16. Evaluate these quantities.

a) -17 **mod** 2   **b)** 144 **mod** 7
c) -101 **mod** 13   **d)** 199 **mod** 19

17. Evaluate these quantities.

a) 13 **mod** 3   **b)** -97 **mod** 11
c) 155 **mod** 19   **d)** -221 **mod** 23

18. List five integers that are congruent to 4 modulo 12.

19. Decide whether each of these integers is congruent to 5 modulo 17.

a) 80   **b)** 103
c) -29 **d)** -122

20. Show that if $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, where a, b, c, d, and m are integers with $m \geq 2$, then $a - c \equiv b - d \pmod{m}$.

Find gcd(1000, 625) and lcm(1000, 625) and verify that gcd(1000, 625) · lcm(1000, 625) = 1000 · 625.

Find gcd(92928, 123552) and lcm(92928, 123552), and verify that gcd(92928, 123552) · lcm(92928, 123552) = 92928 · 123552. [*Hint:* First find the prime factorizations of 92928 and 123552.]

Use the Euclidean algorithm to find

a) gcd(12, 18).   **b)** gcd(111, 201).
c) gcd(1001, 1331).   **d)** gcd(12345, 54321).
e) gcd(1000, 5040).   **f)** gcd(9888, 6060).

II.

Find all solutions to the system of congruences.

$x \equiv 2 \pmod{3}$
$x \equiv 1 \pmod{4}$
$x \equiv 3 \pmod{5}$

Find all solutions to the system of congruences.

$x \equiv 1 \pmod{2}$
$x \equiv 2 \pmod{3}$
$x \equiv 3 \pmod{5}$
$x \equiv 4 \pmod{11}$

Find all solutions, if any, to the system of congrue

III

Use the Extended Euclidean algorithm to compute the following multiplicative inverses:

(a) $17^{-1} \bmod 101$
(b) $357^{-1} \bmod 1234$
(c) $3125^{-1} \bmod 9987$.

IV

a) Show that $2^{340} \equiv 1 \pmod{11}$ by Fermat's Little Theorem and noting that $2^{340} = (2^{10})^{34}$.

b) Show that $2^{340} \equiv 1 \pmod{31}$ using the fact that $2^{340} = (2^5)^{68} = 32^{68}$.

c) Conclude from parts (a) and (b) that $2^{340} \equiv 1 \pmod{341}$.