No.

Date

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1. Dik: $f(x) = x^2$

$g(x) = 1 - \sqrt{x}$

Dit: a. $\frac{f}{g}(x)$

b. $\frac{g}{f}(x)$

c. $D_{f/g}$

d. $D_{g/f}$

Jwb:

a. $\frac{f}{g}(x) = \frac{x^2}{1-\sqrt{x}}$

b. $\frac{g}{f}(x) = \frac{1-\sqrt{x}}{x^2}$

c. $D_{f/g} = [0, \infty), x \neq 1$

d. $D_{g/f} = [0, \infty)$

e. $g(f(2)) = 1 - \sqrt{x^2} = 1 - x = -1$

d. $f(g(0)) = (1-\sqrt{x})^2 = (1-0)^2 = 1$

2. Dik: $f(x) = \sqrt{x-2}$

$g(x) = x^2 - 3$

Dit: a. $f \circ g$ terdef ?

b. $f \circ g(x)$, $D_{f \circ g}$, $R_{f \circ g}$

Jwb:

a. $D_f = [2, \infty)$

$R_f = [0, \infty)$

$D_g = (-\infty, \infty)$

$R_g = [-3, \infty)$

$D_f \cap R_g = [2, \infty) \cap [-3, \infty)$

$= [2, \infty)$

b. $f \circ g(x) = f(g(x))$

$= \sqrt{(x^2-3)-2}$

$= \sqrt{x^2-5}$

$D_{f \circ g} = (-\infty, -\sqrt{5}] \cup [\sqrt{5}; \infty)$

$R_{f \circ g} = [0, \infty)$

3. Dik:

$$f(x) = \begin{cases} x^2+1, & x < 1 \\ \frac{x^2-1}{x-1}, & 1 \leq x \leq 2 \\ 5-x, & x > 2 \end{cases}$$

Dit: a. apakah $f(x)$ kontinu di $x = 1$ dan $x = 2$?

b. apakah $f(x)$ bisa diturunkan di $x = 2$?

c. gambar grafik fungsi.

Jwb:

a. limit kiri

$\lim_{x \to 1^-} x^2 + 1$

$= 2$

limit kanan

$\lim_{x \to 1^+} \frac{x^2-1}{x-1}$

$= \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$

$= 2$

$f(1) = \frac{x^2-1}{x-1} = \frac{1^2-1}{1-1} = \frac{0}{0}$

$f(1) \neq \lim_{x \to 1^-} f(1) = \lim_{x \to 1^+} f(1)$

$f(x)$ tidak kontinu pada $x = 1$

limit kiri

$\lim\limits_{x\to 2^-} \frac{x^2-1}{x-1}$

$= \frac{4-1}{2-1}$

$= 3$

limit kanan

$\lim\limits_{x\to 2^+} 5-x$

$= 3$

$F(2) = \frac{x^2-1}{x-1} = \frac{4-1}{2-1} = 3$

$F(2) = \lim\limits_{x\to 2^-} F(x) = \lim\limits_{x\to 2^+} F(x)$

$F(x)$ kontinu di $x = 2$

b. $f'(x) = \lim\limits_{x\to 2^-} \frac{F(x)-F(2)}{x-2}$

$= \lim\limits_{x\to 2^-} \frac{\frac{x^2-1}{x-1} - \frac{4-1}{2-1}}{x-2}$

$= \lim\limits_{x\to 2^-} \frac{x^2-3x+2}{(x-2)(x-1)}$

$= \lim\limits_{x\to 2^-} \frac{\cancel{(x-2)}\cancel{(x-1)}}{\cancel{(x-2)}\cancel{(x-1)}} = 1$

$f'(x) = \lim\limits_{x\to 2^+} \frac{F(x)-F(2)}{x-2}$

$= \lim\limits_{x\to 2^+} \frac{5-x-3}{x-2}$

$= \lim\limits_{x\to 2^+} \frac{2-x}{x-2}$

$= \lim\limits_{x\to 2^+} \frac{-\cancel{(x-2)}}{\cancel{x-2}} = -1$

$\lim\limits_{x\to 2^-} \frac{F(x)-F(2)}{x-2} \neq \lim\limits_{x\to 2^+} \frac{F(x)-F(2)}{x-2}$

