

Q1 Z-tests are statistical calculations that can be used to compare population means to a sample's. T-tests are calculations used to test a hypothesis

Ztest: Large sample size ($n > 30$), $(\text{Sample mean} - \text{Population mean}) / (\text{Population SD} / \sqrt{n})$, Test for a population mean or proportion

t-test: -Population standard deviation is unknown, Small sample size ($n < 30$), Test for a population mean, Used when the population standard deviation is unknown or the sample size is small, EX; -Testing whether a new teaching method improves student test scores compared to the old method

Q2 A one-tailed test is based on a uni-directional hypothesis where the area of rejection is on only one side of the sampling distribution. It determines whether a particular population parameter is larger or smaller than the predefined parameter. It uses one single critical value to test the data.

A two-tailed test is also called a nondirectional hypothesis. For checking whether the sample is greater or less than a range of values, we use the two-tailed. It is used for null hypothesis testing.

Q3 A Type I error means rejecting the null hypothesis when it's actually true. It means concluding that results are statistically significant when, in reality, they came about purely by chance or because of unrelated factors. The risk of committing this error is the significance level (alpha or α) you choose.

A type II error is a statistical term used within the context of hypothesis testing that describes the error that occurs when one fails to reject a null hypothesis that is actually false. A type II error produces a false negative, also known as an error of omission.

Q4 Bayes' Theorem describes the probability of an event, based on precedent knowledge of conditions which might be related to the event. In other words, Bayes' Theorem is the addition of Conditional Probability.

Bayes theorem is also known as the formula for the probability of "causes". For example: if we have to calculate the probability of taking a blue ball from the second bag out of three different bags of balls, where each bag contains three different colour balls viz. red, blue, black.

Q5 A confidence interval displays the probability that a parameter will fall between a pair of values around the mean. Confidence intervals measure the degree of uncertainty or certainty in a sampling method. They are also used in hypothesis testing and regression analysis.

For example, if you construct a confidence interval with a 95% confidence level, you are confident that 95 out of 100 times the estimate will fall between the upper and lower values specified by the confidence interval.

Q6 The Bayes law is a mathematical inference of the conditional probability—an event A will happen provided event B has already occurred. Here, the occurrence of even B was the condition. It is computed as: $P(A|B) = [P(B|A) P(A)] / P(B)$.

Q7

Given mean = 40 and SD = 5 and $SE = 5/(81)^{0.5} = 5/9 = 0.555$

The 95% confidence Interval can be given as mean $\pm 1.96 \cdot SE$

mean - $1.96SE = 40 - 1.96 \cdot 0.555 = 40 - 1.088 = 38.912$

mean + $1.96SE = 40 + 1.96 \cdot 0.555 = 40 + 1.088 = 41.088$

So, the 95% confidence limit of mean

$38.912 < \mu < 41.088$ where μ = population mean

Q8 The margin of error is half the confidence interval (also, the radius of the interval). The larger the sample, the smaller the margin of error. Also, the further from 50% the reported percentage, the smaller the margin of error.

The margin of error at 95% confidence is about equal to or smaller than the square root of the reciprocal of the sample size. Thus, samples of 400 have a margin of error of less than around 1/20 at 95% confidence.

Q9

The margin of error at 95% confidence is about equal to or smaller than the square root of the reciprocal of the sample size. Thus, samples of 400 have a margin of error of less than around 1/20 at 95% confidence.

Step 1: The margin of error is 5% or 0.05. The critical z-value for a 95% confidence interval is -1.96 because we want the middle 95% so the 2.5% on either side will be excluded. Divide the margin of error, 0.05, by the critical z-value, -1.96.

Q10

The standard deviation of a sample is the quantity, which indicates the variation amongst individual member value from the mean value of that sample. The favorable value for the standard deviation should be close to the mean of the sample.

thus, the confidence interval at 95% is (0.5570, 0.6429).

Q11

In this formula we know \bar{x} , σ and n , the sample size. (In actuality we do not know the population standard deviation, but we do have a point estimate for it, s , from the sample we took. More on this later.) What we do not know is μ or Z_1 . We can solve for either one of these in terms of the other. Solving for μ in terms of Z_1 gives: The confidence interval estimate has the format $(\bar{x} - EBM, \bar{x} + EBM)$ or the formula: $\bar{x} - Z\alpha(\sigma/\sqrt{n}) \leq \mu \leq \bar{x} + Z\alpha(\sigma/\sqrt{n})$

The graph gives a picture of the entire situation.

$$CL + \alpha/2 + \alpha/2 = CL + \alpha = 1.$$

Q12

$\hat{p} \pm z^* (\hat{p}(1 - \hat{p})/n)^{0.5}$. Here the value of z^* is determined by our level of confidence C . For the standard normal distribution, exactly C percent of the standard normal distribution is between $-z^*$ and z^* . *Common values for z* include 1.645 for 90% confidence and 1.96 for 95% confidence interval. Given mean = 40 and SD = 5 and $SE = 5/(81)^{0.5} = 5/9 = 0.555$

The 95% confidence Interval can be given as mean $\pm 1.96*SE$

$$\text{mean} - 1.96SE = 40 - 1.96(0.555) = 40 - 1.088 = 38.912$$

$$\text{mean} + 1.96SE = 40 + 1.96(0.555) = 40 + 1.088 = 41.088$$

So, the 95% confidence limit of mean

$$38.912 < \mu < 41.088 \text{ where } \mu = \text{population mean}$$

Q13 Two closest values in the z-table.

$$P(z < 1.64) = 0.94950$$

$$P(z < 1.65) = 0.95053$$

Interpolating gives :

$$1.64 + (0.95 - 0.9495) * (0.01) / (0.95053 - 0.9495) = 1.6449 \text{ (rounded to 4 digits)}$$

$$CI_{\text{low}} = 25 - 1.6449 * 1.5 = \text{approx. } 22.53265$$

$$CI_{\text{High}} = 25 + 1.6449 * 1.5 = \text{approx. } 27.46735$$

Then, the CI would be a little wider (22.3705 , 27.6295) based on the t-table only if your textbook says you must use t-table since you only have the sample standard deviation.

Q14

The p-value (0.003) is small so the decision is to reject H_0 and conclude that the mean recall for sleep ($\bar{x}_s = 15.25$) is different from the mean recall for caffeine ($\bar{x}_c = 12.25$). Since the

mean for the sleep group is higher than the mean for the caffeine group, we have sufficient evidence to conclude that mean recall after sleep is in fact better than after caffeine. Yes, sleep is really better for you than caffeine for enhancing recall ability. (b) The p-value (0.06) is not less than 0.05 so we would not reject H_0 at a 5% level, but it is less than 0.10 so we would reject H_0 at a 10% level. There is some moderate evidence of a difference in mean recall ability between sleep and a placebo, but not very strong evidence. (c) The p-value (0.22) is larger than any common significance level, so do not reject H_0 . The placebo group had a better mean recall in this sample ($\bar{x}_p = 13.70$ compared to $\bar{x}_c = 12.25$), but there is not enough evidence to conclude that the mean for the population would be different for a placebo than the mean recall for caffeine.