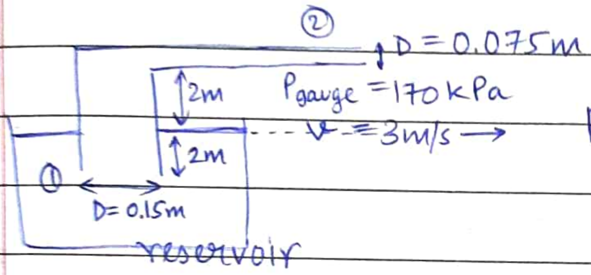
Tutorial 3.

Energy eqn: $\dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{CV} e\rho dV + \int_{CS}\left(u + pv + \frac{v^2}{2} + gz\right) \rho \vec{v} \cdot d\vec{A}$

1.



pump efficiency = 75%

$\dot{Q} - \dot{W}_s - \cancel{\dot{W}_{shear}} - \cancel{\dot{W}_{other}} = \cancel{\frac{\partial}{\partial t}\int_{CV} e\rho dV} + \int_{CS}\left(e + \frac{P}{\rho}\right)\rho \vec{v} \cdot d\vec{A}$

$e = u + \frac{v^2}{2} + gz$

assump^{ns} $\rightarrow$ $\dot{W}_{shear} = \dot{W}_{other} = 0$, steady state

$\rightarrow$ $v_1, z_1, P_1 = 0$

$\rightarrow$ incompressible flow.

$$\dot{Q} - \dot{W}_s = u_1(-\dot{m}) + \left(u_2 + \frac{v_2^2}{2} + gz_2 + \frac{P_2}{\rho}\right)(\dot{m})$$

Get $\dot{W}_{s\,ideal}$ considering min power input $\because$ neglect thermal effects

$$-\dot{W}_{s,ideal} = \dot{m}\left(\frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2\right)$$

$$= (\rho \times V_2 A_2)\left(\frac{170\text{ kPa}}{\rho} + \frac{(3\text{ m/s})^2}{2} + 9.8 \times 2\text{m}\right).$$

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$$= \left(999 \times 3 \times \frac{\pi}{4} \times (0.075)^2\right) \times \left(\frac{1.7 \times 10^8}{999} + \frac{1}{2} \times (3)^2 + 9.8 \times 2\right)$$

$$= -2.56 \text{ kW}.$$

$$\dot{W}_{s,actual} = \frac{W_{s,ideal}}{\eta} = \frac{-2.56}{0.75} = \underline{-3.41 \text{ kW}}$$

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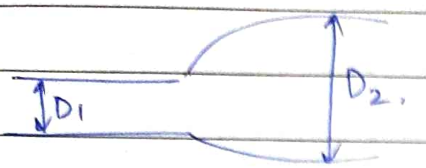
Tutorial 3. contd.

2. 

Energy eqn for pipe

$$\frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + g z_1 = \frac{P_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + g z_2 + h_{lt}$$

$$h_{lt} = \frac{k \bar{V}_1^2}{2}$$

$$\therefore P_2 - P_1 = \frac{\rho}{2}\left(V_1^2 - V_2^2\right) - \rho h_{lt}$$

From continuity, $A_1 V_1 = A_2 V_2$. $\therefore V_2^2 = V_1^2 \left(\frac{D_1}{D_2}\right)^4$

$A_1 V_1 = A_2 V_2$

$$\therefore P_2 - P_1 = \frac{1}{2}\rho V_1^2 \left(1 - \left(\frac{D_1}{D_2}\right)^4 - k\right)$$

To maximise pressure, $D_2 = \sqrt{2} D_1$.

~~flow~~ energy dissipation $\frac{\rho}{2}\left(V_1^2 - V_2^2\right) = \frac{\rho}{2}\left(V_1^2 - \frac{A_1^2 V_1^2}{A_2^2}\right)$

$$= \frac{\rho U^2}{2}\left(1 - \frac{1}{4}\right) = \frac{\rho U^2}{8} \Rightarrow \frac{\rho a U^3}{8}$$

3. $V_z = V_{max}\left(1 - \frac{r^2}{R^2}\right)$

$$\nabla \cdot \bar{V} = \frac{1}{r}\frac{\partial(rV_r)}{\partial r} + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{\partial V_z}{\partial z} = 0$$

$\therefore$ linear deformation in any dirxn $= 0$.

Angular deformation.

r-θ plane $\Rightarrow \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{V_\theta}{r}\right) + \frac{\partial V_r}{\partial \theta} = 0$.

θ-z plane $\Rightarrow \frac{\partial}{\partial z}V_\theta + \frac{1}{r}\frac{\partial}{\partial \theta}V_z = 0$

r-z plane $\Rightarrow \frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r} = -V_{max}\frac{2r}{R^2}$.

