USC University of Southern California

EE675
Data analysis and control techniques for neurotechnology design

Decoding Spatial Position from Hippocampal Place Cells: An Analysis of Encoding Model and Recursive Filter

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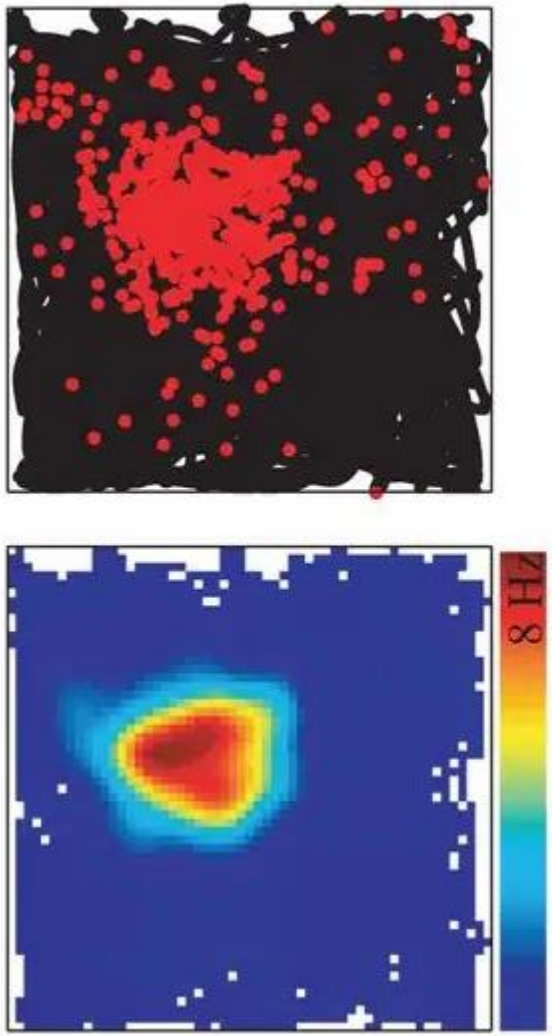
Exploration of Spatial Memory: The Place Cell's Role



Place Cells: Key to Spatial Navigation?

- **Discovered in 1971 by O'Keefe and Dostrovsky**
- **Unique neurons in the rat's hippocampus regions of the brain.**
- **Fire intensively in specific areas ("place fields")**

Context: A rat navigating a square room.

Top Image: Rat's paths (**black lines**) and place cell activation points/spikes (**red dots**).

Bottom Image: Firing rate intensity indicated by color gradients.

Project Objective:

- **Analyze hippocampal place cell activity in a rat navigating a linear track.**
- **Reconstruct the rat's spatial trajectory using neural signals from hippocampal place cell in the CA1 area.**

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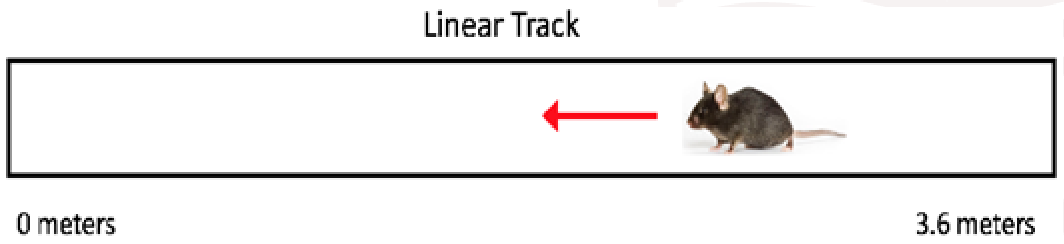
*Credit: Royal Society (2013)

