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Financial Engineering

Portfolio Construction and Management in QWIM

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Abstract

This study presents three innovative approaches to portfolio construction and optimization to investigate potential improvements on classical portfolio construction models. First, it explores network-based approaches to address issues with classic Markowitz optimization, focusing on improving the covariance matrix estimation using interconnected matrices derived from correlation networks. Various pruning techniques are employed to reduce graph complexity, leading to the generation of a semi-positive interconnected matrix used to optimize the portfolio. Secondly, it introduces a novel convex risk measure called "negative quadratic skewness" to enhance portfolio skewness, thus improving the risk-return profile. By decomposing portfolio skewness into spectral positive and negative components, the study defines negative quadratic skewness as the disparity between two convex positive semidefinite quadratic forms. A heuristic approach is proposed to minimize this measure, facilitating practical enhancement of portfolio skewness. Lastly, it investigates machine learning techniques for the asset selection problem in portfolio construction, drawing from risk-controlled statistical arbitrage methods employing an ensemble of robust regression algorithms. The efficacy of this asset allocation technique is measured by the general performance of long-short and long-only quadratic portfolios built by the selected assets. Ensembling the predictions provides the benefit of reducing the bias of any particular model while maintaining high accuracy. These approaches offer various improvements in the success of portfolios, particularly portfolio risk, risk-adjusted return, and the level of diversification.
Keywords: networks, skewness, statistics, machine learning, portfolio construction, portfolio management

1 Introduction

In recent decades, portfolio management has been a subject of much research in economic and financial mathematics. Some of the oldest and most cited works belong to Sharpe (1964) and Lintner (1965), both of which proposed the capital asset pricing model as a way for investors to reach a given expected return by taking on more risk, as well as how to select investments in a portfolio. However, with CAPM being one of the earliest comprehensive theories in portfolio management, it can be seen that the theoretical approaches do not adapt well to real markets.

Markowitz (1991) is credited for additional, major contributions in portfolio theory such as the earliest form of mean-variance portfolios and the 'Markowitz Frontier' as a graphical view of optimal points of portfolio volatility and return. Fama and French (1993), another popularly cited model, contributes the identification of factors influencing the periodic and mean return of assets, through which investors can determine the level of influence from both stock and bond markets. Contemporary research vastly improves on these models, incorporating advanced quantitative and computational techniques to select assets and fit the constraints of popular portfolio functions. This paper investigates three such methods aiming to improve some aspect of the portfolio construction process, beit diversification, asset selection or allocation, or optimizing risk-adjusted returns. The ultimate purpose of this paper is the study of different methods meant to offer some risk-control to portfolio management using quantitative and computational techniques.

First, we consider a network-based approach to improve the covariance matrix in the optimization problem introduced in Markowitz (1991). As opposed to a pairwise correlation covariance matrix, network-based approaches instead use an interconnectedness matrix, which for each element measures its embedding in the whole system. We consider for the network approach both a linear correlation (Pearson correlation) and a non-linear correlation (Kendall correlation). From these correlations we use two networks: Pearson Network Approach and Kendall Network Approach, or PNA and KNA respectively. We also incorporate pruning to reduce the complexity of the graph, such as the PMFG method proposed by Pozzi et al. (2013).

Second, we consider a novel risk measure known as "negative quadratic skewness", which aims to increase portfolio skewness through minimizing an approximation of its negative component, done so through a disciplined convex programming method. Konno and Suzuki (1995) and Konno and Yamamoto (2005) propose piece-wise linear approximations of skewness using mixed integer linear programming. This makes the model suitable for large-scale problems and enables the incorporation of negative quadratic skewness into various optimization objectives such as minimizing risk, maximizing risk-averse utility functions, and risk parity.

Third, we consider an asset selection and allocation method using the ensemble of four robust regression algorithms. This approach is directly influenced by Carta et al. (2021) who propose a statistical arbitrage method for pairs trading, in order to capture the spread between long and short positions of assets with strong association. This methodology is applied similarly by creating an asset selection algorithm to pick assets with the least "unpredictability". After the asset selection process is complete, we experiment on our results with two quadratic portfolios - one traded both long and short, and one traded only long.

2 Literature review

This paper, Clemente et al. (2021), proposed KNA(Kendall network approach) and PNA(Pearson network approach) to construct and optimize portfolio. It backtested this strategy on banks and insurance companies data and showed improvement in risk adjusted return. This paper, Pozzi et al. (2013), showed a step-to-step guidance how to use MST and PMFG method to prune a correlation based graph. It also illustrates the advantage of visualizing portfolio choices directly over the graphic layout of the network.

Jurczenko et al. (2012) introduces a novel non-parametric optimization criterion for portfolio selection in the mean-variance-skewness-kurtosis space. This approach enables the simultaneous exploration of multiple competing asset allocation objectives within a unified framework. By defining a shortage function in the four-moment space, the methodology allows for improvements in expected return, variance, skewness, and kurtosis directions, ensuring a comprehensive evaluation of portfolio efficiency. Importantly, the global optimality of resulting optimal portfolios is guaranteed, enhancing the robustness and reliability of the optimization process. Building upon prior works by Briec et al. (2004), Briec et al. (2007), and Luenberger (1995), the paper formulates a shortage optimization framework tailored for the mean-variance-skewness-kurtosis space. This framework facilitates the simultaneous consideration of multiple portfolio objectives, leveraging the shortage function to navigate the non-convex nature of the optimization problem. By establishing a link to an indirect four-moment utility function, the methodology enhances interpretability and applicability. Empirical validation is conducted through an application on funds of hedge funds, demonstrating the computational tractability and effectiveness of the approach.

The proposed methodology offers a versatile tool for portfolio optimization, particularly in scenarios where traditional mean-variance approaches may be inadequate. The empirical application on funds of hedge funds provides tangible insights into the effectiveness of the approach, showcasing its computational tractability and robustness in real-world investment scenarios. Furthermore, the framework's flexibility allows for the optimization of various asset allocation objectives, including return-maximization, skewness-maximization, variance-minimization, or kurtosis-minimization. This versatility makes it applicable across a wide range of investment contexts, with potential applications in gauging the performance of hedge funds and other investment vehicles.

Cajas (2023) this work significantly contributes to portfolio optimization methodology by introducing a novel approach that integrates higher moments, specifically L-moments, into the optimization process. By extending traditional portfolio optimization models to include higher moments, the study expands the toolkit available to portfolio managers, allowing for a more comprehensive consideration of risk factors and investment objectives. Furthermore, the development of a convex risk measure that combines higher L-moments provides practitioners with greater flexibility in portfolio construction and optimization, enabling them to tailor portfolios to specific risk preferences and investment goals.

The paper outlines two key methodologies for incorporating higher moments into portfolio optimization. Firstly, it presents an approach where a utility function based on higher L-moments is maximized, thereby providing a straightforward means of integrating higher moment considerations into the optimization process. Secondly, it develops a convex risk measure that combines L-moments higher than two, offering greater flexibility in modeling various portfolio objectives, including risk constraints and risk-adjusted return optimization. Leveraging convex optimization techniques ensures the tractability of the resulting risk measure, allowing for efficient optimization using state-of-the-art solvers. These methodologies have broad applications in portfolio construction, risk management, and asset allocation. By considering factors such as L-skewness and skewness alongside traditional risk measures, practitioners can construct portfolios that better reflect their risk preferences and investment objectives. Additionally, the developed convex risk measure enables more effective risk management and asset allocation strategies, allowing for the optimization of risk-adjusted returns while accounting for higher moment considerations. Empirical validation through numerical experiments demonstrates the practical implications of incorporating higher moments into portfolio optimization, informing investment decision-making processes and enhancing portfolio construction strategies.

Konno and Suzuki (1995) this work introduces a novel mean-variance-skewness (MVS) portfolio optimization model, extending the classical mean-variance model to incorporate the skewness of asset returns and the third-order derivative of a utility function. By accounting for these additional factors, the MVS model offers a more comprehensive framework for selecting an optimal portfolio, allowing for the calculation of an approximate mean-variance-skewness efficient surface. This advancement enables the computation of portfolios with maximal expected utility across a range of decreasingly risk-averse utility functions, enhancing portfolio decision-making in the presence of non-negligible skewness. The proposed MVS portfolio optimization model represents a natural extension of the classical mean-variance model to incorporate higher moments of asset returns. One advantage of this approach is its ability to maximize the third-order approximation of expected utility, providing greater flexibility for portfolio optimization under varying risk preferences. To solve the associated nonconcave maximization problem, three computational schemes are proposed, offering different approaches to optimizing the MVS model. These computational schemes pave the way for efficient portfolio optimization, ensuring robust and reliable solutions in the presence of non-negligible skewness. The MVS portfolio optimization model has broad applications in portfolio management and decision-making. By considering skewness and higher-order moments alongside traditional risk measures, practitioners can construct portfolios that better align with their risk preferences and investment goals. However, it remains to be seen whether the MVS model outperforms existing models such as the MY model or MADS model, highlighting the need for extensive numerical simulations and comparative analyses. Ongoing efforts in conducting extensive numerical simulations aim to derive definitive conclusions regarding the relative advantages of the MVS model and the computational schemes proposed in this study, with the results expected to inform future portfolio optimization strategies.

Zhou and Palomar (2021) this paper addresses the limitations of traditional portfolio optimization methods by proposing an efficient algorithm framework that incorporates high-order moments, such as skewness and kurtosis, into portfolio design. Unlike approaches that focus solely on mean and variance, the proposed framework considers the asymmetry and heavy-tailedness of real-world asset return distributions, thereby providing investors with a more comprehensive assessment of portfolio risk and return. The key contribution lies in the development of a convergence-provable algorithm based on successive convex approximation (SCA), which efficiently solves high-order portfolio optimization problems while mitigating computational complexity.

