By [illegible] / By [illegible] Most plot lib

1995 IIT Kanpur

IIT Kharagpur

Robotics

→ ROBOT : automatically controlled, reprogrammable and multipurpose machine.

→ In Robotics, we try to copy HAND, HEAD, HEART (3 H's) of Human beings.

→ Robots are available in the form of MANIPULATOR, WHEELED robots, MULTI-LEGGED robots, TRACKED vehicle, DRONE

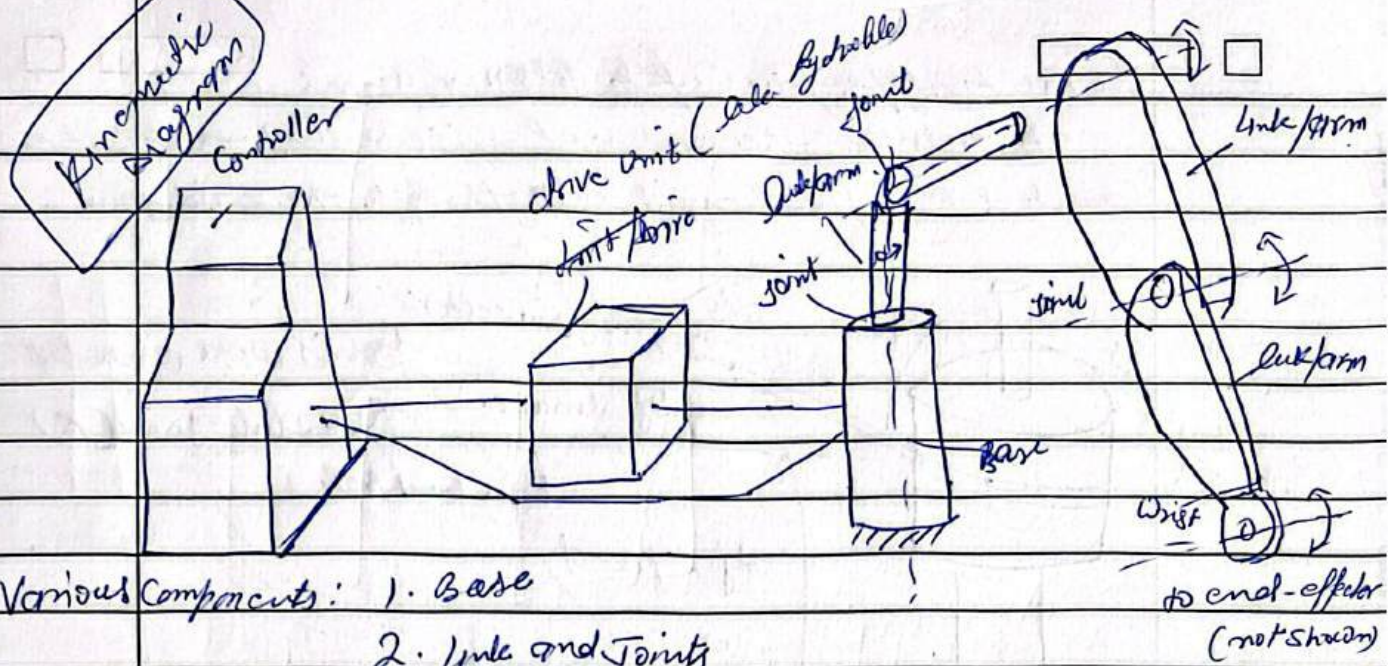
→ four modules of Robotics : KINEMATICS, DYNAMICS (Mechanical Engineering), CONTROL SCHEMES (Electrical / Electronics Engineering), INTELLIGENCE ISSUES (Computer Science)

* Robotics by Fu, Gonzalez and Lee.
* Introduction to Robotics by J.J. Craig

BE79 9670 4078 5533

P10583647

Scanned with CamScanner



Various Components: 1. Base
2. Link and Joints
3. end-effector / Gripper
4. Wrist
5. Drive / Actuator
6. Controller
7. Sensors

Mechanical Engineering
→ Kinematic: motion of robot arm without considering the force and/or moments.
→ Dynamics: Study of the forces and/or moments
→ Sensing: collecting information of the environment

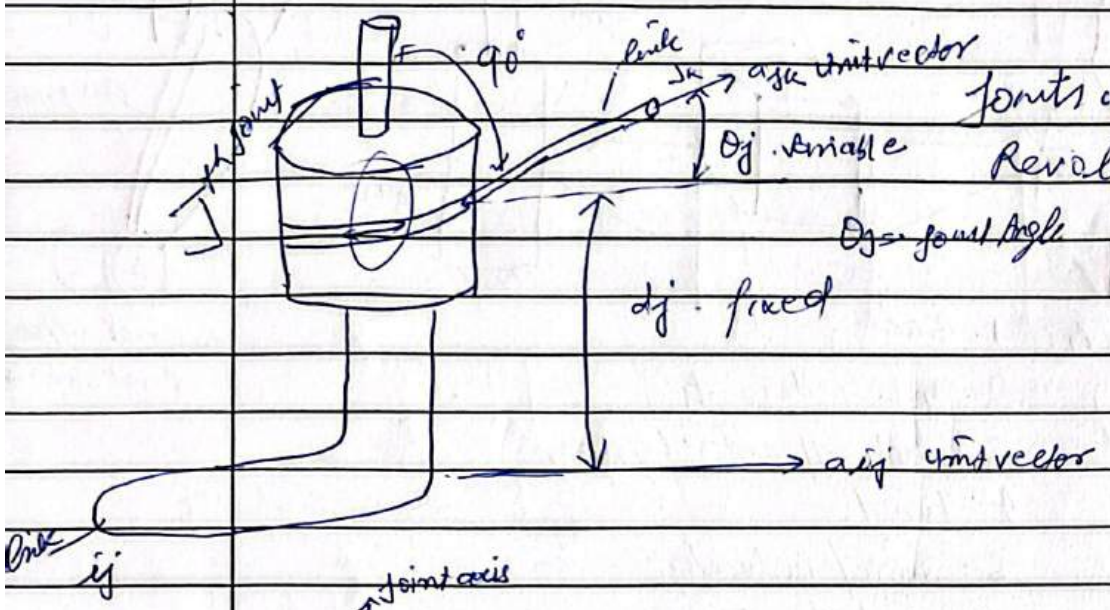
Computer Science
→ motion planning: planning the course of action
→ Artificial Intelligence: To design and develop suitable brain for the robots
→

Electrical and Electronics Engineering
→ Control schemes and hardware implementations

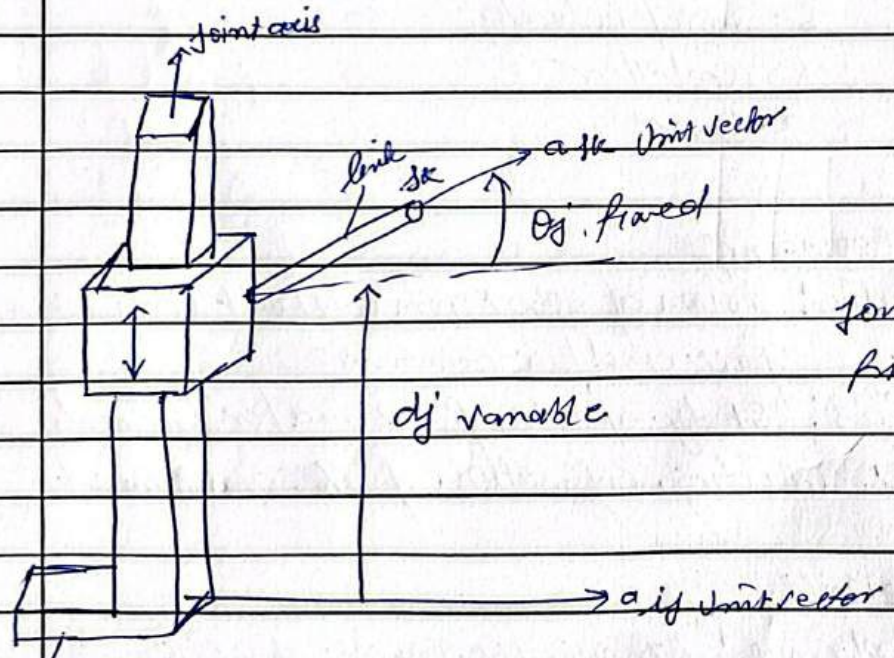
General Sci Sciences:
→ physics.
→ mathematics

