Essential Mathematics for Data Science / Machine Learning

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Learning Outcomes Live Session - 1

- Vectors
- Vector Operations
- Matrix Algebra

Role of Mathematics in DS/ML

- Mathematics is an essential part of many engineering and science branches
- Modern techniques in data science and machine learning are no exceptions
- “Mathematics is the language with which God has written the universe.” -Galileo Galilei
- AI is not magic, it is just mathematics .. !!
- The concepts of Linear Algebra, Calculus, game theory, Optimization and Gradient Descent are frequently used in many DS/ML techniques

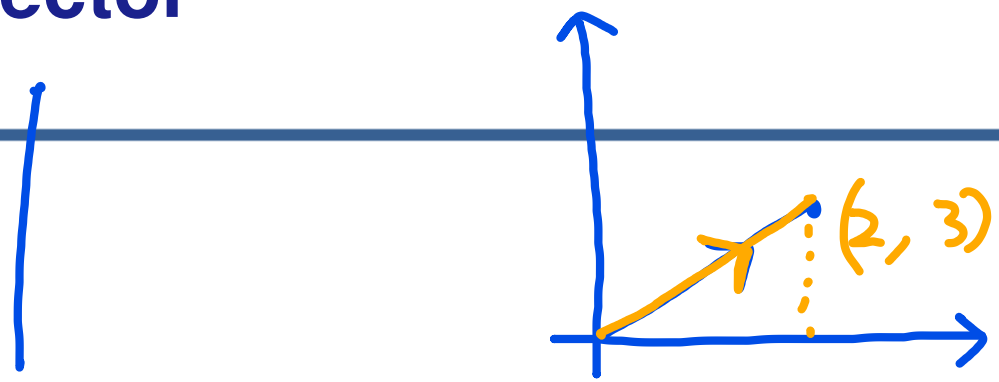
Mathematical skills in DS/ML

- Linear Algebra
- Gradient Calculus
- Probability and Statistics
- Optimization

Linear Algebra

- branch of applied mathematics related to linear equations, linear transformations and their representations through matrices and vector spaces.
- We will use Linear Algebra in techniques like PCA, neural network algorithms

Vector



$2\hat{i} + 3\hat{j}$

$2 + 3i$

(2, 3)

(x_1, x_2)

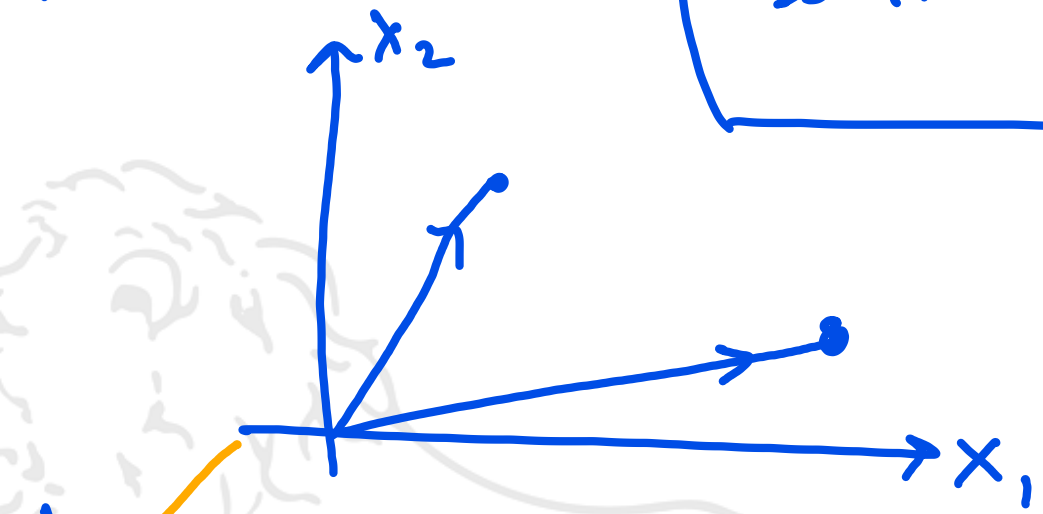
- a mathematical object that encodes a length and direction
- A vector is often represented as a 1-dimensional array of numbers, referred to as components and is displayed either in column form or row form
- Represented geometrically, vectors typically represent coordinates within a n-dimensional space

	Age	Exp.	Income
1.	24	2	7.5L
2.	35	14	20L
3.			

