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Slide : 06

1. a $y'' - 3y' + 2y = \cos x$

• PK : $r^2 - 3r + 2 = 0$

$(r-2)(r-1)$

$r_1 = 2, \; r_2 = 1$

• Solusi homogen : $y_h = c_1 e^{2x} + c_2 e^{x}$

$y_p = A\cos x + B\sin x$

$y_p' = -A\sin x + B\cos x$

$y_p'' = -A\cos x - B\sin x$

$(-A\cos x - B\sin x) - 3(-A\sin x + B\cos x) + 2(A\cos x + B\sin x) = \cos x$

$-A\cos x - B\sin x + 3A\sin x - 3B\cos x + 2A\cos x + 2B\sin x = \cos x$

$A\cos x + B\sin x + 3A\sin x - 3B\cos x = \cos x$

$(A - 3B)\cos x + (3A + B)\sin x = \cos x$

$3A + B = 0 \Rightarrow 3A + B = 0$

$A - 3B = 3 \quad \underline{3A - 9B = 9} \; -$

$10B = -3$

$B = -3/10$

$3A - 3/10 = 0$

$3A = 3/10$

$A = 1/10$

SU : $y = y_h + y_p$

$= (c_1 e^{2x} + c_2 e^{x}) + 1/10 \cos x - 3/10 \sin x$

b. $y'' - 9y = x + 2$

• PK : $r^2 - 9 = 0$

$(r+3)(r-3)$

$r_1 = 3, \; r_2 = -3$

• Solusi homogen : $y_h = c_1 e^{3x} + c_2 e^{-3x}$

$y_p = A_1 x + A_2$

$y_p' = A_1$

$y_p'' = 0$

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$0 - 9(A_1 x + A_2) = x + 2$

$-9A_1 x + 9A_2 = x + 2$

$-9A_1 x = x \qquad -9A_2 = 2$

$A_1 = -1/9 \qquad A_2 = -2/9$

• SU : $y = y_h + y_p$

$= C_1 e^{3x} + C_2 e^{-3x} - x/9 - 2/9$

c. $y'' - 3y' - 4y = 3x^2 + 2$

• PK : $R^2 - 3R - 4 = 0$ • Solusi homogen : $y_h = C_1 e^{4x} + C_2 e^{-x}$

$(R-4)(R+1)$

$R_1 = 4, R_2 = -1$

$y_p = A_2 x^2 + A_1 x + A_2$

$y_p' = 2A_2 x + A_1$

$y_p'' = 2A$

$2A_2 - 3(2A_2 x + A_1) - 4(A_2 x^2 + A_1 x + A_0) = 3x^2 + 2$

$2A_2 - 6A_2 x - 3A_1 - 4A_2 x^2 - 4A_1 x - 4A_0 = 3x^2 + 2$

$-4A_2 x^2 + (-6A_2 - 4A_1) x + 2A_2 - 4A_0 - 3A = 3x^2 + 2$

$-4A_2 x^2 = 3x^2 \qquad -3/2 - 4A_0 - 27/8 = 2$

$A_2 = -3/4 \qquad -4A_0 = 2 + 3/2 + 27/8$

$= \frac{16 + 12 + 27}{8} = \frac{-55}{32}$

$(-6A_2 - 4A_1) x = 0$

$\frac{18}{4} - 4A_1 = 0$

$-4A_1 = -\frac{12}{4}$

$A_1 = \frac{18}{16} = \frac{9}{8}$

SU : $y = C_1 e^{4x} + C_2 e^{-4} - \frac{3x^2}{4} + \frac{9x}{8} - \frac{55}{32}$