Connectivity/degrees of freedom of a joint

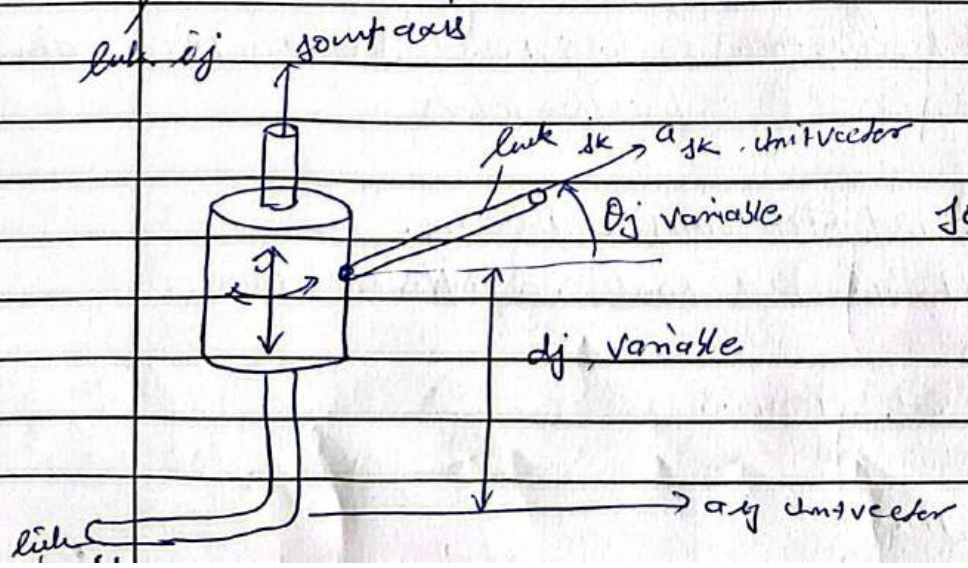
It indicates the number of rigid (bodies) that can be connected to a fixed rigid body through the said joint.



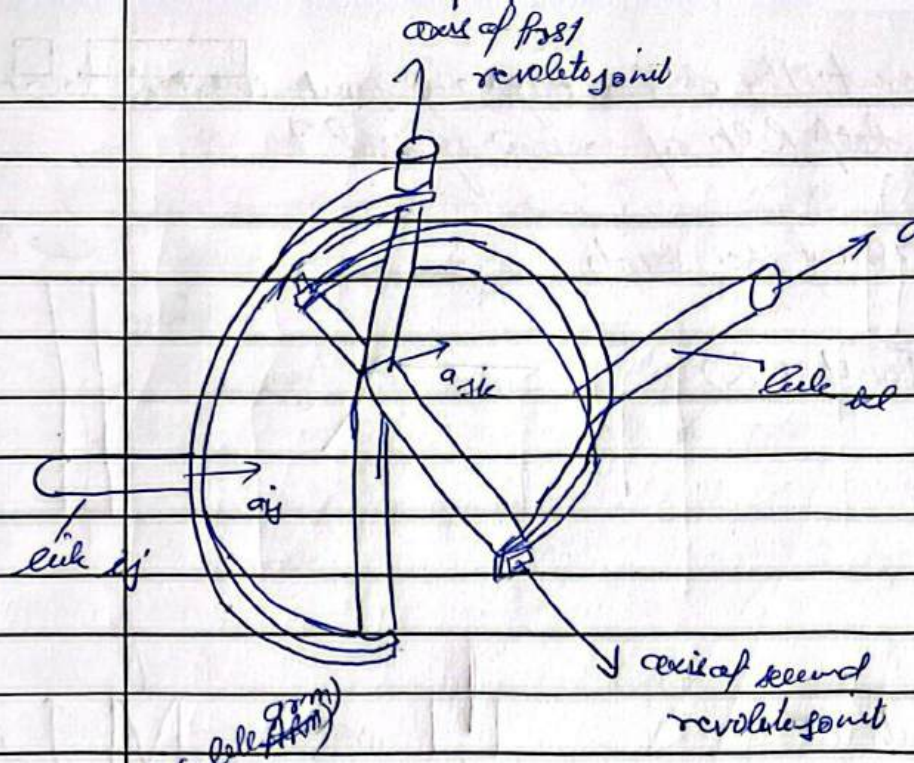
Joints with one dof
Revolute joint (R)



Joints with one dof
Prismatic Joint (P)



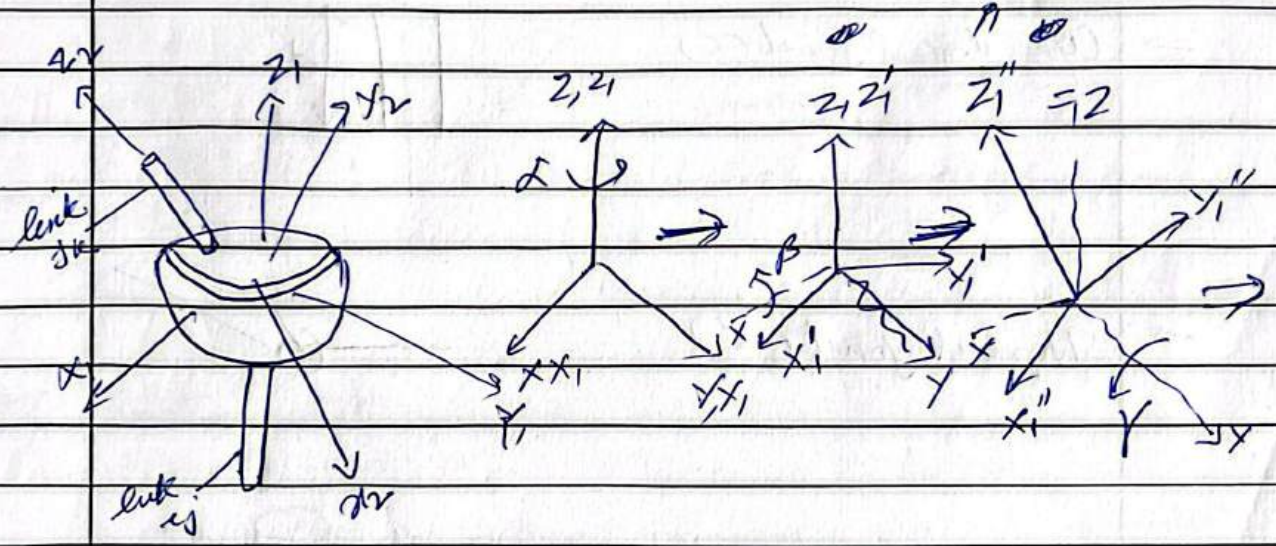
Joint with two dof
Cylindrical joint (C)



joints with two dof

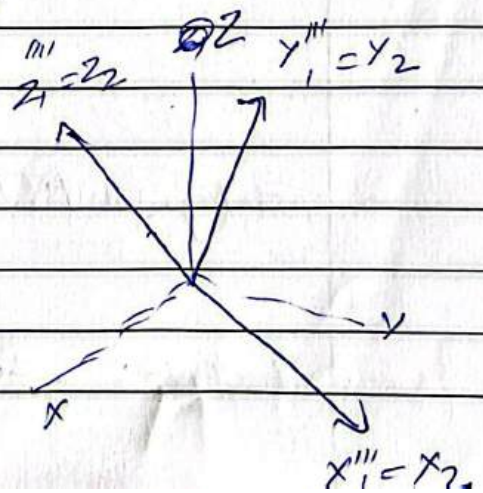
Hooke joint or Universal joint (U)

Snap & Parallel manipulator



joints with three dof

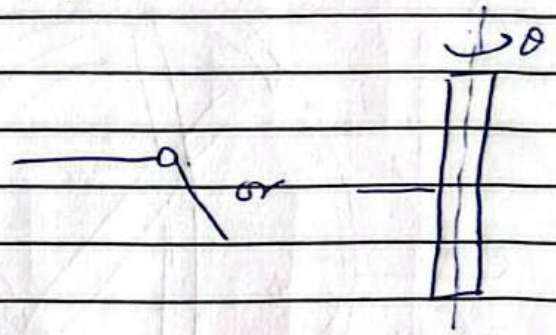
Ball and Socket joint / spherical joint (S)



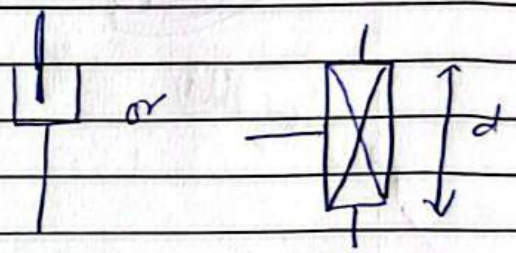
How to represent the different types of joint used in Robot with the ~~help~~ help of a few symbol?

Representation of the joints

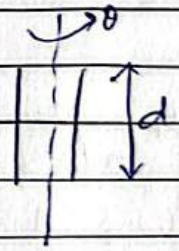
→ Revolute Joint (R)



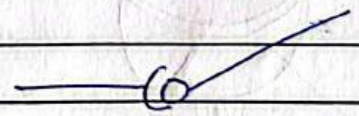
→ Prismatic Joint (P)



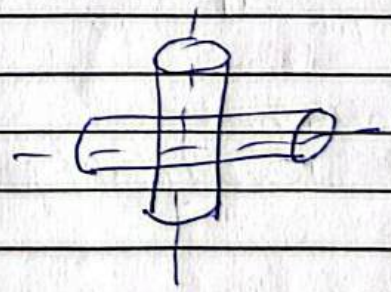
→ Cylindrical Joint (C)



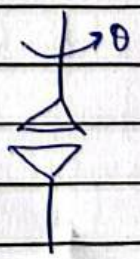
→ Spherical joint (S)



→ Hooke joint (U)



→ Twisting Joint (T)



→ End effector

Degrees of freedom of a System

→ It is defined as the minimum number of independent parameters/variable/coordinates needed to describe a system completely.

