\pi}}{2}erfi(x)=\int_0^x e^{t^2}dt\ \ => Solution$$

$$^{267}erf(x)=\frac{2x}{\sqrt{\pi}}\,{}_2F_1\left(-,\frac{1}{2};\frac{3}{2};-x^2\right)||erf(x)=1-\frac{\Gamma(\frac{1}{2},x^2)}{\sqrt{\pi}}\ \ => Solution$$

$$^{268}\int_0^\infty \sin(x^2)erfc(x)dx=\frac{\pi-2\coth^{-1}(\sqrt{2})}{4\sqrt{2}\pi}\ \ => Solution$$

$$^{269}\int_0^\infty erfc(x)e^{-2x^2}dx\ \ => Solution$$

$$^{270}\int_0^\infty erfc(x)dx||\int_0^\infty erfc^2(x)dx||\int_0^\infty erfc^3(x)dx.....\ \ => Solution$$

$$^{271}\int_0^\infty e^{-\ln^2(x)}dx=\sqrt[4]{e}\sqrt{\pi}\ \ => Solution$$

$$^{272}\int_0^\infty e^{-W(x)^2}dx=e^{\frac{1}{4}}\left[\frac{3\sqrt{\pi}}{4}+\frac{e^{\frac{-1}{4}}}{2}-\frac{3\sqrt{\pi}}{4}erf\left(\frac{-1}{2}\right)\right]\ \ => Solution$$

$$^{273}E(z)=\int_z^\infty \frac{e^{-t}}{t}dt=\int_1^\infty \frac{e^{-zt}}{t}dt\ \ => Solution$$

[266] Introduction and relation between these function
[267] Relation of error function with hypergeometric functions and incomplete beta function
[268] Trig+error=Trigger function
[269] A somehow unpopular technique
[270] How far can we go?
[271] Modified Gaussian Integral-I
[272] Modified Gaussian Integral - II
[273] Sum and Integral Representation for Exponential Integral Function

$$E(z) = -\gamma - \ln(z) + \int_0^z \frac{1-e^{-u}}{u} du || E(z) = -\gamma - \ln(z) + \sum_{k=1}^{\infty} \frac{(-1)^{k+1} z^k}{k!k} \quad => Solution$$

[274] $$\lim_{x \to 0} [\log(x) + E(x)] = -\gamma \quad => Solution$$

[275] $$\int_0^{\infty} x^{p-1} E(ax) dx = \frac{\Gamma(p)}{pa^p} \quad => Solution$$

[276] $$\int_0^{\infty} x^{p-1} e^{ax} E(ax) dx = \frac{\pi}{\sin(p\pi)} . \frac{\Gamma(p)}{a^p} \quad => Solution$$

[277] $$\int_0^{\infty} e^z E^2(z) dz = \zeta(2) \quad => Solution$$

[278] $$\int_0^1 \frac{x \ln^2(x)}{x^3 + x\sqrt{x} + 1} dx = \frac{8}{729} \left(\psi''(\frac{7}{9}) - \psi''(\frac{4}{9}) \right) \quad => Solution$$

[279]

4970 : Proposed by Isabel Dıaz-Iriberri and Jose Luis Dıaz-Barrero, Barcelona, Spain.

Let f : [0, 1] → R be a continuous convex function. Prove that:

$$\frac{3}{4} \int_0^{\frac{1}{5}} f(t) dt + \frac{1}{8} \int_0^{\frac{2}{5}} f(t) dt \geq \frac{4}{5} \int_0^{\frac{1}{4}} f(t) dt \quad => Solution$$

[274] A nice limit problem for Exponential Integral Function Credit: Advanced Integration Techniques by Zaid Alyafeai

[275] Try this simple integral involving Exponential Integral Function Credit: Advanced Integration Techniques by Zaid Alyafeai

[276] This is a good problem to review Exponential Integral Function Credit: Advanced Integration Techniques by Zaid Alyafeai

[277] Surprise!!!!

[278] This is a regular standard boring integral, but still I am doing it. Why?

[279] Jensen Inequality problem from School of Science and Math Journal November 2007 (school science and math journal)

[280]

4983 : Proposed by Ovidiu Furdui, Kalamazoo, MI.
Let k be a positive integer. Evaluate:

$$\int_0^1 \left\{\frac{k}{x}\right\} dx \quad => \textit{Solution}$$

where {a} is the fraction part of a.

[281]

4996 : Proposed by Kenneth Korbin, New York, NY
Simplify:

$$\sum_{i=1}^{N} \binom{N}{i} (2^{i-1})(1+3^{N-i}) = \frac{5^N - 1}{2} \quad => \textit{Solution}$$

[282]

- *5006 : Proposed by Ovidiu Furdui, Toledo, OH*
Find the sum :

$$\sum_{k=2}^{\infty} (-1)^k \ln\left(1 - \frac{1}{k^2}\right) = \ln\left(\frac{8}{\pi^2}\right) \quad => \textit{Solution}$$

[283]

- *5068 : Proposed by Kenneth Korbin, New York, NY*
Find the value of

[280] You might enjoy this integral (school science and math journal)
[281] A simple sum from Binomial Expansion (school science and math journal)
[282] Do you see Wallis Product in this sum (school science and math journal)
[283] This problem involves Ramanujan's nested Radical (school science and math journal)

$$\sqrt{1+2009\sqrt{1+2010\sqrt{1+2011\sqrt{1+....}}}} \;\; => Solution$$

[284]

- *5073: Proposed by Ovidiu Furdui, Campia-Turzii, Cluj, Romania.*

Let m > - 1 be a real number. Evaluate:

$$\int_0^1 \{\ln(x)\} x^m dx \;\; => Solution$$

where {a} = a - [a] denotes the fractional part of a.

[285] $$\int_0^1 \{-\ln(x)\} dx$$

[286] $$\int_0^1 \int_0^1 \{y^2 - x\} dx dy = \frac{1}{2} \;\; => Solution$$

[287]

- *5118: Proposed by David E. Manes, Oneonta, NY*

Find the value of :

$$\sqrt{2011+2007\sqrt{2012+2008\sqrt{2013+2009\sqrt{2014+.....}}}} = 2009 \;\; => Solution$$

[288]

$$\sqrt{1^2+\sqrt{2^2+\sqrt{4^2+\sqrt{8^2+\sqrt{16^2+.....}}}}} = 2 \;\; => Solution$$

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[284] Integrals involving fractional part function are so beautiful
[285] The answer is irrational fraction
[286] You have to take care of this integral geometrically
[287] Ramanujan's nested radical returns
[288] Similar ramanujan's problem created by myself

$$\sqrt{1+\sqrt{1+2^2+\sqrt{2+3^2+\sqrt{3+4^2+\sqrt{4+5^2+....}}}}} = 2 \;\; => \textit{Solution}$$

$$\sqrt{1^2+\sqrt{2^2+\sqrt{3^2+\sqrt{4^2+\sqrt{5^2+.....}}}}} = ? \;\; => \textit{Solution}$$

289

- *5139: Proposed by Ovidiu Furdui, Cluj, Romania*

Prove:

$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\zeta(m+n)-1}{m+n} = \gamma, \;\; => \textit{Solution}$$

where ζ denotes the Riemann zeta function.

