

# EDUCATIVE JEE (MATHEMATICS)

## ERRATA FOR SECOND EDITION

Last updated on April 3, 2015

All the errors noticed in the first edition before the second edition went to press have been corrected in the second edition. Also, the alternate proofs given in the Errata to the first edition have been incorporated in the second edition. The present errata list the errors that came to light after the second edition went to press.

Any errors still remaining may be brought to the attention of the author by e-mail ([kdjoshi@math.iitb.ac.in](mailto:kdjoshi@math.iitb.ac.in) or [kdjoshi314@gmail.com](mailto:kdjoshi314@gmail.com)) or by phone (9819961036 or 9713612285). Alternate solutions are also welcome.

On p. 37, the guess about the sum  $\sum_{i=1}^n i^r$  can be proved by induction on  $r$ . Consider the  $n$  equations  $i^{r+1} - (i-1)^{r+1} = i^r + i^{r-1}(i-1) + i^{r-2}(i-1)^2 + \dots + i^2(i-1)^{r-2} + i(i-1)^{r-1} + (i-1)^r$  for  $i = n, n-1, n-2, \dots, 3, 2, 1$ . If we add these  $n$  equations, the left hand sides add up to  $n^{r+1}$ . The sum of right hand sides contains  $(r+1)\sum_{i=1}^n i^r$ . The remaining terms

contain sums of the form  $\sum_{i=1}^n i^k$  for  $k < r$  and hence, by the induction hypothesis, add up to a polynomial in  $n$  of degree less than  $r$ .

On p. 122, the given answer to Exercise (3.12) is wrong. There are, in fact, no possible values of  $k$  because the quadratic  $x^2 - 3x + 7$  has no real roots. (Pointed out by Deepanshu Rajvanshi.)

On p. 176, a shorter, albeit trickier, proof of the identity (15) can be given by recognising the series on the L.H.S. as a telescopic series. Specifically, for  $k \geq 0$ , rewrite  $2n - 4k + 1$  in the numerator as  $(2n - 2k + 1) - 2k$ . Then note that the expression  $\frac{\binom{2n-k}{k} 2^{n-2k}}{\binom{2n-k}{n}}$  is

simply  $\frac{\binom{n}{k} 2^{n-2k}}{\binom{2n-2k}{n-k}} 2^{n-2k}$  while the expression  $\frac{\binom{2n-k}{k} 2k 2^{n-2k}}{\binom{2n-k}{n} (2n - 2k + 1)}$ , upon simplification, equals  $\frac{\binom{n}{k-1} 2^{n-2k+2}}{\binom{2n-2k+2}{n-k+1}} 2^{n-2k+2}$ . Hence the  $k$ -th term of the

L.H.S. can be expressed as  $A_k - A_{k-1}$  where  $A_k = \frac{\binom{n}{k} 2^{n-2k}}{\binom{2n-2k}{n-k}} 2^{n-2k}$ . Since  $A_{-1} = 0$ , the sum equals  $A_m$  which is precisely the R.H.S. (Contributed

by Gaurav Bhatnagar.)

On p. 341 the last term in Equation (16) should be dropped. The equation  $\cos 2x = -\cos x$  reduces to a quadratic in  $\cos x$  with  $\cos x = -1$  and  $\cos x = \frac{1}{2}$  as solutions. The corresponding values of  $x$  in  $[-\pi, \pi]$  are  $\pm\pi$  and  $\pm\frac{\pi}{3}$ . Hence the answer to the problem is  $x = \pm\pi, \pm\frac{\pi}{3}, -\frac{\pi}{2}$ .

On p. 418, in the answer to Q.8.8(e), in the degenerate case when  $a = 1$ , every  $x > 0$  is a solution. (Pointed out by Deepanshu Rajvanshi.)

On p. 436, the answer to (6.25) (f) is the union of the two parabolas  $y^2 = \pm x$ .

On p. 509, in Exercise (13.19), the condition  $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$  should be replaced by  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ . However, then there are no values of  $\lambda$  for which  $f$  has a (global) maximum and also a (global) minimum. If the problem is taken to mean a local maximum and a local minimum, then the answer is  $\lambda \in (-\frac{3}{2}, 0) \cup (0, \frac{3}{2})$ . This is obtained by putting  $u = \sin x$ , and noting that the derivative of the function  $g(u) = u^3 + \lambda u^2$  must have two distinct roots in the interval  $(-1, 1)$ . (Pointed out by Sameer Kulkarni.)