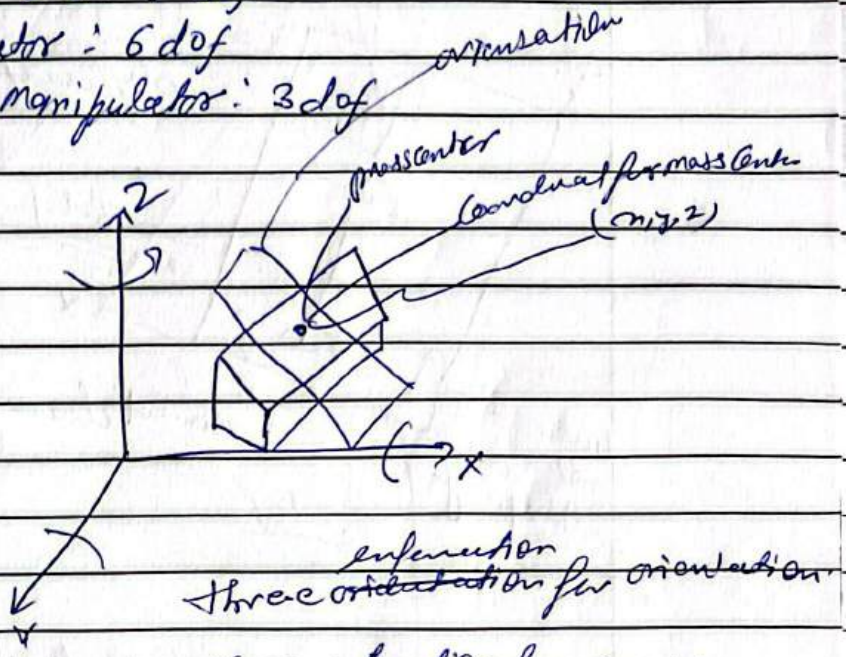
Notes:-

→ ~~A point~~ A point in 2-D dof; in 3-D space: 3dof

→ A rigid body in 3-D: 6 dof

→ spatial manipulator: 6 dof

→ ~~Planer~~ planer manipulator: 3dof



rigid body

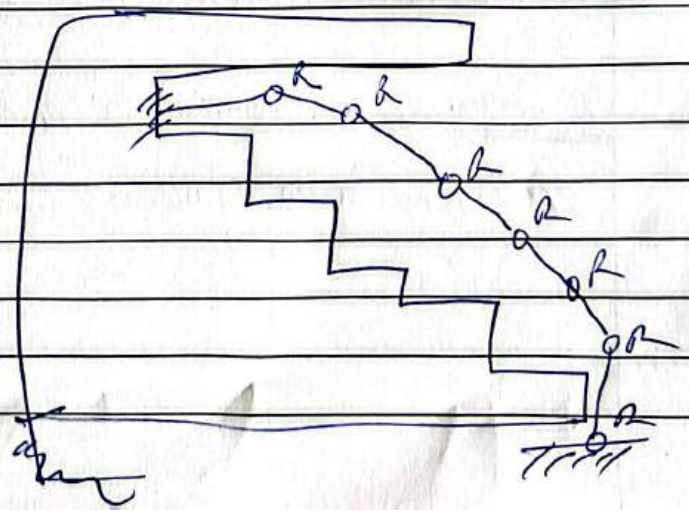
three information for orientation

three information for position

3d object in space
get 6 dof

Redundant Manipulator

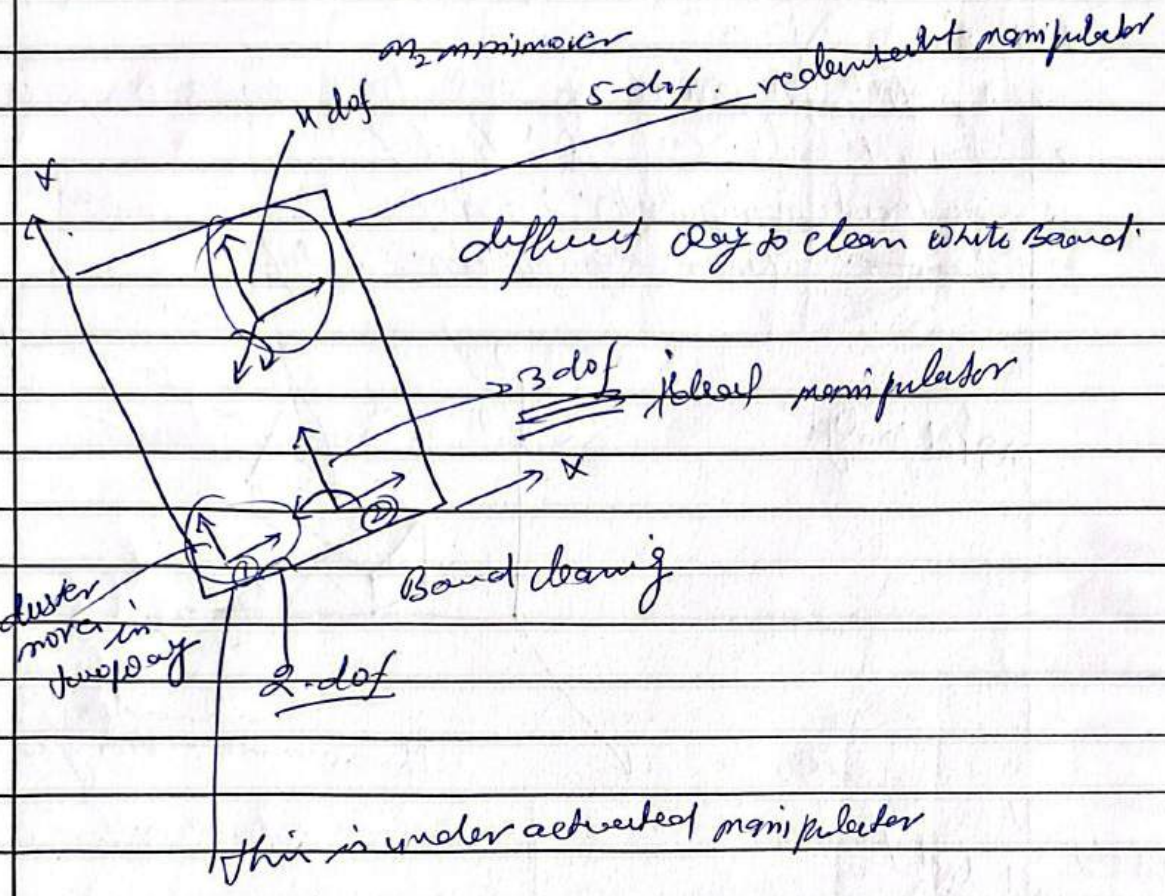
→ Either a spatial manipulator with more than 6 dof or a planar manipulator with more than 3dof



Under-actuated Manipulator

→ Either a spatial manipulator with less than 6 dof or a planar manipulator with less than 3 dof.

#



Mobility / dof of spatial manipulator

→ Let us consider a manipulator with n rigid moving links and m joints

c_i: Connectivity of i-th joint; $i = 1, 2, 3 \ldots, m$

No. of constraints put by i-th joint $= (6 - c_i)$

Total no. of constraints $= \sum_{i=1}^{m} (6 - c_i)$

Mobility of the manipulator $M = 6n - \sum_{i=1}^{m} (6 - c_i)$

It is known as Grubler's Criterion

mobility / dof of Planar Manipulator

→ let us consider a manipulator with n rigid moving links and m joints

c_i : connectivity of ith joint ; $i = 1, 2, 3 \cdots, m$

No. of constraints put by i-th joint $= (3 - c_i)$

Total no. of constraints $= \sum_{i=1}^{m} (3 - c_i)$

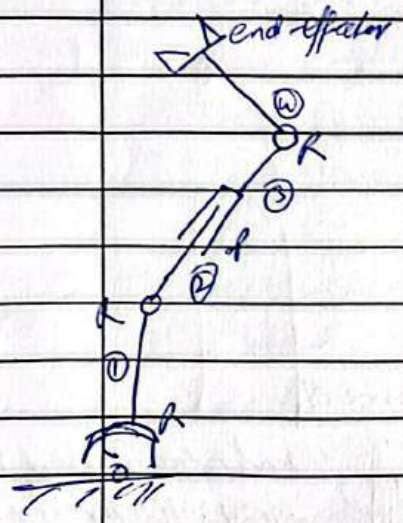
implies dof

mobility of a manipulator $M = 3n - \sum_{i=1}^{m} (3 - c_i)$

It is known as Grubler's criterion.