290

- *5174: Proposed by Jose Luis Dıaz-Barrero, Barcelona, Spain*

Let n be a positive integer. Compute:

$$\lim_{n\to\infty}\frac{n^2}{2^n}\sum_{k=0}^{n}\frac{(k+4)}{(k+1)(k+2)(k+3)}\binom{n}{k} \;\; => \textit{Solution}$$

291

- *5175: Proposed by Ovidiu Furdui, Cluj-Napoca, Romania*

[289] The math never lies
[290] A pretty standard and easy problem to know
[291] This sum involves Riemann Definition of double integral

Find the value of:

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i,j=1}^{n} \frac{i+j}{i^2+j^2} \quad => \textit{Solution}$$

292

- *5181: Proposed by Ovidiu Furdui, Cluj, Romania*
Calculate:

$$\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{n.m}{(m+n)!} \quad => \textit{Solution}$$

293 $$\int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx = \frac{22}{7} - \pi \quad => \textit{Solution}$$

294 $$\text{Area of Circle} = \pi r^2 \quad => \textit{Solution}$$

295 $$\int_0^{\frac{\pi}{2}} \ln(\sin(x)) dx = -\frac{\pi}{2} \ln(2)$$

$$\int \ln(\sin(x)) dx = ? \quad => \textit{Solution}$$

296 $$\int_{-\infty}^{\infty} \binom{n}{x} dx = \sum_{x=0}^{\infty} \binom{n}{x} \quad => \textit{Solution}$$

[292] Feels like using beta function?
[293] Happy Pi day guys
[294] Deriving the area of circle using l'hopital's rule
[295] You can solve the first integral but can you solve the second one?
[296] What an extra-ordinary result?

[297] $$\int_0^\infty e^{-c(y+y^{-1})} y^{-\frac{1}{2}} dy \ \ => Solution$$

[298] $$\int_{-\infty}^\infty \frac{\sin(x-\frac{1}{x})}{x-\frac{1}{x}}(1+\frac{1}{x^2})dx \neq \pi \text{ but } = 2\pi \ \ => Solution$$

[299] $$\int_0^\infty sech^2(x+\tan(x))dx \ \ => Solution$$

[300] $$\int_{-\infty}^\infty \frac{2x^2}{x^4+2x^2+5}dx = \frac{\pi}{\sqrt{\phi}} \ \ => Solution$$

[301] $$\cot(x) = \sum_{k\in z} \frac{1}{x+k\pi} \ \ => Solution$$

$$cosec(x) = \sum_{k\in z} \frac{(-1)^k}{x+k\pi} \ \ => Solution$$

[302] $$\int_0^{\frac{\pi}{2}} e^{-tan^2(x)}dx \ \ => Solution$$

$$special functions \ \ => Solution$$

[303] $$\int_0^{\frac{\pi}{2}} e^{-\cot^2(x)}dx \ \ => Solution$$

[297] This Cambridge integral involves Glasser's Master Theorem
[298] I bet you got this wrong
[299] Grand Glasser's Master theorem is so powerful
[300] What a beautiful answer to have?
[301] Proof of this amazing series
[302] Modified Gaussian Integral- III
[303] Modified Gaussian Integral - IV

304 $$\int_0^{\frac{\pi}{2}} e^{-\sec^2(x)dx} \Rightarrow Solution$$

305 $$\int_0^{\frac{\pi}{2}} e^{-\csc^2(x)}dx \Rightarrow Solution$$

306 $$\int_0^{\frac{\pi}{2}} e^{-\sin^2(x)}dx \Rightarrow Solution$$

307 $$\int_0^{\frac{\pi}{2}} e^{-\cos^2(x)}dx \Rightarrow Solution$$

308 $$\int_0^{1} e^{-\arcsin^2(x)}dx \Rightarrow Solution$$

309 $$\int_0^{1} e^{-\arccos^2(x)}dx \Rightarrow Solution$$

[310] $$K(k) = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-k^2\sin^2(\theta)}} = \int_0^1 \frac{dx}{\sqrt{1-k^2x^2}\sqrt{1-x^2}} \Rightarrow Solution$$

$$E(k) = \int_0^{\frac{\pi}{2}} \sqrt{1-k^2\sin^2(\theta)}d\theta = \int_0^1 \frac{\sqrt{1-k^2x^2}}{\sqrt{1-x^2}}dx \Rightarrow Solution$$

[304] Modified Gaussian Integral - V
[305] Modified Gaussian Integral - VI
[306] Modified Gaussian Integral - VII
[307] Modified Gaussian Integral - VIII
[308] Modified Gaussian Integral - IX
[309] Modified Gaussian Integral - X
[310] An introduction to Complete Elliptic Integral of 1st kind and 2nd kind

[311] $$K(k)=\frac{\pi}{2}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2};1;k^2\right) \quad => Solution$$

$$E(k)=\frac{\pi}{2}\,{}_2F_1\left(\frac{1}{2},-\frac{1}{2};1;k^2\right) \quad => Solution$$

[312] $$\int_0^{\infty} e^{-arcsinh(x)^2}dx=\frac{\sqrt{\pi}}{2}e^{\frac{1}{4}} \quad => Solution$$

[313] $$\int_0^{\infty} e^{-arccosh(x)^2}dx \quad => Solution$$

[314] $$\int_0^{1} K(k)dk=2G \quad => Solution$$

where G is the catalan's constant and K(k) is complete Elliptic Integral of first kind.

[315] $$K\left(\sqrt{\frac{k}{k-1}}\right)=K(\sqrt{k})\sqrt{1-k} \quad => Solution$$

$$E\left(\sqrt{\frac{k}{k-1}}\right)=\frac{E(\sqrt{k})}{\sqrt{1-k}} \quad => Solution$$

[316] $$K\left(\frac{2\sqrt{k}}{1+k}\right)=\frac{1+k}{1-k}K\left(\frac{2\sqrt{-k}}{1-k}\right) \quad => Solution$$

[311] Deriving the Hyper-Geometric Series for Complete Elliptic Integrals of First and second kind using both sum and integral definition of hyper-geometric function

[312] Modified Gaussian Integral - XI // A Beautiful. Crazy Girlfriend of Gaussian Integral

[313] Modified Gaussian Integral - XII

[314] The celebrity return of Catalan's Constant

[315] Proof of the identities involving Complete Elliptic Integral of First and Second Kind

[316] Proof of the identities involving Complete Elliptic Integral of First and Second Kind - II

$$E\left(\frac{2\sqrt{k}}{1+k}\right) = \frac{1-k}{1+k}E\left(\frac{2\sqrt{-k}}{1-k}\right) \quad => Solution$$

$$^{317}K(i) = \frac{1}{4\sqrt{2\pi}}\Gamma^2(\frac{1}{4}) \quad => Solution$$

$$E(i) = \frac{1}{4\sqrt{2\pi}}\Gamma^2(\frac{1}{4}) + \frac{1}{\sqrt{2\pi}}\Gamma^2(\frac{3}{4}) \quad => Solution$$

$$^{318}K(\frac{1}{\sqrt{2}}) = \frac{1}{4\sqrt{\pi}}\Gamma^2(\frac{1}{4}) \quad => Solution$$

$$E(\frac{1}{\sqrt{2}}) = \frac{1}{8\sqrt{\pi}}\Gamma^2(\frac{1}{4}) + \frac{1}{2\sqrt{\pi}}\Gamma^2(\frac{3}{4}) \quad => Solution$$

$$^{319}\frac{d}{dk}(E(k)) = \frac{1}{k}\left[E(k) - K(k)\right] \quad => Solution$$

$$\frac{d}{dk}(K(k)) = \frac{1}{k}\left[\frac{E(k)}{1-k^2} - K(k)\right] \quad => Solution$$

$$^{320}\zeta(0) = -\frac{1}{2} \quad => Solution$$

$$^{321}A = \pi r^2 \quad => Solution$$

[317] Special Values of Complete Elliptic Integal of first and second Kind
[318] Special Values of Complete Elliptic Integral of first and second kind
[319] The Legendary Derivatives of Elliptic Integrals
[320] Come, let me prove my claim
[321] Using green's theorem to derive area of circle formula

$$^{322}\int (x^6+x^3)\sqrt[3]{x^3+2}\,dx \;\; => Solution$$

$$^{323}\int \frac{1}{(1-x^2)\sqrt[4]{2x^2-1}}dx \;\; => Solution$$

$$^{324}\int_a^b \frac{e^{\frac{x}{a}}-e^{\frac{b}{x}}}{x}dx \;\; => Solution$$

$$^{325}\int_0^{\frac{\pi}{2}} \frac{x\cos(x)-\sin(x)}{x^2+\sin^2(x)}dx \;\; => Solution$$

$$^{326}\int_0^{\pi} \frac{1-\cos(nx)}{1-\cos(x)}dx \;\; => Solution$$

$$^{327}\int_{-\pi}^{\pi} \frac{\sin(nx)}{(1+2^x)\sin(x)}dx, n \geq 0 \;\; => Solution$$

$$^{328}\int_0^1 \frac{\ln(1+x)}{1+x^2}dx || \int_0^1 \frac{\ln(x)}{1+x^2}dx \;\; => Solution$$

$$^{329}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{m^2 n}{3^m(n3^m+m3^n)} \;\; => Solution$$