The paper presents an algorithmic framework for high-order portfolio optimization based on successive convex approximation. Specifically, efficient algorithms are proposed for solving both mean-variance-skewness-kurtosis (MVSK) portfolio optimization and mean-variance-skewness-kurtosis tilting (MVSKT) portfolio optimization problems. These algorithms leverage the SCA framework to ensure convergence to a stationary point, overcoming the non-convexity and computational challenges associated with high-order moments. The theoretical soundness of the algorithms is established, providing confidence in their efficacy for solving complex portfolio optimization problems. The proposed algorithms have broad applications in portfolio management, particularly in scenarios where traditional mean-variance optimization falls short due to the limitations of Gaussian assumptions. By incorporating high-order moments, investors can better capture the asymmetry and heavy-tailedness of asset return distributions, leading to more robust portfolio designs. Numerical experiments demonstrate the efficiency of the proposed algorithms, notably outperforming existing methods and general solvers. The results highlight the practical utility of the proposed framework in addressing real-world portfolio optimization challenges, paving the way for enhanced decision-making and risk management strategies in investment portfolios.

Carta et al. (2021) holds significant influence on how we designed our machine learning strategy. This paper implements a robust regression ensemble featuring four robust machine learning models: random forests, light-gradient boosted trees, support vector regressors, and ARIMA. The approach is designed for statistical arbitrage between stocks in the S&P 500 index, where the results of the regression ensemble are used to trade assets in a long-short portfolio each period, with k pairs. In addition to providing a ranking for different assets, it also features a possible pruning strategy, making use of the predictiveness of the ensemble model and the variance present in each asset to control risk.

Although more specific to arbitrage trading, this paper provides insight to asset selection most of all. As noted in the paper, trading assets long and short in this way helps to reduce exposure to the effects of broader market, and even asset-level volatility. This paper makes up a significant portion of the machine learning approach, providing the basis for a regression ensemble and asset selection criteria, accomplished through ranking and pruning. One change for the consideration of wealth and investment management is the rebalancing frequency changed to monthly, whereas the original paper featured daily rebalancing.

Jaeger et al. (2021) introduces a dynamic asset allocation strategy based on an implement with machine learning, in which two agents are employed to construct portfolios dynamically. More specifically, two agents are tasked with portfolio construction and measured performance wise based on each portfolio's Calmar ratio and Shapley values. The Calmar and Shapley values are regressed by the novel XGBoost model across multiple markets to investigate several investment strategies. It is found that the most significant features in the regression are of max drawdowns for both futures and fixed income instruments.

Above all, this paper contributes documented use of the XGBoost model, which has grown in popularity in recent years due to both its computational speed and prediction accuracy. It also provides additional insight into factor models for regressing with XGBoost and dynamic asset allocation, which will help with asset allocation for the machine learning approach.

Denault and Simonato (2022) offers advice towards another paper in the space of goals-based wealth management. Similar to the methods of Carta et al. (2021), the model studies the discrete-time wealth management of an investor with a set profit goal, who tries to maximize their probability of achieving said profit goal. The original paper, Das et al. (2020), models the investors dynamics with geometric Brownian motion and employs dynamic programming to show how the investor can meet multiple wealth goals. This paper introduces a smoothing function on the results of the original, and moving backwards in time, determines if a goal was reached and maximizes the expected value of the next point using the Bellman equation.

This paper and Das et al. (2020) both provide basic approaches to considering an investor's wealth goals and how the probability of achieving said goals may be optimized. Furthermore, the approach of dynamic programming in the 2022 paper serves as a reference for both backtesting models and optimizing portfolio construction models at each period with set goals in mind.

3 Methodology

3.1 Method 1: Network Based Approach

3.1.1 Dependency Structure

To capture dependency, it is common to use Pearson correlation, though only for a linear relationship. As such, to capture nonlinear relationship, we use Kendall's Tau, which is a non-parametric and rank-based measure defined as:

$$\tau(R_i, R_j) = \frac{\sum_{k=1}^{n} \sum_{h \neq k}^{n} \operatorname{sgn}(r_{i,h} - r_{i,k}) \operatorname{sgn}(r_{j,h} - r_{j,k})}{n(n-1)} \tag{1}$$

where $(r_{j,1}, r_{j,1}), ..., (r_{i,n}, r_{j,n})$ is observation of joint random variables R_i, R_j

3.1.2 Interconnected Matrix

For a graph, we use methods proposed in McAssey and Bijma (2015) to calculate clustering coefficient for node i.

$$C_i = \int_{-1}^{1} C_i(t) dt. \tag{2}$$

where $C_i(t)$ is defined as $\frac{[\mathbf{A_t}^3]_{ii}}{[\mathbf{A_t O A_t}]_{ii}}$ and $\mathbf{A_t}$ is the adjacent matrix of a network when an edge is assigned between every pair of nodes having a weight at or above the threshold t.

From clustering coefficients, we construct an interconnected matrix $\mathbf{C}$ whose elements are defined:

$$c_{ij} = \begin{cases} C_i C_j & \text{if } i \neq j \\ 1 & \text{otherwise} \end{cases} \tag{3}$$

3.1.3 Optimization Formulation

We start from the Markowitz minimum variance optimization problem:

$$\begin{aligned} \text{minimize} \quad & \mathbf{w}^\top \mathbf{\Omega} \mathbf{w} \\ \text{subject to} \quad & \mathbf{1}^\top \mathbf{w} = 1 \\ & \mathbf{w} \geq \mathbf{0} \end{aligned}$$

where:

- $\mathbf{w} = (w_1, w_2, \ldots, w_n)^\top$ is the vector of portfolio weights, with $w_i \geq 0$ for all $i = 1, 2, \ldots, n$, representing the long-only constraint.
- $\mathbf{\Omega}$ is the normalized covariance matrix: $\mathbf{\Omega} = \mathbf{\Delta}^\top \mathbf{\Pi} \mathbf{\Delta}$. Here $\mathbf{\Pi}$ is the correlation matrix and $\mathbf{\Delta} = diag(\frac{\sigma_i}{\sqrt{\sum_{i=1}^N \sigma_i^2}})$. $s\sigma_i$ represents standard deviation of $asset_i$
- $\mathbf{1}$ is a vector of ones, representing the constraint that the portfolio weights must sum to 1, ensuring that the portfolio is fully invested.

In network based approach, we replace $\mathbf{\Omega}$ with $\mathbf{H} = \mathbf{\Delta}^\top \mathbf{C} \mathbf{\Delta}$ to get a new but simliar optmiztion problem.

$$\begin{aligned} \text{minimize} \quad & \mathbf{w}^\top \mathbf{H} \mathbf{w} \\ \text{subject to} \quad & \mathbf{1}^\top \mathbf{w} = 1 \\ & \mathbf{w} \geq \mathbf{0} \end{aligned}$$

3.1.4 Performance Metrics

One issue for the classic Markowitz optimization problem is overconcentration in the portfolio. To evaluate how the network based approach improves this issue, we introduce a modified HHI index:

$$HHI_t = \frac{\mathbf{w_t^*}^\top \mathbf{w_t^*} - \frac{1}{N}}{1 - \frac{1}{N}} \tag{4}$$

where:

- $\mathbf{w_t^*}$ represents optimal weight vector on window t
- N represents investment universe size

To proxy transaction cost, we use portfolio turnover rate, which is expressed at time t:

$$\sum_{i=1}^{N} |\mathbf{w_t^*} - \mathbf{w_{t-1}^*}| \tag{5}$$

3.2 Method 2: Skewness Based Approach

3.2.1 Spectral Decomposition of Portfolio Skewness

We can express portfolio skewness:

$$\sigma_3^3(x) = x'\mathbf{M}_3(x \otimes x) \tag{6}$$

$$\mathbf{M}_3 = [S_1|S_2|...|S_n] \tag{7}$$

$$\mathbf{S}_i = \begin{bmatrix} s_i11 & s_i21 & \cdots & s_i1n \\ s_i21 & s_i22 & \cdots & s_i2n \\ \vdots & \vdots & \ddots & \vdots \\ s_in1 & s_in2 & \cdots & s_inn \end{bmatrix} \tag{8}$$

[1] where M_3 is the coskewness tensor of shape $n \times n^2$, Si are the slices of coskewness tensor that are symmetric by construction, x is portfolio weights vector of shape $n \times 1$ and $\otimes$ is the kronecker product. We can transform the formula above using the block diagonalization operator block.diag$^n(\cdot)$ over the matrixes S_i and the ones column vector 1_n of size n as follows:

$$\sigma_3^3(x) = (1_n \otimes x)' \textstyle\sum_3 (x \otimes x) \tag{9}$$

$$\textstyle\sum_3 = \text{block.diag}^n(\mathbf{M}_3) \tag{10}$$

$$\textstyle\sum_3 = \begin{bmatrix} S_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & S_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & Sn \end{bmatrix} \tag{11}$$

$$\mathbf{1}_n = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}' \tag{12}$$

if we develop the expresion above we get :

$$\sigma_3^3(x) = (1_n \otimes x)' \textstyle\sum_3 (x \otimes x) \tag{13}$$

$$\sigma_3^3(x) = \begin{bmatrix} x' & x' & \cdots & x' \end{bmatrix} \textstyle\sum_3 \begin{bmatrix} x_1x & x_2x & \cdots & x_nx \end{bmatrix} \tag{14}$$

$$\sigma_3^3(x) = x_1(x'S_1x) + x_2(x'S_2x) + \cdots + x_n(x'S_nx) \tag{15}$$

$$\textstyle\sum_3 = \sum_{i=1}^{n} x'(x_iS_i)x \tag{16}$$

Then, if we split the matrices S_i using spectral decomposition as the sum of a positive semi-definite S_i^+ and a negative semi-definite matrix S_i^-, this means that $S_i = S_i^+ + S_i^{-2}$, we can split portfolio skewness into two components:

$$\sigma_3^3(x) = \sum_{i=1}^{n} x'(x_iS_i^+)x - \sum_{i=1}^{n} x'(-x_iS_i^-)x \tag{17}$$

$$\sigma_3^3(x) = \underbrace{x'\left(\sum_{i=1}^{n} x_iS_i^+\right)x}_{\text{Positive Spectral Skewness}} - \underbrace{x'\left(\sum_{i=1}^{n} -x_iS_i^-\right)x}_{\text{Negative Spectral Skewness}} \tag{18}$$

[1]The spectral or eigenvalue decomposition of $S_i = Q_i\Lambda Q_i$, where Q_i is the matrix of eigenvectors and Λ is a diagonal matrix of associated eigenvalues. $S_i^+ = Q_i\Lambda^+Q_i$ and $S_i^- = Q_i\Lambda^-Q_i$ where Λ^+ and Λ^- are the diagonal matrices of positive and negative eigenvalues.

