PROGRAMMING ANALYSIS OF PRINCIPAL STRESS FOR A SIMPLY SUPPORTED BEAM DUE TO THE COMBINED EFFECT OF BENDING & TORSION

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PROGRAMMING ANALYSIS OF PRINCIPAL STRESS FOR A SIMPLY SUPPORTED BEAM DUE TO THE COMBINED EFFECT OF BENDING & TORSION

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ABSTRACT

The skeleton of this paper is to study and analyze the principal stress for a simply supported beam due to the effect of bending and torsion. Here the load is applied whole over the span and also uniform in nature. Flexural rigidity is one of the important parameters for calculating the bending stress for the simply supported beam where we are using the equation of bending, and also the deflection equation with radius of curvature. Torsional rigidity is another vital parameter for calculating the torsion for the same beam with rectangular cross section by using the equation of torsion. We can also determine the slope and deflection of the beam using some methods which are discussed in this paper. However, the main aim of the paper is to calculate the maximum and minimum principal stress due to the combined effect of bending and torsion for a simply supported steel I-beam with rectangular cross section.

Keywords: Bending stress, Shear stress, Torsional rigidity, Flexural rigidity, Maximum and Minimum principal stresses.

I. INTRODUCTION

Deflection is the degree to which a structural member undergoes bending when supposed to load (due to its deformation). It may refer to bending i.e., slope. Generally, the word deflection is used in the case of a beam. The beam is a straight structural member which undertakes load on it and mostly used in the field of civil engineering. When the weight is applied on the beam, it undergoes bending (deformation) and on applying load, the resistance to bear the load by the beam fails and may lead to failure of the beam. A beam can be made of same material (homogeneous) or different materials (composite). The beam is a flat physical member which resists the load upon it.

Generally, a beam bends when load is applied and upon applying load continuously, failure occurs. In this paper, we will calculate the maximum and minimum principal stress based on the bending & the torsion equations. Also, we can know the variations of flexural rigidity, torsional rigidity, shear stress based on load applied and deflection on the beam.

One of the important aspects in this paper is, programming “C” language has been used for the derivation of the equation for bending and torsion. According to the code, algorithm has also been mentioned so that one can easily understand the code which was written.

Objective: The main objective of this paper is to determine the maximum and minimum principal stress due to the effect of bending and torsion for statically determinate steel beam with both ends hinged having rectangular cross section.

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II. MATERIALS AND METHODS

The method to calculate the deflection is mentioned below; we can calculate deflection based on the following methods:

1. Moment Area Method

This method is used to find slant (slope) &bent (deflection) at a particular point on the beam due to bending from bending moment diagram.

$slope = \int [M/(E*I)] *dx$

$deflection= slope*x$

Where x=distance from centroidal axis

For a simply supported beam:

1) Point load at its mid span

$slope = [(W*L^2) /16*E*I]$

$deflection = [(W*L^3)/48*E*I]$

2) Uniformly distributed load whole over the span

$slope = [(W*L^3) /24*E*I]$

$deflection = [(5*W*L^4)/384*E*I]$

2. Double Integration Method

This method is useful for solving slope and deflection of a beam:

$$[(E*I) *(d^2*y/d*x^2)] = \iint(-M)$$

3. Conjugate-Beam Method

In this method, we will consider an imaginary beam which should be a replica for the original beam i.e., all the dimensions are approximately equal. The load applied at any point on the beam can be determined by the ratio of bending moment to the flexural rigidity.

$W = M/(E*I)$

4. Method of Superposition

According to this theory, total deflection of a beam is the algebraic sum of individual deflections.

$$y=y_1+y_2+y_3+y_4+\ldots\ldots\ldots\ldots +y_n$$

From these the first two are commonly used.

Consider a beam which is subjected to intensity of loading WkN/m acting whole over the span which has a radius(R) and change in length(y).

