

19/04/24

MIDL ASSIGNMENT 3.2

Question 3.2

→ Given probability = 0.7, perpendicular prob = $0.15 \left(\frac{1-0.7}{2} \right)$

→ Initial Grid = $\begin{bmatrix} 0 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Grid[0][1] = Penalty

Grid[0][2] = Goal

Grid[2][1] = Wall

$$U_e(i, j) = \max_A [\text{cost}(i, j, A) + \gamma \sum_j P_{ij}(A, j) U_e(j)]$$

$$\text{Grid}(i, j) = U_e(i, j)$$

Step cost = -0.04
(i, j) \equiv state

γ = discount Rate $\equiv 0.95$
A \equiv Action

So for Iteration 1

[Right
up
down
left]

$$Q_{mid}[0][0] = -0.04 + 0.95 \times \max \begin{cases} 0.7 \times (-1) + 0.15 \times 0 + 0.15 \times 0 \\ 0.7 \times 0 + 0.15 \times 0 + 0.15 \times 0 \\ 0.7 \times 0 + 0.15 \times 1 + 0.15 \times 0 \\ 0.7 \times 0 + 0.15 \times 0 + 0.15 \times 0 \end{cases}$$

$$= -0.04 + 0.95 \times 0 = -0.04$$

$$Q_{mid}[0][0] = -0.04 + 0.95 \times \max \begin{cases} 0.7 \times 0 + 0.15(0+0) \\ 0.7 \times 0 + 0.15(0+0) \\ 0.7 \times 0 + 0.15(0+0) \\ 0.7 \times 0 + 0.15(0+0) \end{cases}$$

$$= -0.04$$

$$Q_{mid}[1][1] = -0.04 + 0.95 \times \max \begin{cases} 0.7 \times 0 + 0.15(0+0) \\ 0.7 \times 0 + 0.15 \times (0+0) \\ 0.7 \times 0 + 0.15(0+0) \\ 0.7 \times 0 + 0.15(0+0) \end{cases}$$

$$= -0.04$$

$$Q_{mid}[1][2] = -0.04 + 0.95 \times \max \begin{cases} 0.7 \times 0 + 0.15(1+0) \\ 0.7 \times 0 + 0.15(0+0) \\ 0.7 \times 0 + 0.15(0+0) \\ 0.7 \times 0 + 0.15(1+0) \end{cases}$$

$$= -0.04 + 0.665$$

$$= 0.625$$

$$Q_{mid}[2][0] = -0.04 + 0.95 \times \max \begin{cases} 0.7 \times 0 + 0.15(0+0) \\ 0.7 \times 0 + 0.15(0+0) \\ 0.7 \times 0 + 0.15(0+0) \\ 0.7 \times 0 + 0.15(0+0) \end{cases}$$

$$= -0.04$$

$$Grid[2][2] = -0.04 + 0.95 \times \max \begin{bmatrix} 0.7 \times 0 + 0.15(0+0) \\ 0.7 \times 0 + 0.15(0+0) \\ 0.7 \times 0 + 0.15(0+0) \\ 0.7 \times 0 + 0.15(0+0) \end{bmatrix}$$

$$= -0.04$$

$$Grid[3][0] = -0.04 + 0.95 \times \max \begin{bmatrix} 0.7 \times 0 + 0.15(0+0) \\ 0.7 \times 0 + 0.15(0+0) \\ 0.7 \times 0 + 0.15(0+0) \\ 0.7 \times 0 + 0.15(0+0) \end{bmatrix}$$

$$= -0.04$$

$$Grid[3][1] = -0.04 + 0.95 \times \max \begin{bmatrix} 0.7 \times 0 + 0.15(0+0) \\ 0.7 \times 0 + 0.15(0+0) \\ 0.7 \times 0 + 0.15(0+0) \\ 0.7 \times 0 + 0.15(0+0) \end{bmatrix}$$

$$= -0.04$$

$$Grid[3][2] = -0.04 + 0.95 \times \max \begin{bmatrix} 0.7 \times 0 + 0.15(0+0) \\ 0.7 \times 0 + 0.15(0+0) \\ 0.7 \times 0 + 0.15(0+0) \\ 0.7 \times 0 + 0.15(0+0) \end{bmatrix}$$

$$= -0.04$$

$$Grid = \begin{bmatrix} -0.04 & -1 & 1 \\ -0.04 & -0.04 & 0.625 \\ -0.04 & 0 & -0.04 \\ -0.04 & -0.04 & -0.04 \end{bmatrix}$$

Iteration 1:

Utility Values:

```
[[-0.04  -1.      1.    ]
 [-0.04  -0.04   0.625]
 [-0.04   0.     -0.04 ]
 [-0.04  -0.04  -0.04 ]]
```


