

# GRAFIKA KOMPUTER D10K-5C01

Semester Ganjil 2023-2024

**GK05: Transformasi 3D** 

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# Program Studi S-1 Teknik Informatika FMIPA Universitas Padjadjaran



### **TRANSFORMASI 3 DIMENSI**

- Pendahuluan
- Skala
- Rotasi
- Refleksi
- Transformasi Majemuk



#### **PENDAHULUAN**

- Menggunakan koordinat 3 sumbu yaitu x, y dan z
- Sebuah titik pada ruang 3 dimensi dituliskan sebagai [ x y z 1]
- Transformasi 3 dimensi dituliskan sebagai [x' y' z' 1] = [x y z 1] [T]
- MTU 3 dimensi

$$\begin{bmatrix} a & b & c & p \\ d & e & f & q \\ g & h & i & r \\ l & m & n & s \end{bmatrix}$$

a, b, c, d, e, f, g, h, i adalah elemen yang a, b, c, d, e, f, g, h, i adalah elemen yang berpengaruh terhadap transformasi linier

berpengaruh terhadap transformasi linier

p, q, r adalah elemen yang untuk proyeksi dan perspektif

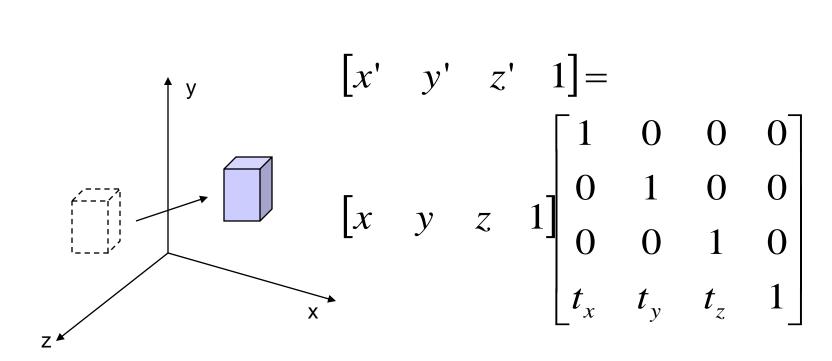
l, m, n adalah elemen untuk translasi pada sumbu x y dan z sumbu x, y dan z s adalah elemen untuk overall scaling



### **Translasi**

Translasi sebuah titik

$$x' = x + t_x$$
,  $y' = y + t_y$ ,  $z' = z + t_z$ 





#### **SKALA 3D**

- Terdapat 2 jenis skala yaitu local scaling dan overall scaling. Local scaling dipengaruhi oleh elemen a, e, dan i.
   Sedangkan overal scaling dipengaruhi oleh elemen s
- Contoh di bawah adalah MTU local scaling untuk faktor 1/3, 1/2 dan 1 serta overall scaling dengan faktor 2. Ingat bahwa nilai overall scaling adalah 1/s

$$\begin{bmatrix} 1/3 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$
(a) 
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$
(b)



# 3D Scaling

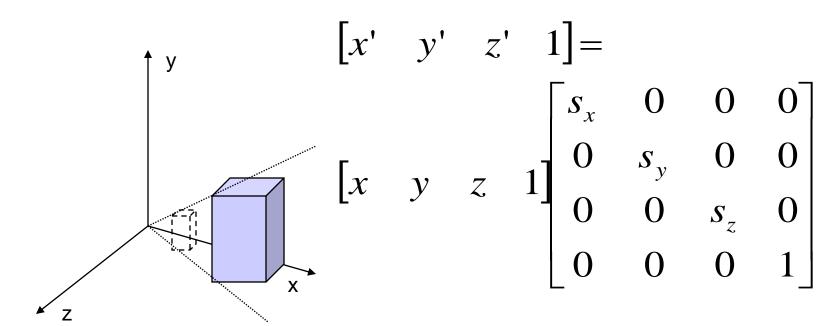
Global Scaling



## 3D Scaling

Local Scaling

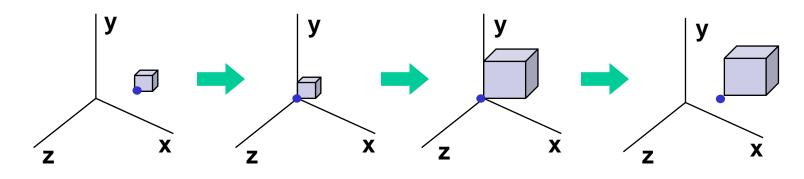
$$x'=x\cdot s_x$$
,  $y'=y\cdot s_y$ ,  $z'=z\cdot s_z$ 