Algorithm for bending moment equation:

Step 1: Start

Step 2: Take the value of load W

Step 3: Take the value of length of the beam L

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Step 4: Take the value of width of the beam b

Step 5: Take the value of depth of the beam d

Step 6: Take the value of elasticity E

Step 7: Calculate the depth from centroidal axis y=(d/2)

Step 8: Calculate the moment of inertia I = $(b*d^3)/12$

Step 9: Calculate the moment M = $(W*L^2)/4$

Step 10: Calculate bending stress sigma = (M*y)/I

Step 11: Calculate radius of curvature R = (E*I)/M

Step 12: Take A = (M/I), B=(sigma/y), C=(E/R)

Step 13: Use if conditional statement to check whether the bending moment equation is satisfied or not

Step 14: Display the statement

Step 15: Stop

Code for equation of bending:

```
#include<stdio.h>

int main ()

{

float W, L, b, d, sigma, E, R, A, B, C, I, y, M;

printf ("Enter the values of W, L, b, d, E");

scanf ("%f%f%f%f%f", &W, &L, &b, &d, &E);

y=d/2;

I=(b*d*d*d)/12;

M=(W*L*L)/4;

sigma=(M*y)/I;

R=(E*I)/M;

A=M/I;

B=sigma/y;

C=E/R;

if (A==B && B==C)

{printf ("The bending moment equation is True");}

Else
```

```
{printf ("The bending moment equation is False");}

return 0;

}
```

Algorithm for equation of torsion:

Step 1: Start

Step 2: Take the value of load W

Step 3: Take the value of length of the beam L

Step 4: Take the value of width of the beam b

Step 5: Take the value of depth of the beam d

Step 6: Take the value of modulus of rigidity c

Step 7: Take the value of angle of twist Theta

Step 8: Calculate the polar moment of inertia Ip= $(b*d^3)/36$

Step 9: Calculate the torsional moment T = (W*L)/2

Step 10: Calculate shear stress Tou = (T*R)/Ip

Step 11: Take A = (T/Ip), B=(Tou/R), C=(c*Theta)/L

Step 12: Use if conditional statement to check whether the torsion equation is satisfied or no

Step 13: Display the statement

Step 14: Stop

Code for equation of torsion:

```
#include<stdio.h>

int main ()

{

  float W, L, b, d, Tou, c, Theta, R, A, B, C, Ip, T;

printf ("Enter the values of load W, L, b, d, c, Theta");

scanf ("%f%f%f%f%f%f", &W, &L, &b, &d, &c, &Theta);

   Ip=(b*d*d*d)/36;

   T=(W*L)/2;

Tou = (T*R)/Ip;

   A=T/Ip;

   B= Tou/R;
```