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d $y'' - 3y' - 4y = e^{2x}$

• PK : $R^2 - 3R - 4 = 0$

$(R-4)(R+1)$

$R_1 = 4, \quad R_2 = -1$

• Solusi homogen : $y_h = C_1 e^{4x} + C_2 e^{-x}$

$y_p = Ce^{2x}$

$y_p' = 2Ce^{2x}$

$y_p'' = 4Ce^{2x}$

$4Ce^{2x} - 6Ce^{2x} - 4Ce^{2x} = e^{2x}$

$(4C - 6C - 4C)e^{2x} = e^{2x}$

$(-6C)e^{2x} = e^{2x}$

$C = -1/6$

• SU : $C_1 e^{4x} + C_2 e^{-x} - e^{2x}/6$

E $y'' + 4y = 2\sin x$

• PK : $R^2 + 4 = 0$

$R^2 = -4$

$R_{12} = \pm 2\sqrt{-1}$

$R_1 = 2i \quad , R_2 = -2i$

• Solusi homogen : $C_1 \cos 2x + C_2 \sin 2x$

$y_p = A\cos x + B\sin x$

$y_p' = -A\sin x + B\cos x$

$y_p'' = -A\cos x - B\sin x$

$-A\cos x - B\sin x + 4A\cos x + 4B\sin x = 2\sin x$

$3A\cos x + 3B\sin x = 2\sin x$

$3A\cos x = 0 \qquad 3B\sin x = 2\sin x$

$A = 0 \qquad B = 2/3$

• SU : $y = C_1 \cos 2x + C_2 \sin 2x + 2/3 \sin x$

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F $y'' + 4y = 2\cos 2x$

• PK : $r^2 + 4 = 0$ • Solusi homogen : $c_1 \cos 2x + c_2 \sin 2x$

$r^2 = -4$

$r_{12} = \pm 2\sqrt{-1}$

$r_1 = 2i$ , $r_2 = -2i$

$y_p = Ax\cos 2x + Bx\sin x$

$y_p' = (A\cos 2x - 2Ax\sin 2x) + (B\sin 2x + 2Bx\cos 2x)$

$y_p'' = (-2A\sin 2x - 2A\sin 2x - 4Ax\cos 2x) + (2B\cos 2x + 2B\cos 2x - 4Bx\sin 2x)$

$-2A\sin 2x - 2A\sin 2x - 4Ax\cos 2x + 2B\cos 2x + 2B\cos 2x - 4Bx\sin 2x +$

$4Ax\cos 2x + 4Bx\sin 2x = 2\cos 2x$

$-4A\sin 2x + 4B\cos 2x = 2\cos 2x$

$-4A = 0 \quad 4B = 2$

$A = 0 \quad B = 1/2$

• SU : $y = c_1 \cos(2x) + c_2 \sin(2x) + \frac{x \sin(2x)}{2}$

G $y'' + y' = 3x^2 + 2$

• PK : $r^2 + 2r = 0$ • Solusi homogen : $c_1 + c_2 e^{-2x}$

$r(r+2) = 0$

$r_1 = 0$ , $r_2 = -2$

$y_p = Ax^3 + Bx^2 + Cx$

$y_p' = 3Ax^2 + 2Bx + C$

$y_p'' = 6A + 2B$

$6Ax + 2B + 2(3Ax^2 + 2Bx + C) = 3x^2 + 2$

$6Ax + 2B + 6Ax^2 + 4Bx + 2C = 3x^2 + 2$

$6Ax^2 + (6A + 4B)x + 2B + 2C = 3x^2 + 2$

$6Ax^2 = 3x^2 \qquad 3 + 4B = 0 \qquad -3/2 + 2C = 2$

$A = 1/2 \qquad B = -3/4 \qquad 2C = \frac{4+3}{2} \Rightarrow C = \frac{7}{4}$

• SU : $c_1 + c_2 e^{-2x} + x^3/2 - 3x^2/4 + 7x/4$

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H. $y'' + 4y' + 4y = 9\cos x$

