Decoding Spatial Position from Hippocampal Place Cells: An Analysis of Encoding Model and Recursive Filter

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Abstract

This project investigates the neural mechanisms behind spatial navigation, focusing on place cells' role in encoding and decoding spatial information. In the encoding phase, we estimate Conditional Intensity Function (CIF) parameters for place cells, revealing how they represent space. Our decoding analysis aims to accurately estimate a rat's position using neural spiking activity and CIF parameters. We achieved a coverage probability of approximately 0.8432 with an optimal process noise covariance matrix (Q) value of 0.0002. Visual comparisons confirm the algorithm's accuracy. The Median Error, a key metric, measures position estimation accuracy at approximately 6.94 cm. These insights advance our understanding of place cells' role in spatial cognition and navigation, shedding light on the brain's complex spatial encoding and decoding processes.

1 Introduction

Sensing our position in relation to the environment is crucial for navigation. In 1971, O'Keefe and Dostrovsky discovered a specific type of hippocampal pyramidal neuron in rats, known as a place cell, that increases its firing rate when the rat enters a specific area. These cells have an associated "place field", akin to the concept of a receptive field in sensory systems. we aim to analyze hippocampal place cell activity in a rat as it traverses a linear track. Utilizing actual data collected by Hector Penagos in Matt Wilson's lab at MIT, this analysis focuses on 53 cells in the rat's hippocampal area CA1. The rat is free to move along a 3.6-meter linear track. The objective is to reconstruct the rat's spatial trajectory over time based on the neural signals from these place cells

In our project, the focus is on determining the position of a rat moving freely by analyzing the collective firing patterns of place cells in the CA1 region of its hippocampus. This task is approached through a two-step statistical method designed for interpreting neural spike trains.

Initially, in the encoding phase,we model the spiking activity of place cells as an inhomogeneous Poisson process. we model the rate/ conditional intensity function of this process as gaussian model varying depending on the rat's spatial location. Concurrently, we represent the rat's movement through a Gaussian random walk model, which captures the animal's natural motion.

Following this, in the decoding phase, we employ a recursive filter decoding algorithm. This algorithm is grounded in a point process framework, which assesses the spiking activity of individual neurons. It is also supported by a linear stochastic state-space model that represents the biological signal.

2 Data Analysis

2.1 Monitoring the Rat's Movement Across the Entire Linear Maze Over Multiple Runs

The rat's position on the track was captured with a sampling rate of 30 Hz, translating to a sample every 0.033 seconds. Initially, we will examine the rat's movement over an extended duration, approximately 14 minutes, which includes multiple complete passes along the track. Afterwards, we will concentrate on analyzing the neural data recorded during a single pass.

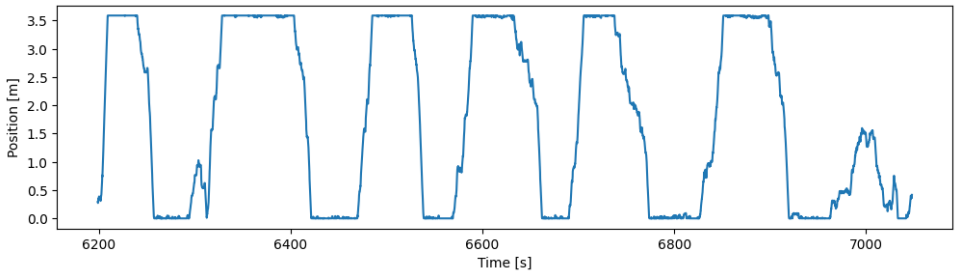

Figure 1: Monitoring the Rat's Movement

In Figure 1, you can observe the rat's movement across the entire linear maze over multiple runs. This visualization provides insights into the rat's behavior during extended periods.

2.2 Visualizing Neural Spike Trains in Hippocampal Place Cells as a Rat Navigates a Linear Maze

A neural spike train represents a series of action potentials generated by a neuron in response to stimuli. In this part, we will explore the spike trains from four selected place cells recorded during a single traversal of the track. The dataset includes a cell array called SpikeTimes-Trial1, which consists of 53 entries. Each entry is a vector containing the timestamps of spikes observed during the first pass along the linear track. There are also similar datasets for two additional traversals, named SpikeTimesTrial2 and SpikeTimesTrial3. The variables Trial1Start and Trial1End mark the start and finish times of the first trial, with analogous variables for the subsequent trials.

