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Matkul: Kalkulus II

I. 1. $\int_0^1 x^{-1/3}\,dx$ Fungsi $f(x)$ terdefinisi pada selang $(0,1]$

$$\int_0^1 x^{-1/3}\,dx = \lim_{a\to 0^+}\int_a^1 x^{-1/3}\,dx$$
$$= \lim_{a\to 0^+} -3x^{-1/3}\Big|_a^1$$
$$= \lim_{a\to 0^+}\left(-3(1)^{-1/3} + 3(a)^{-1/3}\right) = -3+\infty = \infty \text{ (Divergen)}$$

2. $\int^{10} \frac{2}{\sqrt{x-5}}\,dx$ Fungsi $f(x)$ terdefinisi pada selang $(5,10]$

$$\int_5^{10}\frac{2}{\sqrt{x-5}}\,dx = \lim_{b\to 5^+}\int_b^{10}\frac{2}{\sqrt{x-5}}\,dx$$
$$= \lim_{b\to 5^+}\int_b^{10} 4\sqrt{x-5}\Big|_b^{10}$$
$$= \lim_{b\to 5^+}\left(4\sqrt{10-5} - 4\sqrt{b-5}\right) = 4-0 = 4 \text{ (konvergen)}$$

3. $\int_0^1 \frac{2}{x\sqrt{1-x^2}}\,dx$ Fungsi $f(x)$ terdefinisi pada selang $(0,1)$

$$\int_0^1 \frac{2}{x\sqrt{1-x}}\,dx = \int_0^{1/2}\frac{2}{x\sqrt{1-x^2}} + \int^1 \frac{2}{x\sqrt{1-x^2}}$$
$$\int_0^{1/2}\frac{2}{x\sqrt{1-x^2}}\,dx = \lim_{a\to 0^+}\int_a^{1/2}\frac{2}{x\sqrt{1-x^2}}\,dx = 2\ln\left(\left|\csc(\sin^{-1}(x)) - \cot(\sin^{-1}(x))\right|\right)\Big|_a^{1/2}$$
$$= \lim_{a\to 0^+} 2\ln\left(\left|\csc(\sin^{-1}(\tfrac{1}{2})) - \cot(\sin^{-1}(\tfrac{1}{2}))\right|\right) -$$
$$2\ln\left(\left|\csc(\sin^{-1}(a)) - \cot(\sin^{-1}(a))\right|\right)$$
$$= -\infty \text{ (Divergen)}$$

4. $\int_1^2 \frac{dx}{(x-1)^{2/3}}$ Fungsi $f(x)$ terdefinisi pada selang $(1,2]$

$$\int_1^2 \frac{dx}{(x-1)^{2/3}} = \lim_{c\to 1^+}\int^2 \frac{dx}{(x-1)^{2/3}}$$
$$= \lim_{c\to 1^+} \frac{3}{2}(x-1)^{2/3}\Big|_c^2$$
$$= \lim_{c\to 1^+}\left(\frac{3}{2}(2-1)^{2/3} - \frac{3}{2}(c-1)^{2/3}\right) = \frac{3}{2} \text{ (konvergen)}$$

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5. $\int_0^1 \frac{\ln x}{x}\,dx$ fungsi $F(x)$ terdefinisi pada selang $(0,1]$

$$\int_0^1 \frac{\ln x}{x}\,dx = \lim_{R \to 0^+} \int_R^1 \frac{\ln x}{x}\,dx$$

$$= \lim_{R \to 0^+} \left. \frac{(\ln x)^2}{2} \right|_R^1$$

$$= \lim_{R \to 0^+} \left( \frac{(\ln 1)^2}{2} - \frac{(\ln R)^2}{2} \right) = -\infty \text{ (Divergen)}$$

6. $\int_3^7 \frac{dx}{\sqrt{x-3}}$ fungsi $F(x)$ terdefinisi pada selang $(3,7]$

$$\int_3^7 \frac{dx}{\sqrt{x-3}} = \lim_{D \to 3^+} \int_D^7 \frac{dx}{\sqrt{x-3}}$$

$$= \lim_{D \to 3^+} \left. 2\sqrt{x-3} \right|_D^7$$

$$= \lim_{D \to 3^+} \left( 2\sqrt{7-3} - 2\sqrt{D-3} \right) = 4 \text{ (konvergen)}$$