Vectors in R^n/Concept of Vector Space

$\mathbb{R}, \mathbb{R}^2, \mathbb{R}^3,$

(Is $\mathbb{R}^2$ a vector space??



S = vector space

s_1, s_2

$s_1 + s_2$ ←

$\alpha s_1, \beta s_2$ ←

$\alpha s_1 + \beta s_2$ ←

Operations with Vectors in R^n

- Addition/Subtraction
- Dot Product (Inner Product)
- Length/Magnitude / Norm
- Angle between two vectors

Linearly Independent and Dependent Vec

$\mathbb{R}^n$.

$\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m$ are linearly independent if

$$\alpha_1 \vec{v}_1 + \alpha_2 \vec{v}_2 + \ldots + \alpha_n \vec{v}_m = \vec{0}$$

$\Rightarrow \alpha_1 = \alpha_2 = \ldots = \alpha_n = 0$ is the only solution.

Orthogonal Vectors/Orthonormal Vectors

$\vec{v}_1$ $\qquad$ $\vec{v}_2 = \dfrac{\vec{v}_1}{\|\vec{v}_1\|}$

$\mathbb{R}^2$ $\qquad$ $(0,1)$ $\qquad$ $(1,0)$ $\qquad$ $(0,2)$ $\qquad$ $(4,0)$

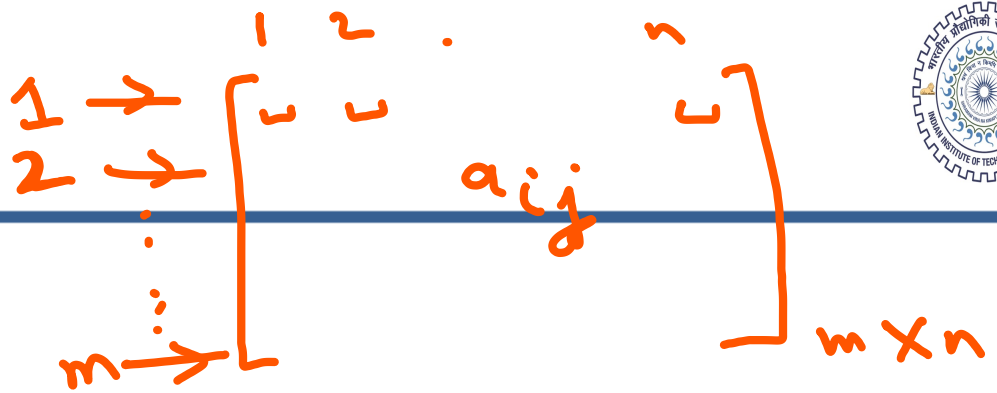
$\mathbb{R}^{10}$ $\qquad$ 3 vectors which are orthonormal

$(1,0,0,-\!-\!-)$

$(0,1,0,\ldots)$

$(0,0,1,\ldots)$

Matrices



In mathematics, a matrix (plural matrices) is a rectangular array or table of numbers, symbols, or expressions, arranged in rows and columns, which is used to represent a mathematical object or a property of such an object. - Wikipedia

$M = \{a_{ij}\}_{m \times n}$

- Matrix Size
- Representing any element of a matrix
- (1,n) and (m,1) matrices

(Size) (Row) (Column)

a_{47}

$a_{12,8}$

"Matrix as Linear Transformation"

$m \times n$, "$m = n$" Square Matrix.

Eigenvalues and Eigenvectors

Let M be a $n \times n$ matrix. A non-zero vector X is said to be a eigenvector of M corresponding to eigenvalue λ if

$$M . \underline{X} = \lambda . X$$

Python Implementation