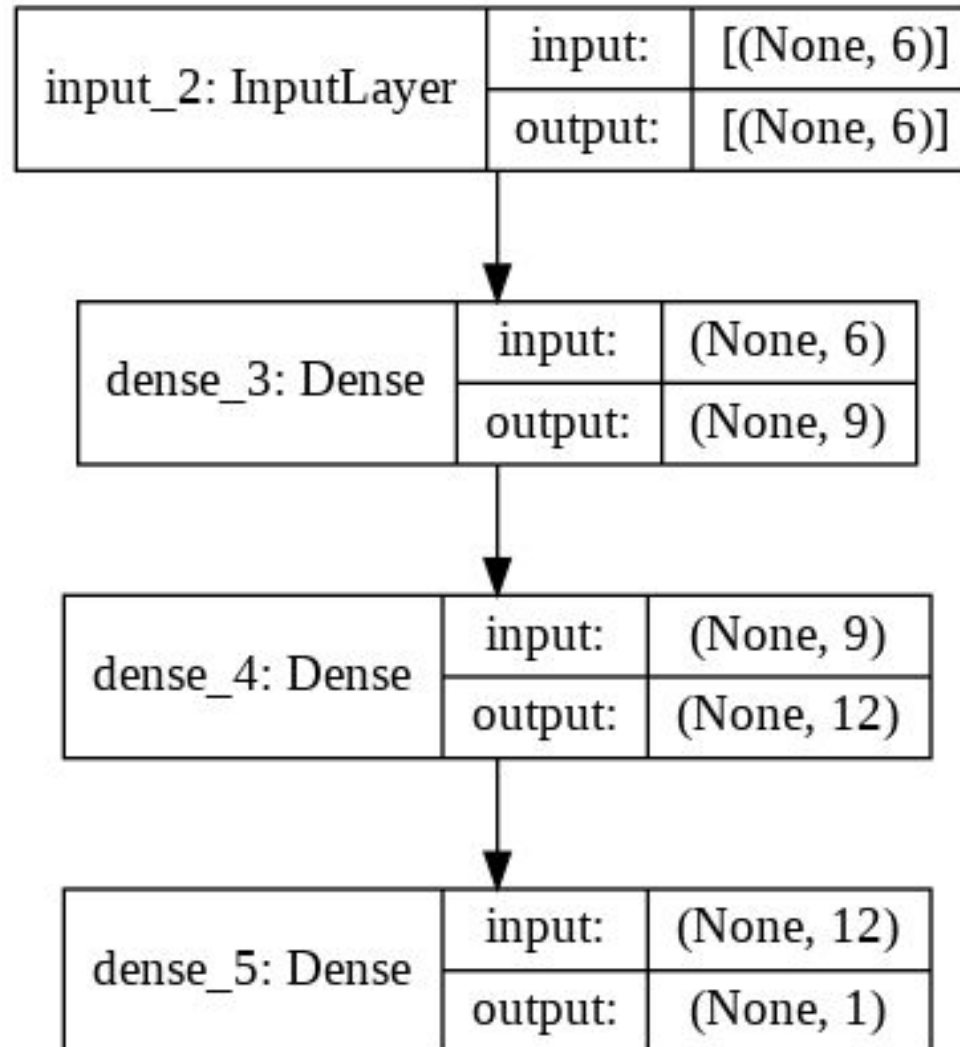


ANALYSIS OF DATA AND ENVIRONMENT

- Number of train data points = 120435
- Number of Test data points = 54235 (predicting High data points and low data points)
- input features: $x1(n-2)$, $x1(n-1)$, $x1(n)$, $x2(n-2)$, $x2(n-1)$, $x2(n)$
- Output feature = **Force (F1)**
- Optimizer = Adam
- Loss function = Mean Squared Error
- Version of TensorFlow used = 2.6.0

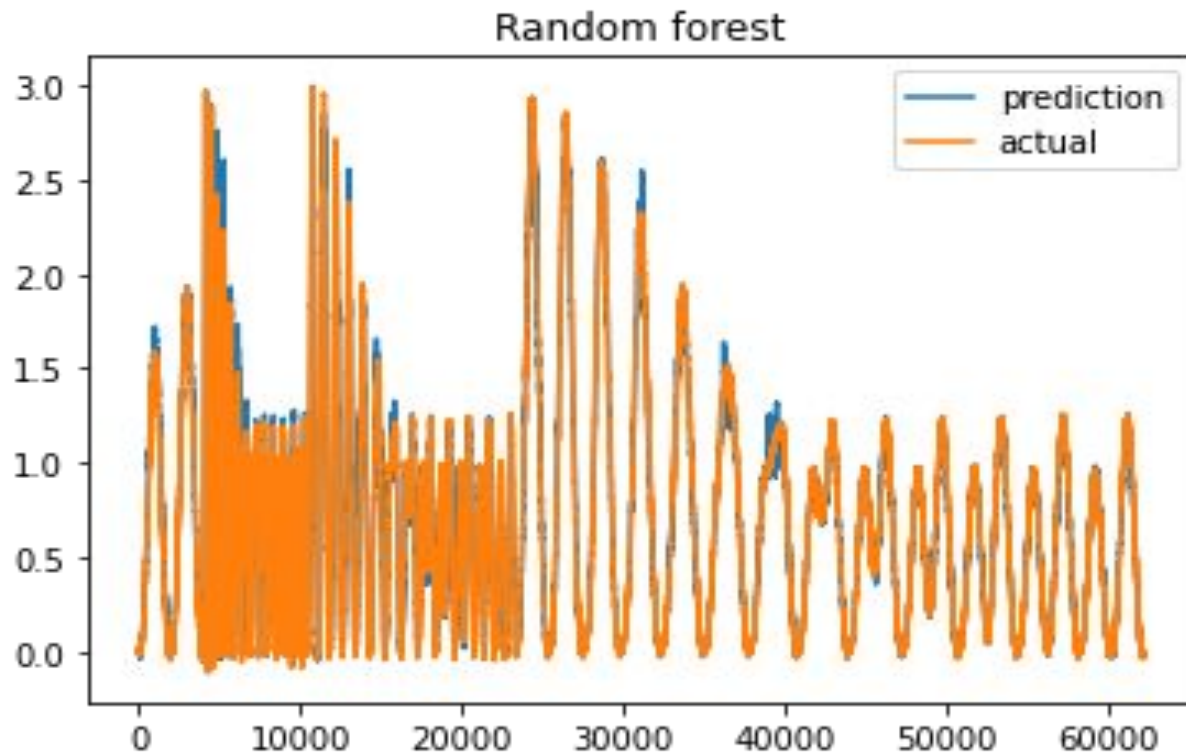
DEEP LEARNING MODEL STRUCTURE



Predicting Middle Amplitude points(39-78 Interaction points) including past appearances

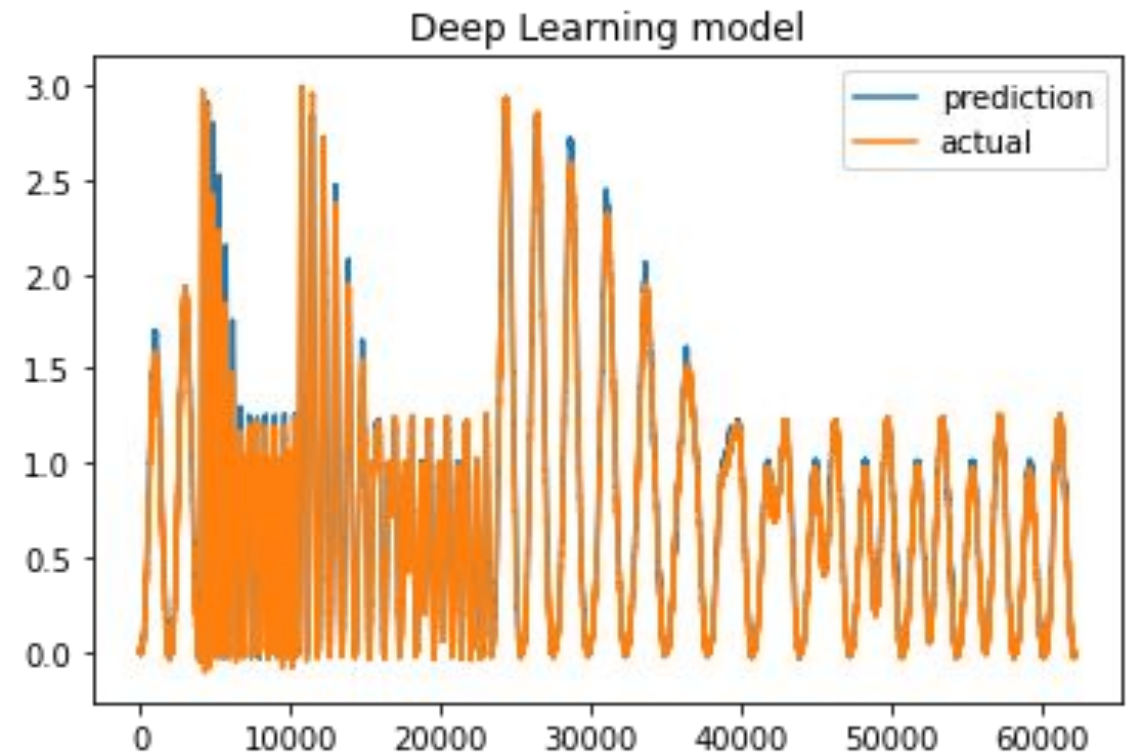
Random Forest

Root Mean squared error = 0.062257



Deep Learning Model

Root Mean squared error = 0.055043

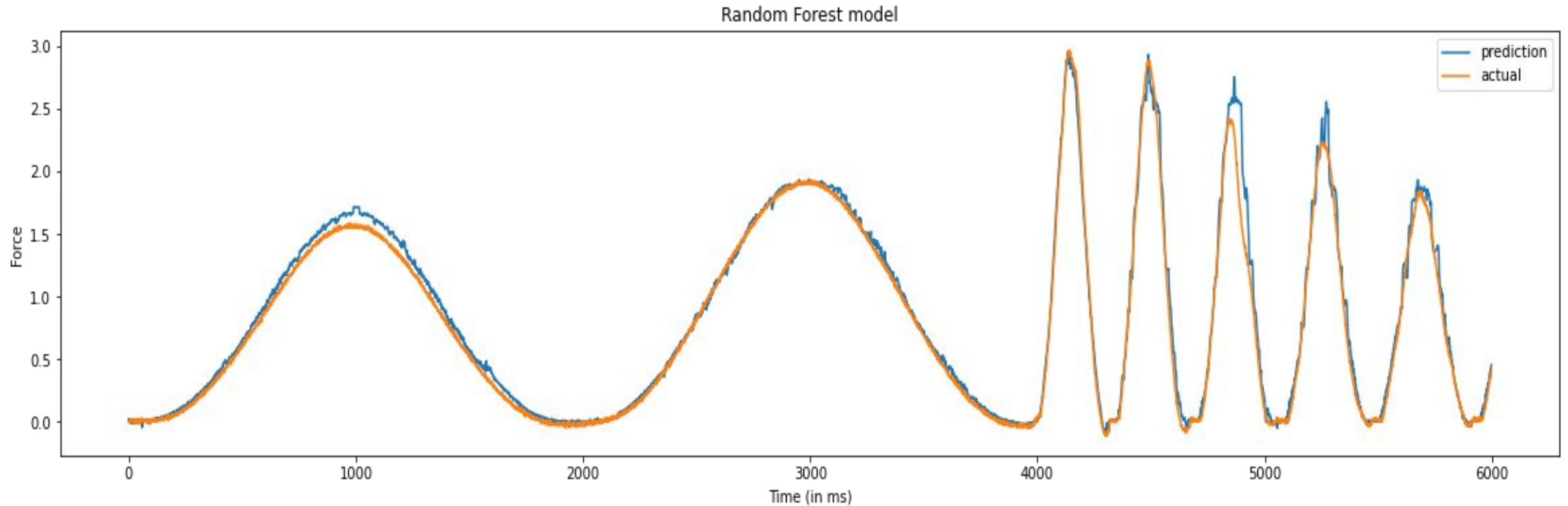


Predicting Middle Amplitude points(39-78 Interaction points) for a lower time period including past appearances

Random Forest

(CLOSER VIEW OF INITIAL POINTS)

Root Mean squared error = 0.062257

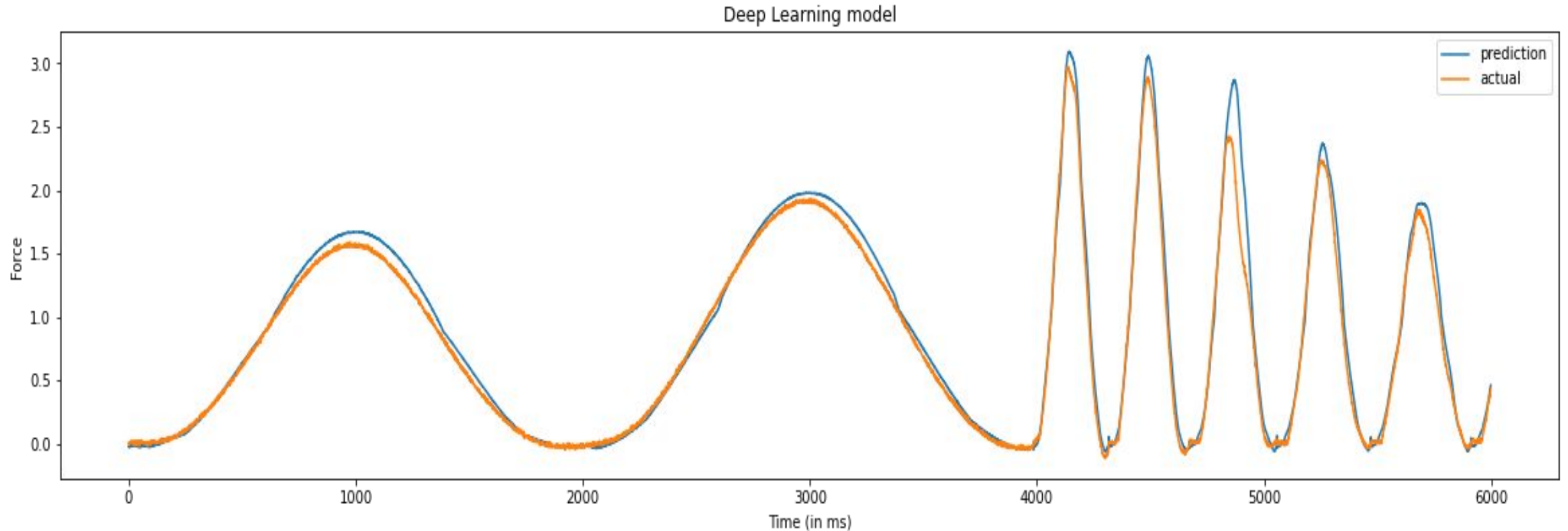


Predicting Middle Amplitude points(39-78 Interaction points) for a lower time period including past appearances

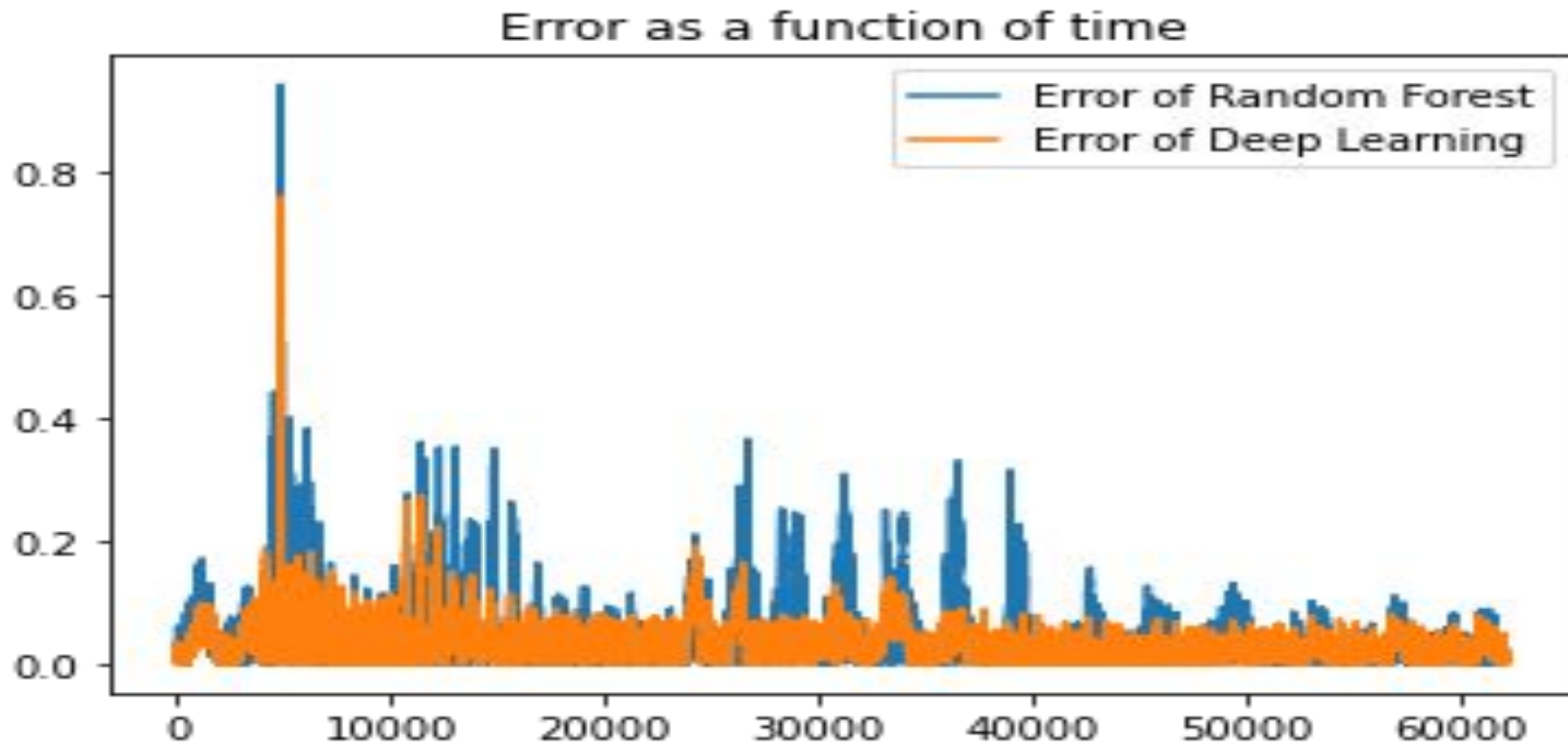
Deep Learning Model

(CLOSER VIEW OF INITIAL POINTS)

