KALKULUS II

Tim dosen Matematika
FMIPA UNPAD

1

Materi Kalkulus II

- Integral Tak Wajar
- Barisan dan Deret
- Persamaan Diferensial Biasa
- Fungsi Bernilai Vektor
- Fungsi Dua Peubah
- Integral Lipat Dua

2

Pustaka

1. Purcell,"Kalkulus dan Geometri Analitis", jilid 2
2. Koko Martono,"Kalkulus"
3. Stewart,"Kalkulus",jilid 2
4. Kreyszig,"Advanced Engineering Mathematic"

3

kalkulus 2-Unpad

SISTEM PERKULIAHAN

- Kuliah di kelas (2x2 jam pelajaran)
- Tutorial / Responsi (1x2 jam)

EVALUASI

- QUIS
- TUGAS
- UTS
- UAS

Rentang Nilai :

$A \geq 80$

$68 \leq B < 80$

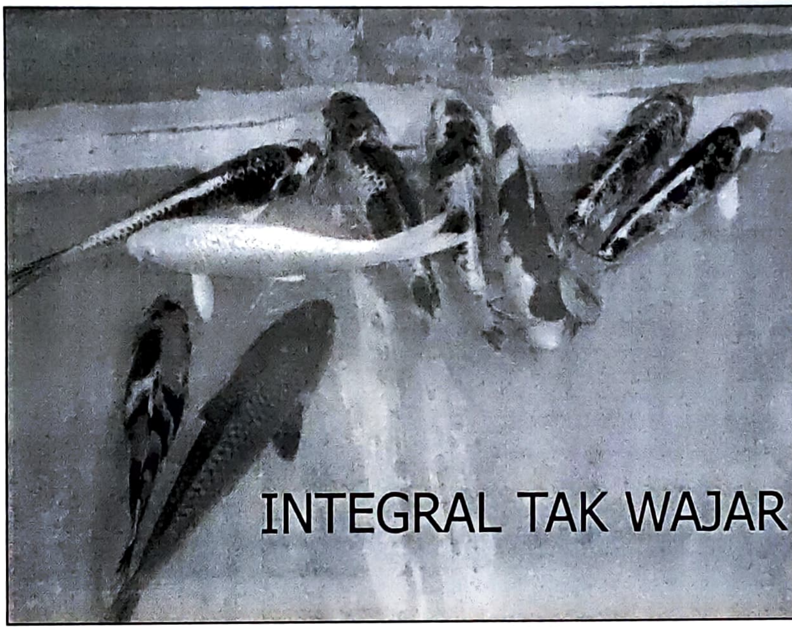
$56 \leq C < 68$

$45 \leq D < 56$

$E < 45$

4

kalkulus 2-Unpad



Integral Tak Wajar

Dalam mendefinisikan integral tentu $\int_a^b f(x)dx$ sebagai limit jumlah reiman ada dua syarat yang harus dipenuhi, yaitu :

a. Batas pengintegralan berhingga

b. Integran($f(x)$) kontinu pada selang pengintegralan

Jika paling kurang salah satu syarat diatas tidak dipenuhi maka integral itu disebut **integral tak wajar**

Jenis-jenis integral tak wajar

a. Integral tak wajar dengan batas pengintegralan tak hingga

b. Integral tak wajar dengan integran tak kontinu

Integral TAK wajar (Improper Integral)

Integral tentu $\int_a^b f(x)\,dx$ dikatakan integral tak wajar bila:

1. Fungsi $f(x)$ terdefinisi pada selang $(a, b]$, $[a, b)$ dan himpunan $[a, c) \cup (c, b]$.
2. Fungsi $f(x)$ terdefinisi pada selang $[a, +\infty)$; $(-\infty, \;\;)$; $(-\infty, +\infty)$

Integral tak wajar pada selang hingga

Ada 3 definisi dalam kasus ini :

Definisi I. jika fungsi $f(x)$ kontinu pada selang $(a, b]$ dan $|f(x)| \to \infty$ untuk $x \to a^+$, maka $\int_a^b f(x)\,dx$ didefinisikan sebagai

$$\int_a^b f(x)\,dx = \lim_{R \to a^+} \int_R^b f(x)\,dx .$$

jika limit di ruas kanan $= L$ (ada), maka $\int_a^b f(x)dx$ konvergen ke L dan jika limit di ruas kanan tak ada atau ∞, maka $\int_a^b f(x)\,dx$ divergen.