[322] Try this easy Romanian College entrance exam problem
[323] A Clever u-substitution Credit: @mathematician6124
[324] A cutie integral
[325] Guys, I crafted an amazing solution for this integral
[326] The standard approach for this integral is to compute recursively
[327] Integral from 3rd International Mathematics Competition for University Students, 1996
[328] An easy modification of famous PUTNAM problem
[329] PUTNAM 1999 A4 series problem

$$^{330}\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}\sum_{n=0}^{\infty}\frac{1}{k2^n+1} \quad => Solution$$

$$^{331}\lim_{n\to\infty}\left[\frac{1}{n^5}\sum_{h=1}^{n}\sum_{k=1}^{n}(5h^4-18h^2k^2+5k^4)\right] \quad => Solution$$

$$^{332}\int_0^{\frac{\pi}{2}}\frac{\sin(x)}{\sin(x)+\cos(x)}dx||\int_1^2\frac{\ln(x)}{x^2-2x+2}dx \quad => Solution$$

$$^{333}\sin\left(\frac{\pi}{n}\right)\sin\left(\frac{2\pi}{n}\right)......\sin\left(\frac{(n-1)\pi}{n}\right)=\frac{2n}{2^n} \quad => Solution$$

$$^{334}Given:\int_x^1 f(t)dt\geq\frac{1-x^2}{2}\ \ Prove:\int_0^1 f^2(t)dt\geq\frac{1}{3} \quad => Solution$$

$$^{335}S_{p^r,q}=\sum_{k=1}^{\infty}\frac{(H_k^{(p)})^r}{k^q} \quad => Solution$$

$$^{336}\sum_{k=1}^{\infty}H_k^{(p)}x^k=\frac{Li_p(x)}{1-x} \quad => Solution$$

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[330] PUTNAM 2016 B6 Double Summation Problem

[331] PUTNAM 1981 B1 A Simple Sum limit

[332] Use King's Rule and extended King's Rule

[333] Product of Sines is easily proved using Complex Number

[334] Solving an average IMC inequality

[335] A short introduction to Euler Sums Credit: Advanced Integration Techniques by Zaid Alyafeai

[336] Generating function associated with the Harmonic number of order p Credit: Advanced Integration Techniques by Zaid Alyafeai

$$^{337}\sum_{n=1}^{\infty}\frac{H_n}{n^2}=2\zeta(3) \ \ => Solution$$

$$^{338}\sum_{k=1}^{\infty}\frac{H_k}{k^2}x^k=Li_3(x)-Li_3(1-x)+\log(1-x)Li_2(1-x)+\frac{1}{2}\log(x)\log^2(1-x)+\zeta(3) \ \ => Solution$$

$$^{339}\sum_{n=1}^{\infty}\frac{H_n}{n^q}=\left(1+\frac{q}{2}\right)\zeta(q+1)-\frac{1}{2}\sum_{k=1}^{q-2}\zeta(k+1)\zeta(q-k) \ \ => Solution$$

$$^{340}S_{p,q}+S_{q,p}=\zeta(p)\zeta(q)+\zeta(p+q) => S_{p,p}=\frac{1}{2}(\zeta^2(p)+\zeta(2p)) \ \ => Solution$$

$$^{341}M_x(ln(x+1))(s)=\frac{\pi cosec(\pi s)}{s} \ \ => Solution$$

$$^{342}M_x(erfc(x))(s)=\frac{\Gamma(\frac{s+1}{2})}{\sqrt{\pi}s} \ \ => Solution$$

$$^{343}\sum_{n=1}^{\infty}\frac{H_n}{n2^n}=\frac{\zeta(2)}{2} \ \ => Solution$$

[337] Using the integral representation of Harmonic number to solve this elegant sum Credit: Advanced Integration Techniques by Zaid Alyafeai

[338] Can you use the generating function of Harmonic Series to derive this? Credit: Advanced Integration Techniques by Zaid Alyafeai

[339] To be proved, someday in future Credit: Advanced Integration Techniques by Zaid Alyafeai

[340] To be proved, someday in future , symmetricity of Euler sums

[341] Mellin Transform of ln(x+1)

[342] Mellin Transform of erfc(x)

[343] A nice usage of Poly-logarithm

[344] $$\int_0^1 \frac{\log^2(1-x)\log(x)}{x}dx = -\frac{\pi^4}{180} \ \ => Solution$$

[345] $$\int_0^{\infty} e^{-t}\sin(t)\ln(t)\frac{1}{t}dt \ \ => Solution$$

[346] $$\lim_{n\to\infty}\left(\frac{2^{\frac{1}{n}}}{n+1}+\frac{2^{\frac{2}{n}}}{n+\frac{1}{2}}+...+\frac{2^{\frac{n}{n}}}{n+\frac{1}{n}}\right) \ \ => Solution$$

[347] $$\int_0^{\pi} \ln(1-2a\cos(x)+a^2)dx \ \ => Solution$$

[348] continuous $f:[0,1]\to\mathbb{R}$. Find $max\left(\int_0^1 (x^2f(x)-xf^2(x))dx\right)=\frac{1}{16}$ $=> Solution$

[349] $$\int_0^{\frac{1}{2}}\sum_{n=0}^{\infty} {}^{n+3}C_n x^n dx \ \ => Solution$$

[350] $$\zeta(s)=\frac{1}{1^s}+\frac{1}{2^s}+\frac{1}{3^s}+\frac{1}{4^s}+\frac{1}{5^s}+......... \ \ => Solution$$

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[344] Harmonic number and poly-logarithm function might make maniplation easier Credit: Advanced Integration Techniques by Zaid Alyafeai

[345] The ultimate combination of exponential, trigonometric, logarithmic and rational function Credit: Advanced Integration Techniques by Zaid Alyafeai

[346] Love you if you can see Reimann sum here. Problem from Soviet Union University Student Mathematical Olympiad, 1976

[347] Believe me, this problem will be easier using Riemann sum definition of integration

[348] A tricky inequality from 49th W.L. Putnam Mathematical Competition 2006, proposed by Titu Andresscu

[349] The true solution of this problem by @mathematician6124

[350] Do you know the integral representation of all these series?

$$\eta(s) = \frac{1}{1^s} - \frac{1}{2^s} + \frac{1}{3^s} - \frac{1}{4^s} + \frac{1}{5^s} + ... \implies Solution$$

$$\beta(s) = \frac{1}{1^s} - \frac{1}{3^s} + \frac{1}{5^s} - \frac{1}{7^s} + \frac{1}{9^s} + ... \implies Solution$$

$$(1 - \frac{1}{2^s})\zeta(s) = \frac{1}{1^s} + \frac{1}{3^s} + \frac{1}{5^s} + \frac{1}{7^s} + \frac{1}{9^s} + ... \implies Solution$$

[351] $$\int_1^\infty \frac{dt}{\lfloor t \rfloor^3 + 9\lfloor t \rfloor^2 + 26\lfloor t \rfloor + 24} \implies Solution$$

[352] $$\frac{d}{dx}(\frac{1}{x}) \implies Solution$$

[353] $$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{(\frac{1}{4})^{m+n}}{(2m+1)(m+n+1)} \implies Solution$$

[354] $$\int_0^\infty \frac{\sin(x)}{x + \frac{1}{x}} dx = \frac{\pi}{2e} \implies Solution$$

[355] $$\int_0^\infty \frac{e^{-x^2}}{(x^2 + \frac{1}{2})} dx = \pi\sqrt{\frac{e}{2}} erfc(\frac{1}{\sqrt{2}}) \implies Solution$$