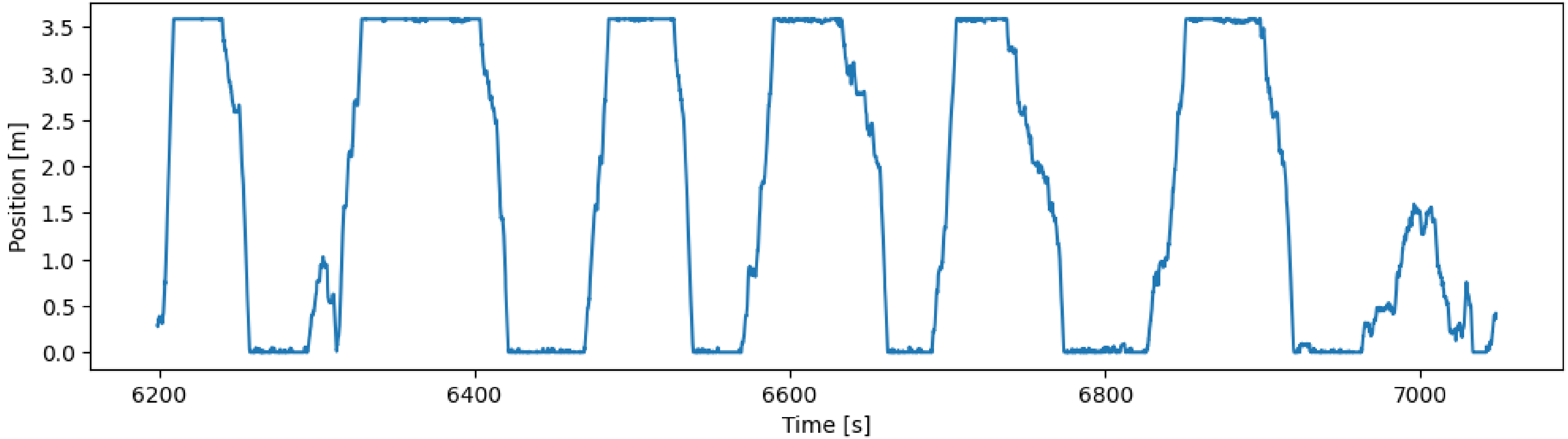
Data Overview

- Rat free to move along a 3.6-meter linear track.
- Place cell activity captured simultaneously from 53 cells in the rat's hippocampal area CA1.
- Sampling rate: 30 Hz (Position of rat tracked for every 0.033 seconds).
- Total recording duration: ~14 minutes, including multiple track passes.



Trial1Start: 6240sec **Trial1End:** 6259sec
Trial2Start: 6526sec **Trial2End:** 6539sec
Trial3Start: 6620sec **Trial3End:** 6665sec

Monitoring the Rat's Movement



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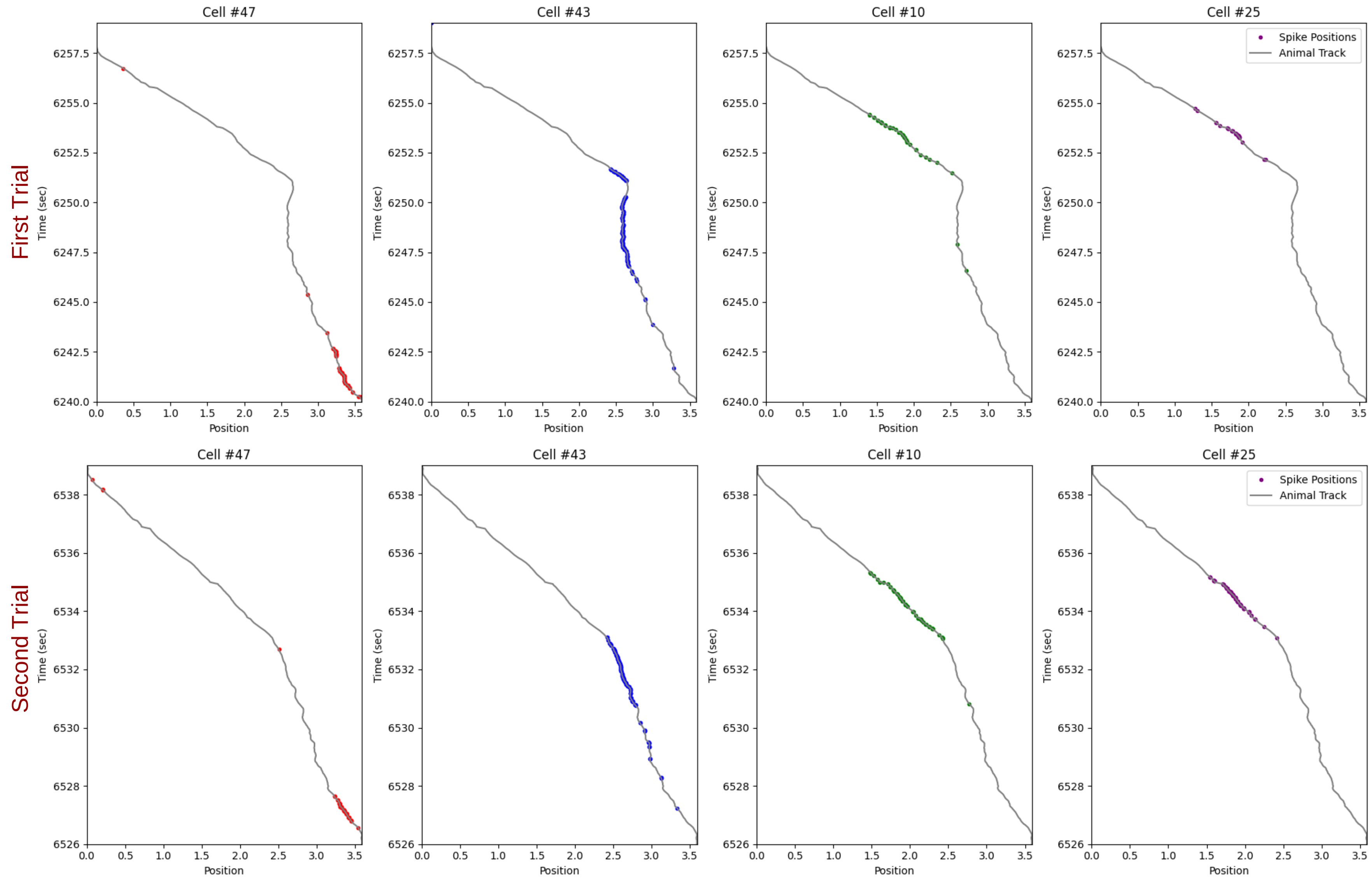
Spike trains from four place cells recorded during a single traversal of the track:



Exploring Place Cell Activity

- Observing rats in a linear maze.
- Highlights how hippocampal cells encode spatial information.