$F(x)$ tidak dapat diturunkan untuk $x = 2$

c. $x < 1$

$(-2,5)$

$(-1,2)$

$(0,1)$

$(1,2)$

$1 \leq x \leq 2$

$(1^+,\infty)$

$(2,3)$

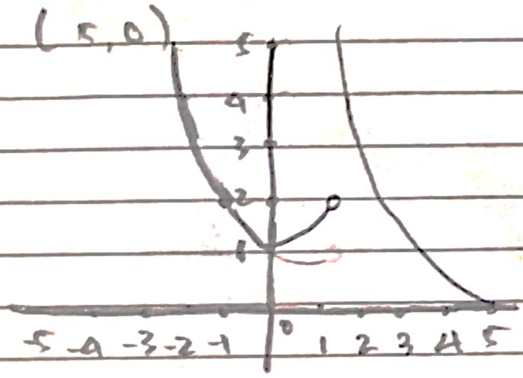
$x > 2$

$(2^+,3)$

$(3,2)$

$(4,1)$

$(5,0)$



4.) a. $\lim_{x \to 3^+} \frac{2x}{9-x^2} = \frac{2(3)}{9-(3)^2} = \frac{6}{0} = \infty$

b. $\lim_{x \to 0} \frac{3x^2}{1-\cos^2 \frac{1}{2}x}$

=

=

=

c. $\lim_{x \to -\infty} \frac{\sqrt{4x^2+1}}{x+1}$

$\lim_{x \to -\infty} \frac{\frac{1}{x}\sqrt{4x^2+1}}{\frac{1}{x}(x+1)}$

$\lim_{x \to -\infty} \frac{\sqrt{\frac{4x^2}{x^2}+\frac{1}{x^2}}}{\frac{x}{x}+\frac{1}{x}} = \frac{\sqrt{4-0}}{1-0}$

$= 2$

d. $\lim_{x \to 2^-} \frac{|x-2|}{(x-2)} + 2x$

$\lim_{x \to 2^-} \frac{2-x}{x-2} + \frac{2x(x-2)}{x-2}$

$\lim_{x \to 2^-} \frac{2x^2-5x+2}{x-2}$

$\lim_{x \to 2^-} \frac{(2x-1)\cancel{(x-2)}}{\cancel{(x-2)}}$

$\lim_{x \to 2^-} (2x-1) = 2(2)-1 = 5$

e. $\lim_{x \to 0^+} [\![x]\!] + |x-1|$

$\lim_{x \to 0^+} [\![x]\!] + 1 - x = 0+1-0 = 1$

5. b. y' secara implisit dari

$2x^3y + \cos xy = 2x$

$6x^2y - y\sin(xy) - 2 + \frac{dy}{dx} x(2x^2 - \sin(xy))$

$\frac{dy}{dx} = \frac{2-6x^2y+y\sin(xy)}{x(2x^2-\sin(xy))}$

c. Dari soal b, tentukan persamaan garis singgung di $(\frac{1}{2}, 0)$

$m = \frac{dy}{dx}$

$= \frac{2-6x^2y+y\sin(xy)}{x(2x^2-\sin(xy))}$

$= \frac{2}{\frac{1}{2}(2(\frac{1}{2})^2-0)} = \frac{2}{\frac{1}{4}} = 8$

$y - y_1 = m(x-x_1)$

$y = 8(x-\frac{1}{2})$

$= 8x - 4$

a. $\frac{d^2y}{dx^2}$ dari $y = ((x+1)^3 \sin 2x)$

$\frac{dy}{dx} = (3)(1)(x+1)^2 . \sin 2x + (x+1)^3 . 2\sin 2x \cos 2x$

$= 3(x+1)^2 . \sin 2x + (x+1)^3 . \sin 4x$

$\frac{d^2y}{dx^2} = (6)(1)(x+1)\sin 2x + 3(x+1)^2 . 2\cos 2x + (3)(1)(x+1)^3 . 4\cos 4x$

$= 6(x+1) . \sin 2x + 3(x+1)^2 . 2\cos 2x + 3(x+1)^2 . \sin 4x + (x+1)^3 . 4\cos 4x$

$= (6x+6)\sin 2x + 3(x+1)^2 (2\cos 2x + \sin 4x) + (x+1)^3$