[351] Hope this doesnot scare you. Problem credit :poser. • 5575: Proposed by José Luis Díaz-Barrero, Barce, school science and math journal

[352] differentiating geometrically

[353] Problem from Stanford Math Tournament 2024

[354] This integral will help you get matured

[355] The art of introducing double integrals

[356]
$$\int_0^{\infty} \frac{e^{-x^2}}{(x^2+\frac{1}{2})^2} dx = \sqrt{\pi} \quad => Solution$$

[357]
$$\int_0^{\infty} \frac{e^{-x^2}}{(x^2+\frac{1}{2})^3} dx = \pi\sqrt{\frac{e}{2}} erfc(\frac{1}{\sqrt{2}}) + \sqrt{\pi} \quad => Solution$$

[358]
$$\int_{-\pi}^{\pi} \frac{x^2}{1+\sin(x)+\sqrt{1+\sin^2(x)}} dx = \frac{\pi^3}{3} \quad => Solution$$

$$f(x) = \frac{x^3 e^{x^2}}{1-x^2} \quad f^7(0) = ? \quad 12600 \quad => Solution$$

[359]
$$\int_0^1 \frac{\ln(1+x)}{1+x^2} dx \quad \int_0^1 \frac{\ln(1+x^2)}{1+x} dx \quad => Solution$$

[360]
$$\int_0^1 \frac{\ln(1+x)\ln(1+x^2)}{1+x} dx \quad => Solution$$

[361]
$$\frac{(2020)^2}{0!} + \frac{(2021)^2}{1!} + \frac{(2022)^2}{2!} + \frac{(2023)^2}{3!} + \frac{(2024)^2}{4!} + ... \quad => Solution$$

[362]
$$\int_0^1 \frac{\ln^2(1-x)\ln(x)}{x} dx = -\frac{\pi^4}{180} \quad => Solution$$

[356] The art of introducing double integrals
[357] The art of introducing double integrals
[358] Stanford Maths Tournament 2011
[359] Everyone can solve the first one. Can you solve the second one?
[360] Problem proposed by @mathematician6124
[361] Again back with an standard idea
[362] Let's do one more integral involving poly-logarithm

363 $$\int_0^1 x^{x^2} dx \;\; => Solution$$

364 $$\int_0^1 x^{\sqrt{x}} dx \;\; => Solution$$

365 $$\int_0^1 \frac{Li_p(x)Li_q(x)}{x} dx \;\; => Solution$$

366 $$\sum_{n=1}^{k} \frac{1}{n^p} = H_k^{(p)} = \zeta(p) + (-1)^{p-1} \frac{\psi_{p-1}(k+1)}{(p-1)!} \;\; => Solution$$

367 $$S_{p^r,q} = \sum_{k=1}^{\infty} \frac{(H_k^{(p)})^r}{k^q} \;\; => Solution$$

368 $$\sum_{k=1}^{\infty} \frac{H_k^{(p)}}{k^q} + \sum_{k=1}^{\infty} \frac{H_k^{(q)}}{k^p} = \zeta(p)\zeta(q) + \zeta(p+q) \;\; => Solution$$

369 $$\sum_{k=1}^{\infty} \frac{H_k^{(3)}}{k^2} = \frac{11\zeta(5)}{2} - 2\zeta(2)\zeta(3) \;\; => Solution$$

370 $$\mathrm{Si}(z) = \int_0^z \frac{\sin(x)}{x} dx \quad \mathrm{si}(z) = -\int_z^{\infty} \frac{\sin(x)}{x} dx \quad \mathrm{Si}(z) = \mathrm{si}(z) + \frac{\pi}{2} \;\; => Solution$$

[363] Exotic Integral - I
[364] Exotic Integral - I
[365] I am getting s surge of poly-logarithms
[366] Relation between Generalised Harmonic Number and Poly-Gamma function
[367] Integral Representation for r=1
[368] A nice and beautiful symmetric formula
[369] Finding the value of a Euler sum
[370] A short and sweet introduction to Sine Integral function

$$^{371}\operatorname{sinc}(x) = \begin{cases} 1 & \text{if } x = 0, \\ \frac{\sin(x)}{x} & \text{if } x \neq 0 \end{cases} \quad => Solution$$

$$^{372}\frac{d}{dx}\operatorname{Si}(x) = \operatorname{sinc}(x) \qquad \int \operatorname{Si}(x)dx = \cos(x) + x\operatorname{Si}(x) + C \quad => Solution$$

$$^{373}\int_0^\infty \sin(x)\operatorname{si}(x)dx = -\frac{\pi}{4} \quad => Solution$$

$$^{374}\int_0^\infty x^{\alpha-1}\operatorname{si}(x)dx = -\frac{\Gamma(\alpha)}{\alpha}\sin(\frac{\pi\alpha}{2}) \quad => Solution$$

$$^{375}\int_0^\infty e^{-\alpha x}\operatorname{si}(x)dx = -\frac{\arctan(\alpha)}{\alpha} \quad => Solution$$

$$^{376}\int_0^\infty \operatorname{si}(x)\ln(x)dx = \gamma + 1 \quad => Solution$$

$$^{377}\int_0^\infty \operatorname{si}(x)\sin(px)dx \quad => Solution$$

$$^{378}\text{ For a} \neq 1, \int_0^\infty \operatorname{si}(x)\cos(ax)dx = \frac{1}{2a}\ln(\frac{a-1}{a+1}) \quad => Solution$$

[371] Definition of sinc function
[372] Derivative and antiderivative of sine integral function
[373] Doesn't this combination look nice?
[374] Mellin Transform of si(x)
[375] Laplace Tranform of si(x)
[376] Just one more than euler mascheroni constant
[377] An integration of sin and its sister
[378] Not the normal sine function

[379] $$\mathrm{ci}(x) = -\int_x^{\infty} \frac{\cos(t)}{t} dt \quad \mathrm{Cin}(x) = \int_0^x \frac{1-\cos(t)}{t} dt \;\; => \textit{Solution}$$

$$\frac{d}{dx}\mathrm{ci}(x) = \frac{\cos(x)}{x} \quad \int \mathrm{ci}(x) dx = x\mathrm{ci}(x) - \sin(x) + C \;\; => \textit{Solution}$$

[380] $$\lim_{z\to\infty} H_z - \ln(z) = \gamma \qquad \lim_{z\to\infty} (\mathrm{Cin}(z) - \log(z)) = \gamma \;\; => \textit{Solution}$$

[381] $$\mathrm{Cin}(x) = \gamma + \log(x) - \mathrm{ci}(x) \;\; => \textit{Solution}$$

[382] $$\int_0^{\infty} \mathrm{ci}(x)\cos(px)\, dx \;\; => \textit{Solution}$$

[383] $$\int_0^{\infty} \mathrm{ci}(px)\mathrm{ci}(x)\, dx \;\; \& p > 1 \;\; => \textit{Solution}$$

[384] $$\int_0^{\infty} x^{\alpha-1}\mathrm{ci}(x)\, dx = -\frac{\Gamma(\alpha)}{\alpha}\cos\left(\frac{\pi\alpha}{2}\right) \;\; => \textit{Solution}$$

[385] $$\int_0^{\infty} \mathrm{ci}(x)\log(x)\, dx = \frac{\pi}{2} \;\; => \textit{Solution}$$