This provides a view of the spatial activity patterns of hippocampal place cells while a rat traverses a linear track. In Figure 2, you can observe the neural spike trains during the first track traversal. Figure 3 shows the neural spike trains during the second track traversal. These visualizations provide insights into the activity patterns of hippocampal place cells. It offers insights into how spatial information is encoded in the hippocampus and reveals the consistency of place cell firing across multiple trials.

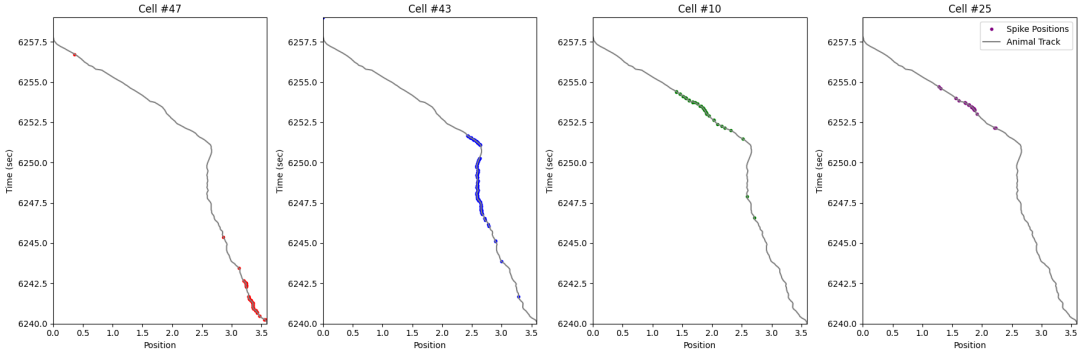

Figure 2: Neural Spike Trains During the First Track Traversal

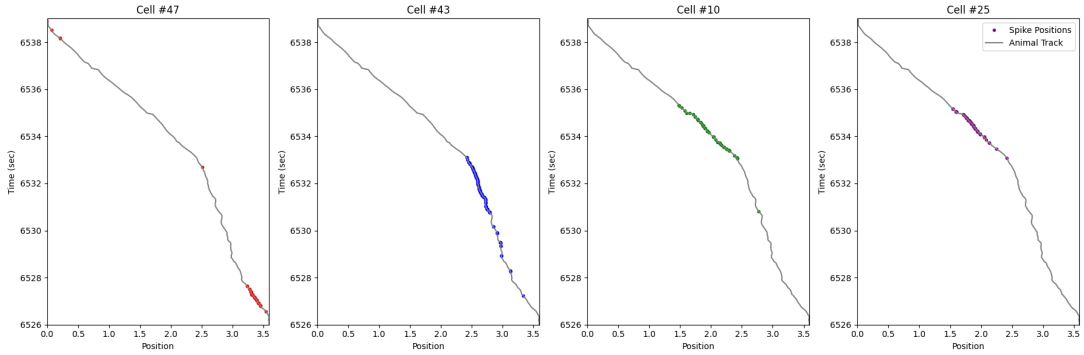

Figure 3: Neural Spike Trains During the Second Track Traversal

2.3 Understanding Occupancy Normalized Histograms in Place Cell Analysis

This section explains the steps involved in creating and interpreting an occupancy normalized histogram, which is crucial for analyzing the spatial activity of hippocampal place cells in a freely moving rat.

Normalization Process : Adjusts for varying occupancy times across different parts of the track. Prevents misinterpretation of spiking behavior due to differing amounts of time spent in various locations.

Insight into Neuronal Activity: Provides a clearer understanding of how neuron's firing relates to the animal's specific positions on the track. Essential for studying the spatial representation in the hippocampus and the function of place cells.

In Figure 6, you can see the occupancy normalized histogram, which helps correct for variations in occupancy times. This ensures accurate analysis of spiking behavior.