Vorticity $\Rightarrow \nabla \times \bar{V} = \frac{1}{r}\left(\frac{\partial V_z}{\partial \theta} - \frac{\partial V_\theta}{\partial z}\right)\hat{e}_r + \left(\frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r}\right)\hat{e}_\theta$

$$+ \left(\frac{1}{r}\frac{\partial r V_\theta}{\partial r} - \frac{1}{r}\frac{\partial V_r}{\partial \theta}\right)\hat{k}$$

$$= -V_{max}\frac{2r}{R^2}\hat{e}_\theta.$$

4. $V_x = iU\left(1 - \frac{x}{2L}\right)\tanh\left(\frac{Ut}{L}\right)$

$$\bar{a}_p = \frac{D\bar{V}}{Dt} = u\frac{\partial \bar{V}}{\partial x} + v\frac{\partial \bar{V}}{\partial y} + z\frac{\partial \bar{V}}{\partial z}$$

$$= \left[iU\left(1 - \frac{x}{2L}\right)\tanh\left(\frac{Ut}{L}\right)\right]\left[iU\tanh\left(\frac{Ut}{L}\right)\left(\frac{-1}{2L}\right)\right]$$

$$= -\frac{U^2}{2L}\left(1 - \frac{x}{2L}\right)\tanh^2\left(\frac{Ut}{L}\right).$$

at $(L, L/U)$

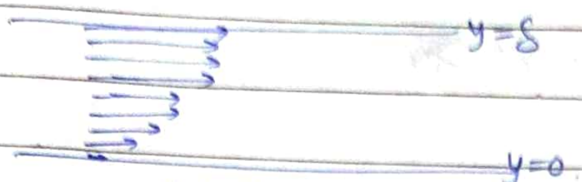
$$= \frac{U^2}{2L}\left(\frac{1}{2}\right)\tan^2h(1).$$

$$= \frac{U^2(\tanh(1))^2}{4L}$$

$$0 = \frac{U^2}{2L}\left(\frac{1}{2}\right)\tanh^2\left(\frac{Ut}{L}\right)$$

$\tanh(x)$ is 0 at $x=0$ $\therefore$ $t=0$

5.



$u = U\dfrac{y}{\delta}$ $\quad\quad$ $\delta = cx^{0.5}$

$v_y = \dfrac{Uy}{4x}$ (show) $\quad$ rotation, max rate

angular deformation

Rotation $\bar{\omega} = \dfrac{\nabla \times \bar{V}}{2} = \dfrac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ Uy/\delta & Uy^2/4x\delta & 0 \end{vmatrix}$

$$= \frac{\partial}{\partial x}\left(\frac{Uy^2}{4x\delta}\right) - \frac{\partial}{\partial y}\left(\frac{Uy}{\delta}\right)\hat{k}$$

$$= \frac{U}{2\delta}\left(1 + \frac{3}{8}\left(\frac{y}{x}\right)^2\right)$$

Max value $\longrightarrow y=\delta$.

Angular deform^n: $\dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y} = \dfrac{U}{\delta}\left(1 - \dfrac{3}{8}\left(\dfrac{y}{x}\right)^2\right)$

Max value $\longrightarrow y=0$

EULER EQN

6.

$\bar{V} = (4x+2y)\hat{i} + (2x-4y)\hat{j}$ $\quad$ $\rho = 1500\ kg/m^3$

$P_0 = 200\ kPa$ at $(0,0)$

$P(x,y)$ at $(2,2)$

$$\overset{(u)}{V_x} = 4x+2y \qquad \overset{(v)}{V_y} = 2x-4y \qquad w=0.$$

$$\rho\left(u(4)+v(2)\right) = \cancel{\rho g_x} - \frac{\partial p}{\partial x}.$$

$$\rho\left(u(2)+v(-4)\right) = \cancel{\rho g_y} - \frac{\partial p}{\partial y}.$$

$\cancel{\rho(}$

$$\frac{\partial p}{\partial x} = \rho\left(16x + \cancel{8y} + 4x - \cancel{8y}\right)$$

$$\frac{\partial p}{\partial x} = \cancel{16x\rho+4}\ 20x\rho$$

$$P = 10x^2\rho + f(y).$$

$$\frac{\partial p}{\partial y} = f'(y) = -\rho\left(\cancel{8x} + 4y - \cancel{8x} + 16y\right)$$

$$f'(y) = -20y\rho$$

$$f(y) = -10y^2\rho + c$$

$$P = 10\rho\left(x^2-y^2\right) + 200$$

$\therefore$ at 2,2 $\underline{P = 200}$

(w)