Slide 23

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A
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Transcenden

A. Tentukan $y'$

1. $y = \sec e^{2x} + e^{2\sec x}$

$D_x(y) = D_x(\sec e^{2x} + e^{2\sec x})$

$y' = (2e^{2x})(\sec e^{2x})(\tan e^{2x}) + (2)(\sec(x))(\tan x)(e^{2\sec x})$

2. $y = x^5 e^{-3\ln x}$

$D_x(y) = D_x(x^5 e^{-3\ln x})$

$y' = (5x^4)(e^{-3\ln x}) + \left(\frac{1}{x^3}\right)(e^{-3\ln x})(x^5)$

3. $y = \tan e^{\sqrt{x}}$

$= \tan (e)^{1/2}$

$\left(\frac{1}{2\sqrt{x}}\right)(e^{\sqrt{x}})(\sec^2(e^{\sqrt{x}}))$

4. $y^2 e^{2x} + xy^3 = 1$

$y^2(e^{2x} + xy) = 1$

$y^2 = \frac{1}{e^{2x}+xy}$

$2yy' = (e^{2x} + xy)^{-1}$

$2yy' = (2e^{2x}) \cdot (y + xy')(e^{2x} + xy)^{-2}$

$y' = \frac{(2e^{2x})(y+xy')(e^{2x}+xy)^{-2}}{2y}$

5. $e^y = \ln(x^3+3y)$

$D_x(e^y) = D_x(\ln(x^3+3y))$

$y'e^y = \dfrac{(3x^2+3y')}{(x^3+3y)}$

$y'e^y = \dfrac{3x^2}{(x^3+3y)} + \dfrac{3y'}{(x^3+3y)}$

$y'e^y - \dfrac{3y'}{(x^3+3y)} = \dfrac{3x^2}{(x^3+3y)}$

$\dfrac{(y'e^y)((x^3+3y)-3y'}{(x^3+3y)} = \dfrac{3x^2}{(x^3+3y)}$

$\dfrac{y'(e^y\ln(x^3+3y)-3)}{(x^3+3y)} = \dfrac{3x^2}{\ln(x^3+3y)}$

$y' = \dfrac{(3x^2)(\ln(x^3+3y))}{(\ln(x^3+3y)(e^y(x^3+3y)-3))}$

6. $y = \ln(x^2-5x+6)$

$y' = \dfrac{(2x-5)}{(x^2-5x+6)}$

7. $y = \ln(\cos 3x)$

$y' = \dfrac{-3\sin 3x}{\cos 3x}$

$y' = -3\tan 3x$

8. $y = \left(\frac{1}{x^2}\right)(\ln(x))$

$y' = (-2x^{-3})(\ln(x)) + \left(\frac{1}{x^2}\,\frac{1}{x}\right)$

$y' = \dfrac{-2\ln x}{x^3} + \dfrac{1}{x^3} \Rightarrow \dfrac{-\ln x^2+1}{x^3}$

9. $y = \ln(\sin x)$

$y' = \dfrac{\cos x}{\sin x}$

$y' = \cot x$

10. $y = \sin(\ln(2x+1))$

$y' = \dfrac{1 \cdot 2}{2x+1} \cos(\ln(2x+1))$

$= \dfrac{2\cos(\ln(2x+1))}{2x+1}$

Slide 34 A. Tentukan $y'$

1. $y = 3^{2x^4 - 4x}$

$y' = (3^{2x^4 - 4x})(8x^3 - 4)(\ln 3)$

2. $y = {}^{10}\log(x^2+9)$

$y' = \dfrac{(1)(2x)}{(x^2+9)\ln 10}$

$y' = \dfrac{2x}{(x^2+9)\ln 10}$

3. $(x)({}^{3}\log xy) + y = 2$

$(x)({}^{3}\log xy) - 2 = -y$

$(1)({}^{3}\log xy) + (x)\left(\dfrac{y + xy'}{xy\ln 3}\right) = -y'$

$({}^{3}\log xy) + \dfrac{xy + x^2y'}{xy\ln 3} = -y'$

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$$^3\log xy + \frac{xy}{xy\ln 3} + \frac{x^2y'}{xy\ln 3} = -y'$$

$$^3\log xy + \frac{xy}{xy\ln 3} = -y' - \frac{x^2y'}{xy\ln 3}$$

$$^3\log xy + \frac{xy}{xy\ln 3} = \frac{(-y')(xy\ln 3) - x^2y'}{xy\ln 3}$$

$$\frac{(^3\log xy)(xy\ln 3) + xy}{xy\ln 3} = \frac{y'(-xy\ln 3 - x^2)}{xy\ln 3}$$

$$\frac{\left[(^3\log xy)(xy\ln 3) + xy\right] . xy\ln 3}{(xy\ln 3)(-xy\ln 3 - x^2)} = y'$$

