Nama: Prames Ray Lapian
NPM : 140810210059

1. Periksa kekonvergenan dari barisan berikut :

   a. $$a_n = \frac{3n^2 + 4n - 1}{4n - 2n^2 + 4}$$

   Answer:

   lim n →∞ (a_n) = $\lim_{n\to\infty} \frac{3n^2+4n-1}{4n-2n^2+4} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{3+\frac{4}{n}-\frac{1}{n^2}}{\frac{4}{n}-2+\frac{4}{n^2}} = \frac{3}{-2}$

   Karena hasilnya **-1,5**, maka $\frac{3n^2+4n-1}{4n-2n^2+4}$ konvergen ke **-1,5**

   b. $$a_n = \frac{\sqrt{n}}{4n - 5}$$

   Answer:

   lim n →∞ (a_n) = $\lim_{n\to\infty} \frac{\sqrt{n}}{4n-5} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{\frac{1}{\sqrt{n}}}{4-\frac{5}{n}} = \frac{0}{4} = 0$

   Karena hasilnya **0**, maka $\lim_{n\to\infty} \frac{\sqrt{n}}{4n-5}$ konvergen ke **0**

2. Selidiki kekonvergenan deret berikut :

   a. $a_n = \frac{2}{\sqrt{3n+1}}$

   lim n →∞ (a_n) = $\lim_{n\to\infty} \frac{2}{\sqrt{3n+1}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\frac{2}{\sqrt{n}}}{\sqrt{3+\frac{1}{n}}} = \frac{0}{\sqrt{3}} = 0$

   Karena hasilnya **0**, maka $\sum_{n=0}^{\infty} \frac{2}{\sqrt{3n+1}}$ konvergen

   b. $a_n = \left(1 - \frac{1}{n}\right)^n$

   a = $\lim_{n\to\infty} (a_n)^{\frac{1}{2}} = \lim_{n\to\infty} (\left(1-\frac{1}{n}\right)^n)^{\frac{1}{n}} = \lim_{n\to\infty} 1 - \frac{1}{n} = 1$

   Karena hasilnya 1, maka tidak ada kesimpulan menggunakan uji akar.

   lim n →∞ (a_n) = $\lim_{n\to\infty} (1-\frac{1}{n})^n = \lim_{n\to\infty} e^{n\ln(1-\frac{1}{n})}$

   karena e kontinu pada interval [1, ∞), maka persamaan dapat ditulis ulang sebagai berikut:

   $$= e \lim_{n\to\infty} n.\ln\left(1-\frac{1}{n}\right) = e \lim_{n\to\infty} \ln\left(1-\frac{1}{n}\right) + n.\left(\frac{1}{n(n-1)}\right)$$

   $$= e \lim_{n\to\infty} \ln\left(1-\frac{1}{n}\right) + \lim_{n\to\infty} \frac{1}{n-1} = e^{0+0} = 1$$

   Karena hasilnya **1** =/= **0**, maka deret $\sum_{n=0}^{\infty} \left(1-\frac{1}{n}\right)^n$ divergen

3. Selidiki kekonvergenan deret berikut dengan uji hasil bagi :

Nama: Prames Ray Lapian
NPM : 140810210059

a. Misal $a_n = \frac{4^n+n}{n!}$ dan $a_{n+1} = \frac{4\,.4^n+n+1}{(n+1)!}$

Sehingga:

$$\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$$

$$\rho = \lim_{n\to\infty} \frac{4\,.\,4^n + n + 1}{(n+1)!} \cdot \frac{n!}{4^n + n}$$

$$\rho = \lim_{n\to\infty} \frac{4\,.\,4^n + n + 1}{n+1} \cdot \frac{1}{4^n + n}$$

$$\rho = \lim_{n\to\infty} \frac{4\,.\,4^n + n + 1}{n\,.\,4^n + n^2 + 4^n + n}$$

(penyebut memiliki pangkat n yang lebih tinggi daripada pembilang) = 0

Karena ρ = 0 < 1, maka $\sum_{n=1}^{\infty} \frac{4^n+n}{n!}$ Konvergen

b. Misal $a_n = \frac{n^3}{(2n)!}$ dan $a_{n+1} = \frac{(n+1)^3}{(2n+2)!}$

Sehingga:

$$\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$$

$$\rho = \lim_{n\to\infty} \frac{(n+1)^3}{(2n+2)!} \cdot \frac{(2n)!}{n^3}$$

$$\rho = \lim_{n\to\infty} \frac{(n+1)^3}{(2n+2)(2n+1)} \cdot \frac{1}{n^3}$$

(penyebut memiliki pangkat n yang lebih tinggi daripada pembilang) = 0

Karena ρ = 0 < 1, maka $\sum_{n=1}^{\infty} \frac{n^3}{(2n)!}$ Konvergen

4. Selidiki kekonvergenan deret berikut dengan uji akar :

a. $a_n = \left(1 - \frac{1}{n}\right)^n$

a = $\lim_{n\to\infty} (a_n)^{\frac{1}{2}}$

a = $\lim_{n\to\infty} \left(\left(\frac{3n+5}{n-1}\right)^n\right)^{\frac{1}{n}}$

a = $\lim_{n\to\infty} \frac{3n+5}{n-1} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}$

