No. :

Date :

Nama : Pramos Pay Lapran
NPM : 1908102100059 -A

Slide :23

1. $x^2+y^2=c^2$

$y^2=c^2-x^2$

$y=\sqrt{c^2-x^2}$

$y'=\frac{1}{2(c^2-x^2)^{1/2}}\,2x=\frac{-x}{(y^2)^{1/2}}=\frac{-x}{y}$

$\frac{-1}{y}=\frac{y}{x}$

$\frac{dy}{dx}=\frac{y}{x}$

$\int\frac{dy}{y}=\int\frac{dx}{x}\Rightarrow \ln|y|=\ln|x|+c$

$=\ln|x|+\ln|c|$

$y=xc\Rightarrow TO$

2. $x^2-y^2=c^2$

$y^2=x^2-c$

$y=\sqrt{x^2-c^2}$

$y'=\frac{x}{(x^2-c^2)^{1/2}}=\frac{x}{(y^2)^{1/2}}=\frac{x}{y}$

$\frac{-1}{y'}=\frac{-y}{x}$

$\frac{dy}{dx}=-\frac{y}{x}\Rightarrow\int\frac{dy}{y}=\int-\frac{dx}{x}$

$\ln|y|=-\ln|x|+c$

$=-\ln|x|+\ln|c|$

$y=\frac{c}{x}\Rightarrow TO$

No. :

Date :

3. $y = cx \Rightarrow c = y/x$

$y' = c$

TO: $-\frac{1}{y'} = \frac{-1}{c} \Rightarrow \frac{-x}{y}$

$\frac{dy}{dx} = -\frac{x}{y}$

$\int y\,dy = \int -x\,dx$

$\frac{1}{2} y^2 = -\frac{1}{2} x^2 + c$

$y^2 = -x^2 + 2c$

$y = (-x^2 + 2c)^{1/2}$

$y' = \pm (-x^2 + c)^{1/2} \Rightarrow$ TO

4. $xy = c$

$y = c/x$

$y' = \frac{-c}{x^2} \Rightarrow y' = \frac{xy}{x^2} = \frac{-y}{x}$

TO: $\frac{-1}{y} = \frac{x}{y}$

$\frac{dy}{dx} = \frac{x}{y} \Rightarrow \int y\,dy = \int x\,dx$

$1/2\, y^2 = 1/2\, x^2 + c$

$y^2 = x^2 + 2c$

$y = \pm\sqrt{x^2 + c} \Rightarrow$ TO

5. $y = \sqrt{x + c}$

$y^2 = x + c$

$y^2 - x = c$

$y' = \frac{1}{2\sqrt{x+c}} = \frac{1}{2\sqrt{y^2}} = \frac{1}{2y}$

TO: $\frac{-1}{y'} = -2y$

No. :

Date :

$\frac{dy}{dx} = -2y$

$\int \frac{dy}{dx} = \int dx$

$\frac{1}{2}\ln|y| = x + C$

$\ln|y|^{1/2} = -x - C$

$y = (e^{-x+C})^2 = e^{-2x+C} \Rightarrow$ CV

6

$4x^2 + y^2 = C$

$y = \sqrt{C - 4x^2}$

$y' = \frac{1}{2\sqrt{C-4x^2}} \cdot -8x = \frac{-4x}{\sqrt{C-4x^2}} = \frac{-4x}{\sqrt{4x^2+y^2-4x^2}} = \frac{-4x}{\sqrt{y^2}} = \frac{-4x}{y}$

TO: $\frac{-1}{y'} = \frac{y}{4x}$

$\frac{dy}{dx} = \frac{y}{4x}$

$\int \frac{dy}{y} = \int \frac{dx}{4x}$

$\ln|y| = \frac{1}{4}\ln|x| + C$

$= \frac{1}{4}\ln|x| + \ln|c|$

$= \ln|x^{1/4} c|$

$= |x^{1/4} c| \Rightarrow y = (x)^{1/4} c$

7.