Key Insights

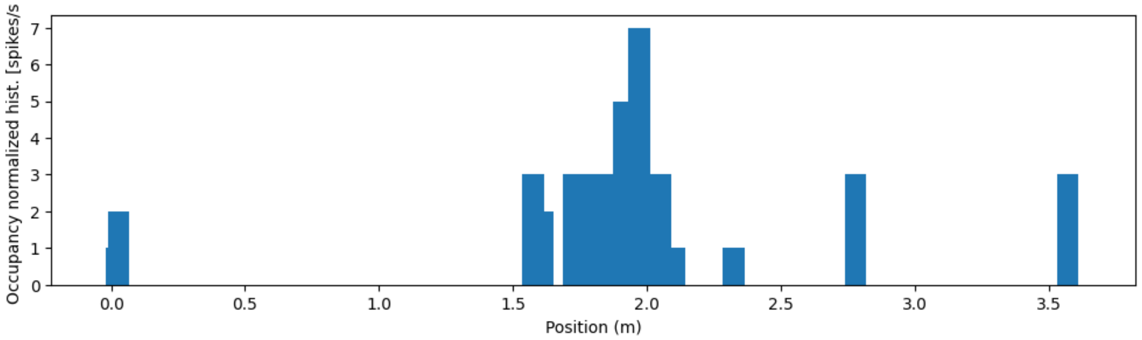
- **Place cells activated in specific track regions.**
- **Consistent firing patterns across multiple trials.**

Occupancy Normalized Histograms In Place Cell Analysis:

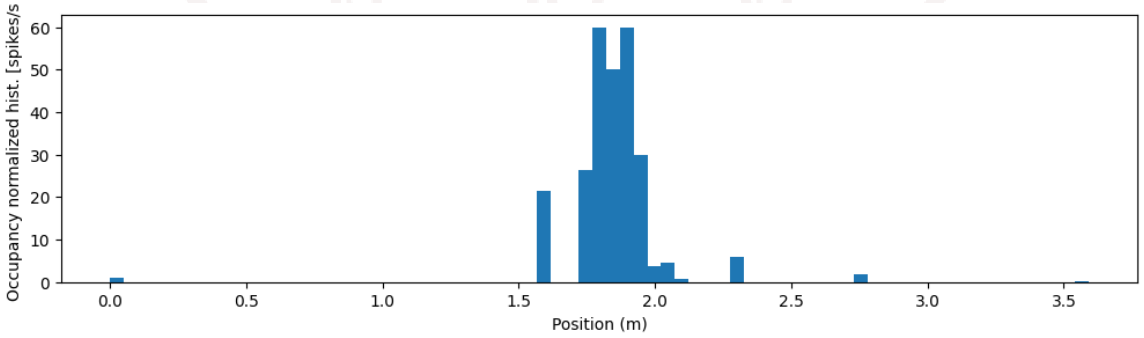
Explains the steps involved in creating and interpreting an occupancy normalized histogram, which is crucial for analyzing the spatial activity of hippocampal place cells in a freely moving rat.

- **Normalization Process:**
  Adjusts for varying occupancy times across different parts of the track. Prevents misinterpretation of spiking behavior due to differing amounts of time spent in various locations.

- **Insight into Neuronal Activity:**
  Provides a clearer understanding of how neuron's firing relates to the animal's specific positions on the track. Essential for studying the spatial representation in the hippocampus and the function of place cells.



Histogram of Spike data after binning



Occupancy Normalized Histogram of Spike data after binning

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Encoding Analysis - Place Cell Model

- To define the point process observation model, we consider **inhomogeneous Poisson models for representing the place cell spiking activity**.
- This model captures the likelihood of a neuron firing at a given position.
- In the first, we model the **rate or conditional intensity function** $\lambda_G^c(t \mid x(t), \zeta_G)$

$$\lambda_G^c(t \mid x(t), \zeta_G^c) = \exp\left\{\alpha^c - \frac{1}{2}(x(t) - \mu^c)'(Q^c)^{-1}(x(t) - \mu^c)\right\}$$

$x(t) = (x_1(t), x_2(t))$ are the coordinates of the animal's position at time(t)

$(\mu^c = (\mu_1^c, \mu_2^c))$ are the coordinates of the place field's center for neuron ‘c’

α_c is the log maximum firing rate for neuron

$\zeta_G^c = (\alpha^c, \mu^c, Q^c)'$ are the parameters of the CIF for neuron ‘c’

$$Q^c = \begin{bmatrix} (\sigma_1^c)^2 & 0 \\ 0 & (\sigma_2^c)^2 \end{bmatrix}$$

Q^c is a scalar matrix, with $(\sigma_1^c)^2$ and $(\sigma_2^c)^2$ being the standard deviations along the x_1 and x_2 axes, respectively.

