2025

Booklet Number: **10671** TEST CODE: **UGA**

Forenoon **Time: 2 hours**

- This test contains thirty (30) multiple-choice questions (MCQs).
- The questions are to be answered on a separate *Optical Mark Recognition* (OMR) Answer Sheet.
- Please write your *Name, Registration Number, Test Centre, Test Code* and the *Number of this Question Booklet* in the appropriate places on the OMR Answer Sheet. Please do not forget to put your signature in the designated place.
- For each of the questions there are four suggested answers, of which only one is correct. For each question, indicate your choice of the correct answer by darkening the appropriate circle (●) completely on the OMR Answer Sheet, using ball-point pen with BLACK ink only.
- You will score
  - ☞ 4 marks for each correctly answered question,
  - ☞ 0 mark for each incorrectly answered question, and
  - ☞ 1 mark for each unattempted question.
- ALL ROUGH WORK MUST BE DONE ONLY IN THE SPACE AVAILABLE ON THIS QUESTION BOOKLET.
- USE OF CALCULATORS, MOBILE PHONES AND ALL TYPES OF ELECTRONIC DEVICES IS STRICTLY PROHIBITED.

STOP! WAIT FOR THE SIGNAL TO START.

1. Let A be an $m \times n$ matrix with the (i,j)th entry given by the real number a_{ij}, $1 \le i \le m$, $1 \le j \le n$. Let

$$\alpha = \max_{1\le j\le n}\left\{\min_{1\le i\le m} a_{ij}\right\} \quad \text{and} \quad \beta = \min_{1\le j\le n}\left\{\max_{1\le i\le m} a_{ij}\right\}.$$

Then

(A) $\alpha \le \beta$ but not necessarily $\alpha = \beta$
(B) $\beta \le \alpha$ but not necessarily $\alpha = \beta$
(C) $\alpha = \beta$
(D) nothing can be said in general

2. The coefficient of x^8 in $(1-3x)^6(1+9x^2)^6(1+3x)^6$ is

(A) $-3^9 \times 5$ (B) $3^9 \times 5$ (C) $-3^8 \times 5$ (D) $3^8 \times 5$

3. Consider the cyclic quadrilateral $ABCD$ given below.

Assume that $AB = BC$, $AD = CD$, and $\frac{AB}{AD} = \frac{1}{3}$. Let $\theta = \angle ADC$. Then $\cos\theta$ is equal to

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{3}{5}$ (D) $\frac{4}{5}$

4. In the xy-plane, the curve $3x^3y + 6xy + 2xy^3 = 0$ represents

(A) a pair of straight lines
(B) an ellipse
(C) a pair of straight lines and an ellipse
(D) a hyperbola

5. Let $I = \int_3^5 \frac{1}{1+x^3}\,dx$. Then

(A) $I < \frac{1}{64}$
(B) $I > \frac{1}{13}$
(C) $\frac{1}{63} < I < \frac{1}{14}$
(D) $I > \frac{1}{2}\left(\frac{1}{14} + \frac{1}{63}\right)$

6. Consider two events A and B with probabilities $P(A)$ and $P(B)$ respectively such that $0 < P(A), P(B) < 1$. Define

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

Consider the following statements.

(I) $P(A|B^c) + P(A|B) = 1$.

(II) $P(A^c|B) + P(A|B) = 1$.

Then, in general,

(A) (I) is true and (II) is false
(B) (I) is false and (II) is true
(C) both (I) and (II) are true
(D) both (I) and (II) are false

7. Let $f(x) = 7x^{11} + 4x^5 - 3$. Then f has

(A) exactly 1 real root
(B) exactly 3 real roots
(C) exactly 5 real roots
(D) 11 real roots

8. Twelve boxes are placed along a circle. In each box, $1, 2, 3$ or 4 balls are put such that the total number of balls in any 4 consecutive boxes is same. The number of ways this can be done is

(A) $4!$ (B) 4^4 (C) $(4!)^3$ (D) $(4!)^4$

9. Let $A = \{(x, y) : x, y \in [0, 1]\}$ and $B = \{(x, y) : x, y \in [0, 2]\}$. Define $f : A \to B$ by $f(x, y) = (x^2 + y, x + y^2)$. Then f is

(A) one-to-one but not onto
(B) onto but not one-to-one
(C) both one-to-one and onto
(D) neither one-to-one nor onto

10. The number of ordered pair (a, b) of positive integers with $a < b$ satisfying $a^2 + b^2 = 2025$ is

(A) 0 (B) 1 (C) 2 (D) 6

11. For each $n \geq 1$, let a_n and b_n be real numbers such that $a_n \neq 0$ and $b_n \neq 0$. Let

$$(a_n + ib_n)^n = n(a_n + ib_n) \quad \text{for all } n \geq 6.$$

Then

(A) no such a_n, b_n exist

(B) $a_n = n^{\frac{1}{n+1}} \cos \frac{2\pi}{n+1}$, $b_n = n^{\frac{1}{n+1}} \sin \frac{2\pi}{n+1}$