• PK : $r^2 + 4r + 4 = 0$ • Solusi homogen : $C_1 e^{-2x} + C_2 x e^{-2x}$

$(r+2)(r+2)$

$r_{1,2} = -2$

$y_p = A\sin x + B\cos x$

$y_p' = A\cos x - B\sin x$

$y_p'' = -A\sin x - B\cos x$

$-A\sin x - B\cos x + 4A\cos x - 4B\sin x + 4A\sin x + 4B\cos x = 9\cos x$

$3A\sin x + 3B\cos x + 4A\cos x - 4B\sin x = 9\cos x$

$(3A - 4B)\sin x + (3B + 4A)\cos x = 9\cos x$

$3A - 4B = 0$ $\quad (3B + {}^{16}/_{3}B)\cos x = 9\cos x$

$A = {}^{4}/_{3}B$ $\quad {}^{25}/_{9}B = 9$

$= {}^{108}/_{75}$ $\quad B = {}^{27}/_{25}$

$= {}^{36}/_{25}$

• SU : $C_1 e^{-2x} + C_2 x e^{-2x} + 36\sin x/25 + 27\cos x/25$

I. $y'' - 4y' + 4y = e^{2x}$

• PK : $r^2 - 4r + 4 = 0$ • Solusi homogen : $C_1 e^{2x} + C_2 x e^{2x}$

$(r-2)(r-2)$

$r_{1,2} = 2$

$y_p = Cx^2 e^{2x}$

$y_p' = C(2xe^{2x} + e^{2x} \cdot 2x^2)$

$y_p'' = C(4e^{2x}x^2 + 8e^{2x}x + 2e^{2x})$

$4Ce^{2x}x^2 + 8Ce^{2x} + 2Ce^{2x} - 8Cxe^{2x} - 8Ce^{2x}x^2 + 4Cx^2e^{2x} = e^{2x}$

$2Ce^{2x} = e^{2x}$

$C = {}^{1}/_{2}$

• SU : $C_1 e^{2x} + C_2 x e^{2x} + \dfrac{e^{2x}x^2}{2}$

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1. $y'' + 3y' - 4y = 3x^2 + 2$

• PK : $R^2 + 3R - 4 = 0$ • Solusi homogen : $y_h = C_1 e^{-4x} + C_2 e^{x}$

$(R+4)(R-1)$

$R_1 = -4$ , $R_2 = 1$

$y_p = Ax^2 + Bx + C$

$y_p' = 2Ax + B$

$y_p'' = 2A$

$2A + 6Ax + 3B - 4Ax^2 - 4Bx - 4C = 3x^2 + 2$

$-4Ax^2 + (6A - 4B)x + 2A + 3B - 4C = 3x^2 + 2$

$-4Ax^2 = 3x^2$ $\quad -18/4 - 4B = 0$ $\quad -6/4 - 54/16 = 4C = 2$

$A = -3/4$ $\quad B = -18/16$ $\quad \frac{-24-54}{16} + 4C = 2$

• SU : $y = C_1 e^{-4x} + C_2 e^{x} - 3x^2/4 - 18x/16 - 55/64$ $\quad 4C = \frac{32+73}{16}$

$-C = -55/64$

2. A $y'' - y' - 2y = 3e^{2x}$

$y(0) = 0 \; y'(0) = -2$

• PK : $R^2 - R - 2 = 0$ • Solusi homogen : $y_h = C_1 e^{-x} + C_2 e^{2x}$

$(R-2)(R+1)$

$R_2 = 2$ , $R_1 = -1$

$y_p = Cx e^{2x}$

$y_p' = 2Cx e^{2x} + Ce^{2x}$

$y_p'' = 4Cx e^{2x} + 4Ce^{2x}$

$4Cx e^{2x} + 4Ce^{2x} - 2Cx e^{2x} - Ce^{2x} - 2Cx e^{2x} = 3e^{2x}$

$3Ce^{2x} = 3e^{2x}$

$Ce^{2x} = e^{2x}$

$C = 1$

• SU : $y = C_2 e^{2x} + C_1 e^{-x} + e^{2x} x -$

$y' = 2C_2 e^{2x} - C_1 e^{-x} + 2xe^{2x} + e^{2x}$

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$0 = C_1 + C_2$ ... (1)

$C_1 = -C_2$

$2C_2 + C_2 = -3$

$C_2 = -1$

$C_1 = 1$

$y = -e^{2x} + e^{-x} + e^{2x}x$

B. $y'' - 4y' + 3y = 10e^{-2x}$

$y(0) = -3$

• PK : $R^2 - 4R + 3 = 0$

$(R-3)(R-1)$

$R_1 = 1 \quad R_2 = 3$

• Solusi homogen : $C_1e^x + C_2e^{3x}$

$y_p = Ce^{-2x}$

$y_p' = -2Ce^{-2x}$

$y_p'' = 4Ce^{-2x}$

$4Ce^{-2x} + 8Ce^{-2x} + 3Ce^{-2x} = 10e^{-2x}$

$15Ce^{-2x} = 10e^{-2x}$

$C = 2/3$

• SU : $y = C_1e^x + C_2e^{3x} + 2/3\,e^{-2x}$

$y' = C_1e^x + 3C_2e^{3x} - 4/3\,e^{-2x}$

$1 = C_1 + C_2 + 2/3$

$1/3 = C_1 + C_2$ ... (1)