Gaussian CIF Model:

$$\lambda^c(t \mid x(t), \zeta^c) = \alpha^c \exp\left(-\frac{(\mathbf{x(t)} - \mu^c)^2}{2(\sigma^c)^2}\right)$$

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* Dynamic Analyses of Information Encoding in Neural Ensembles (Barbieri et al, 2004)

CIF Model Transformation:

The use of $(\log(\lambda(t)))$ link function expression in a GLM is justified for modeling place cell activity. It conforms to GLM principles, captures nonlinear firing patterns, and provides interpretability and adaptability in analyzing neuronal behavior.

$$log(\lambda(t)) = (\beta_0 + \beta_1 x(t) + \beta_2 x(t)^2)$$

$$\beta_2 x(t)^2 + \beta_1 x(t) = \beta_2 \left(x(t)^2 + \frac{\beta_1}{\beta_2} x(t) \right)$$

$$= \beta_2 \left(\left(x(t) + \frac{\beta_1}{2\beta_2} \right)^2 - \left(\frac{\beta_1}{2\beta_2} \right)^2 \right)$$

Rewriting the quadratic term in the exponent to resemble the form of a Gaussian distribution. Therefore, we expand and rearrange the quadratic term, and use completing the square

Transforming to Gaussian Form:

$$\lambda(t) = \exp\left(\beta_0 - \beta_2 \left(\frac{\beta_1}{2\beta_2} \right)^2 + \beta_2 \left(x(t) + \frac{\beta_1}{2\beta_2} \right)^2 \right)$$

This can be rewritten as

$$\lambda(t) = \exp\left(\beta_0 - \frac{\beta_1^2}{4\beta_2} \right) \exp\left(-\beta_2 \left(x(t) + \frac{\beta_1}{2\beta_2} \right)^2 \right)$$

$$\lambda(t) = \exp\left(\beta_0 - \frac{\beta_1^2}{4\beta_2} \right) \exp\left(\frac{\left(x(t) - \left(\frac{-\beta_1}{2\beta_2} \right) \right)^2}{2\left(\frac{1}{-2\beta_2} \right)} \right)$$

Comparing and Identifying the Gaussian Parameters

$$\lambda^c (t \mid x(t), \zeta^c) = \alpha^c \exp\left(-\frac{(\mathbf{x(t)} - \mu^c)^2}{2(\sigma^c)^2} \right)$$

$\mu = -\frac{\beta_1}{2\beta_2}$ is the center of the place field

$\sigma^2 = -\frac{1}{2\beta_2}$ is the size of the place field

$\alpha = \exp\left(\beta_0 - \frac{\beta_1^2}{4\beta_2}\right)$ is the maximum firing rate.

Given the estimates $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2$, we can compute:

$$\hat{\mu} = -\frac{\hat{\beta}_1}{2\hat{\beta}_2}$$

$$\hat{\sigma} = \sqrt{-\frac{1}{2\hat{\beta}_2}}$$

$$\hat{\alpha} = \exp\left(\hat{\beta}_0 - \frac{\hat{\beta}_1^2}{4\hat{\beta}_2}\right)$$

These are the ML Estimates of the center, size, and maximum firing rate of the place field.

Invariance Property

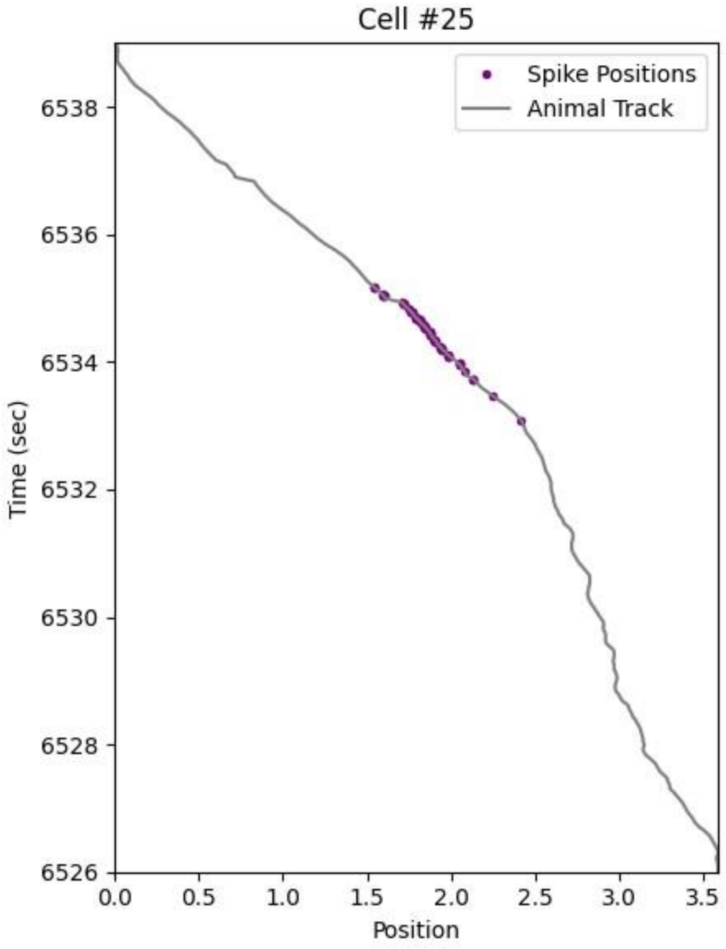
Fitting Gaussian CIF Model to Cell 25 (Trial 3)

