TUGAS PERTEMUAN 11

MATEMATIKA DISKRIT

Disusun Oleh:

Rifqi Muhadzib Ahdan - 14010210029

PROGRAM STUDI S-1 TEKNIK INFORMATIKA

FAKULTAS MATEMATIKA DAN ILMU PENGETAHUAN ALAM

UNIVERSITAS PADJADJARAN

JATINANGOR

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HALAMAN 144

7. What is the probability ofthese events when we randomly select a permutation of {I, 2, 3, 4)?

a) 1 precedes 4.
b) 4 precedes 1.
c) 4 precedes 1 and 4 precedes 2.
d) 4 precedes 1, 4 precedes 2, and 4 precedes 3.
e) 4 precedes 3 and 2 precedes 1.

JAWABAN

Total semua kemungkinan : 4P1 = 4! = 24 cara

a) 1 mendahului 4
   - 1 di urutan pertama ➔ 1 (3) (2) (1) ➔ 3.2.1 = 6
   - 1 di urutan kedua ➔(2) 1 (2) (1) ➔ 2.2.1 = 4
   - 1 di urutan ketiga ➔ (2) (1) 1 (1) ➔2.1.1 = 2

   Sehingga todal 1 precedes 4 = 6 + 4 + 2 = 12

   Sehingga probabilitynya adalah 12/24 = ½

b) 4 mendahului 1
   - 4 di urutan pertama ➔ 4 (3) (2) (1) ➔ 3.2.1 = 6
   - 4 di urutan kedua ➔(2) 4 (2) (1) ➔ 2.2.1 = 4
   - 4 di urutan ketiga ➔ (2) (1) 4 (1) ➔2.1.1 = 2

   Sehingga todal 1 precedes 4 = 6 + 4 + 2 = 12

   Sehingga probabilitynya adalah 12/24 = ½

c) 4 mendahului 1 dan 4 mendahului 2
   - 4 di urutan pertama ➔ 4 (3) (2) (1) ➔ 3.2.1 = 6
   - 4 di urutan kedua ➔ (1) 4 (2) (1) ➔ 1.2.1 = 2

   Sehingga totalnya 6 + 2 = 8

   Dan probabilitynya ➔ 8/24 = 1/3

d) 4 mendahului 1, 4 mendahului 2, dan 4 mendahului 3
   - 4 diurutan pertama ➔ 4 (3) (2) (1) = 3.2.1 = 6

   Sehignga totalnya 6 dan probabilitynya 6/24 = 1/4

e) 4 mendahului 3 dan 2 mendahului 1
   - 4 di pertama dan 2 di kedua ➔ 4 2 (2) (1) = 2.1 = 2
   - 3 di kedua dan 2 di pertama ➔2 4 (2) (1) = 2.1 = 2
   - 4 di pertama dan 2 di ketiga ➔ 4 (1) 2 (1) = 1.1 = 1
   - 4 di ketiga dan 2 di pertama ➔ 2 (1) 4 (1) = 1.1 = 1

   Sehinga totalnya dalah 2+2+1+1 = 6

   Porbabilitynya ➔ 6/24 = 1/4

HALAMAN 415

11. Suppose that E and F are events such that pee) = 0.7 and p(F) = 0.5. Show that p(E U F) >= 0.7 and p(E ∩ F) >= 0.2

JAWABAN

p(E U F) memiliki probability = 1 karena merupakan total dari semua kemungkinan

Dan diketahui persamaan di bawah ini

$p(E \cup F) = p(E) + p(F) - p(E \cap F)$

$1 >= 0.7 + 0.5 - p(E \cap F)$

$1 >= 1.2 - p(E \cap F)$

$p(E \cap F) >= 0.2$

Sehingga terbukti bahwa p(E U F) >= 0.7 (di mana di sini 1) dan p(E ∩ F) >= 0.2

12. Suppose that E and F are events such that pee) = 0.8 and p(F) = 0.6. Show that p(E U F) >= 0.8 and p(E ∩ F) >= 0.4

JAWABAN

p(E U F) memiliki probability = 1 karena merupakan total dari semua kemungkinan

Dan diketahui persamaan di bawah ini

$p(E \cup F) = p(E) + p(F) - p(E \cap F)$

$1 >= 0.8 + 0.6 - p(E \cap F)$

$1 >= 1.4 - p(E \cap F)$

$p(E \cap F) >= 0.4$

Sehingga terbukti bahwa p(E U F) >= 0.8 (di mana di sini 1) dan p(E ∩ F) >= 0.4

HALAMAN 416

28. Assume that the probability a child is a boy is 0.51 and that the sexes of children born into a family are independent. What is the probability that a family of five children has

a) exactly three boys?
b) at least one boy?
c) at least one girl?
d) all children of the same sex?