$-3 = C_1 + 3C_2 - 4/3$

$-5/3 = C_1 + 3C_2$ ... (2)

$C_1 + C_2 = 1/3$

$C_1 + 3C_2 = -5/3$

$-2C_2 = 2$

$C_2 = -1$

$C_1 = 4/3$

• SK : $-e^x + 4e^{3x}/3 + 2e^{-2x}/3$

c. $y'' - 3y' + 2y = e^{2x} + e^{3x}$

$y(0) = 1$

$y'(0) = 2$

• PK : $r^2 - 3r + 2 = 0$

$(r-2)(r-1)$

$r_1 = 2 \quad r_2 = 1$

• Solusi homogen : $C_2 e^{x} + C_1 e^{2x}$

$y_p = Ae^{2x} + Be^{3x}$

$y_p' = 2Ae^{2x} + 3Be^{3x}$

$y_p'' = 4Ae^{2x} + 9Be^{3x}$

$4Ae^{2x} + 9Be^{3x} - 6Ae^{2x} - 9Be^{3x} + 2Ae^{2x} + 2Be^{3x} = e^{2x} + e^{3x}$

$2Be^{3x} = e^{3x}$

$B = 1/2$

$6Ae^{2x} = e^{2x}$

$A = 1/6$

• SU : $y = C_2 e^{x} + C_1 e^{2x} + \frac{e^{2x}}{6} + \frac{e^{3x}}{2}$

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$y' = C_2 e^{x} + 2C_1 e^{2x} = \frac{2e^{4x}}{5} + \frac{3e^{3x}}{2}$

Sk: $C_1 + C_2 + 1/6 + 1/2 = 1$

$C_1 + C_2 + \frac{1+3}{6} = 1$

$C_2 + 2C_1 + 2/3 + 3/2 = 2$

$C_2 + 2C_1 + \frac{4+9}{6} = 2$

$C_2 + 2C_1 = 2 - 13/6 \Rightarrow C_2 + 2C_1 = -1/6$

$C_2 + C_1 = 2/6$

$C_1 = -3/6$

$= -1/2$

$C_2 = 5/6$

• Sk: $y = \frac{5e^{x}}{6} - \frac{e^{2x}}{2} + \frac{e^{4x}}{6} + \frac{e^{3x}}{2}$

D. $y'' - 4y = 4\sin x$

$y(0) = 4$

$y'(0) = 0$

• Plk: $R^2 - 4 = 0$

$(R-2)(R+2)$

$R_1 = 2, \; R_2 = -2$

• Solusi homogen: $y_h = C_1 e^{2x} + C_2 e^{-2x}$

$y_p = A\sin x + B\cos x$

$y_p' = A\cos x - B\sin x$

$y_p'' = -A\sin x - B\cos x$

$-A\sin x - B\cos x - 4A\sin x - 4B\cos x = 4\sin x$

$-5A\sin x - 5B\cos x = 4\sin x$

$-5B = 0 \qquad -5A = 4$

$B = 0 \qquad A = -4/5$

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SU : $y = C_1 e^{2x} + C_2 e^{-2x} - 4/5 \sin x$

$y' = 2C_1 e^{2x} - 2C_2 e^{-2x} - 4/5 \cos x$

$C_1 + C_2 = 4 \ldots (1)$ $\quad 2C_1 - 2C_2 - 4/5 = 0$

$2C_1 - 2C_2 = 4/5$

$C_1 - C_2 = 4/10 \ldots (2)$

$C_1 + C_2 = 4$

$C_1 - C_2 = 4/10$ .

$2C_2 = 36/10$

$C_2 = 36/20$

$C_1 = 56/20$

SK : $y = \frac{56}{20} e^{2x} + \frac{36}{20} e^{-2x} - \frac{4}{5} \sin x$

$= \frac{11}{20} e^{2x} + \frac{9}{5} e^{-2x} - \frac{4}{5} \sin x$