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Slide : 41

1. $\sum_{n=1}^{\infty}\left(\frac{1}{\ln(n)}\right)^n \Rightarrow \lim_{n\to\infty}\left(\left(\frac{1}{\ln(n)}\right)^n\right)^{1/n}$

$= \lim_{n\to\infty}\frac{1}{\infty} = 0$ konvergen

2. $\sum_{n=1}^{\infty}\left(\frac{n}{3n+2}\right)^n \Rightarrow \lim_{n\to\infty}\left(\left(\frac{n}{3n+2}\right)^n\right)^{1/n}$

$= \lim_{n\to\infty}\left(\frac{n}{3n+2}\right) = \frac{n}{n(3+\frac{2}{n})} = \frac{1}{3}$ Konvergen

3. $\sum_{n=1}^{\infty}\left(\frac{1}{2}+\frac{1}{n}\right)^n \Rightarrow \lim_{n\to\infty}\left(\left(\frac{n+2}{2n}\right)^n\right)^{1/n}$

$= \lim_{n\to\infty}\frac{\cancel{n}(1+2/n)}{2\cancel{n}} = \frac{1+2/\infty}{2} = \frac{1+0}{2} = \frac{1}{2}$ konvergen

4. $\sum_{n=1}^{\infty}\left(\frac{3n+2}{2n-1}\right)^n \Rightarrow \lim_{n\to\infty}\left(\left(\frac{3n+2}{2n-1}\right)^n\right)^{1/n} = \lim_{n\to\infty}\frac{3n+2}{2n-1} = \lim_{n\to\infty}\frac{n(3+2/n)}{n(2-1/n)}$

$= \frac{3+0}{2-0} = \frac{3}{2}$ Divergen

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1. $\sum_{n=3}^{\infty}\frac{1}{n^2-5}$

$a_n = \frac{1}{n^2-5}$ $\quad b_n = \frac{1}{n^2}$

$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{\frac{1}{n^2-5}}{\frac{1}{n^2}} = \lim_{n\to\infty}\frac{n^2}{n^2(1-\frac{5}{n^2})} = \frac{1}{(1-\frac{5}{\infty})} = \frac{1}{1-0} = 1$

karena $L = 1$ dan $\sum_{n=3}^{\infty}\frac{1}{n^2}$ konvergen, maka $\sum_{n=3}^{\infty}\frac{1}{n^2-5}$ konvergen

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2) $\sum_{n=1}^{\infty} \frac{5^n}{n^2+5}$

$a_n = \frac{n}{n^2+5} < b_n = \frac{n}{n^2}$, $\sum_{n=1}^{\infty} \frac{1}{n}$ Divergen

karena $b_n > a_n$ dan b_n Divergen, maka $\sum_{n=1}^{\infty} \frac{n}{n^2+5}$ Divergen

3.) $\sum_{n=1}^{\infty} \frac{5^n}{n!}$

$a_n = \frac{5^n}{n!}$ $\quad a_{n+1} = \frac{5^{n+1}}{(n+1)!}$.

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{5^{n+1}}{(n+1)!} \cdot \frac{n!}{5^n} = \frac{5}{n+1} = \frac{5}{\infty} = 0$

karena $P \cdot (P = 0)$, $P < 1$, maka $\sum_{n=1}^{\infty} \frac{5^n}{n!}$ konvergen

4.) $\sum_{n=3}^{\infty} \frac{1}{(n-2)^2} = \frac{1}{n^2-4n+4}$

$a_n = \frac{1}{n^2-4n+4}$, $b_n = \frac{1}{n^2}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{n^2}{n^2\left(1-\frac{4}{n}+\frac{4}{n^2}\right)} = \frac{1}{\left(1-\frac{4}{\infty}+\frac{4}{\infty}\right)} = \frac{1}{(1+0-0)} = \frac{1}{1} = 1$

karena $L = 1$ dan $\sum_{n=3}^{\infty} \frac{n}{n^2}$ konvergen, maka $\sum_{n=3}^{\infty} \frac{1}{(n-2)^2}$ konvergen

5.) $\sum_{n=1}^{\infty} \frac{3e^n + e^{2n}}{2e^{2n}}$

$\lim_{n\to\infty} \frac{e^{2n}(3e^{1/2}+1)}{2e^{2n}} = \frac{3e^{1/2}+1}{2}$

karena $\sum_{n=1}^{\infty} \frac{3e^n + e^{2n}}{2e^{2n}} \neq 0$, maka $\sum_{n=1}^{\infty} \frac{3e^n + e^{2n}}{2e^{2n}}$