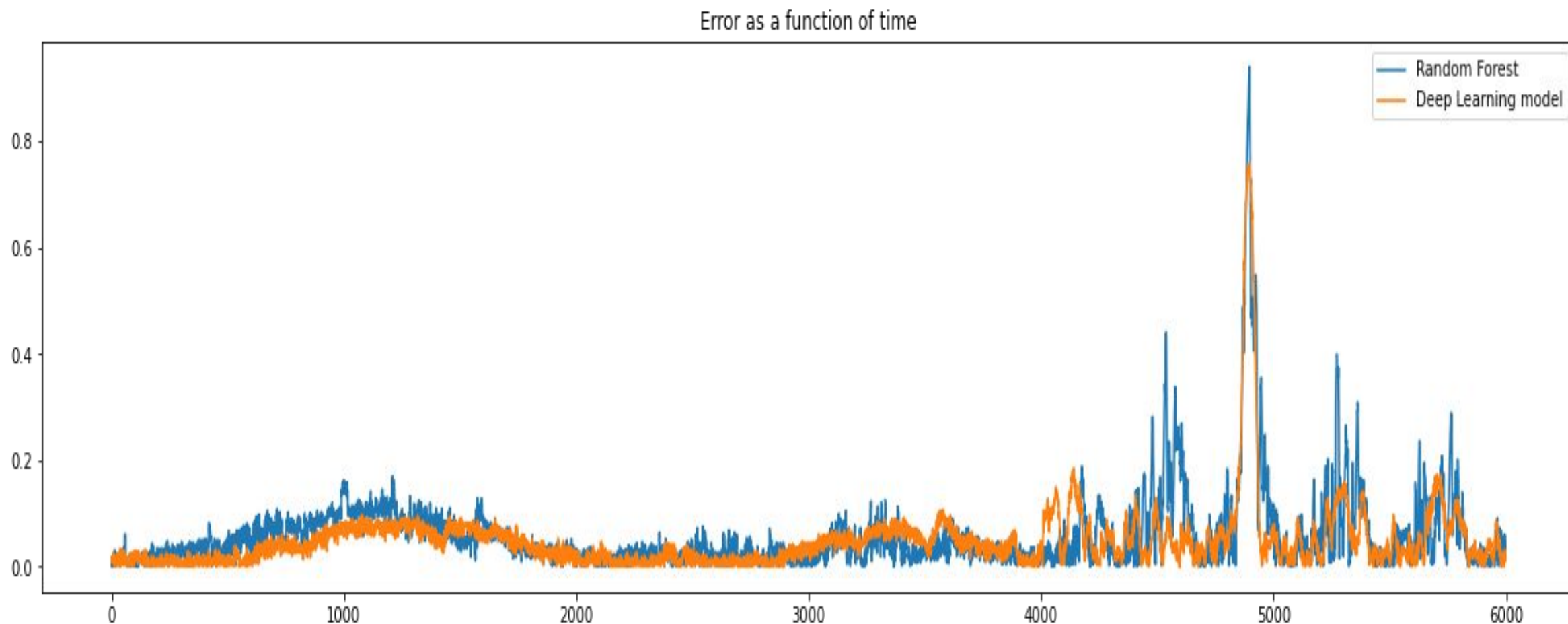
Root Mean squared error = 0.055043



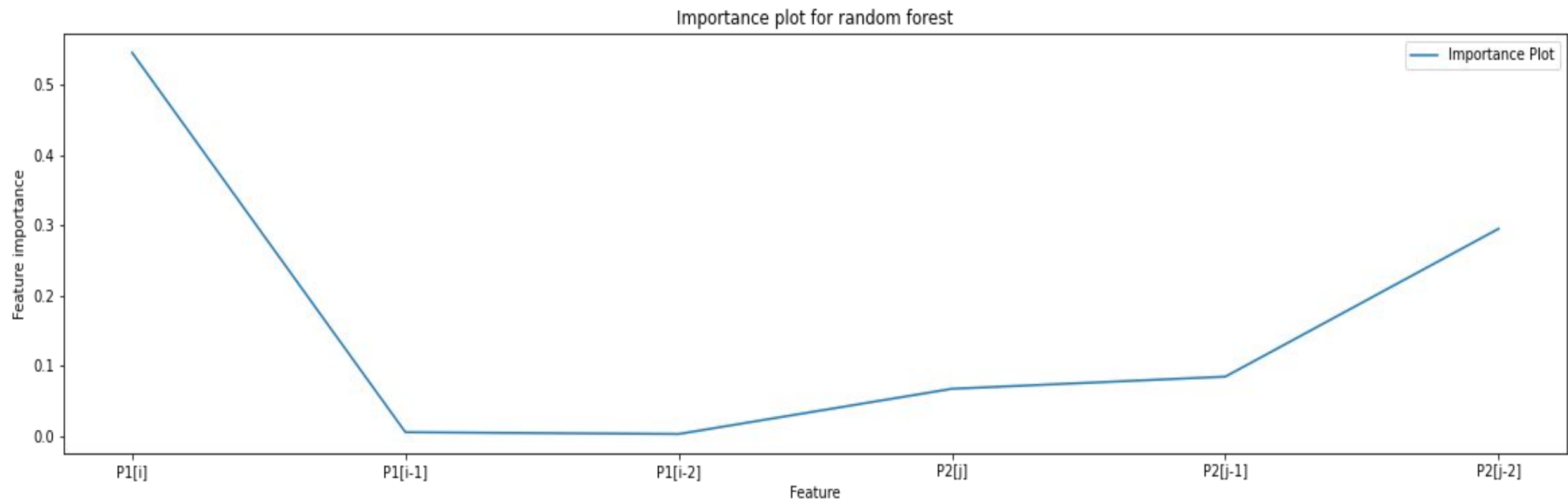
Error as a function of time using past appearances



Error as a function of time using past appearances (closer view of initial points)



Feature Importance plot for random forest using past appearances



ANALYSIS OF DATA AND ENVIRONMENT

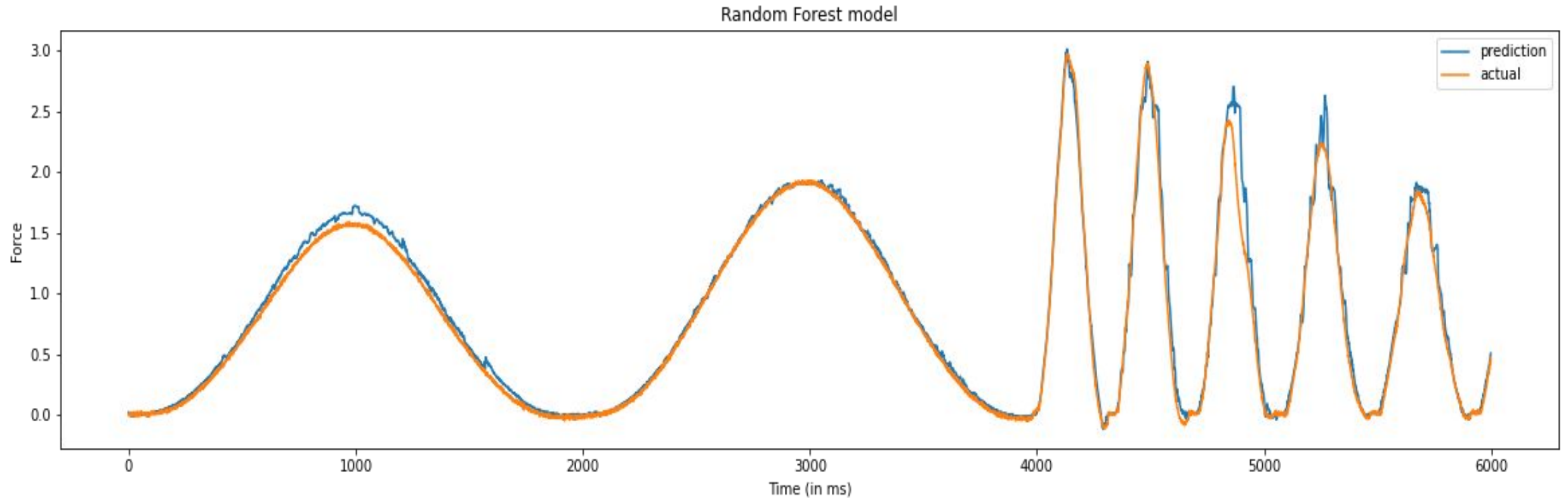
- Number of train data points = 120435
- Number of Test data points = 54235 (predicting High data points and low data points)
- input features: $x1(n-2*\tau)$, $x1(n-\tau)$, $x1(n)$, $x2(n-2*\tau)$, $x2(n-\tau)$, $x2(n)$
- Output feature = **Force (F1)**
- Optimizer = Adam
- Loss function = Mean Squared Error
- Version of TensorFlow used = 2.6.0

Predictions

Random Forest

(CLOSER VIEW OF INITIAL POINTS)

Root Mean squared error = 0.060872

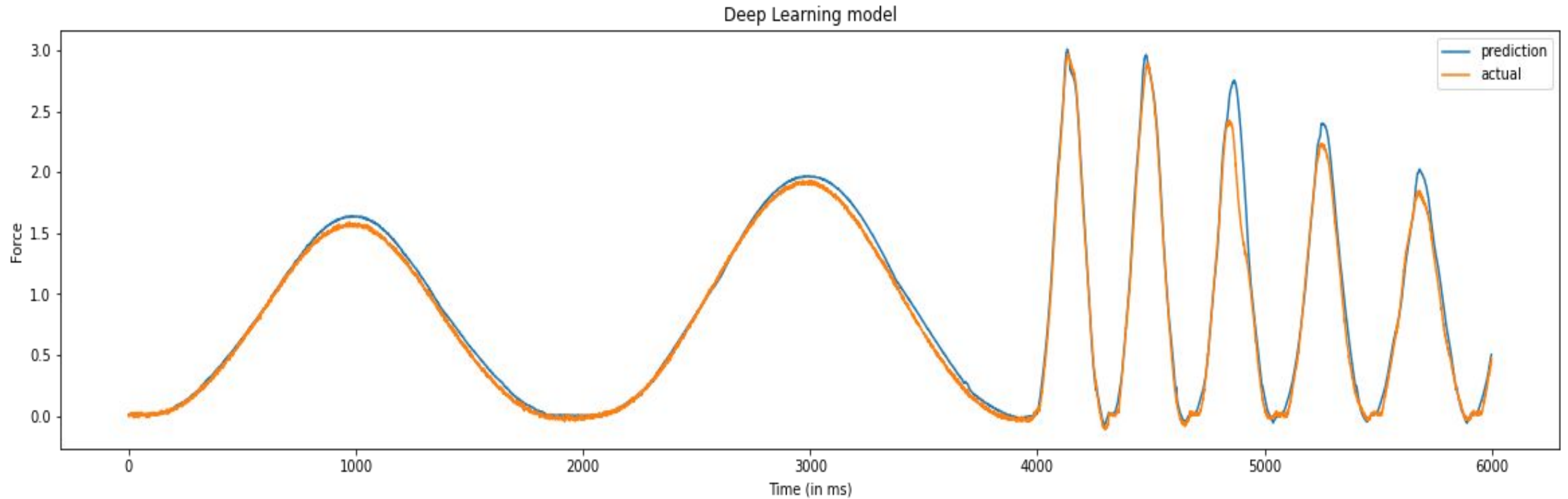


Predictions

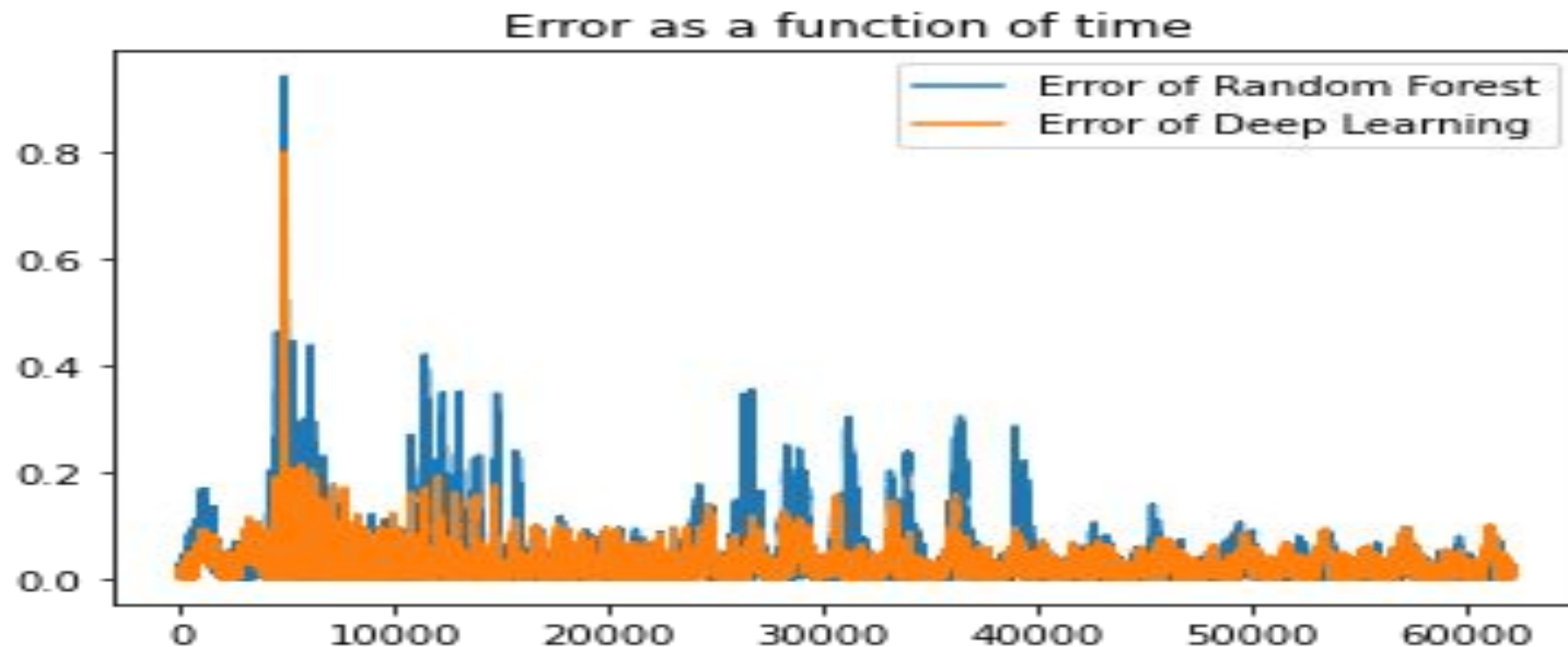
Deep Learning Model

(CLOSER VIEW OF INITIAL POINTS)

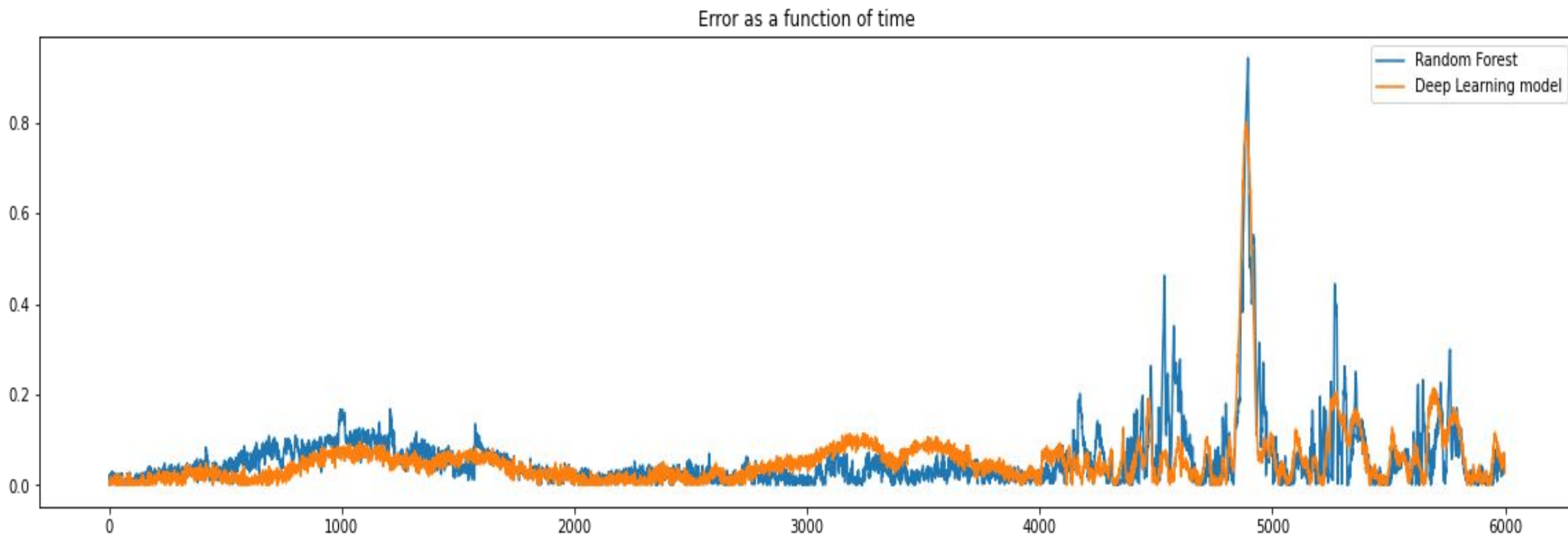
Root Mean squared error = 0.047670



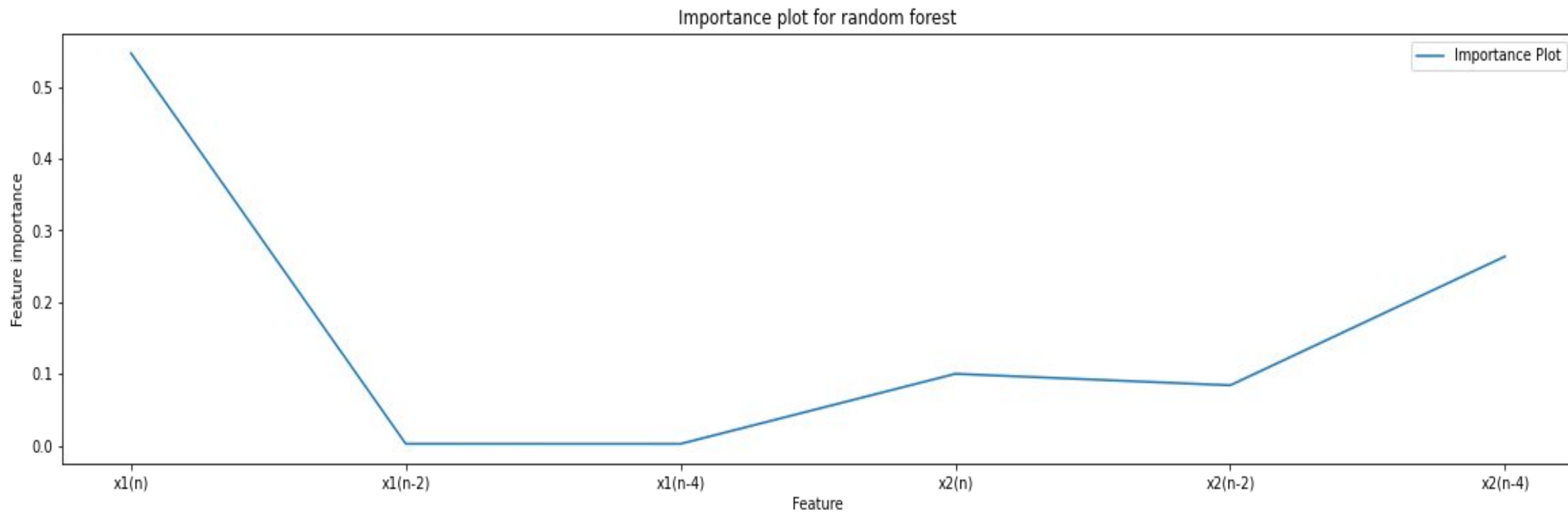
Error as a function of time using past appearances for large samples