Example

Serial Planar manipulator



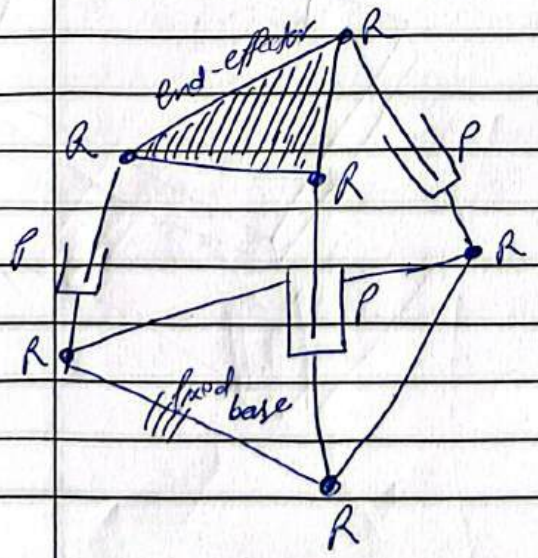
$n = 4$, $m = 4$

$c_1 = c_2 = c_3 = c_4 = 1$ | $3 - c_i$, $3 - 2 = 1$

mobility / dof

$M = 3n - \sum_{i=1}^{m} (3 - c_i) = 3 \times 4 - 8 = 4$

Parallel pole planar manipulator



$n = 7$, $m = 9$

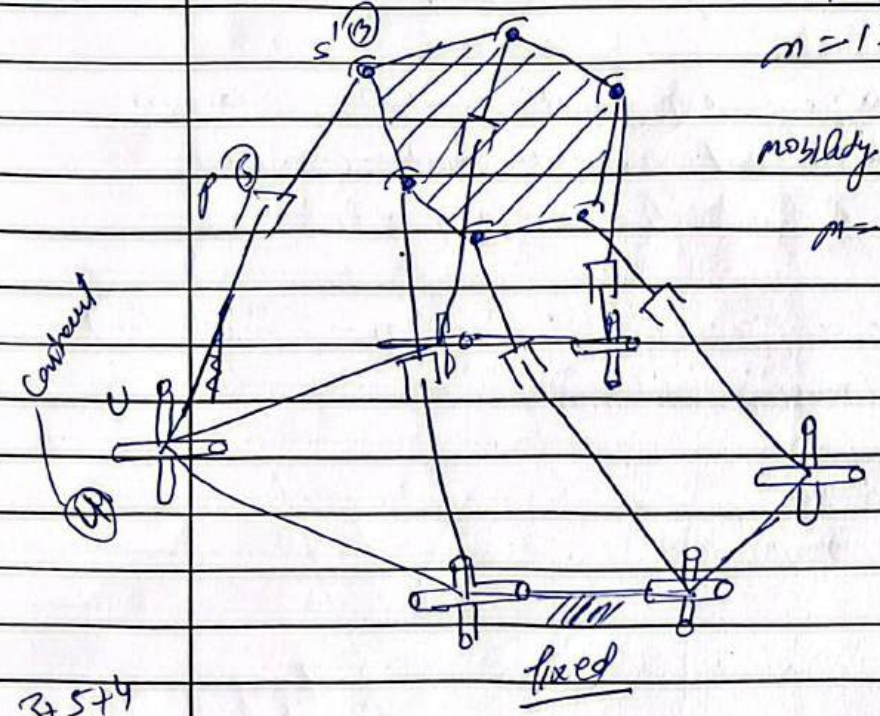
$c_i = 2$ $c_i = 1$ when $i = 1, \cdots, 9$

mobility / dof

$M = 3n - \sum_{i=1}^{m} (3 - c_i) = 3 \times 7 - 18$

$= 3$

Parallel Spatial manipulator.



$n = 13$, $m = 18$

mobility / dof

$$M = 6n - \sum_{i=1}^{m} (6 - C_i)$$

$$= 6 \times 13 - 72 = 6$$

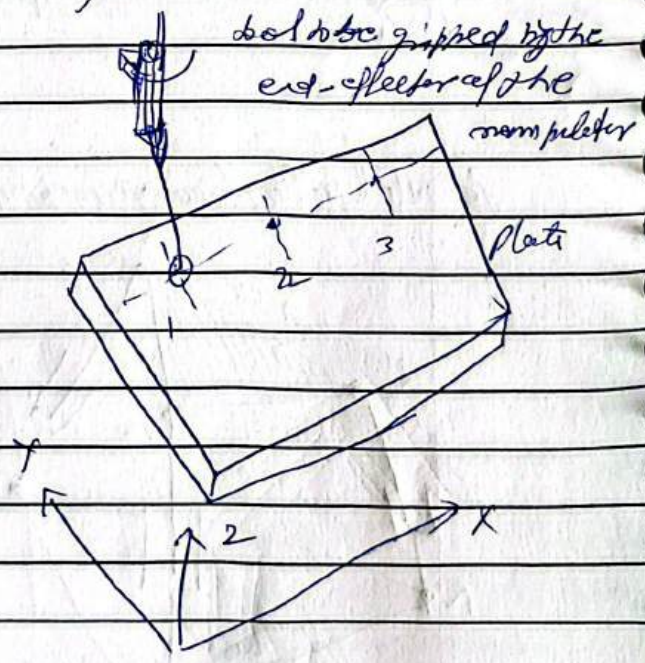
$3 + 5 + 4$
$= 12$
$= 12 \times 6 = 72$

Classification of robots.

→ Based on the Type of Task performed.

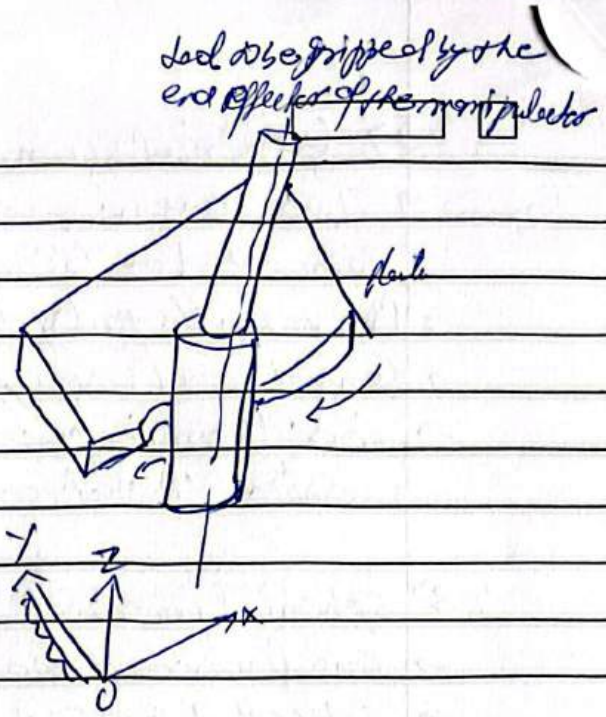
1.) Point-to-Point robots

examples: Unimate zero
T^3



② Contd

2) Continuous Path Robots
example: PUMA
CRS

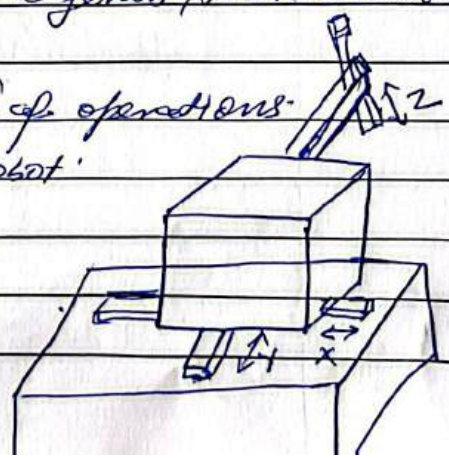


Continuous Path Robot can be used as point to point Path Robot.

* Based on the Type of Controllers:
1) Non-Servo-Controlled Robots.
→ Open-loop Control System
Example: Seiko PN-100
· Less accurate and less expensive

2) Servo-controlled Robots.
→ Closed-loop Control System
Example: Unimate 2000, PUMA
T^3
· More accurate and more expensive

* Based on Configuration (Coordinate system) of the Robot
1) Cartesian Coordinate Robot:
→ Linear movement along three different axes.
→ Have either sliding or prismatic joints, that is SSS or PPP.
→ Rigid and accurate.
→ Suitable for pick and place type of operations.
→ Examples: IBM's RS-1, Sigma robot.



2) Cylindrical Coordinate Robot.

→ Two linear and one rotary movement.

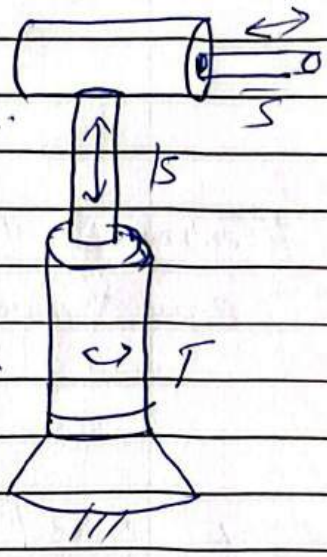
→ Represented as TPP, TSS

→ Used to handle parts/objects in manufacturing

→ Cannot reach the objects lying on the floor

→ Poor dynamic performance.

