1. $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

$\Rightarrow$ Here, $a = 3$, $b = 2$

$\log_b a = \log_2 3 = 1.58$

$n^{\log_b a} = n^{1.58}$

Here, $f(n) = n^2$

$\because n^2 > n^{1.58}$

$\therefore T.C = \Theta(n^2)$

2. $T(n) = 4(T(n/2)) + n^2$

$\Rightarrow$ $a = 4$, $b = 2$

$\log_b a = \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2$

$\therefore n^{\log_b a} = n^2$

$\therefore f(n) = n^2$

$\because n^2 = n^2$

$\therefore T.C = \Theta(n^2 \log n)$

3. $T(n) = T(n/2) + 2^n$

Here $a = 1$ $b = 2$

$\log_b a = \log_2 1 = 0$

$n^c = n^{\log_b a} = n^0 \Rightarrow 1$

$\because 2^n > 1$

$\therefore T.C = \Theta(2^n)$

4. $T(n) = 2^n T(n/2) + n^n$

$\Rightarrow$ $a = 2^n$

which cannot be possible

Since $a \geq 1$ & a ~~cannot~~ is not ~~be~~ constant

Thus, master's method do not apply.

5. $T(n) = 16\,T\left(\frac{n}{4}\right) + n$

$\Rightarrow$ $a = 16 \quad b = 4$

$\log_b a = \log_4 16 = \log_4 (4)^2 = 2\log_4 4 = 2$

$n^{\log_b a} = n^2$

$f(n) = n \quad \therefore \quad n^{\log_b a} > f(n)$

$\therefore \quad T.C = \Theta(n^2)$

6. $T(n) = 2T(n/2) + n\log n$

$\Rightarrow$ $a = 2 \quad b = 2$

$\log_b a = \log_2 2 = 1$

$f(n) = n\log n$

$\therefore \quad n^c = n^{\log_b a} = n^1 = n$

$\therefore \quad f(n) > n^c$

$\therefore \quad TC = \Theta(n\log n)$

7. $T(n) = 2T(n/2) + \frac{n}{\log n}$

$a = 2 \quad b = 2$

$n^c = n^{\log_b a} = n^{\log_2 2} = n^1$

$f(n) = \frac{n}{\log n}$

$\therefore \quad \frac{n}{\log n} < n$

$\therefore \quad f(n) < n^c$

$\therefore \quad T(n) = \Theta(n)$

8. $T(n) = 2T(n/4) + n^{0.51}$

$\Rightarrow$ $a = 2 \quad b = 4$

$c = \log_b a = \log_4 2 = 0.5$

$n^c = n^{0.5}$

$f(n) = n^{0.51}$

$\therefore f(n) > n^c$

$T(n) = \Theta(n^{0.51})$

9. $T(n) = 0.5T(n/2) + 1/n$

$\Rightarrow$ $a = 0.5 \quad b = 2$

$a \geq 1$ Therefore we cannot apply Master's Theorem.

10. $T(n) = 16T(n/4) + n!$

$\Rightarrow$ $a = 16 \quad b = 4$

$c = \log_4 16 = 2\log_4 4 = 2$

$n^c = n^2$

As $n! > n^2$

$\therefore T(n) = \Theta(n!)$

11. $4T(n/2) + \log n$

$\Rightarrow$ $a = 4 \quad b = 2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) = \log n$

$\therefore \log n < n^2$

$T(n) = \Theta(n^2)$

12. $T(n) = \mathrm{sqrt}(n)\, T(n/2) + \log n$

$a = \sqrt{n} \quad b = 2$

$c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2}\log_2 n$

$\frac{1}{2}\log_2 n < \log(n) \qquad \therefore T(n) = \Theta(\log(n))$

13. $T(n) = 3T(n/2)+n$

$\Rightarrow$ $a = 3 \quad b = 2$

$c = \log_b a = \log_2 3 = 1.5849$

$n^c = n^{1.5849}$

$f(n) = n$

$\therefore n < n^{1.58}$

$\therefore T(n) = \Theta(n^{1.5849})$

Q14. $T(n) = 3T(n/3) + sqrt(n)$

$\Rightarrow$ $a = 3 \quad b = 3$

$c = \log_3 3 = 1$

$n^c = \cancel{1}\, n' = n$

$sqrt(n) < 1$

$f(n) < n^c$

$\therefore T(n) = \Theta(n)$

Q15. $T(n) = 4T(n/2)+n$

$a = \cancel{3}4 \quad b = 2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) = n$

$\therefore n^2 > n$

$n^c > f(n)$

$T(n) = \Theta(n^2)$

Q16. $T(n) = 3T(n/4) + n\log n$

$a = 3 \quad b = 4 \quad f(n) = n\log n$

$c = \log_b a = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$n^{0.792} < n\log n$

$T(n) = \Theta(n\log n)$

Q17. $T(n) = 3T(n/3) + n/2$

$\Rightarrow$ $a = 3$, $b = 3$

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$f(n) = n/2$

$\because \frac{n}{2} < n$

$T(n) = \Theta(n)$

Q18. $T(n) = 6T(n/3) + n^2 \log n$

$\Rightarrow$ $a = 6$ $b = 3$

$c = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

$f(n) = n^2 \log n$

$\therefore T(n)$ $\Theta(n^2 \log n)$

19. $T(n) = 4T(n/2) + n/\log n$

$\Rightarrow$ $a = 4$ $b = 2$

$c = \log_2 4 = 2$

$n^c = n^2$

$f(n) = \frac{n}{\log n}$

$\because \frac{n}{\log n} < n^2$ $\therefore T(n) = \Theta(n^2)$

20. $T(n) = 64T(n/8) - n^2 \log n$

$a = 64$ $b = 8$

$c = \log_8 64 = 2$

$n^c = n^2$ $f(n) = n^2 \log n$

$T(n) = \Theta(n^2 \log n)$

Q1. $T(n) = 7T(n/3) + n^2$

$\Rightarrow$ $a = 7$ $b = 3$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$f(n) = n^2$

$\because$ $f(n) > n^c$

$\therefore$ $T(n) = \theta(n^2)$

Q2. $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, b = 2$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$f(n) = n(2 - \cos n)$

$\because$ $n(2 - \cos n) > n^c$

$\therefore$ $T(n) = \theta(n(2 - \cos n))$