[379] A short and sweet introduction to Cosine Integral Function
[380] You knew the first one. But did you know the second one?
[381] One formula that connects them all
[382] Integrals involving two brothers
[383] Integrals with two rivals
[384] Mellin Transform of ci function
[385] Sweet trick of Feynman

$$ {}^{386}\int_0^\infty e^{-\alpha x}\mathrm{ci}(x)\,dx = -\frac{1}{2\alpha}\log(1+\alpha^2) \;\; => \textit{Solution} $$

$$ {}^{387}\int_0^\infty \mathrm{si}(qx)\mathrm{ci}(x)dx \;\; => \textit{Solution} $$

$$ {}^{388}\int_0^\infty \frac{\mathrm{ci}(\alpha x)}{x+\beta}dx = -\frac{1}{2}\{\mathrm{si}(\alpha\beta)^2 + \mathrm{ci}(\alpha\beta)^2\} \;\; => \textit{Solution} $$

$$ {}^{389}\mathrm{li}(x) = \int_0^x \frac{dt}{\log(t)}dt \;\; => \textit{Solution} $$

$$ {}^{390}\frac{d}{dx}\mathrm{li}(z) = \frac{1}{\log(z)} \qquad \int \mathrm{li}(z)dz = z\mathrm{li}(z) - \mathrm{Ei}(2\log(z)) \;\; => \textit{Solution} $$

$$ {}^{391}\int_0^1 \mathrm{li}(z)dz = -\log(2) \;\; => \textit{Solution} $$

$$ {}^{392}\int_0^1 x^{p-1}\mathrm{li}(x)dx = -\frac{1}{p}\log(p+1) \;\; => \textit{Solution} $$

$$ {}^{393}\sum_{n=0}^{N-1} cos(n\theta) \qquad \sum_{n=1}^{N} cos((2n-1)\theta) \;\; => \textit{Solution} $$

[386]Laplace Transform of ci function
[387]Combination of Sine Integral Function and Cosine Integral Function
[388]The answer is a cool combination of sine and cosine integral function
[389]A short and cozy introduction to Logarithm Integral Function li(x)
[390]Differential and Integral of Logarithm Integral Function
[391]A mysteriously simple integral
[392]Mellin Transform of Logarithm Integral Function
[393]Such sums become easier with Complex Summation Technique

394 $$\sum_{n=1}^{N} 2^n \sin(n\theta) \;\; => Solution$$

395 $$\sum_{n=0}^{\infty} 4^{-n} \cos(\frac{n\pi}{3}) \;\; => Solution$$

396 $$\sum_{n=0}^{\infty} 2^{-n} \sin(\frac{n\pi}{3}) \;\; => Solution$$

397 $$\sum_{n=0}^{\infty} 2^{-n} \sin(\frac{n\pi}{2}) \;\; => Solution$$

398 $$\int_0^1 \mathrm{li}(\frac{1}{x}) \sin(a \log(x)) dx \;\; => Solution$$

399 $$\int_0^1 \frac{\mathrm{li}(x)}{x} \log^{p-1}(\frac{1}{x}) dx \;\; => Solution$$

400 $$\int_1^{\infty} \mathrm{li}(\frac{1}{x}) \log^{p-1}(x) dx \;\; => Solution$$

401 $$\int_0^1 \mathrm{li}(x) \log(x) dx = \log(2) - \frac{1}{2} \;\; => Solution$$

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[394]Problems involving Complex Summation Technique - I
[395]Problems involving Complex Summation Technique - II
[396]Problems involving Complex Summation Technique - III
[397]Problems involving Complex Summation Technique - IV
[398]The general trick to deal with Logarithm Integral Function
[399]Will Feynman be useful in this integral
[400]Combination of Logarithm and Logarithm integral function
[401]Same same but different

$$^{402}Cl_m(\theta) = \begin{cases} \sum_{k=1}^{\infty} \frac{\sin(k\theta)}{k^m} & \text{if m is even,} \\ \sum_{k=1}^{\infty} \frac{\cos(k\theta)}{k^m} & \text{if m is odd} \end{cases} => Solution$$

$$Sl_m(\theta) = \begin{cases} \sum_{k=1}^{\infty} \frac{\cos(k\theta)}{k^m} & \text{if m is even,} \\ \sum_{k=1}^{\infty} \frac{\sin(k\theta)}{k^m} & \text{if m is odd} \end{cases} => Solution$$

$$Li_m(e^{i\theta}) = \begin{cases} Sl_m(\theta) + iCl_m(\theta) & \text{if m is even,} \\ Cl_m(\theta) + iSl_m(\theta) & \text{if m is odd} \end{cases} => Solution$$

$$\frac{d}{d\theta}(Cl_2(\theta)) = -\log(2\sin(\frac{\theta}{2})) \qquad Cl_2(\theta) = -\int_0^{\theta} \ln|2\sin(\frac{x}{2})|dx \ => Solution$$

$$Cl_2(\theta + 2m\pi) = Cl_2(\theta)||Cl_2(-\theta) = -Cl_2(\theta) \ => Solution$$

$$^{403}Cl_m(2\theta) = 2^{m-1}(Cl_m(\theta) - (-1)^m Cl_m(\pi - \theta))||Cl_2(2\theta) = 2(Cl_2(\theta) - Cl_2(\pi - \theta)) \ => Solution$$

$$^{404}\int_0^{\pi} Cl_m(\theta)d\theta : //youtu.be/lUFB3NDjCiM?si = weYX_g iY8QmwG2C4 => Solution$$

$$^{405} \text{ If m is even, find: } \int_0^{\infty} Cl_m(\theta)e^{-n\theta}d\theta \ => Solution$$

$$^{406}Cl_2(x) = \sum_{k=1}^{\infty} \frac{\sin(kx)}{k^2} \quad ||Cl_2(\theta) = -\int_0^{\theta} \log(2\sin(\frac{x}{2}))dx \quad ||Cl_2(\frac{\pi}{2}) = G \ => Solution$$

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[402] A short introduction to Clausen Function
[403] Reflection formula of Clausen Functions
[404] Integral of Clausen Function
[405] Laplace Transform of Clausen Function
[406] Introduction to Clausen Integral Function

[407] $$Cl_2(\theta) = \sum_{k=1}^{\infty} \frac{\sin(k\theta)}{k^2} = -\int_0^{\theta} \log(2\sin(\frac{x}{2}))dx \ \ => Solution$$

[408] $$Cl_2(2\theta) = 2Cl_2(\theta) - 2Cl_2(\pi - \theta) \ \ => Solution$$

[409] $$\int_0^{2\pi} Cl_2(x)^2 dx = \frac{\pi^5}{90} \ \ => Solution$$

[410] $$\int_0^{\frac{\pi}{2}} x\log(\sin(x))dx = \frac{7}{16}\zeta(3) - \frac{\pi^2}{8}\log(2) \ \ => Solution$$

[411] $$\int_0^{\frac{\pi}{4}} x\cot(x)dx = \frac{1}{8}(\pi\ln(2) + 4G) \ \ => Solution$$

[412] $$\beta(1), \beta(2), \beta(3), \beta(4), \beta(5), \beta(6), \beta(7), \beta(8), \beta(9), \beta(10), \beta(11), \beta(12), \beta(13), \beta(14), \beta(15), ... \ \ =>$$

[413] $$\beta(s) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^s} \ \ => Solution$$

$$\beta(s) = \frac{1}{\Gamma(s)} \int_0^{\infty} \frac{x^{s-1}}{e^{-x} + e^{x}} dx \ \ => Solution$$

[407] Two different derivations for this identity

[408] Derivation of reflection formula of Clausen Integral Function using Sum and Integral Definition

[409] Square of Clausen Integral Function

[410] This is when Clausen Integral Function can be useful

[411] This is the best match to use Clausen Integral Function

[412] Finding all the beta values

[413] Sum and Integral representation of beta function

$$\beta(s)=\frac{1}{\Gamma(s)}\int_0^1\frac{(-\ln(x))^{s-1}}{1+x^2}dx \;\; => Solution$$

[414] $$\zeta(s,\alpha)=\sum_{n=0}^{\infty}\frac{1}{(n+\alpha)^s} \quad \beta(s)=\frac{1}{4^s}[\zeta(s,\frac{1}{4})-\zeta(s,\frac{3}{4})] \;\; => Solution$$

[415] $$\Phi(z,s,\alpha)=\sum_{n=0}^{\infty}\frac{z^n}{(n+\alpha)^s} \quad \beta(s)=2^{-s}\Phi(-1,s,\frac{1}{2}) \;\; => Solution$$