Error as a function of time using past appearances (closer view of initial points)



Feature Importance plot for random forest using past appearances



ANALYSIS OF DATA AND ENVIRONMENT

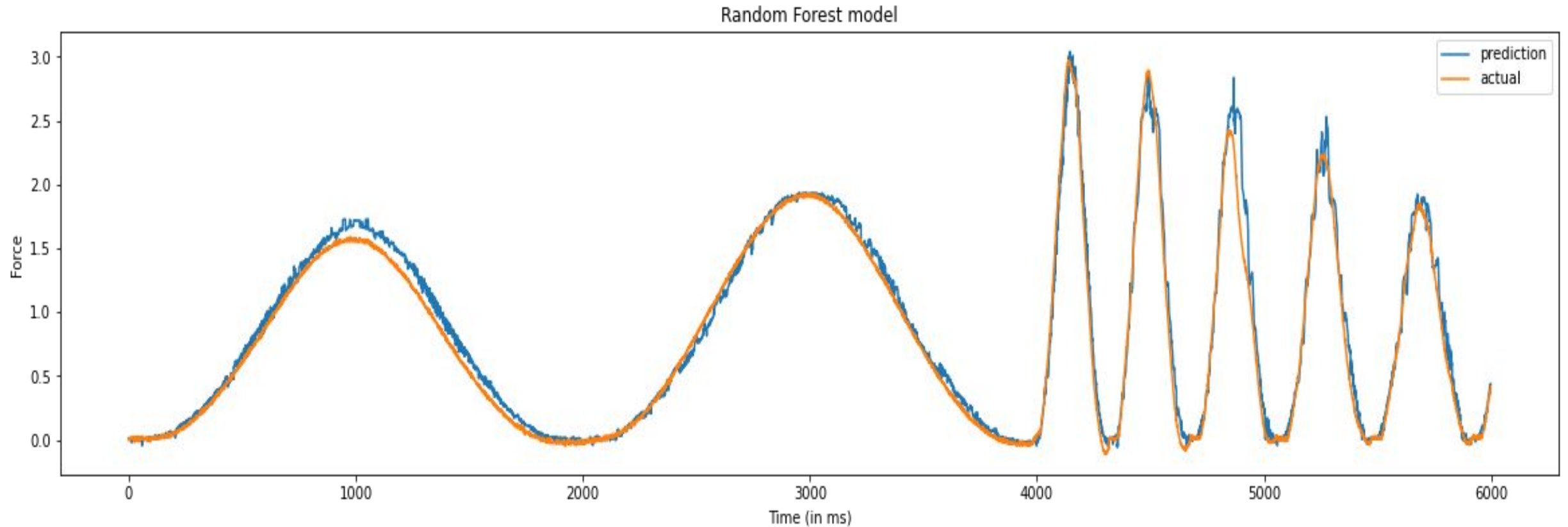
- Number of train data points = 120435
- Number of Test data points = 54235 (predicting High data points and low data points)
- input features: **x1,x2**
- Output feature = **Force (F1)**
- Optimizer = Adam
- Loss function = Mean Squared Error
- Version of TensorFlow used = 2.6.0

Predicting Middle Amplitude points(39-78 Interaction points) for a lower time period including past appearances

Random Forest

(CLOSER VIEW OF INITIAL POINTS)

Root Mean squared error = 0.075171

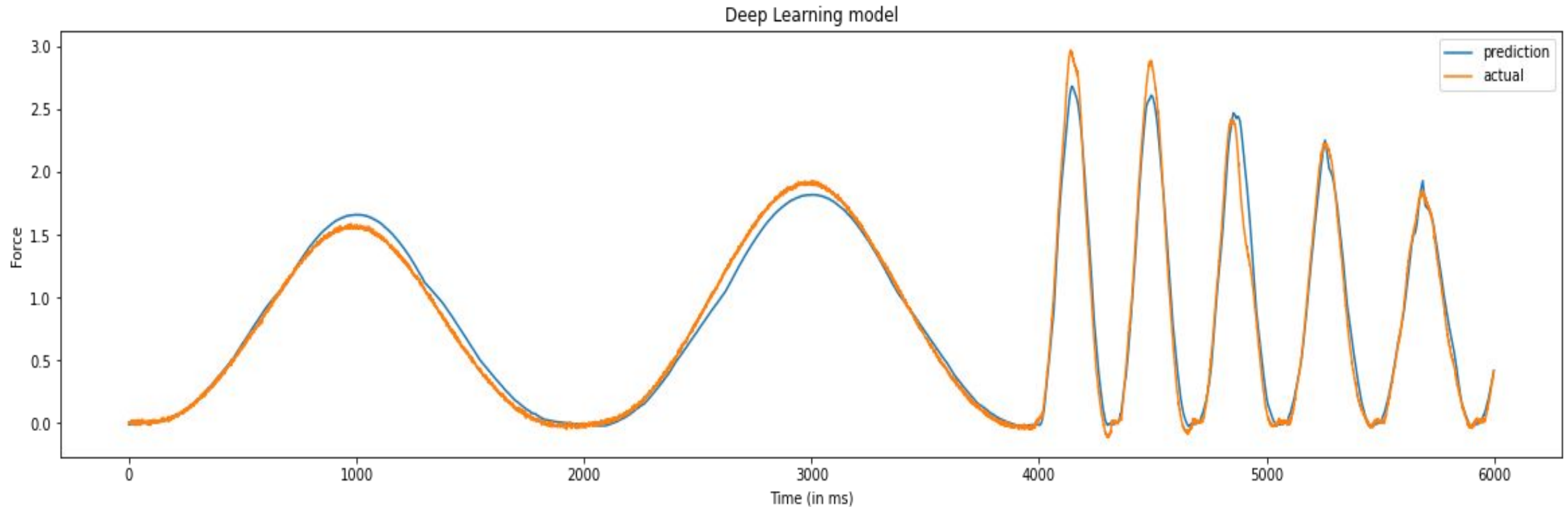


Predicting Middle Amplitude points(39-78 Interaction points) for a lower time period including past appearances

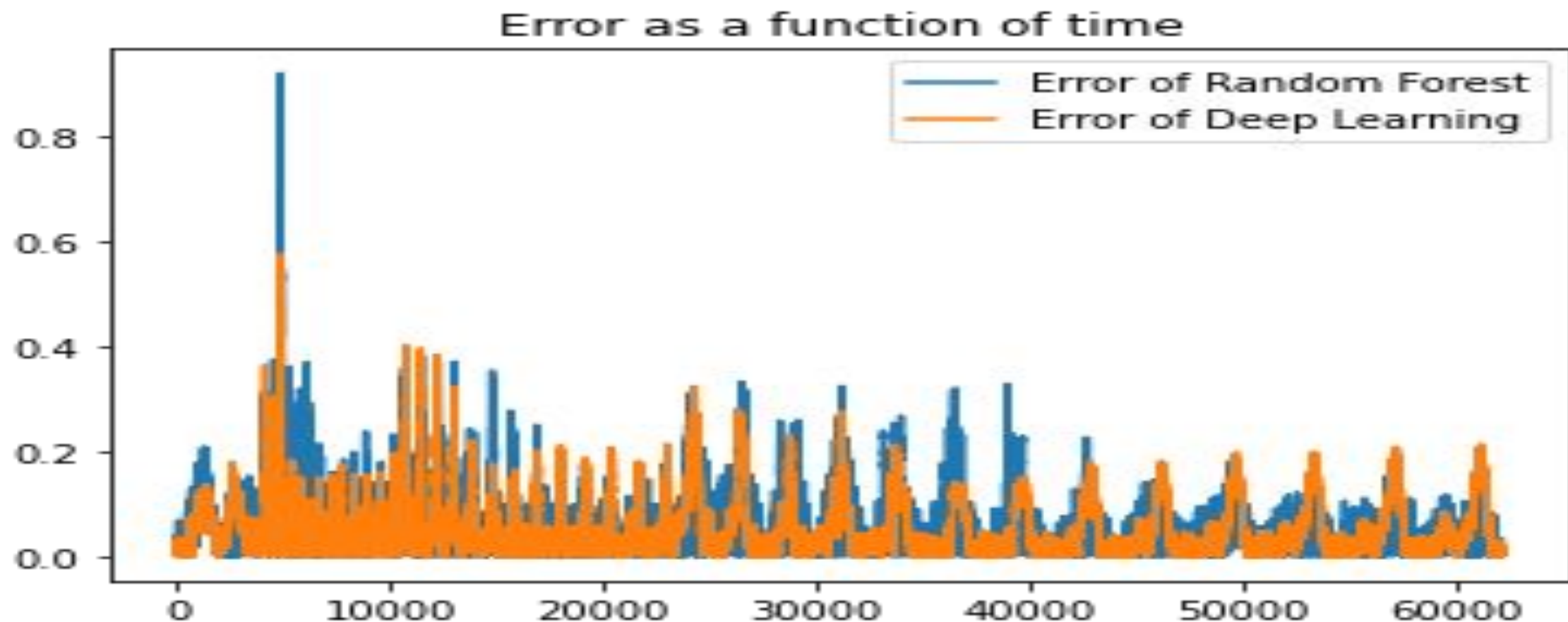
Deep Learning Model

(CLOSER VIEW OF INITIAL POINTS)

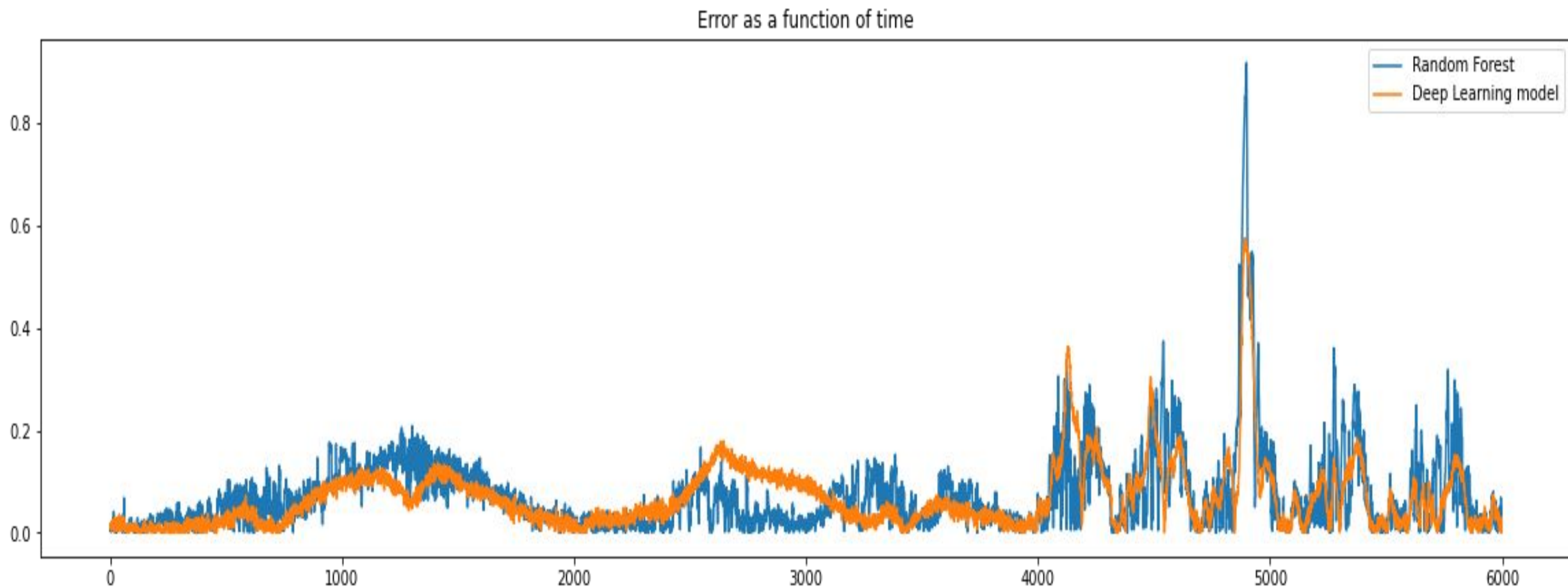
Root Mean squared error = 0.081841



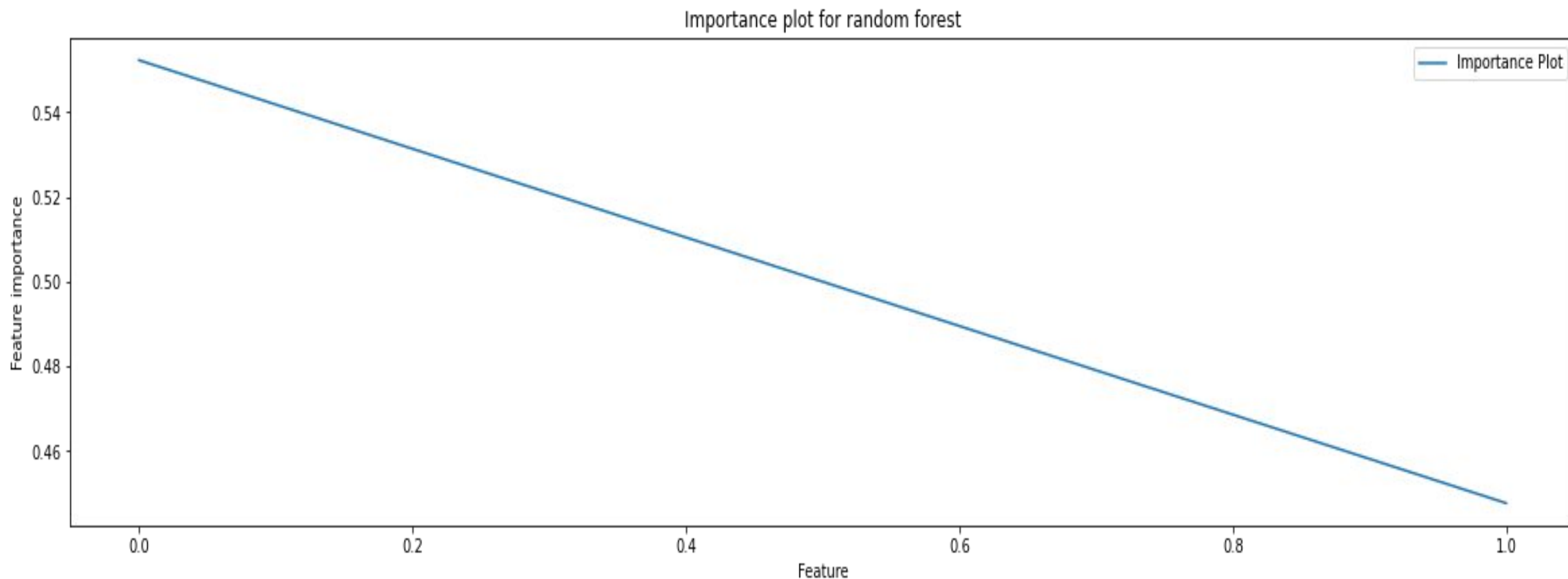
Error as a function of time using past appearances for large samples



Error as a function of time using past appearances for small samples



Feature Importance plot for random forest using past appearances



Random Forest Vs Deep Learning

Models' performances on Middle amplitude points

Input feature vs output feature	p1,p2 f1		p1[i+1] - p1[i], p2[i+1] - p2[i] f1		p1[i],p1[i- τ],p1[i-2* τ] p2[i],p2[i- τ],p2[i-2* τ] f1	
Model	DL model	RF	DL model	RF	DL model	RF
“RMSE”	<i>0.081841</i>	<i>0.075171</i>	<i>0.055043</i>	<i>0.060872</i>	<i>0.047670</i>	<i>0.060872</i>
Max error	<i>0.575535</i>	<i>0.918093</i>	<i>0.798003</i>	<i>0.923737</i>	<i>0.800174</i>	<i>0.94290</i>
Min error	<i>4.17*10⁻⁷</i>	<i>5 * 10⁻⁷</i>	<i>1.192*10⁻⁶</i>	<i>2.42*10⁻¹⁷</i>	<i>5.96*10⁻⁸</i>	<i>1.98*10⁻⁶</i>
Medin error	<i>0.034598</i>	<i>0.034598</i>	<i>0.021189</i>	<i>0.021189</i>	<i>0.019451</i>	<i>0.019451</i>
Time taken by the model(in sec)	<i>0:00:27.6</i>	<i>0:01:22.6</i>	<i>0:01:15.9</i>	<i>0:01:12.7</i>	<i>0:01:15.0</i>	<i>0:01:22.3</i>