II 1. $\int_{-1}^2 \frac{dx}{(x-2)^2}$ fungsi $F(x)$ terdefinisi pada selang $[-1,2)$

$$\int_{-1}^2 \frac{dx}{(x-2)^2} = \lim_{a \to 2^-} \int_{-1}^a \frac{dx}{(x-2)^2}$$

$$= \lim_{a \to 2^-} \left. \frac{1}{(x-2)} \right|_{-1}^a$$

$$= \lim_{a \to 2^-} \left( \frac{1}{a-2} - \frac{1}{-1-2} \right) = -\infty \text{ (Divergen)}$$

2. $\int_{-2}^{-1} \frac{dx}{(x+1)^{4/3}}$ fungsi $F(x)$ terdefinisi pada selang $[-2,-1)$

$$\int_{-2}^{-1} \frac{dx}{(x+1)^{4/3}} = \lim_{P \to -1^-} \int_{-2}^{P} \frac{dx}{(x+1)^{4/3}}$$

$$= \lim_{P \to -1^-} \left. \frac{-3}{(x+1)^{1/3}} \right|_{-2}^{P}$$

$$= \lim_{P \to -1^-} \frac{-3}{(P+1)^{1/3}} - 3 = -\infty \text{ (Divergen)}$$

3. $\int_0^9 \frac{dx}{\sqrt{9-x}}$ fungsi $f(x)$ terdefinisi pada selang $[0,9)$

$$\int_0^9 \frac{dx}{\sqrt{9-x}} = \lim_{x \to 9^-} \int_0^k \frac{dx}{\sqrt{9-x}}$$

$$= \lim_{x \to 9^-} 2\sqrt{9-x} \Big|_0^k$$

$$= \lim_{x \to 9^-} \left(2\sqrt{9-0} - 2\sqrt{9-k}\right) = 6 \text{ (konvergen)}$$

4. $\int_0^3 \frac{x}{9-x^2}\, dx$ fungsi $f(x)$ terdefinisi pada selang $[0,3)$

$$\int_0^3 \frac{x}{9-x^2}\, dx = \lim_{Q \to 3^-} \int_0^Q \frac{x}{9-x^2}\, dx$$

$$= \lim_{Q \to 3^-} -\frac{1}{2} \ln(9-x^2) \Big|_0^Q$$

$$= \lim_{Q \to 3^-} -\frac{1}{2} \ln(9-Q^2) - \left(-\frac{1}{2} \ln(9-0)\right) = -\infty \text{ (Divergen)}$$

5. $\int_0^2 \frac{dx}{\sqrt{4-x^2}}$ fungsi $f(x)$ terdefinisi pada selang $[0,2)$

$$\int_0^2 \frac{dx}{\sqrt{4-x^2}} = \lim_{c \to 2^-} \int_0^c \frac{dx}{\sqrt{4-x^2}}$$

$$= \lim_{c \to 2^-} \sin^{-1}\left(\frac{1}{2}x\right) \Big|_0^c$$

$$= \lim_{c \to 2^-} \sin^{-1}\left(\frac{1}{2}c\right) - \sin^{-1}(0) = \frac{\pi}{2} \text{ (konvergen)}$$

6. $\int_0^3 \frac{x}{\sqrt{9-x^2}}\, dx$ fungsi $f(x)$ terdefinisi pada selang $[0,3)$

$$\int_0^3 \frac{x}{\sqrt{9-x^2}} = \lim_{A \to 3^-} \int_0^A \frac{x}{\sqrt{9-x^2}}\, dx$$

$$= \lim_{A \to 3^-} -\sqrt{9-x^2} \Big|_0^A$$

$$= \lim_{A \to 3^-} -\sqrt{9-A^2} - \left(-\sqrt{9-0}\right) = 3 \text{ (konvergen)}$$