We call the first component as positive spectral skewness and the second negative spectral skewness. In the case of long only portfolios, we have a portfolio with positive (negative) skewness when the positive spectral skewness is higher (lower) than negative spectral skewness. In the case of long short portfolios, there are cases when the spectral positive (negative) skewness is negative (positive) and the total skewness of portfolio is negative (positive). We can notice that this behavior is due x_i variables multiply each S_i^+ and S_i^- matrices. We can interpret $\sum_{i=1}^n x_i S_i^+$ and $\sum_{i=1}^n x_i S_i^-$ as weighted averages of matrices S_i^+ and S_i^- respectively, due to a common constraint is that portfolio weights sum one, $\sum_{i=1}^n x_i = 1$. We choose to make $x_i = 1$[2] to transform both components into quadratic forms. Using this idea we propose an alternative measure of portfolio skewness:

$$v_3^2(x) = x'\left(\sum_{i=1}^n S_i^+\right)x - x'\left(-\sum_{i=1}^n S_i^-\right)x \tag{19}$$

$$v_3^2(x) = x'V^+x - x'V^-x \tag{20}$$

$$v_3^2(x) = \underbrace{v_3^{2+}(x)}_{\text{Positive Quadratic Skewness}} - \underbrace{v_3^{2-}(x)}_{\text{Negative Quadratic Skewness}} \tag{21}$$

where the matrices V^+ and V^- are positive semidefinite. This new measure of skewness,that we call quadratic skewness v_3^2, is expressed as the difference of two positive convex expressions: the first represents the positive component of skewness and the second is the negative component of skewness. Due that in practice v_3^2 is the sum of a convex $v_3^{2+}(x)$ and concave $v_3^{2-}(x)$ expressions, it is hard to optimize v_3^2. However, one approach to increase the quadratic skewness is to minimize $v_3^{2-}(x)$, an idea that we develop in the following section.

3.2.2 Minimization of Quadratic Negative Skewness

The negative component of quadratic skewness $v_3^{2-}(x) = x'V^-x$ is convex and positive, so one approach to reduce the impact of this component on quadratic skewness is to minimize the negative quadratic skewness (approximate to zero). If we posed this idea as an optimization problem we get the following problem:

$$\begin{aligned} &\text{minimize} \quad x'V^-x \\ &\text{subject to} \quad \sum_{i=1}^n x_i = 1 \\ &x \geq x_L \end{aligned} \tag{1}$$

where x_L is a lower bound for the portfolio weights. The model above is a quadratic problem similar to the Markowitz model. However, taking advantage of second order

[2] Taking the average $x_i = \frac{1}{n}$ is equivalent to the sum $x_i = 1$ because the sum is n times the average.

cone we can represent the minimization of negative quadratic skewness as follows:

$$
\begin{aligned}
& min_{x,v} v \\
& \text{s.t.} \|V^{1/2}x\| \leq v \\
& \sum_{i=1}^{n} x_i = 1 \\
& \quad x \geq x_L
\end{aligned} \tag{2}
$$

The model above allow us to minimize the square root of negative quadratic skewness. This formulation is very practical because allow us to increase the skewness of portfolio solving a second order cone problem similar to the minimization of standard deviation. Also, we can use this formulation in combination with other convex risk measures in order to add positive skewness to our custom portfolios.

3.3 Method 3: Machine Learning Ensemble

We break down the machine learning ensemble approach into four steps: (1) feature engineering; (2) asset return prediction and ensembling; (3) calculating mean-directional accuracy, pruning unpredictable assets, and ranking by their associated MDA; and (4) assigning trading designations to each asset and fitting into portfolios. The first 3 steps are direct implementations taken from Carta et al. (2021), with some influence from Jaeger et al. (2021), particularly the application of the novel XGBoost package as the decision tree model. Step 4 takes the assets selected previously and fits them into two quadratic portfolios, which are discussed in further detail in 4.

3.3.1 Feature Engineering

The regression algorithms are fitted with two different explanatory variables: lagged returns - the autoregressive model - and technical trading indicators - the multi-variable regressive model. Lagged returns are calculated as the daily simple return:

$$LR = \frac{C_t}{C_{t-1}} - 1. \tag{22}$$

Besides lagged returns, we consider 9 technical indicators: exponential moving average, moving average convergence-divergence, the stochastic oscillator (or more simple $\%K$), William's $\%R$, the accumulation-distribution oscillator, relative strength indicators, rate of change, and disparities. Except when otherwise noted, each indicator is computed with a ten-day lookback period.

Exponential moving average is the ten-day average of the asset's price; mathematically expressed:

$$EMA(n) = (C_t a) + (EMA_{t-1}(n)(1-a)) \text{ where } a = \frac{2}{n+1}; \tag{23}$$

moving average convergence-divergence line (henceforth $MACD$), is used to identify signals to enter or exit positions; it is the difference between 12-day and 26-day exponential moving averages:

$$MACD = EMA(12) - EMA(26). \tag{24}$$

The stochastic oscillator $\%K$ and William's $\%R$ are both used to find strength or weakness in prices over a n-day period and are calculated very similarly:

$$\%K = \frac{C_t - LL_{t-n}}{HH_{t-n} - LL_{t-n}} \times 100, \tag{25}$$

$$\%R = \frac{HH_{t-n} - C_t}{HH_{t-n} - LL_{t-n}} \times 100, \tag{26}$$

where LL_{t-n} and HH_{t-n} are the lowest-lows and highest-highs of the last n days.

The accumulation distribution oscillator (henceforth AccDO) shows the strength of buying or selling volume and its likelihood to continue. Its simplified form is the difference between $\%K$ and $\%R$ over 100, multiplied by traded volume for the time t:

$$AccDO = \frac{\%K - \%R}{100} V_t. \tag{27}$$

Relative strength indicators (RSI), similar to AccDO, detail the strength of a price swing, in this case based on the disparity between total gain and loss over a period of n days:

$$RSI = 100 - \frac{100}{1 + (U/T_n)}, \tag{28}$$

where U_n is the total gain in the last n days, and T_n the total loss.

Price rate of change is calculated similarly to lagged returns, except for a period of n days:

$$RoC = \frac{C_t - C_{t-n}}{C_{t-n}} \times 100. \tag{29}$$

Lastly, disparity windows - both 5 and 10-day windows - show the price position relative to an underlying moving average:

$$Disp(n) = \frac{C_t}{MA_n} \times 100. \tag{30}$$

3.3.2 Model Selection

The asset selection criteria was derived from Carta et al. (2021), truncated to only focus only on individual securities: for each asset, we evaluate a total of 7 regression models. Each regression is fitted with lagged returns as the response variable and either technical indicators or lagged returns as the explanatory variables. We evaluate each model through mean-squared error, and for each algorithm we take the model with better predictive power, or lower MSE. The ARIMA model is the exception in that only one model is computed each period, due to it being purely autoregressive. In summary, we select 4 models (one of each algorithm) for each asset, in each period. This selection is crucial in maximizing predictive power each period.

With model selection complete, the ensemble is computed with a simple average of the forecasts generated by the models. This ensemble is known to reduce model bias while maintaining generally high accuracy, thanks especially to the predictive power of the random forest and XGBoost algorithms. In fact, Carta et al. (2021) states that a simple average performs better for an ensemble compared to more sophisticated weightings and rankings, which perform poorly in other studies. In 13 we see the impact of the ensemble on average mean-squared error compared to individual algorithms for predictions made over the whole period of returns.

3.3.3 Dynamic Asset Selection

Trading decisions are made using a measure known as the mean-directional average. In essence, this is the proportion of return forecasts which match the direction of actual returns for the given period, expressed:

$$MDA_D = \frac{1}{D}\sum_{i=1}^{D-1} \mathbf{1}_{sng(r_i^*)=sgn(r_i)} \tag{31}$$

where D is the number of days evaluated, $\mathbf{1}$ is the identity function, r_i^* is the predicted return on a given day, r_i is the actual return, and $sng(\cdot)$ is a function which tells the sign (positive or negative) of either return.

After calculating the MDA value for each asset, we prune based on a predesignated threshold of the same value. Carta et al. (2021) performs experiments testing different levels of thresholds, but for the purposes of this analysis we only consider assets whose MDA each period is above 0.5. Once the list of tradeable assets is pared down, we trade the remaining in two different portfolios. In the long-short portfolio, we long the top k_p ranked assets and short the bottom k_p assets (resulting in $2k_p$ trading operations each period p). This produces two "portfolios" - one long-only and one short-only - which we find the weights for by solving a quadratic programming problem. In the long-only portfolio, we trade the $2k_p$ selected assets long, again solving a quadratic programming problem for each.