## **Relative Scaling**

Scaling with a Selected Fixed Position



**Original position** 

**Translate** 

**Scaling** 

**Inverse Translate** 

$$T(-t_{x},-t_{y},-t_{z})\cdot S(s_{x},s_{y},s_{z})\cdot T(t_{x},t_{y},t_{z}) = \begin{bmatrix} x' & y' & z' & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -t_{x} & -t_{y} & -t_{z} & 1 \end{bmatrix} \begin{bmatrix} s_{x} & 0 & 0 & 0 \\ 0 & s_{y} & 0 & 0 \\ 0 & 0 & s_{z} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_{x} & t_{y} & t_{y} & 1 \end{bmatrix}$$



### 3D Rotation

- Coordinate-Axes Rotations
  - X-axis rotation
  - Y-axis rotation
  - Z-axis rotation
- General 3D Rotations
  - Rotation about Origin
  - Rotation about an axis that is parallel to one of the coordinate axes
  - Rotation about an arbitrary axis



### **ROTASI 3D**

- Rotasi pada sumbu utama
- MTU  $[T_x]$  untuk rotasi pada sumbu x sebesar  $\theta^{\circ}$
- ullet MTU  $[T_v]$  untuk rotasi pada sumbu y sebesar o°
- MTU  $[T_z]$  untuk rotasi pada sumbu z sebesar  $\psi^{\circ}$

$$[T_x] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[T_y] = \begin{bmatrix} \cos\phi & 0 & -\sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ \sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[T_z] = \begin{bmatrix} \cos\psi & \sin\psi & 0 & 0 \\ -\sin\psi & \cos\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$[T_z] = \begin{bmatrix} \cos\psi & \sin\psi & 0 & 0 \\ -\sin\psi & \cos\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



### **Coordinate-Axes Rotations**

#### Z-Axis Rotation

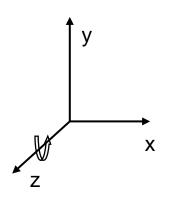
#### X-Axis Rotation

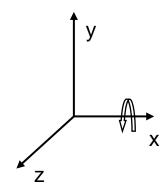
#### Y-Axis Rotation

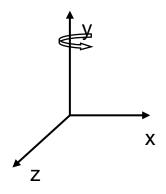
$$\begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



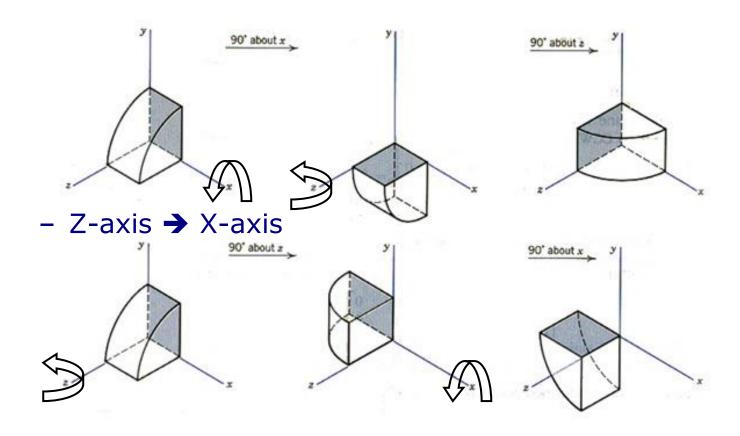






### **Urutan Rotasi**

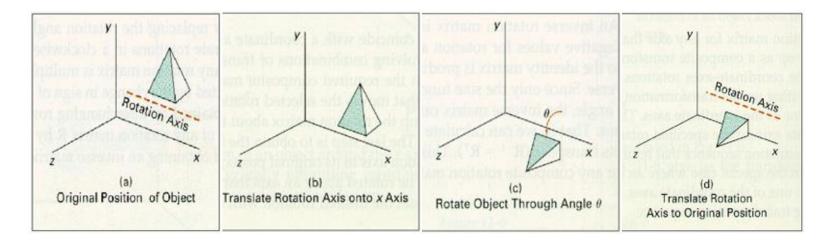
- Urutan rotasi mempengaruhi hasil akhir
  - X-axis → Z-axis