B

1. $\int 10^{5x-1}\, dx \Rightarrow \int 10^{v} \frac{1}{5} dv$

$v = 5x-1$ $\qquad \Rightarrow \frac{1}{5}\int 10^{v} dv$

$dv = 5dx$

$\frac{1}{5}dv = dx$ $\qquad \Rightarrow \frac{10^{v}}{5\ln 10} \Rightarrow \frac{10^{5x-1}}{5\ln 10} + C$

2. $\int x\, 2^{x^2} dx \Rightarrow \int 2^{v} \frac{1}{2} dv$

$v = x^2$ $\qquad \frac{1}{2}\int 2^{v} dv$

$dv = 2x\,dx$

$\frac{1}{2}dv = x\,dx$ $\qquad \Rightarrow \frac{2^{v}}{2\ln 2}$

$$\Rightarrow \frac{2^{x^2}}{2\ln 2} + C$$

c. Hitung $\lim_{x \to \infty} (3^x + 5^x)^{1/x}$

$$\lim_{x \to \infty} \exp \ln (3^x + 5^x)^{1/x}$$

$$\exp \lim_{x \to \infty} \frac{\ln(3^x + 5^x)}{x}$$

$$\exp \lim_{x \to \infty} \left( \frac{3^x \ln(3) + 5^x \ln(5)}{(3^x + 5^x)} \right)$$

$$\frac{\frac{3^x \ln 3}{5^x} + \frac{5^x \ln(5)}{5^x}}{\frac{3^x}{5^x} + \frac{5^x}{5^x}}$$

$$\exp \lim_{x \to \infty} \frac{0 + \ln(5)}{0 + 1}$$

$$e^{\ln 5} = 5$$

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1. $\int \frac{4}{2x+1}\,dx \Rightarrow \int \frac{1}{v}\,2\,dv$

$v = 2x+1$ $\quad = 2\int \frac{1}{v}\,dv$

$dv = 2\,dx$ $\quad = 2\ln|v|$

$2dv = 4\,dx$ $\quad = 2\ln|2x+1| + C$

2. $\int \frac{\ln^2 3x}{x}\,dx \Rightarrow \int v^2\,dv$

$v = \ln 3x$ $\quad \Rightarrow \frac{v^3}{3}$

$dv = \frac{1\cdot 3}{3x}\,dx$ $\quad \Rightarrow \frac{\ln^3(3x)}{3} + C$

$dv = \frac{1}{x}\,dx$

3. $\int \frac{x^3}{x^2+1}\,dx \Rightarrow \int \frac{x^2 \cdot x}{v}\,dx$

$\Rightarrow \int \frac{v-1}{v}\,\frac{1}{2}\,dv$

$\Rightarrow \frac{1}{2}\int \frac{v-1}{v}\,dv$

$\Rightarrow \frac{1}{2}\int 1 - \frac{1}{v}\,dv$

$v = x^2+1$ $\quad \Rightarrow \frac{1}{2}(v - \ln|v|)\,dv$

$dv = 2x\,dx$ $\quad \frac{1}{2}(x^2+1 - \ln(x^2+1)) + C$

$\frac{1}{2}dv = x\,dx$

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4. $\int \frac{\tan(\ln x)}{x} dx \Rightarrow \int \tan(v)\, dv$