[416] $$Li_p(z)=\sum_{n=1}^{\infty}\frac{z^n}{n^p} \quad \beta(s)=\frac{i}{2}\left(Li_s(-i)-Li_s(i)\right) \;\; => Solution$$

[417] $$\sum_{n=0}^{\infty}\frac{1}{(n+\alpha)^s}=\frac{(-1)^s\psi^{s-1}(\alpha)}{(s-1)!} \quad \beta(s)=\frac{1}{(-4)^s(s-1)!}[\psi^{s-1}(\frac{1}{4})-\psi^{s-1}(\frac{3}{4})] \;\; => Solution$$

[418] $$\int_0^1 cot^{-1}(1-x+x^2)dx=\frac{\pi}{2}-\log(2) \;\; => Solution$$

[419] $$\int_0^{\pi}\frac{x\sin(x)}{1+\cos^2(x)}dx \quad \int_0^{\frac{\pi}{2}}\frac{x\sin(x)}{1+\cos^2(x)}dx \;\; => Solution$$

[420] $$Cl_2\theta=-\sin(\theta)\int_0^1\frac{\log(x)}{x^2-2\cos(\theta)x+1}dx \;\; => Solution$$

[414] Representation of beta function in terms of Hurwitz zeta function
[415] Representation of beta function in terms of Lerch transcendent function
[416] Representation of beta function in terms of Poly-Logarithm Function
[417] Representation of beta function in terms of Poly-gamma function
[418] A cool use of King's Rule
[419] Everyone can do the first integral. Can you do the second one? Solution Credit: @SussySusan-lf6fk
[420] Strange but useful integral representation of Clausen Integral Function

$$^{421}Cl_2(0) = 0 || Cl_2(\pi) = 0 || Cl_2(\frac{\pi}{2}) = G || Cl_2(\frac{-\pi}{2}) = -G || Cl_2(\frac{3\pi}{2}) = -G \quad => Solution$$

$$Cl_2(\frac{2\pi}{3}) = -\frac{1}{6\sqrt{3}}\left(\psi'(\frac{2}{3}) - \psi'(\frac{1}{3})\right) \quad => Solution$$

$$Cl_2(\frac{\pi}{3}) = -\frac{1}{24\sqrt{3}}[-\psi'(\frac{1}{6}) - \psi'(\frac{1}{3}) + \psi'(\frac{2}{3}) + \psi'(\frac{5}{6}) \quad => Solution$$

$$^{422}\int_0^T \frac{e^{\frac{-a}{T-\tau}-\frac{b}{\tau}}}{(T-\tau)^{\frac{1}{2}}\tau^{\frac{3}{2}}}d\tau = \sqrt{\frac{\pi}{bT}}e^{-\frac{1}{T}(\sqrt{a}+\sqrt{b})^2} => Solution$$

$$^{423}\int_0^\infty \sin(t^2)dt = \int_0^\infty \cos(t^2)dt = \frac{\sqrt{\pi}}{2\sqrt{2}} \quad => Solution$$

$$\int_0^\infty \sin(t^2 - \frac{1}{t^2})dt = \int_0^\infty \cos(t^2 - \frac{1}{t^2})dt = \frac{\sqrt{\pi}}{2\sqrt{2}e^2} \quad => Solution$$

$$\int_0^\infty \sin(t^2+\frac{1}{t^2})dt = \frac{\sqrt{\pi}}{2}\sin(\frac{\pi}{4}+2) || \int_0^\infty \cos(t^2+\frac{1}{t^2})dt = \frac{\sqrt{\pi}}{2}\cos(\frac{\pi}{4}+2) \quad => Solution$$

$$\int_0^\infty e^{-pt^2-\frac{q}{t^2}}dt = \frac{1}{2}\sqrt{\frac{\pi}{p}}e^{-2\sqrt{pq}} \quad => Solution$$

$$^{424}\int_0^\infty \frac{2e^{-x^2\sqrt{3}}\sin(3x^2)}{x}dx = \frac{\pi}{3} \quad => Solution$$

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[421] Some special values of $\mathrm{Cl}_2(\theta)$

[422] Doing this Feynman-Hibbs Integral will make you an quantum theory expert

[423] A single integral will solve all these integrals

[424] MAZ identity is back

[425] $$\int_0^\infty \frac{e^{-x}tanh(x)}{x}dx = \ln(\frac{\varpi^2}{\pi}) \ \ => Solution$$

[426] $$\int_0^{\frac{1}{2}} \frac{\ln(1+x)\ln(x)}{x}dx = \ln(2)Li_2(-\frac{1}{2}) + Li_3(-\frac{1}{2}) \ \ => Solution$$

[427] $$\int_0^1 x^5 \ln(1+x)dx = \frac{74}{720} \ \ => Solution$$

[428] $$\frac{\sqrt{3}}{2}x - \frac{1}{2}y = 1$$

$$\frac{1}{2}x + \frac{\sqrt{3}}{2}y = 2 \ \ => Solution$$

[429] $$\sum_{k=1}^{\infty} \frac{k}{2^k} = 2 || \sum_{k=1}^{\infty} \frac{k^2}{2^k} = 6 || \sum_{k=1}^{\infty} \frac{k^3}{2^k} = 26 || \sum_{k=1}^{\infty} \frac{k^4}{2^k} = 150 \ \ => Solution$$

[430] $$\text{Calculate } S = \sum_{n=1}^{\infty}(2n-1)\left(\sum_{k=0}^{\infty} \frac{(-1)^k}{(n+k)^2}\right)^2 = \frac{\pi^2}{12} \ \ => Solution$$

[431] $$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + ... = Diverges \ \ => Solution$$

[425] An integral with answer as Lemniscate constant
[426] There is a nice little idea involved in this integral
[427] A problem from the PREFACE section of In Pursuit of Zeta 3
[428] An interesting way to solve these system of equations
[429] Proving the results stated by Jacob Bernoulli
[430] Problem from the School science and Math Journal : Problem proposed by Ovidiu Furdui and Alina Sînt˘ am˘arian
[431] I bet you know this .... But what about this?

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{11}+\frac{1}{13}+\frac{1}{17}+\frac{1}{19}+......=? \ => Solution$$

$$^{432}\sum_{n=0}^{\infty}\binom{2n}{n}=\frac{-1}{\sqrt{3}}i \ => Solution$$

$$\sum_{n=0}^{\infty}(-1)^n\binom{2n}{n}=\frac{1}{\sqrt{5}} \ => Solution$$

$$^{433}\sum_{n=1}^{\infty}\frac{(n+1)}{(-4)^{n+1}}\zeta(n+1)=\frac{G}{2}+\frac{\pi^2}{16}-\frac{\pi}{8}-\frac{3}{4}\ln(2) \ => Solution$$

$$^{434}\psi(1)\psi(\frac{1}{2})\psi(\frac{1}{3})\psi(\frac{2}{3})\psi(\frac{1}{4})\psi(\frac{3}{4})\psi(\frac{1}{6})\psi(\frac{5}{6}) \ => Solution$$

$$^{435}This can be used to prove that fact that F_{(n+1)}F_{(n-1)}-F_n^2=(-1)^n \ => Solution$$

$$^{436}If L_n be Lucas number with L_n=L_{n-1}+L_{n-2}; L_0=2, L_1=1$$

$$show that L_{n+1}L_{n-1}-L_n^2=5(-1)^{n-1}. \ => Solution$$

$$If P_n be pell number with P_n=2P_{n-1}+P_{n-2} with P_0=0 and P_1=1$$

$$show that P_{n+1}P_{n-1}-P_n^2=(-1)^n. \ => Solution$$

[432] Can we make sense of this ?
[433] I love how the simple tool of differentiation can become powerful at times
[434] Finding these frequently used Digamma Values
[435] Using Matrix Multiplication to find the nth fibonacci Number
[436] Some more oneliner proofs

$$^{437} if F_n = k.F_{n-1} + F_{n-2}; F_0 = 0, F_1 = 1$$

$$then prove that F_{n+1}.F_{n-1} - F_n^2 = (-1)^n \;\; => Solution$$

$$^{438} If F_n be the nth fibonacci number defined by F_n = F_{n-1}+F_{n-1}; F_0 = 0, F_1 = 1, then \;\; => Solution$$

