UTS MATDISK

100 MNT

1. **Gambar grfik fungsi dari**

$$f(x) = \lceil \lfloor x - \tfrac{1}{2} \rfloor + \tfrac{1}{2} \rceil$$

2.

Let $g(x) = \lfloor x \rfloor$. Find

a) $g^{-1}(\{0\})$. **b)** $g^{-1}(\{-1, 0, 1\})$.
c) $g^{-1}(\{x \mid 0 < x < 1\})$.

3.

Give an example of a relation on the set $\{1, 2, 3, 4\}$ that is

a) reflexive, symmetric, and not transitive.
b) not reflexive, symmetric, and transitive.
c) reflexive, antisymmetric, and not transitive.

4.

Prove that for every positive integer n,

$$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots + n(n+1)(n+2) = n(n+1)(n+2)(n+3)/4.$$

5.

Use a merge sort to sort $b, d, a, f, g, h, z, p, o, k$. Show all the steps used by the algorithm.

6.

Solve the following system of congruences:

First use the Extended Euclidean algorithm, and then apply the Chinese remainder theorem.

$$13x \equiv 4 \pmod{99}$$

$$15x \equiv 56 \pmod{101}.$$

7.

Answer these questions for the poset $(\{2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72\}, |)$.

a) Find the maximal elements.
b) Find the minimal elements.
c) Is there a greatest element?
d) Is there a least element?
e) Find all upper bounds of $\{2, 9\}$.
f) Find the least upper bound of $\{2, 9\}$, if it exists.
g) Find all lower bounds of $\{60, 72\}$.
h) Find the greatest lower bound of $\{60, 72\}$, if it exists.