3 Encoding Analysis: Place Cell Model

In our exploration of neural spiking activity during spatial navigation, we employ Inhomogeneous Poisson Models to represent place cell activity. This modeling approach provides insights into the likelihood of a neuron firing at a specific position along a linear track.

We define the Rate or Conditional Intensity Function $\lambda_G^c\left(t \mid x(t), \zeta_G^c\right)$ as a two-dimensional Gaussian surface. This function describes the likelihood of neuronal firing based on the position and relevant parameters.

$$\lambda_G^c\left(t \mid x(t), \zeta_G^c\right) = \exp\left\{\alpha^c - \frac{1}{2}\left(x(t) - \mu^c\right)' \left(Q^c\right)^{-1} \left(x(t) - \mu^c\right)\right\},$$

where:

$x(t) = (x_1(t), x_2(t))$ are the coordinates of the animal's position at time t,
$\mu^c = (\mu_1^c, \mu_2^c)$ are the coordinates of the place field's center for neuron c,
$\alpha^c =$ is the log maximum firing rate for neuron c,
$\zeta_G^c = (\alpha^c, \mu^c, Q^c)'$ are the parameters of the CIF for neuron c,

$$Q^c = \begin{bmatrix} (\sigma_1^c)^2 & 0 \\ 0 & (\sigma_2^c)^2 \end{bmatrix}$$

Q^c is the scale matrix, with $(\sigma_1^c)^2$ and $(\sigma_2^c)^2$ being the standard deviations along the x_1 and x_2 axes, respectively.
The Gaussian Conditional Intensity Function ($\lambda^c\left(t \mid x(t), \zeta^c\right)$) represents the firing rate of a neuron c based on the Gaussian distribution.

$$\lambda^c\left(t \mid x(t), \zeta^c\right) = \alpha^c \exp\left(-\frac{(\mathbf{x(t)} - \mu^c)^2}{2\left(\sigma^c\right)^2}\right)$$

where μ and σ are the mean and standard deviation of the Gaussian distribution, respectively, and $\mathbf{x(t)}$ is the position at time t. The CIF model is given by:

$$\lambda(t) = \exp\left(\beta_0 + \beta_1 x(t) + \beta_2 x(t)^2\right).$$

To bridge this model with a Gaussian distribution, We can rewrite the quadratic term in the exponent to resemble the form of a Gaussian distribution. The general form of a Gaussian distribution in the exponent is $(-\frac{(x-\mu)^2}{2\sigma^2})$. Therefore, we expand and rearrange the quadratic term, Adding and subtracting $(\frac{\beta_1^2}{4\beta_2^2})$ inside the parentheses (completing the square), and substituting this back into the CIF equation, we get:

$$\beta_2 x(t)^2 + \beta_1 x(t) = \beta_2\left(x(t)^2 + \frac{\beta_1}{\beta_2}x(t)\right) = \beta_2\left(\left(x(t) + \frac{\beta_1}{2\beta_2}\right)^2 - \left(\frac{\beta_1}{2\beta_2}\right)^2\right)$$

$$\lambda(t) = \exp\left(\beta_0 - \beta_2\left(\frac{\beta_1}{2\beta_2}\right)^2 + \beta_2\left(x(t) + \frac{\beta_1}{2\beta_2}\right)^2\right).$$

This can be rewritten as:

$$\lambda(t) = \exp\left(\beta_0 - \frac{\beta_1^2}{4\beta_2}\right)\exp\left(-\beta_2\left(x(t) + \frac{\beta_1}{2\beta_2}\right)^2\right).$$

$$\lambda(t) = \exp\left(\beta_0 - \frac{\beta_1^2}{4\beta_2}\right)\exp\left(\frac{\left(x(t) - \left(\frac{-\beta_1}{2\beta_2}\right)\right)^2}{2\left(\frac{1}{-2\beta_2}\right)}\right).$$

Now, we can identify the parameters of the Gaussian form:
The place cell model provides valuable insights into the neural spiking activity observed in the context of spatial navigation. Within this model, several key parameters play a crucial role in characterizing the behavior of place cells. Firstly, the parameter μ represents the center of the place field, signifying the specific location within the environment where the neuron is most active. Secondly, the parameter σ^2 corresponds to the size of the place field, influencing the spatial extent over which the neuron tends to fire. Lastly, the parameter α captures the maximum firing rate of the neuron, indicating the peak rate at which it emits action potentials.