4 Numerical Experiments

We select 11 assets (i.e.,'S5COND', 'S5CONS', 'S5ENRS', 'S5FINL','S5HLTH', 'S5INDU', 'S5INFT','S5MATR', 'S5TELS', 'S5UTIL', 'SPXT') from the S&P 500 (NYSE) and download Monthly adjusted closed price from Bloomberg terminal for the period from January 1, 1990 to January 1, 2024. From this data, we consider three 9-10 year periods: 1994 to 2004; 2004 to 2014; and 2014 to 2024.

4.1 Method 1: Network Based Approach

Aligned with Clemente et al. (2021), we consider monthly stepped window of 2 years and use classic Markowitz minimum variance portfolio as benchmark. At each rebalancing point, here is what we do: estimate correlation, create a correlation based graph(nodes represent securities and edge represent correlation), prune the graph, get internconnectedness matrix, plug it into our optimization problem to solve weights and hold this position until next rebalancing point.

4.2 Method 2 : Skewness Based Approach

4.2.1 Minimization of Quadratic Negative Skewness

We calculated monthly returns building a returns matrix of size $T = 410$ and $N = 11$. To calculate the portfolio we use Python 3.9, CVXPY[3] solver.

To show how this new risk measure adds positive skewness to other risk measures we are going to compare three portfolios: minimum standard deviation $\sigma(x)$, minimum square root quadratic negative skewness $v_3^-(x)$, and minimization of a combined risk measure of standard deviation and square root negative quadratic skewness $\sigma(x) + 8v_3^-(x)$ [4] .

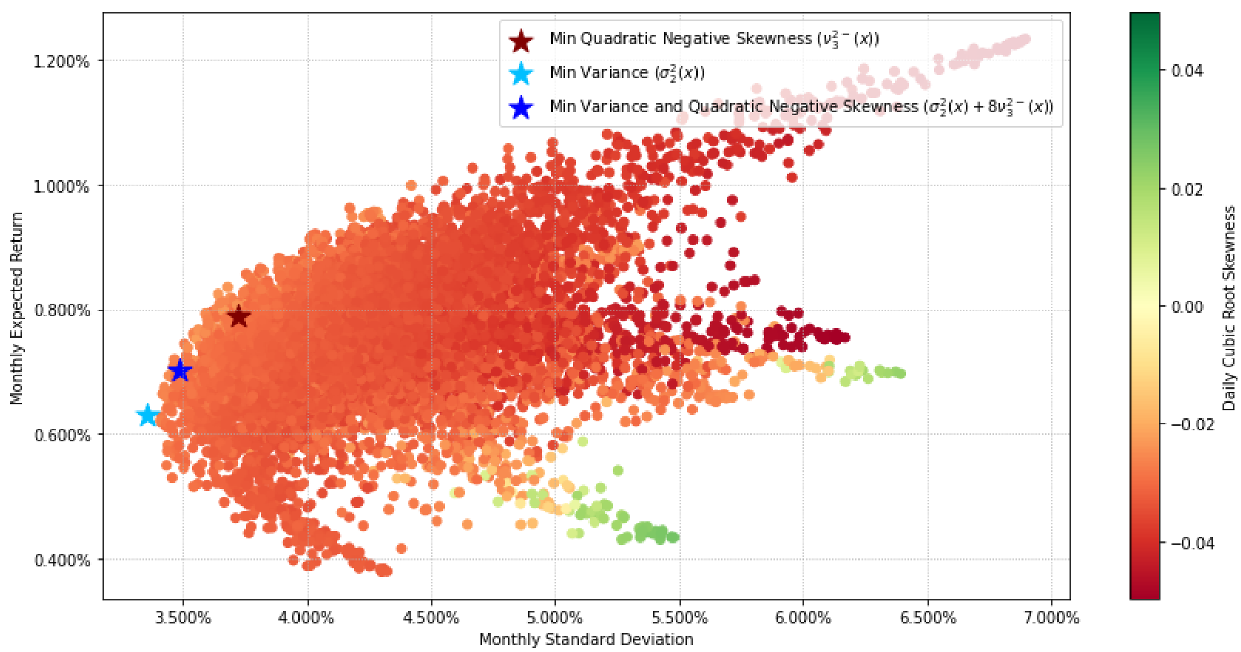


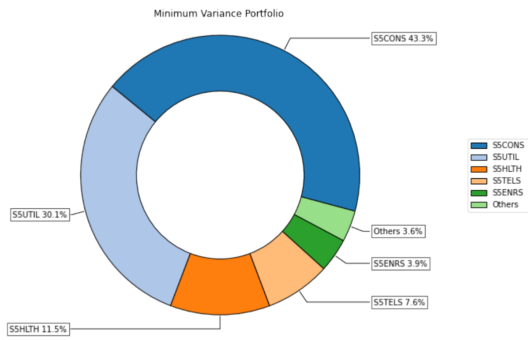
Figure 1: Optimal Portfolio in Mean Standard Deviation Plane

In figure 1 we can see that in the mean standard deviation plane, the minimum square root negative quadratic skewness and the minimum combined risk measure are inside the efficient frontier. Also, we can notice that the skewness of portfolios are

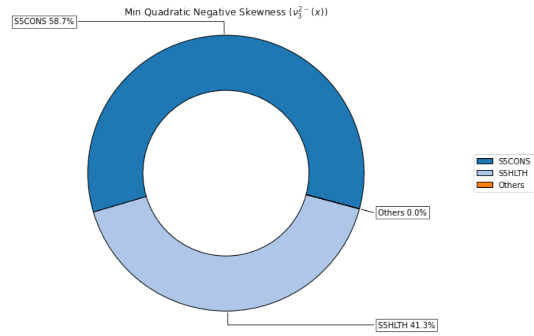
[3] Diamond and Boyd (2016)and Agrawal et al. (2018)

[4] We choose 8 as the weight for the negative quadratic skewness in the objective function to give more weight to the reduction of the negative component of the skewness. For other samples, this value may differ and must be adjusted by users according to their needs.

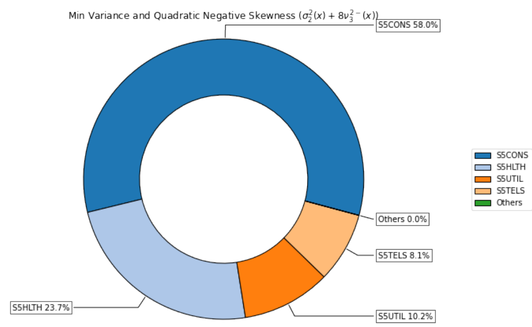
distributed in a unordered way along the mean standard deviation plane.



(a) Minimum Variane



(b) Quadratic Negative skewness



(c) Min Variance + Negative Quadratic skewness

Figure 2: Portfolio Allocation

In the figure 2, the portfolio allocation of each model and their respective weight distributions are shown. In the minimum variance portfolio figure 2a, we can see that:

- S5CONS: S&P 500 Consumer Staples Index (43.3%)
- S5UTIL: S&P 500 Utilities Index (30.1%)
- S5HLTH: S&P 500 Health Care Index (11.5%)
- S5TELS: S&P 500 Communication Services Index (7.6%)
- S5ENRS: S&P 500 Energy Index (3.9%)
- Others: (3.6%)

In the minimum quadratic negative skewness portfolio figure 2b:

- S5CONS: S&P 500 Consumer Staples Index (58.7%)
- S5HLTH: S&P 500 Health Care Index (41.3%)

In the combined minimum variance and quadratic negative skewness model figure 2c:

- S5CONS: S&P 500 Consumer Staples Index (58%)
- S5UTIL: S&P 500 Utilities Index (23.7%)

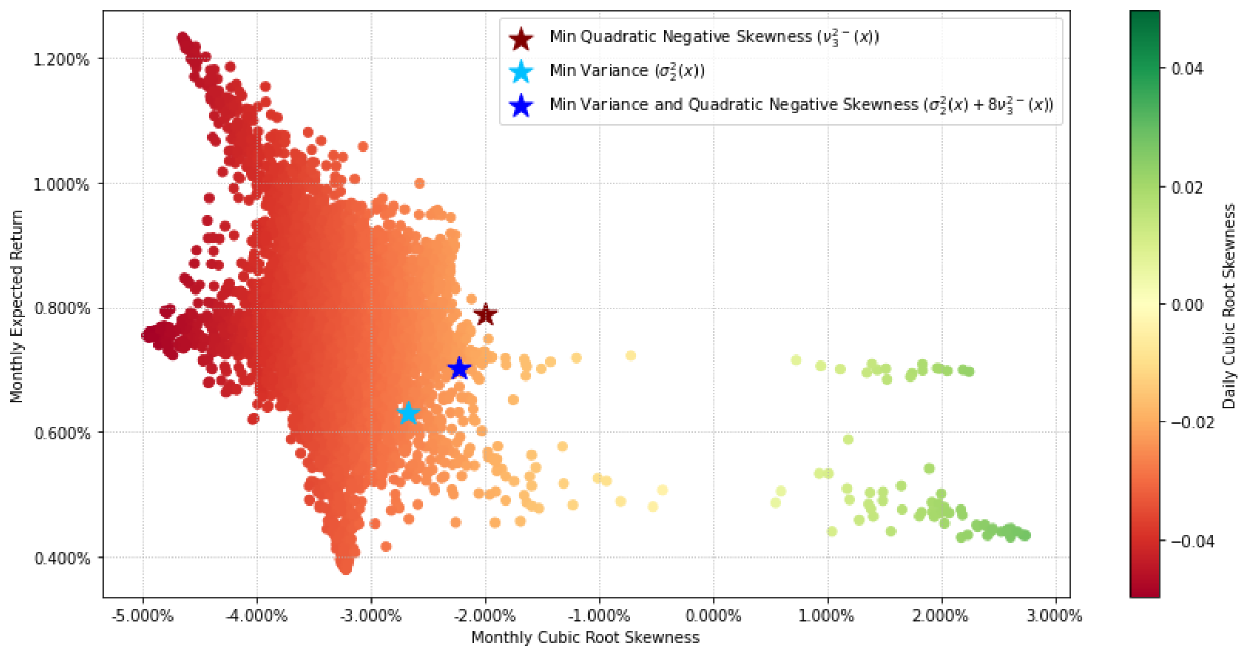


Figure 3: Optimal Portfolio in Mean Cubic Root Skewness Plane

- S5HLTH: S&P 500 Health Care Index (10.2%)
- S5TELS: S&P 500 Communication Services Index (8.1%)

In figure 3 we can see that in the mean skewness plane, the minimum square root quadratic negative skewness and the minimum combined risk measure have a higher skewness than minimum standard deviation portfolios. This means that the addition of the square root negative quadratic skewness in the objective function increase the skewness of the minimum standard deviation portfolio.