On p. 668, it is mentioned that the integral  $\int_0^{\pi/2} \frac{\sin((2m-1)u)}{\sin u} du$  can be evaluated using Chebychev polynomials. Actually, an easy evaluation is possible. Call this integral as  $J_m$  for  $m \geq 1$ . Then the integrand of  $J_{m+1} - J_m$  can be written as  $\frac{2\cos(2mu)\sin u}{\sin u} = 2\cos(2mu)$ .

But then  $J_{m+1} - J_m$  equals  $\int_0^{\pi/2} 2\cos 2m\pi du = \frac{1}{m} \sin(2mu) \Big|_0^{\pi/2} = 0$ . Thus we have shown that  $J_{m+1} = J_m$  for all  $m \geq 1$ . A direct calculation gives  $J_1 = \pi/2$ . This proves (12) on p. 668 and gives an alternate (and a more direct) solution to the problem.

On p. 989, in the solution to Exercise 27(iii), the substitution  $x = \cos^2 \theta$  was discouraged as leading to complicated partial fractions. Actually, an easy solution is possible with this substitution too because in the new integrand, the expression  $\tan \frac{\theta}{2} \sin \theta \cos \theta$  simplifies to  $(1 - \cos \theta) \cos \theta$ .

The following table lists the minor errors. (A negatively numbered line is to be counted from the bottom.)

| <i>Page (line)</i> | <i>For</i>   | <i>Read</i>  |
|--------------------|--|--|
| 63 (-18)           | $\sqrt{2}(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}})$                        | $\sqrt{2}(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}})$                        |
| 183 (2)            | $(1 - x + x^2)(1 + x + x^2)$   | $(1 - x + x^2)^n(1 + x + x^2)^n$   |
| 188 (-8)           | $(-1)^n(n + 2)$  | $(-1)^{n/2}(n + 2)$  |
| 304 (-14)          | $\lambda$ must equal 1.  | $\lambda$ must equal -1.   |
| 305 (10)           | $\lambda = 1.$   | $\lambda = -1.$  |
| 356 (-10)          | $B'y^2$  | $B'Y^2$  |
| 366 (8)            | $r_1 r_2 r_3 r_1 + r_2 + r_3$  | $\frac{r_1 r_2 r_3}{r_1 + r_2 + r_3}$                                      |
| 368 (-4)           | is far too   | is too   |
| 368 (-4)           | useful.  | useful all by itself.  |
| 436 (-6)           | $[1, \infty)$  | $\{-1\} \cup [1, \infty)$  |
| 455 (-19)          | $(at_1^2, 2at_i)$  | $(at_i^2, 2at_i)$  |
| 456 (13)           | (c) (A)  | (c) (C)  |
| 484 (-1)           | $\frac{\pi}{3}$  | $\frac{\pi}{6}$  |
| 485 (3, 14)        | $\frac{\pi}{3}$  | $\frac{\pi}{6}$  |
| 503 (-7)           | $y_2 = f(x_3)$   | $y_2 = f(x_2)$   |
| 518 (-11)          | $-\cos A_0$  | $-\sin A_0$  |
| 518 (-10)          | $-\cos B_0$  | $-\sin B_0$  |
| 518 (-9)           | $-\cos C_0$  | $-\sin C_0$  |
| 547 (-5)           | $g(x_0)$   | $f(x_0)$   |
| 564 (-17)          | $g(\alpha) = f'(c)$  | $g(\alpha) = f'(\alpha)$   |
| 579 (-15, -9)      | Comment No. 21   | Comment No. 19   |
| 597 (-2)           | but that $g$   | and that $g'$  |
| 635 (-17)          | $-\frac{1}{a} \sin(ax + b)$  | $-\frac{1}{a} \cos(ax + b)$  |
| 709 (-17)          | continuous   | non-negative and continuous  |
| 941 (-10)          | system (10)  | system (11)  |
| 942 (2)            | (10)   | (11)   |
| 971 (-20)          | 16   | 1/16   |
| 977 (-2)           | derivatives  | limits   |
| 978 (7)            | $1 + 4 \log_2 x$   | $1 + \sqrt{4 \log_2 x}$  |
| 979 (2)            | $g(0)$ and $g(1)$  | $g(0)$ and $g(\frac{1}{2})$  |
| 986 (-14)          | (c) 1  | (c) $2 \ln 2 - 1$  |
| 990 (10)           | $1 - \frac{\sqrt{3}}{2}$   | $\frac{1}{2} - \frac{\pi\sqrt{3}}{12}$                                     |
| 1001 (-8)          | (d) 0.   | (d) 14.  |
| 1024 (2)           | $2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$ | $2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$ |
| 1046 (11)          | 977  | 1034   |
| 1052 (18)          | Yaglom, 976  | Yaglom, 1034   |