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```
    C=(c*Theta)/L;
    if (A==B && B==C)
{
printf ("The torsion equation is True");
}
else
{
printf ("The torsion equation is False");
}
  return 0;
}
```

Formulae:

1. Deflection = $(W*L^3)/(48*E*I)$

➔$(E*I) = (W*L^3)/(48*Deflection)$

2. Deflection = $(L^2)/8R$

➔$R = L^2/(8*Deflection)$

3. sigma= (E*y)/R

4. I= $(b*d^3)/12$

5. $I_p = (b*d^3)/36$

6. b=d/2

7. Strain = L/del L

8. Tou = (W/2) *dL

9. bending moment of the beam is = $(W*L^2)/4$

10. $sigma_{max} =\{(sigma/2) +[(sigma/2)^2+(Tou^2)]^{1/2}\}$

$=\{32(M)/ (2*(3.14) *d^3) +[(32*M/2*pi*d^3)^2) +(16*T/ (3.14) *d^3)^2]^{(1/2)}\}$

$= \{(16/(3.14*d^3) *[M+(M^2+Tou^2)^{(1/2)}]\}$

11. $sigma_{min} = \{(16/ (3.14) *d^3) *[M-(M^2+Tou^2)^{(1/2)}]\}$

In this paper, we will calculate the maximum and minimum principal stress with varying load and deflection. From the graph strain vs flexural rigidity & bending stress vs radius of curvature, firstly

1) Calculate the strain from load and deflection.

2) Take 3 conditions

(i) Varying load and deflection

(ii) Constant load and varying deflection

(iii)Varying load and constant load

3) To compute flexural rigidity, take moment of inertia i.e. $(b*d^3)/12$ and multiply with E i.e. 200GPA (as the beam is steel).

4) Use bending moment equation and calculate radius of curvature for the beam and from that calculate bending stress (sigma) and draw a graph for sigma vs radius of curvature.

III. RESULTS AND DISCUSSION

1. The graph for flexural rigidity vs strain, is linear i.e., parallel to X-axis. With the variation in strain, the flexural rigidity is constant due to the variation in both load and deflection. The graph for radius of curvature (R) vs bending stress is a downward slope. As (R) is decreasing, the bending stress is increasing with the increase in load on the beam.

Figure 1.1: The values for L/DELTA(L) & EI1

X-Axis	Y-Axis
L/DELTA(L)	EI1
1000	20833.33333
500	20833.33333
333.3333333	20833.33333
250	20833.33333
200	20833.33333
166.6666667	20833.33333
142.8571429	20833.33333
125	20833.33333
111.1111111	20833.33333
100	20833.33333

Graph 1.1: The graph for L/DELTA(L) vs EI1

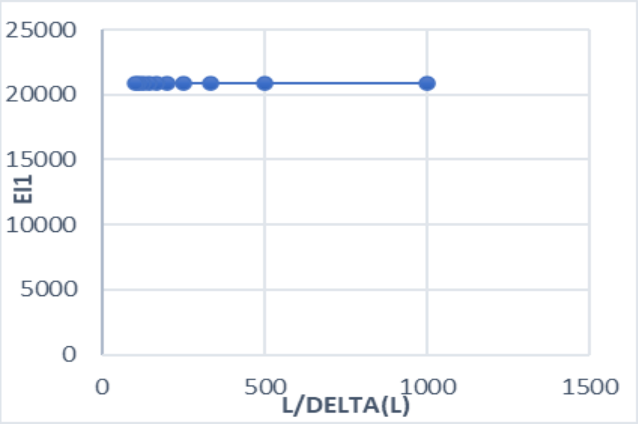

Figure 1.2: The values for L/DELTA(L) & EI1

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X-Axis	**Y-Axis**
L/DELTA(L)	EI2
1000	13020.83333
500	13020.83333
333.3333333	13020.83333
250	13020.83333
200	13020.83333