Contoh Hitunglah :

1. $\int_0^1 \frac{1}{\sqrt{x}}\,dx$

Jawab: fungsi $f(x) = \frac{1}{\sqrt{x}}$ terdefinisi pada selang $(0, 1]$

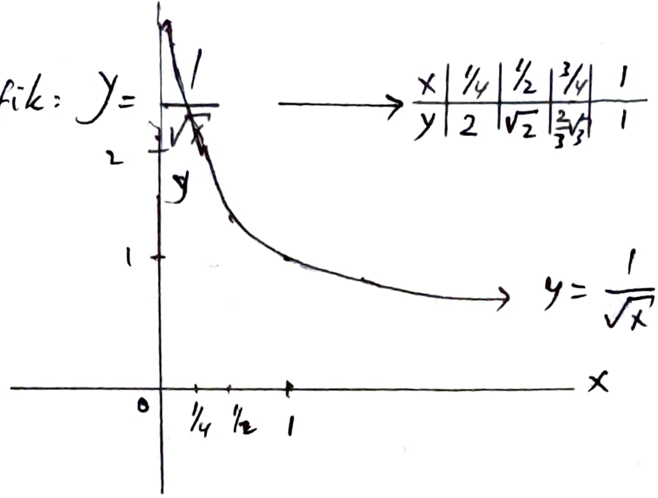
$$\int_0^1 \frac{1}{\sqrt{x}}\,dx = \lim_{R \to 0^+} \int_R^1 \frac{1}{\sqrt{x}}\,dx = \lim_{R \to 0^+} 2\sqrt{x}\Big|_R^1$$

$$= \lim_{R \to 0^+} \left(2\sqrt{1} - 2\sqrt{R}\right) = 2 - 2\sqrt{0} = 2$$

Jadi $\int_0^1 \frac{1}{\sqrt{x}}\,dx$ konvergen ke 2.

Grafik: $y = \frac{1}{\sqrt{x}}$ $\longrightarrow$

x	$1/4$	$1/2$	$3/4$	1
y	2	$\sqrt{2}$	$\frac{2}{3}\sqrt{3}$	1



Contoh 2 $\displaystyle\int_1^2 \frac{1}{x-1}\,dx$

Jawab: Fungsi $f(x) = \frac{1}{x-1}$ terdefinisi pada selang $(1, 2]$

$$\int_{+1}^{2} \frac{1}{x-1}\,dx = \lim_{R \to 1^+} \int_R^2 \frac{1}{x-1}\,dx = \lim_{R \to 1^+} \ln|x-1| \Big|_R^2$$

$$= \lim_{R \to 1^+} \left(\ln|2-1| - \ln|R-1| \right)$$

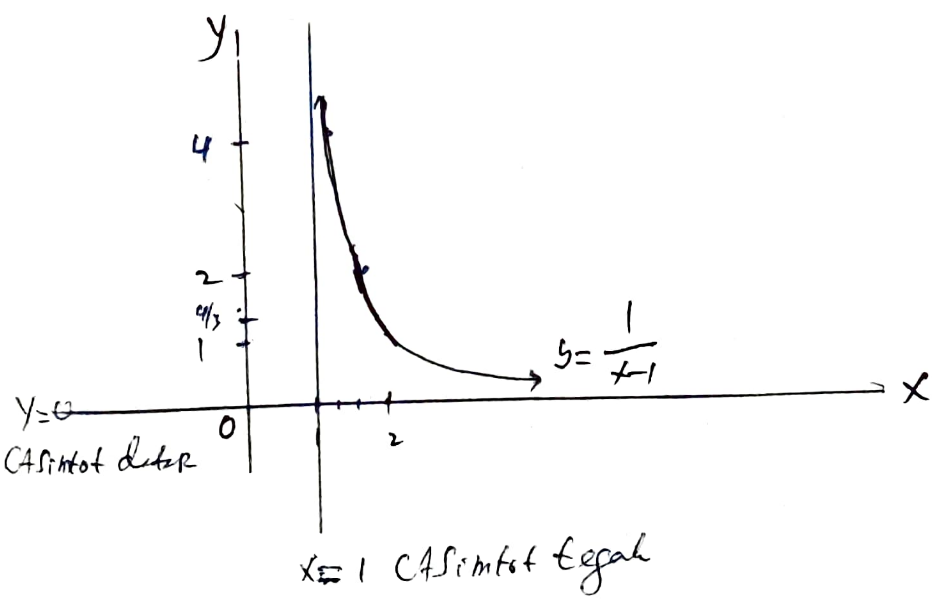
$$= \lim_{R \to 1^+} \left(0 - \ln|R-1| \right) = -\ln|1-1|$$

$$= -(-\infty) = \infty$$

Jadi $\displaystyle\int_1^2 \frac{1}{x-1}\,dx$ divergen.