### Rotasi 3D Secara Umum

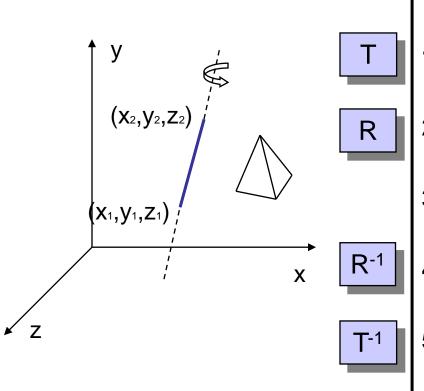
- Rotasi pada sebuah sumbu yang paralel dengan sumbu utama
  - Translate objek sehingga sumbu rotasi <u>berimpit dengan</u> sumbu koordinat yang paralel
  - Lakukan rotation yang diinginkan pada sumbu tsb
  - Translate objek <u>kembali ke posisi semula</u>





### **Rotasi 3D Secara Umum**

Rotasi pada Sumbu Sembarang



#### **Basic Idea**

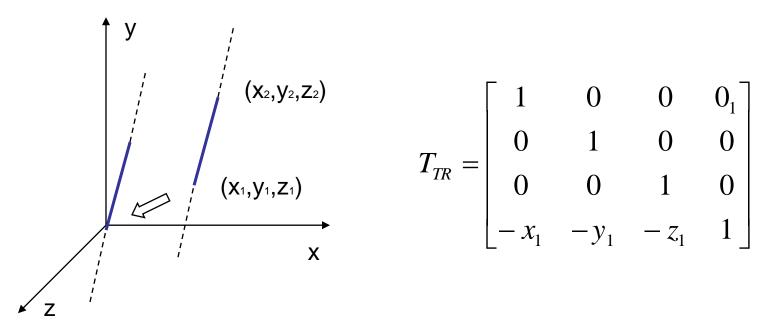
- 1. Translate (x1, y1, z1) to the origin
- 2. Rotate (x'2, y'2, z'2) on to the z axis
- 3. Rotate the object around the z-axis
- 4. Rotate the axis to the original orientation
- 5. Translate the rotation axis to the original position





### **General 3D Rotations**

Step 1. Translation

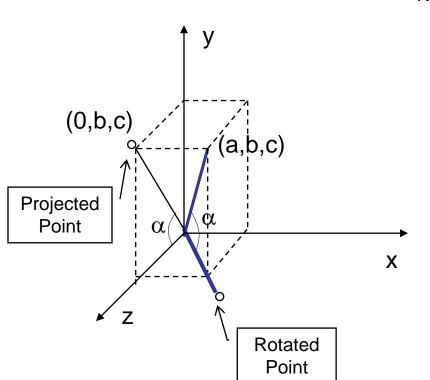


$$T_{TR} = \begin{bmatrix} 1 & 0 & 0 & 0_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -x_1 & -y_1 & -z_1 & 1 \end{bmatrix}$$



### **General 3D Rotations**

• Step 2. Establish  $[T_R]^{\alpha}_{x}$  x axis



$$\sin \alpha = \frac{b}{\sqrt{b^2 + c^2}} = \frac{b}{d}$$

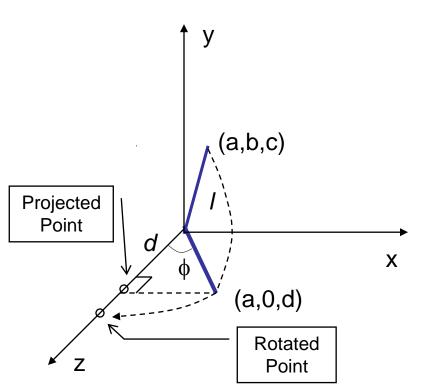
$$\cos\alpha = \frac{c}{\sqrt{b^2 + c^2}} = \frac{c}{d}$$

$$\begin{bmatrix} \mathbf{X} & \begin{bmatrix} T_R \end{bmatrix}_x^{\alpha} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c/d & -b/d & 0 \\ 0 & b/d & c/d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