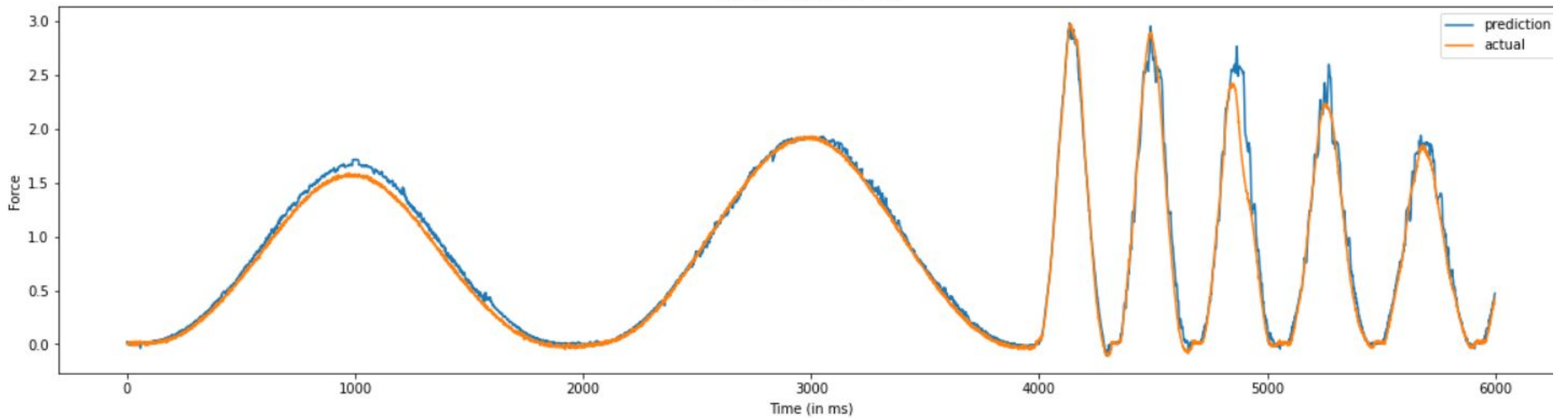
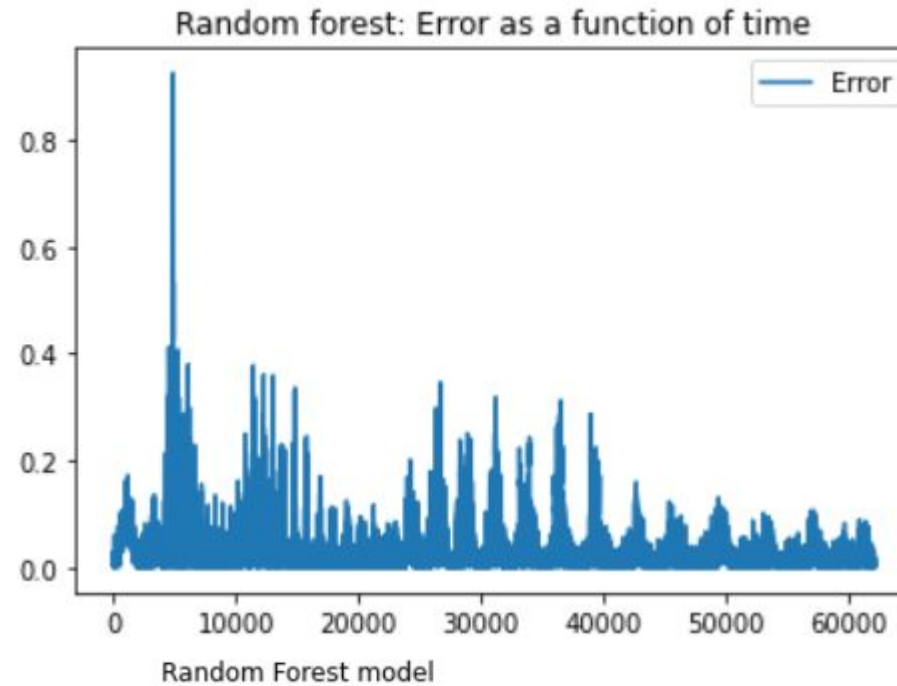
Including Past Inputs

$x_1(n), x_1(n-t), x_1(n-2t), x_2(n), x_2(n-t), x_2(n-2t)$

$t = 1$

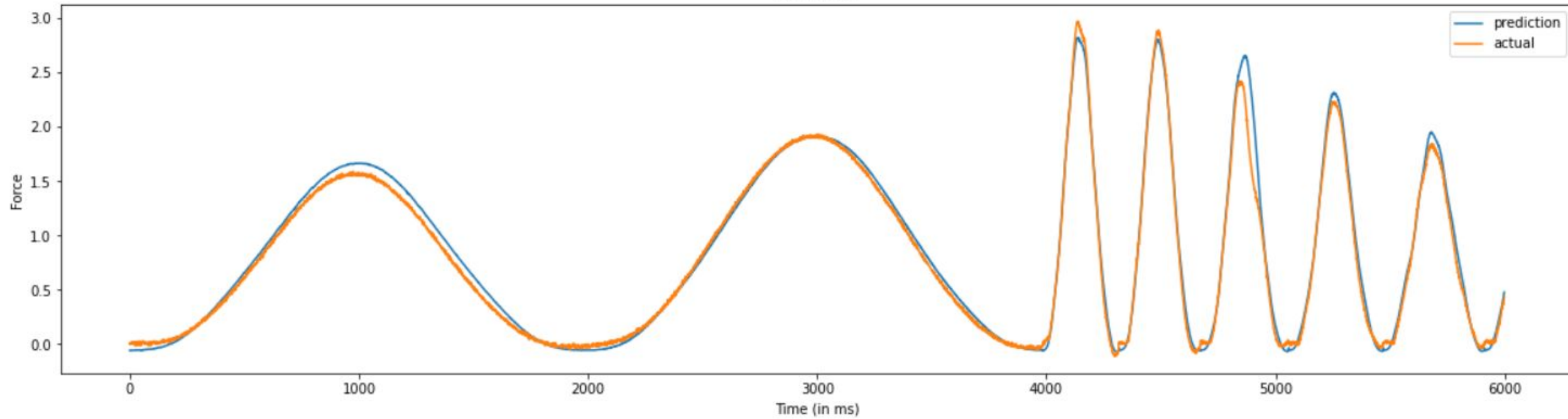
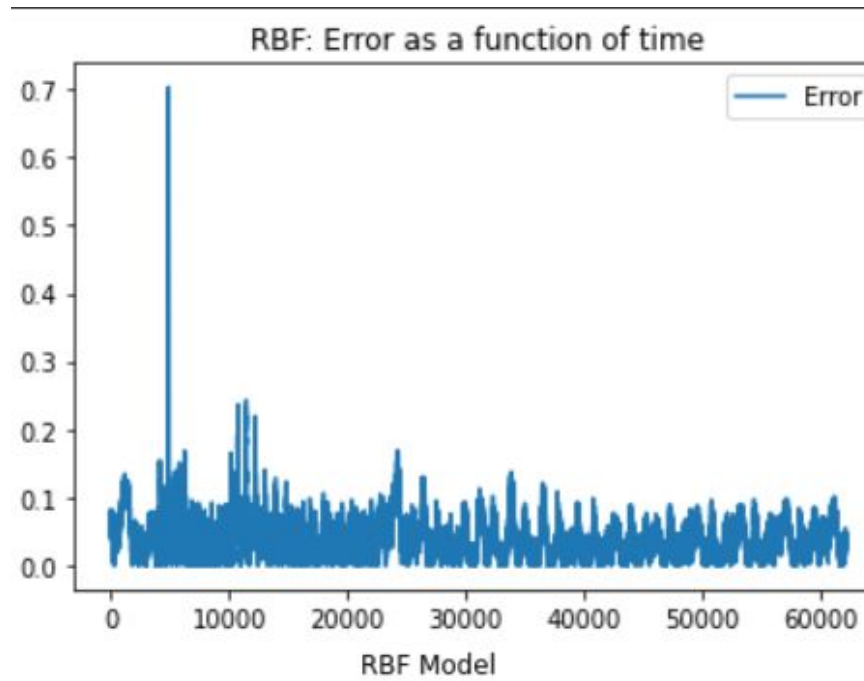
Random Forest

RMSE : 0.062569



RBF

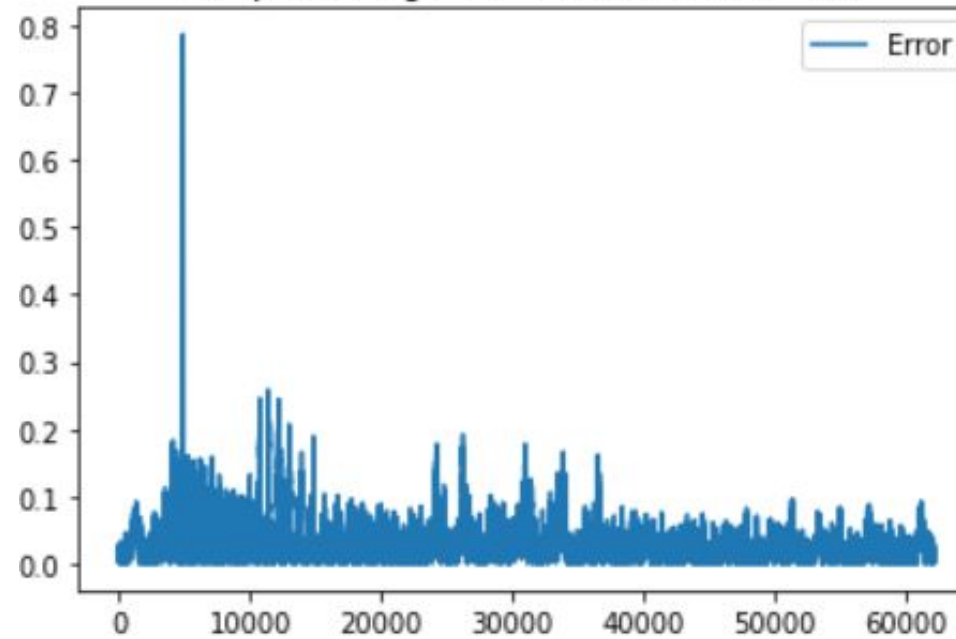
RMSE : 0.0519104



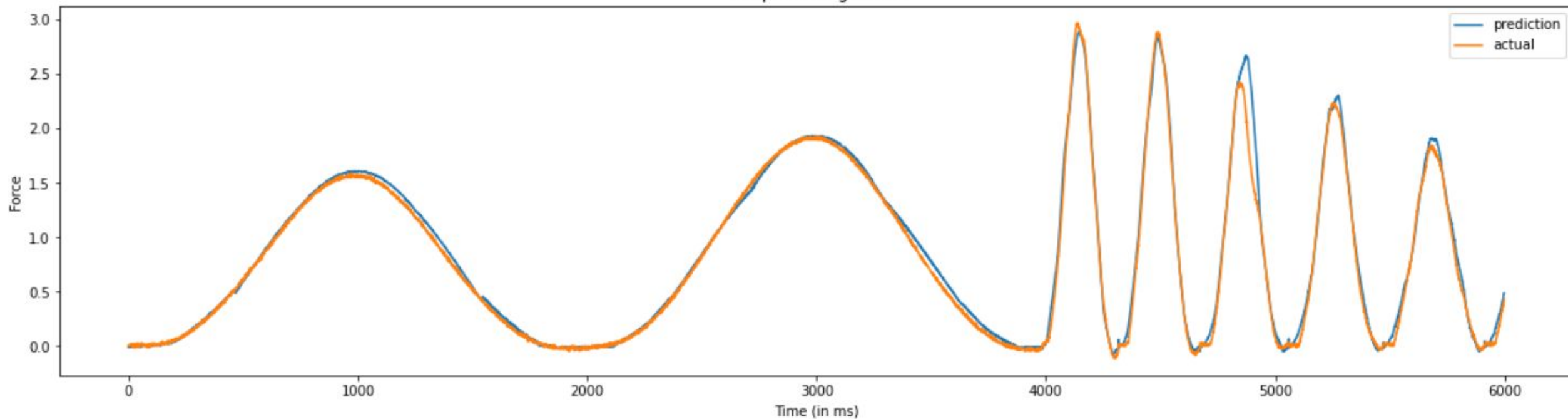
Deep Learning

RMSE: 0.04770387

Deep Learning: Error as a function of time

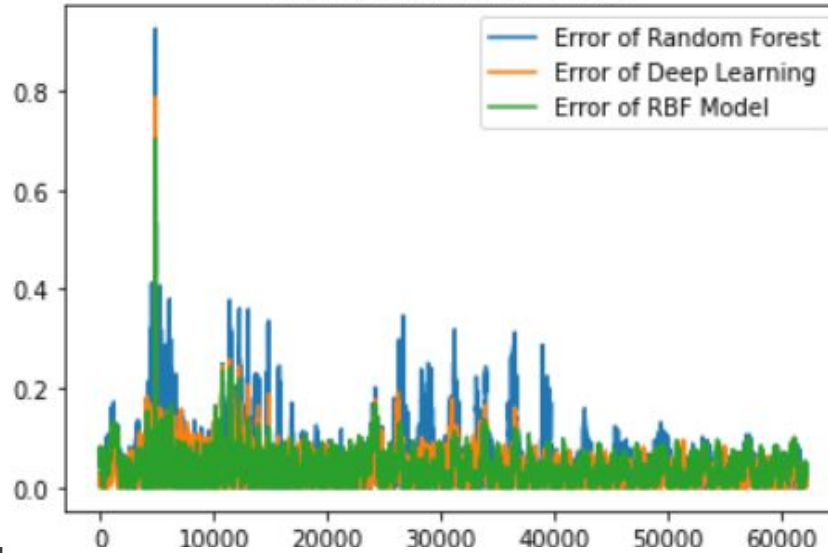


Deep Learning model

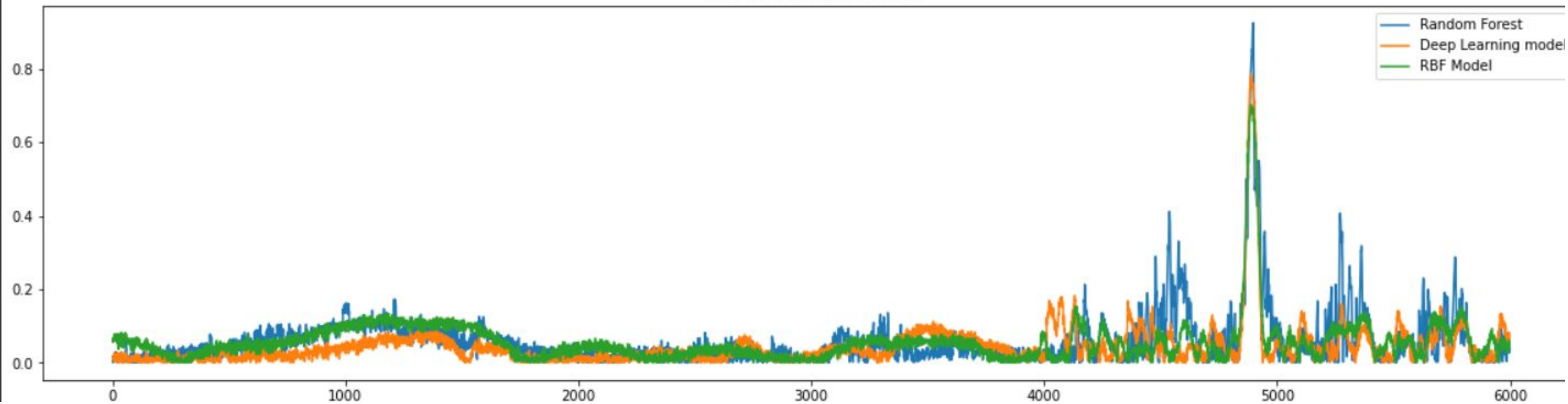


Error comparison among the three models

Error as a function of time

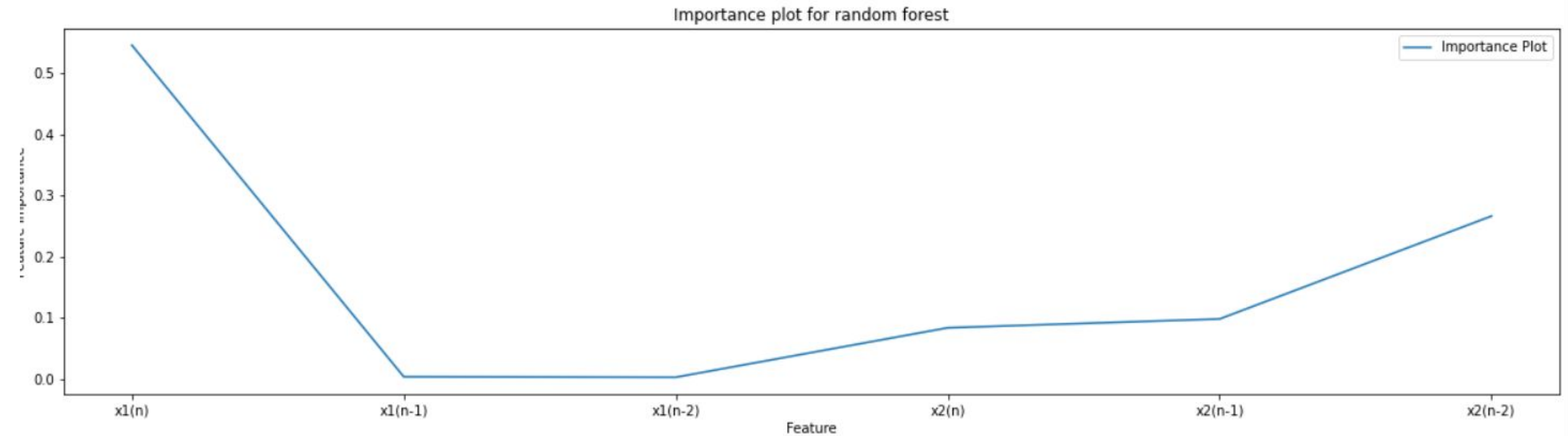


Error as a function of time



Feature Importance

$x1(n)$, $x1(n-1)$, $x1(n-2)$, $x2(n)$, $x2(n-1)$, $x2(n-2)$



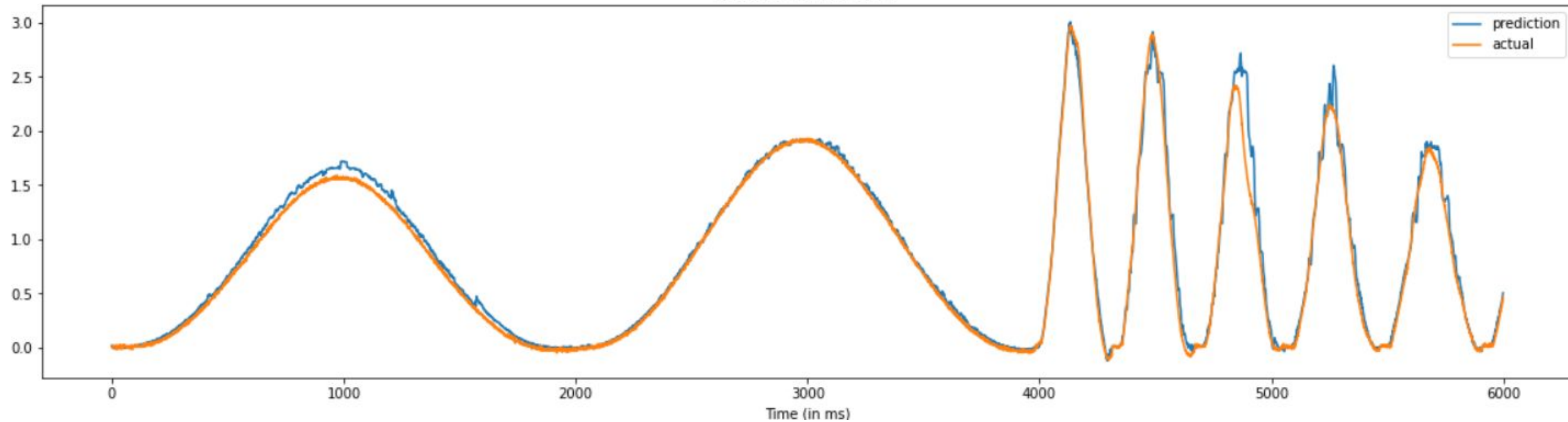
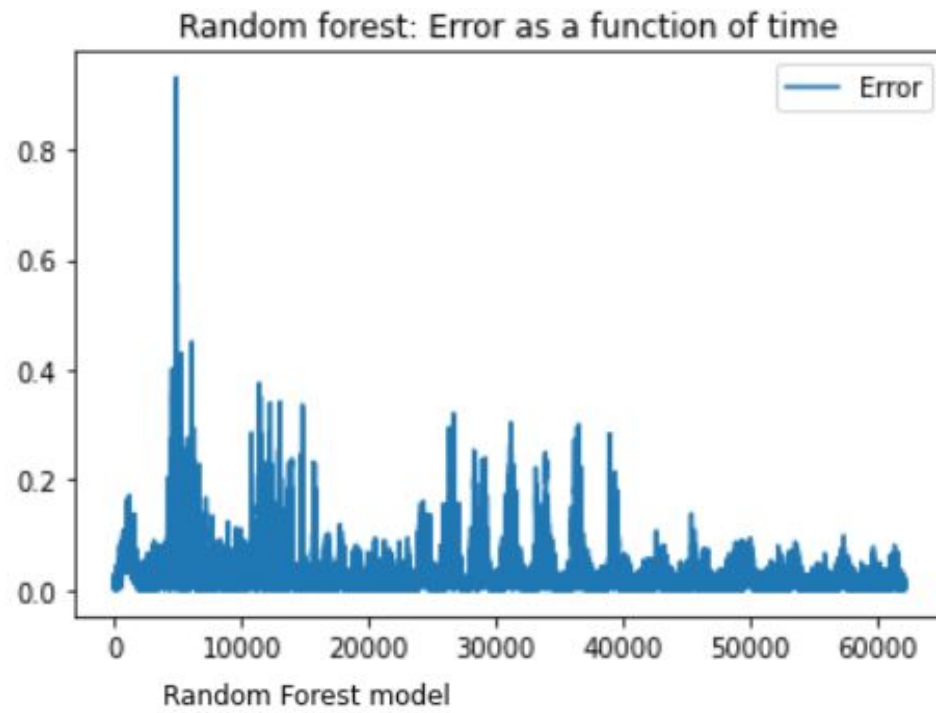
Including Past Inputs