Grafik. $y = \frac{1}{x-1}$ $\longrightarrow$

x	$5/4$	$3/2$	$7/4$	2
y	4	2	$4/3$	1



Contoh 3. $\int_1^5 \frac{dx}{\sqrt{x-1}}$

Jawab: Fungsi $f(x) = \frac{1}{\sqrt{x-1}}$ terdefinisi pada selang $(1, 5]$

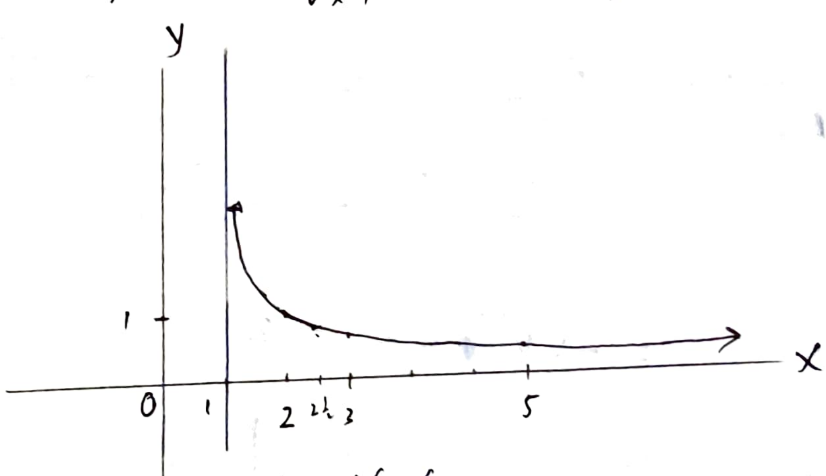
$$\int_1^5 \frac{dx}{\sqrt{x-1}} = \lim_{R \to 1^+} \int_R^5 \frac{dx}{\sqrt{x-1}} = \lim_{R \to 1^+} 2\sqrt{x-1}\Big|_R^5$$

$$= \lim_{R \to 1^+} \left(2\sqrt{5-1} - 2\sqrt{R-1}\right) = 4 - 2\sqrt{1-1} = \underline{\underline{4}}$$

$\therefore \int_1^5 \frac{dx}{\sqrt{x-1}}$ konvergen ke 4.

Grafik $y = \frac{1}{\sqrt{x-1}} \longrightarrow$

x	2	$2\frac{1}{2}$	3	5
y	1	$\frac{\sqrt{6}}{3}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$



$x = 1$ asimtot tegak

Soal: Hitunglah:

1. $\int_0^2 x^{-\frac{2}{5}}\,dx$
2. $\int_0^1 x^{-\frac{4}{3}}\,dx$
3. $\int_1^3 \frac{1}{\sqrt{x-1}}\,dx$
4. $\int_5^{10} \frac{2}{\sqrt{x-5}}\,dx$
5. $\int_0^1 \frac{2}{x\sqrt{1-x^2}}\,dx$

× 6. $\int_0^1 \frac{dx}{1-x}$

7. $\int_1^2 \frac{dx}{(x-1)^{1/3}}$

8. $\int_0^1 \frac{\ln x}{x}\,dx$

9. $\int_3^7 \frac{dx}{\sqrt{x-3}}$

10. $\int_1^4 \frac{dx}{\sqrt{x-1}}$

11. $\int_0^{16} \frac{1}{\sqrt[4]{x}}$

12.

Definisi II Jika fungsi $f(x)$ kontinu pada interval $[a, b)$ dan $|f(x)| \to \infty$ untuk $x \to b^-$, maka $\int_a^b f(x)\,dx$ didefinisikan sebagai $\int_a^b f(x)\,dx = \lim_{R \to b^-} \int_a^R f(x)\,dx$

Jika limit ini ada (L), maka $\int_a^b f(x)\,dx$ dikatakan konvergen ke L. Jika limit ini tak ada (∞), maka $\int_a^b f(x)\,dx$ dikatakan divergen.

Contoh Hitunglah:

1. $\int_0^1 \frac{1}{\sqrt{1-x}}\,dx$

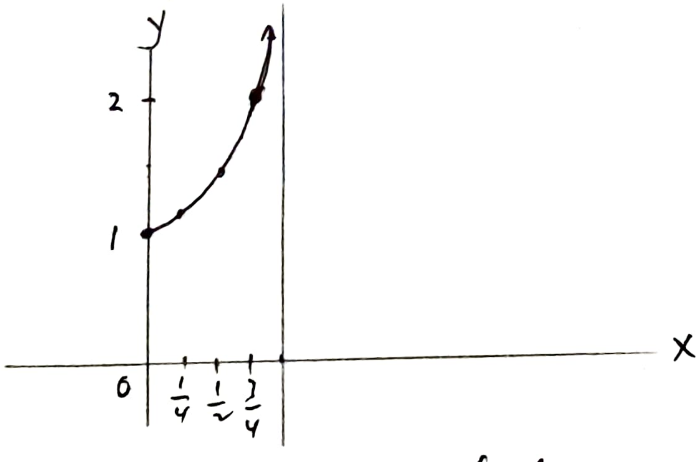
Jawab: $\int_0^1 \frac{1}{\sqrt{1-x}}\,dx = \lim_{R \to 1^-} \int_0^R \frac{1}{\sqrt{1-x}}\,dx = \lim_{R \to 1^-} -2\sqrt{1-x}\Big|_0^R$

