

Assignment - 1

Prakriti Sahu - SM21MTECH12009

1 PROBLEM

- 1.1. Find the areas of the triangles formed by the triads of points (4,3), (1,-3), (-3,1), and (4,3), (-3,1), (1,-3) and explain the difference of signs in the two cases.

Solution: Let the points be-

$$\mathbf{A} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \quad (1.1.1)$$

$$\mathbf{P} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \quad (1.1.2)$$

Area of a Δ with the vertices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ is

$$\Delta = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{vmatrix} \quad (1.1.3)$$

$$\text{For } \mathbf{A} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} x_3 \\ y_3 \end{pmatrix}, \quad (1.1.4)$$

$$\Delta = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} \quad (1.1.5)$$

$\therefore \Delta ABC$

$$\begin{aligned} &= \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ 4 & 1 & -3 \\ 3 & -3 & 1 \end{vmatrix} \xrightarrow{C1 \leftrightarrow C1 - C3} \frac{1}{2} \begin{vmatrix} 0 & 1 & 1 \\ 7 & 1 & -3 \\ 2 & -3 & 1 \end{vmatrix} \\ &\xrightarrow{C2 \leftrightarrow C2 - C3} \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 7 & 4 & -3 \\ 2 & -4 & 1 \end{vmatrix} \xrightarrow{C2 \leftrightarrow C2/4} \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 7 & 1 & -3 \\ 2 & -1 & 1 \end{vmatrix} \end{aligned} \quad (1.1.6)$$

Expanding along the first row,

$$\begin{aligned} \Delta ABC &= 2 [1(7(-1) - 2(1))] \\ &= 2(-9) = -18 \end{aligned} \quad (1.1.7)$$

For any matrix X , product with I (identity matrix), gives matrix X itself:

$$XI = IX = X \quad (1.1.8)$$

Also, exchanging the columns of I in the product, will exchange columns of X too.

Let J be the matrix obtained by exchanging $C2$ and $C3$ of I , so that

$$J = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \text{ where } |J| = 1(0 - 1) = -1 \quad (1.1.9)$$

$\therefore \Delta ABC$ can be transformed to ΔPQR :

$$\begin{pmatrix} 1 & 1 & 1 \\ 4 & 1 & -3 \\ 3 & -3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 4 & -3 & 1 \\ 3 & 1 & -3 \end{pmatrix} = PQR \quad (1.1.10)$$

We know,

$$|X||Y| = |XY| \quad (1.1.11)$$

From equations (1.1.7) and (1.1.9),

$$|ABC||J| = |(ABC)J| = (-18)(-1) = 18 \quad (1.1.12)$$

$$\therefore \Delta PQR = |PQR| = 18 \quad (1.1.13)$$

Hence, the difference in the sign in the areas of ΔABC and ΔPQR .