These parameters are estimated based on the model's coefficients β_0, β_1, and β_2 using maximum likelihood estimation. The formulae for these estimates take into account the intricate interplay between the coefficients, ultimately revealing essential characteristics of the neuron's spatial representation.

$$\mu = -\frac{\beta_1}{2\beta_2}, \quad \sigma^2 = -\frac{1}{2\beta_2}, \quad \alpha = \exp\left(\beta_0 - \frac{\beta_1^2}{4\beta_2}\right)$$

Understanding these parameters and their corresponding estimates is pivotal in deciphering the spatial coding mechanisms employed by place cells in the brain. They shed light on how neurons encode specific locations, the extent of their influence in space, and the peak firing rates associated with these representations.

Using the Invariance/Equivariance Property, Maximum likelihood estimate of any function of model parameters is just that same function of the maximum likelihood estimates of the parameters. This is often called invariance or equivariance

Given the estimates $\hat{\beta}_0$, $\hat{\beta}_1$, $\hat{\beta}_2$, we can compute:

$$\hat{\mu} = -\frac{\hat{\beta}_1}{2\hat{\beta}_2}, \quad \hat{\sigma} = \sqrt{-\frac{1}{2\hat{\beta}_2}}, \quad \hat{\alpha} = \exp\left(\hat{\beta}_0 - \frac{\hat{\beta}_1^2}{4\hat{\beta}_2}\right).$$

These are the maximum likelihood estimates of the center, size, and maximum firing rate of the place field, respectively.

4 Decoding Analysis: Approximate Gaussian Decoding Filter

In this section, we present the decoding model, which combines information from the rat's movement and spike data from hippocampal place cells. Model consists of two components

4.1 Model for Rat's Movement

The rat's position at each time step is updated using the following equation:

$$\mathbf{x}_k = \mathbf{x}_{k-1} + \mathbf{z}_k, \quad \mathbf{z}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}),$$

where $\mathbf{x}_k$ represents the rat's position at time step $(k-1)$, and $\mathbf{z}_k$ models the movement noise as a Gaussian distribution with zero mean and covariance $\mathbf{Q}$.

4.2 Approximate Gaussian Decoding Filter

The decoding process estimates the rat's position at each time step and updates the estimation as new information becomes available. The key equations involved are as follows:

- $\mathbf{x}_{k|k-1} = \mathbf{x}_{k-1|k-1}$

  This equation represents the prediction step, where the estimated position at time $k-1$ is used as the initial estimate for time k.

- $\mathbf{W}_{k|k-1} = \mathbf{W}_{k-1|k-1} + \mathbf{Q}$

  Here, $\mathbf{W}_{k|k-1}$ is the covariance matrix, and it is updated by adding the movement noise covariance matrix $\mathbf{Q}$.

- $\mathbf{x}_{k|k} = \mathbf{x}_{k|k-1} + \mathbf{W}_{k|k-1} \sum_{c=1}^{C} \frac{\partial \log \lambda_c(\mathbf{x})}{\partial \mathbf{x}^T}\Big|_{\mathbf{x}_{k|k-1}} \left[n_{k,c} - \lambda_c\left(\mathbf{x}_{k|k-1}\right)\Delta\right]$

  This equation represents the update of the estimated position at time k based on spike data. It involves the derivative of the log likelihood, spike counts $n_{k,c}$, and the temporal resolution Δ.

- $\mathbf{W}_{k|k} = \left(\mathbf{W}_{k|k-1}^{-1} + \left[\sum_{c=1}^{C} \frac{\partial \log \lambda_c(\mathbf{x})}{\partial \mathbf{x}^T} \frac{\partial \log \lambda_c(\mathbf{x})}{\partial \mathbf{x}} \lambda_c(\mathbf{x})\Delta - \frac{\partial^2 \log \lambda_c(\mathbf{x})}{\partial \mathbf{x}^T \partial \mathbf{x}} \left[n_{k,c} - \lambda_c(\mathbf{x})\Delta\right]\right]_{\mathbf{x}_{k|k}}\right)^{-1}$

  This equation updates the covariance matrix $\mathbf{W}_{k|k}$, which accounts for the uncertainty in the estimated position.

