1. Using the axioms of probability, show that if A_1, A_2 are two events of interest with $A_1 \subset A_2$, then $P(A_1) \leq P(A_2)$.
Hint: Write A_2 as a union of two disjoint sets involving A_1 and $\bar{A}_1$ (A_1 complement).

2. An experiment consists of observing the voltage $\mathbf{x}$ of the parity bit in a word in computer memory. If the bit is *on*, then $\mathbf{x} = 1$; if *off*, then $\mathbf{x} = 0$. Assume that the *off* state has probability q and *on* state has probability 1-q. Generate the probability distribution function $F_{\mathbf{x}}(x)$.

3. Random variable correctness illustration: Consider an experiment where the sample space is an interval on the real line: $S = (0, 10]$. We are interested in distinguishing whether ω taks on a value in the interval $I_1 = (0, 5]$ or in $I_2 = (5, 10]$. The Borel Field could then be: $\mathbf{B} = \{\emptyset, S = (0, 10], I_1 = (0, 5], I_2 = (5, 10]\}$. Do the following:

   (a) Define random variable as: $\mathbf{x}(\omega) = \omega,\ \forall \omega \in (0, 10]$. Verify that this is not a suitable choice by checking the sets of the form $A = \{\omega : \mathbf{x}(\omega) \leq x\}$ for various values of x are not part of the Borel Field $\mathbf{B}$.

   (b) Define random variable as: $\mathbf{x}(\omega) = 5$ if $\omega \in I_1$ and $\mathbf{x}(\omega) = 10$ if $\omega \in I_2$. For this definition show that the sets $A = \{\omega : \mathbf{x}(\omega) \leq x\}$ for all values of $x \in \mathbf{R}$ are part of $\mathbf{B}$.

4. Consider a toss of a die with the sample space being $S = \{1, 2, 3, 4, 5, 6\}$. Suppose that, for some reason, you are interested only in the occurrence of one of two events,

$$A_1 = \{\text{a 1 or 2 was thrown}\} = \{1, 2\}, \text{ and } A_2 = \{\text{a 3 was thrown}\} = \{3\}$$

   (a) Define the Borel Field by taking appropriate complements, unions, intersections of A_1, A_2 and the sets resulting from these operations.

   (b) It is given that $P(A_1) = p_1$ and $P(A_2) = p_2$. Using this information, assign probabilities to all the elements of the Borel Field $\mathbf{B}$ constructed in the above part.

   (c) Define a random variable as

$$\mathbf{x}(\omega) = \begin{cases} 0, & \text{if } \omega \notin A_1 \text{ or } A_2 \\ 1, & \text{if } \omega \in A_1 \\ 2, & \text{if } \omega \in A_2 \end{cases}$$

   Construct the probability distribution function $F_{\mathbf{x}}(x)$ for this random variable. Use $p_1 = 1/3, p_2 = 1/6$.

5. The probability distribution function $F_{\mathbf{x}}(x)$ of the random variable $\mathbf{x}$ is defined as,

$$F_{\mathbf{x}}(x) = \begin{cases} 0, & x < 0 \\ K(1 - e^{-2x}), & x \geq 0 \end{cases}$$

   (a) For what value of K is the function a valid probability distribution function?

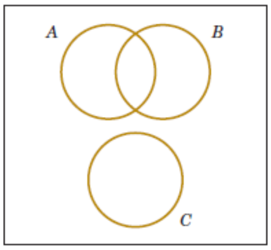
   (b) With the above value of K, what is $P\{2 < \mathbf{x} < \infty\}$?

6. Find the probability mass function of a discrete random variable $\mathbf{x}$ whose probability distribution function is given as,

$$F_{\mathbf{x}}(x) = \begin{cases} 0, & x < 0 \\ 1/6, & 0 \leq x < 2 \\ 1/2, & 2 \leq x < 4 \\ 5/8, & 4 \leq x < 6 \\ 1, & x \geq 6 \end{cases}$$

7. Consider two tosses of a fair (probability of getting head = probability of getting tail = $1/2$) coin. We are interested in the number of heads in the two tosses. Define the appropriate probability space i.e. the triplet $(\Omega, \mathbb{F}, P)$. Define an appropriate random vector $x(\omega)$ to consider the number of heads appearing in the two tosses. Obtain and sketch the probability distribution function for this x.

2-20. Three events are shown on the Venn diagram in the following figure:



Reproduce the figure and shade the region that corresponds to each of the following events.