4.3 Method 3: Machine Learning Approach

We consider two primary portfolios for evaluation, as discussed in 3.3. The long-short portfolio really consists of two dynamically weighted long-only and short-only "tranches"; the assets themselves are equal weighted in each "tranche", but we determine the weight of each tranche in the portfolio with quadratic programming:

$$\begin{aligned} &\min h^\top Q h + h\mu \\ &\text{s.t. } \sum h = 1, \\ &0 < h_p < 1 \text{ for all } p \end{aligned}$$

The long-only portfolio works more traditionally, where we again solve a quadratic programming problem to assign weights to each tradeable asset in the portfolio. We evaluate three separate 10-year periods, where we consider a 60-day lookback window in each period, with monthly rebalancing. [5] The first two months of each period are used to initially train and test the models; trading begins in the third month of the dataset (so for example if we use November and December of 1993 for initial model fitting, then trades are expected to be placed in January 1994). The performance of each strategy is measured through annualized return, annualized volatility, the Sharpe ratio, and the Sortino ratio. We compare these results against an equal weight portfolio of all assets as our baseline.

5 Results and Analysis

5.1 Network Based Approach Results

5.1.1 Return Metrics

As seen in table 1, all six network approaches yields higher risk adjusted returns compared to the chosen benchmark, a Global Minimum Variance Portfolio, particularly when judged through the Sortino ratio. Compared to the equal weight portfolio, network based approaches fail to match the overall return but have lower volatility, improving the risk adjusted returns. Furthermore, observing the results indicates that, regardless of using Pearson correlation or Kendall correlation, there is no significant impact on the rank of Sortino ratios. However, this does have a significant impact on graph pruning. Figure 4 shows cumulative return of the different portfolio construction techniques based on changes in correlation and pruning methods. Network based approaches boost greatly from Global Minimum Variance portfolio.

Portfolios	**Return (%)**	**Volatility (%)**	**Sharpe Ratio**	**Sortino Ratio**
PNA Complete Graph	6.22	14.15	0.44	0.68
KNA Complete Graph	6.22	14.18	0.44	0.68
KNA PMFG	6.08	14.63	0.42	0.64
PNA PMFG	6.04	14.59	0.41	0.64
PNA MST	6.31	15.81	0.40	0.61
KNA MST	6.31	15.81	0.40	0.61
Equal Weight	6.59	17.19	0.38	0.59
GMV	5.25	13.94	0.38	0.58

Table 1: Performance metrics of different portfolio techniques.

5.1.2 Robust Test

We perform a robustness test to evaluate the networks' ability to beat benchmarks, performed over 5000 simulations. For each simulation: a sample from 1994 to 2024 is taken to start, and a distance of 5 years is considered for backtesting. This is compared

[5] Carta et al. (2021) experiments with different lengths of rolling window, with $T \in \{30, 40, 60\}$, with daily rebalancing. Since this implementation involves monthly rebalancing, the window size is selected to be 60 in order to maximize the amount of data available in the model fitting stage.

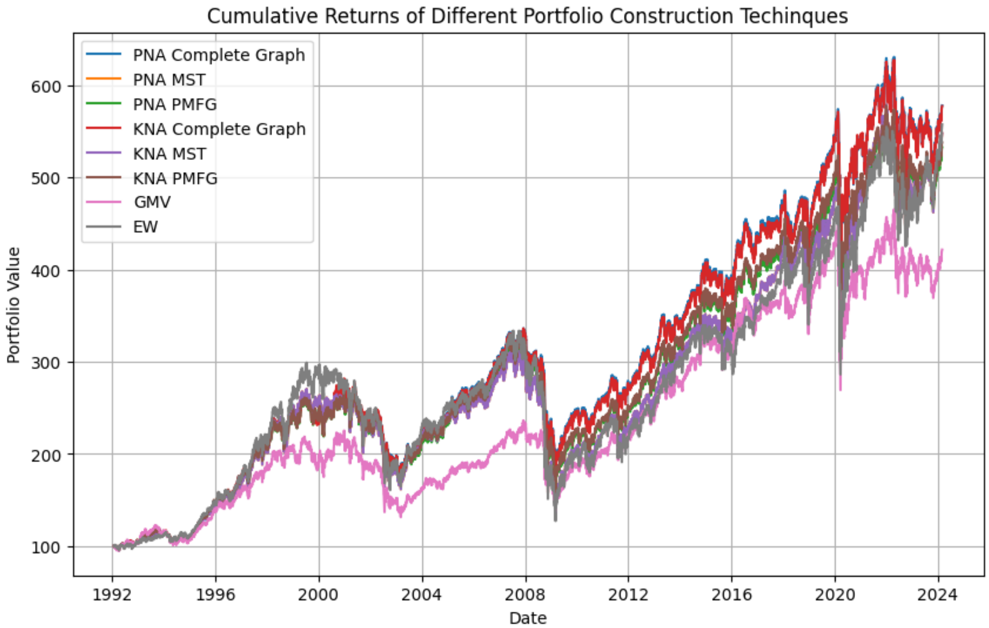


Figure 4: Cumulative Return

to the results of the benchmark in each period and aggregated to obtain each portfolio method's probability of beating the benchmark across metrics.

We use a GMV portfolio as benchmark in table 2 and an equal weight portfolio as benchmark in table 3. Table 2 shows that network based approaches have approximately a 65% of chance to beat the benchmarks in terms of Sortino ratio. Again, we note that the choice of the correlation calculation does not significantly impact performance; rather, the graph pruning method greatly improves the chance of success against the benchmark.

In table 3, network based approaches have lower volatility than the benchmark 100% of the time. In addition, the best performing portfolio - PNA with a complete graph - has a chance of 80% to beat the equal weight benchmark in Sortino ratio.

This test is performed in the period 1992 to 2024. As seen in table 1, all six network approaches yield higher risk adjusted return compared to the benchmark, Global Minimum Variance Portfolio, in terms of the Sortino ratio.

5.1.3 Diversification

It is known that a signficant drawback for the Markowitz construction technique is over-concentration, as it tends to provide a corner solution; a benefit of the network approach is its ability to more effectively spread investor's wealth between assets. Figure 5 shows the portfolio concentration of different portfolio construction techniques, where a higher HHI indicates a higher degree of portfolio concentration. Here, we

Portfolios	Return	Volatility	Sharpe Ratio	Sortino Ratio
KNA_PMFG	65.38%	22.02%	65.38%	65.38%
PNA_Complete_Graph	64.80%	34.34%	64.80%	64.80%
KNA_Complete_Graph	61.24%	34.34%	64.80%	61.24%
PNA_PMFG	61.02%	22.62%	61.02%	61.02%
PNA_MST	69.70%	0.00%	57.54%	57.54%
KNA_MST	69.70%	0.00%	57.54%	57.54%

Table 2: Robustness Test: Probability to beat the GMV benchmark in 5000 simulations

Portfolios	Return	Volatility	Sharpe Ratio	Sortino Ratio
PNA Complete Graph	44.96%	100.00%	73.16%	80.22%
KNA Complete Graph	44.96%	100.00%	69.36%	76.42%
PNA MST	52.92%	100.00%	77.34%	73.36%
KNA MST	52.92%	100.00%	77.34%	73.36%
PNA PMFG	37.34%	100.00%	68.62%	72.34%
KNA PMFG	37.86%	100.00%	65.22%	68.94%

Table 3: Robustness Test: Probability to beat the equal weight benchmark in 5000 simulations

see different levels of concentration across portfolios, with Global Minimum Variance portfolio having the highest concentration, whereas the network approaches offer better diversification. In particular, the KNA approach with MST pruning method has the best diversification effect. We note this feature in graphs with different pruning methods. Figure 10 and Figure 11 shows allocation for both a simple threshold and the PMFG method; both do not choose all available assets. However as Figure 12 shows, MST invests in all assets and is more balanced compared to the other two methods. From a graph complexity perspective, PMFG retains many edges from the simple threshold method while MST has pruned the graph significantly to resemble a tree structure.

