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Nama: Drames Ray Lapian
NPM: 140810210059 - A
Matkul: Matematika Diskrit

16. a) $-17 \bmod 2 \rightarrow$

$-17 = 2(-9) + \underline{\underline{1}}$

c) $-101 \bmod 13 \rightarrow$

$-101 = 13(-8) + \underline{\underline{3}}$

b) $144 \bmod 7 \rightarrow$

$144 = 7(20) + \underline{\underline{4}}$

d) $199 \bmod 19 \rightarrow$

$199 = 19(10) + \underline{\underline{9}}$

17. a) $13 \bmod 3 \rightarrow$

$13 = 3(4) + \underline{\underline{1}}$

c) $155 \bmod 19 \rightarrow$

$155 = 19(8) + \underline{\underline{3}}$

b) $-97 \bmod 11 \rightarrow$

$-97 = 11(-9) + \underline{\underline{2}}$

d) $-221 \bmod 23 \rightarrow$

$-221 = 23(-10) + \underline{\underline{9}}$

18. A modulo 12 = 4

$4x \equiv 4 \pmod{12} \rightarrow 4x = 4 + 12k$

$x = \frac{4 + 12k}{4}$

$x = 1 \rightarrow \frac{4+12}{4} \rightarrow x = \frac{16}{4} = 16$

$x = 2 \rightarrow \frac{4+24}{4} \rightarrow x = \frac{28}{4} = 28$

$x = 3 \rightarrow \frac{4+36}{4} \rightarrow x = \frac{40}{4} = 40$

$x = -1 \rightarrow \frac{4-12}{4} \rightarrow x = \frac{-8}{4} = -8$

$x = -2 \rightarrow \frac{4-24}{4} \rightarrow x = \frac{-20}{4} = -20$

19. a) 80

$80 \not\equiv 5 \pmod{17} \rightarrow$ Tidak kongruen, karena 17 tidak habis membagi $80 - 5 = 75$

b) 103

$103 \not\equiv 5 \pmod{17} \rightarrow$ Tidak kongruen, karena 17 tidak habis membagi $103 - 15 = 98$

c) -29

$-29 \equiv 5 \pmod{17} \rightarrow$ kongruen, karena 17 habis membagi $-29 - 5 = -34$

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d) -122

$-122 \equiv 5 \pmod{17}$ → kongruen, karena 17 habis membagi $-122 - 5 = -127$

20 $a \equiv b \pmod m$ → $a = b + k_1 m$

$c \equiv d \pmod m$ → $c = d + k_2 m$

$(a-c) = (b-d) + (k_1 . m - k_2 . m)$

$= (b-d) + m(k_1 - k_2)$

$(a-c) \equiv (b-d) \pmod m$

GCD (1000, 625)

$1000 = 625(1) + 375$

$625 = 375(1) + 250$

$375 = 250(1) + 125$

$250 = 125(2) + 0$

→ GCD (1000, 625) = 125

lcm (1000, 625)

$1000 = (2^3)(5^3)$

$625 = 5^4$

→ lcm (1000, 625) $= (2^3)(5^4)$

$= 5000$

gcd (1000, 625) . lcm (1000, 625) = 125 . 5000 = 625000

GCD (92928, 123552)

$m = 123552$ $n = 92928$

$123552 = 92928(1) + 30624$

$92928 = 30624(1) + 1056$

$30624 = 1056(29) + 0$

→ GCD (92928, 123552) = 29

lcm (92928, 123552)

$92928 = (2^8)(3)(11^2)$

$123552 = (2^5)(3^3)(11)(13)$

→ lcm (92928, 123552)

$= (2^8)(3^3)(11^2)(13)$

$= 10872576$

gcd (92928, 123552) . lcm (92928, 123552) = 92928 . 123552

1056 . 10872576 = 11481440256

11481440256 = 11481440256

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1) Gunakan Algoritma Euclidean:

a) gcd(12, 18)

