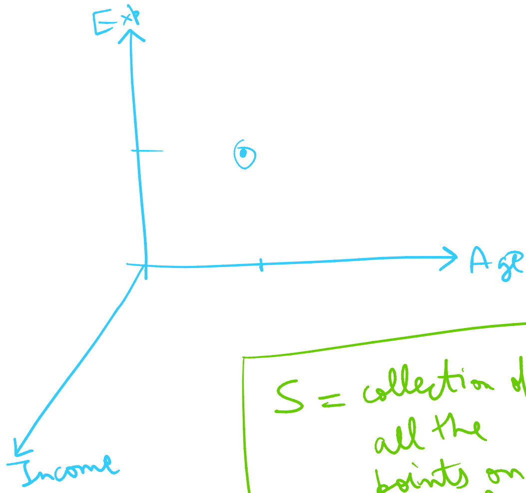
15 January 2022 14:03



$s_1 = (2, 3)$

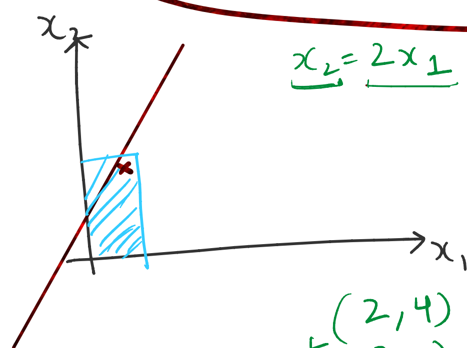
$s_2 = (-1, 4)$

$s_3 = s_1 + s_2 = (1, 7)$

$s_4 = 4 \cdot s_1 = 4 \cdot (2,3) = (8, 12)$

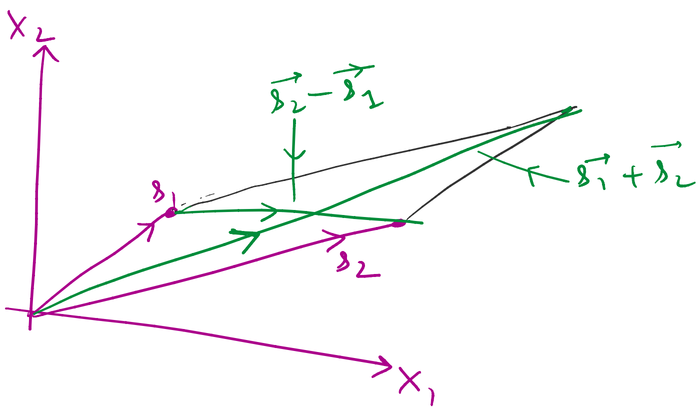
$s_1 - s_2 = s_1 + (-1) \cdot s_2$

S = collection of all the points on this line



$x_2 = 2x_1$

$$\begin{array}{r} (2,4) \\ + \ (3,6) \\ \hline = (5,10) \end{array}$$



$s_1 = (1, 0)$

$s_2 = (0, 1)$

$\alpha s_1 + \beta s_2$

$(-10, 4)$

$\alpha = -10$

$\beta = 4$

$\vec{x} = (x_1, x_2, \ldots, x_n)$

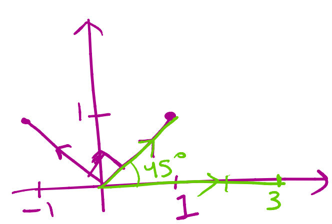
$\vec{y} = (y_1, y_2, \ldots, y_n)$

$\vec{x} \cdot \vec{y} = x_1 y_1 + x_2 y_2 + \ldots + x_n y_n$

$\|\vec{x}\| = \sqrt{x_1^2 + x_2^2 + \ldots + x_n^2}$

$\|\vec{y}\| = \sqrt{y_1^2 + y_2^2 + \ldots + y_n^2}$

$(1, 1)$ and $(1, -1)$ (orthogonal vectors)



$(3, 0)$ and $(1, 1)$

$\cos\theta = \frac{3}{3 \cdot \sqrt{2}} = \frac{1}{\sqrt{2}}$

$\cos\theta = \frac{\vec{x} \cdot \vec{y}}{}$

$$\boxed{\cos\theta = \frac{\vec{x}\cdot\vec{y}}{\|\vec{x}\|.\|\vec{y}\|}}$$

$$\cos\theta = \frac{3}{3.\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\theta = 45^\circ$$