$x_1(n)$, $x_1(n-t)$, $x_1(n-2t)$, $x_2(n)$, $x_2(n-t)$, $x_2(n-2t)$

$t = 2$

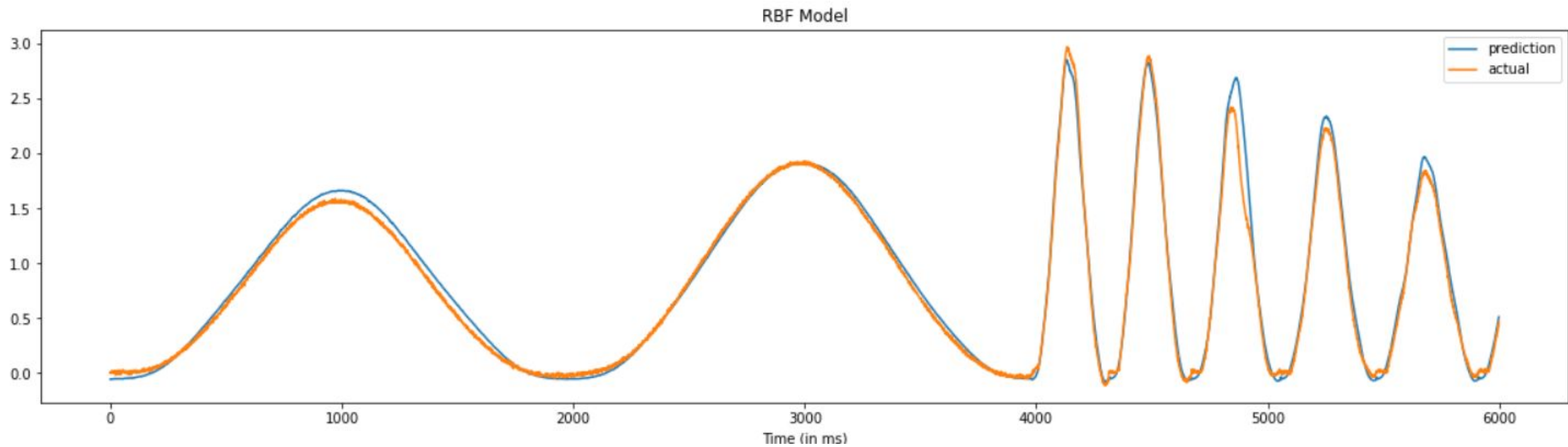
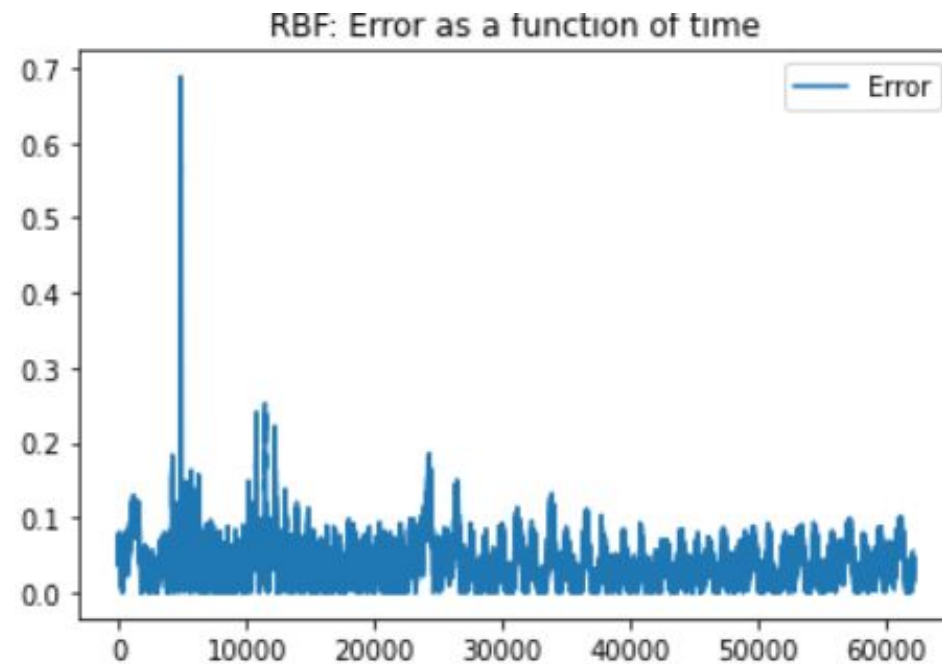
Random Forest

RMSE : 0.061248481



RBF

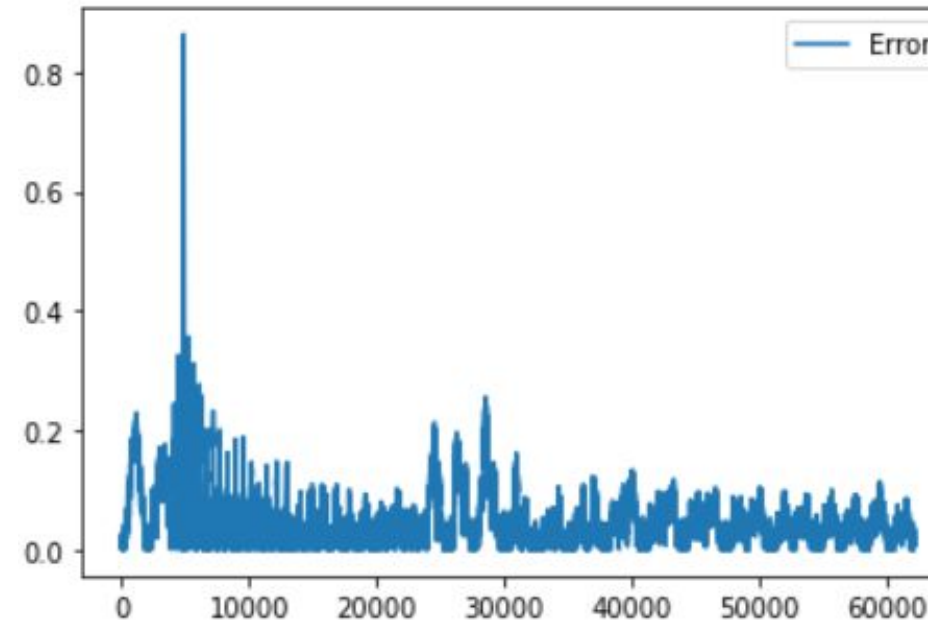
RMSE : 0.05222988



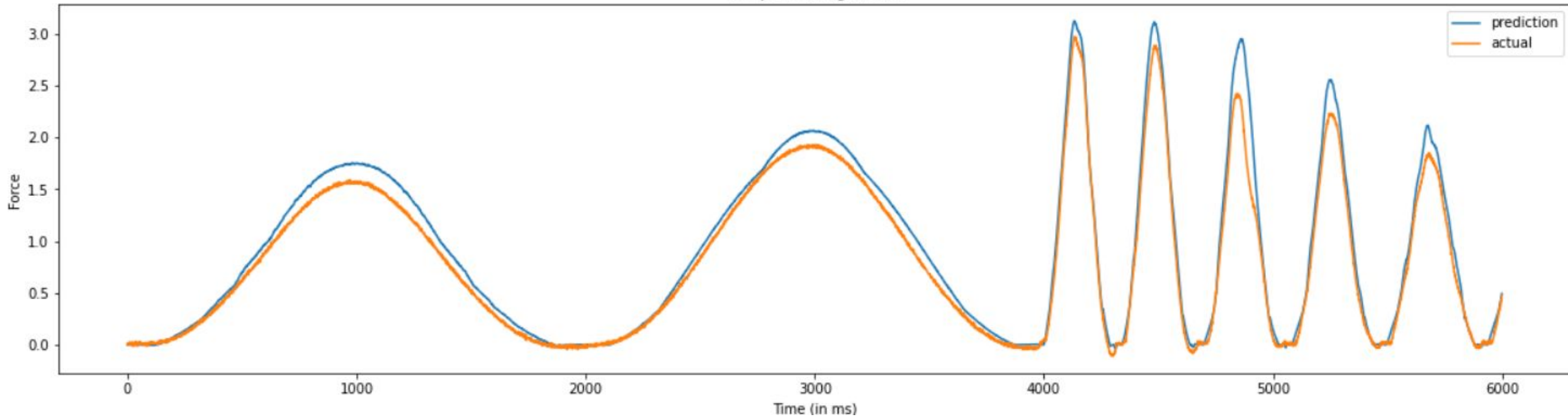
Deep Learning

RMSE : 0.0698973

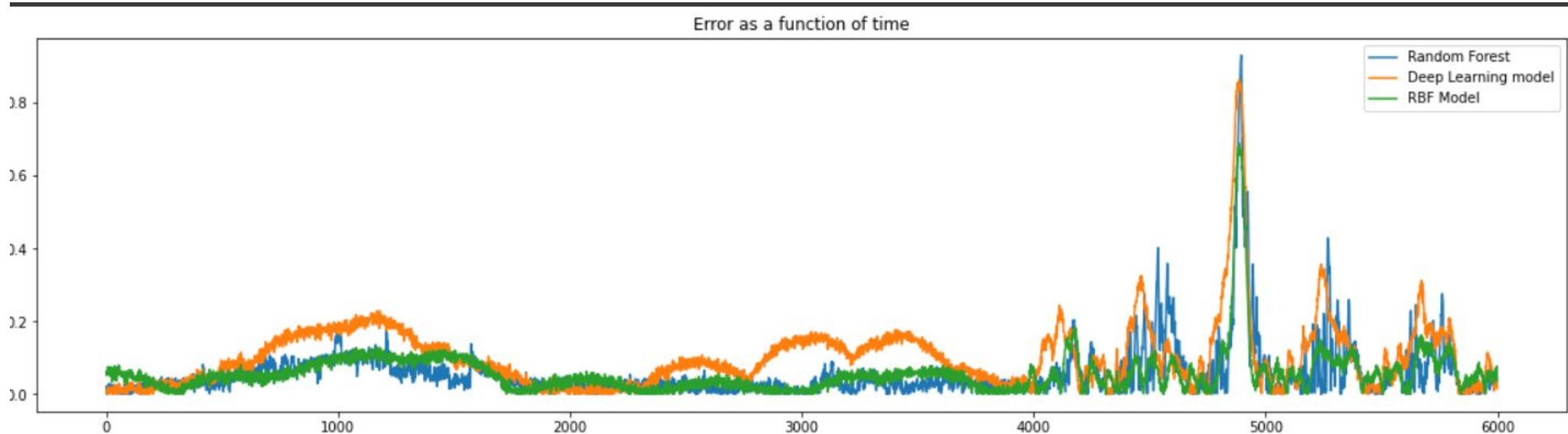
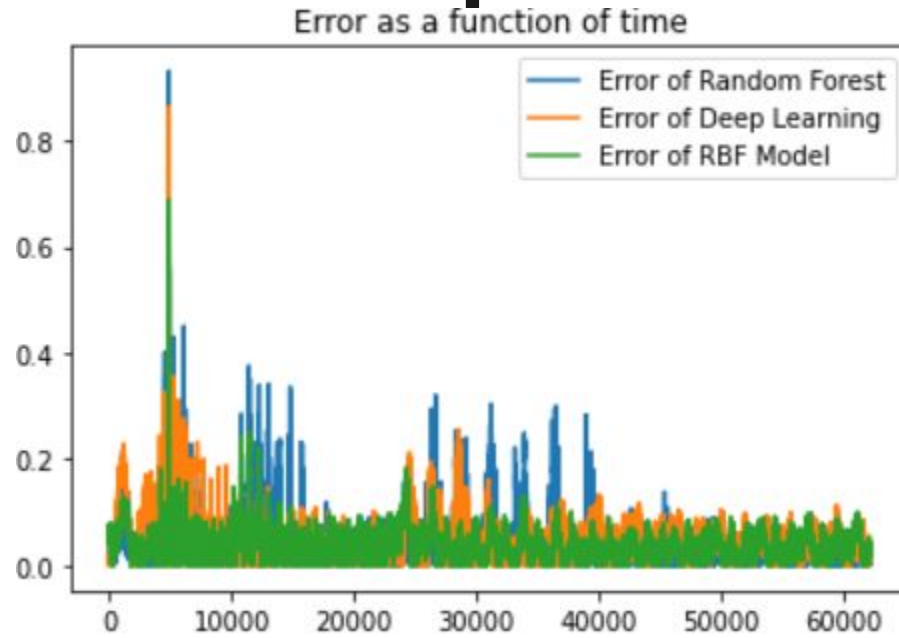
Deep learning model error as a function of time



Deep Learning model

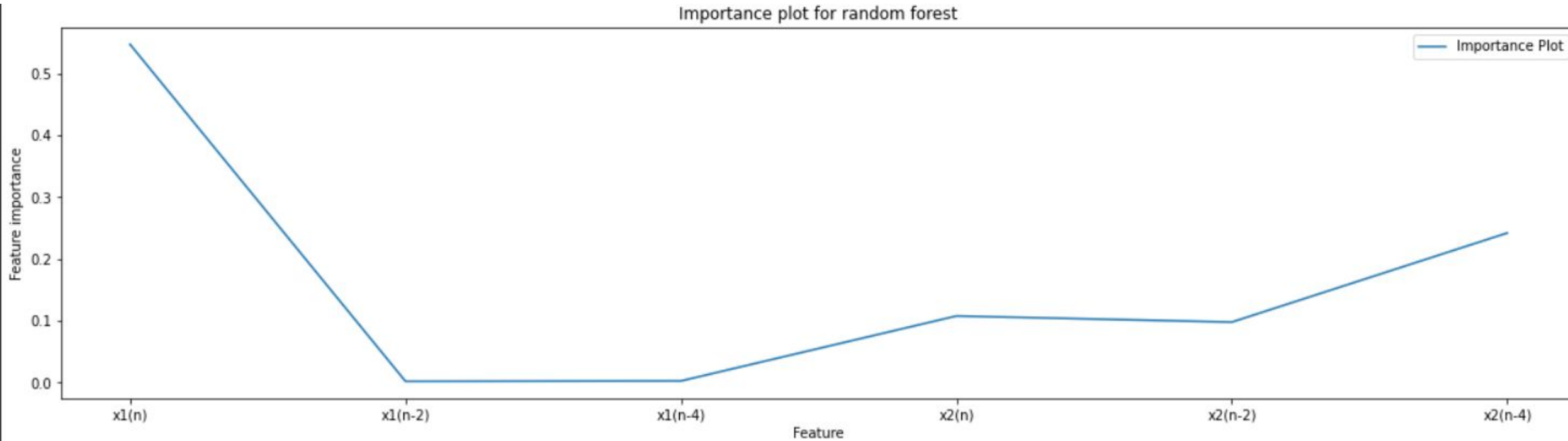


Error comparison of the three models



Feature importance

$x1(n)$, $x1(n-2)$, $x1(n-4)$, $x2(n)$, $x2(n-2)$, $x2(n-4)$



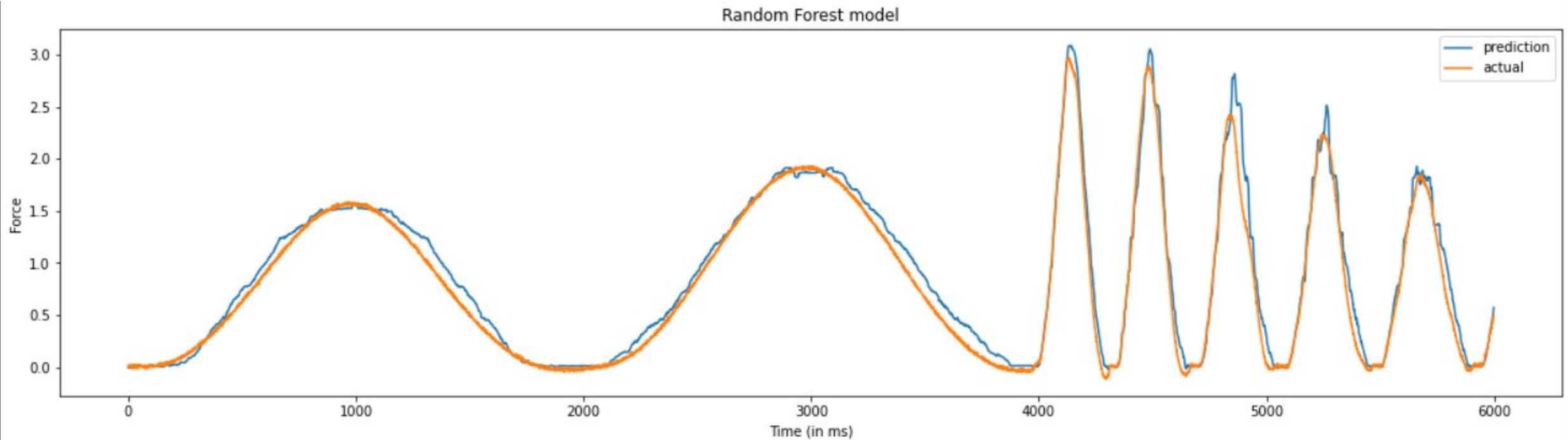
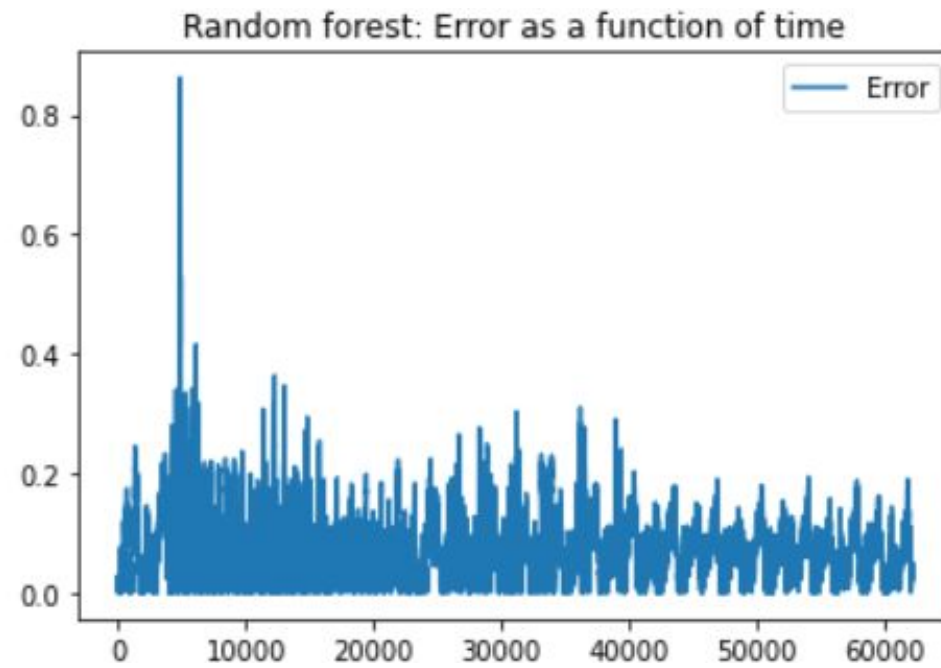
5 past inputs