Example: Versatran 600



3) Spherical Coordinate or Polar Coordinate Robots.

→ One linear and two rotary movement

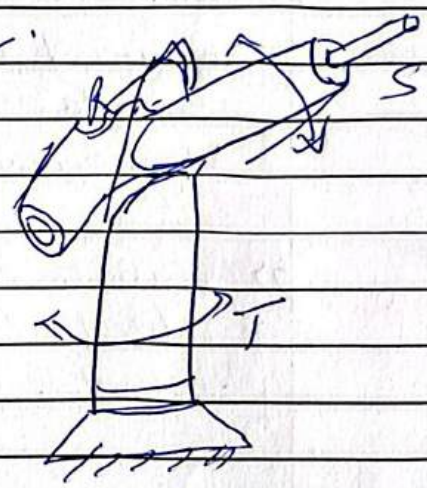
→ Represented as TRP, TRS

→ Suitable for handling parts/objects in manufacturing

→ Can pick up objects lying on the floor

→ Poor dynamic performance

example: Unimate 2000B



4) Revolute Coordinate or Articulated Coordinate Robots.

→ Rotary movement about three independent axes.

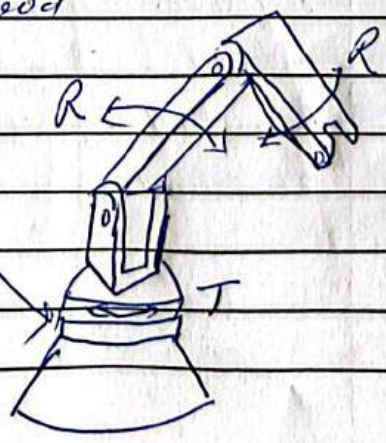
→ Represented as TRR

→ Suitable for handling parts/components in manufacturing system

→ Rigid and accuracy may not be good enough.

Example: T3, 2000

Dynamic performance may not good because of



* Based on mobility levels:

1) Robots with fixed base (also known as manipulators)

manipulators
- Serial — PUMA, CRS
- Parallel — Stewart platform

Loading Carry Capability of parallel is more than Serial.

2.) Mobile Robots.

mobile robots.
- Wheeled Robot
- Tracked robots
- multi-legged robots.

for smooth ~~terrain~~ terrain go for wheeled robot.

if perfectly rough → go for multi-legged robot.

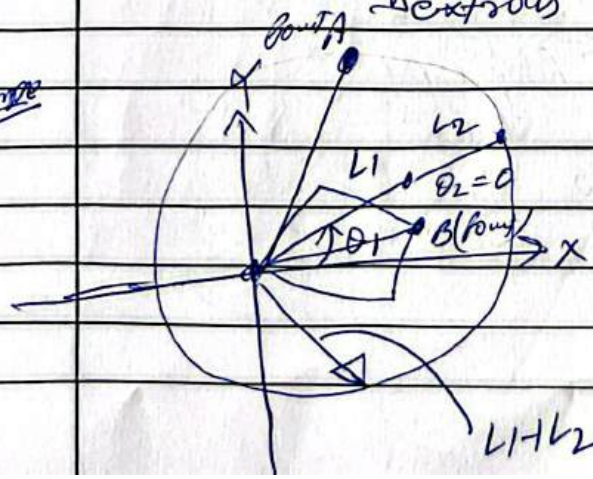
if terrain in between - not ~~not~~ neither smooth nor rough then go for Tracked robot.

Workspace of manipulators.

It is the volume of space that the end-effector of a manipulator can reach.

Workspace
- Dextrous
- Reachable

Example

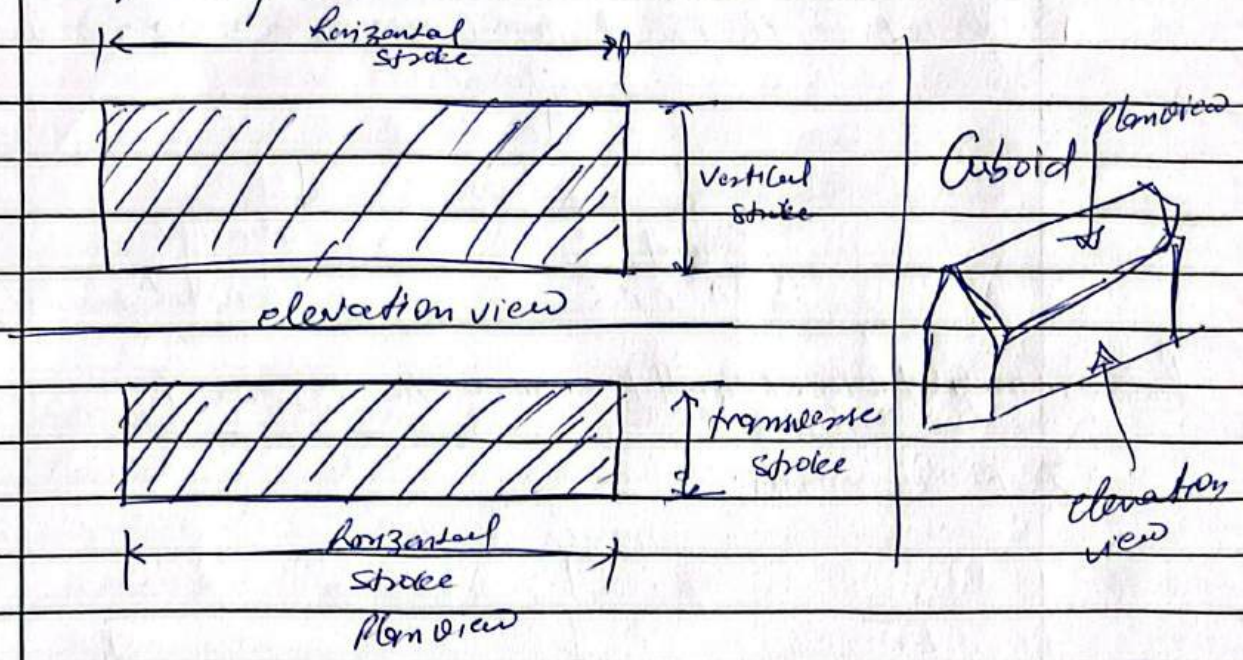


Dextrous workspace: It is the volume of space, which the robot's end-effector can reach with various orientations

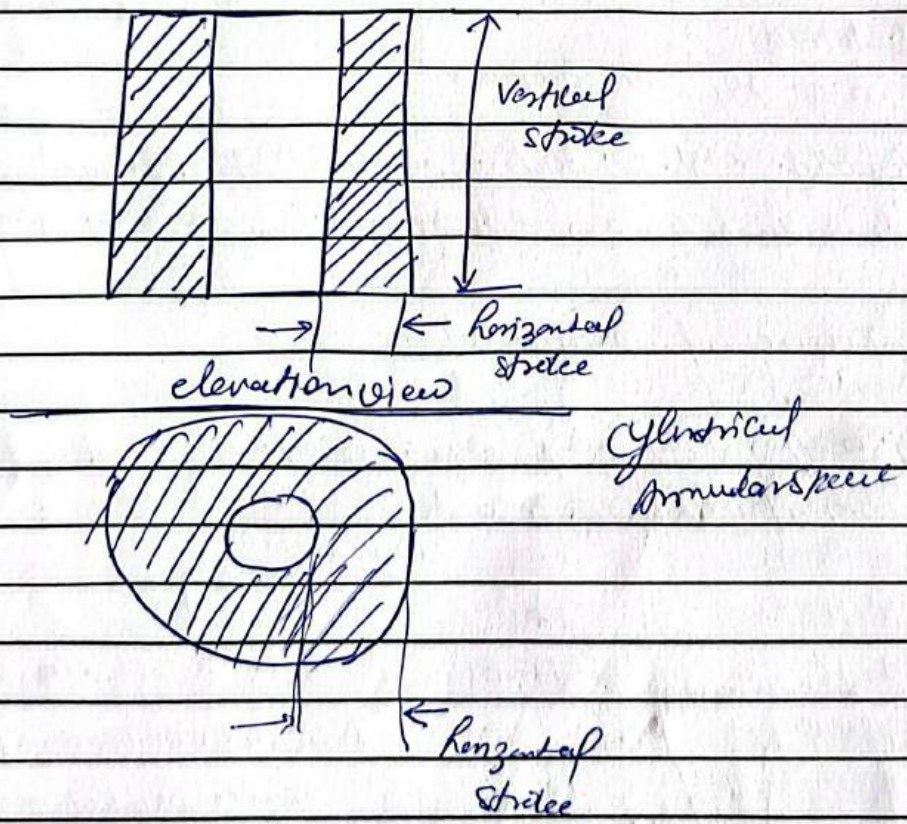
Reachable workspace: It is the volume of space that the end-effector can reach with one orientation

Note: Dextrous workspace is a subset of the reachable workspace.