(C) $a_n = n^{\frac{1}{n}} \cos \frac{2\pi}{n}$, $b_n = n^{\frac{1}{n}} \sin \frac{2\pi}{n}$

(D) $a_n = n^{\frac{1}{n-1}} \cos \frac{2\pi}{n-1}$, $b_n = n^{\frac{1}{n-1}} \sin \frac{2\pi}{n-1}$

12. Let $a_0 = 0$, $a_1 = 1$ and $a_n = 5a_{n-1} + a_{n-2}$ for $n \geq 2$. Then the value of the determinant

$$\begin{vmatrix} a_{101} & a_{100} \\ a_{100} & a_{99} \end{vmatrix}$$

is

(A) -1 (B) -5^{101} (C) 1 (D) 5^{101}

13. The lengths of the three sides of a right angled triangle are in geometric progression. The smallest angle of the triangle is

(A) $\tan^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$ (B) $\cos^{-1}\left(\frac{2}{\sqrt{5}-1}\right)$

(C) $\sin^{-1}\left(\frac{2}{\sqrt{5}-1}\right)$ (D) $\sin^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$

14. The lengths of the two adjacent sides of a parallelogram are 2 cm and 3 cm. The length of one diagonal is $\sqrt{19}$ cm. Then the length of the other diagonal is

(A) $\sqrt{5}$ cm (B) $\sqrt{7}$ cm (C) $\sqrt{15}$ cm (D) $\sqrt{21}$ cm

15. Consider the following statements about two similar triangles Δ_1 and Δ_2.

S_1: Lengths of the sides of Δ_1 are in arithmetic progression.

S_2: Lengths of the sides of Δ_1 are in geometric progression.

S_3: Lengths of the sides of Δ_2 are in arithmetic progression.

S_4: Lengths of the sides of Δ_2 are in geometric progression.

Then

(A) S_1 implies S_3, but S_2 does not imply S_4
(B) S_1 does not imply S_3, but S_2 implies S_4
(C) S_1 implies S_3, and S_2 implies S_4
(D) S_1 does not imply S_3, and S_2 does not imply S_4

16. Let $[x]$ denote the greatest integer less than or equal to $x \in \mathbb{R}$ and $|x|$ has its usual meaning, that is, $|x| = x$ if $x \geq 0$, and $|x| = -x$, if $x < 0$. Then the value of the integral

$$\int_{-2}^{1} ([x] + 2)^{|x|}\, dx$$

is

(A) $1 + \dfrac{1}{\log_e 2}$

(B) $1 + \log_e \dfrac{1}{2}$

(C) $2 - \log_e 2$

(D) none of the above

17. Let $f(x) = \dfrac{1}{1 + |x-1|} + \dfrac{1}{1 + |x+1|}$. Then the function f has

(A) neither a local maximum nor a local minimum

(B) a local minimum at $x = 0$, but no local maximum

(C) local maxima at $x = \pm 1$, but no local minimum

(D) a local minimum at $x = 0$ and local maxima at $x = \pm 1$

18. Let

$$f(x) = \begin{cases} -1, & \text{if } x < 0, \\ 0, & \text{if } x = 0, \\ 1, & \text{if } x > 0. \end{cases}$$

Then the function F defined by $F(x) = \displaystyle\int_{-5}^{x} f(t)\, dt$ is

(A) not continuous

(B) continuous, but nowhere differentiable

(C) differentiable everywhere

(D) differentiable everywhere except at 0

19. For a real number x, let $f(x) = \int_{-20}^{20} g(t)g(x-t)\,dt$, where

$$g(x) = \begin{cases} 1, & \text{if } x \in [0,1], \\ 0, & \text{otherwise.} \end{cases}$$

Then $f(x)$ is equal to

(A) $\begin{cases} x, & \text{if } x \in [0,1], \\ 2-x, & \text{if } x \in [1,2], \\ 0, & \text{otherwise} \end{cases}$

(B) $\begin{cases} 1+x, & \text{if } x \in [0,1), \\ 1-x, & \text{if } x \in [1,2), \\ 0, & \text{otherwise} \end{cases}$

(C) $\begin{cases} 1, & \text{if } x \in (-20,20), \\ 0, & \text{otherwise} \end{cases}$

(D) none of the above

20. Let

$$L = \lim_{n\to\infty} (n+100)^{\frac{5}{\log_e(n-50)}}.$$

Then

(A) $2 \le L \le 16$

(B) $16 \le L \le 32$

(C) $32 \le L \le 243$

(D) $L > 243$

21. Let a, b, c, d be positive integers such that the product $abcd = 999$. Then the number of different ordered 4-tuples (a, b, c, d) is

