16 January 2022 13:19

"Agenda"

1. Inverse of a Matrix
2. Trace of a Matrix
3. Relationship of Eigenvalues with trace and determinant
4. Symmetric matrix
5. Orthogonal Matrix

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6. "Principal Component Analysis"

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1. Inverse

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$\boxed{A.B \neq B.A}$ in general

$\boxed{A \cdot I = I.A = A}$

$4 \cdot \left(\frac{1}{4}\right) = 1$

If $A.B = I$, then B is called inverse of A. $\boxed{A^{-1}}$
$B.A = I$

$\boxed{A^{-1} = \frac{1}{\det(A)} \text{Adj}(A)}$

$B = A^{-1}$
$B^{-1} = A$

$A \begin{bmatrix} 4 & 3 \\ 1 & 1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 1 & -3 \\ -1 & 4 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(I) $(AB)^{-1} = B^{-1} \cdot A^{-1}$
$(ABC)^{-1} = C^{-1} B^{-1} A^{-1}$

(II.) If the eigenvalues of A are $\lambda_1, \lambda_2, \ldots \lambda_n$, then the eigenvalues of A^{-1} are $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \ldots \frac{1}{\lambda_n}$

$A = \begin{bmatrix} 0 & -3 & -2 \\ 1 & -4 & -2 \\ -3 & 4 & 1 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} 4 & -5 & -2 \\ 5 & -6 & -2 \\ -8 & 9 & 3 \end{bmatrix}$

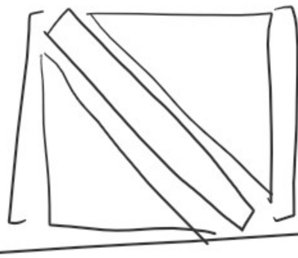
$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

II. Trace of a matrix. = Sum of diagonal elements.

III. $\boxed{\text{Sum of eigenvalues = Trace of the matrix}}$

Product of eigenvalues = Determinant of the matrix

IV. "$a_{ij} = a_{ji}$" Symmetric



V. Orthogonal Matrix

$$AA^T = A^T \cdot A = I \qquad A^{-1} = A^T$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & \cdots & & a_{nn} \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} & & a_{n1} \\ a_{12} & a_{22} & & a_{n2} \\ a_{13} & & & \vdots \\ a_{1n} & a_{2n} & & a_{nn} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \cdots & \\ & & & & \\ 0 & \cdots & & & 1 \end{bmatrix}$$

VI. Positive Semi-Definite Matrix

$(A_{n \times n})$ for any non-zero vector $x \in \mathbb{R}^n$

$X - (n \times 1)$

$(X^T A X) \geq 0$,

then A is said to be PSD.

$\boxed{\rightarrow \text{All the eigenvalues are } \geq 0}$

$$[x_1, x_2] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= [x_1 \quad x_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= (x_1^2 + x_2^2)$$

