Assignment - 1

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1 PROBLEM

1.1. Find the areas of the triangles formed by the triads of points (4,3), (1,-3), (-3,1), and (4,3), (-3,1), (1,-3) and explain the difference of signs in the two cases.

Solution: Let the points be-

$$\mathbf{A} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \qquad (1.1.1)$$

$$\mathbf{P} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \qquad (1.1.2)$$

We know area of a △ with the vertices

$$(x_1, y_1), (x_2, y_2), (x_3, y_3) \qquad (1.1.3)$$

can be given by:

$$\mathbf{\Delta} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \qquad (1.1.4)$$

which can be expanded as

$$\mathbf{\Delta} = \frac{1}{2} \{(x_1 y_2 - x_2 y_1) + (x_2 y_3 - x_3 y_2) + (x_3 y_1 - x_1 y_3)\} \qquad (1.1.5)$$

∴ the area of △ABC is

$$\Delta ABC = \frac{1}{2} \{[(4)(-3) - (1)(3)] + [(1)(1) - (-3)(-3)] + [(-3)(3) - (4)(1)]\} = -18 \qquad (1.1.6)$$

And, the area of △PQR is

$$\Delta PQR = \frac{1}{2} \{[(4)(1) - (-3)(3)] + [(-3)(-3) - (1)(1)] + [(1)(3) - (4)(-3)]\} = 18 \qquad (1.1.7)$$

1.2. Reason for difference in signs in the two cases: If we take a look at the matrix representation of the two triangles:

$$ABC = \begin{bmatrix} 4 & 3 & 1 \\ 1 & -3 & 1 \\ -3 & 1 & 1 \end{bmatrix} \qquad (1.2.1)$$

$$PQR = \begin{bmatrix} 4 & 3 & 1 \\ -3 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix} \qquad (1.2.2)$$

The area of the triangle is half of $3X3$ determinant. If we exchange the 2nd and 3rd rows of △ABC, we will get the representation for △PQR. And we know that exchanging two rows or columns of a matrix changes the sign of its determinant. Hence, the difference in signs of the areas of the two triangles.