$$prove that : \sum_{i=0}^{n} F_i = F_{n+2} - 1 \;\; => Solution$$

$$^{439} \sum_{i=0}^{n} F_{2i+1} = F_{2n+2} \& \sum_{i=0}^{n} F_{2i} = F_{2n+1} - 1 \;\; => Solution$$

$$^{440} F_{m+n} = F_{m+1}F_n + F_mF_{n-1} \;\; => Solution$$

$$^{441} \sum_{i=1}^{n} i.F_i = nF_{n+2} - F_{n+3} + 2 \;\; => Solution$$

$$^{442} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{\sin^2(x)(\sin^2(x)+1)(\sin^2(x)+2)} \;\; => Solution$$

$$^{443} \int_0^{\infty} \frac{dx}{(x+\sqrt{1+x^2})^n}, n > 1 \;\; => Solution$$

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[437] one liner proof of the extended Fibonacci type theorem
[438] Proving this with Matrix // I can't believe how satisfied I am right now
[439] I am just loving this stuff: proving fibonacci theorems using Matrix Multiplication
[440] d'Ocagne's Identity:
[441] Sum of fibonaccci numbers with weighted index
[442] Don't get tricked to using Trig - sub
[443] You know, Trigs are really helpful sometimes

[444] $$\int_1^\infty \frac{dx}{(1+x)\sqrt{1+2ax+x^2}}, a > 1 \;\; => Solution$$

[445] $$0+1+1+2+3+5+8+13+$$

$$21+34+55+89+144+....+F_n+.. = -1 \;\; => Solution$$

[446] $$\sum_{k=1}^{n}(-1)^k k^2 = (-1)^n \frac{n(n+1)}{2} \;\; => Solution$$

[447] $=> Solution$

[448] $$1^4+2^4+3^4+....+(n-1)^4 = \frac{1}{5}\left[\binom{5}{0}B_0 n^5 + \binom{5}{1}B_1 n^4 + \binom{5}{2}B_2 n^3 + \binom{5}{3}B_3 n^2 + \binom{5}{4}B_4 n\right] \;\; => S$$

[449] $$\ln(ab) = \ln(a) + \ln(b) \;\; => Solution$$

[450] $$\sqrt{2} \;\; => Solution$$

[451] $https://www.youtube.com/watch?v = 5 - pXwWNcsbc \;\; => Solution$

[444] My friend's Homework Question

[445] let's prove this interesting result:

[446] This result is satisfying to prove

[447] Everyone knows about Bernoulli numbers but do you know the Euler numbers ?

[448] Three ways to find Bernoulli Numbers

[449] How people prove this in late transcendentals method without using exponential function?

[450] Inventing math to prove the irrationality of the root(2)

[451] A Modern Solution to Basel Problem

$$^{452}3x^2yz + 3e^x z + 3\ln(y)z = 0, \frac{\partial z}{\partial x} =? \frac{\partial z}{\partial y} =? \ => Solution$$

$$^{453}\sum_{n=1}^{\infty}\frac{1}{n^2} = \int_0^1\int_0^1 \frac{1}{1-xy}dxdy = \frac{\pi^2}{6} \ => Solution$$

$$^{454}\frac{d^n}{dx^n}(e^x \sin x) \ => Solution$$

$$^{455}1.Volume(DiscMethod, ShellMethod, IcecreamMethod)$$

$$2.SurfaceArea(RingMethod, SurfaceAreaelementmethod) \ => Solution$$

$$^{456}1.Surfaceareaelementmethodandvolumeelementmethod \ => Solution$$

$$^{457}dV = \left[\left(\frac{\partial x}{\partial u}du, \frac{\partial y}{\partial u}du, \frac{\partial z}{\partial u}du\right) \times \left(\frac{\partial x}{\partial v}dv, \frac{\partial y}{\partial v}dv, \frac{\partial z}{\partial v}dv\right)\right] . \left(\frac{\partial x}{\partial w}dw, \frac{\partial y}{\partial w}dw, \frac{\partial z}{\partial w}dw\right) \ => Solution$$

$$^{458}Finding4 - Dvolumeof4 - Dparallelopipedspannedby\,\vec{a}, \vec{b}, \vec{c}, \vec{d}$$

$$< 1, 1, 2, 3 >, < 1, 2, 3, 5 >, < 2, 3, 5, 8 >, < 3, 5, 8, 13 > => Solution \ => Solution$$

$$^{459}\int_0^{\infty} x^m e^{-x}\sin(x)dx = \frac{m!}{\sqrt{2^{m+1}}}\sin\frac{(m+1)\pi}{4} \ => Solution$$

[452]Finding these partial derivatives in 3 seconds, I am not lying
[453]I taught this in my class today
[454]Complex Numbers are the best friends of mankind!!
[455]Finding formula for surface area and volume of sphere using Calculus
[456]Using calculus to find volume and surface area of Dough nut (Torus)
[457]Chill Guys chill, Jacobian is just a scalar Triple product
[458]n-D volume of n-D parallelopiped spanned by n- linearly independent vectors
[459]Complex Numbers are best friends of humanity - II

[460] $$\int_0^{\pi} \frac{1 - a\cos\theta}{1 - 2a\cos\theta + a^2} d\theta \ \ => Solution$$

[461] $$\text{If} f(x,y,z) = 0 \,\&\, g(x,y,z) = 0$$
$$\frac{dy}{dx} = \frac{\frac{\partial g}{\partial x}\frac{\partial f}{\partial z} - \frac{\partial f}{\partial x}\frac{\partial g}{\partial z}}{\frac{\partial f}{\partial y}\frac{\partial g}{\partial z} - \frac{\partial g}{\partial y}\frac{\partial f}{\partial z}} \ \ => Solution$$

[462] $$\int_0^{\frac{\pi}{2}} \frac{d\theta}{(x^2\cos^2\theta + y^2\sin^2\theta)^2} = \frac{\pi}{4xy}\left(\frac{1}{x^2} + \frac{1}{y^2}\right) \ \ => Solution$$

[463] $$x = rcosh\theta, y = rsinh\theta, \frac{\partial r}{\partial x}, \frac{\partial r}{\partial y}, \frac{\partial \theta}{\partial x}, \frac{\partial \theta}{\partial y} \ \ => Solution$$

[464] $$x = rcosh\theta, y = rsinh\theta, V = V(x,y)$$
$$\frac{\partial^2 V}{\partial x^2} - \frac{\partial^2 V}{\partial y^2} = \frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} - \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} \ \ => Solution$$

[465] $$x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi, z = r\cos\theta; \frac{\partial r}{\partial x}, \frac{\partial \theta}{\partial x}, \frac{\partial \phi}{\partial x}, \frac{\partial r}{\partial y}, \frac{\partial \theta}{\partial y}, \frac{\partial \phi}{\partial y} \ \ => Solution$$

[466] $$[A \,|\, I] \ \ => Solution$$

[467] $$z = \frac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{5}} + y^{\frac{1}{5}}}.Find : x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} =? \ \ => Solution$$

