No.
Date

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Skala : 9

1. $\{a_n\} = \left\{\dfrac{4n^2+1}{n^2-2n+3}\right\}$

$f(x) = \dfrac{4x^2+1}{x^2-2x+3}$

$\lim\limits_{x \to \infty} \dfrac{4x^2+1}{x^2-2x+3} = \dfrac{x^2(4+\frac{1}{x^2})}{x^2(1-\frac{2}{x}+\frac{3}{x^2})} = \dfrac{4+0}{1-0+0} = 4$

karena $\lim\limits_{x \to \infty} f(x) = \lim\limits_{n \to \infty} a_n = 4$, maka $\left\{\dfrac{4n^2+1}{n^2-2n+3}\right\}$ konvergen ke 4.

2. $\{a_n\} = \left\{\dfrac{3n^2+2}{n+1}\right\}$

$f(x) = \dfrac{3x^2+2}{x+1}$

$\lim\limits_{x \to \infty} \dfrac{3x^2+2}{x+1}$

$\lim\limits_{x \to \infty} \dfrac{6x}{1} = \infty$

karena $\lim\limits_{x \to \infty} f(x) = \lim\limits_{n \to \infty} a_n = \infty$, maka $\left\{\dfrac{3n^2+2}{n+1}\right\}$ divergen

3. $\{a_n\} = \left\{\dfrac{\sqrt{n}}{n+1}\right\}$

$f(x) = \dfrac{\sqrt{x}}{x+1}$

$\lim\limits_{x \to \infty} \dfrac{\sqrt{x}}{x+1}$

$\lim\limits_{x \to \infty} \dfrac{1}{2\sqrt{x}} = 0$

karena $\lim\limits_{x \to \infty} f(x) = \lim\limits_{n \to \infty} a_n = 0$, maka $\left\{\dfrac{\sqrt{n}}{n+1}\right\}$ konvergen ke 0

No.

Date

4.) $\{a_n\} = \left\{ \frac{\pi^n}{4^n} \right\}$

$f(x) = \left\{ \frac{\pi^x}{4^x} \right\}$

$\lim_{x \to \infty} \frac{\pi^x}{4^x} = \left(\frac{\pi}{4}\right)^x$

$= \lim_{x \to \infty} \exp \ln \left(\frac{\pi}{4}\right)^x$

$= \lim_{x \to \infty} x \left(\frac{\pi}{4}\right)^x$

$= \lim_{x \to \infty} \frac{\pi/4}{1/x} = 0 \qquad e^0 = 1$

karena $\lim_{x \to \infty} f(x) = \lim_{n \to \infty} a_n = 1$, maka $\left\{ \frac{\pi^n}{4^n} \right\}$ konvergen ke 1

5.) $\{a_n\} = \left\{ \frac{\ln(n)}{n} \right\}$

$f(x) = \frac{\ln(x)}{x}$

$\lim_{x \to \infty} \frac{\ln(x)}{x}$

$\lim_{x \to \infty} \frac{1}{x} = 0$

karena $\lim_{x \to \infty} f(x) = \lim_{n \to \infty} a_n = 0$, maka $\left\{ \frac{\ln(n)}{n} \right\}$ konvergen ke 0

6. $\left\{ \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots \right\}$ rumus $a_n = \frac{n}{n+1}$

$f(x) = \frac{x}{x+1}$

$\lim_{x \to \infty} \frac{x}{x+1} = \frac{x}{x(0 + \frac{1}{x})} = 1$

karena $\lim_{x \to \infty} f(x) = \lim_{n \to \infty} a_n = 1$, maka $\left\{ \frac{n}{n+1} \right\}$ konvergen ke 1

No.

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7. $\{a_n\} = \left\{ n \sin \frac{\pi}{n} \right\}$

$f(x) = x \sin \frac{\pi}{x}$

$\lim\limits_{x \to \infty} x \sin \frac{\pi}{x}$

$\lim\limits_{x \to \infty} \frac{\sin \frac{\pi}{x}}{\frac{1}{x}}$

$\lim\limits_{x \to \infty} \frac{-\pi x^{-2} \cos \frac{\pi}{x}}{-x^{-2}} = \pi \cos \frac{\pi}{x} = \pi \cos 0 = \pi$

karena $\lim\limits_{x \to \infty} f(x) = \lim\limits_{n \to \infty} a_n = 0$, maka $\left\{ n \sin \frac{\pi}{n} \right\}$ konvergen ke π

8. $\{a_n\} = \{n^2 - n\}$

$f(x) = x^2 - x$

$\lim\limits_{x \to \infty} x^2 - x = x^2 \left(1 - \frac{1}{x}\right)$

$\lim\limits_{x \to \infty} \frac{\left(1 - \frac{1}{x}\right)}{\frac{1}{x^2}} = \frac{1-0}{0} = \infty$

karena $\lim\limits_{x \to \infty} f(x) = \lim\limits_{n \to \infty} a_n = \infty$, maka $\{n^2 - n\}$ divergen

9. $\{a_n\} = \left\{ \frac{n^2}{2n+1} \sin \frac{\pi}{n} \right\}$

$f(x) = \frac{x^2}{2x+1} \sin \frac{\pi}{x}$

$\lim\limits_{x \to \infty} \frac{(x^2) \sin \frac{\pi}{x}}{2x+1} = \frac{(x^2) \sin \frac{\pi}{x}}{x\left(2 + \frac{1}{x}\right)}$

