Assignment - 1

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1 Problem

1.1. Find the areas of the triangles formed by the triads of points (4,3), (1,-3), (-3,1), and (4,3), (-3,1), (1,-3) and explain the difference of signs in the two cases.

Solution: Let the points be-

$$\mathbf{A} = \begin{pmatrix}4\\3\end{pmatrix}, \mathbf{B} = \begin{pmatrix}1\\-3\end{pmatrix}, \mathbf{C} = \begin{pmatrix}-3\\1\end{pmatrix} \tag{1.1.1}$$

$$\mathbf{P} = \begin{pmatrix}4\\3\end{pmatrix}, \mathbf{Q} = \begin{pmatrix}-3\\1\end{pmatrix}, \mathbf{R} = \begin{pmatrix}1\\-3\end{pmatrix} \tag{1.1.2}$$

We know area of a $\triangle$ with the vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ can be given by:

$$\mathbf{\Delta} = \frac{\mathbf{1}}{\mathbf{2}}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix} \tag{1.1.3}$$

$\therefore$ the area of ΔABC is

$$\Delta ABC = \frac{1}{2}\begin{vmatrix}4 & 3 & 1\\ 1 & -3 & 1\\ -3 & 1 & 1\end{vmatrix}$$

$$\xrightarrow{R1\leftarrow R1+R2} \frac{1}{2}\begin{vmatrix}5 & 0 & 2\\ 1 & -3 & 1\\ -3 & 1 & 1\end{vmatrix}$$

$$\xrightarrow{R2\leftarrow R2-R3} \frac{1}{2}\begin{vmatrix}5 & 0 & 2\\ 4 & -4 & 0\\ -3 & 1 & 1\end{vmatrix}$$

$$\xrightarrow{C1\leftarrow C1+C2} \frac{1}{2}\begin{vmatrix}5 & 0 & 2\\ 0 & -4 & 0\\ -2 & 1 & 1\end{vmatrix}$$

Expanding along the first row,

$\Delta ABC = \frac{1}{2}[5((-4)1-0)-0+2(0-(-4)(-2))]$
$= \frac{1}{2}[5(-4)+2(-8)]$
$= \frac{1}{2}[-20-16]$
$= \frac{1}{2}(-36)$

$$\therefore \Delta ABC = -18 \tag{1.1.4}$$

And, the area of ΔPQR is

$$\Delta PQR = \frac{1}{2}\begin{vmatrix}4 & 3 & 1\\ -3 & 1 & 1\\ 1 & -3 & 1\end{vmatrix}$$

$$\xrightarrow{R2\leftrightarrow R3} \frac{-1}{2}\begin{vmatrix}4 & 3 & 1\\ 1 & -3 & 1\\ -3 & 1 & 1\end{vmatrix}$$

We notice that we get the exact same determinant as ΔABC, except for the difference in sign. $\therefore$ Substituting from (1.1.4), we get

$$\Delta PQR = 18 \tag{1.1.5}$$

1.2. Reason for difference in signs in the two cases:

From (1.1.4) and (1.1.5), it is clear that the areas of both triangles have equal magnitude. The difference lies in the sign. This is because exchanging the 2^{nd} and 3^{rd} rows of determinant form of ΔABC will get us the determinant form of ΔPQR. And we know that exchanging two rows or columns of a determinant changes the sign.