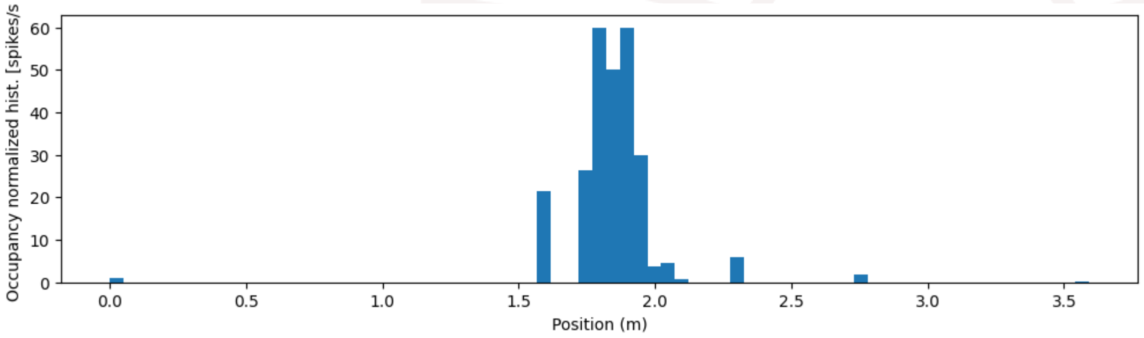
Using Generalized Linear Model (GLM) to fit the data. Retrieved GLM β parameters as a basis for the CIF model.

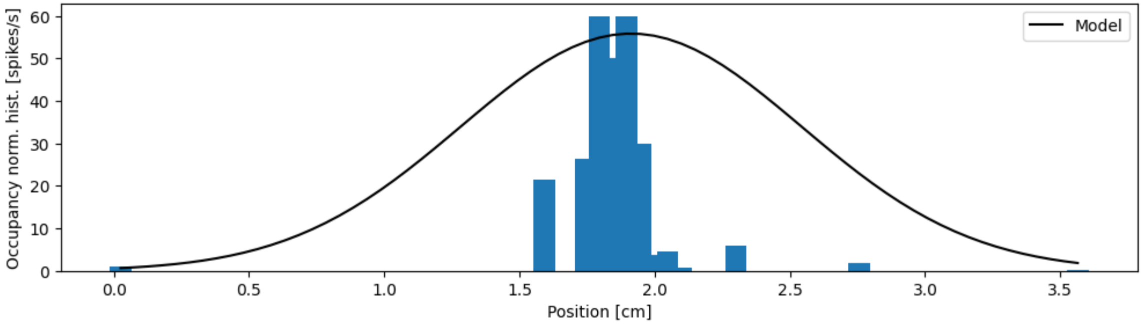
$\lambda(t) = \exp\left(\beta_0 + \beta_1 x(t) + \beta_2 x(t)^2\right)$ ⟶ $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2$ ⟶ $\hat{\mu} = -\frac{\hat{\beta}_1}{2\hat{\beta}_2}, \hat{\sigma} = \sqrt{-\frac{1}{2\hat{\beta}_2}}, \hat{\alpha} = \exp\left(\hat{\beta}_0 - \frac{\hat{\beta}_1^2}{4\hat{\beta}_2}\right)$

Model Parameters

ML Estimates







β_0= -3.965 β_1 = 4.795 β_2 = -1.253 ⟶ $\hat{\mu}_{25}$ = 1.913 $\hat{\sigma}_{25}$ = 0.631 $\hat{\alpha}_{25}$= 1.861

Time step is 1/30s, so the maximum firing rate $\hat{\alpha}_{25}$ = 1.861*30 = 55.83 spikes/sec

We see that the estimated place field center for cell 25 is about 1.93m down the track. The estimated place field size, $\hat{\sigma}$ = 0.63m, suggests that the firing rate decreases about 40% when the rat is about 0.63m from the place field center, The neuron spikes at a rate near $\hat{\alpha} = 56$ spikes/s when the rat at 1.93m down the track

Decoding Analysis: Approximate Gaussian Decoding Filter

The decoding model is based on the movement of the rat and the spike data from hippocampal place cells. The model includes the following components

Model for Rat's Movement:

$$\mathbf{x}_k = \mathbf{x}_{k-1} + \mathbf{z}_k, \quad \mathbf{z}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$$

This equation models the rat's movement as a random walk. At each time step (k), the rat's new position $(\mathbf{x}_k)$ is the sum of its previous position $\mathbf{x}_{k-1}$ and some movement noise $(\mathbf{z}_k)$.