$\lim\limits_{x \to \infty} \frac{x \sin \left(\frac{\pi}{x}\right)}{2 + \frac{1}{x}} = \frac{\frac{\pi x^{-2} \left(\cos \pi/x\right)}{-x^{-2}}}{2 + \frac{1}{8}} = \frac{\pi}{2}$

10. $\{a_n\} = \left\{ \frac{e^n + e^{2n}}{2e^{2n}} \right\}$

$f(x) = \frac{e^x + e^{2x}}{2e^{2x}}$

$\lim\limits_{x \to \infty} \frac{e^x + e^{2x}}{2e^{2x}} = \frac{e^x(1 + e^x)}{2e^x e^x} = \frac{1 + e^x}{2e^x}$

$= \frac{e^x\left(\frac{1}{e^x} + 1\right)}{2e^x} = \frac{\left(\frac{1}{0} + 1\right)}{2} = \frac{1}{2}$

karena $\lim\limits_{x \to \infty} f(x) = \lim\limits_{n \to \infty} a_n = \frac{1}{2}$, maka $\left\{ \frac{e^n + e^{2n}}{2e^{2n}} \right\}$ konvergen ke $\frac{1}{2}$

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Slide : 23

1. $\sum_{n=3}^{\infty} \frac{1}{(n-2)^2}$

$\int_3^{\infty} \frac{dx}{(x-2)^2} = \lim_{b \to \infty} \int_3^b \frac{dx}{(x-2)^2}$

$\lim_{b \to \infty} \int_3^b \frac{du}{(u)^2} = \lim_{b \to \infty} -\frac{1}{x} \Big]_3^b$

$\lim_{b \to \infty} -\frac{1}{b} - \left(-\frac{1}{3}\right) = \frac{1}{3}$ konvergen

$\int_3^b \frac{dx}{(x-2)^2}$ konvergen, maka $\sum_{n=3}^{\infty} \frac{1}{(n-2)^2}$ konvergen

3. $\sum_{n=2}^{\infty} \frac{1}{n \ln^2 n}$

$\int_2^{\infty} \frac{1}{x \ln^2 x} = \lim_{b \to \infty} \int_2^b \frac{1}{x \ln^2 x} dx$

$\lim_{b \to \infty} \int_2^b \frac{1}{u^2} du = \lim_{b \to \infty} -\frac{1}{\ln x} \Big]_2^b$

$\lim_{b \to \infty} -\frac{1}{\ln b} - \left(-\frac{1}{\ln 2}\right) = 0 + \frac{1}{\ln 2} \to$ konvergen

karena $\int_2^{\infty} \frac{1}{x \ln^2 x}$ konvergen, maka $\sum_{n=2}^{\infty} \frac{1}{n \ln^2 n}$ konvergen

2. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{2n+1}}$

$\int_1^{\infty} \frac{1}{\sqrt{2x+1}} = \lim_{b \to \infty} \int_1^b \frac{1}{\sqrt{2x+1}} dx$

$\lim_{b \to \infty} \int_1^b du = u \Big]_1^b = \sqrt{2x+1} \Big]_1^b$

$\lim_{b \to \infty} \sqrt{2b+1} - \sqrt{3} = \infty - \sqrt{3} = \infty$ divergen

karena $\int^{\infty} \frac{1}{\sqrt{2x+1}}$ divergen, maka $\sum_{n=1}^{\infty} \frac{1}{\sqrt{2n+1}}$ divergen

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4. $\sum_{n=1}^{\infty} \frac{1}{4n^2+1}$

$\int_1^{\infty} \frac{1}{4x^2+1} = \lim_{b \to \infty} \int_1^{b} \frac{1}{4x^2+1} = \lim_{b \to \infty} \frac{1}{2} \arctan 2x \Big|_1^b$

$\lim_{b \to \infty} \frac{1}{2} \arctan 2b - \frac{1}{2} \arctan 1$

$\infty - \ldots = \infty$

karena $\int_1^{\infty} \frac{1}{4x^2+1}$ divergen, maka $\sum_{n=1}^{\infty} \frac{1}{4n^2+1}$ divergen

6. Syarat k agar $\sum_{n \geq 0}^{\infty} \frac{1}{n \ln^k n}$, $k > 0$ konvergen:

$\int_2^{\infty} \frac{dx}{n \ln^k n} = \lim_{b \to \infty} \int_2^{\infty} \frac{dx}{x \ln^k x} = \lim_{b \to \infty} \int_2^{\infty} \frac{du}{u^k} = \lim_{b \to \infty} \int_2^{\infty} \lim_{b \to \infty} u^{-k}$

$\lim_{b \to \infty} \frac{1}{-k+1} (\ln x)^{-k+1} \Big]_2^b \quad \lim_{b \to \infty} \frac{1}{-k+1} (\ln b)^{-k+1} - \frac{1}{-k+1} (\ln 2)^{-k+1}$

$\frac{1}{-k+1} \ln (\infty)^{-k+1}$

$\frac{1}{(-k+1)} (\infty)^{-k+1}$

TIARA SHAKTI MAKMUR