Assignment - 1

Prakriti Sahu - SM21MTECH12009

1 Problem

1.1. Find the areas of the triangles formed by the triads of points (4,3), (1,-3), (-3,1), and (4,3), (-3,1), (1,-3) and explain the difference of signs in the two cases.

Solution: Let the points be-

$$\mathbf{A} = \begin{pmatrix}4\\3\end{pmatrix}, \mathbf{B} = \begin{pmatrix}1\\-3\end{pmatrix}, \mathbf{C} = \begin{pmatrix}-3\\1\end{pmatrix} \tag{1.1.1}$$

$$\mathbf{P} = \begin{pmatrix}4\\3\end{pmatrix}, \mathbf{Q} = \begin{pmatrix}-3\\1\end{pmatrix}, \mathbf{R} = \begin{pmatrix}1\\-3\end{pmatrix} \tag{1.1.2}$$

Area of a $\triangle$ with the vertices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ is

$$\mathbf{\Delta} = \frac{\mathbf{1}}{\mathbf{2}} \begin{vmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{vmatrix} \tag{1.1.3}$$

For $\mathbf{A} = \begin{pmatrix}x_1\\y_1\end{pmatrix}, \mathbf{B} = \begin{pmatrix}x_2\\y_2\end{pmatrix}, \mathbf{C} = \begin{pmatrix}x_3\\y_3\end{pmatrix}$,

$$\mathbf{\Delta} = \frac{\mathbf{1}}{\mathbf{2}} \begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} \tag{1.1.4}$$

$\therefore$ the area of ΔABC is

$$\Delta ABC = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ 4 & 1 & -3 \\ 3 & -3 & 1 \end{vmatrix}$$

$$\xrightarrow{C1 \leftarrow C1 - C3} \frac{1}{2} \begin{vmatrix} 0 & 1 & 1 \\ 7 & 1 & -3 \\ 2 & -3 & 1 \end{vmatrix}$$

$$\xrightarrow{C2 \leftarrow C2 - C3} \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 7 & 4 & -3 \\ 2 & -4 & 1 \end{vmatrix}$$

$$\xrightarrow{R2 \leftarrow R2 + R3} \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 9 & 0 & -2 \\ 2 & -4 & 1 \end{vmatrix}$$

Expanding along the first row,

$\Delta ABC = \frac{1}{2}[1(9(-4) - 2(0))]$

$= \frac{1}{2}[-36 - 0]$

$= \frac{1}{2}(-36)$

$$\therefore \Delta ABC = -18 \tag{1.1.5}$$

And, the area of ΔPQR is

$$\Delta PQR = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ 4 & -3 & 1 \\ 3 & 1 & -3 \end{vmatrix}$$

$$\xrightarrow{C2 \leftrightarrow C3} \frac{-1}{2} \begin{vmatrix} 1 & 1 & 1 \\ 4 & 1 & -3 \\ 3 & -3 & 1 \end{vmatrix}$$

which is the same as that of ΔABC, except for the difference in sign. $\therefore$ Substituting from (1.1.5), we get

$$\Delta PQR = 18 \tag{1.1.6}$$

1.2. Reason for difference in signs in the two cases: From (1.1.5) and (1.1.6), it is clear that the areas of both triangles have equal magnitude. The difference lies in the sign. This is because exchanging the 2^{nd} and 3^{rd} columns of determinant form of ΔPQR will get us the determinant form of ΔABC. And we know that exchanging two rows or columns of a determinant changes the sign.