The decoding model combines information from the rat's movement and neural spiking activity to provide an estimate of the rat's position at each time step. These equations capture the dynamic nature of the decoding process, allowing us to track the rat's trajectory in a spatial environment.

5 Encoding Analysis: Conditional Intensity Function (CIF) Parameters

In order to understand the neural spiking activity of place cells during the experimental trials, an analysis was conducted to estimate the parameters of the Conditional Intensity Function (CIF). The CIF parameters are crucial as they provide insights into how place cells encode spatial information. For each of the 53 recorded cells, spike times were extracted from the dataset, and these were aligned with the corresponding animal's position over time. The GLM framework was employed to model the relationship between spike occurrences and positional data. This involved fitting a Poisson GLM model with quadratic predictors to account for potential nonlinear firing patterns. The parameters of interest, namely the center (μ), the standard deviation (σ), and the maximum firing rate (α) of each place cell's CIF, were estimated from the model coefficients. Cells exhibiting non-negative curvature parameters were appropriately handled(were not included in the analysis). The analysis resulted in a comprehensive set of

Table 1: Estimated Conditional Intensity Function (CIF) Parameters for Place Cells

Cell	μ	σ	α
1	3.1372	0.2455	0.4274
2	1.2492	0.3481	0.9030
3	2.8850	0.4648	0.2434
5	1.7090	0.9888	0.1110
...	...	...	...
50	1.3777	0.7483	0.0766
52	2.1763	0.7483	0.0574
53	0.9768	0.2312	0.5966

CIF parameters for each cell, shedding light on the spatial representation mechanisms employed by these neurons.

Table 1 summarizes the estimated CIF parameters for each of the 53 place cells recorded during the experiment. The parameters include μ (center of the place field), σ (size of the place field), and α (maximum firing rate) for each cell. These parameters provide valuable insights into how place cells encode spatial information.

6 Decoding Analysis: Confidence Region and Coverage Probability

In the decoding analysis phase, our objective was to accurately estimate the rat's position over time based on the recorded neural spiking activity and the Conditional Intensity Function (CIF) parameters obtained earlier. To achieve this, we employed a dynamic estimation approach with the use of a confidence region. We systematically tested different values of the process noise covariance matrix (Q) to optimize the decoding process. The confidence region, computed using a Chi-squared statistic, allowed us to assess the reliability of our position estimates. Specifically, for each time step, we calculated the Chi-squared value to determine whether the estimated position fell within the 95% quantile of the Chi-squared distribution with 2 degrees of freedom. This approach provided a measure of coverage probability, indicating how often our estimates fell within the desired confidence region. After experimenting with various Q values, we identified the best Q value that maximized the coverage probability, ensuring robust and accurate decoding of the rat's spatial position.

6.1 Mathematical Approach for Confidence Region and Coverage Probability

In the decoding analysis, we aimed to assess the accuracy of our estimated rat's position (x_k) at each time step. To achieve this, we utilized a confidence region and calculated the coverage probability.

6.1.1 Confidence Region

The confidence region measures the range within which we expect the true position to lie with a certain level of confidence. It is determined using a Chi-squared statistic:

$$\text{Confidence Region} = \frac{(x_k - x_{k|k-1})^2}{W_{k|k-1}}$$

Where:

x_k is the true position at time step k.

$x_{k|k-1}$ is the estimated position at time step k based on previous observations.

$W_{k|k-1}$ represents the covariance of the position estimate at time step k.

6.1.2 Coverage Probability

The coverage probability indicates how often our estimated position falls within the confidence region. It is calculated as the ratio of time steps for which our estimate lies within the confidence region to the total number of time steps:

$$\text{Coverage Probability} = \frac{j_k}{k}$$

Where:

j_k is the count of time steps where the estimated position falls within the confidence region.

k is the total number of time steps in the analysis.