$= \lim_{R \to 1^-} \left\{-2\sqrt{1-R} - (-2\sqrt{1-0})\right\}$

$= -2\sqrt{1-1} + 2 = 2$

Grafik: $y = \frac{1}{\sqrt{1-x}}$ terdefinisi pada interval $[0, 1)$

x	0	1/4	1/2	3/4
y	1	$\frac{2}{3}\sqrt{3}$	$\sqrt{2}$	2



$x = 1$ asimtot tegak

2. $\int_{-1}^{0} \frac{1}{x^2}\, dx$

Jawab: fungsi $f(x) = \frac{1}{x^2}$ terdefinisi pada interval $[-1, 0)$

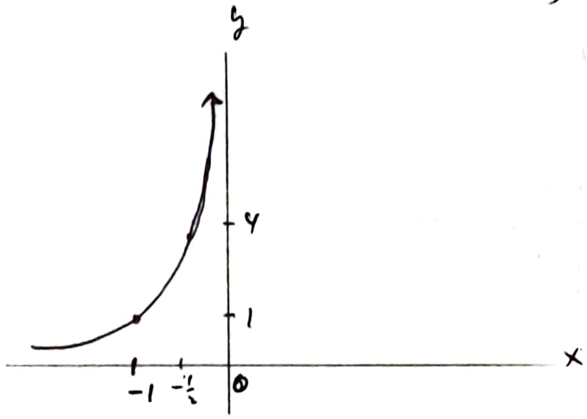
$$\int_{-1}^{0} \frac{1}{x^2}\, dx = \lim_{R \to \bar{0}} \int_{-1}^{R} \frac{1}{x^2}\, dx = \lim_{R \to \bar{0}} -\frac{1}{x} \Big|_{-1}^{R}$$

$$= \lim_{R \to \bar{0}} \left\{ -\frac{1}{R} - \left(-\frac{1}{-1} \right) \right\} = \lim_{R \to \bar{0}} \left(-\frac{1}{R} - 1 \right)$$

$$= -(-\infty) = \infty$$

Grafik: $y = \frac{1}{x^2}$ $\longrightarrow$

x	-1	$-1/2$	$-1/4$
y	1	4	16



Contoh 3. $\int_{0}^{1} \frac{dx}{1-x}$

Jawab: Fungsi $f(x)$ terdefinisi pada selang $[0, 1)$

$$\int_{0}^{1} \frac{dx}{1-x} = \lim_{R \to \bar{1}} \int_{0}^{R} \frac{dx}{1-x} = \lim_{R \to \bar{1}} -\ln|1-x| \Big|_{0}^{R}$$

$$= \lim_{R \to \bar{1}} \left\{ -\ln|1-R| - (-\ln|1-0|) \right\}$$

$$= \lim_{R \to \bar{1}} \left(-\ln|1-R| + 0 \right) = -\ln|1-1| = -\ln 0$$

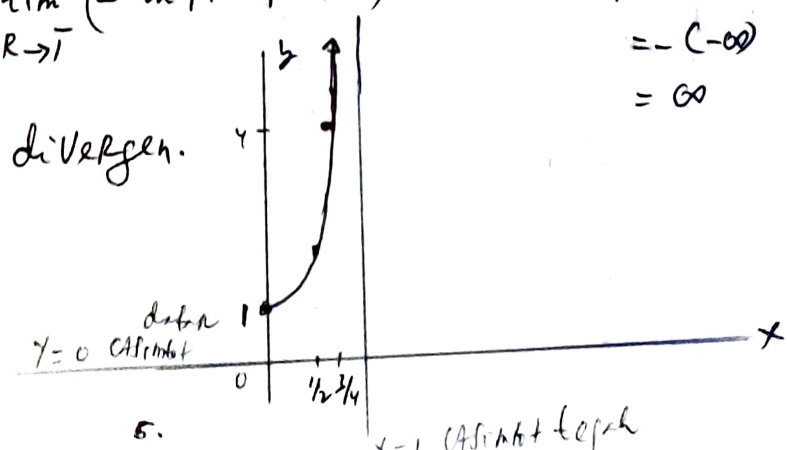
$$= -(-\infty)$$

$$= \infty$$

Jadi $\int_{0}^{1} \frac{dx}{1-x}$ divergen.

Grafik: $y = \frac{1}{1-x}$

x	0	1/2	3/4
y	1	2	4



Contoh 4. $\int_0^1 \frac{2}{\sqrt{1-x^2}} dx$

Rumus: $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\frac{x}{a}$

Jawab: Fungsi f(x) terdefinisi pada interval [0, 1)

$$\int_0^1 \frac{2}{\sqrt{1-x^2}} dx = \lim_{R \to 1^-} \int_0^R \frac{2}{\sqrt{1-x^2}} dx$$

$$= \lim_{R \to 1^-} 2 . \sin^{-1}\frac{x}{1} \Big|_0^R$$

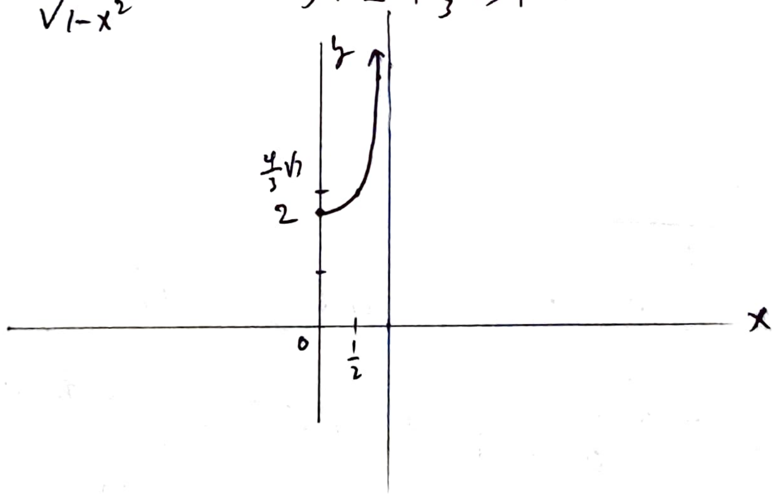
$$= 2 . \lim_{R \to 1^-} \left(\sin^{-1} R - \sin^{-1} 0\right)$$

$$= 2\left(\sin^{-1} 1 - 0\right) = 2\left(\frac{\pi}{2}\right) = \underline{\underline{\pi}}$$