Approximate Gaussian Decoding Filter:

The decoding process updates the estimated position of the rat at each time step using the following equations:

Prediction Step:

$$\mathbf{x}_{k|k-1} = \mathbf{x}_{k-1|k-1}$$
$$\mathbf{W}_{k|k-1} = \mathbf{W}_{k-1|k-1} + \mathbf{Q}$$

Predicts the rat's position at time (k) based on the information up to time $(k-1)$.
$\mathbf{W}_{k|k-1}$ is the predicted error covariance, updating the previous error covariance $\mathbf{W}_{k-1|k-1}$ by adding the movement noise covariance (Q)

Update Step:

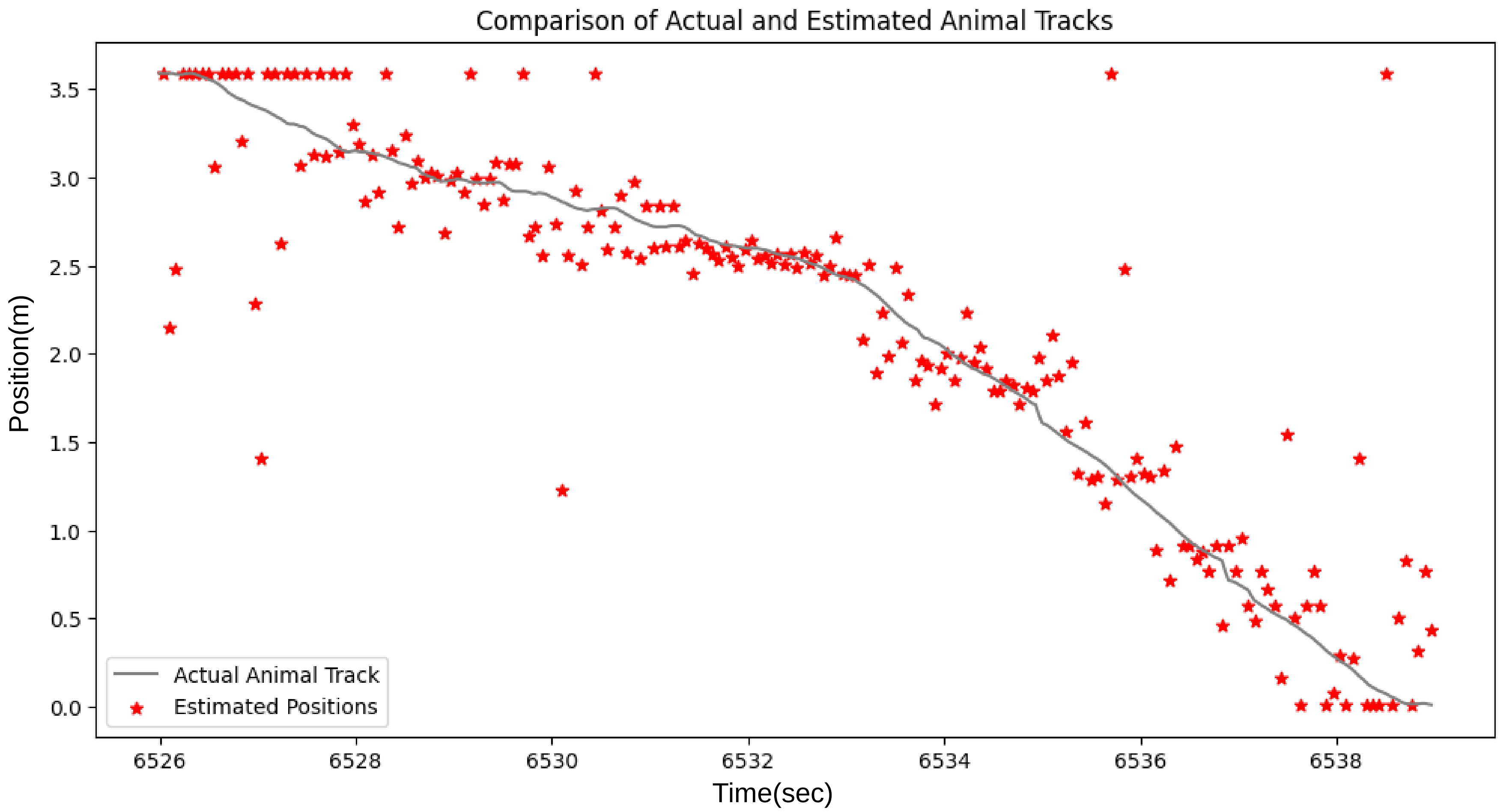
$$\mathbf{x}_{k|k} = \mathbf{x}_{k|k-1} + \mathbf{W}_{k|k-1} \sum_{c=1}^{C} \frac{\partial \log \lambda_c(\mathbf{x})}{\partial \mathbf{x}^T}\bigg|_{\mathbf{x}_{k|k-1}} \left[n_{k,c} - \lambda_c\left(\mathbf{x}_{k|k-1}\right)\Delta\right]$$

