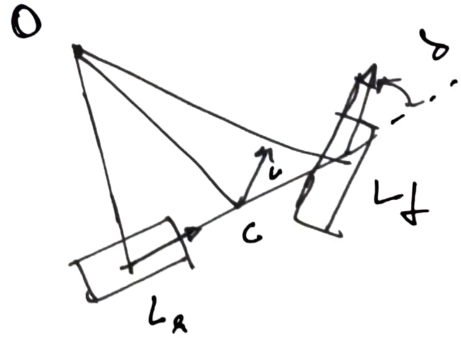
Kinematic bicycle model

↳ Simple model of four wheel car ~~b~~model.

→ ~~One~~ Two wheel is joined with a single wheel..

→ Slip of all wheel are zero,

Equation of motion.



O → instaneous center of rotion.

$L = L_f + L_r$ (wheel base).

δ = steering angle.

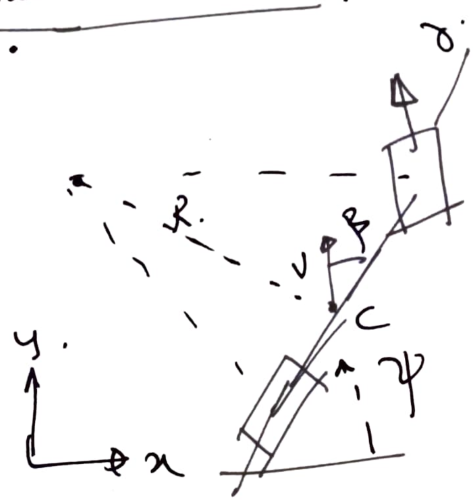
heading angle (angle model is moving).

L_f → distance b/w front wheel and center.

L_r → distance b/w rear wheel and center.

V = is velocity of the model and is perpendicular to the line b/w instaneous center and center.

EOM derivation continued



Understanding angle (δ, β, ψ)

$\beta \to$ side slip angle / slip angle.
angle b/w heading of the vehicle and velocity vector $\vec{v}$:

$$\boxed{\text{if } l_r = 0 \quad L_f = L \quad \text{Then } \beta = \delta}$$

[Not relevant for me since our car only moves front steering]

if $l_r \neq l_f$

Then velocity is given by.

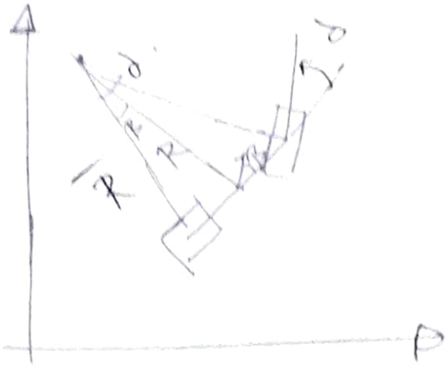
$$\dot{x}(t) = v\cos(\psi + \beta)$$
$$\dot{y}(t) = v\sin(\psi + \beta)$$
$$\dot{\psi}(t) = v/R$$

Need to find R (instantaneous radius of the car)

$\Downarrow$

Turning radius.

& $\beta \to$ slip angle.



Calculating β.

$$\tan(\delta) = \cancel{B}\,\frac{L}{\bar{R}}$$

$$= \frac{L_f + L_r}{\cancel{B}\bar{R}}$$

Using $1/R$.

$$\cos\beta = \frac{\bar{R}}{R} \Rightarrow 1/R = \frac{1}{\bar{R}}\cos\beta$$

$$= \frac{1}{L_r + L_f}\tan\delta\cos\beta.$$

$$\tan(\beta) = \frac{l_r}{\bar{R}} = \frac{l_r}{L_f + L_r}\cdot\tan\delta$$

$$\tan(\beta) = \frac{l_r}{L}\cdot\tan(\delta)$$

$$\boxed{\beta = \tan^{-1}\left(\frac{l_r}{L}\tan(\delta)\right)}.$$

Final EOM

$$\boxed{\begin{aligned} \dot{x}(t) &= v\cos(\psi + \underline{\beta}) \\ \dot{y}(t) &= v\sin(\psi + \underline{\beta}) \\ \beta &= \tan^{-1}\left(\frac{L_r}{L}\tan\delta\right) \\ \dot{\psi}(t) &= \frac{V}{R}. \\ &= \frac{V}{L_f + l_r}\cos\underline{\beta}\tan\delta. \end{aligned}}$$

In our case we have $\dot{x}(t)$
$\dot{y}(t)$
$\dot{\psi}(t)$.

i.e.

$$0 = \underline{v}\cos(\psi + \beta) - \dot{x}(t).$$
$$0 = \underline{v}\sin(\psi + \beta) - \dot{y}(t).$$
$$0 = \frac{\underline{v}}{L}\cos(\beta)\cdot\tan\delta - \dot{\psi}(t).$$

$\Downarrow$ adding β.

$$0 = \underline{v}\cos\left(\psi + \tan^{-1}\left(\frac{L_r}{L}\tan\underline{\delta}\right)\right) - \dot{x}(t)$$
$$0 = \underline{v}\sin\left(\psi + \tan^{-1}\left(\frac{L_r}{L}\tan\underline{\delta}\right)\right) - \dot{y}(t)$$
$$0 = \frac{\underline{v}}{L}\cdot\cos\left(\tan^{-1}\left(\frac{L_r}{L}\cdot\tan\underline{\delta}\right)\right)\cdot\tan(\underline{\delta}) - \dot{\psi}(t)$$

$\underline{v}$ & $\underline{\delta}$ is not given.