$y = Ce^{-2x} \Rightarrow ye^{2x} = C$

$y' = \frac{-2C}{e^{2x}} = \frac{-2(ye^{2x})}{e^{2x}} = -2y$

TO: $\frac{-1}{y'} = \frac{1}{2y}$

No. :

Date :

$\frac{dy}{dx} = \frac{1}{2y}$

$\int 2y\,dy = \int dx$

$y^2 = x + c$

$y = \pm\sqrt{x+c}$

8. $y = cx^3$

$y' = 3cx^2 = 3\left(\frac{y}{x^3}\right)x^2 = \frac{3y}{x}$

$\frac{-1}{y'} = \frac{-x}{3y}$

$\frac{dy}{dx} = \frac{-x}{3y}$

$\int 3y\,dy = \int -x\,dx$

$\frac{3y^2}{2} = \frac{-x^2}{2} + c$

$y = \pm\sqrt{\frac{-x^2+c}{2}}$

slide : 35

1. A $y'' + 5y' + 4y = 0$

$PK = R^2 + 5R + 4 = 0$

$(R+4)(R+1)$

$R_1 = -4, \; R_2 = -1 \rightarrow y_1 = e^{-4x}, \; y_2 = e^{-x}$

SU : $y = c_1 e^{-4x} + c_2 e^{-x}$

B $y'' - 2y' + y = 0$

$PK : R^2 + 2R + 1 = 0$

$(R-1)(R-1)$

$R_1 = 1, \; R_2 = 1 \rightarrow y_1 = e^x, \; y_2 = xe^x$

SU : $y = c_1 e^x + c_2 x e^x$

No. :

Date :

C $y'' + 2y' + 6y = 0$

PK: $r^2 + 2r + 5 = 0$

$r_{12} = \frac{-2 \pm \sqrt{4 - 4(1)(6)}}{2} = \frac{-2 \pm \sqrt{4-20}}{2} = \frac{-2 \pm 4\sqrt{-1}}{2} = -1 \pm 2i$

$r_1 = -1 + 2i$, $r = -1 - 2i \rightarrow y_1 = e^{-x}\cos 2x$, $y_2 = e^{-x}\sin 2x$

$y = e^{-x}(c_1 \cos 2x + c_2 \sin 2x) \Rightarrow SU$

D $y'' + 4y = 0$

PK: $r^2 + 4 = 0$

$r^2 = -4$

$r_{12} = \pm 2\sqrt{-1}$

$r_1 = 2i$, $r_2 = -2i \rightarrow y_1 = \cos 2x$, $y_2 = \sin 2x$

$y = c_1 \cos 2x + c_2 \sin 2x \Rightarrow SU$

2. A $4y'' - 4y' - 3y = 0$

PK: $4r^2 - 4r - 3 = 0$

$r_{12} = \frac{4 \pm \sqrt{16 - 4(4)(-3)}}{8} = \frac{4 \pm \sqrt{16 + 48}}{8} = \frac{4 \pm 8}{8}$

$r_1 = 12/8 = 3/2$, $r = -1/2 \rightarrow y_1 = e^{3/2x}$, $y_2 = e^{1/2x}$

$y = c_1 e^{3x/2} + c_2 e^{-x/2} \Rightarrow SU$

$y' = \frac{3}{2} c_1 e^{3x/2} - \frac{1}{2} c_2 e^{-x/2}$

$2y = 3c_1 e^{3x/2} - c_2 e^{-x/2}$

$-e = 3c_1 e^{3} - c_2 e$

$\frac{e^3(-e + c_2 e)}{3} = c_1$

$c_1 = \frac{-e^4 + c_2 e^4}{3}$

$y = e^{3x/2}\left(\frac{-e^4 + c_2 e^4}{3}\right) + c_2 e^{-x/2} \Rightarrow SK$

No. :

Date :

B $y'' - 2y' + 2y = 0$

PK: $r^2 - 2r + 2 = 0$

$r_{12} = \frac{2 \pm \sqrt{4 - 4(1)(2)}}{2} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$

$r_1 = 1+i$, $r_2 = 1-i \Rightarrow y_1 = e^x \cos x$, $y_2 = e^x \sin x$

$y = e^x(C_1 \cos x + C_2 \sin x) \Rightarrow$ SU

$y' = (e^x \cos x - e^x \sin x) C_1 + (e^x \sin x + e^x \cos x) C_2$

$0 = (e^{r/2}\, 0 - e^{r/2}\, 1) C_1 + (e^{r/2}\, 1 + e^{r/2}\, 0) C_2$

$0 = -e^{r/2} C_1 + e^{r/2} C_2$

$-e^{r/2} C_2 = -e^{r/2} C_1$

$C_2 = C_1$

$y = e^x(C_2 \cos x + C_2 \sin x)$

$= e^x C_2 (\cos x + \sin x) \Rightarrow$ SK