Assignment Week 5
(Social Networks)

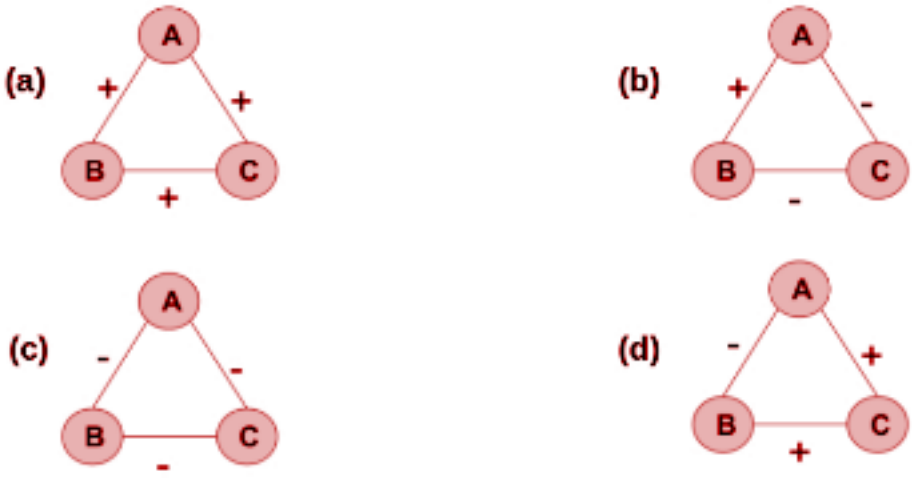
1. In a graph having ‘n’ nodes, how many possible triangles can be present?
    a. n^3
    b. 3n
    c. (n(n−1)(n−2))/6
    d. None of the above

    Explanation: It is ‘nC3’.

2. What can we say about the stability/balance of K6 (complete graph on 6 nodes) having all edges -ve, with one random edge being positive? (Choose the most appropriate):
    a. It is a stable graph.
    b. It is an unstable graph but it will have exactly four balanced triangles.
    c. It is an unstable graph but it will have exactly six balanced triangles.
    d. It is an unstable graph but It will have only one balanced triangle.

3. In a graph having 6 nodes, how many possible triangles can be present?
    a. 10
    b. 20
    c. 60
    d. 40

4. Which of the following graphs are stable?



    a. (b) & (d) ONLY
    b. (a) & (c) ONLY
    c. (c) & (d) ONLY
    d. (a) & (b) ONLY

5. Which of the triangles shown in Figure displayed in Ques 3 follows the social belief that 'Enemy of my enemy is my friend' ?
    a. (a)
    b. (b)
    c. (c)
    d. (d)

    Explanation: In triangle (b), 'A' seems to be friends with 'B' because 'B' is an enemy of his enemy, i.e. 'C'. The same is true for why 'B' seems to be friends with 'A'.
    Due to the social belief 'Enemy of my enemy is my friend', this sort of triangle stays stable.

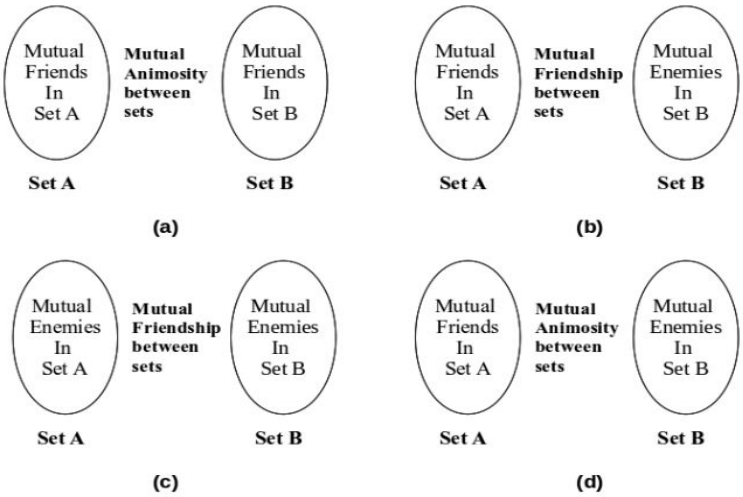
6. A friend's friend tends to become a friend, and so does an enemy's enemy. The reasons for the same are
    a. Social influence and clustering respectively
    b. Social influence and structural balance respectively
    c. Triadic closure and structural balance respectively
    d. Triadic closure and clustering respectively

    Explanation: Social influence refers to the phenomenon of an individual's decision being affected by his/her friends. Clustering is the same as triadic closure that refers to the increased likelihood of two individuals with a common friend becoming friends themselves. Structural balance of a network represents the elimination of all unbalanced triangles in the network. Hence, a friend's friend becomes friend because of triadic closure, while an enemy's enemy becomes friend in order for the network to achieve stability.

7. A signed graph is balanced if and only if it contains no cycle with
    a. An even number of negative edges
    b. An odd number of negative edges
    c. An even number of positive edges
    d. An odd number of positive edges

    Explanation:
    A signed graph is balanced if and only if it contains no cycle with an odd number of negative edges.

