Problem Proposed to SSMJ

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Geometrical Integrals

Statement of the Problem: Show that:

$$\int_0^1 \int_0^1 \{y^2 - x\} dxdy = \frac{1}{2}$$

Solution of the problem:
From the geometrical interpretation of double integral, we can see that:

$$\int_0^1 \int_0^1 \{y^2 - x\} dxdy = \int_0^1 \int_0^{y^2} \{y^2 - x\} dxdy + \int_0^1 \int_0^{\sqrt{x}} \{y^2 - x\} dydx$$

Notice that in the first region, $0 < \{y^2 - x\} < 1$ and in the second region, $0 > \{y^2 - x\} > -1$. which is why we can write

$$= \int_0^1 \int_0^{y^2} (y^2 - x) dxdy + \int_0^1 \int_0^{\sqrt{x}} (y^2 - x + 1) dydx$$

$$= \int_0^1 \frac{y^4}{2} dy + \int_0^1 x^{\frac{1}{2}} - \frac{2x^{\frac{3}{2}}}{3} dx$$

$$= \frac{1}{10} + \frac{2}{3} - \frac{2}{3} \times \frac{2}{5}$$

$$= \frac{1}{2}$$