$x_1(n), x_1(n-t), x_1(n-2t), x_1(n-3t), x_1(n-4t), x_2(n), x_2(n-t), x_2(n-2t), x_2(n-3t), x_2(n-4t)$

$t = 2$

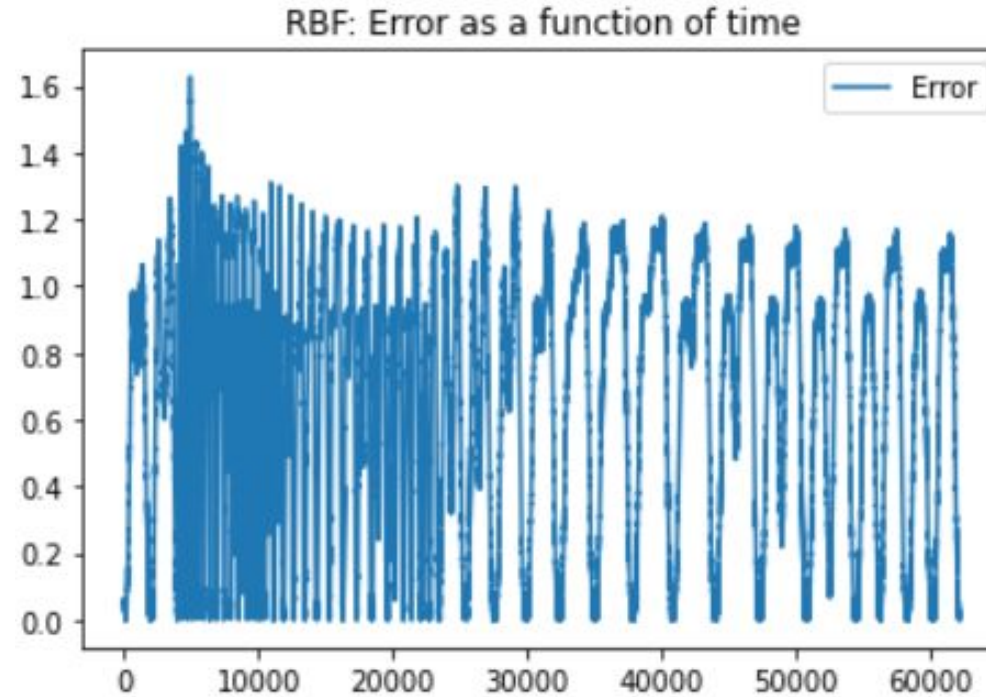
Random Forest

RMSE : 0.09221858

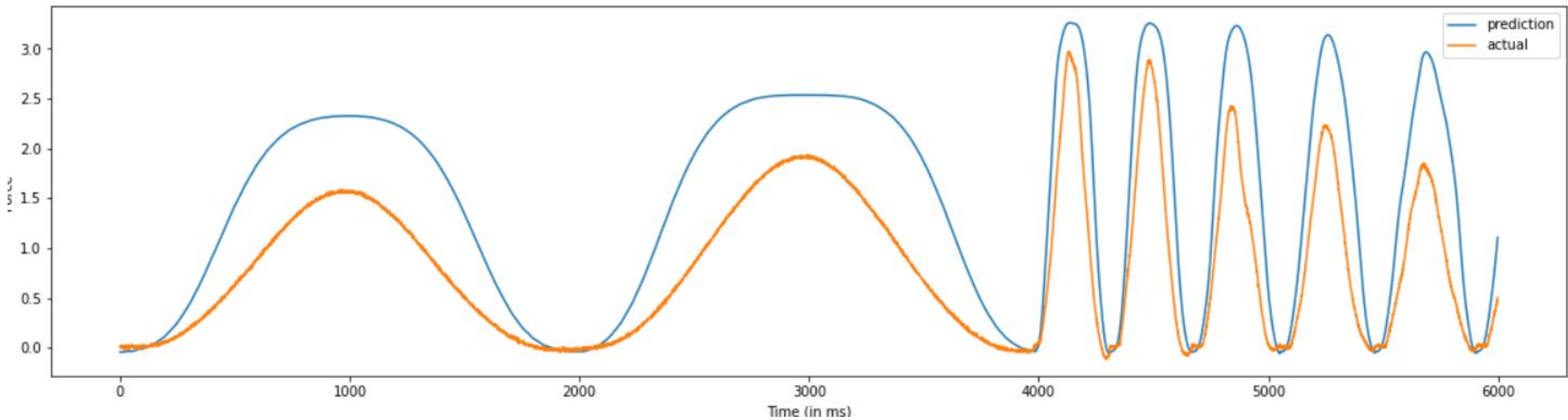


RBF

RMSE: 0.77797065



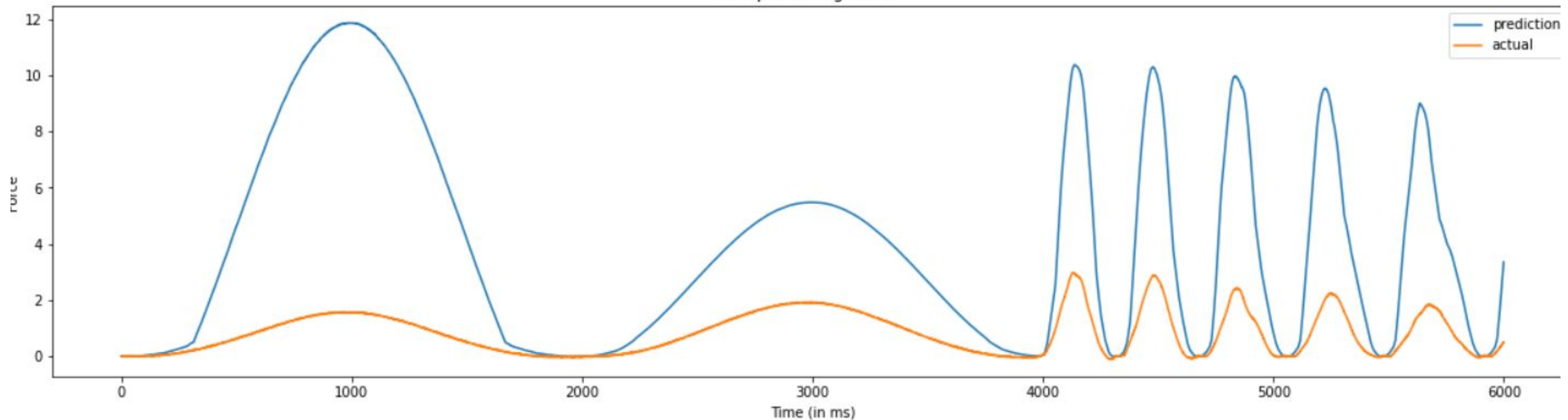
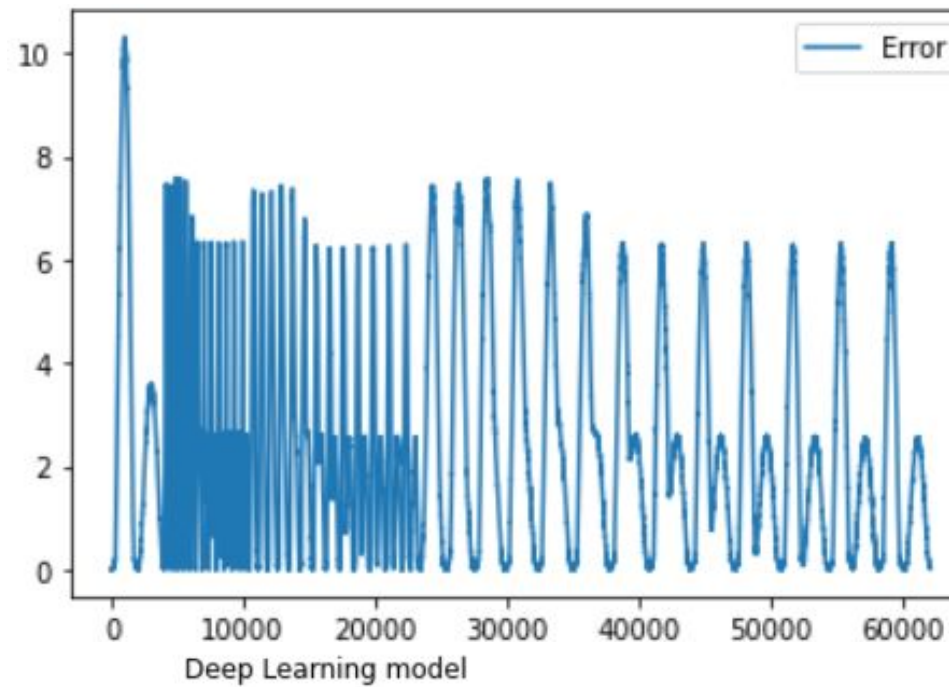
RBF Model