(a) A'
(b) $(A \cap B) \cup (A \cap B')$
(c) $(A \cap B) \cup C$
(d) $(B \cup C)'$
(e) $(A \cap B)' \cup C$

2-22. In an injection-molding operation, the length and width, denoted as X and Y, respectively, of each molded part are evaluated. Let

A denote the event of $48 < X < 52$ centimeters
B denote the event of $9 < Y < 11$ centimeters

Construct a Venn diagram that includes these events. Shade the areas that represent the following:

(a) A
(b) $A \cap B$
(c) $A' \cup B$
(d) $A \cap B$
(e) If these events were mutually exclusive, how successful would this production operation be? Would the process produce parts with $X = 50$ centimeters and $Y = 10$ centimeters?

2-27. Samples of a cast aluminum part are classified on the basis of surface finish (in microinches) and edge finish. The results of 100 parts are summarized as follows:

		Edge Finish	
		Excellent	Good
Surface	Excellent	80	2
Finish	Good	10	8

(a) Let A denote the event that a sample has excellent surface finish, and let B denote the event that a sample has excellent edge finish. Determine the number of samples in $A' \cap B$, B' and in $A \cup B$.
(b) Assume that each of two samples is to be classified on the basis of surface finish, either excellent or good, and on the basis of edge finish, either excellent or good. Use a tree diagram to represent the possible outcomes of this experiment.

2-47. In a chemical plant, 24 holding tanks are used for final product storage. Four tanks are selected at random and without replacement. Suppose that six of the tanks contain material in which the viscosity exceeds the customer requirements.

(a) What is the probability that exactly one tank in the sample contains high-viscosity material?
(b) What is the probability that at least one tank in the sample contains high-viscosity material?
(c) In addition to the six tanks with high viscosity levels, four different tanks contain material with high impurities. What is the probability that exactly one tank in the sample contains high-viscosity material and exactly one tank in the sample contains material with high impurities?

2-63. An injection-molded part is equally likely to be obtained from any one of the eight cavities on a mold.

(a) What is the sample space?
(b) What is the probability that a part is from cavity 1 or 2?
(c) What is the probability that a part is from neither cavity 3 nor 4?

2-86. Strands of copper wire from a manufacturer are analyzed for strength and conductivity. The results from 100 strands are as follows:

	Strength	
	High	Low
High conductivity	74	8
Low conductivity	15	3

(a) If a strand is randomly selected, what is the probability that its conductivity is high and its strength is high?
(b) If a strand is randomly selected, what is the probability that its conductivity is low or its strength is low?
(c) Consider the event that a strand has low conductivity and the event that the strand has low strength. Are these two events mutually exclusive?

2-88. Cooking oil is produced in two main varieties: mono- and polyunsaturated. Two common sources of cooking oil are corn and canola. The following table shows the number of bottles of these oils at a supermarket:

		Type of oil	
		Canola	Corn
Type of Unsaturation	Mono	7	13
	Poly	93	77

(a) If a bottle of oil is selected at random, what is the probability that it belongs to the polyunsaturated category?
(b) What is the probability that the chosen bottle is monounsaturated canola oil?

2-103. The following table summarizes the analysis of samples of galvanized steel for coating weight and surface roughness:

		Coating Weight	
		High	Low
Surface	High	12	16
Roughness	Low	88	34

(a) If the coating weight of a sample is high, what is the probability that the surface roughness is high?
(b) If the surface roughness of a sample is high, what is the probability that the coating weight is high?
(c) If the surface roughness of a sample is low, what is the probability that the coating weight is low?

2-108. A batch of 500 containers for frozen orange juice contains 5 that are defective. Two are selected, at random, without replacement from the batch.

(a) What is the probability that the second one selected is defective given that the first one was defective?
(b) What is the probability that both are defective?
(c) What is the probability that both are acceptable?

Three containers are selected, at random, without replacement, from the batch.

(d) What is the probability that the third one selected is defective given that the first and second ones selected were defective?
(e) What is the probability that the third one selected is defective given that the first one selected was defective and the second one selected was okay?
(f) What is the probability that all three are defective?

2-124. Suppose 2% of cotton fabric rolls and 3% of nylon fabric rolls contain flaws. Of the rolls used by a manufacturer, 70% are cotton and 30% are nylon. What is the probability that a randomly selected roll used by the manufacturer contains flaws?

2-154. A credit card contains 16 digits. It also contains the month and year of expiration. Suppose there are 1 million credit card holders with unique card numbers. A hacker randomly selects a 16-digit credit card number.
(a) What is the probability that it belongs to a user?
(b) Suppose a hacker has a 25% chance of correctly guessing the year your card expires and randomly selects 1 of the 12 months. What is the probability that the hacker correctly selects the month and year of expiration?