Zero-vector in $\mathbb{R}^3 = (0,0,0)$

$$\boxed{(2,4,8) - 2(1,2,4) = (0,0,0)}$$

$\vec{v}_1 = (1,1,1)$

$\vec{v}_2 = (1,1,0)$

$\vec{v}_3 = (0,1,0)$

$$\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 = \vec{0}$$

$$(\alpha_1, \alpha_1, \alpha_1) + (\alpha_2, \alpha_2, 0) + (0, \alpha_3, 0) = (0,0,0)$$

$$\Rightarrow (\alpha_1 + \alpha_2, \underline{\alpha_1 + \alpha_2 + \alpha_3}, \underline{\alpha_1}) = (0,0,0)$$

$$\boxed{\begin{array}{c}(a,b,c) = (d,e,f) \\ a=d,\ b=e,\ c=f\end{array}}$$

$\alpha_1 = 0$

$\alpha_1 + \alpha_2 = 0 \Rightarrow \alpha_2 = 0$

$\alpha_1 + \alpha_2 + \alpha_3 = 0 \Rightarrow \alpha_3 = 0$

$\vec{v}_1 = (1, 2, -1)$

$\vec{v}_2 = (4, 3, 0)$

$\vec{v}_3 = (2, -1, 2)$

$$\boxed{-2\vec{v}_1 + \vec{v}_2 = \vec{v}_3 + \vec{0}}$$

$$\boxed{-2\vec{v}_1 + \vec{v}_2 - \vec{v}_3 = \vec{0}}$$

$$\boxed{\alpha_1 = -2,\ \alpha_2 = 1,\ \alpha_3 = -1}$$

"Linearly dependent"

III. $\vec{v}_1 = (1,1,0)$ $\quad \vec{v}_2 = (1,0,0)$

IV. $\vec{v}_1 = (1,2,3)$ $\quad \vec{v}_2 = (2,4,6)$ $\quad \Rightarrow 2\vec{v}_1 - \vec{v}_2 = \vec{0}$

Addition of Two Matrices

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 3 \end{bmatrix}_{2\times3} \qquad B = \begin{bmatrix} 4 & -3 & 7 \\ 5 & -1 & 0 \end{bmatrix}_{2\times3}$$

$A + B = [5 \quad -1 \quad 6]$

Transpose of a Matrix

$$A^T = \begin{bmatrix} 1 & 0 \\ 2 & 4 \\ -1 & 3 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 5 & -1 & 6 \\ 5 & 3 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}_{3\times 2}$$

4. $A = \begin{bmatrix} 4 & 8 & -4 \\ 0 & 16 & 12 \end{bmatrix}_{2\times 3}$

"Multiplication of Matrices"

① If $\underbrace{AB}$ has to be defined, then "number of columns of A = number of rows of B."

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & -1 \end{bmatrix}_{2\times 3} \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ -1 & 2 \end{bmatrix}_{3\times 2}$$

$A \to (m \times n) \qquad B \to (n \times p) \qquad AB = m \times p$

$\vec{v}_1 = (2, 1, 3)$
$\vec{v}_2 = (1, 0, -1)$
$= -1$

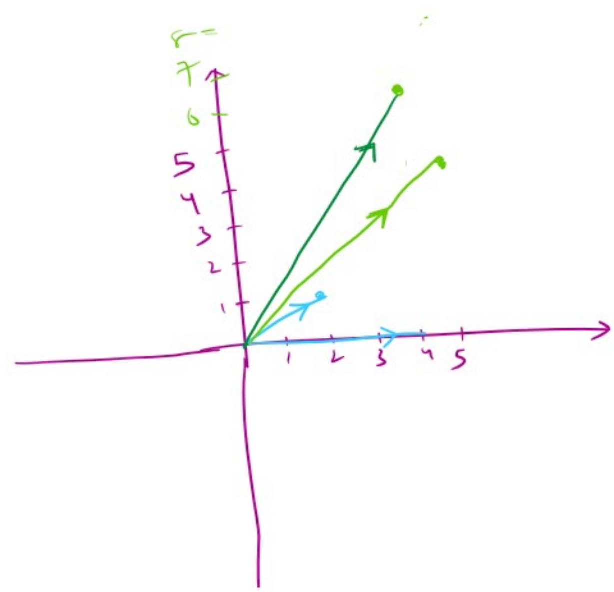
$$\begin{bmatrix} -1 & 8 \\ 2 & 2 \end{bmatrix}$$

$$\underset{(2\times 2)}{\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}} \underset{(2\times 1)}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} = \underset{2\times 1}{\begin{bmatrix} \quad \end{bmatrix}}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