JAWABAN

a) Gunakan rumus distribusi binomial, yaitu
   Jumlah sample = 5, jumlah laki-laki = 3, probability laki-laki = 0.51, probability perempuan = 1 – 0.51 = 0.49
   ➔P(a, b) = C(a, b) . p^a . q^{b-a}
   ➔P(5, 3) = C(5, 3) . $(0.51)^3$. $(0.49)^2$
   = 10 x 0,132651 x 0,2401
   = 0.32
b) Setidaknya ada 1 laki-laki
   1 – (Probabilitas kelima anak perempuan)
   ➔P(5, 0) = C(5, 0) . $(0.51)^0$. $(0.49)^5$
   = 5!/(0!.5!) x 1 x 0.02824
   = 1 x 1 x 0.02824
   = 0.02824

   ➔1 – 0.02824 = 0.97176 kemungkinan setidakanya ada 1 laki-laki
c) Setidaknya ada 1 perempuan
   1 – (Probabilitas kelima anak cowo)
   ➔ P(5, 5) = C(5, 5) . $(0.49)^0$. $(0.51)^5$
   = 5!/(0!.5!) x 1 x 0.034502
   = 1 x 1 x 0.0345
   = 0.0345
   ➔1 - 0.0345 = 0.97175 kemungkiann setidaknya ada 1 perempuan
d) Semua anak memiliki sex yang sama
   P = kemungkinan semua laki + kemungkinan semua perempuan
   = P(5, 5) + P(5, 0)
   = 0.0345 + 0.02824
   = 0.6273

HALAMAN 424

1. Suppose that E and F are events in a sample space and p(E) = 1/3, p(F) = 1/2, and p(E | F) = 2/5. Find p(F | E).

JAWABAN

Gunakan teorema bayes 1

P (F | E) = $\frac{p(E\mid F)p(F)}{p(E)}$

$= \frac{\frac{2}{5} * \frac{1}{2}}{\frac{1}{3}} = \frac{2}{5} * \frac{3}{2} = \frac{3}{5}$

2. Suppose that E and F are events in a sample space and p(E) = 2/3, p(F) = 3/4, and p(F | E) = 5/8. Find p(E | F).

JAWABAN

Gunakan teorema bayes 1

P (E | F) = $\frac{p(F\mid E)p(E)}{p(F)}$

$= \frac{\frac{5}{8} * \frac{2}{3}}{\frac{3}{4}} = \frac{5}{8} * \frac{8}{9} = \frac{5}{9}$

3. Suppose that Frida selects a ball by first picking one of two boxes at random and then selecting a ball from this box at random. The first box contains two white balls and three blue balls, and the second box contains four white balls and one blue ball. What is the probability that Frida picked a ball from the first box if she has selected a blue ball?

JAWABAN

Gunakan bayes therom 1

$$p(A\mid B) = \frac{p(A)p(B\mid A)}{p(B\mid A)p(A)\ + p(B\mid A')p(A')}$$

- Diketahui bahwa probability mengambil di kotak pertama = p(A) = ½

  Probability mengambil di kotak kedua p(A’) = ½

- Karena kita tahu tiap bola di masing-masing box, maka probability mengambil bole biru di kotak pertama = p(B | A) = 3/5

  dan probability mengambil bole biru di kotak kedua = p(B | A’) = 1/5.

Sehingga $p(A\mid B) = \frac{\frac{1}{2}\cdot\frac{3}{5}}{\frac{1}{2}\cdot\frac{3}{5}+\frac{1}{2}\cdot\frac{1}{5}} = \frac{3}{4}$

13. Suppose that E, F_1, F_2, and $F_3$are events from a sample space S and that F_1, F_2, and F_3 are mutually disjoint and their union is S. Find p(F_1 | E)

if p(E | F_1) = 1/8, p(E | F_2) = 1 /4, p(E | F_3)= 1/6, p(F_1)= 1/4, P(F_2) = 1/4, and p(F_3) = 1/2.

JAWABAN

$$p(Fj \mid E) = \frac{p(Fj)p(E \mid Fj)}{\sum_{i=1}^{n} p(E \mid Fj)p(Fj)}$$

$$p(F1 \mid E) = \frac{p(F1)p(E \mid F1)}{p(E \mid F1)p(F1) + p(E \mid F2)p(F2) + p(E \mid F3)p(F3)} = \frac{\frac{1}{8}\cdot\frac{1}{4}}{\frac{1}{8}\cdot\frac{1}{4}+\frac{1}{4}\cdot\frac{1}{4}+\frac{1}{6}\cdot\frac{1}{2}} = \frac{3}{17} = 0.1765$$

14. Suppose that E, F_1, F_2, and F_3 are events from a sample space S and that F_1, F_2, and F_3 are mutually disjoint and their union is S. Find p(F_2 | E)

if p(E | F_1) = 2/7, p(E | F_2) = 3/8, p(E | F_3)= 1/2, p(F_1)= 1/6, P(F_2) = 1/2, and p(F_3) = 1/3.

JAWABAN

$$p(Fj \mid E) = \frac{p(Fj)p(E \mid Fj)}{\sum_{i=1}^{n} p(E \mid Fj)p(Fj)}$$

$$p(F1 \mid E) = \frac{p(F1)p(E \mid F1)}{p(E \mid F1)p(F1) + p(E \mid F2)p(F2) + p(E \mid F3)p(F3)} = \frac{\frac{3}{8}\cdot\frac{1}{2}}{\frac{2}{7}\cdot\frac{1}{6}+\frac{3}{8}\cdot\frac{1}{2}+\frac{1}{2}\cdot\frac{1}{3}} = \frac{7}{15} = 0.46$$