$\Rightarrow -\ln|\cos|\ln x|| + C$

$\Rightarrow$

$v = \ln x$

$dv = \frac{1}{x} dx$

5. $\int \frac{2}{x(\ln x)^2} dx \Rightarrow \int \frac{2}{v^2} dv$

$v = \ln x$

$dv = \frac{1}{x} dx$

$2 \int \frac{1}{v^2} dv$

$2\left(-\frac{1}{v}\right)$

$\frac{-2}{\ln|x|} + C$

6. $\int \frac{4x+2}{x^2+x+5} dx \Rightarrow \int \frac{2(2x+1)}{x^2+x+5} dx$

$v = x^2+x+5$

$dv = 2x+1\, dx$

$\Rightarrow \int \frac{2}{v} dv$

$\Rightarrow 2 \int \frac{1}{v} dv$

$\Rightarrow 2\ln|v| + C$

$\Rightarrow 2\ln|x^2+x+5| + C$

7. $\int (x+3)\, e^{x^2+6x}\, dx \Rightarrow \frac{1}{2} \int e^{v}\, dx$

$v = x^2+6x$ $\Rightarrow \frac{1}{2} e^{x^2+6x} + C$

$dv = 2x+6\, dx$

$\frac{1}{2} dv = (x+3) dx$

8. $\int e^{-x} \sec^2(2-e^{-x})\, dx \Rightarrow \int -\sec^2(2-v)\, dv$

$v = e^{-x}$ $\quad + \tan(2-e^{-x}) + C$

$dv = -e^{-x} dx$

$-dv = e^{-x} dx$

9. $\int (\cos x)\, e^{\sin x}\, dx \Rightarrow \int e^{v} dv$

$v = \sin x$ $\quad \Rightarrow e^{\sin x} + C$

$dv = \cos x\, dx$

10. $\int e^{2\ln x}\, dx$

$\int e^{\ln x^2}\, dx$

$\int x^2\, dx$

$\frac{1}{3} x^3 + C$

11. $\int x^2 e^{2x^3} dx \Rightarrow \frac{1}{6}\int e^{v} dx$

$v = 2x^3$

$dv = 6x^2 dx$

$\frac{1}{6} dv = x^2 dx$

$\frac{e^{2x^3}}{6} + C$

12. $\int \frac{e^{2x}}{e^x + 3} dx \Rightarrow \int \frac{e^x \cdot e^x}{e^x + 3} dx$

$v = e^x + 3$

$dv = e^x dx$

$\Rightarrow \int \frac{v - 3}{v} dv$

$\int 1 - \frac{3}{v} dv$

$v - \frac{3}{\ln|v|} + C$

$3 + e^x - 3\ln|e^x + 3| + C$

13. $\int \frac{e^{3x}}{(1-2e^{3x})^2} dx \Rightarrow -\frac{1}{6}\int \frac{1}{v^2} dx$

$v = 1 - 2e^{3x}$

$dv = -6e^{3x} dx$

$-\frac{1}{6} dv = e^{3x} dx$

$\frac{1}{6v} + C$

$\frac{1}{6(1-2e^{3x})} + C$

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1. $\int_1^4 \frac{3}{1-2x}\,dx \Rightarrow \int_1 \frac{1}{1-2x}\,3\,dx$

$V = 1-2x$

$\int_1 \frac{1}{1-2x}\left(-\frac{3}{2}\right)-2\,dx$

$dv = -2\,dx$

$-\frac{3}{2}\int \frac{1}{v}\,dv$

$-\frac{3}{2}\ln|1-2x|\Big]_1^4$

$-\frac{3}{2}\ln 7$

2. $\int_1^4 \frac{1}{\sqrt{x}(1+\sqrt{x})} \Rightarrow 2\int \frac{1}{v}\,dv$

$\Rightarrow 2\ln|1+\sqrt{x}|\Big]_1^4$

$V = 1+\sqrt{x}$

$\Rightarrow 2\ln|3| - 2\ln|2|$

$dv = \frac{1}{2\sqrt{x}}\,dx$

$\Rightarrow 2(\ln|3| - \ln|2|)$

$2dv = \frac{1}{\sqrt{x}}\,dx$

3. $\int_{-\ln 3}^{\ln 3} \frac{e^x}{e^x+4}\,dx \Rightarrow \int \frac{1}{v}\,dv$