8. Which of the following structures of graphs is/are balanced?

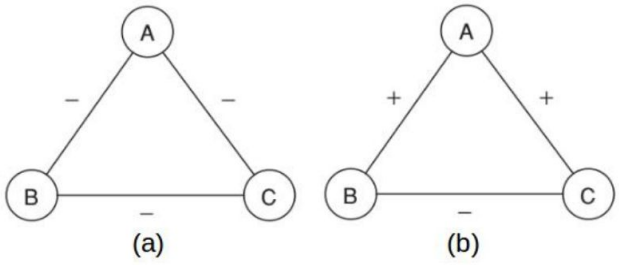


a. Both (a) and (c)
b. Only (a)
c. Only (b)
d. Both (b) and (d)

Explanation: A graph that is balanced, can be divided into two components such that all the nodes inside one component are friends to each other, all the nodes in the second component are also friends to each other, however, the nodes in the first component are enemies to the nodes in the second component. Hence, only the first graph is balanced.

9. Can we have a complete signed graph on 4 nodes (K4) and 5 nodes (K5) respectively, each having exactly one unstable triangle?
   a. K4 - Yes, K5 - Yes
   b. K4 - Yes, K5 - No
   c. K4 - No, K5 - Yes
   d. K4 - No, K5 - No

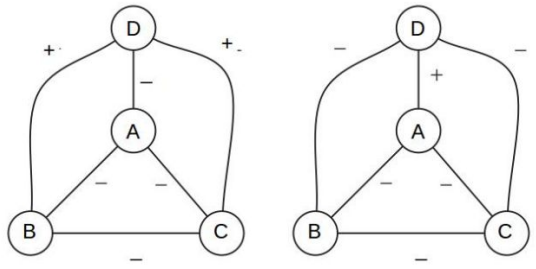
Explanation:
The proof structure is as follows:
We begin by first showing that it is not possible to have a single unstable triangle in K4 . This would imply that the same holds good for K5 as well, since K4 is a proper subgraph of a K5 graph. Therefore, a possibility of such a construction for K5 would imply a possibility for K4, which we would have proved to be impossible.
Part A: Showing the impossibility in K4. Let us assume that it is possible to have a labeled K4 graph with only one unstable triangle. Without loss of generality, let us assume that the triangle ABC is the only unstable one. The triangle could either be labeled as (a) or (b) as shown in the following Figure:



Now the aim would be to introduce the last node D and label the remaining edges DA, DB, DC such that no more unstable triangles are created in both the cases.

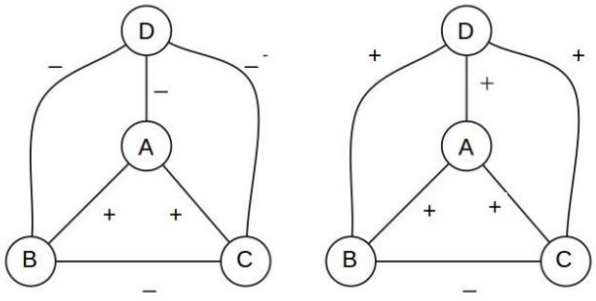
Case 1: triangle ABC is labeled as in Figure (a).

Let us consider the triangle DAB. Since one of the edges is negative (AB), for the triangle to be stable, one of the remaining edges must be labeled positive while the other negative. Let us say DA was negative and DB positive. Then a similar argument in triangle DAC would ensure that the edge DC is labeled positive. This would make the triangle DBC also unstable, increasing the number of unstable triangles. It can be seen that the same problem persists in case we had begun by labeling DA positive instead. Both these sub-cases are illustrated in the following Figure:

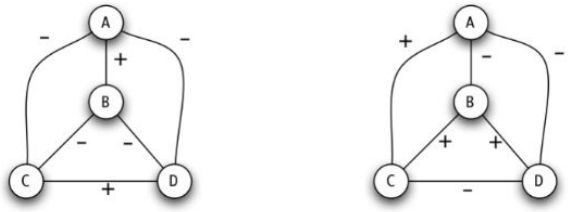


Case 1: triangle ABC is labeled as in Figure (b).

An argument similar to Case 1 can be made. Let us consider the triangle DAB. Since one of the edges is positive (AB), for the triangle to be stable, both of the remaining edges must be labeled positive or both negative. Let us say DA, DB were negative. Then a similar argument in triangle DAC would ensure that the edge DC is labeled negative. This would make the triangle DBC also unstable (with all edges negative), increasing the number of unstable triangles. It can be seen that the same problem persists in case we had begun by labeling DA, DB positive instead. Both these sub-cases are illustrated in the following Figure.



10. Which of the following graphs is/are balanced?



(a) (b)

a. Only (b) and not (a)
b. Neither (a) nor (b)
c. Both (a) and (b)
d. Only (a) and not (b)

Explanation:

A graph is said to be balanced if all its triangles are balanced.

In graph (a): the three triangles have parity (-, +, -), (-, +, -) and (-, -, +) respectively. This indicates that all the triangles have one positive edge, which means they all are balanced. Hence graph (a) is balanced.

In graph (b): the three triangles have parity (+, -, +), (-, -, +) and (+, +, -) respectively. This indicates that two out of three triangles have 2 positive edges, which means they are not balanced. Since all the triangles of graph (b) are not balanced, this graph will be imbalanced.