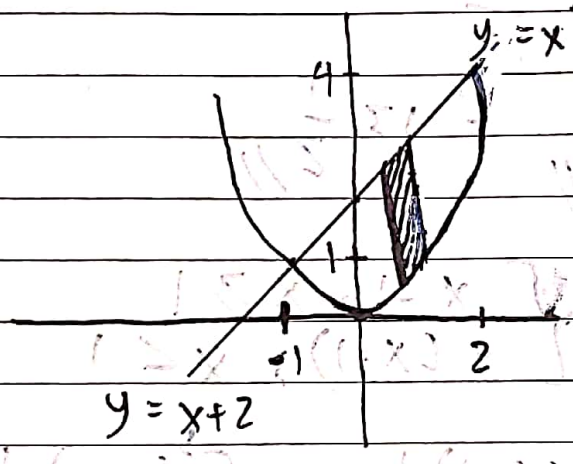
Nama : Ardes Zulka Putra
NPM : 1408102100009
Kelas : A

TUGAS 11
Aplikasi Integral

A. Gambarkan dan hitung luas daerah yang dibatasi oleh

1. $y = x^2$ dan $y = x + 2$



$y = y$

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1)$

$x = 2 \quad x = -1$

$$L = \int_{-1}^{2} (x+2) - x^2 \, dx$$

$$= \left[\frac{1}{2}x^2 + 2x - \frac{1}{3}x^3\right]_{-1}^{2}$$

$$= \left[2 + 4 - \frac{8}{3}\right] - \left[\frac{1}{2} - 2 + \frac{1}{3}\right]$$

$$= \left[\frac{10}{3}\right] - \left[-\frac{7}{6}\right]$$

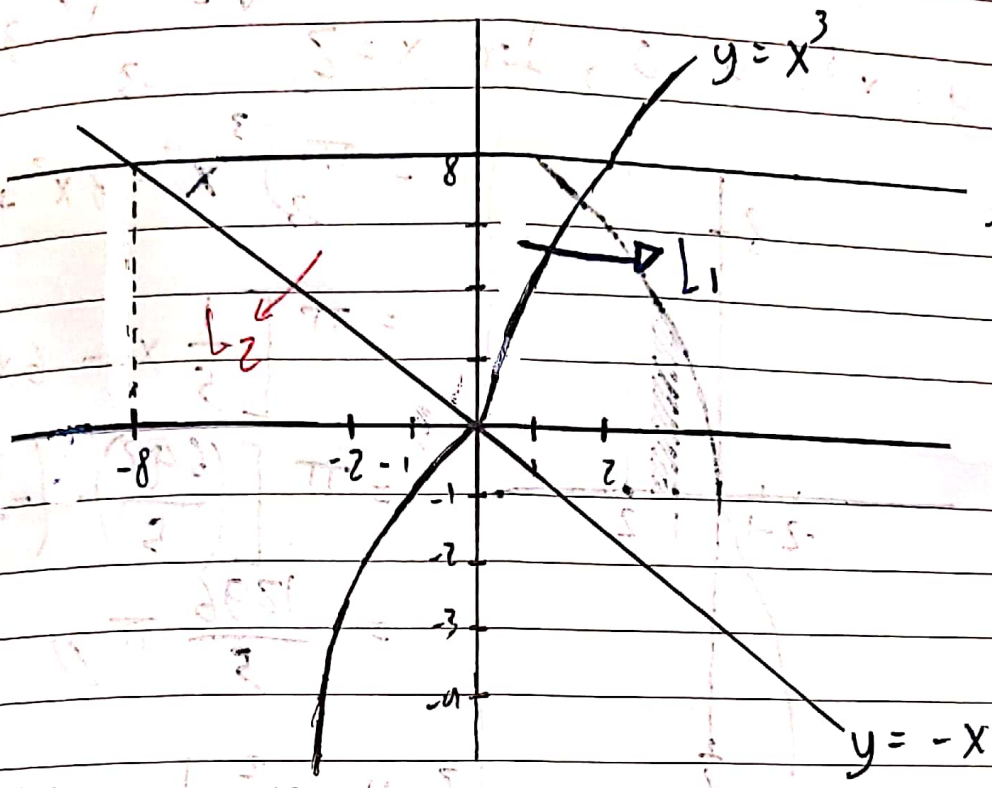
$$= \frac{27}{6}$$

$$= 4\frac{1}{2} \text{ //}$$

Dipindai dengan CamScanner

Date:

2. $y = x^3$, $y = -x$, dan $y = 8$



$$L_1 = \int_0^2 8 - x^3 \, dx$$

$$= \left[8x - \frac{1}{4} x^4 \right]_0^2$$

$$= 16 - 4$$

$$= 12$$

$$L_2 = \int_{-8}^0 8 + x \, dx$$

$$= \left[8x + \frac{1}{2} x^2 \right]_{-8}^0$$

$$= 0 - (-32)$$

$$= 32$$

Luas total $= L_1 + L_2$

$= 12 + 32$

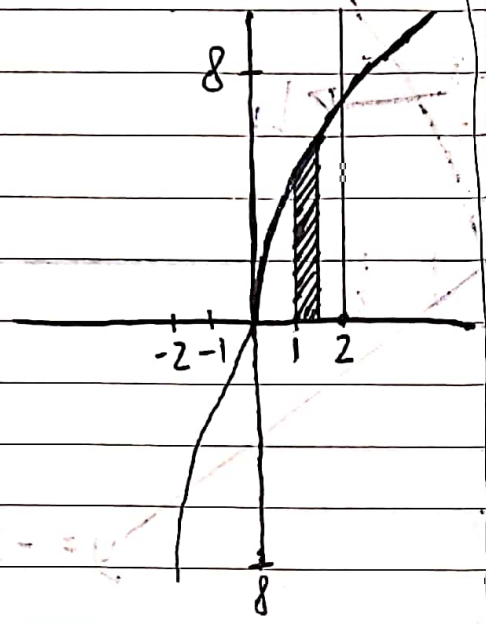
$= 44$ //

Dipindai dengan CamScanner

Date:

B. Hitung volume benda putar

1. $y = x^3$, $y = 0$, dan $x = 2$

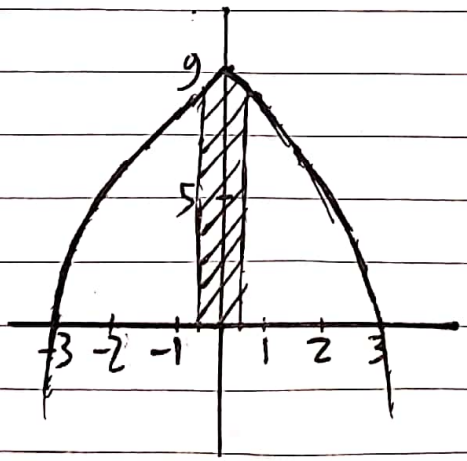


$$V = \pi \int_0^2 (x^3)^2 \, dx$$

$$= \pi \left[\frac{1}{7} x^7 \right]_0^2$$

$$= \frac{128}{7} \pi \; //$$

2. $y = 9 - x^2$ dan $y = 0$



$$V = \pi \int_{-3}^{3} (9 - x^2)^2 \, dx$$

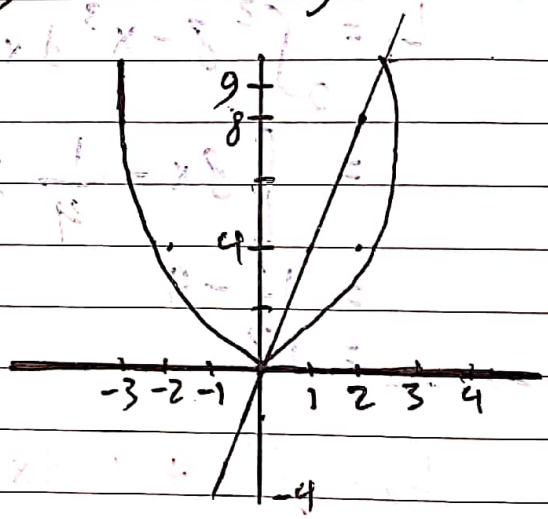
$$= \pi \int_{-3}^{3} x^4 - 18x^2 + 81$$

$$= \pi \left[\frac{1}{5} x^5 - 6x^3 + 81x \right]_{-3}^{3}$$

$$= \pi \left[\left(\frac{698}{5} \right) - \left(-\frac{698}{5} \right) \right]$$

$$= \frac{1296}{5} \pi \; //$$

3. $y = x^2$ dan $y = 4x$



$y_1 = y_2$

$x^2 = 4x$

$x^2 - 4x = 0 \rightarrow x = 0, x = 4$

$$V = \pi \int_0^4 ((4x)^2 - (x^2)^2) \, dx$$

$$= \pi \int_0^4 (16x^2 - x^4) \, dx$$

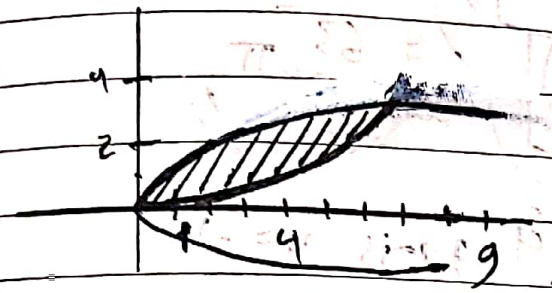
$$= \pi \left[\frac{16}{3} x^3 - \frac{1}{5} x^5 \right]_0^4$$

$$= \frac{2048}{15} \pi \; //$$

Dipindai dengan CamScanner

Date:

6. Daerah dibatasi kurva $y=\sqrt{x}$ dan garis $x=2y$, D diputar terhadap



1. Sumbu x

$y_1 = y_2$

$\sqrt{x} = \frac{1}{2}x$

$x = \frac{1}{4}x^2$

$\frac{1}{4}x^2 - x = 0$

$x^2 - 4x = 0$

$x = 0, x = 4$

$V = \pi \int_0^4 (\sqrt{x})^2 - \left(\frac{1}{2}x\right)^2$

$= \pi \int_0^4 x - \frac{1}{4}x^2$

$= \pi \left[\frac{1}{2}x^2 - \frac{1}{12}x^3\right]_0^4$

$= \pi \left[8 - \frac{16}{3}\right]$

$= \frac{8}{3}\pi$ //

4. Sumbu y

$x_1 = x_2$

$y^2 - 2y = 0$

$y = 0 \vee y = 2$

$V = \pi \int_0^2 (2y)^2 - (y^2)^2 \, dy$

$= \pi \int_0^2 4y^2 - y^4 \, dy$

$= \pi \left[\frac{4}{3}y^3 - \frac{1}{5}y^5\right]_0^2$

$= \pi \left[\frac{32}{3} - \frac{32}{5}\right]$

$= \frac{64}{15}\pi$ //

2. garis $x = -1$

$\Delta V = \pi \left((2y+1)^2 - (y^2+1)^2\right)$

$V = \pi \int_0^2 4y^2 - y^4 + 4y$

$= \pi \left[\frac{4}{5}y^3 - \frac{1}{5}y^5 + 2y^2\right]_0^2$

$= \frac{104}{15}\pi$ //

3. garis $y = 4$

$V = \pi \int_0^4 \left(4 - \frac{x}{2}\right)^2 - (4 - \sqrt{x})^2 \, dx$

$= \pi \int_0^4 \frac{x^2}{4} - 4x + 16 - x + 8\sqrt{x} - 16$

Dipindai dengan CamScanner

No.

Date:

$= \pi \int_0^4 \frac{x^2}{4} - 5x + 8\sqrt{x}$

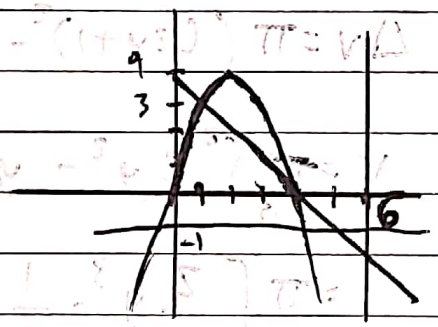
$= \pi \left[\frac{1}{12}x^3 - \frac{5}{2}x^2 + \frac{16}{3}x^{3/2} \right]_0^4$

$= \pi \left[\frac{16}{3} - 40 + \left(\frac{16}{3} \cdot 8 \right) \right]$

$= \pi \left[\frac{29}{3} \right]$

$= 8\pi$ //

D. Daerah dibatasi parabol $y = 4x - x^2$ dan garis $x = 4 - y$. Hitung volume benda putar, jika diputar terhadap



2. garis $x = 6$

$V = 2\pi \int_1^4 (6-x)(4x - x^2 - (4-x))\,dx$

$= 2\pi \int_1^4 (6-x)(5x - x^2 - 4)$

$= 2\pi \int_1^4 x^3 - 11x^2 + 34x - 24$

$= 2\pi \left[\frac{x^4}{4} - 11\frac{x^3}{3} + 39\frac{x^2}{2} - 24x \right]_1^4$

$= 2\pi \left[\frac{x^4}{4} - \frac{11x^3}{3} + 17x^2 - 24x \right]_1^4$

$= \frac{63}{2}\pi$ //

4. garis $y = -1$

$y_1 = y_2$

$4 - x = 4x - x^2$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1)$

$x = 4 \vee x = 1$

$V = \pi \int_0^4 (4x - x^2 + 1)^2 - (5-x)^2$

$= \pi \left[\frac{1}{5}x^5 - 2x^4 + \frac{14}{3}x^3 + 4x^2 + x + \frac{1}{3}(5-x)^3 \right]_1^4$

$= \left[\frac{897}{15} - \frac{438}{15} \right]$

$= \frac{153}{5}\pi$ //

Dipindai dengan CamScanner