$\ln|e^x+4| + C\Big]_{-\ln 3}^{\ln 3}$

$v = e^x+4$

$\ln|e^{\ln 3}+4| - \ln|e^{-\ln 3}+4|$

$dv = e^x\,dx$

$\ln 7 - \ln|e^{-\ln 3}+4|$

4. $\int_0^{\ln 5} e^x(3-4e^x)\,dx \Rightarrow -\frac{1}{4}\int (v)\,dv$

$\left(\frac{-1}{4}\right)\left(\frac{v^2}{2}\right)\Big]_0^{\ln 5}$

$v = 3-4e^x$

$dv = -4e^x dx$

$-\frac{1}{4}dv = e^x dx$

$\frac{-v^2}{8}\Big]_0^{\ln 5}$

$-\frac{(3-4e^{\ln 5})^2}{8} + \frac{(3-4e^0)^2}{8}$

$-\frac{(17)^2}{8} + \frac{1}{8} = \frac{-288}{8} = -36$

5. $\int_0^1 e^{2x+3}\,dx \Rightarrow \frac{1}{2}\int e^v\,dv$

$\frac{e^{2x+3}}{2}\Big]_0^1$

$v = 2x+3$

$\frac{e^5-e^3}{2}$

$dv = 2\,dx$

$\frac{1}{2}dv = dx$

6. $\int_0^{\ln 2} e^{-3x}\,dx \Rightarrow -\frac{1}{3}\int e^v\,dv$

$\frac{-e^{-3x}}{3}\Big]_0^{\ln 2}$

$v = -3x$

$dv = -3\,dx$

$-\frac{1}{3}dv = dx$

$-\frac{e^{-3\ln 2}}{3} + \frac{1}{3}$

$-\frac{1}{24} + \frac{1}{3} = \frac{-1+8}{24} = \frac{7}{24}$

7. $\int_1^2 \frac{e^{3/x}}{x^2}\,dx \Rightarrow -\frac{1}{3}\int e^v\,dv$

$v = \frac{3}{x}$

$dv = 3(x)^{-1}\,dx$

$= -3(x)^{-2}\,dx$

$dv = \frac{-3}{x^2}\,dx$

$-\frac{1}{3}dv = \frac{1}{x^2}\,dx$

$\left.\frac{-e^{3/x}}{3}\right]_1^2$

$\frac{-e^{3/2}+e^3}{3} + C$

8. $\int_0^2 x e^{4-x^2}\,dx \Rightarrow -\frac{1}{2}\int e^v\,dv$

$v = 4-x^2$

$dv = -2x\,dx$

$-\frac{1}{2}dv = x\,dx$

$\left.\frac{-e^{4-x^2}}{2}\right]_0^2$

$\frac{-1+e^4}{2} + C$

9. $\int_e^{e^2} \frac{1}{x(\ln x)^2}\,dx \Rightarrow \int \frac{1}{v^2}\,dv$

$v = \ln x$

$dv = \frac{1}{x}\,dx$

$\left.\frac{-1}{\ln x}\right]_e^{e^2}$

$\frac{-1}{\ln e^2} + \frac{1}{\ln e} = -\frac{1}{2}+1$

$= \frac{1}{2} + C$

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1. $\lim_{x \to 1} x^{\frac{1}{1-x}}$

$\lim_{x \to 1} \exp \ln x^{\frac{1}{1-x}}$

$\exp \lim_{x \to 1} \left(\frac{1}{1-x}\right) \ln(x)$

$\exp \lim_{x \to 1} \left(\frac{\ln x}{1-x}\right)$

$\exp \lim_{x \to 1} \frac{\frac{1}{x}}{-1}$

$\exp \lim_{x \to 1} \frac{1}{-x}$

$e^{-1}$

2. $\lim_{x \to 0} (1+\sin 2x)^{\frac{1}{x}}$

$\lim_{x \to 0} \exp \ln (1+\sin 2x)^{\frac{1}{x}}$

$\exp \lim_{x \to 0} \frac{\ln(1+\sin 2x)}{x}$

$\exp \lim_{x \to 0} 2\cos 2x \frac{1}{(1+\sin 2x)}$

$\exp \; 2(1) \frac{1}{(1+0)}$

$e^2$

5. $\lim_{x\to\infty} (1+x^2)^{\frac{1}{\ln x}}$

$$\exp \lim_{x\to\infty} \frac{\ln(1+x^2)}{\ln x}$$

$$\exp \lim_{x\to\infty} \frac{\frac{2x}{1+x^2}}{\frac{1}{x}}$$

$$\exp \lim_{x\to\infty} \frac{2x^2}{1+x^2}$$

$$\frac{\frac{2\cancel{x^2}}{\cancel{x^2}}}{\frac{1}{\cancel{x^2}} + \frac{\cancel{x^2}}{\cancel{x^2}}} = \frac{2}{0+1} = 2$$

$$e^2$$

6. $\lim_{x\to\infty} (\ln x)^{\frac{1}{x}}$

$$\exp \lim_{x\to\infty} \frac{1}{x} \ln(\ln x)$$

$$\exp \lim_{x\to\infty} \frac{\frac{1}{x}}{\ln x} \Rightarrow \frac{1}{x\ln x} = \frac{1}{\infty} = 0$$

$$e^0 = 1$$

7. $\lim_{x \to \infty} \left( \frac{x+1}{x+2} \right)^x$

$\exp \lim_{x \to \infty} x \ln \left( \frac{x+1}{x+2} \right)$

$\frac{1}{\frac{x+1}{x+2}}$ $\frac{1}{(x+2)^2}$

$\exp \lim_{x \to \infty} \frac{\ln \left( \frac{x+1}{x+2} \right)}{\frac{1}{x}}$