$18 = 12 \cdot 1 + 6$

$12 = 6 \cdot 2 + 0$

$\rightarrow \gcd(12, 18) = 6$

b) gcd(111, 201)

$201 = 111 \cdot 1 + 90$

$111 = 90 \cdot 1 + 21$

$90 = 21 \cdot 4 + 6$

$21 = 6 \cdot 3 + 3$

$6 = 3 \cdot 2 + 0$

$\rightarrow \gcd(111, 201) = 3$

c) gcd(1001, 1331)

$1331 = 1001 \cdot 1 + 330$

$1001 = 330 \cdot 3 + 11$

$330 = 11 \cdot 30 + 0$

$\rightarrow \gcd(1001, 1331) = 11$

d) gcd(12345, 54321)

$54321 = 12345 \cdot 4 + 4941$

$12245 = 4941 \cdot 2 + 2463$

$4941 = 2463 \cdot 2 + 15$

$2463 = 15 \cdot 164 + 3$

$15 = 3 \cdot 5 + 0$

$\rightarrow \gcd(12345, 54321) = 3$

e) gcd(1000, 5040)

$5040 = 1000 \cdot 5 + 40$

$1000 = 40 \cdot 25 + 0$

$\rightarrow \gcd(1000, 5040) = 40$

f) gcd(9288, 6060)

$9288 = 6060 \cdot 1 + 3828$

$6060 = 3828 \cdot 1 + 2232$

$3828 = 2232 \cdot 1 + 1596$

$2232 = 1596 \cdot 1 + 636$

$1596 = 636 \cdot 2 + 324$

$636 = 324 \cdot 1 + 312$

$324 = 312 \cdot 1 + 12$

$312 = 12 \cdot 26 + 0$

$\rightarrow \gcd(9288, 6060) = 12$

2) Tentukan solusi untuk sistem kongruensi berikut:

$x = 2 \pmod 3$ $\quad N = 3 \cdot 4 \cdot 5 = 60$

$x = 1 \pmod 4$ $\quad N_1 = 20$

$x = 3 \pmod 5$ $\quad N_2 = 15$

$N_3 = 12$

$20x_1 = 0 \pmod 3 \quad x_1 = 2$

$2x_1 = 1 \pmod 3$

$15x_2 = 1 \pmod 4 \quad x_2 = 3$

$3x_2 = 1 \pmod 4$

$12x_3 = 1 \pmod 5 \quad x_3 = 8$

$2x_3 = 1 \pmod 5$

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$x = \sum x_i \, N_i \, b_i$

$= 2.20.2 + 3.15.1 + 8.12.3$

$= 80 + 45 + 288$

$= 413$

$= 53 \pmod{60}$

b. Tentukan solusi untuk sistem kongruensi berikut

$x = 1 \pmod 2$ $N = 2.3.5 = 330$

$x = 2 \pmod 3$ $N_1 = 3.5.11 = 165$

$x = 3 \pmod 5$ $N_2 = 2.5.11 = 110$

$x = 4 \pmod{11}$ $N_3 = 2.3.11 = 66$

$N_4 = 2.3.5 = 30$

$165 x_1 = 1 \pmod 2$ $x_1 = 1$ $66 x_3 = 1 \pmod 5$ $x_3 = 1$

$x_1 = 1 \pmod 2$ $x_3 = 1 \pmod 5$

$110 x_2 = 1 \pmod 3$ $x_2 = 2$ $30 x_4 = 1 \pmod{11}$

$2 x_2 = 1 \pmod 3$ $8 . x_4 = 1 \pmod{11}$

$x = \sum x_i \, N_i . b_i$

$= 1.165.1 + 2.110.2 + 1.66.3 + 7.30.4$

$= 165 + 440 + 198 + 840$

$= 1643$

$= 323 \pmod{330}$

b. Gunakan Algoritma Euclidean untuk masalah Invers:

a) $17^{-1} \bmod 101$

$101 = 17(5) + 16 \rightarrow 16 = 101 - (1)(5)$

$17 = 16(1) + 1 \rightarrow 1 = 17 - 6$

$1 = 17 - 16$

$= 17 - (101 - (17(5)))$

$= 17 - 101 + 17(5)$

$= 17(6) - 101.1$

$17(6) = 1 \pmod{101} \rightarrow 17^{-1} \bmod 101 = 6$

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b) $357^{-1} \bmod 1234$

$1234 = 357(3) + 163 \rightarrow 163 = 1234 - 357(3)$

$357 = 163(2) + 31 \rightarrow 31 = 357 - 163(2)$

$163 = 31(5) + 8 \rightarrow 8 = 163 - 31(5)$

$31 = 8(3) + 7 \rightarrow 7 = 31 - 8(3)$

$8 = 7(1) + 1 \rightarrow 1 = 8 - 7(1)$

$1 = 4(163) - 21(31)$

$= 4(163) - 21(357 - 163(2))$

$= 46(163) - 21(357)$

$= 46(1234 - 357(3)) - 21(357)$

$= 46(1234) - 159(357)$

$-159(357) + 46(1234) = 1$

$-159(357) \equiv 1 \pmod{1234} \rightarrow 357^{-1} \bmod 1234 = -159$

c) $3125^{-1} \bmod 9987$

$9987 = 3125(3) + 612 \rightarrow 612 = 9987 - 3125(3)$

$3125 = 612(5) + 65 \rightarrow 65 = 3125 - 612(5)$

$612 = 65(9) + 27 \rightarrow 27 = 612 - 65(9)$

$65 = 27(2) + 11 \rightarrow 11 = 65 - 27(2)$

$27 = 11(2) + 5 \rightarrow 5 = 27 - 11(2)$

$11 = 5(2) + 1 \rightarrow 1 = 11 - 5(2)$

$1 = 11 - 5(2)$

$= 11 - 2(27 - 11(2))$

$= 11(5) - 2(27)$

$= 5(65 - 27(2)) - 2(27)$

$= 5(65) - 12(27)$

$= 5(65) - 12(612 - 65(9))$

$= 113(65) - 12(612)$

$= 113(3125 - 612(5)) - 12(612)$

$= 113(3125) - 577(612)$

$= 113(3125) - 577(9987 - 3125(3))$

$= 1844(3125) - 577(9987)$

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$1844(3125) - 577(9987) = 1$

$1844(3125) \equiv 1 \pmod{9987}$ → $3125^{-1} \bmod 9987$

$= \underline{\underline{1844}}$

a) $2^{340} \equiv 1 \pmod{11}$

By Fermat's Little Theorem

$2^{10} \equiv 1 \pmod{11}$

$(2^{10})^{34} \equiv 1^{34} \pmod{11}$

$2^{340} \equiv 1 \pmod{11}$

→ Terbukti $2^{340} \equiv 1 \underline{\underline{\pmod{11}}}$

b) $2^{340} \equiv 1 \pmod{31}$

$(2^5)^6 \equiv 1 \pmod{31}$

$2^5 \equiv 1^{1/6} \pmod{31}$ → $2^5 \equiv 1 \pmod{31}$

$(2^5)^{68} \equiv 1^{68} \pmod{31}$

$(2^5)^{68} \equiv 1 \pmod{31}$

→ Terbukti $2^{340} \equiv 1 \underline{\underline{\pmod{31}}}$

c) Persamaan ① $\dfrac{11}{2^{340}-1}$

Persamaan ② $\dfrac{31}{2^{340}-1}$

→ $\dfrac{11}{2^{340}-1}$ $\dfrac{31}{2^{340}-1} = \dfrac{341}{2^{340}-1}$

→ $2^{340} \equiv 1 \pmod{\underline{\underline{341}}}$