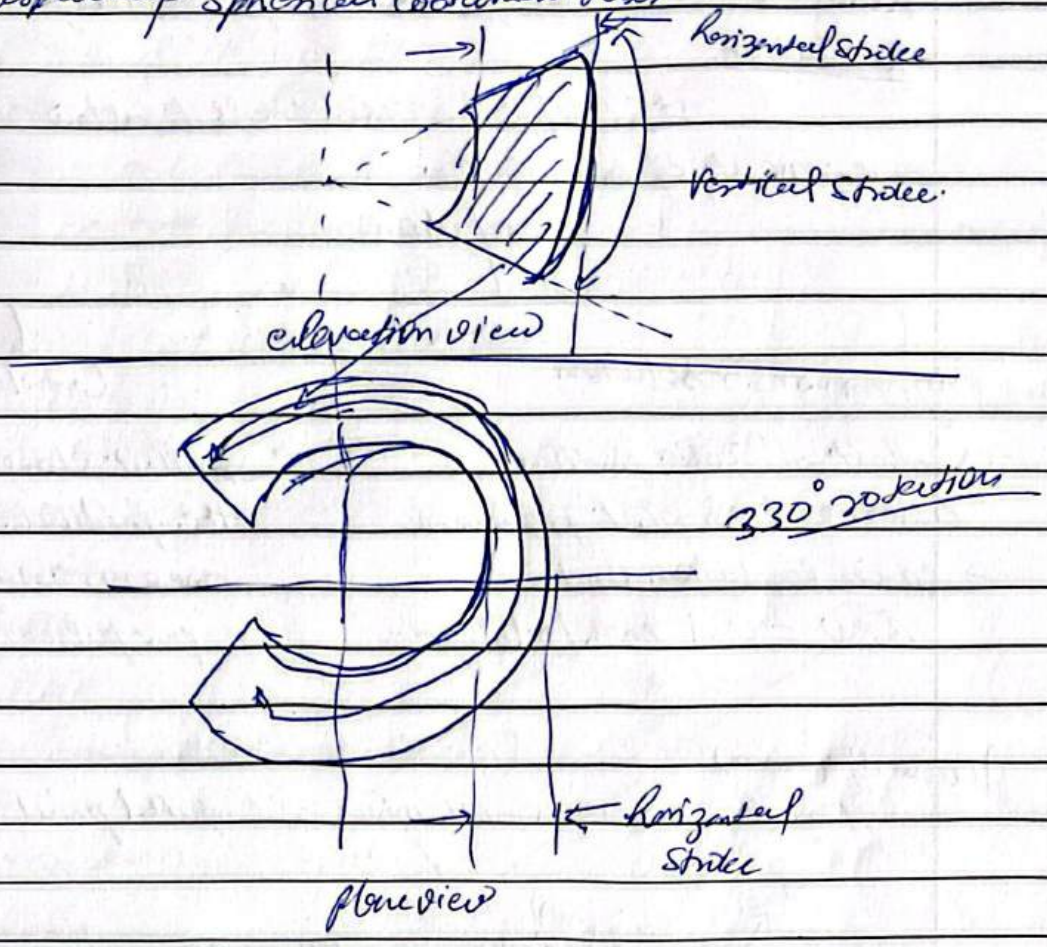
Workspace of Cartesian Coordinate Robot :



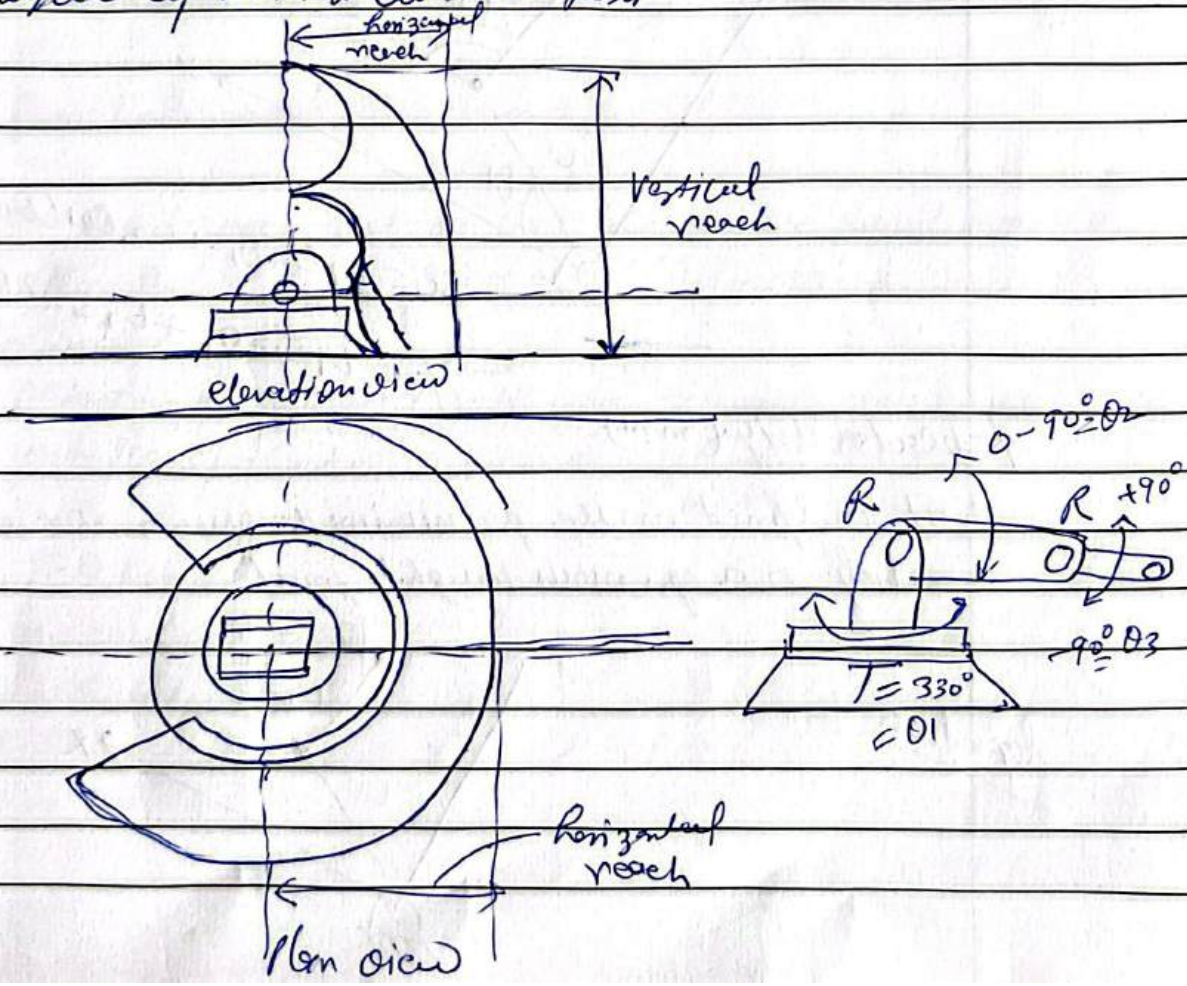
Workspace of Cylindrical Coordinate Robot :



Workspace of spherical Coordinate Robot.



Workspace of Revolute Coordinate Robot.



Resolution, Accuracy and Repeatability:

Resolution:

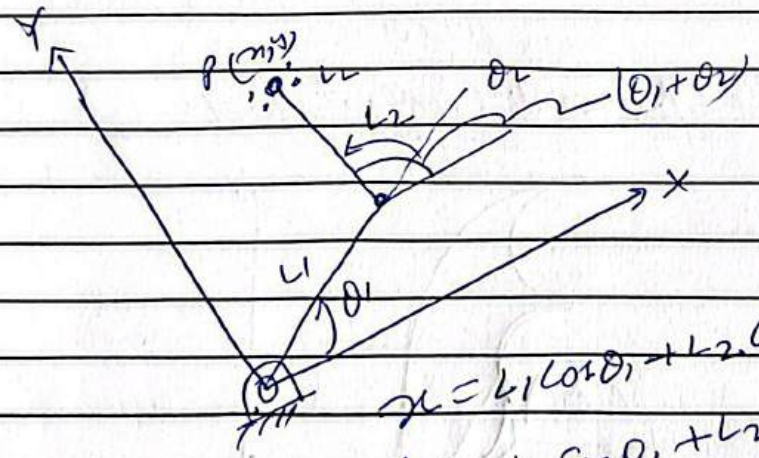
It is defined as the smallest allowable position increment of a robot.

Resolution

- Programming resolution
  - → Smallest allowable position increment in robot programme.
  - → Basic Resolution Unit BRU = 0.01 inch / 0.1 degree
- Control resolution
  - → Smallest change in position that the feedback device can measure say 0.36 degree per pulse.

Accuracy (mm)

It is the precision with which a computed point can be reached.