$$\mathbf{W}_{k|k} = \left(\mathbf{W}_{k|k-1}^{-1} + \left[\sum_{c=1}^{C} \frac{\partial \log \lambda_c(\mathbf{x})}{\partial \mathbf{x}^T}\frac{\partial \log \lambda_c(\mathbf{x})}{\partial \mathbf{x}}\lambda_c(\mathbf{x})\Delta - \frac{\partial^2 \log \lambda_c(\mathbf{x})}{\partial \mathbf{x}^T \partial \mathbf{x}}\left[n_{k,c} - \lambda_c(\mathbf{x})\Delta\right]\right]_{\mathbf{x}_{k|k}}\right)^{-1}$$

Decoding the Rat's Position Using Maximum Likelihood Estimation

Computing Likelihood Across Position Bins: For each time bin, calculate the likelihood of the rat being at each position. Use the combined spike counts from all place cells.

Finding the Maximum-Likelihood Position: Identify the position with the highest likelihood for each time bin. This position represents the decoded or estimated location of the rat.



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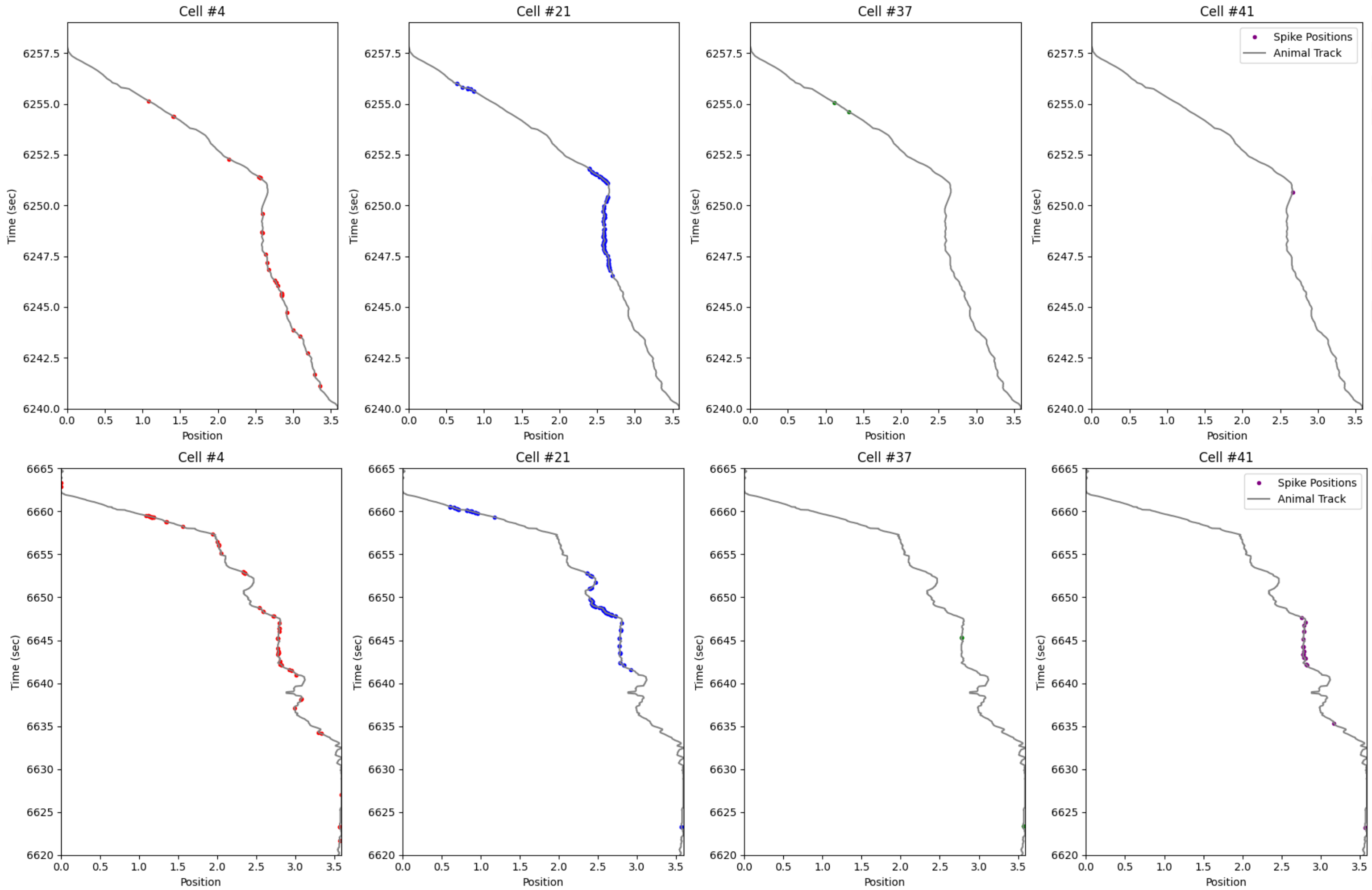
Data Inconsistencies and Spike Absences

Inconsistencies in Neuronal Data:

- Not all neurons exhibit consistent spiking activity throughout the trials

Challenges in Model Fitting:

- Our goal is to refine the model for a more accurate representation of the data.
- Due to gaps in spiking data, perfect model accuracy may not be achievable.