[468] $$\cos\begin{pmatrix} 7 & 3 \\ 3 & -1 \end{pmatrix} \ \ => Solution$$

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[460] Complex Numbers are best friends of Humanity - III
[461] Love you to infinity!!! Intense Satisfaction
[462] Partial differentiation comes to rescue
[463] Find these partials in two different ways
[464] WTF ???
[465] Trick to find reciprocated partials
[466] Gauss Jordan Method / Easiest Method to calculate Matrix Inverse
[467] Our Professor gave this as classwork, WTF
[468] This will be one of your unforgettable maths journey

$$^{469}\det\begin{pmatrix}1 & 2 & 3\\ 1 & 1 & 3\\ 2 & 1 & 2\end{pmatrix} \quad => Solution$$

$$^{470}(uv)''' = \binom{3}{0}u'''v + \binom{3}{1}u''v' + \binom{3}{2}u'v'' + \binom{3}{3}uv''' \quad => Solution$$

$$(\frac{u}{v})''' = -\frac{1}{v^4}\det\begin{vmatrix}u & v & 0 & 0\\ u' & v' & v & 0\\ u'' & v'' & 2v' & v\\ u''' & v''' & 3v'' & 3v'\end{vmatrix} \quad => Solution$$

$$^{471}\vec{a} =< a_1, a_2, a_3 >, \vec{b} =< b_1, b_2, b_3 >$$

$$a_1b_1 + a_2b_2 + a_3b_3 = ||\vec{a}||\,||\vec{b}||\cos\theta \quad => Solution$$

$$\left\|\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}\right\| = ||\vec{a}||\,||\vec{b}||\sin\theta \quad => Solution$$

$$^{472}\det\begin{vmatrix}(y+z)^2 & x^2 & x^2\\ y^2 & (z+x)^2 & y^2\\ z^2 & z^2 & (x+y)^2\end{vmatrix} \quad => Solution$$

[469] Five different ways to compute determinant
[470] In search of gold, we lost diamond
[471] Just in case if you are still wondering why this was true in the first place
[472] do it without doing

[473] $$\begin{vmatrix} 1 & a & a^2 \\ \cos((n-1)x) & \cos(nx) & \cos((n+1)x) \\ \sin((n-1)x) & \sin(nx) & \sin((n+1)x) \end{vmatrix} \;=> Solution$$

[474] $$\frac{dx}{dt} = 5x + y$$

$$\frac{dy}{dt} = x + 5y \;=> Solution$$

[475] $$\begin{cases} \lambda x + y + \sqrt{2}z = 0, \\ x + \lambda y + \sqrt{2}z = 0, \\ \sqrt{2}x + \sqrt{2}y + (\lambda - 2)z = 0. \end{cases} \qquad => Solution$$

[476] $$y'' - 2y' - 8y = 0$$

$$y'' - 2y' - 8y = e^{-x}$$

$$y'' - 2y' - 8y = e^{-2x} \;=> Solution$$

[477] $$a > 0, b > 0 \int_0^\infty \frac{\log(1 + a^2x^2)}{1 + b^2x^2} dx \;=> Solution$$

[478] $$\lim_{n\to\infty} \frac{1}{n^2} \sum_{i=1}^{n} i \sin^3\left(\frac{i\pi}{4n}\right) \;=> Solution$$

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[473] Answer won't have n !!

[474] Solving these system of differential equations in two different ways i) Exponential Ansatz Method ii) Using Matrix

[475] Find non trivial solutions for x, y and z

[476] Third is slightly different!

[477] University of Ibadan, ODE Integration Bee, Finals, problem 9

[478] University of Ibadan, ODE Integration Bee Finals, Problem 8

$$^{479}\int_0^\infty cos((\frac{x}{\pi}-\frac{e}{x})^2)dx \;\; => Solution$$

$$^{480}Giventhatforanidealgas: PV = nRT, provethat: \frac{\partial P}{\partial V}.\frac{\partial V}{\partial T}.\frac{\partial T}{\partial P} = -1 \;\; => Solution$$

$$^{481}\int_{-\infty}^\infty erfc((\frac{x}{e}-\frac{\zeta(3)\pi\gamma}{x})^2)dx = \frac{2e}{\sqrt{\pi}}\Gamma(\frac{3}{4}) \;\; => Solution$$

$$^{482}\frac{1}{1^2}-\frac{2}{3^2}+\frac{3}{5^2}-\frac{4}{7^2}+.... \;\; => Solution$$

$$^{483}1+\frac{\cos(x)}{1!}+\frac{\cos(2x)}{2!}+\frac{\cos(3x)}{3!}+......... = e^{\cos x}\cos(\sin x) \;\; => Solution$$

$$^{484}x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-xz) \;\; => Solution$$

$$^{485} = (x+y+z)(x+\omega^2 y+\omega z)(x+\omega y+\omega^2 z) \;\; => Solution$$

$$^{486}Factorize: x^2+xy+y^2 \;\; => Solution$$

[479] University of Ibadan, ODE Integration Bee Finals, Problem 7
[480] This textbook problem is misleading
[481] The absolute perfectness
[482] Summation Notation isn't just a tool, it's an emotion
[483] Isn't this beautiful?
[484] Hehe
[485] x$^3 + y^3 + z^3 - 3xyz$
[486] Explicit Content

$$^{487}D_vf = \cos\theta\frac{\partial f}{\partial x} + \sin\theta\frac{\partial f}{\partial y} \;\; => Solution$$

$$D_v^2f = \cos^2\theta\frac{\partial^2 f}{\partial x^2} + 2\cos\theta\sin\theta\frac{\partial^2 f}{\partial x\partial y} + \sin^2\theta\frac{\partial^2 f}{\partial y^2} \;\; => Solution$$

$$D_uD_vf = (\cos\phi\partial_x + \sin\phi\partial_y)(\cos\theta\partial_x + \sin\theta\partial_y)f \;\; => Solution$$

$$D_wD_uD_vf = (\cos\alpha\partial_x{+}\sin\alpha\partial_y)(\cos\phi\partial_x{+}\sin\phi\partial_y)(\cos\theta\partial_x{+}\sin\theta\partial_y)f \;\; => Solution$$

$$^{488}\sqrt[n]{2} : n \geq 2 isirrational. \;\; => Solution$$

$$^{489}e + \pi, e\pi \;\; => Solution$$

$$^{490}(x+y)^p = x^p + y^p \;\; => Solution$$

$$^{491}\int_0^{\frac{\pi}{4}} \sin(2x) \prod_{n=0}^{\infty} \left(e^{(-1)^n(\tan(x))^{2n}}\right) dx \;\; => Solution$$

[487] Proof of the second order directional derivatives and more
[488] Overkilling this proof using Fermat's last theorem
[489] A really elegant proof to prove at least one of e+pi or e.pi is irrational
[490] This is true in some world
[491] It's a prank

$$^{492}\int \sum_{k=0}^{2024} \sin\left(x + \frac{2k.\pi}{2024}\right) dx \;\; => Solution$$

$$^{493}\begin{vmatrix} a_1^2+k & a_1a_2 & a_1a_3 & a_1a_4 & \cdots & a_1a_n \\ a_2a_1 & a_2^2+k & a_2a_3 & a_2a_4 & \cdots & a_2a_n \\ a_3a_1 & a_3a_2 & a_3^2+k & a_3a_4 & \cdots & a_3a_n \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_na_1 & a_na_2 & a_na_3 & a_na_4 & \cdots & a_n^2+k \end{vmatrix} => Solution$$

$$^{494}y'' - y' = \ln(t); y(0) = 0; y'(0) = 0 \;\; => Solution$$

$$y(t) = -\gamma e^t - \ln(t) - t\ln(t) + t + e^t Ei(-t) \;\; => Solution$$

$$^{495}For - \pi < x < \pi,$$

$$x^2 = \frac{\pi^2}{3} + \sum_{k=1}^{\infty} \frac{4(-1)^k}{k^2}\cos(kx) \;\; => Solution$$

$$^{496}f(x) = \sum_{k=0}^{\infty} a_n \cos(\frac{n2\pi x}{T}) + \sum_{k=0}^{\infty} b_n \sin(\frac{n2\pi x}{T})$$

[492] Austria Integration Bee 2024 Quarter Finals
[493] Wohoo, Computing determinant with Induction, this is so fun
[494] Hmm, isn't that interesting
[495] Solving the Basel Problem using Fourier series of x^2
[496] Introducing fourier transform from the fourier series

$$F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx \ \ => Solution$$

$$^{497}\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$$

$$-\infty < x < \infty, t \geq 0$$

$$u(x,0) = f(x), u(0,t) = 0, u(L,t) = 0 \ \ => Solution$$

[497] Solving Heat Equation PDE: which one is easier? i) Variable Separation Method ii) Fourier Transform Method