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Soal:

Misalkan $p = 31$, $a = 1$ dan $b = 6$ sehingga didapat kurva elips: $y^2 \equiv x^3 + x + 6 \pmod{31}$

Lakukan proses enkripsi dan dekripsi menggunakan kriptografi kurva elips

Menezes-Vanstone untuk plainteks = $(7, 8)$ dan fungsi pembangkit $\alpha = (3, 6)$ dengan $q = 2$ dan $r = 3$.

Jawaban:

Enkripsi:

Menghitung y_0 :

$$\begin{aligned}y_0 &= q \cdot \alpha \\&= 2(3, 6) \\2\alpha &= \alpha + \alpha \\&= (3, 6) + (3, 6) \\\lambda &= (3x_1^2 + a)(2y_1)^{-1} \pmod{31} \\&= 3(3^2 + 1)(2(6))^{-1} \pmod{31} \\&= (3(9) + 1)(12)^{-1} \pmod{31} \\&= (3(9) + 1)(13) \pmod{31} \\&= 28(13) \pmod{31} \\&= 364 \pmod{31} \\&= 23\end{aligned}$$

$$x_3 = (\lambda^2 - x_1 - x_2) \pmod{31}$$

$$x_3 = (23^2 - 3 - 3) \pmod{31}$$

$$x_3 = (529 - 6) \pmod{31}$$

$$x_3 = 27$$

$$y_3 = (\lambda(x_1 - x_3) - y_1) \pmod{31}$$

$$y_3 = (23(3 - 27) - 6) \pmod{31}$$

$$y_3 = (-558) \pmod{31}$$

$$y_3 = 0$$

Maka didapat $y_0 = 2\alpha = (27, 0)$

Menghitung $r\alpha$ yang dapat didefinisikan menjadi β , dimana $\beta = 3\alpha$. $3\alpha = 2\alpha + \alpha = (27, 0) + (3, 6)$

$$\begin{aligned}\lambda &= (y_2 - y_1)(x_2 - x_1)^{-1} \pmod{31} \\&= (6 - 0)(3 - 27)^{-1} \pmod{31} \\&= (6)(-24)^{-1} \pmod{31} \\&= (6)(7)^{-1} \pmod{31} \\&= (6)(9) \pmod{31} \\&= 54 \pmod{31} \\&= 23\end{aligned}$$

$$\begin{aligned}
 x_3 &= (\lambda^2 - x_1 - x_2) \bmod 31 \\
 &= (23^2 - 27 - 3) \bmod 31 \\
 &= (529 - 30) \bmod 31 \\
 &= 3
 \end{aligned}$$

$$\begin{aligned}
 y_3 &= (\lambda (x_1 - x_3) - y_1) \bmod 31 \\
 &= (23(27 - 3) - 0) \bmod 31 \\
 &= (552 - 0) \bmod 31 \\
 &= 25
 \end{aligned}$$

Maka $\beta = 3\alpha = (3, 25)$

Menghitung $(c_1, c_2) = q * \beta = 2(3, 25)$

$$2\beta = \beta + \beta = (3, 25) + (3, 25)$$

$$\begin{aligned}
 \lambda &= (3x_1^2 + a)(2y_1)^{-1} \bmod 31 \\
 &= 3(3^2 + 1)(2(25))^{-1} \bmod 31 \\
 &= (3(9) + 1)(19)^{-1} \bmod 31 \\
 &= (3(9) + 1)(18) \bmod 31 \\
 &= 28(18) \bmod 31 \\
 &= 504 \bmod 31 \\
 &= 8
 \end{aligned}$$

$$\begin{aligned}
 x_3 &= (\lambda^2 - x_1 - x_2) \bmod 31 \\
 &= (8^2 - 3 - 3) \bmod 31 \\
 &= (64 - 6) \bmod 31 \\
 &= 27
 \end{aligned}$$

$$\begin{aligned}
 y_3 &= (\lambda (x_1 - x_3) - y_1) \bmod 31 \\
 &= (8(3 - 27) - 25) \bmod 31 \\
 &= (-217) \bmod 31 \\
 &= 0
 \end{aligned}$$

Maka didapat $(c_1, c_2) = 2\beta = (27, 0)$

Menghitung y_1 dan y_2

$$\begin{aligned}y_1 &= c_1 x_1 \bmod 31 \\&= 27(7) \bmod 31 \\&= 189 \bmod 31 \\&= 3\end{aligned}$$

$$\begin{aligned}y_2 &= c_2 x_2 \bmod 31 \\&= 0(8) \bmod 31 \\&= 0 \bmod 31 \\&= 0\end{aligned}$$

Jadi didapat ciphertext $y = (y_0, y_1, y_2) = ((27, 0), 3, 0)$.

Dekripsi:

Menghitung $(c_1, c_2) = r * y_0 = 2(3, 25)$

$$(c_1, c_2) = 3(27, 0).$$

$$\begin{aligned}2\alpha &= \alpha + \alpha \\&= (27, 0) + (27, 0)\end{aligned}$$

$$\begin{aligned}\lambda &= (3x_1^2 + a)(2y_1)^{-1} \bmod 31 \\&= 3(27)^2 + 1(2(0))^{-1} \bmod 31 \\&= (3(729) + 1)(0)^{-1} \bmod 31 \\&= (2187 + 1)(0) \bmod 31 \\&= 2188(0) \bmod 31 \\&= 0 \bmod 31 \\&= 0\end{aligned}$$

$$\begin{aligned}y_3 &= (\lambda (x_1 - x_3) - y_1) \bmod 31 \\&= (0(27 - 8) - 0) \bmod 31 \\&= (0) \bmod 31 \\&= 0\end{aligned}$$

Maka didapat $2\alpha = (8, 0)$

$$\begin{aligned}3\alpha &= 2\alpha + \alpha \\&= (8, 0) + (27, 0)\end{aligned}$$

$$\begin{aligned}\lambda &= (y_2 - y_1)(x_2 - x_1)^{-1} \bmod 31 \\&= (0 - 0)(27 - 8)^{-1} \bmod 31 \\&= (0)(19)^{-1} \bmod 31 \\&= (0)(18) \bmod 31 \\&= 0 \bmod 31 \\&= 0\end{aligned}$$

$$\begin{aligned}
 x_3 &= (\lambda^2 - x_1 - x_2) \bmod 31 \\
 &= (0^2 - 8 - 27) \bmod 31 \\
 &= -35 \bmod 31 \\
 &= 27
 \end{aligned}$$

$$\begin{aligned}
 y_3 &= (\lambda (x_1 - x_3) - y_1) \bmod 31 \\
 &= (0(8 - 27) - 0) \bmod 31 \\
 &= (0) \bmod 31 \\
 &= 0
 \end{aligned}$$

Maka didapat $(c_1, c_2) = 3\alpha = (27, 0)$

Menghitung x sebagai berikut:

$$x = (y_1 c_1^{-1} \bmod 31, y_2 c_2^{-1} \bmod 31)$$

$$\begin{aligned}
 x_1 &= y_1 c_1^{-1} \bmod 31 \\
 &= 3(27)^{-1} \bmod 31 \\
 &= 3(23) \bmod 31 \\
 &= 69 \bmod 31 \\
 &= 7
 \end{aligned}$$

$$\begin{aligned}
 x_2 &= y_2 c_2^{-1} \bmod 31 \\
 &= 0(0)^{-1} \bmod 31 \\
 &= 0(0) \bmod 31 \\
 &= 0
 \end{aligned}$$

Maka didapat plaintext $x = (7, 0)$.