5.1.4 Transaction Cost

Transaction cost is a crucial metric to consider for investment strategies in wealth management. To compare different strategies' transaction costs, we consider the portfolio turnover rate. Figure 6 shows the turnover rate of different portfolio construction techniques: a higher turnover rate corresponds to a higher transaction cost and can also serve as proxy for robustness. Overall, network based approaches have a lower turnover rate than benchmark portfolios. We note that an anomaly occurs for benchmark portfolios around 2021, where a corner allocation is generated at that time. However, the allocation is generally smooth for network approaches and no similar spike in turnover rate is observed.

In table 4, we have average of turnover rate. We can see MST method has the most

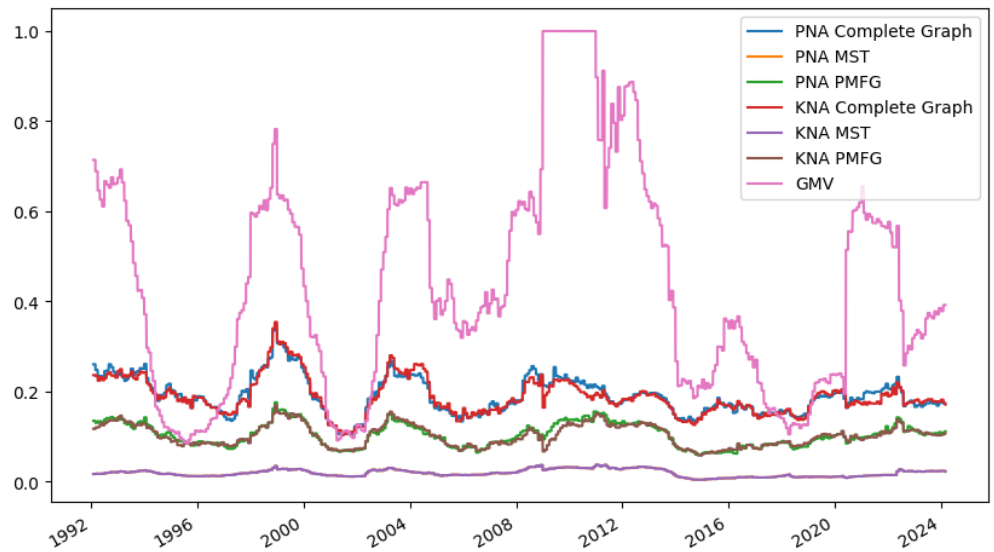


Figure 5: Modified HHI: Proxy for Portfolio Concentration

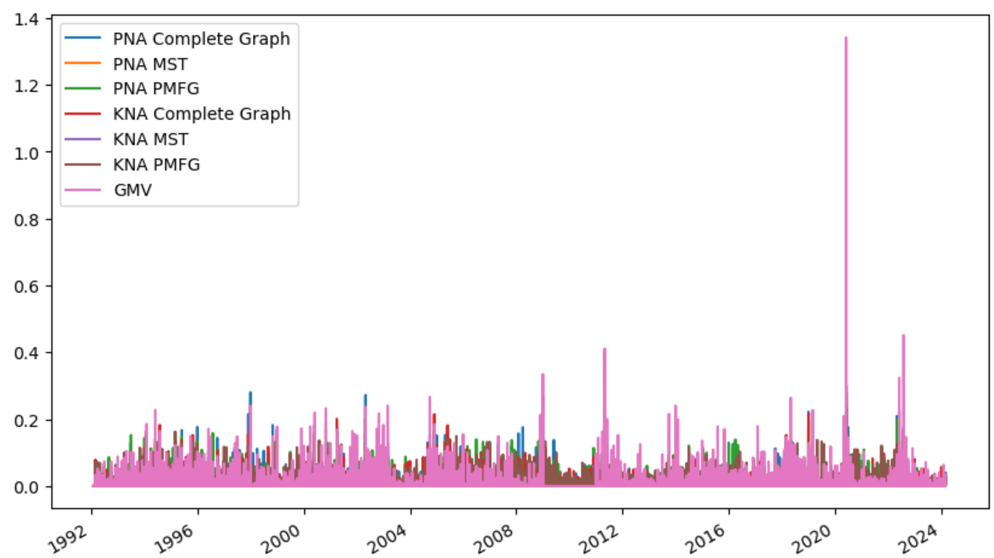


Figure 6: Portfolio Turnover Rate: Proxy for Transaction Cost

superior turnover rate performance. It has the least turnover rate while PMFG and PNA are close and still better than Global Minimum Variance portfolio.

Portfolio	**Turnover Rate**
PNA MST	0.09%
KNA MST	0.09%
KNA Complete Graph	0.28%
PNA Complete Graph	0.29%
PNA PMFG	0.29%
KNA PMFG	0.29%
GMV	0.35%

Table 4: Turnover Rate for Different Pruning Methods

5.2 Skewness-Based Approach Results

Table 6, the data was sliced from 1994 to 2004, and the results were obtained by conducting monthly re-balancing with a skewness weight of 5. This involved calculating the weights for the last 30 days and using these weights for the next 30 days without changing or recalculating daily. It is evident from the table that both the negative quadratic skewness and the combined minimum variance and negative quadratic skewness models have outperformed the minimum variance model. In table 7, data is sliced from 2004 to 2014. Here, we can observe that the mean variance has been outperformed when compared with the other two models, namely the negative quadratic skewness and the combined minimum variance and negative quadratic skewness models. In table 8, data is sliced from 2014 to 2024. Here, it is evident from the table that both the negative quadratic skewness and the combined minimum variance and negative quadratic skewness models have outperformed the minimum variance model, holding the expected results.

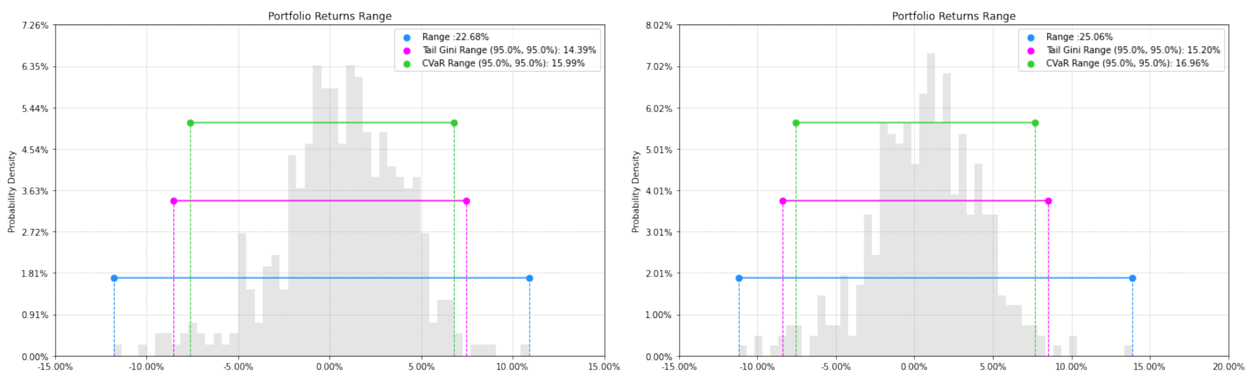
For table 5, returns data was analyzed on a monthly timeframe, with the weights calculated using all available data. This table shows the performance metrics for all three methods: minimum variance, negative quadratic skewness, and combined minimum variance and negative quadratic skewness. The results indicate that both methods have outperformed, successfully enhancing positive skewness in the risk model while reducing drawdown and enhancing risk parity.

In Figure 7, we observe the return distribution for both the minimum variance and combined minimum variance and negative quadratic skewness models. It is evident that the addition of a utility function to the risk model has led to an increase in positive skewness within the model.

Overall, investors who prioritize downside risk management or favor investments with asymmetric returns can customize their portfolios to better navigate adverse market conditions. This tailored approach may help mitigate losses during market downturns, enhancing the resilience of the investment portfolio.

Method	Mean Variance	Negative Quadratic Skewness	MV+QV
Annual Returns	7.12%	8.99%	7.62%
Volatility	11.61%	12.88%	11.88%
Max Drawdown	37.66%	32.997%	34.03%
Sharpe Ratio	0.65	0.73	0.67
Sortino Ratio	0.99	1.19	1.06
Calmar Ratio	0.18	0.27	0.22

Table 5: Overall Monthly Performance Table



(a) MV Distribution

(b) MV+QV Distribution

Figure 7: Returns Distribution

5.3 Machine Learning Results

Trading results between each period were mixed overall between the quadratic-ensemble portfolios and the benchmark. In graph 8, we observe that for the trading period 1994-2004, the ensemble long-only portfolio outperforms both the ensemble long-short portfolio and the equal weight portfolio by pure profit-and-loss. The former attained a peak cumulative return over 400% [6] while the long-short portfolio ultimately underperformed against the equal weight portfolio. Table 16 shows that despite its relative shortcomings, the long-short portfolio maintains a lower annualized volatility than either the long only or equal weight portfolios. In addition, both ensemble portfolios bear high risk-adjusted returns, with the Sortino ratios of both being over 1, and the ensemble long-only portfolio having a higher sortino ratio than for the equal weight portfolio.

[6] These results were obtained over a 10-year trading period.

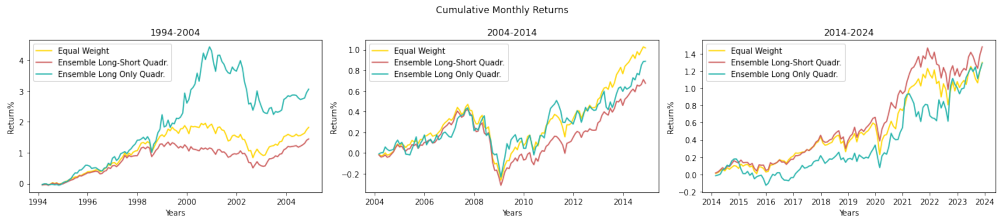


Figure 8: Cumulative monthly returns of ensemble portfolios, compared to an equal weight portfolio.

