MDL ASSIGNMENT 3 Q2

Prakhar Jain
2022115006

April 2024

Consider a coin that comes up heads with probability p and tails with probability $1-p$. Let q_n be the probability that after n independent tosses, there have been an even number of heads. Derive a recursion that relates q_n to q_{n-1}, and solve this recursion to establish the formula $q_n = (1+(1-2p)^n)/2$.

Answer:
q_n is the probability of an even number of heads in n tosses. Then, we would need a head on the n^{th} toss if there were odd heads in $n-1$ tosses and similarly, we wouldn't need a head if we had even heads in $n-1$ tosses.
So, we write it as:

$$q_n = p(1-q_{n-1}) + (1-p)q_{n-1}$$

$$q_n = p - p.q_{n-1} - p.q_{n-1} + q_{n-1}$$

$$q_n = p + q_{n-1}(1-2p)$$

$$q_n = p + q_{n-2}(p + q_{n-2}(1-2p))(1-2p)$$

$$q_n = p + p(1-p) + q_{n-2}(1-2p)^2$$

$\vdots$

$\vdots$

$$q_n = p + p(1-2p) + p(1-2p)^2 + \cdots + p(1-2p)^{n-1}$$

$$q_n = p\tfrac{1-(1-2p)^n}{1-(1-2p)}$$

$$q_n = p\tfrac{1-(1-2p)^n}{2p}$$

$$q_n = \tfrac{1-(1-2p)^n}{2}$$