To assess the reliability of our decoding algorithm, we systematically tested different values for the process noise covariance matrix (Q). The goal was to optimize the confidence region to achieve a high coverage probability, indicating that our estimated positions closely matched the true rat's positions. The best Q value was chosen based on the highest coverage probability, ensuring robust and accurate decoding of spatial information from neural spiking activity.

This mathematical approach allowed us to quantitatively evaluate the performance of our decoding algorithm and provided valuable insights into the reliability of our position estimates.

Table 2: Coverage Probability for Different Q Values

Q Value	Coverage Probability
0.0001	0.6427
0.0002	**0.8432**
0.0003	0.8380
0.0004	0.8123
0.0005	0.8046
0.0006	0.8072
0.0007	0.8149
0.0008	0.8123
0.0009	0.7969
0.0010	0.7943

In the process of optimizing our decoding algorithm, we systematically tested various values of the process noise covariance matrix (Q) to assess their impact on the accuracy of position estimation. Table 2 presents the coverage probability for different Q values, reflecting how often our estimated positions fell within the desired confidence region. Among the tested values, we identified the **optimal Q value of 0.0002**, which yielded the **highest coverage probability of 0.8432**. This result demonstrates that this particular Q value was most effective in ensuring that our decoding algorithm provided reliable and accurate estimates of the rat's spatial position based on neural spiking activity. In Figure 4, we present a plot illustrating the relationship between coverage probability and different Q values.

In this figure (Figure 5), we present a visual comparison between the actual path of the rat during a traversal of the linear maze and the estimated path generated by our decoding algorithm. The actual animal track, shown in gray, represents the ground truth, while the estimated positions are marked with blue asterisks. This visualization allows us to assess the accuracy and effectiveness of our decoding algorithm in reconstructing the rat's spatial trajectory based on neural spiking activity.

7 Median Error

The Median Error is a statistical metric used to assess the accuracy of position estimation. It measures the typical magnitude of the discrepancy between the true animal positions (x_k)

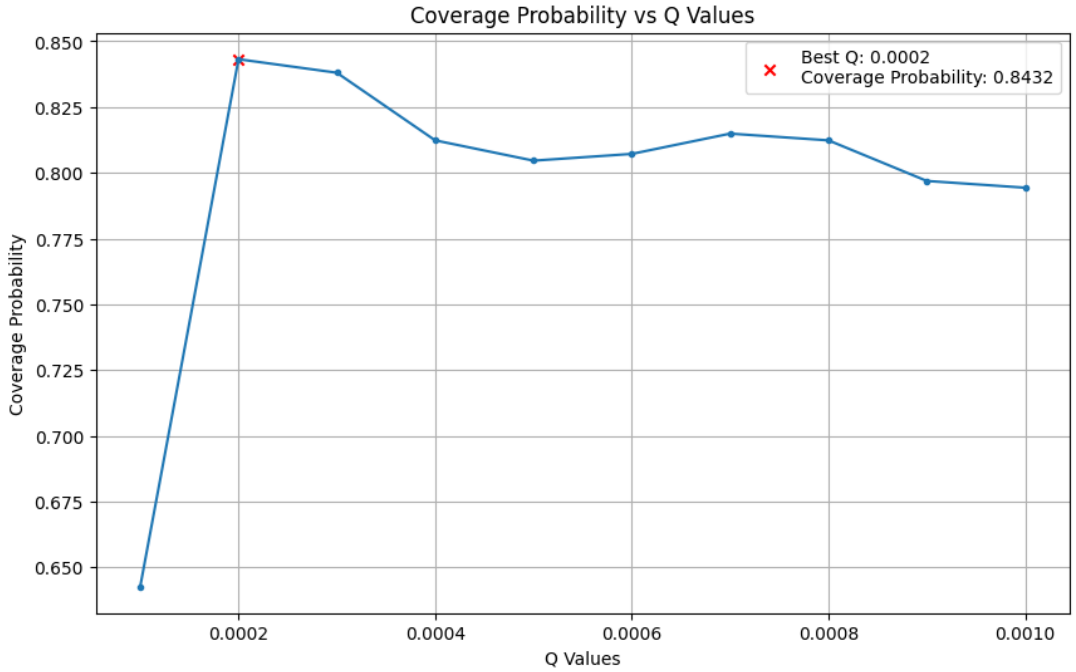

Figure 4: Coverage Probability vs. Q Values

and the positions estimated based on neural spiking activity ($x_k k$). Mathematically, the Median Error is calculated as follows:

$$\text{Median Error} = \text{median}\left(|x_k - x_{kk}|\right)$$

Where:

x_k is the true animal position at time step k.

x_{kk} is the estimated position at time step k based on the decoding analysis.