$\therefore \int_0^1 \frac{2}{\sqrt{1-x^2}} dx$ konvergen ke π.

$y = \frac{2}{\sqrt{1-x^2}} \longrightarrow$

x	0	$\frac{1}{2}$	3/4
y	2	$\frac{4}{3}\sqrt{3}$	4



x = 1 Asimtot tegak

Soal: Hitunglah:

1. $\int_{-1}^{2} \frac{dx}{(x-2)^2}$
2. $\int_{-2}^{-1} \frac{dx}{(x+1)^{4/3}}$
3. $\int_0^9 \frac{dx}{\sqrt{9-x}}$
4. $\int_0^3 \frac{x}{9-x^2} dx$
5. $\int_0^2 \frac{dx}{\sqrt{4-x^2}}$
6. $\int_0^3 \frac{x}{\sqrt{9-x^2}} dx$
7.

$\frac{1}{x}$ Definisi III Misalkan fungsi f kontinu pada interval $[a,b]$ kecuali di $c \in (a,b)$ dan $|f(x)| \to \infty$ untuk $x \to c$.

Integral $\int_a^b f(x)\,dx$ adalah integral tak wajar dan ditulis sebagai

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

Jika kedua integral $\int_a^c f(x)\,dx$ dan $\int_c^b f(x)\,dx$ konvergen masing-masing ke L_1 dan L_2, maka integral tak wajar $\int_a^b f(x)\,dx$ konvergen ke (L_1+L_2). Jika salah satu dari integral tak wajar $\int_a^c f(x)\,dx$ atau $\int_c^b f(x)\,dx$ divergen maka integral tak wajar $\int_a^b f(x)\,dx$ juga divergen.

Contoh 1. Tentukan apakah integral tak wajar $\int_{-1}^{2} \frac{1}{x^2}\,dx$ konvergen atau divergen?

Jawab:

$$\int_{-1}^{2} \frac{1}{x^2}\,dx = \int_{-1}^{0} \frac{1}{x^2}\,dx + \int_{0}^{2} \frac{1}{x^2}\,dx$$

$$\int_{-1}^{0} \frac{1}{x^2}\,dx = \lim_{R \to 0^-} \int_{-1}^{R} \frac{1}{x^2}\,dx = \lim_{R \to 0^-} \left(-x^{-1}\right)\Big|_{-1}^{R}$$

$$= \lim_{R \to 0^-} \left(-\frac{1}{x}\right)\Big|_{-1}^{R}$$

$$= \lim_{R \to 0^-} \left(-\frac{1}{R} + \frac{1}{-1}\right)$$

$$= \infty$$

Karena $\int_{-1}^{0} \frac{1}{x^2} dx$ divergen, maka $\int_{-1}^{2} \frac{1}{x^2} dx$ juga divergen

Contoh 2 Hitunglah $\int_{0}^{2} \frac{x}{1-x} dx$

Jawab :

$$\int_{0}^{2} \frac{x}{1-x} dx = \int_{0}^{1} \frac{x}{1-x} dx + \int_{1}^{2} \frac{x}{1-x} dx$$

$$\int_{0}^{1} \frac{x}{1-x} dx = \lim_{R \to 1^-} \int_{0}^{R} \frac{x}{1-x} dx = \lim_{R \to 1^-} \int_{0}^{R} \left(-1 + \frac{1}{1-x}\right) dx$$

$$= \lim_{R \to 1^-} \left(-x - \ln|1-x|\right)\Big|_0^R$$

$$= \lim_{R \to 1^-} \left[-R - \ln|1-R| - \{-0 - \ln|1-0|\}\right]$$

$$= \lim_{R \to 1^-} \left(-R - \ln|1-R|\right)$$

$$= -1 - \ln|1-1|$$

$$= -1 - \ln 0 = \underline{\underline{\infty}}$$

$\therefore \int_{0}^{2} \frac{x}{1-x} dx$ divergen.

Soal 2: Tentukan apakah integral tak wajar berikut konvergen atau divergen!