(A) 20

(B) 48

(C) 80

(D) 84

22. In a certain test there are n questions. At least i questions were wrongly answered by 2^{n-i} students, where $i = 1, 2, \ldots, n$. If the total number of wrong answers given by all students is 2047, then n is equal to

(A) 10 (B) 11 (C) 12 (D) 13

23. Let $n \geq 3$. There are n straight lines in a plane, no two of which are parallel and no three pass through a common point. Their points of intersection are joined. Then the number of **fresh** line segments thus created is

(A) $\dfrac{n(n-1)(n-2)}{8}$ (B) $\dfrac{n(n-1)(n-2)(n-3)}{6}$

(C) $\dfrac{n(n-1)(n-2)(n-3)}{8}$ (D) none of the above

24. Let d be the side length of the largest possible equilateral triangle that can be put inside a square of side length 1. Then

(A) $d < 1$ (B) $d = 1$

(C) $1 < d < \dfrac{2}{3^{1/4}}$ (D) $d \geq \dfrac{2}{3^{1/4}}$

25. Let

$$a_n = \frac{n^2}{\sqrt{n^6+1}} + \frac{n^2}{\sqrt{n^6+2}} + \cdots + \frac{n^2}{\sqrt{n^6+n}}, \quad n \geq 1.$$

Then $\lim\limits_{n\to\infty} a_n$

(A) does not exist (B) is equal to 1

(C) is equal to e (D) is equal to $\frac{1}{e}$

26. Let n be a positive integer. The value of $\sum_{k=0}^{n} \tan^{-1} \frac{1}{k^2+k+1}$ is

(A) $\tan^{-1}(n+1)$

(B) $\tan^{-1}(\frac{1}{n+1})$

(C) $\tan^{-1} n$

(D) $\tan^{-1}(\frac{1}{n})$

27. Let $f(x) = (x^2+18)(x-4)x(x+4) - 2$. Then

(A) f has exactly one real root

(B) f has exactly 3 distinct real roots

(C) f has 5 distinct real roots

(D) f has a repeated root

28. Let k be a positive integer and $f(x) = e^x - 1$. Then

$$\lim_{x \to 0} \frac{f(x) + f(\frac{x}{2}) + f(\frac{x}{2^2}) + \cdots + f(\frac{x}{2^k})}{x}$$

is equal to

(A) $2 - \frac{1}{2^k}$

(B) $2 - \frac{1}{2^{k+1}}$

(C) k

(D) $2^{k+1} - 1$

29. Let

$$a_n = \frac{2^3-1}{2^3+1} \times \frac{3^3-1}{3^3+1} \times \cdots \times \frac{n^3-1}{n^3+1}, \quad n \geq 2.$$

Then $\lim_{n \to \infty} a_n$

(A) does not exist

(B) is equal to $\frac{2}{3}$

(C) is equal to 1

(D) is equal to $\frac{1}{2}$

30. A subset $\{u_1, u_2, u_3, u_4, u_5\}$ of the first 90 positive integers can be selected in $\binom{90}{5}$ ways. Let $u_{\max} = \max\{u_1, u_2, u_3, u_4, u_5\}$ and $u_{\min} = \min\{u_1, u_2, u_3, u_4, u_5\}$. Then the arithmetic mean of $u_{\max} + u_{\min}$ over all such subsets is

(A) 45 (B) 46 (C) 89 (D) 91

$x(x-1) = y(y-1)$

$x = \frac{1}{2}$ $y = \frac{1}{2}$

9×111

$(1-3x)^6 (1+3x)^6 (1+9x^2)^6$

$(1 - 81x^4)^6$

$1 - {}^6C_1\, 81x^4 + {}^6C_2\, 81^2 x^8$

$37 \times 3 \times 9$

$3 \times 3 \times 3 \times 37$

$\frac{6!}{2!4!} \times 3^8$

$9 \times 111 \times 1 \times 1$

$\frac{4!}{2!}$

$3 \cdot 3 \cdot 3 \cdot 37$

$3\,3\,7\,3$ $\quad 3x^3y + 6xy + 2xy^3 = 0$

$3\,3\,3\,7\,3$

$xy(3x^2 + 2y^2 + 6) = 0$

$x = 0$

$y = 0$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$3x^2 + 2y^2 = -6$

$\frac{n^3 - 1}{n^3 + 1}$

$\frac{n^2 + n + 1}{n^2 - n + 1} \frac{(n-1)}{(n+1)}$

$\tan\theta/2 = 1/3$

$f(a) \le$

$\frac{1 - \frac{1}{9}}{1 + \frac{1}{9}}$ $\quad \frac{9-1}{9+1}$ $\quad \frac{8}{10}$

$f(x) = 7x^{11} + 4x^5 - 3$

$\frac{9-1}{9+1}$ $\quad \frac{8}{10} = \frac{4}{5}$

$-7i + 4i = 3$