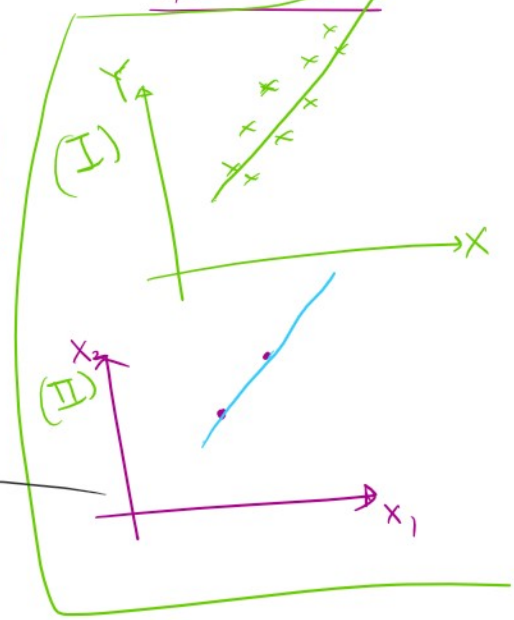
+ Principal Component Analysis

	Gender	# Runs	# wickets	# Strike Rate	# Age
1	Male	10327	0		

5 dimensions

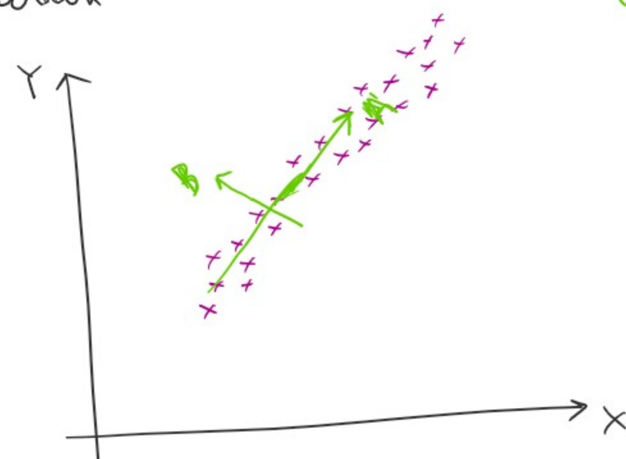
	Gender	Runs	wickets	Strike Rate	Age
1	Male	10327	0		
2	Male	9231	47		
n	Male				

5 dimensions



"Curse of Dimensionality"

1. Feature Selection
2. Feature Extraction



"5-dimensional"

	f_1	f_2	f_3	f_4	f_5
r_1					
r_2					
r_3					
⋮					
r_{1000}					

"(1000×5)"

=

(5×2)

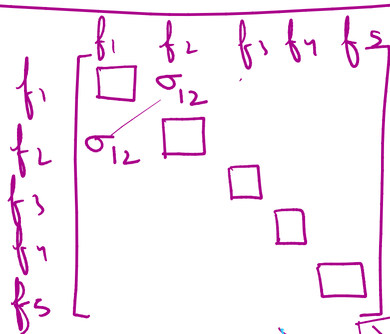
2-dimensional

	F_1	F_2
r_1		
r_2		
r_3		
⋮		
r_{1000}		

"(1000×2)"

$$F_1 = 0.8 f_1 + 0.6 f_3 + 0. f_4 + 0.1 f_1$$
$$F_2 = 0.1 f_1 + 0.9 f_5$$

"Variance – Covariance Matrix"



σ_i^2 = Variance in the i^{th} feature

σ_{ij} = "Covariance between i^{th} and j^{th} feature"

$$Cov(X,Y) = \frac{\sum_{i=1}^{n}(X_i-\overline{X})(Y_i-\overline{Y})}{n}$$

$$\boxed{\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n \geq 0}$$

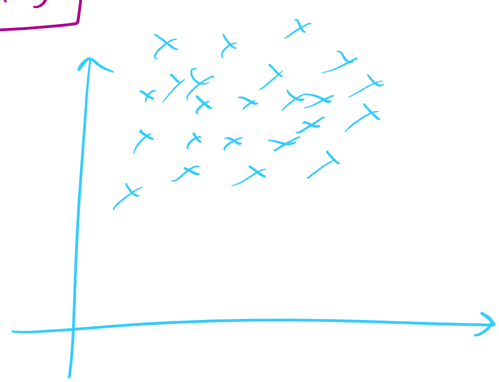
(1) Compute the eigenvalues and eigenvectors of variance-covariance matrix.

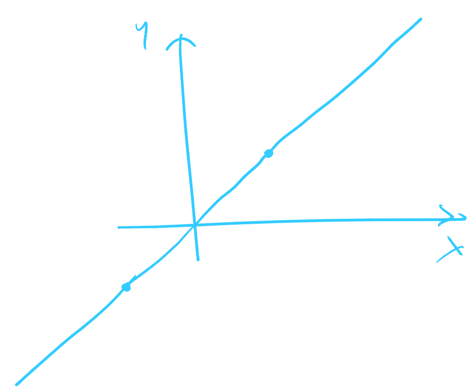
(2) Pick the k largest (if we want k-dimensions)

$$\boxed{\frac{\lambda_1+\lambda_2}{\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5}}$$

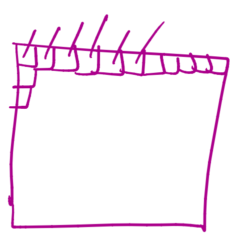
(2 × 5)

$$\boxed{\text{"}\sigma_1^2+\sigma_2^2+\sigma_3^2+\sigma_4^2+\sigma_5^2\text{"} = \lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5}$$