1. $\int_{-1}^{5} \frac{dx}{(x-2)^2}$
2. $\int_{-1}^{1} \frac{dx}{x^2}$
3. $\int_{-2}^{2} \frac{3}{x} dx$
4. $\int_{0}^{3} \frac{2}{x^2-1} dx$
5. $\int_{5}^{10} \frac{2}{\sqrt{x-6}} dx$
6. $\int_{0}^{2} \frac{x}{x^2-1} dx$
7. $\int_{-3}^{2} \frac{1}{x^4} dx$
8. $\int_{2}^{4} \frac{dx}{(3-x)^{2/3}}$
9. $\int_{-3}^{0} \frac{x}{(x^2-4)^{2/3}} dx$
10. $\int_{3}^{7} \frac{dx}{\sqrt{x-4}}$
11. $\int_{-2}^{0} \frac{dx}{2x+3}$
12. $\int_{3}^{5} \frac{dx}{(4-x)^{2/3}}$
13. $\int_{\frac{1}{2}}^{2} \frac{dx}{x(\ln x)^{1/5}}$

Integral Tak Wajar Pada selang tak hingga

Definisi I : Jika fungsi f kontinu pada interval $[a, +\infty)$ maka integral tak wajar $\int_a^\infty f(x)\,dx$ didefinisikan sebagai: $\int_a^\infty f(x)\,dx = \lim_{R \to \infty} \int_a^R f(x)\,dx$

Jika limit di ruas kanan ada (L), maka $\int_a^\infty f(x)\,dx$ dikatakan konvergen ke L. Jika limit di ruas kanan tak ada (∞), maka $\int_a^\infty f(x)\,dx$ dikatakan divergen.

Contoh Hitunglah :

1. $\int_1^\infty \frac{1}{x^2}\,dx$

Jawab:

$$\int_1^\infty \frac{1}{x^2}\,dx = \lim_{R \to \infty} \int_1^R \frac{1}{x^2}\,dx = \lim_{R \to \infty} -\frac{1}{x}\Big|_1^R$$

$$= \lim_{R \to \infty} \left(-\frac{1}{R} - \left(-\frac{1}{1}\right)\right)$$

$$= -\frac{1}{\infty} + 1 = -0 + 1 = \underline{\underline{1}}$$

$\therefore \int_1^\infty \frac{1}{x^2}\,dx$ konvergen ke 1.

2. $\int_1^\infty \frac{1}{\sqrt{x}}\,dx$

Jawab:

$$\int_1^\infty \frac{1}{\sqrt{x}}\,dx = \lim_{R \to \infty} \int_1^R x^{-1/2}\,dx = \lim_{R \to \infty} 2\sqrt{x}\Big|_1^R$$

$$= \lim_{R \to \infty} \left(2\sqrt{R} - 2\sqrt{1}\right)$$

$$= 2\sqrt{\infty} - 2 = \underline{\underline{\infty}}$$

$\therefore \int_1^\infty \frac{1}{\sqrt{x}}\,dx$ divergen.

Soal: Hitunglah:

1. $\int_2^\infty \frac{3}{x}\,dx$
2. $\int_0^\infty x^{2/5}\,dx$
3. $\int_1^\infty x^{-\frac{1}{3}}\,dx$
4. $\int_1^\infty x^{-4/3}\,dx$
5. $\int_0^\infty t\,e^{t}\,dx$
6. $\int_1^\infty x^{-\frac{4}{5}}\,dx$
7. $\int_1^\infty x^{-\frac{6}{5}}\,dx$
8. $\int_0^\infty x\,e^{-2x}\,dx$
9. $\int_0^\infty \frac{1}{(x-2)^2}\,dx$
10. $\int_0^\infty \frac{1}{\sqrt{x}\,.\,e^{\sqrt{x}}}\,dx$
11. $\int_0^\infty \cos x\,dx$
12. $\int_0^\infty \tan x\,dx$
13. $\int_0^\infty \cos x\,.\,e^{-\sin x}\,dx$
14. $\int_1^\infty \frac{x}{1+x^3}\,dx$
15. $\int_2^\infty \frac{x}{x^{3/2}-1}\,dx$
16. $\int_0^\infty \frac{3}{x+e^x}\,dx$
17. $\int_0^\infty \frac{\sin^2 x}{1+e^x}\,dx$
18. $\int_2^\infty \frac{x^2 e^x}{\ln x}\,dx$
19. $\int_1^\infty \frac{x^2-2}{x^4+3}\,dx$
20. $\int_1^\infty \frac{2+\sec^2 x}{x}\,dx$
21. $\int_1^\infty e^{-x^3}\,dx$
22. $\int_2^\infty \frac{\ln x}{e^x+1}\,dx$
23. $\int_1^\infty e^{x^2+x+1}\,dx$
24. $\int_0^\infty x\,e^{-2x}\,dx$
25. $\int_m^\infty n\,.\,e^{-ax}\,dx = \frac{1}{2}$
26. $\int_0^\infty \frac{1}{1+x^2}\,dx$
27. $\int_1^\infty e^{x}\,dx$
28. $\int_4^\infty x\,e^{-x^2}\,dx$
29. $\int_3^\infty \frac{x\,dx}{\sqrt{9+x^2}}$
30. $\int_1^\infty \frac{dx}{x^{1,09}}$
31. $\int_2^\infty \frac{dx}{x\ln x}$
32. $\int_2^\infty \frac{dx}{x(\ln x)^2}$
33. $\int_1^\infty \frac{dx}{\sqrt{3x}}$
34. $\int_2^\infty \frac{x}{1+x^2}\,dx$
35. $\int_2^\infty \frac{x}{(1+x^2)^2}\,dx$
36. $\int_1^\infty \frac{\ln x}{x}\,dx$
37. $\int_0^\infty x\,e^{-x}\,dx$
38. $\int_2^\infty \frac{dx}{(1-x)^{2/3}}$
39. $\int_0^\infty e^{-x}\cos x\,dx$
40. $\int_0^\infty e^{-x}\sin x\,dx$
41. $\int_1^\infty \cosh x\,dx$
42. $\int_1^\infty \frac{dx}{x^2+x^4}$
43. $\int_2^\infty x\,e^{-x^2}\,dx$
44. $\int_0^\infty \frac{dx}{e^{x/2}}$
45. $\int_3^\infty \frac{\ln x}{x}\,dx$