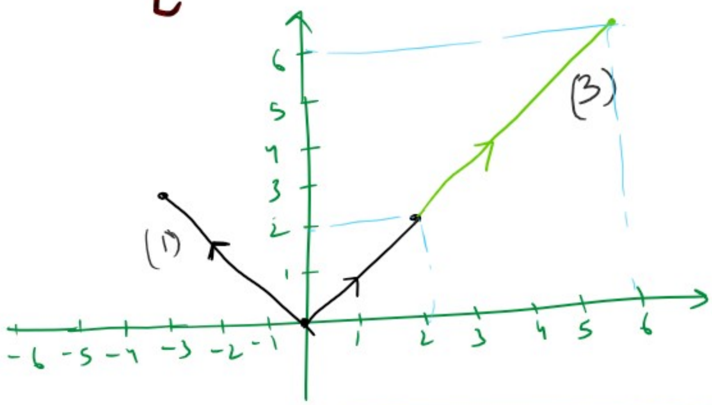
$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -3 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} 2 \\ \cdot \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \end{bmatrix}$$



$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$$



$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$A\vec{X} = \lambda \vec{X}$$

$$\Rightarrow AX - \lambda X = \vec{0}$$

$$\Rightarrow AX - (\lambda . I)X = \vec{0}$$

$$\Rightarrow \underline{(A - \lambda I)} X = \vec{0}$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$

$$= \underline{\begin{pmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{pmatrix}} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

"Identity Matrix"

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$(x_1, x_2, x_3)$$

$$\to (x_3, x_2, x_1)$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_2 \\ x_1 \end{bmatrix}$$

determinant $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = a[ei - fh] - b[di - fg] + c.[dh - eg]$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}$$

$$(2-\lambda)^2 - 1 = 0 \Rightarrow \lambda^2 - 4\lambda + 4 - 1 = 0$$

$$\Rightarrow \lambda^2 - 4\lambda + 3 = 0$$

$$\Rightarrow (\lambda - 1)(\lambda - 3) = 0$$

$$\Rightarrow \lambda = 1, \ \underline{\lambda = 3}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 3 \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

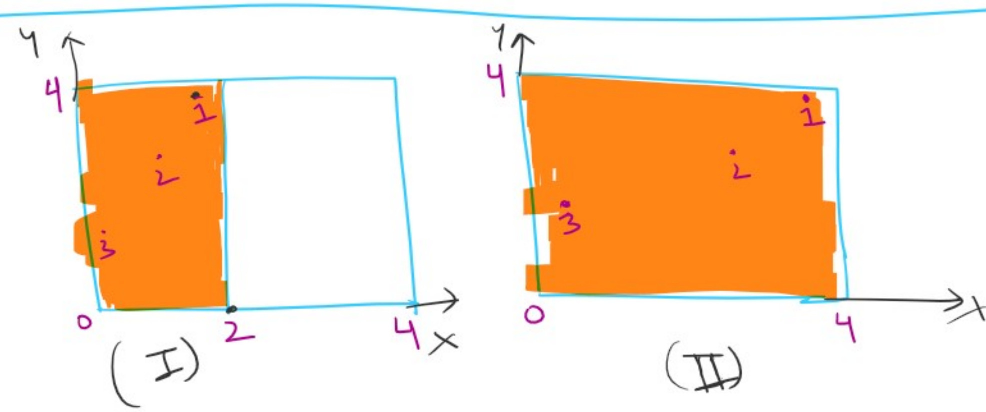
$$\Rightarrow \begin{matrix} 2x_1 + x_2 = 3x_1 \\ x_1 + 2x_2 = 3x_2 \end{matrix} \Rightarrow \boxed{\begin{matrix} x_1 = x_2 \\ x_1 = x_2 \end{matrix}}$$

$(-1,-1)$
$(1,1)$
$(2,2)$
$(3,3)$
$\rightarrow \lambda = 3$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 1 \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

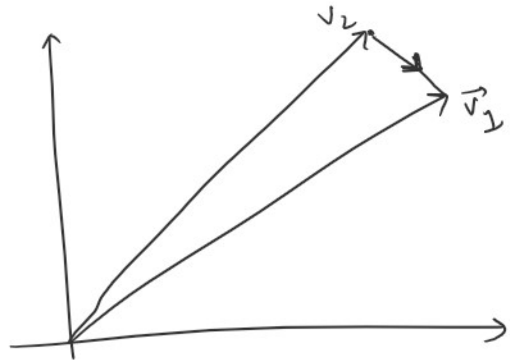
$$\Rightarrow \begin{matrix} 2x_1 + x_2 = x_1 \\ x_1 + 2x_2 = x_2 \end{matrix} \Rightarrow \begin{matrix} x_1 + x_2 = 0 \\ x_1 + x_2 = 0 \end{matrix}$$

$(1,-1)$
$(-4,4)$
$(3,-3)$
$(-100,100)$
$\rightarrow \underline{\lambda = 1}$



$$\begin{matrix} x \to 2x \\ y \to y \end{matrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ y \end{bmatrix}$$



v_2
$\vec{v}_1$