# **Arbitrary Axis Rotation**

Step 3. Rotate about y axis by ∮



$$\sin \phi = \frac{a}{l}, \quad \cos \phi = \frac{d}{l}$$

$$l^2 = a^2 + b^2 + c^2 = a^2 + d^2$$

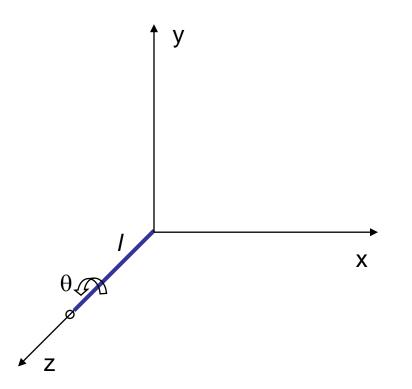
$$d = \sqrt{b^2 + c^2}$$

$$\begin{bmatrix} \mathbf{T}_R \end{bmatrix}_y^{\phi} = \begin{bmatrix} \cos\phi & 0 & \sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} d/l & 0 & a/l & 0 \\ 0 & 1 & 0 & 0 \\ -a/l & 0 & d/l & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



# **Arbitrary Axis Rotation**

ullet Step 4. Rotate about z axis by the desired angle  $\theta$ 



$$[T_R]_z^{\theta} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# **Arbitrary Axis Rotation**

 Step 5. Apply the reverse transformation to place the axis back in its initial position

$$\left[T_{TR}\right]^{-1} \left[T_{R}\right]_{x}^{-\alpha} \left[T_{R}\right]_{y}^{-\phi} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ x_{1} & y_{1} & z_{1} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha & 0 \\ 0 & -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

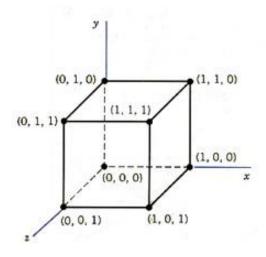
$$\mathbf{x}$$

$$\begin{bmatrix} \cos\phi & 0 & -\sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ \sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[T_R]_{ARB} = [T_{TR}]^{-1} [T_R]_x^{-\alpha} [T_R]_y^{-\phi} [T_R]_z^{\theta} [T_R]_y^{\phi} [T_R]_x^{\alpha} [T_{TR}]$$



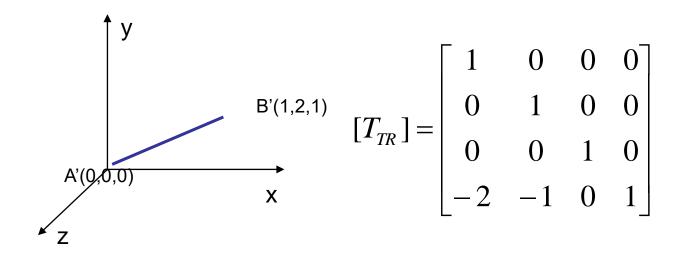
Ex) Find the new coordinates of a unit cube 90°-rotated about an axis defined by its endpoints A(2,1,0) and B(3,3,1).



**A Unit Cube** 

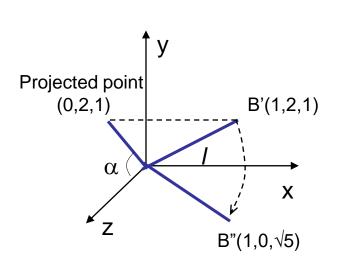


Step1. Translate point A to the origin





• Step 2. Rotate axis A'B' about the x axis by and angle  $\alpha$ , until it lies on the xz plane.



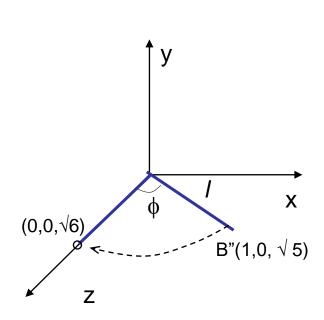
$$\sin \alpha = \frac{2}{\sqrt{2^2 + 1^2}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$
$$\cos \alpha = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$l = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$$

$$[T_R]_x^{\alpha} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{5}}{5} & -\frac{2\sqrt{5}}{5} & 0 \\ 0 & \frac{2\sqrt{5}}{5} & \frac{\sqrt{5}}{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



• Step 3. Rotate axis A'B'' about the y axis by and angle  $\phi$ , until it coincides with the z axis.



$$\sin \phi = \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6}$$
$$\cos \phi = \frac{\sqrt{5}}{\sqrt{6}} = \frac{\sqrt{30}}{6}$$

X
$$[T_R]_y^{\phi} = \begin{bmatrix} \frac{\sqrt{30}}{6} & 0 & -\frac{\sqrt{6}}{6} & 0\\ \frac{0}{6} & 1 & 0 & 0\\ \frac{\sqrt{6}}{6} & 0 & \frac{\sqrt{30}}{6} & 0\\ \frac{0}{6} & 0 & 0 & 1 \end{bmatrix}$$