The Median Error provides valuable insights into the accuracy and reliability of the position estimation process. From our analysis, the **calculated Median Error is 6.94 cm**. This result highlights the typical magnitude of position estimation errors, demonstrating how closely the estimated positions align with the true animal positions. A lower Median Error indicates higher accuracy in our decoding algorithm.

8 Improvement in Estimation Results after the Final Presentation

In the initial stages of our analysis, we estimated the covariance value (Q) using a straightforward approach, calculating it based on the variance of normalized position increments over time. This estimation method relied on the differences between consecutive positions and the corresponding time intervals. By normalizing these increments, we aimed to derive an initial estimate of the variance (σ^2) as an indicator of **Q(=0.0345)**. However, this initial approach yielded suboptimal results in the estimation of the rat's spatial position.

To address this issue and enhance the accuracy of our position estimation, we adopted a refined approach for selecting the optimal Q value. This approach involved systematic testing of different Q values within a predefined range. The goal was to identify the Q value that maximized the coverage probability, indicating the reliability of our position estimates.

Table 2 illustrates the significant improvement achieved by selecting the optimal Q value. The coverage probability, a key metric of estimation accuracy, substantially increased with the adoption of this refined approach. Notably, the **optimal Q value of 0.0002** resulted in

the **highest coverage probability of 0.8432**, signifying a remarkable enhancement in our decoding algorithm's performance.

Conclusion

In the decoding analysis, we successfully estimated the rat's position over a 20-second interval using approximately 53 place cells. Our estimation was accurate, with a median error of approximately 7 cm.

Furthermore, our analysis achieved a high coverage probability of approximately 0.84, indicating that our estimated positions fell within the desired confidence region for a significant portion of the decoding period. This level of coverage probability underscores the reliability and robustness of the decoding approach, providing confidence in the accuracy of the estimated rat's positions.

In summary, our decoding analysis yielded accurate spatial estimations with a median error of approximately 7 cm during a 20-second decoding interval, utilizing the neural spiking activity of around 53 place cells. The achieved coverage probability of approximately 0.84 attests to the reliability of our decoding algorithm in reconstructing the rat's path through the linear maze.