166.6666667	13020.83333
142.8571429	13020.83333
125	13020.83333
111.1111111	13020.83333
100	13020.83333

Graph 1.2: The graph for L/DELTA(L) vs EI2

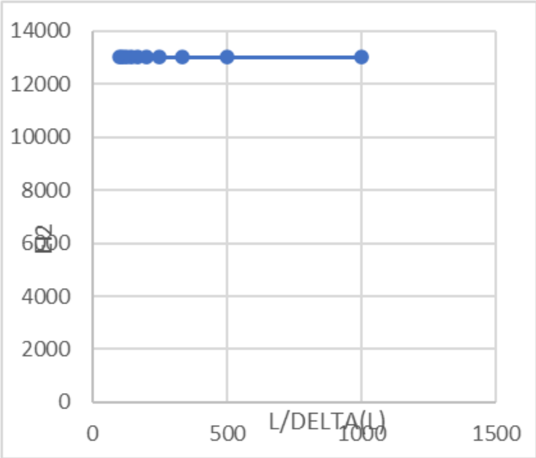

Figure 1.3: The values for bending stress & R

BENDING STRESS(N/M^2)	RADIUS(R)(M)
5656854249	0.125
11313708499	0.0625
16970562748	0.041666667
22627416998	0.03125
28284271247	0.025
33941125497	0.020833333
39597979746	0.017857143
45254833996	0.015625
50911688245	0.013888889
56568542495	0.0125

Graph 1.3: The graph for bending stress vs R

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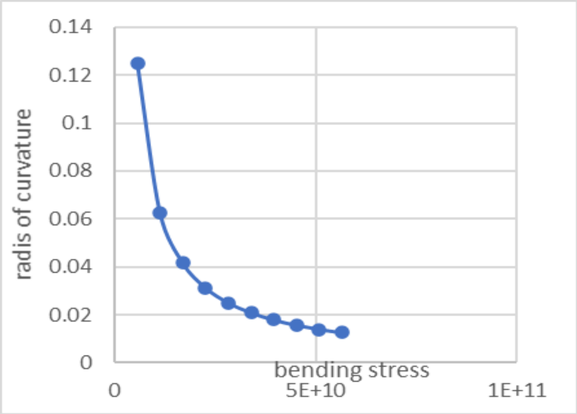


2. The graph for flexural rigidity (EI) vs strain is linear i.e., passing through origin. This is because with the change in strain, the value of flexural rigidity is being reduced as the constant loading condition is applied on the beam. The graph for radius of curvature (R) vs bending stress is a downward slope. At constant load with change in deflection, all the parameters (EI, CI_p, sigma, R) are varying. So, with the change in radius of curvature (R), bending stress also varies.

Figure 2.1: The values for L/DELTA(L) & EI1

X-Axis	Y-Axis
L/DELTA(L)	**EI1**
1000	20833.33
500	10416.67
333.33333	6944.444
250	5208.333
200	4166.667
166.66667	3472.222
142.85714	2976.19
125	2604.167
111.11111	2314.815
100	2083.333

Graph 2.1: The graph for L/DELTA(L) vs EI1

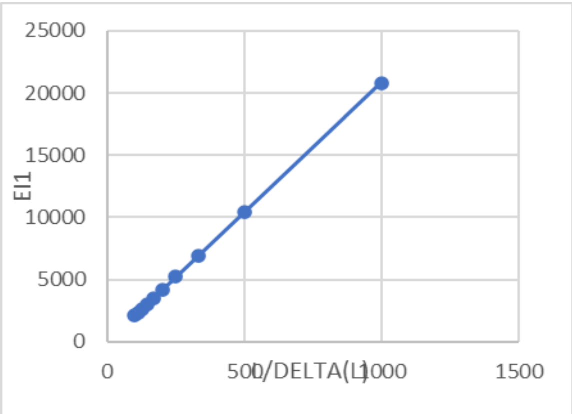


Figure 2.2: The values for L/DELTA(L) & EI2

X-Axis	Y-Axis
L/DELTA(L)	EI2
1000	13020.8333
500	6510.41667
333.33333	4340.27778
250	3255.20833
200	2604.16667
166.66667	2170.13889
142.85714	1860.11905
125	1627.60417
111.11111	1446.75926
100	1302.08333

Graph 2.2: The graphfor L/DELTA(L) vs EI2

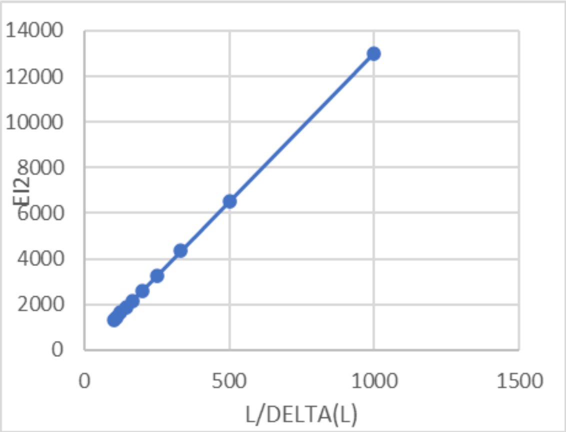
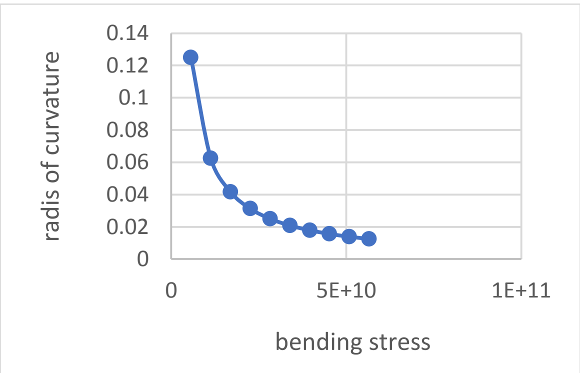


Figure 2.3: The values for bending stress&R

BENDING STRESS(N/M^2)	RADIUS(R)(M)
5656854249	0.125
9513656920	0.0625
12894839182	0.041666667
16000000000	0.03125
18914832180	0.025
21686448087	0.020833333
24344368829	0.017857143
26908685288	0.015625
29393876913	0.013888889
31810829151	0.0125

Graph 2.3: The graphbetweenbending stress vs R



3. As the deflection is constant, the strain is also constant. So, the graph for flexural rigidity (EI) vs strain, is linear i.e., parallel to Y-axis. The graph for radius of curvature (R) vs bending stress is a line parallel to X-axis as the (R) is constant with the change in bending stress.