However, we note in the following 2004 to 2014 period that the equal weight outperformed both ensemble portfolios, though they still attain a strong cumulative return close to the benchmark. It is worth pointing out that the global financial crisis and its apex occurred in this period, hammering PnL for each portfolio. More details of this specific period are covered in 5.3.1. However, 15 shows the lowest annualized returns for each portfolio, as well as risk-adjusted returns, compared to other periods.

Lastly, in 2014 to the beginning of 2024, both ensemble portfolios beat the equal weight benchmark, with the long-only portfolio just barely surpassing it, as can be gleamed from Table 14. Once again, both portfolios consist of high risk-adjusted returns over 1 (with long-short being the highest).

In all periods, the long-short portfolio attains low volatility at or below the level of the equal weight portfolio, whereas the long-only consistently has the highest. A failure of the ensemble model in general is its inability to properly forecast shifting market regimes. Besides optimizing the implementation of the models, an object of further study would be adding functionality to detect market regime changes to alter the strategy. Furthermore, experimenting with different utility functions may provide unique results (though not necessarily better) than those obtained with quadratic programming of the weights.

5.3.1 Stress-test Results

For stress testing, we consider portfolios held during both the global financial crisis (marked as full years, 2007 to 2010) and COVID crisis (FYs 2019 to 2023). Table 17, which holds the results for portfolios held for the former, shows that the ensemble long-only portfolio performs best by overall PnL, as well as risk-adjusted returns through both Sharpe and Sortino ratios. However, it once again boasts higher volatility than either the long-short or equal weight portfolios.

Table 18 gives the results for portfolios constructed during the latter period: interestingly (though not altogether surprising), all portfolios boast high annualized returns in this period, with both ensemble portfolios beating the benchmark through PnL and risk-adjusted returns. This can be explained partly due to the recovery from the COVID crisis, which although strenuous for markets did not leave a terribly long-lasting impact on the assets in this investment universe.

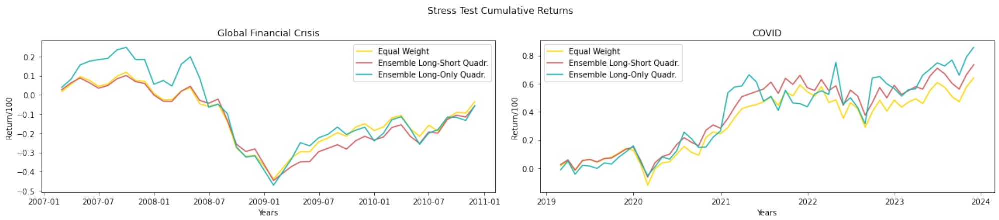


Figure 9: Cumulative returns from stress test of ensemble model.

The stress test helps to highlight the original ensemble model's strength specifically in asset selection, as particularly the long-short portfolio (which is much closer to the original implementation [7]) effectively maintains high returns while keeping the standard deviation of portfolio PnL low, compared to the long-only portfolio and the equal weight benchmark.

6 Conclusion

Studying the effects of each model reveals the overall strengths in each, usually in providing risk control or minimization, or greater asset diversification in a portfolio. As research in portfolio theory becomes more broad, more methods derived from the classic research of Markowitz, Sharpe, Lintner, Fama, and French offer many of the same benefits.

Network approach methods offer great improvements to the Markowitz model and generally better performance than both GMV and an equal weight portfolio, through overall and risk-adjusted returns and portfolio volatility. The most significant factor influencing each method's success in portfolio construction was the choice of pruning method, and not the specific correlation value used. In addition, we note that the MST method will lead to greater diversification and lower transaction cost than other methods. Network approaches as a whole have a greatly boosted performance compared to the benchmark Global Minimum Variance portfolio, with better risk adjusted return and robustness. Although the network approach on average fails to beat the equal weight portfolio in annualized return, it has a significantly improved risk-adjusted return.

The negative quadratic skewness is a novel risk measure allowing us to approximate the negative component of portfolio skewness and provides an alternative in adding positive skewness to our portfolios. The main advantage is that this optimization model can be expressed in quadratic form or by using a second order cone constraint, making this formulation suitable for large scale problems. The resulting model is very similar to the Markowitz model, except instead of using a covariance matrix we use the sum of slices of the co-skewness tensor. Another advantage is that this formulation can be solved using robust convexity solvers which support quadratic and second order

[7] Carta et al. (2021)

cone programming. Finally, this formulation is very flexible since it can be combined with other risk measures or used for other portfolio problems like risk constraints, maximization of return risk ratio or risk parity.

The ensemble method of is an altogether simpler, yet less consistent solution in portfolio construction. As it was originally designed for statistical arbitrage, it performs worst with a portfolio such as a long-only portfolio, particularly in reducing portfolio risk. Despite this, the model is able to deliver strong returns compared to the equal weight portfolio benchmark. In addition, the benefit of risk-control is still present for a long-short portfolio especially thanks to the use of quadratic programming. In both the long-short and long-only portfolios generated with ensembling, the presence of greater diversification greatly accentuates risk-adjusted returns and minimizes the presence of volatiltiy in the long-short portfolio. While both ensemble portfolios deliver high risk-adjusted returns, this implementation has room for further improvement such as by experimenting with different programming problems, look-back periods, and incorporating methodology to detect changes in market regimes.

Among all of the methods discussed, the best performing for risk-control appears to be the network approach, whose methods boasted consistently low volatility which beat equal weight 100% of the time, and mostly beating the risk-adjusted returns of equal weight and GMV portfolios. However in terms of both annualized and risk-adjusted returns, the ensemble portfolio performed very well with higher cumulative PnL in most periods compared to the benchmark. The skewness approach also performed well regarding returns, combining both the benefits of risk control and high PnL from the other two approaches.

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A Additional Tables and Graphs

A.1 Network Analysis Results

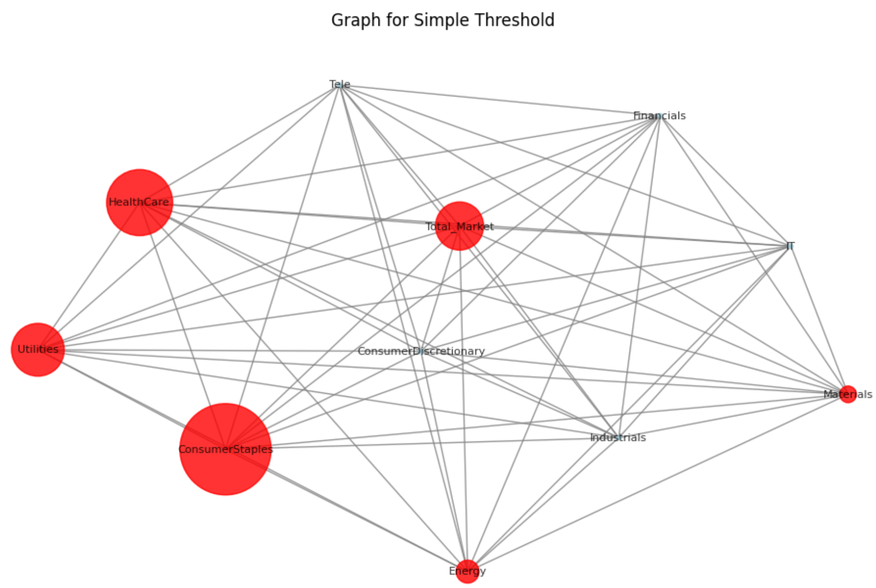

Figure 10: Allocation on Date 2008-11-03 with Simple Threshold pruning. Node size is proportional to allocation weight.

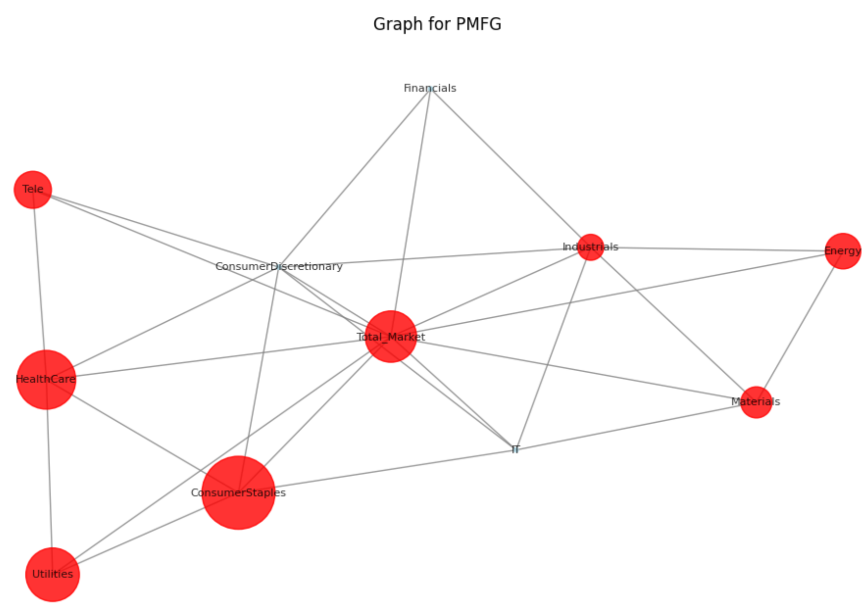

Figure 11: Allocation on Date 2008-11-03 with PMFG pruning method. Node size is proportional to allocation weight.