RANDOM VARIABLES

3-17.

x	-2	-1	0	1	2
$f(x)$	0.2	0.4	0.1	0.2	0.1

(a) $P(X \le 2)$ (b) $P(X > -2)$
(c) $P(-1 \le X \le 1)$ (d) $P(X \le -1 \text{ or } X = 2)$

3-29. The distributor of a machine for cytogenics has developed a new model. The company estimates that when it is introduced into the market, it will be very successful with a probability 0.6, moderately successful with a probability 0.3, and not successful with probability 0.1. The estimated yearly profit associated with the model being very successful is $15 million and with it being moderately successful is $5 million; not successful would result in a loss of $500,000. Let X be the yearly profit of the new model. Determine the probability mass function of X.

3-49. $$F(x) = \begin{cases} 0 & x < 1 \\ 0.5 & 1 \le x < 3 \\ 1 & 3 \le x \end{cases}$$

(a) $P(X \le 3)$ (b) $P(X \le 2)$
(c) $P(1 \le X \le 2)$ (d) $P(X > 2)$

3-52. The thickness of wood paneling (in inches) that a customer orders is a random variable with the following cumulative distribution function:

$$F(x) = \begin{cases} 0 & x < 1/8 \\ 0.2 & 1/8 \le x < 1/4 \\ 0.9 & 1/4 \le x < 3/8 \\ 1 & 3/8 \le x \end{cases}$$

Determine the following probabilities:
(a) $P(X \le 1/18)$ (b) $P(X \le 1/4)$ (c) $P(X \le 5/16)$
(d) $P(X > 1/4)$ (e) $P(X \le 1/2)$

3-65. The range of the random variable X is $[0, 1, 2, 3, x]$ where x is unknown. If each value is equally likely and the mean of X is 6, determine x.

3-68. Trees are subjected to different levels of carbon dioxide atmosphere with 6% of them in a minimal growth condition at 350 parts per million (ppm), 10% at 450 ppm (slow growth), 47% at 550 ppm (moderate growth), and 37% at 650 ppm (rapid growth). What are the mean and standard deviation of the carbon dioxide atmosphere (in ppm) for these trees in ppm?

4-6. Suppose that $f(x) = e^{-(x-4)}$ for $4 < x$. Determine the following:
(a) $P(1 < X)$ (b) $P(2 \le X < 5)$ (c) $P(5 < X)$
(d) $P(8 < X < 12)$ (e) x such that $P(X < x) = 0.90$

4-7. Suppose that $f(x) = 1.5x^2$ for $-1 < x < 1$. Determine the following:
(a) $P(0 < X)$ (b) $P(0.5 < X)$
(c) $P(-0.5 \le X \le 0.5)$ (d) $P(X < -2)$
(e) $P(X < 0 \text{ or } X > -0.5)$ (f) x such that $P(x < X) = 0.05$.

4-11. The probability density function of the length of a metal rod is $f(x) = 2$ for $2.3 < x < 2.8$ meters.
(a) If the specifications for this process are from 2.25 to 2.75 meters, what proportion of rods fail to meet the specifications?
(b) Assume that the probability density function is $f(x) = 2$ for an interval of length 0.5 meters. Over what value should the density be centered to achieve the greatest proportion of rods within specifications?

4-18. Suppose that the cumulative distribution function of the random variable X is

$$F(x) = \begin{cases} 0 & x < -2 \\ 0.25x + 0.5 & -2 \le x < 2 \\ 1 & 2 \le x \end{cases}$$

Determine the following:
(a) $P(X < 1.8)$ (b) $P(X > -1.5)$
(c) $P(X < -2)$ (d) $P(-1 < X < 1)$

Determine the probability density function for each of the following cumulative distribution functions.

4-28. $F(x) = 1 - e^{-2x} \quad x > 0$

4-29.
$$F(x) = \begin{cases} 0 & x < 0 \\ 0.2x & 0 \le x < 4 \\ 0.04x + 0.64 & 4 \le x < 9 \\ 1 & 9 \le x \end{cases}$$

4-37. Suppose that $f(x) = 1.5x^2$ for $-1 < x < 1$. Determine the mean and variance of X.

4-48. The probability density function of the weight of packages delivered by a post office is $f(x) = 70/(69x^2)$ for $1 < x < 70$ pounds.
(a) Determine the mean and variance of weight.
(b) If the shipping cost is $2.50 per pound, what is the average shipping cost of a package?
(c) Determine the probability that the weight of a package exceeds 50 pounds.