Figure 3.1: The values for L/DELTA(L) & EI1

X-Axis	Y-Axis
L/DELTA(L)	**EI1**
1000	20833.33333
1000	41666.66667
1000	62500
1000	83333.33333
1000	104166.6667
1000	125000
1000	145833.3333
1000	166666.6667
1000	187500
1000	208333.3333

Graph 3.1: The graph for L/DELTA(L) vs EI2

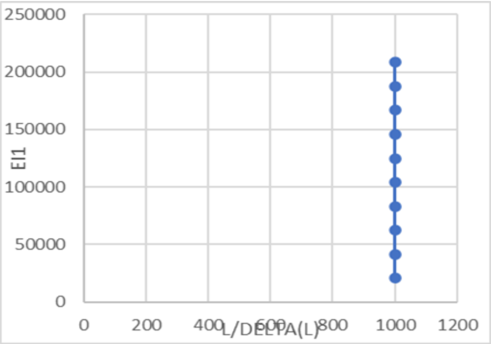


Figure 3.2: The values for L/DELTA(L) & EI2

X-Axis	Y-Axis
L/DELTA(L)	**EI2**
1000	13020.83333
1000	26041.66667
1000	39062.5
1000	52083.33333
1000	65104.16667
1000	78125
1000	91145.83333
1000	104166.6667
1000	117187.5
1000	130208.3333

Graph 3.2: The graph for L/DELTA(L) vs EI2

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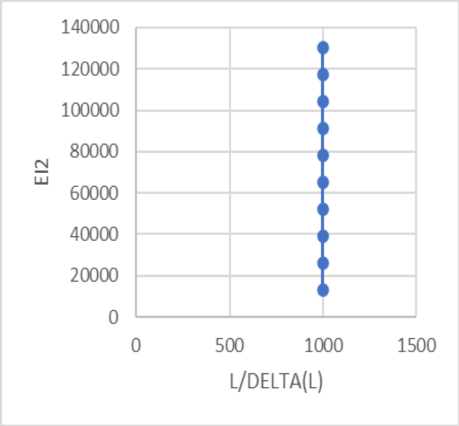
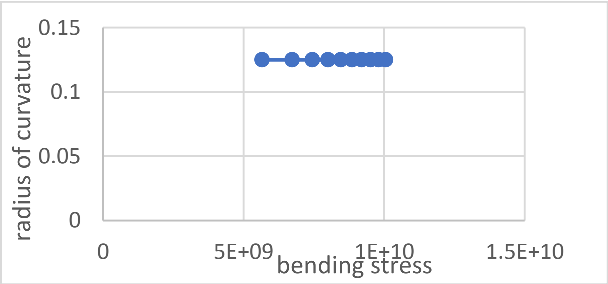


Figure 3.3: The values for bending stress & R

BENDING STRESS(N/M^2)	RADIUS(R)(M)
5656854249	0.125
6727171322	0.125
7444838873	0.125
8000000000	0.125
8458970108	0.125
8853455358	0.125
9201306535	0.125
9513656920	0.125
9797958971	0.125
10059467437	0.125

Graph 3.3: The graph for bending stress vs R



IV. CONCLUSIONS

1. For the simply supported beam of varying load and deflection, the values of flexural rigidity (EI) and torsional rigidity (CIp) are constant. Also, the values of strain and radius of curvature (R) are being reduced upon increasing the load. The value of bending stress is being increased with increase in the load as they are directly proportional to each other. The graph for flexural rigidity (EI) vs strain, is linear i.e., parallel to X-axis. The graph for radius of curvature (R) vs bending stress is a downward slope.

Table 1:

W(kN)	L(m)	DEFLECTION(L)(mm)	SIGMA(MAX)1	SIGMA(MAX)2	SIGMA(MIN)1	SIGMA(MIN)2
1	1	1	0.573949289	787.3104101	-0.09847413	-135.0811108
2	1	2	1.007072548	172.6804781	-0.05612223	-9.623153223
3	1	3	1.465004976	74.42996372	-0.038579498	-1.960041567
4	1	4	1.93116747	41.39162102	-0.029266833	-0.627289812
5	1	5	2.400916456	26.34750569	-0.023540659	-0.258333712
6	1	6	2.872526719	18.24243458	-0.019675764	-0.124954044
7	1	7	3.345221604	13.37837128	-0.01689549	-0.067569255
8	1	8	3.81860228	10.23073742	-0.014801006	-0.039654617
9	1	9	4.292443562	8.076989848	-0.013167129	-0.024776276
10	1	10	4.766608902	6.538558164	-0.011857309	-0.016265171