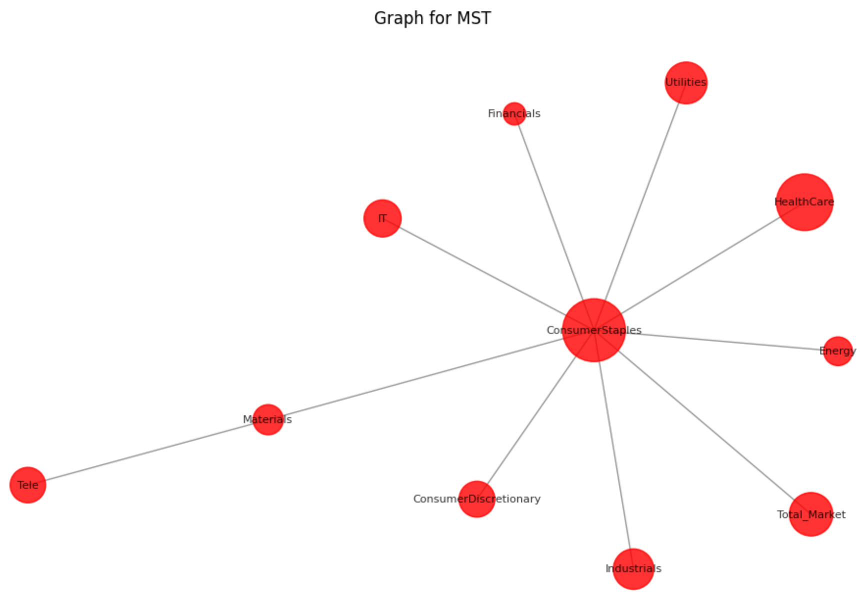

Figure 12: Allocation on Date 2008-11-03 with MST pruning method. Node size is proportional to allocation weight.

A.2 Skewness Approach Results

Method	Mean Variance	Negative Quadratic Skewness	MV+QV
Annual Returns	5.06%	9.25%	5.65%
Volatility	13.40%	17.45%	13.56%
Max Drawdown	36.48%	55.36%	37.42%
Sharpe Ratio	0.44	0.59	0.47
Sortino Ratio	0.61	0.86	0.67
Calmar Ratio	0.13	0.16	0.15

Table 6: 1994-2004 Monthly Rebalance Daily Performance

Method	Mean Variance	Negative Quadratic Skewness	MV+QV
Annual Returns	5.18%	3.35%	3.61%
Volatility	14.11%	20.71%	14.47%
Max Drawdown	42.43%	64.09%	46.04%
Sharpe Ratio	0.42	0.26	0.32
Sortino Ratio	0.60	0.37	0.60
Calmar Ratio	0.12	0.05	0.12

Table 7: 2004-2014 Monthly Rebalance Daily Performance

Method	Mean Variance	Negative Quadratic Skewness	MV+QV
Annual Returns	8.01%	15.38%	9.99%
Volatility	14.27%	17.50%	14.37%
Max Drawdown	26.77%	30.35%	25.90%
Sharpe Ratio	0.61	0.90	0.73
Sortino Ratio	0.85	1.31	1.03
Calmar Ratio	0.29	0.50	0.38

Table 8: 2014-2024 Monthly Rebalance Daily Performance

Method	Mean Variance	Negative Quadratic Skewness	MV+QV
Annual Returns	5.41%	7.52%	4.92%
Volatility	11.91%	15.39%	12.49%
Max Drawdown	40.42%	53.48%	44.30%
Sharpe Ratio	0.50	0.55	0.44
Sortino Ratio	0.74	0.82	0.64
Calmar Ratio	0.13	0.14	0.11

Table 9: Rolling Rebalance (30-days) Monthly Performance

Method	Mean Variance	Negative Quadratic Skewness	MV+QV
Annual Returns	6.56%	3.47%	5.90%
Volatility	14.18%	17.22%	14.27%
Max Drawdown	39.64%	65.50%	40.60%
Sharpe Ratio	0.51	0.28	0.47
Sortino Ratio	0.74	0.40	0.67
Calmar Ratio	0.16	0.05	0.14

Table 10: Long-only Daily Performance

Method	Mean Variance	Negative Quadratic Skewness	MV+QV
Annual Returns	5.14%	7.67%	5.69%
Volatility	13.31%	17.95%	13.65%
Max Drawdown	51.41%	53.82%	52.27%
Sharpe Ratio	0.44	0.50	0.47
Sortino Ratio	0.62	0.72	0.66
Calmar Ratio	0.10	0.14	0.10

Table 11: Rolling Rebalance (30-days) Daily Performance

Method	Mean Variance	Negative Quadratic Skewness	MV+QV
Annual Returns	6.33%	9.10%	6.49%
Volatility	13.51%	17.98%	13.70%
Max Drawdown	42.43%	64.98%	46.04%
Sharpe Ratio	0.52	0.57	0.52
Sortino Ratio	0.73	0.83	0.74
Calmar Ratio	0.14	0.14	0.14

Table 12: Monthly Rebalance Daily Performance

Method	Mean Variance	Negative Quadratic Skewness	MV+QV
Annual Returns	6.64%	5.25%	5.95%
Volatility	13.87%	16.88%	13.93%
Max Drawdown	36.00%	58.33%	39.47%
Sharpe Ratio	0.53	0.38	0.48
Sortino Ratio	0.75	0.55	0.68
Calmar Ratio	0.18	0.09	0.15

Table 13: Yearly Rebalance Daily Performance

A.3 Machine Learning Ensemble Results

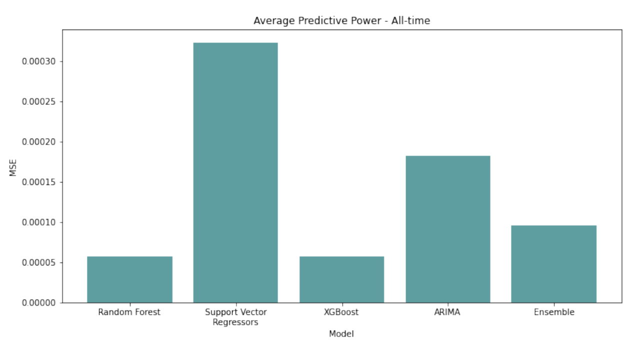

Figure 13: Comparison of predictive power between regression algorithms and prediction ensemble, for whole period 1990-2024.

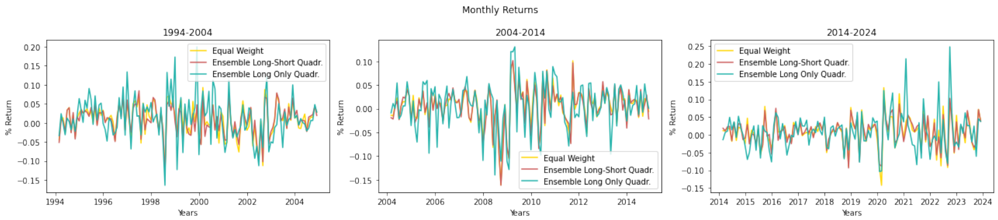

Figure 14: Monthly periodic returns of ensemble portfolios, compared to an equal weight portfolio.

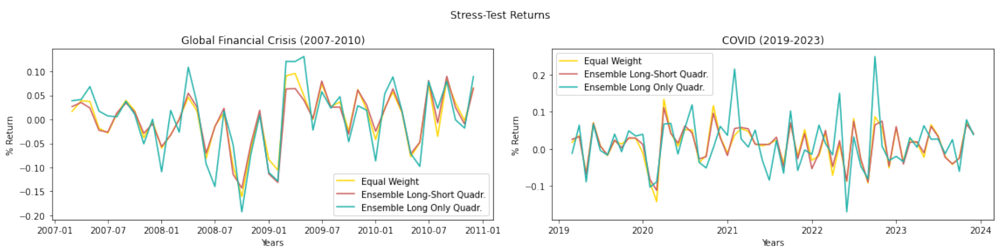

Figure 15: Stress test results of the ensemble model.

Portfolios	Return	Volatility	Sharpe Ratio	Sortino Ratio
Quadratic Long-Short	10.18%	13.52%	0.753	1.335
Quadratic Long-Only	10.10%	18.56%	0.544	1.148
Equal Weight	9.59%	14.81%	0.648	1.117

Table 14: Ensemble portfolio results: 2014-2023

Portfolios	Return	Volatility	Sharpe Ratio	Sortino Ratio
Quadratic Long-Short	5.72%	13.71%	0.417	0.645
Quadratic Long-Only	7.41%	17.38%	0.426	0.689
Equal Weight	7.44%	13.64%	0.545	0.854

Table 15: Ensemble portfolio results: 2004-2014

Portfolios	Return	Volatility	Sharpe Ratio	Sortino Ratio
Quadratic Long-Short	9.17%	13.23%	0.693	1.197
Quadratic Long-Only	14.90%	19.55%	0.762	1.495
Equal Weight	10.61%	13.98%	0.759	1.316

Table 16: Ensemble portfolio results: 1994-2004

Portfolios	**Return**	**Volatility**	**Sharpe Ratio**	**Sortino Ratio**
Quadratic Long-Short	0.35%	19.23%	0.018	0.027
Quadratic Long-Only	1.86%	25.60%	0.073	0.113
Equal Weight	0.98%	19.37%	0.050	0.078

Table 17: Ensemble stress test during the global financial crisis (2007-2010).

Portfolios	**Return**	**Volatility**	**Sharpe Ratio**	**Sortino Ratio**
Quadratic Long-Short	12.90%	17.03%	0.757	1.371
Quadratic Long-Only	15.81%	24.57%	0.643	1.327
Equal Weight	11.96%	18.22%	0.656	1.162

Table 18: Ensemble stress test during the COVID crisis (2019-2023).