Iteration 2

1/1

$$\begin{aligned}
 Grid[0][0] &= -0.04 + 0.95 \times \max \left[\begin{array}{l} 0.7 \times -1 + 0.15(-0.04 + 0.04) \\ 0.7 \times (-0.04) + 0.15[(-0.04) + (-0.04)] \\ 0.7 \times (0.04) + 0.15[(0.04) + (-1)] \\ 0.7 \times (-0.04) + 0.15[(-0.04) + (0.04)] \end{array} \right] \\
 &= -0.04 + (-0.038) \\
 &= -0.078
 \end{aligned}$$

$$\begin{aligned}
 Grid[0][1] &= -0.04 + 0.95 \times \max \left[\begin{array}{l} 0.7 \times (-0.04) + 0.15[(-0.04) + (0.04)] \\ 0.7 \times (-0.04) + 0.15[(0.04) + (-0.04)] \\ 0.7 \times (-0.04) + 0.15[(-0.04) + (-0.04)] \\ 0.7 \times (-0.04) + 0.15[(-0.04) + (0.04)] \end{array} \right] \\
 &= -0.04 + (-0.038) \\
 &= -0.078
 \end{aligned}$$

$$\begin{aligned}
 Grid[1][1] &= -0.04 + 0.95 \times \max \left[\begin{array}{l} 0.7 \times 0.625 + 0.15[-1 + 0] \\ 0.7 \times -1 + 0.15[0.04 - 0.04] \\ 0.7 \times 0 + 0.15[-0.04 - 0.04] \\ 0.7 \times (-0.04) + 0.15[-1 + 0] \end{array} \right] \\
 &= -0.04 + 0.267425 \\
 &= 0.227425
 \end{aligned}$$

$$\begin{aligned}
 Grid[1][2] &= -0.04 + 0.95 \times \max \left[\begin{array}{l} 0.7 \times 0.625 + 0.15(1 - 0.04) \\ 0.7 \times 1 + 0.15(0.625 - 0.04) \\ 0.7 \times (-0.04) + 0.15(0.625 - 0.04) \\ 0.7 \times (-0.04) + 0.15(1 - 0.04) \end{array} \right] \\
 &= -0.04 + 0.7483625 \\
 &= 0.7083625
 \end{aligned}$$

$$\begin{aligned}
 Grid[2][0] &= -0.04 + 0.95 \times \max \left[\begin{array}{l} 0.7 \times (-0.04) + 0.15(-0.04 - 0.04) \\ 0.7 \times (-0.04) + 0.15(1 - 0.04 - 0.04) \\ 0.7 \times (-0.04) + 0.15(-0.04 - 0.04) \\ 0.7 \times (-0.04) + 0.15(-0.04 - 0.04) \end{array} \right] \\
 &= -0.04 + (-0.038) \\
 &= -0.078
 \end{aligned}$$

$$\begin{aligned}
 Grid[2][2] &= -0.04 + 0.95 \times \max \left[\begin{array}{l} 0.7 \times (-0.04) + 0.15(0.625 - 0.04) \\ 0.7 \times (0.625) + 0.15(-0.04 - 0.04) \\ 0.7 \times (-0.04) + 0.15(-0.04 - 0.04) \\ 0.7 \times (-0.04) + 0.15(-0.04 + 0.625) \end{array} \right] \\
 &= -0.04 + 0.46425 \\
 &= 0.36425
 \end{aligned}$$

$$\begin{aligned}
 \text{Grid}[3][0] &= -0.04 + 0.95 \max \left[\begin{array}{l} 0.7 \times (-0.04) + 0.15 (-0.04 - 0.04) \\ 0.7 \times (-0.04) + 0.15 (-0.04 - 0.04) \\ 0.7 \times (-0.04) + 0.15 (-0.04 - 0.04) \\ 0.7 \times (-0.04) + 0.15 (-0.04 - 0.04) \end{array} \right] \\
 &= -0.04 + (-0.038) \\
 &= -0.078
 \end{aligned}$$

$$\begin{aligned}
 \text{Grid}[3][1] &= (-0.04) + 0.95 \max \left[\begin{array}{l} 0.7 (-0.04) + 0.15 (-0.04 - 0.04) \\ 0.7 (-0.04) + 0.15 (-0.04 - 0.04) \\ 0.7 (-0.04) + 0.15 (-0.04 - 0.04) \\ 0.7 (-0.04) + 0.15 (-0.04 - 0.04) \end{array} \right] \\
 &= -0.04 + (-0.038) \\
 &= -0.078
 \end{aligned}$$

$$\begin{aligned}
 \text{Grid}[3][2] &= -0.04 + 0.95 \max \left[\begin{array}{l} 0.7 (-0.04) + 0.15 (-0.04 - 0.04) \\ 0.7 (-0.04) + 0.15 (-0.04 - 0.04) \\ 0.7 (-0.04) + 0.15 (-0.04 - 0.04) \\ 0.7 (-0.04) + 0.15 (-0.04 - 0.04) \end{array} \right] \\
 &= -0.04 + (-0.038) \\
 &= \underline{\underline{-0.078}}
 \end{aligned}$$

$$\text{Grid} = \begin{bmatrix} -0.078 & -1 & 1 \\ -0.078 & 0.227425 & 0.7083625 \\ -0.078 & 0 & 0.364225 \\ -0.078 & -0.078 & -0.078 \end{bmatrix}$$

Iteration 2:

Utility Values:

```

[[-0.078    -1.         1.         ]
 [-0.078    0.227425   0.7083625]
 [-0.078    0.         0.364225   ]
 [-0.078   -0.078     -0.078     ]]

```