

Assignment - 1

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1 PROBLEM

- 1.1. Find the areas of the triangles formed by the triads of points (4,3), (1,-3), (-3,1), and (4,3), (-3,1), (1,-3) and explain the difference of signs in the two cases.

Solution: Let the points be-

$$\mathbf{A} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \quad (1.1.1)$$

$$\mathbf{P} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \quad (1.1.2)$$

Area of a Δ with the vertices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ is

$$\Delta = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{vmatrix} \quad (1.1.3)$$

For $\mathbf{A} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} x_3 \\ y_3 \end{pmatrix},$

$$\Delta = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} \quad (1.1.4)$$

$$\therefore \Delta ABC = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ 4 & 1 & -3 \\ 3 & -3 & 1 \end{vmatrix}$$

$$\xrightarrow{C1 \leftarrow C1 - C3} \frac{1}{2} \begin{vmatrix} 0 & 1 & 1 \\ 7 & 1 & -3 \\ 2 & -3 & 1 \end{vmatrix}$$

$$\xrightarrow{C2 \leftarrow C2 - C3} \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 7 & 4 & -3 \\ 2 & -4 & 1 \end{vmatrix}$$

taking 4 common from C2,

$$\rightarrow \frac{4}{2} \begin{vmatrix} 0 & 0 & 1 \\ 7 & 1 & -3 \\ 2 & -1 & 1 \end{vmatrix}$$

Expanding along the first row,

$$\begin{aligned} \Delta ABC &= 2 [1(7(-1) - 2(1))] \\ &= 2(-7 - 2) \\ &= 2(-9) \end{aligned}$$

$$\therefore \Delta ABC = -18 \quad (1.1.5)$$

For any matrix X , product with I (identity matrix), gives matrix X itself:

$$XI = IX = X \quad (1.1.6)$$

Also, exchanging the columns of I in the product, will exchange columns of X too.

Let J be the matrix obtained by exchanging $C2$ and $C3$ of I , so that

$$J = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \text{ where } |J| = 1(0 - 1) = -1 \quad (1.1.7)$$

$\therefore \Delta ABC$ can be transformed to ΔPQR :

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 1 & -3 \\ 3 & -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 4 & -3 & 1 \\ 3 & 1 & -3 \end{bmatrix} = PQR \quad (1.1.8)$$

We know, $|X||Y| = |XY|$. From (1.1.5), (1.1.7), $|ABC||J| = |(ABC)J| = (-18)(-1) = 18$

$$\therefore \Delta PQR = |PQR| = 18 \quad (1.1.9)$$

Hence, the difference in the sign in the areas of ΔABC and ΔPQR .