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6. $\sum_{n=2}^{\infty} \frac{\ln(n)}{\sqrt{n}} \Rightarrow \lim_{b \to \infty} \int_2^b \frac{\ln x \, dx}{\sqrt{x}}$

$= 2\sqrt{x} \ln(x) - \int_2^b \frac{2\sqrt{x}}{x} \, dx = \left[2x \ln(x) - 4\sqrt{x}\right]_2^{\infty}$ Divergen

$=$

7. $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

$a_n = \frac{n^n}{n!}$, $a_{n+1} = \frac{(n+1)^{n+1}}{(n+1)!}$

$\lim_{n \to \infty} \frac{(n+1)^{n+1}}{(n+1)!} \cdot \frac{n!}{n^n} = \frac{(n+1)^n}{n^n} = \left(\frac{n(1+1/n)}{n}\right)^n = e$

karena $P > 1$, maka deret $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ Divergen

$\boxed{\text{Slide : 48}}$

1. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{3n+1}$

$a_n = \frac{2}{3n+1}$, $a_{n+1} = \frac{2}{3n+1}$

$\frac{a_n}{a_{n+1}} = \frac{2(3n+4)}{2(3n+1)} = \frac{3n+4}{3n+1} > 1$

$\lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{2}{3n+1} = \frac{2}{\infty} = 0$

kedua syarat terpenuhi maka $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{3n+1}$ konvergen

2. $\sum_{n=1}^{\infty} (-1)^n \frac{n+3}{n^2+n}$

$a_n = \frac{n+3}{n^2+n}$, $a_{n+1} = \frac{n+4}{(n+1)^2+n} = \frac{n+4}{n^2+2n+2n+n} = \frac{n+4}{n^2+3n+2}$

$\frac{a_n}{a_{n+1}} = \frac{(n+3)(n^2+3n+2)}{(n^2+n)(n+4)} > 1$

$\lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{n+3}{n^2+n} = \frac{n(1+3/n)}{n(n+1)} = \frac{1+0}{\infty} = 0$

karena kedua syarat terpenuhi maka $\sum_{n=1}^{\infty} (-1)^n \frac{n+3}{n^2+n}$ konvergen

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3. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^n}{n!}$

$a_n = \frac{n^n}{n!}$, $a_{n+1} = \frac{(n+1)^{n+1}}{(n+1)!}$

$\frac{a_n}{a_{n+1}} = \frac{n^n (n+1)!}{n!\ (n+1)^{n+1}} = \frac{n^n}{(n+1)^n} = \left(\frac{n}{n+1}\right)^2 < 1$

karena syarat a sudah tidak terpenuhi, maka $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^n}{n!}$ Divergen

4. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{3^n}$

$a_n = \frac{n}{3^n}$, $a_{n+1} = \frac{n+1}{3^{n+1}}$

$\frac{a_n}{a_{n+1}} = \frac{n!}{3^n} \cdot \frac{3^{n+1}}{n+1} = \frac{3n}{n+1} > 1$

$\lim_{n\to\infty} \frac{n}{3^n} = \lim_{n\to\infty} \frac{1}{3^n \ln(3)} = \frac{1}{\infty} = 0$

karena kedua syarat terpenuhi, maka $\sum_{n=1}^{\infty} (-1)^n \frac{n}{3^n}$ konvergen

5. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n(n+1)}$

$a_n = \frac{1}{n(n+1)}$, $a_{n+1} = \frac{1}{(n+1)(n+2)}$

$\frac{a_n}{a_{n+1}} = \frac{1}{n(n+1)} \cdot \frac{(n+1)(n+2)}{1} = \frac{n+2}{n} > 1$

$\lim_{n\to\infty} \frac{1}{n(n+1)} = 0$

karena kedua syarat sudah terpenuhi maka $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n(n+1)}$ konvergen

6. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln(n)}{\sqrt{n}}$

$a_n = \frac{\ln(n)}{\sqrt{n}}$, $a_{n+1} = \frac{\ln(n+1)}{\sqrt{n+1}}$

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$\frac{a_n}{a_{n+1}} = \frac{[\ln(n)][\sqrt{n+1}]}{[\sqrt{n}][\ln(n+1)]} > 1$

$\lim_{n\to\infty} \frac{\ln n}{\sqrt{n}} = \lim_{n\to\infty} \frac{1/n}{1/2\sqrt{n}} = \frac{2\sqrt{n}}{n} = \frac{2}{\sqrt{n}} = 0$

karena kedua syarat terpenuhi, maka $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln(n)}{\sqrt{n}}$ konvergen