Nama: Prames Ray Lapian
NPM : 140810210059

a = $\lim_{n\to\infty} \frac{3+\frac{5}{n}}{1-\frac{1}{n}}$

a = 3

Karena a = 3 > 1, maka $\sum_{n=1}^{\infty}(\frac{3n+5}{n-1})^n$ divergen

b. a_n = $\left(\frac{2n}{5n+3}\right)^n$

a = $\lim_{n\to\infty} (a_n)^{\frac{1}{2}}$

a = $\lim_{n\to\infty} \left(\left(\frac{2n}{5n+3}\right)^n\right)^{\frac{1}{n}}$

a = $\lim_{n\to\infty} \frac{2n}{5n+3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}$

a = $\lim_{n\to\infty} \frac{2}{5+\frac{3}{n}}$

a = $\frac{2}{5}$

Karena a = $\frac{2}{5}$ < 1, maka $\sum_{n=1}^{\infty}(\frac{2n}{5n+3})^n$ konvergen

5. Perihal kekonvergenan deret ganti tanda berikut :

a. $\sum_{n=1}^{\infty}(-1)^n \cdot \frac{1}{n(n+4)}$

- Apakah a_n monoton turun?

a_n monoton turun apabila $f'(x) < 0$, dimana $f(x) = a_n$

$f(x) = (n^2 + 4n)^{-1}$

$f'(x) = (-1)(n^2 + 4n)^{-2}\ (2n + 4) = -\left(\frac{2n+4}{(n^2+4n)^2}\right) = 0$

$n = -2$

$n = 0$

Menggunakan uji titik, diketahui bahwa $f'(x) < 0$ di (-∞, -2) V (0, ∞). Deret a_n berada di interval [1, ∞). Maka karena itu, a_n monoton turun.

a = $\lim_{n\to\infty} (a_n)$

a = $\lim_{n\to\infty} \frac{1}{(n^2+4n)} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$

a = $\lim_{n\to\infty} \frac{\frac{1}{n}}{1+\frac{4}{n}} = \frac{0}{1} = 0$

Karena a_n monoton turun $\lim_{n\to\infty} (a_n) = 0 < 1$, maka $\sum_{n=1}^{\infty}(-1)^n \cdot \frac{1}{n(n+4)}$ deret konvergen

Nama: Prames Ray Lapian
NPM : 140810210059

b. $\sum_{n=1}^{\infty}(-1)^{n+1} \cdot \frac{\ln n}{\sqrt{n}}$

- Apakah a_n monoton turun?

a_n monoton turun apabila $f'(x) < 0$, dimana $f(x) = a_n$

$$f(x) = \frac{\ln n}{\sqrt{n}}$$

$$f'(x) = \frac{\frac{1}{\sqrt{n}} - \frac{\ln n}{2\sqrt{n}}}{n} = \frac{2 - \ln n}{2n\sqrt{n}} = 0$$

$n = 0$

$n = e^2$ (7,38 ...)

Menggunakan uji titik, diketahui bahwa $f'(x) < 0$ di (-∞, 0) V (e^2, ∞). Deret a_n berada di interval [1, ∞).

Karena a_n tidak selalu turun di interval [1, ∞), $\sum_{n=1}^{\infty}(-1)^{n+1} \cdot \frac{\ln n}{\sqrt{n}}$ divergen

6. Selidiki apakah ferret konvergen mutlak, bersyarat atau divergen :

a. $\sum_{n=1}^{\infty}(-1)^{n} \cdot \frac{n}{4n}$

Dari soal kita dapatkan $U_n = (-1)^n \cdot (\frac{n}{4n})$ dan $|U_n| = (\frac{n}{4n})$

Sehingga dengan uji hasil bagi.

$$|U_{n+1}| = \frac{n+1}{4 \cdot 4^n}$$

$$\rho = \lim_{n\to\infty}(\frac{|U_{n+1}|}{|U_n|}) = \lim_{n\to\infty} \frac{n+1}{4 \cdot 4^n} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{1 + \frac{1}{n}}{4} = \frac{1}{4}$$

Karena $\rho = \frac{1}{4} < 1$, maka deret $\sum_{n=1}^{\infty}(-1)^{n} \cdot \frac{n}{4n}$ konvergen mutlak

b. $\sum_{n=1}^{\infty}(-1)^{n} \cdot \frac{1}{5n+!}$

Dari soal kita dapatkan $U_n = (-1)^n \cdot \frac{1}{5n+1}$ dan $|U_n| = \frac{1}{5n+1}$

Sehingga dengan uji banding limit

Misal a = $|U_n|$ dan b = $\frac{1}{n}$

Nama: Prames Ray Lapian
NPM : 140810210059

$$L = \lim_{n\to\infty} \frac{a}{b} = \lim_{n\to\infty} \frac{1}{5n+1} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{1}{5+\frac{1}{n}} = \frac{1}{5}$$

Karena L = $\frac{1}{5}$, maka deret $\sum_{n=1}^{\infty}(-1)^n \cdot \frac{1}{5n+!}$ konvergen mutlak