∞

($\int \operatorname{sech} x\,dx = \tan^{-1}(\sinh x) + C$

Definisi II Jika fungsi f kontinu pada interval $(-\infty, a]$ maka integral tak wajar $\int_{-\infty}^{a} f(x)\,dx$ didefinisikan sbg

$$\int_{-\infty}^{a} f(x)\,dx = \lim_{R \to -\infty} \int_{R}^{a} f(x)\,dx$$

Jika limit diruas kanan ada (L), maka $\int_{-\infty}^{a} f(x)\,dx$ konvergen ke L, jika tidak ada (∞), maka $\int_{-\infty}^{a} f(x)\,dx$ divergen.

Contoh Hitunglah:

1. $\int_{-\infty}^{-1} \frac{1}{x}\,dx$

Jawab: $$\int_{-\infty}^{-1} \frac{1}{x}\,dx = \lim_{R \to -\infty} \int_{R}^{-1} \frac{1}{x}\,dx = \lim_{R \to -\infty} \ln|x| \Big|_{R}^{-1}$$

$$= \lim_{R \to -\infty} \left(\ln|-1| - \ln|R|\right) = \ln 1 - \ln|-\infty|$$

$$= 0 - \ln\infty = \underline{\underline{-\infty}}$$

$\therefore \int_{-\infty}^{-1} \frac{1}{x}\,dx$ divergen.

2. $\int_{-\infty}^{0} \frac{1}{(x-2)^2}\,dx$

Jawab:

$$\int_{-\infty}^{0} \frac{1}{(x-2)^2}\,dx = \lim_{R \to -\infty} \int_{R}^{0} \frac{1}{(x-2)^2}\,dx = \lim_{R \to -\infty} -(x-2)^{-1} \Big|_{R}^{0}$$

Misal $\frac{1}{(x-2)^{+2}} = (x-2)^{-2}$

$$= \lim_{R \to -\infty} -\frac{1}{x-2} \Big|_{R}^{0}$$

$$= \lim_{R \to -\infty} \left(-\frac{1}{-2} + \frac{1}{R-2}\right)$$

$$= \underline{\underline{\frac{1}{2}}}$$

$\therefore \int_{-\infty}^{0} \frac{1}{(x-2)^2}\,dx$ konvergen ke $\frac{1}{2}$

Defini

Soal : Hitunglah:

1. $\int_{-\infty}^{-1} \frac{3}{x}\, dx$

2. $\int_{-\infty}^{0} x e^{x}\, dx$

3. $\int_{-\infty}^{0} x e^{-2x}\, dx$

4. $\int_{-\infty}^{0} \frac{1}{(x-2)^2}\, dx$

5. $\int_{-\infty}^{-1} x e^{-x^2}\, dx$

6. $\int_{-\infty}^{0} \frac{dx}{(2x-1)^3}$

7. $\int_{-\infty}^{-2} \frac{dx}{x^5}$

8. $\int_{-\infty}^{0} e^{3x}\, dx$

9. $\int_{-\infty}^{1} \frac{dx}{(2-x)^2}$

10.

Definisi III Jika fungsi f kontinu pada interval $(-\infty, \infty)$ tulis $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{a} f(x)\,dx + \int_{a}^{\infty} f(x)\,dx$

untuk konstanta a, dimana $\int_{-\infty}^{\infty} f(x)\,dx$ konvergen jika dan hanya jika kedua integral tak wajar $\int_{-\infty}^{a} f(x)\,dx$ dan $\int_{a}^{\infty} f(x)\,dx$ konvergen.

Jika salah satunya divergen, maka $\int_{-\infty}^{\infty} f(x)\,dx$ divergen.