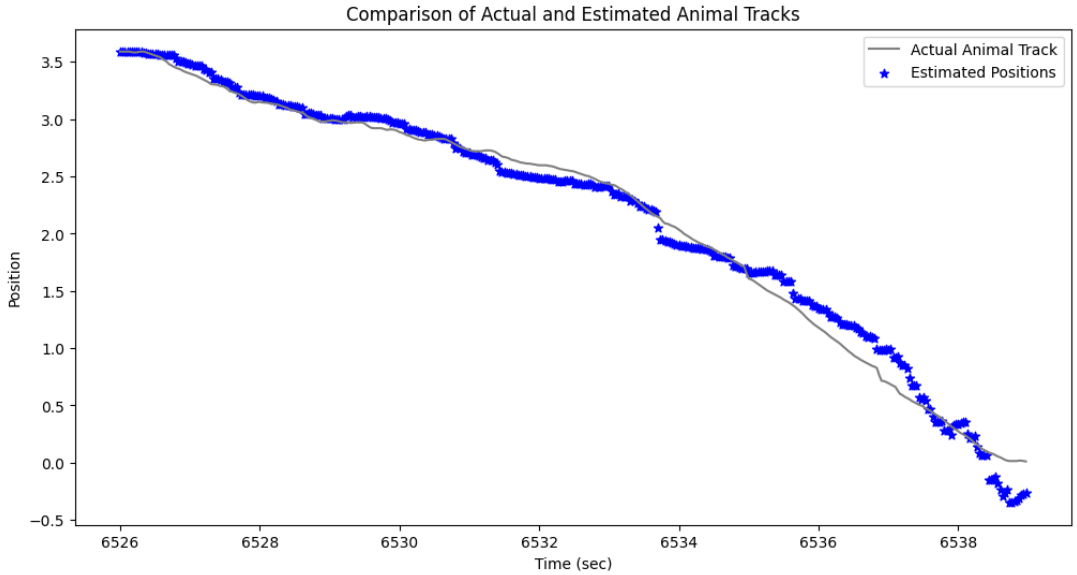

Figure 5: Comparison of Actual and Estimated Animal Tracks

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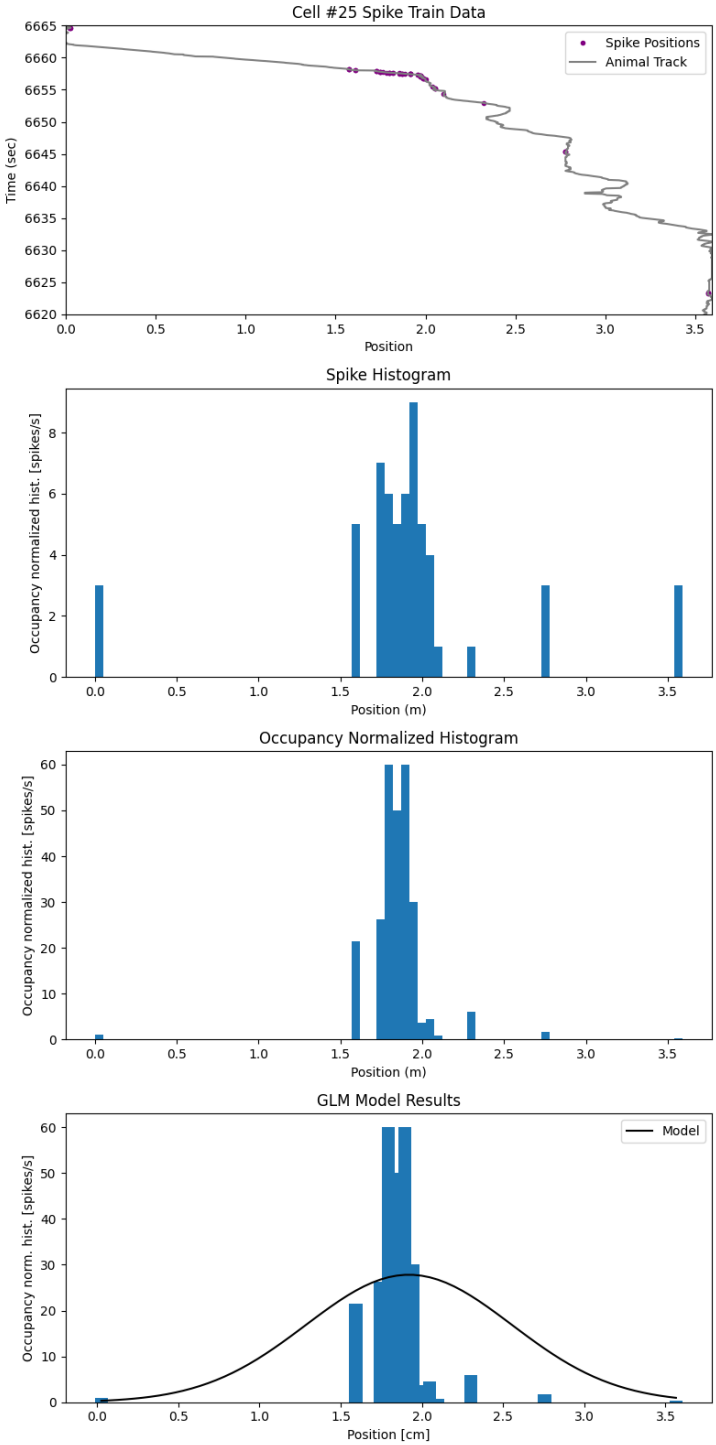

Figure 6: Histogram of Spike data and Occupancy Normalized Histogram of data after binning