Deep Learning

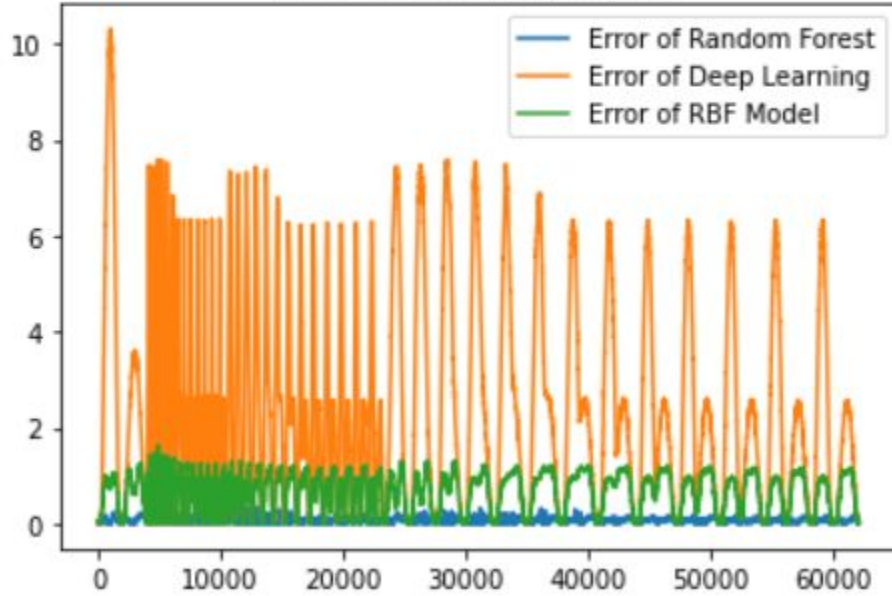
RMSE : 3.400057485

Deep learning model error as a function of time

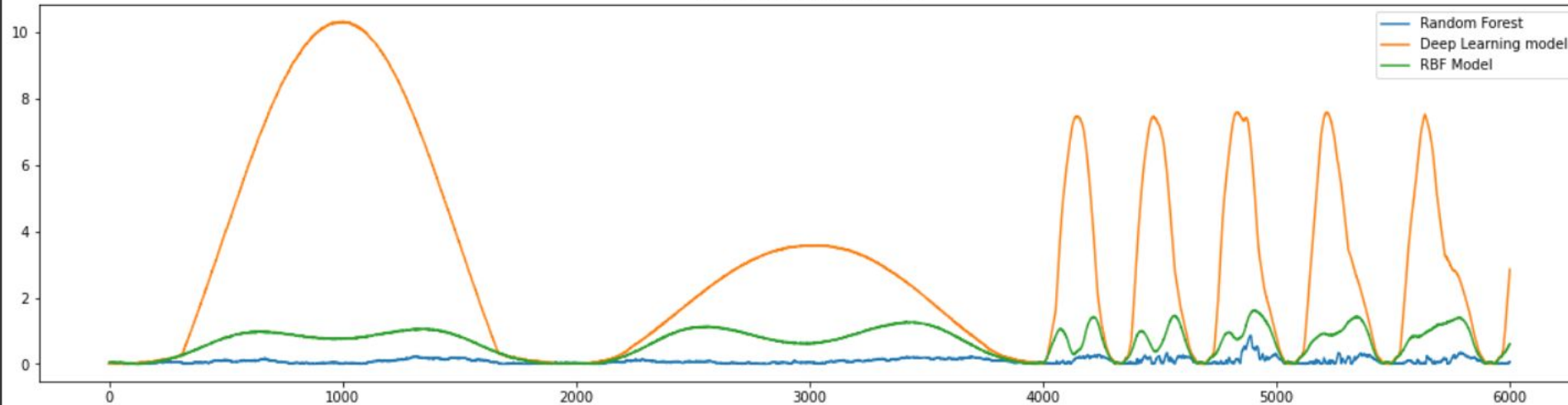


Error comparison of the three models

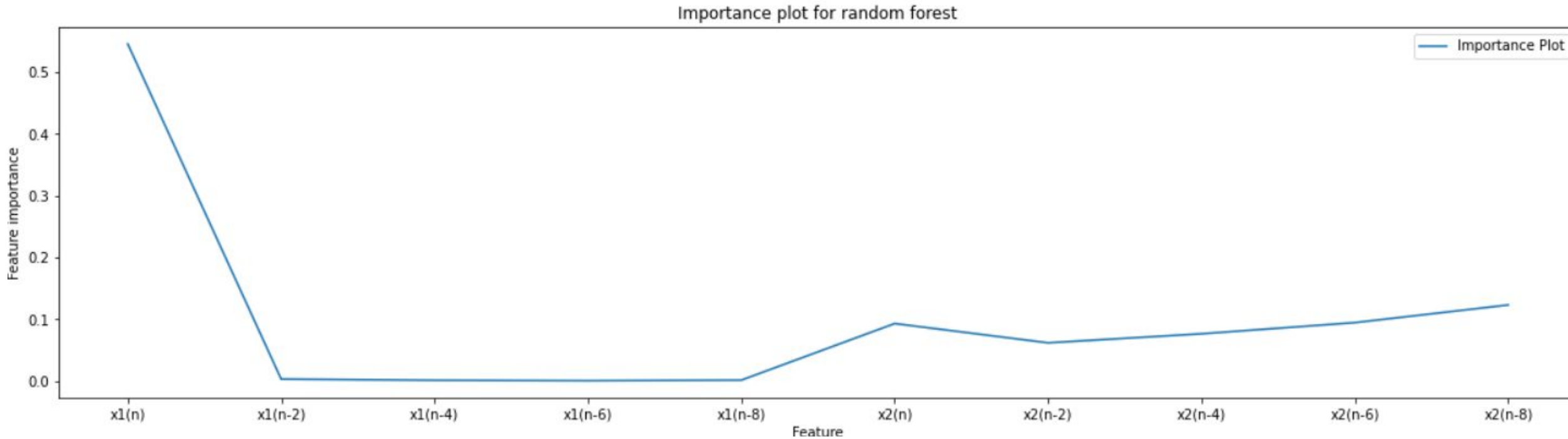
Error as a function of time



Error as a function of time



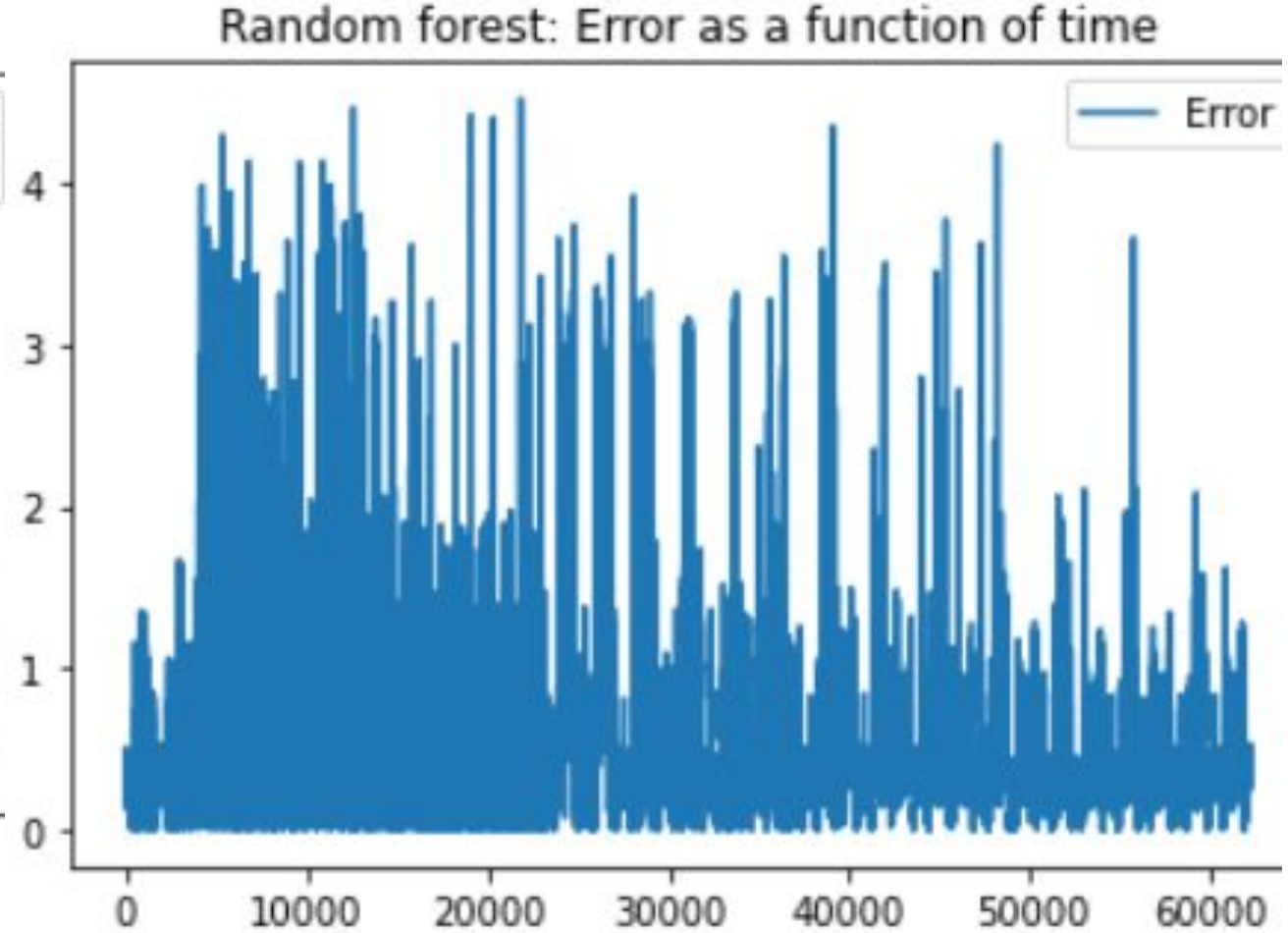
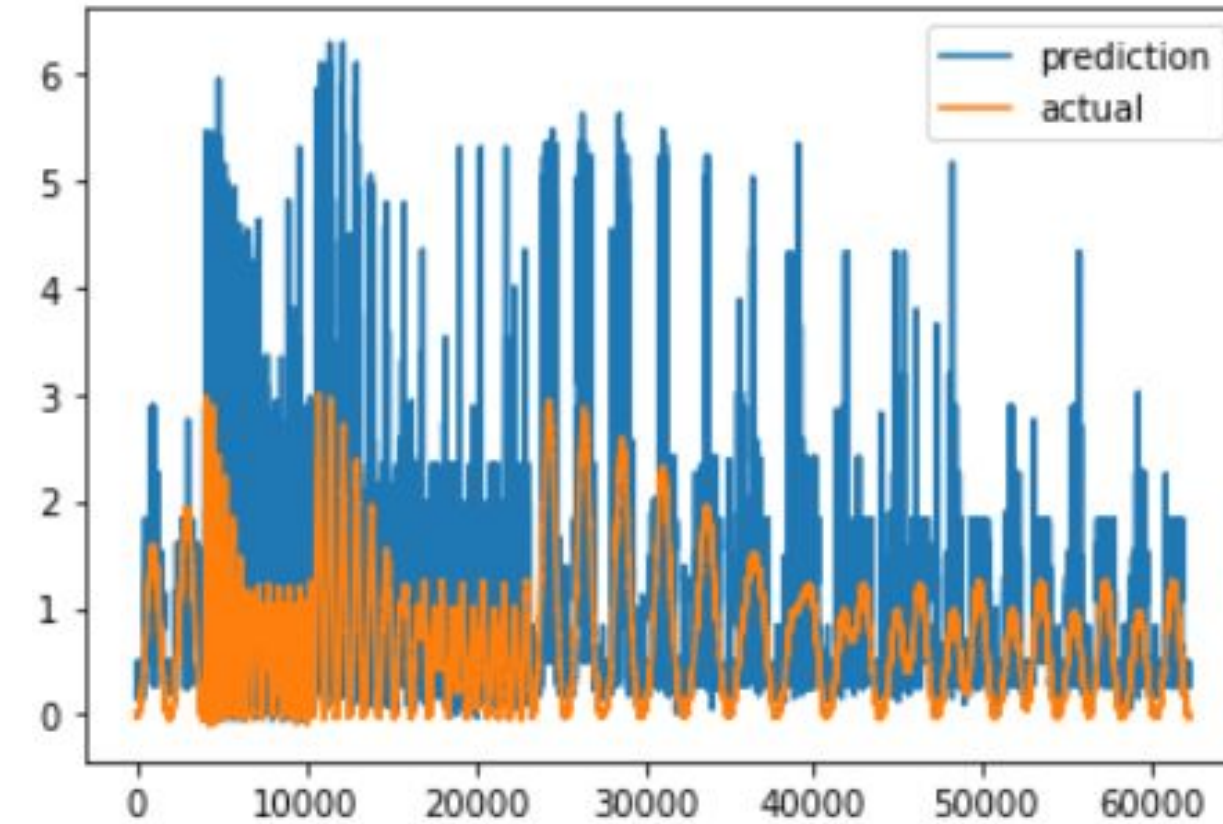
Feature importance (given 5 past inputs)



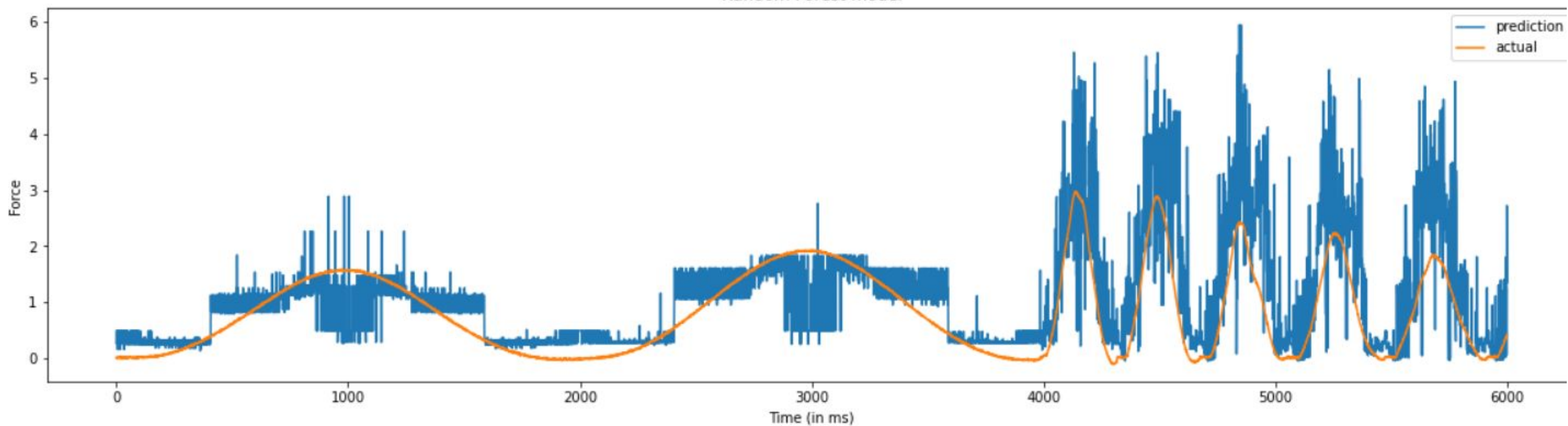
Considering velocity as an input feature (dx/dt)

Random Forest

RMSE : 0.5483032

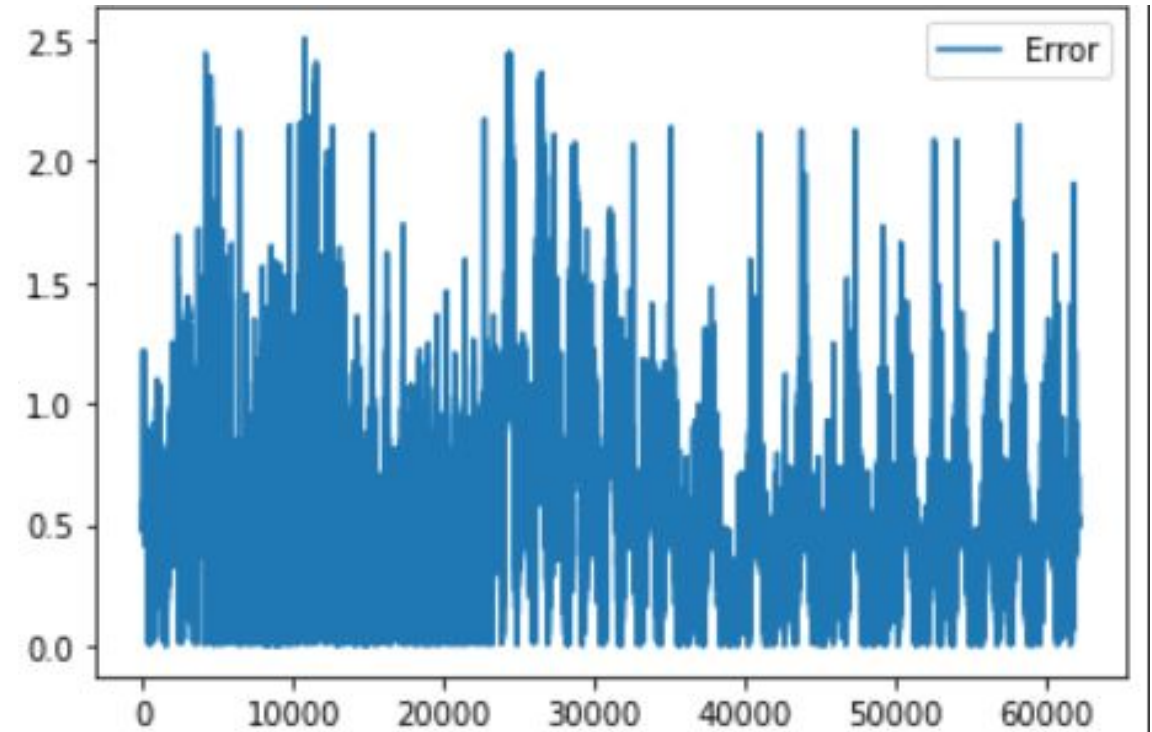
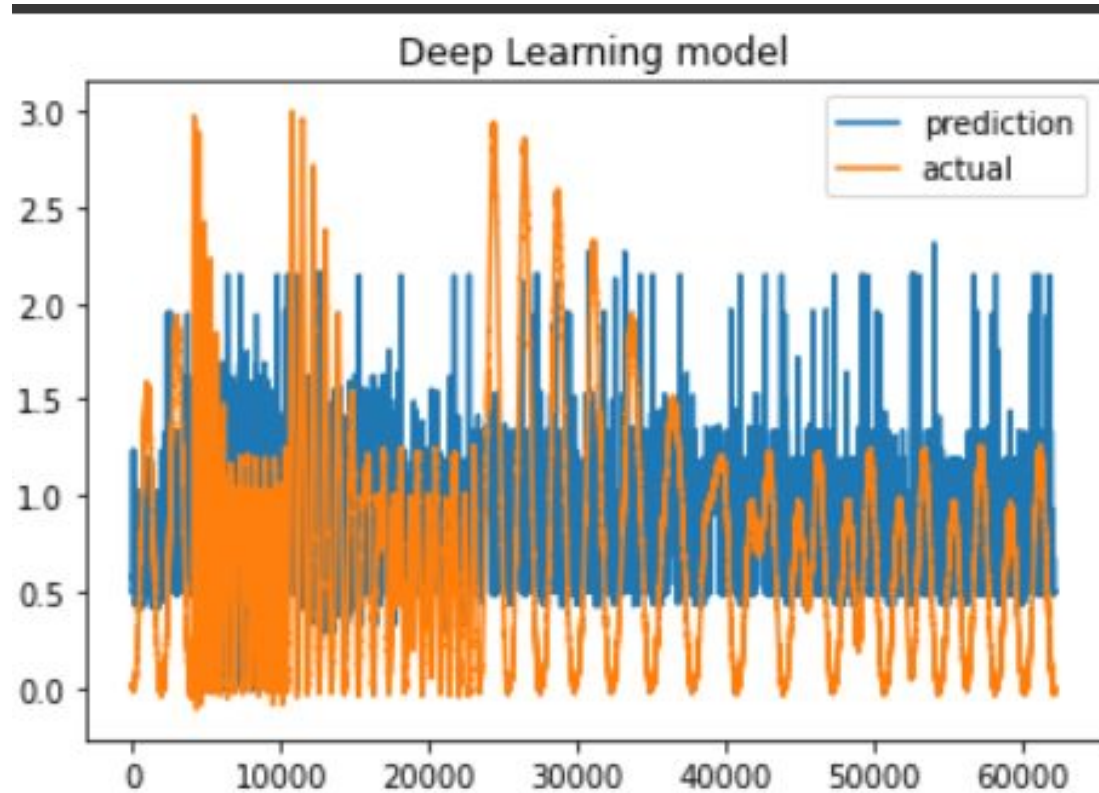