Contoh 1. Hitunglah $\int_{-\infty}^{\infty} x e^{-x^2}\,dx$

Jawab:

$$\int_{-\infty}^{\infty} x e^{-x^2}\,dx = \int_{-\infty}^{0} x e^{-x^2}\,dx + \int_{0}^{\infty} x e^{-x^2}\,dx$$

$$\int_{-\infty}^{0} x e^{-x^2}\,dx = \lim_{R\to-\infty} \int_{R}^{0} x e^{-x^2}\,dx$$

misal $u = -x^2 \to du = -2x\,dx \to x\,dx = -\frac{1}{2}\,du$

$$\int_{-\infty}^{0} x e^{-x^2}\,dx = \cancel{\int_{-\infty}^{0}} \lim_{R\to-\infty} \int_{R}^{0} e^{u} \cdot (-\tfrac{1}{2})\,du$$

$$= \lim_{R\to-\infty} -\frac{1}{2} e^{-x^2} \Big|_{R}^{0}$$

$$= -\frac{1}{2} \lim_{R\to-\infty} (e^{0} - e^{-R^2}) = -\frac{1}{2} \lim_{R\to-\infty} (1 - e^{-R^2})$$

$$= -\frac{1}{2}(1 - e^{-(-\infty)^2}) = -\frac{1}{2}(1 - e^{-\infty})$$

$$= -\frac{1}{2}(1 - 0) = -\frac{1}{2}$$

$$\int_{-\infty}^{0} x e^{-x^2}\,dx = -\frac{1}{2}$$

$$\int_0^{\infty} x e^{-x^2} dx = \lim_{R\to\infty} \int_0^R x e^{-x^2} dx$$

$$= \lim_{R\to\infty} -\frac{1}{2} e^{-x^2} \Big|_0^R = -\frac{1}{2} \lim_{R\to\infty} \left(e^{-R^2} - e^0\right)$$

$$= -\frac{1}{2}\left(e^{-\infty^2} - 1\right) = -\frac{1}{2}(0-1) = \frac{1}{2}$$

Karena $\int_{-\infty}^{0} x e^{-x^2} dx$ dan $\int_0^{\infty} x e^{-x^2} dx$ konvergen

maka $\int_{-\infty}^{\infty} x e^{-x^2} dx$ konvergen.

$$\therefore \int_{-\infty}^{\infty} x e^{-x^2} dx = \int_{-\infty}^{0} x e^{-x^2} dx + \int_0^{\infty} x e^{-x^2} dx =$$

$$= -\frac{1}{2} + \frac{1}{2} = \underline{\underline{0}}$$

<u>Contoh2 Hitunglah</u>

$$\int_{-\infty}^{\infty} e^{-x} dx$$

<u>Jawab:</u> $\int_{-\infty}^{\infty} e^{-x} dx = \int_{-\infty}^{0} e^{-x} dx + \int_0^{\infty} e^{-x} dx$

$$\int_{-\infty}^{0} e^{-x} dx = \lim_{R\to-\infty} \int_R^0 e^{-x} dx = \lim_{R\to-\infty} -e^{-x} \Big|_R^0$$

$$= \lim_{R\to-\infty} \left\{-e^{-0} - (-e^{-R})\right\} = \lim_{R\to-\infty} \left(-1 + e^{-R}\right)$$

$$= -1 + e^{-(-\infty)} = \infty$$

$$\int_0^{\infty} e^{-x} dx = \lim_{R\to\infty} \int_0^R e^{-x} dx = \lim_{R\to\infty} -e^{-x}\Big|_0^R = \lim_{R\to\infty} \left\{-e^{-R} - (-e^{-0})\right\}$$

$$= \lim_{R\to\infty} \left(-e^{-R} + 1\right) = -e^{-\infty} + 1 = 0 + 1 = 1$$

Karena salah satu dari integral tak wajar tsb divergen maka $\int_{-\infty}^{\infty} e^{-x}\,dx$ divergen.

$$\therefore \int_{-\infty}^{\infty} e^{-x}\,dx = \int_{-\infty}^{0} e^{-x}\,dx + \int_{0}^{\infty} e^{-x}\,dx = \infty + 1 = \underline{\underline{\infty}}$$

Soal Hitunglah:

1. $\int_{-\infty}^{\infty} \frac{dx}{x^2+2x+5}$
2. $\int_{-\infty}^{\infty} \frac{1}{x^2}\,dx$
3. $\int_{-\infty}^{\infty} e^{-2x}\,dx$
4. $\int_{-\infty}^{\infty} \frac{1}{\sqrt[3]{x}}\,dx$
5. $\int_{-\infty}^{\infty} e^{3x}\,dx$
6. $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$
7. $\int_{-\infty}^{\infty} \frac{1}{x^2-1}\,dx$
8. $\int_{-\infty}^{\infty} x^3\,dx$
9. $\int_{-\infty}^{\infty} \frac{x}{1-x^2}\,dx$
10. $\int_{-\infty}^{\infty} \frac{x}{\sqrt{x^2+4}}\,dx$
11. $\int_{-\infty}^{\infty} \frac{1}{x^2+2x+5}\,dx$
12. $\int_{-\infty}^{\infty} \operatorname{sech} x\,dx$
13. $\int_{-\infty}^{\infty} \frac{x}{(x^2+4)^2}\,dx$
14. $\int_{-\infty}^{\infty} \frac{x}{e^{|x|}}\,dx$
15. $\int_{-\infty}^{\infty} \frac{x}{x^2+1}\,dx$
16. $\int_{-\infty}^{\infty} \frac{x}{1+x^4}\,dx$