SM5083 - BASICS OF PROGRAMMING

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1 PROBLEM

1.1. Show that the area of the triangle formed by the straight lines:
$y = x\tan a$,
$y = x\tan b$,
$y = x\tan c + d$
is
$$\Delta = \frac{d^2 \sin(a-b)\cos^2 c}{2\sin(b-c)\sin(a-c)}$$

Solution: The given lines are-

$$\begin{aligned} y &= x\tan a \\ x\tan a - y &= 0 \end{aligned} \tag{1.1.1}$$

$$\begin{aligned} y &= x\tan b \\ x\tan b - y &= 0 \end{aligned} \tag{1.1.2}$$

$$\begin{aligned} y &= x\tan c + d \\ x\tan c - y + d &= 0 \end{aligned} \tag{1.1.3}$$

For lines represented by the equations:

$$a_1x + b_1y + c_1 = 0 \tag{1.1.4}$$

$$a_2x + b_2y + c_2 = 0 \tag{1.1.5}$$

$$a_3x + b_3y + c_3 = 0 \tag{1.1.6}$$

The area of the triangle enclosed by these lines is given by:

$$\Delta = \frac{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^2}{2C_1C_2C_3} \tag{1.1.7}$$

where C_1, C_2, C_3 are the co-factors of c_1, c_2, c_3 respectively.

1.2. **Calculating determinant:**
Substituting the values from (1.1.1), (1.1.2), (1.1.3) to find determinant,

$$\begin{vmatrix} \tan a & -1 & 0 \\ \tan b & -1 & 0 \\ \tan c & -1 & d \end{vmatrix} \xleftrightarrow{R_1 \leftrightarrow R_1 - R_2} \begin{vmatrix} \tan a - \tan b & 0 & 0 \\ \tan b & -1 & 0 \\ \tan c & -1 & d \end{vmatrix} \tag{1.2.1}$$

Expanding along the first row,

$$determinant = (\tan a - \tan b)(-d) \tag{1.2.2}$$

1.3. **Calculating Co-factors:**
From (1.2.1),

$$\begin{aligned} C_1 &= (-1)^{(1+3)} \begin{vmatrix} \tan b & -1 \\ \tan c & -1 \end{vmatrix} \\ &= (-1)^4[-\tan b + \tan c] \\ &= \tan c - \tan b \end{aligned} \tag{1.3.1}$$

$$\begin{aligned} C_2 &= (-1)^{(2+3)} \begin{vmatrix} \tan a & -1 \\ \tan c & -1 \end{vmatrix} \\ &= (-1)^5[-\tan a + \tan c] \\ &= \tan a - \tan c \end{aligned} \tag{1.3.2}$$

$$\begin{aligned} C_3 &= (-1)^{(3+3)} \begin{vmatrix} \tan a & -1 \\ \tan b & -1 \end{vmatrix} \\ &= (-1)^6[-\tan a + \tan b] \\ &= \tan b - \tan a \end{aligned} \tag{1.3.3}$$

Substituting the values from (1.2.2), (1.3.1), (1.3.2), (1.3.3) into (1.1.7), we get

$$\Delta = \frac{d^2(\tan a - \tan b)^2}{2(\tan c - \tan b)(\tan a - \tan c)(\tan b - \tan a)} \tag{1.3.4}$$

Using,

$$\tan x = \frac{\sin x}{\cos x} \tag{1.3.5}$$

$$\implies \Delta = \frac{-d^2(\frac{\sin a}{\cos a} - \frac{\sin b}{\cos b})}{2(\frac{\sin c}{\cos c} - \frac{\sin b}{\cos b})(\frac{\sin a}{\cos a} - \frac{\sin c}{\cos c})} \tag{1.3.6}$$

$$\Delta = \frac{d^2(\frac{\sin a}{\cos a} - \frac{\sin b}{\cos b})}{2(\frac{\sin b}{\cos b} - \frac{\sin c}{\cos c})(\frac{\sin a}{\cos a} - \frac{\sin c}{\cos c})} \qquad (1.3.7)$$

$$\Delta = \frac{d^2(\frac{\sin a \cos b - \sin b \cos a}{\cos a \cos b})}{2(\frac{\sin b \cos c - \sin c \cos b}{\cos b \cos c})(\frac{\sin a \cos c - \sin c \cos a}{\cos a \cos c})} \qquad (1.3.8)$$

$$\Delta = \frac{d^2(\sin a \cos b - \sin b \cos a)(\cos^2 c)}{2(\sin b \cos c - \sin c \cos b)(\sin a \cos c - \sin c \cos a)} \qquad (1.3.9)$$

Using identity-

$$\sin(x - y) = \sin x \cos y - \sin y \cos x \qquad (1.3.10)$$

$$\implies \Delta = \frac{d^2 \sin(a - b)(\cos^2 c)}{2 \sin(b - c) \sin(a - c)} \qquad (1.3.11)$$

Hence, proved.