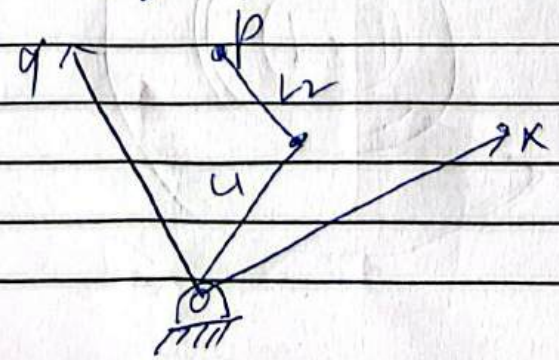
$$x = L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2)$$

$$y = L_1\sin\theta_1 + L_2\sin(\theta_1+\theta_2)$$

Repeatability (mm)

It is defined as the precision with which a robot re-position itself to a previous taught point.

derivation.



Applications of Robots.

In manufacturing units

Advantage of robots:

→ Robot can work in hazardous and dirty environment

→ Can increase productivity after maintaining improved quality

→ Direct labour cost will be reduced.

→ material cost will be reduced.

Application Areas:

→ Arc welding

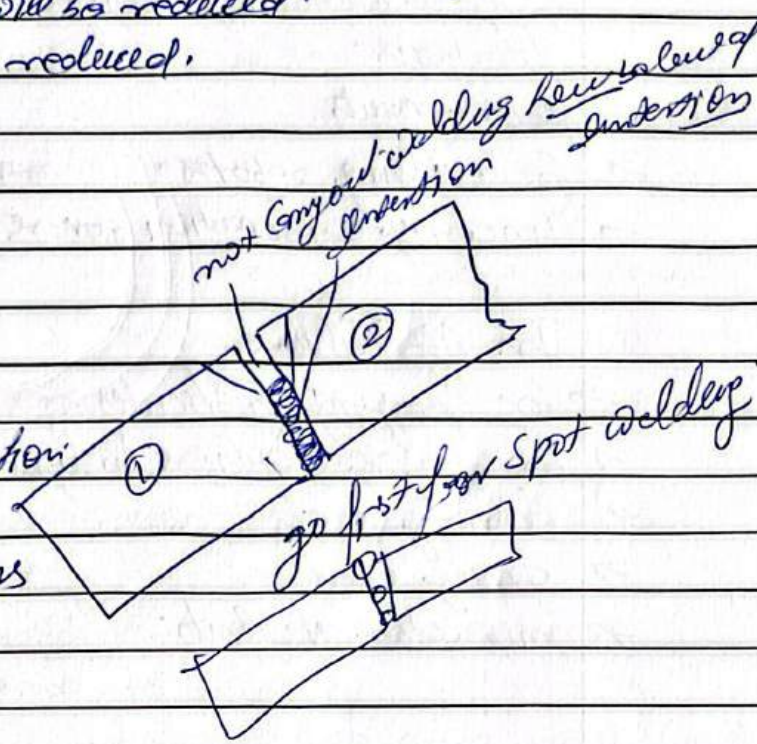
→ spot welding

→ Grinding

→ Drilling

→ milling

→ pick and place operation



Under water applications

Purpose
→ to explore various

different design & development is required.

Note: Robot are developed in the format ROV (Remotely operated vehicle) and UAV (Autonomous under water vehicle).

→ Robot are equipped with navigational sensors, propellers/thrusters, on-board software and others.

medical application

→ Telesurgery

→ micro-capsule multi-legged robots.

→ Prosthetic devices.

Space Applications:

→ For carrying out on-orbit services, assembly jobs and interplanetary missions.

→ Spacecraft deployment and retrieval, survey of outside space shuttle; assembly, testing, ~~manufacturing~~ maintenance of space stations; transport of astronauts to various locations.

→ Robo-nauts

→ Free-flying robots.

→ Planetary exploration rovers.

In Agriculture

→ For spraying pesticides

→ For spraying fertilizers in liquid form.

→ Cleaning weeds.

→ Sowing seeds

→ Inspection of plants.

Scanned with CamScanner

Robot End Effectors:

An end-effector is a device attached to the wrist of a manipulator for the purpose of holding materials, parts, tools to perform a specific task.

End Effectors

- Grippers
  End-effectors used to grasp and hold objects
- Tools
  End-effectors designed to perform some specific tasks
  ex: spot welding electrode, spray gun.

Classification of Grippers:

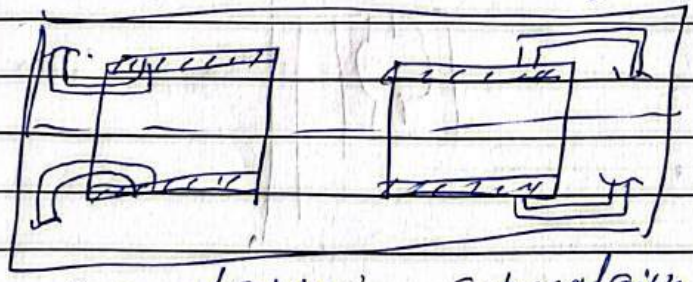
1. Single gripper and double gripper:

→ Single gripper: only one gripping device is mounted on the wrist.

→ Double gripper: Two independent gripping devices are attached to the wrist.

Example: Two separate grippers mounted on the wrist for loading and unloading applications.

2. Internal gripper vs External gripper

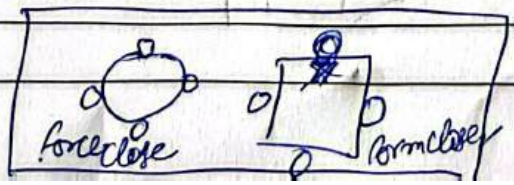


Internal gripper External gripper

3. Soft gripper vs Hard gripper

Hard gripper: Point contact b/w the finger and object.

Soft gripper: Area (surface) contact b/w the finger and object.



4. Active Gripper and Passive Gripper

Active gripper: Gripper equipped with sensor

Passive gripper: Gripper without sensor

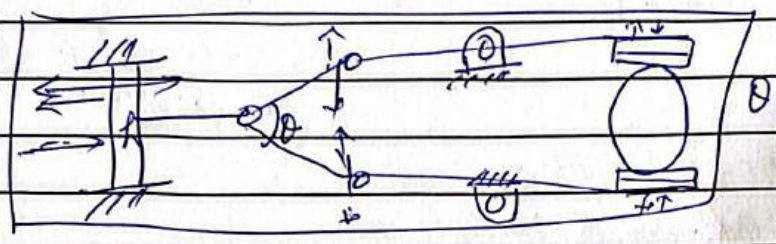
A few Robot Grippers.

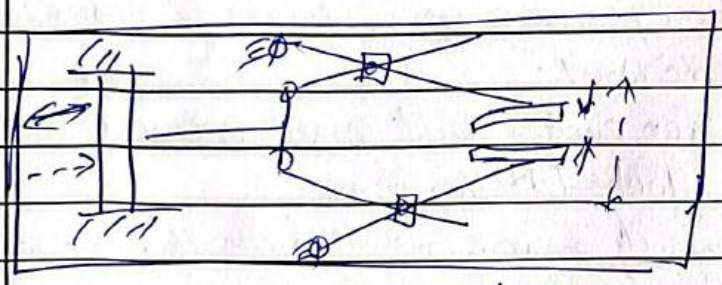
1. Mechanical Grippers.

→ use mechanical fingers (jaws) actuated by some mechanisms.

→ Less versatile, less flexible and less costly.

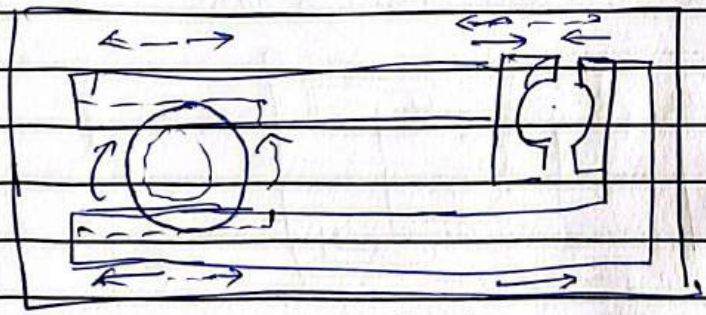
i) Gripper with linkage actuation





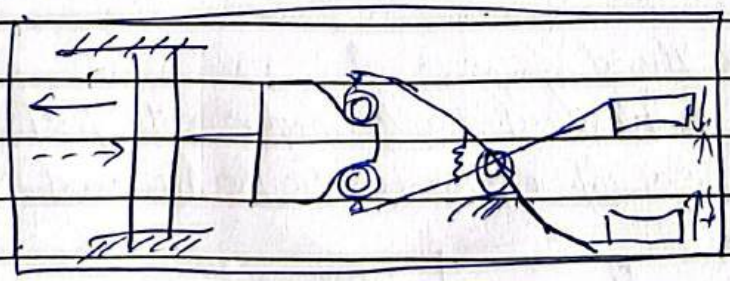
Swing block mechanism

ii) Gripper with rotary actuation



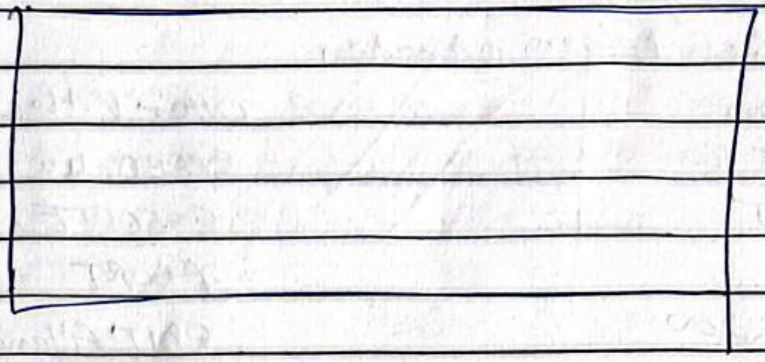
Rack-Pinion

(iii) Gripper with Cam actuation.