$\frac{\frac{1}{(x+1)(x+2)}}{\frac{1}{x^2}} \Rightarrow \frac{-x^2}{x^2+2x+x+2} \Rightarrow \frac{-x^2}{x^2+3x+2}$

$\Rightarrow \frac{-\cancel{x^2}}{\cancel{x^2}(1+\frac{3}{x}+\frac{2}{x^2})} = \frac{-1}{1}$

$\Rightarrow e^{-1} \Rightarrow \frac{1}{e}$

Slide 43

1. $y = (\sin^{-1})^2$

$= 2 \frac{1}{\sqrt{1-x^2}} (\sin^{-1}(x))$

$= \frac{2 \sin^{-1}(x)}{\sqrt{1-x^2}}$

2. $y = \tan^{-1}(e^x)$

$y' = \frac{1}{1+(e^x)^2}$

3. $y = (\tan^{-1} x)(\ln x)$

$y' = \left( \frac{\tan^{-1}(x)}{x} \right) + \left( \frac{\ln x}{1+x^2} \right)$

$y' = \frac{\tan^{-1}(x) + x^2 \tan^{-1}(x) + x\ln(x)}{x+x^3}$

4. $F(t) = e^{\sec^{-1} t}$

$F'(t) = \frac{e^{\sec^{-1} t}}{|t| \sqrt{t^2-1}}$

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6. $y = \tan^{-1}\left(x - \sqrt{1+x^2}\right)$

$$= \frac{1 - \frac{x}{\sqrt{x^2+1}}}{1 + \left(x - \sqrt{1+x^2}\right)^2} = \frac{\sqrt{x^2+1} - x}{\left(\sqrt{x^2+1}\right)\cdot\left(1 + \left(x - \sqrt{1+x^2}\right)^2\right)}$$

Slide 44

1. $\int \frac{dx}{9x^2+16} \Rightarrow \int \frac{dx}{(3x)^2+(4)^2} = \frac{1}{4}\cdot\frac{1}{3} \arctan \frac{3x}{4} + c$

$$= \frac{1}{12} \arctan \frac{3x}{4} + c$$

2. $\int \frac{dx}{4x\sqrt{x^2-16}} \Rightarrow \int \frac{dx}{4x\sqrt{(x)^2-(4)^2}} = \frac{1}{4}\cdot\frac{1}{4} \operatorname{arc\,sec} \frac{1}{4}x + c$

$$= \frac{1}{16} \operatorname{arc\,sec} \frac{1}{4}x + c$$

3. $\int \frac{dx}{\sqrt{2-5x^2}} = \int \frac{dx}{\sqrt{(\sqrt{2})^2 - (\sqrt{5}x)^2}} = \frac{1}{\sqrt{5}} \arcsin \frac{\sqrt{5}x}{\sqrt{2}} + c$

$$= \frac{1}{\sqrt{5}} \arcsin \frac{\sqrt{5}x}{\sqrt{2}} + c$$

4. $$\int_0^{1/2} \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx \Rightarrow \int u\,du$$

$$\frac{1}{2}u^2 + C$$

$u = \sin^{-1}x$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$

$$\frac{1}{2}(\sin^{-1}x)\Big]_0^{1/2}$$

$$\frac{1}{2}\,\frac{1}{\sin 1/2} \Rightarrow \frac{1}{2\sin 1/2}$$

5. $$\int \frac{e^x}{e^{2x}+1}\,dx \Rightarrow \int \frac{1}{u^2+1}\,du$$

$u = e^x$

$du = e^x\,dx$

$$\Rightarrow \frac{1}{1}\arctan\frac{u}{1} + C$$

$$\arctan e^x + C$$

6. $$\int \frac{e^{2x}}{\sqrt{1-e^{4x}}}\,dx \Rightarrow \frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,du$$

$u = e^{2x}$

$du = 2e^{2x}dx$

$\frac{1}{2}du = e^{2x}dx$

$$\frac{1}{2}\arcsin\frac{u}{1}$$

$$\frac{1}{2}\arcsin e^{2x} + C$$

7. $$\int \frac{dx}{x[4+(\ln x)^2]} = \int \frac{1}{(2)^2+u^2}\,du$$

$U = \ln x$

$du = \frac{1}{x}dx$

$$= \frac{1}{2}\arctan\frac{u}{2} + C$$

$$= \frac{1}{2}\arctan\frac{\ln x}{2} + C$$

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