• Step 4. Rotate the cube 90° about the z axis

$$[T_R]_z^{90^\circ} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Finally, the concatenated rotation matrix about the arbitrary axis AB becomes,

$$[T_R]_{ARB} = [T_{TR}]^{-1} [T_R]_x^{-\alpha} [T_R]_y^{-\phi} [T_R]_z^{90^{\circ}} [T_R]_y^{\phi} [T_R]_x^{\alpha} [T_{TR}]$$



$$[T_R]_{ARB} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{5}}{5} & -\frac{2\sqrt{5}}{5} & 0 \\ 0 & \frac{2\sqrt{5}}{5} & \frac{\sqrt{5}}{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{30}}{6} & 0 & -\frac{\sqrt{6}}{6} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\sqrt{6}}{6} & 0 & \frac{\sqrt{30}}{6} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{\sqrt{30}}{6} & 0 & \frac{\sqrt{6}}{6} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{\sqrt{6}}{6} & 0 & \frac{\sqrt{30}}{6} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{5}}{5} & \frac{2\sqrt{5}}{5} & 0 \\ 0 & -\frac{2\sqrt{5}}{5} & \frac{\sqrt{5}}{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.166 & -0.075 & 0.983 & 1.742 \\ 0.742 & 0.667 & 0.075 & -1.151 \\ -0.650 & 0.741 & 0.167 & 0.560 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



•Multiplying  $[T_R]_{AB}$  by the point matrix of the original cube  $[P^*]_{=}[T_R]_{ABR} \cdot [P]$ 

$$[P^*] = \begin{bmatrix} 0.166 & -0.075 & 0.983 & 1.742 \\ 0.742 & 0.667 & 0.075 & -1.151 \\ -0.650 & 0.741 & 0.167 & 0.560 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2.650 & 1.667 & 1.834 & 2.816 & 2.725 & 1.742 & 1.909 & 2.891 \\ -0.558 & -0.484 & 0.258 & 0.184 & -1.225 & -1.151 & -0.409 & -0.483 \\ 1.467 & 1.301 & 0.650 & 0.817 & 0.726 & 0.560 & -0.091 & 0.076 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$



#### **REFLEKSI 3D**

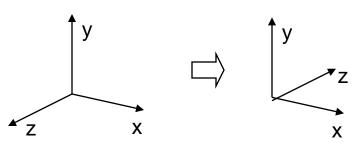
- Refleksi pada bidang utama xy, yz dan xz
- MTU Refleksi pada bidang xy
- MTU Refleksi pada bidang yz
- MTU Refleksi pada bidang xz

$$[T_{xz}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



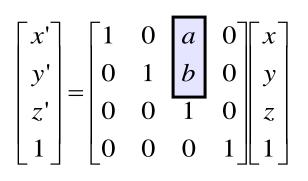
### Refleksi

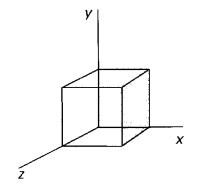
Refleksi pada bidang xy

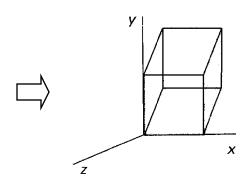


Z-axis Shear

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



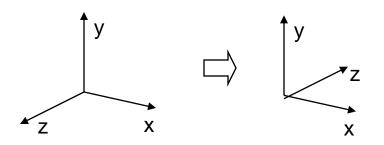


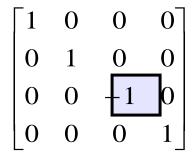




# Refleksi pada bidang utama

Reflection the xy Plane





- Reflection the yz Plane
- Reflection the xz Plane



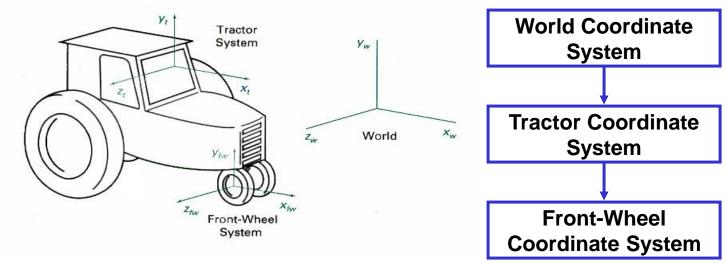
$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