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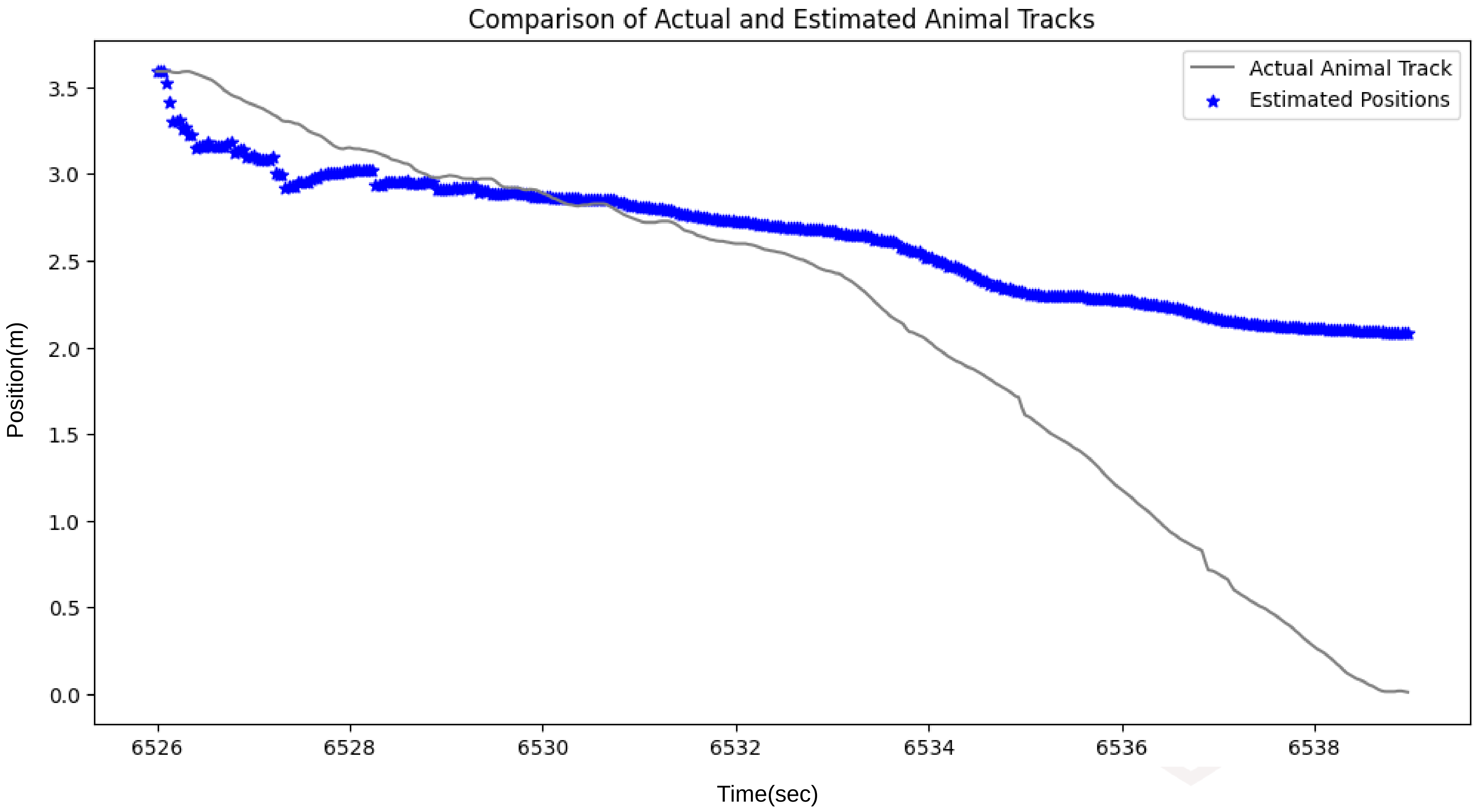
Decoding the Rat's Position Using Approximate Gaussian Decoding Filter



Thank you!

Reference Paper:

Barbieri R, Frank LM, Nguyen DP, Quirk MC, Solo V, Wilson MA, Brown EN. Dynamic analyses of information encoding in neural ensembles. Neural Comput. 2004 Feb;16(2):277-307. doi: 10.1162/089976604322742038. PMID: 15006097.

Invariance/Equivariance Property :

Maximum likelihood estimate of any function of model parameters is just that same function of the maximum likelihood estimates of the parameters. This is often called invariance or equivariance