$\boxed{\text{slide : 53}}$

1. $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n+2}$

$|u_n| = \frac{1}{3n+2}$, $a_n = \frac{1}{3n+2}$

$\lim_{b\to\infty} \int_1^b \frac{1}{3x+2}\, dx$

$\lim_{b\to\infty} \frac{1}{3} \int \frac{1}{u}\, du = \lim_{b\to\infty} \frac{\ln(u)}{3} \Big]_1^b = \lim_{b\to\infty} \frac{\ln(b)}{3} - \frac{\ln(1)}{3} =$

$= \infty - 0 = \infty$ Divergen

• Uji DGT

$a_n = \frac{1}{3n+2}$, $a_{n+1} = \frac{1}{3n+5}$

$\frac{a_n}{a_{n+1}} = \frac{3n+5}{3n+2} > 1$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{3n+2}$

karena $\sum_{n\to\infty}^{\infty} |u_n|$ divergen, tetapi diuji konvergen menyatakan konvergen,

maka $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n+2}$ konvergen bersyarat

3. $\sum_{n=1}^{\infty} (-1)^n \left(\frac{n}{5^n}\right)$

$U_n = \frac{n}{5^n}$, $U_{n+1} = \frac{n+1}{5^{n+1}}$

$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{n+1}{5^{n+1}} \cdot \frac{5^n}{n} = \frac{n+1}{5n} = \frac{n(1+1/n)}{5n} = \frac{1}{5} < 0$ konvergen

karena $\sum^{\infty} |U_n|$ konvergen, maka $(-1)^n \left(\frac{n}{5^n}\right)$ konvergen mutlak

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4. $\sum_{n=1}^{\infty} \frac{(-1)^n e^n}{n}$ $\sum_{n=1}^{\infty} \left|\frac{e^n}{n}\right|$

$a_{n+1} = \frac{e^{n+1}}{n+1}$ $a_n = \frac{e^n}{n}$

$\lim_{n\to\infty} \frac{e^{n+1}}{n+1} \frac{n}{e^n} = \frac{(e)(n)}{n(1+1/n)} = e$

• Uji DGT

$\frac{a_n}{a_{n+1}} = \frac{e^n}{n} \frac{n+1}{e^{n+1}} = \frac{n+1}{(e)(n)}$

karena $\sum_{n=1}^{\infty} |U_n|$ divergen, ~~dan~~ uji DGT divergen, maka $\sum_{n=1}^{\infty} \frac{(-1)^n e^n}{n}$ divergen

5. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n(n+1)}$ $\sum_{n=1}^{\infty} \left|\frac{1}{n(n+1)}\right|$

$a_n = \frac{1}{n^2+n} < b_n = \frac{1}{n^2}$, $\sum_{n=1}^{\infty} \frac{1}{n^2}$ konvergen

Dari uji banding biasa, $\sum_{n=1}^{\infty} |U_n|$ konvergen, maka $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n(n+1)}$ konvergen

6. $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n\ln n}$ $\sum_{n=2}^{\infty} \left|\frac{1}{n\ln n}\right|$

• Uji DGT

$\frac{a_n}{a_{n+1}} = \frac{[n+1]\ln[n+1]}{n\ln[n]} > 1$

$\lim_{n\to\infty} \frac{1}{n\ln(n)} = \frac{1}{\infty} = 0$

$\sum_{n=2}^{\infty} \frac{1}{n\ln n}$ → uji integral

$\lim_{b\to\infty} \int_2^b \frac{1}{x\ln x}\, dx$

$\lim_{b\to\infty} \int_2^b \frac{1}{u}\, d$

$\lim_{b\to\infty} \ln[\ln x]\Big]_2^b$

$\lim_{b\to\infty} \ln(\ln b) - \ln(\ln 2) = \infty$ Divergen

Dari uji integral divergen, namun uji DGT menyatakan konvergen sehingga $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n\ln(n)}$ konvergen bersyarat

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8. $\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{\sqrt{n}} = \sum_{n>2}^{\infty} \left| \frac{\ln n}{\sqrt{n}} \right|$

$\lim_{b \to \infty} \frac{\ln x \, dx}{\sqrt{x}} = 2\sqrt{x} \ln(x) - \int_2^b \frac{2\sqrt{x}}{x} dx = \left[2\sqrt{x} \ln(2) - 4\sqrt{x}\right]_2^b$ divergen

• Uji DGT

$\frac{a_n}{a_{n+1}} = \frac{[\ln(n)][\sqrt{n+1}]}{\sqrt{n} \; \ln(n+1)} > 1$

$\lim_{n \to \infty} \frac{\ln n}{\sqrt{n}} = \frac{\frac{1}{n}}{\frac{2}{\sqrt{n}}} = \frac{2}{\sqrt{n}} = 0$

maka $\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{\sqrt{n}}$ konvergen bersyarat