2. For the simply supported beam of constant load and varying deflection, the values of flexural rigidity (EI) and torsional rigidity (CIp) are decreased. Also, the value of strain and radius of curvature (R) are being reduced upon increasing the load. The value of bending stress is increased on increase in deflection. The graph for flexural rigidity (EI) vs strain is linear i.e., passing through origin. The graph for radius of curvature (R) vs bending stress is a downward slope.

Table 2:

W(kN)	L(m)	DELTA(L)(mm)	SIGMA(MAX)1	SIGMA(MAX)2	SIGMA(MIN)1	SIGMA(MIN)2
1	1	1	0.573949289	787.3104101	-0.09847413	-135.0811108
1	1	2	8.056580385	1381.443825	-0.448977837	-76.98522578
1	1	3	39.55513435	2009.60902	-1.04164645	-52.9211223
1	1	4	123.5947181	2649.063745	-1.873077342	-40.14654797
1	1	5	300.114557	3293.438211	-2.942582437	-32.29171399
1	1	6	620.4657713	3940.36587	-4.249964945	-26.99007357
1	1	7	1147.41101	4588.78135	-5.795152927	-23.17625457
1	1	8	1955.124368	5238.137559	-7.578115273	-20.30316378
1	1	9	3129.191357	5888.125599	-9.598836813	-18.06190492
1	1	10	4766.608902	6538.558164	-11.85730945	-16.26517071

3. For a simply supported beam with increasing load at constant deflection, the values of strain and radius of curvature (R) are constant. The values of flexural rigidity (EI), torsional rigidity (CIp) & bending stress are being increased with increase in load. The graph for flexural rigidity (EI) vs strain, is linear i.e., parallel to Y-axis. The graph for radius of curvature (R) vs bending stress is a line parallel to X-axis.

Table 3:

W(kN)	L(m)	DELTA(L)(mm)	SIGMA(MAX)1	SIGMA(MAX)2	SIGMA(MIN)1	SIGMA(MIN)2
1	1	1	0.573949289	787.3104101	-0.09847413	-135.0811108
2	1	1	0.071743661	98.41380127	-0.012309266	-16.88513885
3	1	1	0.021257381	29.15964482	-0.00364719	-5.003004103
4	1	1	0.008967958	12.30172516	-0.001538658	-2.110642356
5	1	1	0.004591594	6.298483281	-0.000787793	-1.080648886
6	1	1	0.002657173	3.644955603	-0.000455899	-0.625375513
7	1	1	0.001673322	2.295365627	-0.000287097	-0.39382248
8	1	1	0.001120995	1.537715645	-0.000192332	-0.263830295
9	1	1	0.00078731	1.079986845	-0.000135081	-0.185296448
10	1	1	0.000573949	0.78731041	-9.84741E-05	-0.135081111

Future scope: In this paper, the material used is steel. We can also use other materials like wood or composite materials. The methodology is same for any kind of material.

Conflict of interest: This paper is useful for calculation of stress using certain methodologies as mentioned earlier. The material used in this paper is steel. We can also use other composite materials by continuing further research on it.

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