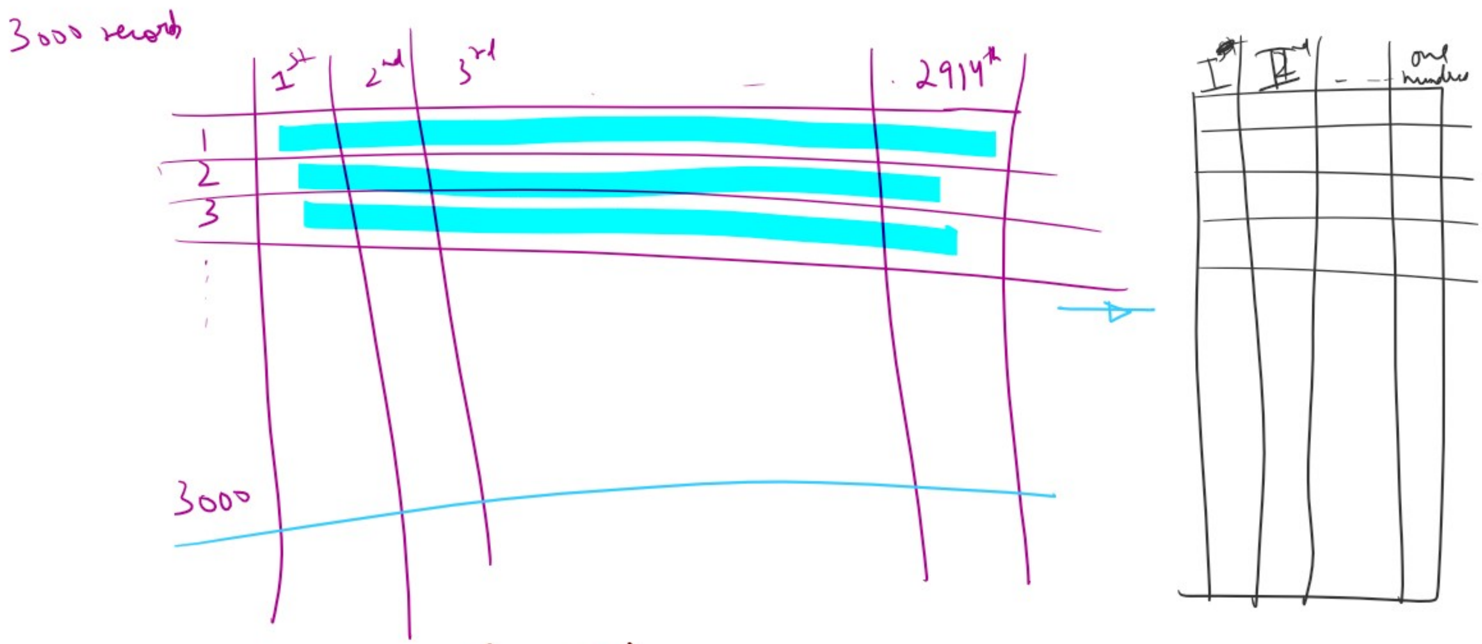




(1) Pete Sampras —



~~6~~ 62×47 = 2914



$$[\underbrace{3000 \times 2914}_{\rightarrow (62 \times 47)}] \text{ — Original Data}$$

$$[3000 \times 100] \text{ — Compressed Data} \rightarrow \underbrace{(100 \times 2914)}_{E^T}$$

$$(3000 \times 2914)$$

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① Original data (D) consists of "N" records and "n" features

$[N \times n]$

② Compute Variance-Covariance Matrix of size $n \times n = \Sigma$

③ Compute eigenvalues and eigenvectors of Σ.

④ Sort the eigenvectors such that eigenvector corresponding to larger eigenvalue should be placed at the top in the eigenvector matrix.

$$E = \begin{bmatrix} e_1, (\lambda_1) \\ (e_2), \lambda_2 \\ (e_3), \lambda_3 \\ \vdots \\ e_n, \lambda_n \end{bmatrix} \qquad \lambda_1 \geq \lambda_2 \geq \lambda_3 \ldots \geq \lambda_n \geq 0$$

⑤ If we want k dimensions, pick the first "k" rows of the matrix (E_k)

(5) If we want k dimensions, pick the first "k" rows of the matrix (E_K)

(E_K) has size $(k \times n)$

(6) Data with new features is $D.(E_K^T) = D_K$

In order to do inverse transform,

reconstruct as $D_K E_K$