No. :

Date :

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Slide : 58

1. SU PD

A $y'' + y = \operatorname{cosec}(x)$

- PD homogen
  $r^2 = -1$
  $r_1 = i$, $r_2 = -i$

- Pd homogen : $C_1 \cos x + C_2 \sin x$

$y_1 = 1 \cos x$
$y_2 = \sin x$

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x - (-\sin^2 x) = \cos^2 x + \sin^2 x = 1$$

$y_p = u y_1 + v y_2$

$$u = -\int \frac{\sin x \cdot \operatorname{cosec}(x)}{1} \, dx = -\int dx = -x$$

$$v = \int \cos x \operatorname{cosec}(x) \, dx = \int \cot(x) \, dx = \ln|\sin(x)|$$

$y_p = -x\cos(x) + \ln|\sin(x)| \sin(x)$

$y = C_1 \cos x + C_2 \sin x - x\cos(x) + \ln|\sin(x)| \sin(x)$

B $y'' - 4y' + 5y = \dfrac{2e^{2x}}{\sin x}$

- PD homogen
  $r^2 - 4r + 5 = 0$
  $$r_{12} = \frac{4 \pm \sqrt{16 - 4(1)(5)}}{2} = \frac{4 \pm \sqrt{-4}}{2}$$

$r_1 = 2 + i \rightarrow y_1 = e^{2x} \cos x$

$r_2 = 2 - i \quad y_2 = e^{2x} \sin x$

- Pd homogen : $e^{2x}(C_1 \cos x + C_2 \sin x)$

$$W = \begin{vmatrix} e^{2x} \cos x & e^{2x} \sin x \\ (2e^{2x} \cos x - e^{2x} \sin x) & (2e^{2x} \sin x + e^{2x} \cos x) \end{vmatrix}$$

No. :

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$2e^{4x}\sin x\cos x + e^{4x}\cos^2 x - 2e^{4x}\sin x\cos x + e^{4x}\sin^2 x$

$= e^{4x}(\sin^2 x + \cos^2 x)$

$= e^{4x}$

$y_p = u y_1 + v y_2$

$u = -\int \frac{e^{2x}\sin x \, 2e^{2x}}{e^{4x}\sin x}\,dx = -2\int 1 = -2x$

$v = \int \frac{e^{2x}\cos x \, 2e^{2x}}{e^{4x}\sin x}\,dx = 2\int \cot(x)\,dx = 2\ln|\sin(x)| + C$

$y = e^{2x}(C_1\cos x + C_2\sin x) - 2x e^{2x}\cos x + \ln|\sin(x)|\, 2e^{2x}\sin(x)$

c $y'' - 2y' + y = e^x \sin x$

• PK : $r^2 - 2r + 1 = 0$ • Pd homogen : $C_1 e^x + C_2 x e^x$

$(r-1)(r-1)$

$r_1 = 1 \quad r_2 = 1$

$y_1 = e^x \quad y_2 = xe^x$

$W = \begin{vmatrix} e^x & xe^x \\ e^x & (xe^x + e^x) \end{vmatrix} = xe^{2x} + e^{2x} - xe^{2x} = e^{2x}$

$y_p = u y_1 + v y_2$

$u = -\int \frac{(xe^x)(e^x \sin x)}{e^{2x}} = -\int x\sin x\,dx$

$u = x \Rightarrow -\left(-x\cos x + \int \cos x\,dx\right)$

$du = dx$

$dv = \sin x \Rightarrow -\left(-x\cos x + \sin x\right) \Rightarrow x\cos x - \sin x$

$v = -\cos x$

$v = \int \frac{(e^x)\, e^x \sin x}{e^{2x}} = -\cos x$

$y = C_1 e^x + C_2 xe^x + (x\cos x - \sin x)e^x + xe^x(-\cos x)$

$= C_1 e^x + C_2 xe^x - e^x \sin x$

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D) $y'' + y = \cot x$

• PK : $r^2 + 1 = 0$

$r = \pm\sqrt{-1}$

$r_1 = i \quad r_2 = -i \to y_1 = \cos x \quad , y_2 = \sin x$

• Pd homogen : $C_1 \cos x + C_2 \sin x$

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1$$

$y_p = v y_1 + V y_2$

$$v = -\int \frac{\sin x \cot x}{1} dx = -\int \sin x \frac{\cos x}{\sin x} dx = -\sin x$$

$$V = \int \frac{\cos x \cot x}{1} dx = \int \frac{\cos^2 x}{\sin x} dx = \int \frac{1 - \sin^2 x}{\sin x} dx - \int \sin x \, dx$$

$$= \ln\left|\tan\left(\frac{x}{2}\right)\right| + \cos x$$

$$y = C_1 \cos x + C_2 \sin x - \sin x \cos x + \left(\ln\left|\tan\left(\frac{x}{2}\right)\right| + \cos x\right) \sin x$$

$$= C_1 \cos x + C_2 \sin x + \ln\left|\tan\left(\frac{x}{2}\right)\right| \sin x$$