### **Transformasi Multi Koordinat**

- Multiple Coordinate System
  - Hierarchical Modeling



- As tractor moves, tractor coordinate system and front-wheel coordinate system move in world coordinate system
- front wheels rotate in wheel coordinate system
- When tractor turns, wheel coordinate system <u>rotates</u> in tractor system



### **Coordinate Transformations**

- Transformation of an Object Description from One Coordinate System to Another
  - Set up a translation that brings the new coordinate origin to the position of the other coordinate origin
  - Rotations that corresponding coordinate axes
  - Scaling transformation, if different scales are used in the two coordinates systems

Example

$$\mathbf{z}$$
 $(0,0,0)$ 
 $\mathbf{z}$ 
 $\mathbf{v}$ 
 $\mathbf$ 

### LATIHAN SCALING

Diketahui sebuah objek P dengan koordinat sebagai berikut :  $\{(0,0,1,1), (2,0,1,1), (2,3,1,1), (0,3,1,1), (0,0,0,1), (2,0,0,1), (2,3,0,1), (0,3,0,1)\}$ .

- 1. Gambarkan objek tersebut!
- 2. Lakukan local scaling terhadap objek P dengan faktor skala  $xyz=\{1/2, 1/3 \text{ dan } 1\}$ .
  - a. Tentukan koordinat baru
  - b. Gambarkan hasilnya
- 3. Lakukan overal scaling terhadap objek asli dengan faktor 2.
  - a. Tentukan koordinat baru
  - b. Gambarkan hasilnya



### **LATIHAN ROTASI**

Diketahui sebuah objek Q dengan koordinat sebagai berikut :  $\{(0,0,1,1), (3,0,1,1), (3,2,1,1), (0,2,1,1), (0,0,0,1), (3,0,0,1), (3,2,0,1), (0,2,0,1)\}$ .

- 1. Gambarkan objek tersebut!
- 2. Lakukan rotasi terhadap Q sebesar  $\theta = -90$  ° pada x
  - a. Tentukan koordinat baru
  - b. Gambarkan hasilnya
- 3.Lakukan rotasi terhadap objek Q sebesar  $\phi = 90^{\circ}$  pada sumbu y
  - a. Tentukan koordinat baru
  - b. Gambarkan hasilnya



### **LATIHAN REFLEKSI**

Diketahui sebuah objek Q dengan koordinat sebagai berikut :  $\{(1,0,-1,1), (2,0,-1,1), (2,1,-1,1), (1,1,-2,1), (1,0,-2,1), (2,0,-2,1), (2,1,-2,1), (1,1,-2,1)\}$ .

- 1. Gambarkan objek tersebut!
- 2. Lakukan refleksi pada bidang xy
  - a. Tentukan koordinat baru
  - b. Gambarkan hasilnya



### LATIHAN TRANSFORMASI GABUNGAN

- 1. Tentukan Matriks Transformasi Umum untuk transformasi berurutan berikut ini:
  - a. Translasi sebesar -1, -1, -1 pada sumbu x, y, z
  - b. Rotasi sebesar +30° pada sumbu x
  - c. Rotasi sebesar + 45° pada sumbu y
- Tentukan koordinat objek baru untuk vektor posisi homogen (3 2 1 1) yang ditransformasikan dengan MTU yang dihasilkan



#### ROTASI PADA SUMBU SEJAJAR SUMBU UTAMA

Diketahui sebuah objek dengan vektor posisi

$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 \\ 2 & 2 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 \\ 2 & 2 & 1 & 1 \\ 1 & 2 & 1 & 1 \end{bmatrix}$$

 Lakukan rotasi sebesar θ=+30° pada sumbu yang sejajar sumbu x dan melalui titik centroid dari objek tersebut. Koordinat centroid adalah [3/2 3/2 3/2 1].





#### Grafika Komputer Menggunakan Python

Sebuah Referensi Pembuatan Gambar Digital 2D dan 3D

Pengarang: Setiawan Hada (Editor) Institusi: Universitas Padjadjaran

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Departemen: Ilms Komputer



You don't have to be amazing to start, BUT you have to start to be amazing

#### Daftar Isi

2425242

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6	Hidden Line Removal	•
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### **Tugas**

- Eksplorasi CABRI 3D
- Buatlah objek 3D dengan aplikasi Cabri 3D
- Tuliskan cara-cara pembuatannya, dan hasilnya
- Kumpulkan di LIVE!