Random Forest model

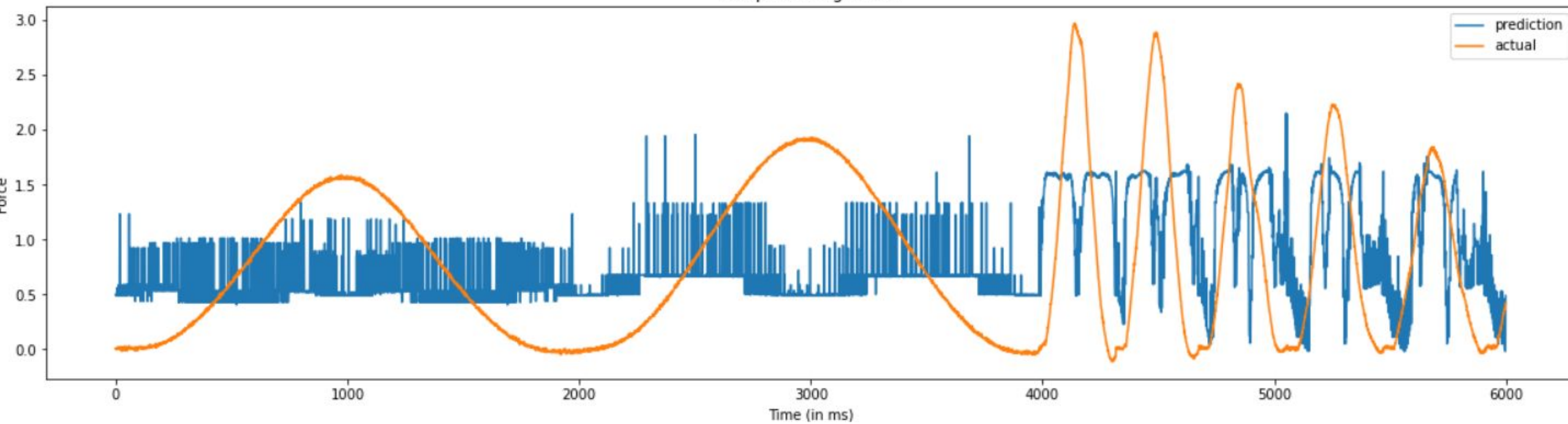


Deep Learning

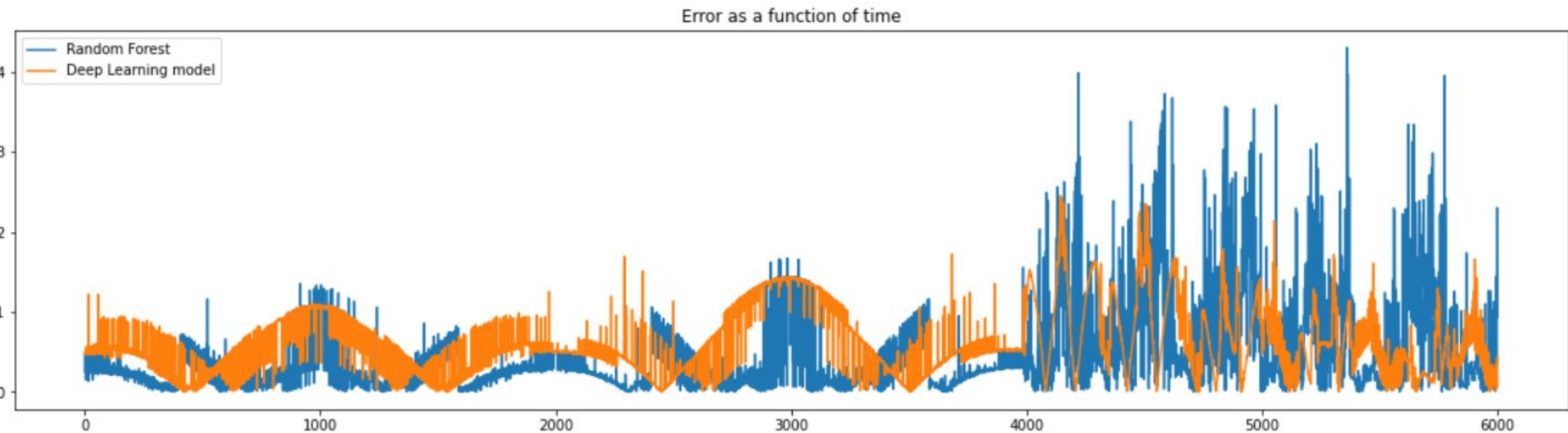
RMSE : 0.39842286



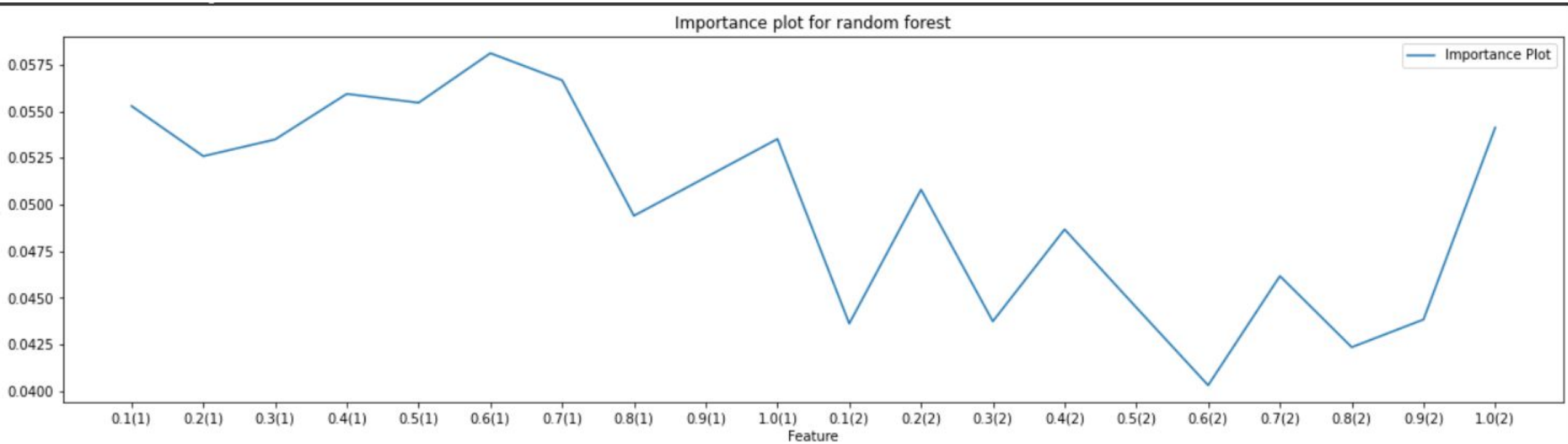
Deep Learning model



Error comparison b/w Random Forest vs Deep learning model



Feature Importance for different values of α

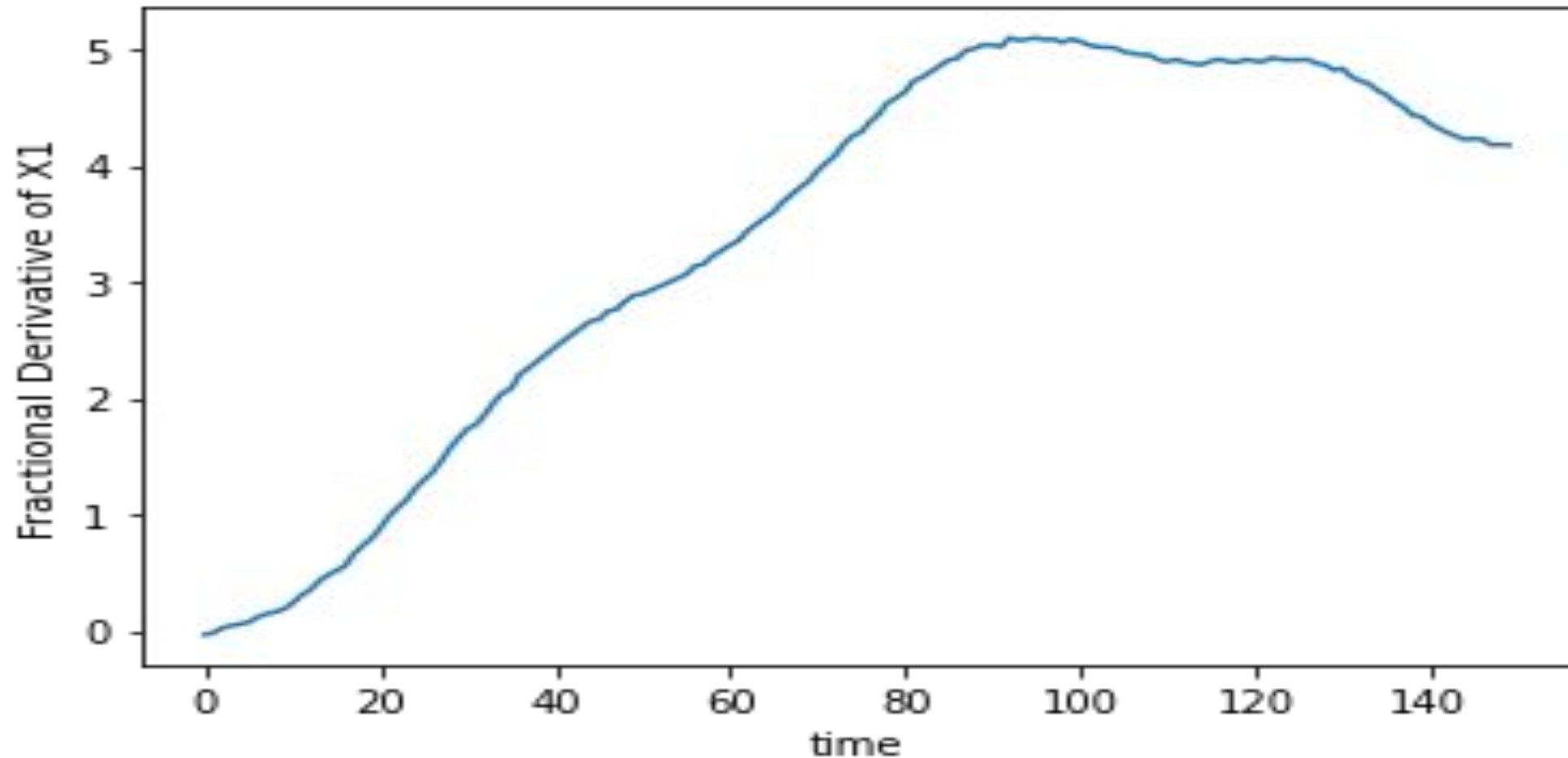


Results:

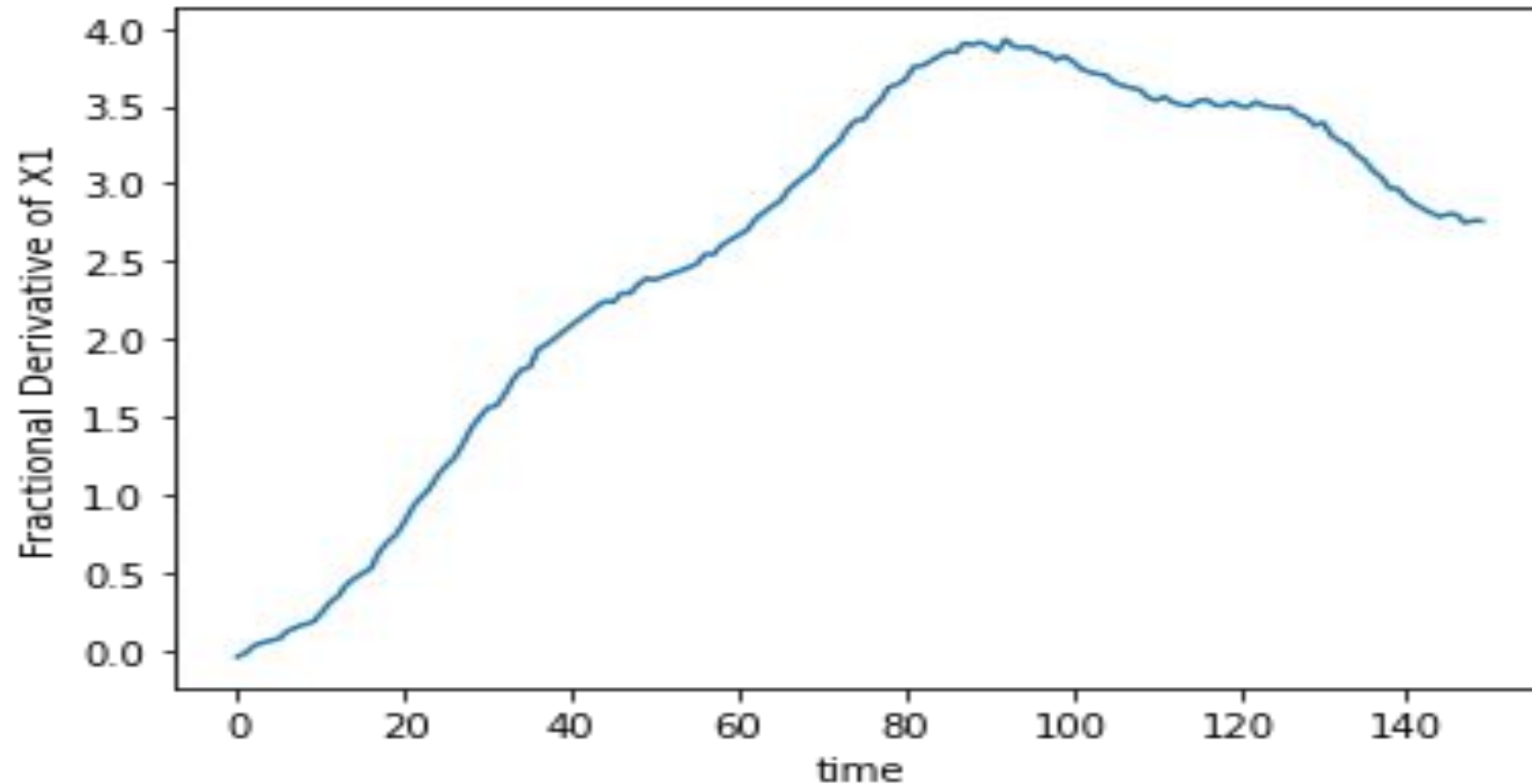
Using Fractional Derivatives of positions as input features with different values of α

$\alpha \in [0.5, 0.6, 0.7, 0.8, 0.9]$

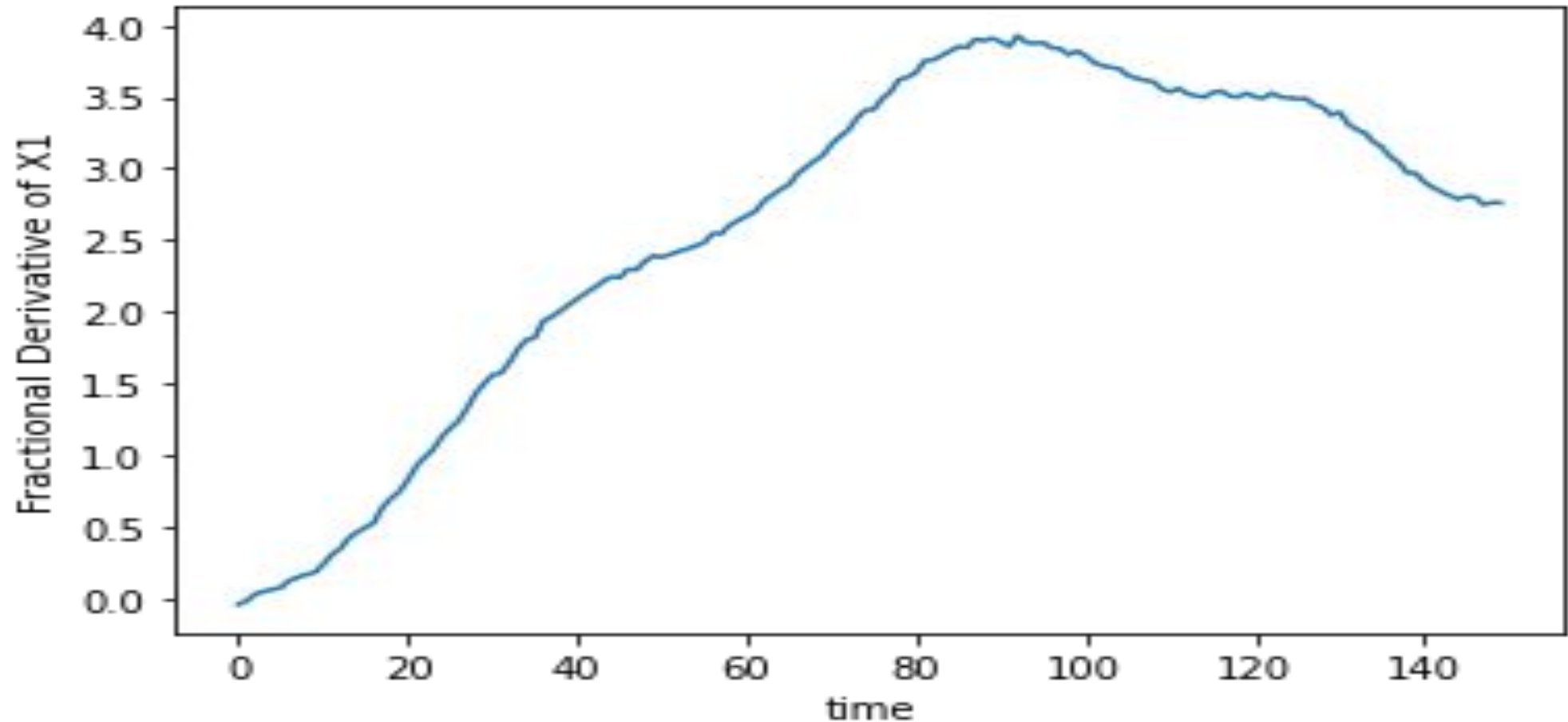
Fractional calculus of X_1 at order = 0.5 using python



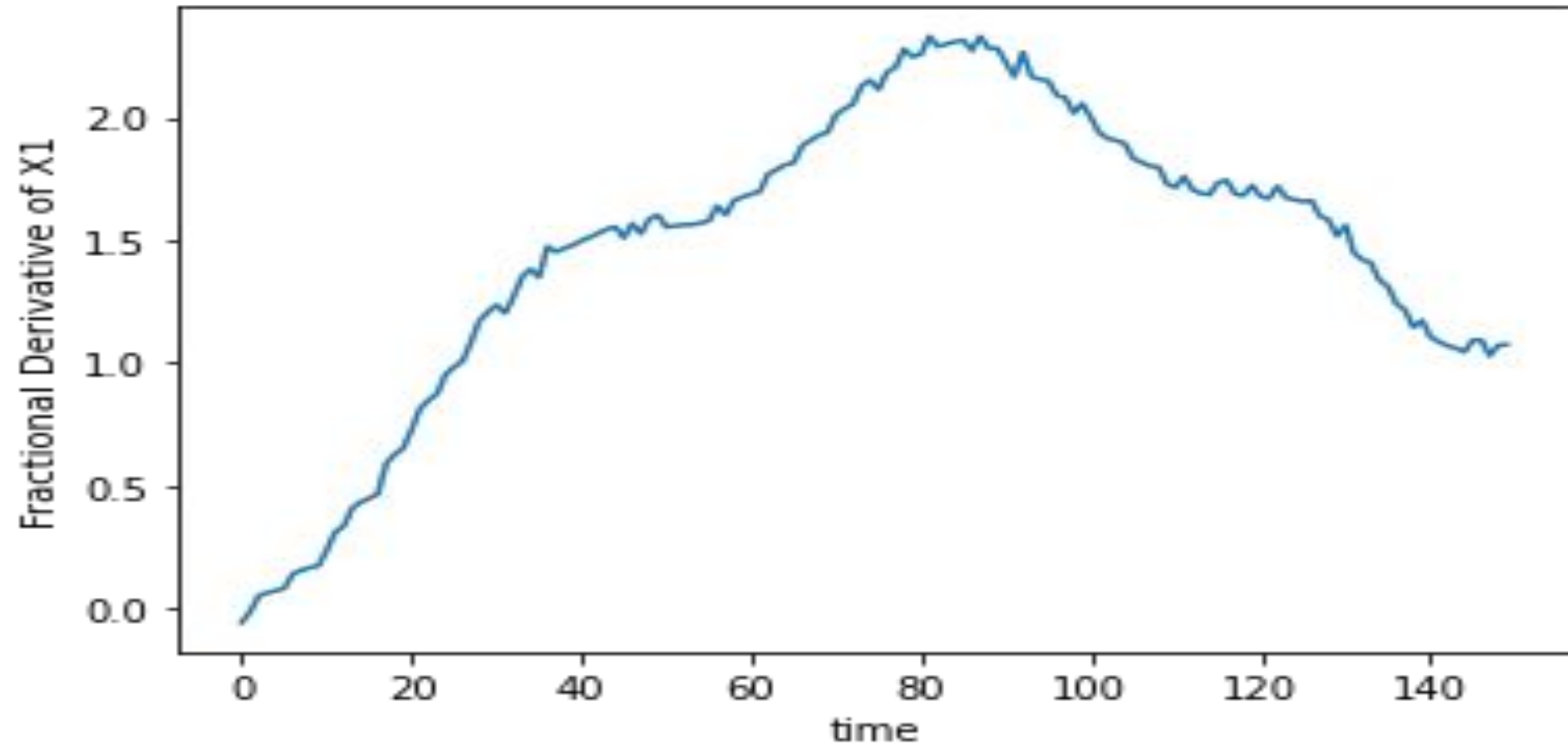
Fractional calculus of X_1 at order = 0.6 using python



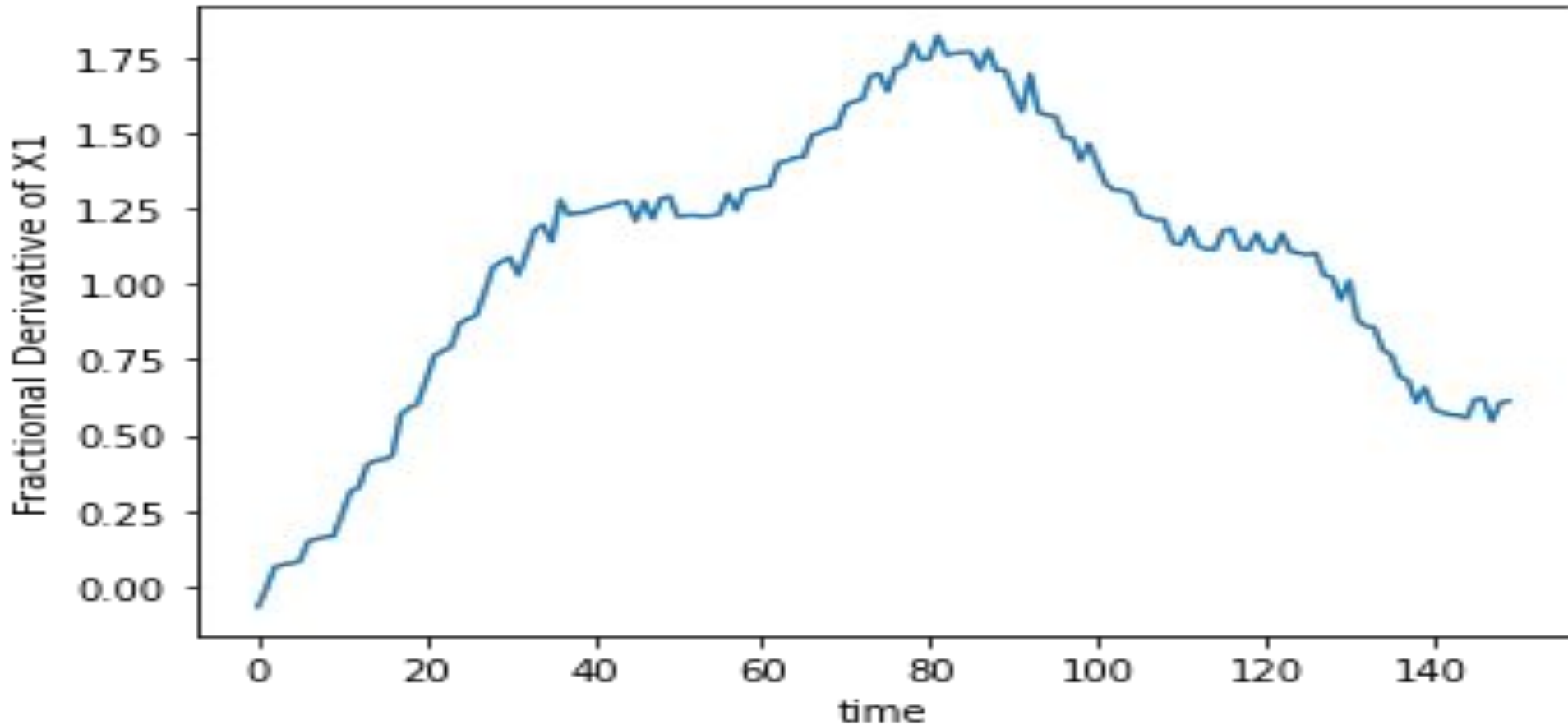
Fractional calculus of X_1 at order = 0.7 using python



Fractional calculus of X_1 at order = 0.8 using python



Fractional calculus of X_1 at order = 0.9 using python



Fractional calculus of X_2 at order = 0.9 using python