Cam-follower

2- Vacuum Copper (used for thin parts)



V.P

$Q = AVcc$

Scanned with CamScanner

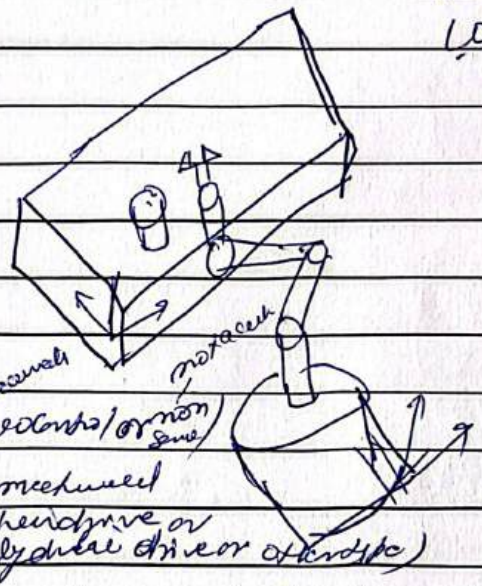
08 09

Off-line method.

VAL Programming for PUMA.

Task: Pick and place operation

VAL Program	Other VAL Commands
APPRO PART, 100	SPEED 100
MOVES PART	EXECUTE
CLOSEI	ABORT
READPARTS 200	EDIT filename
APPROS BIN, 300	LISTE
MOV BIN	STORE
OPENI	DELETE
DEPART 100	LOAD filename



Specification of a robot

→ Control type (Servo control or non servo)

→ Drive System (Pure mechanical or electric drive or hydraulic drive or other type)

→ Coordinate system (Cartesian or other)

→ Teaching (Programming methods.

→ Accuracy, Repeatability, Resolution.

→ Pay-load Capacity

→ Weight of the manipulator

→ Applications

→ Range and speed of arms and wrist.

→ Sensors used.

Scanned with CamScanner

Economic Analysis:

→ Let F: Capital investment to purchase a robot which include its purchasing cost and installation cost.

B: Saving in terms of material and labour cost.

C: Operating and maintenance cost.

D: Depreciation of the robot.

A: Net saving

$$A = B - C - D$$

G: Tax to be paid on the net savings.

Pay-back period, t = (Capital investment, F)/(B − C − G)

Let I: modified net saving after the payment of tax.

→ Rate of return on investment

$$H = (I/F) \times 100\%$$

A company decides to purchase the robot, if:

→ Pay-back period < techno-economic life

→ rate of return on investment > rate of bank interest.

10 Numerical example.

The cost and saving associated with robot installation are given below:

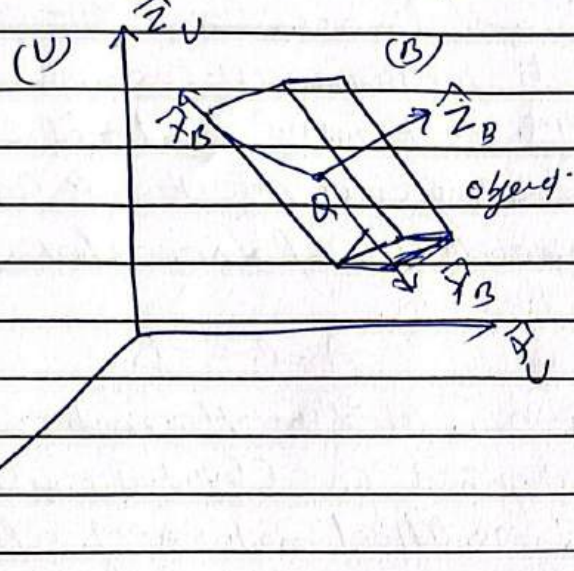
Cost of a robot include accessories	: Rs. 12,00,000
Installation cost	: Rs. 3,00,000
Maintenance & operating cost	: Rs. 20 per hour
Labour saving	: Rs. 100 per hour
Material saving	: Rs. 15 per hour

The shop run 24 hours in a day (in 3 shift) and the effective working days in a year are 300. The tax rate of the company is 30% and techno-economic life of the robot is expected to be equal to six years.

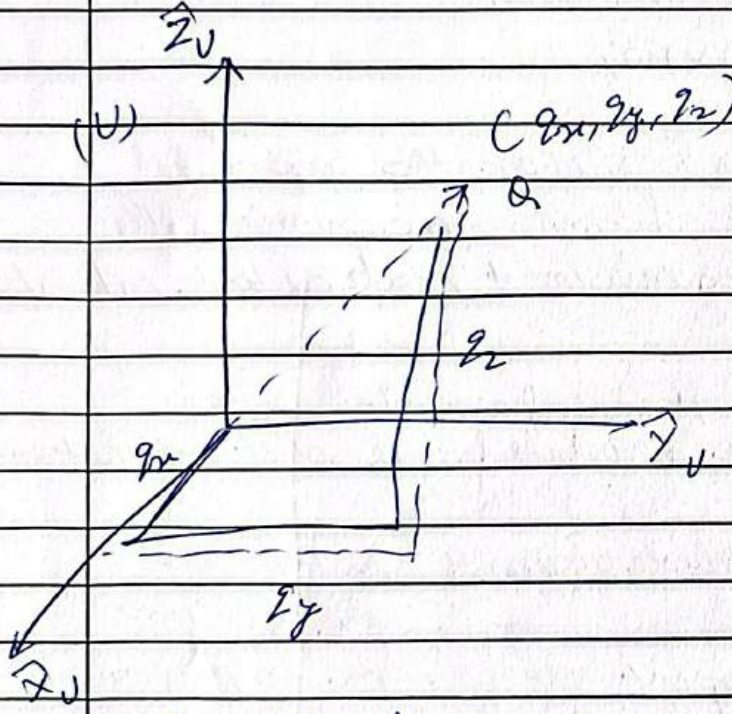
Determine (a) pay-back period of the robot and (b) rate of return on investment.

11 Robot Kinematics

Representation of an object in 3-D space.



Position: 3 information
rotation
object orientation: 3 info
need total 6 information



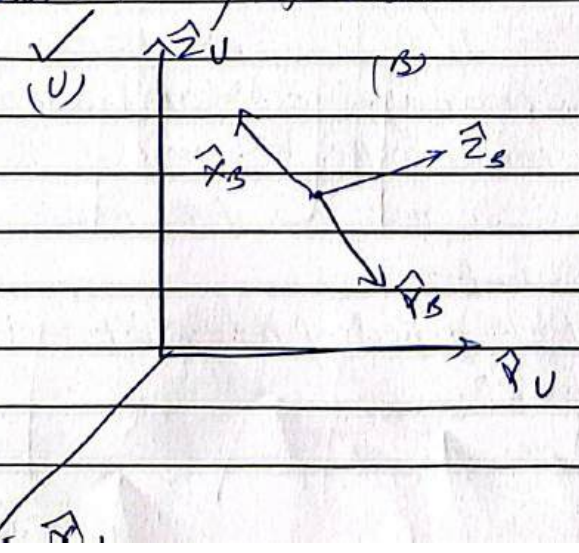
$$U\bar{Q} = \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix}$$ 3x1 matrix or vector

1 col

3 row

Cartesian coordinate system

Representation of the orientation

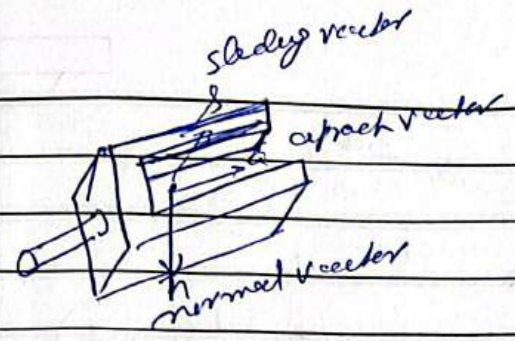


$$^{U}_{B}R = \begin{bmatrix} {}^{U}\hat{X}_B & {}^{U}\hat{Y}_B & {}^{U}\hat{Z}_B \end{bmatrix}_{3\times3 \text{ matrix}}$$

one vector, another vector, another vector

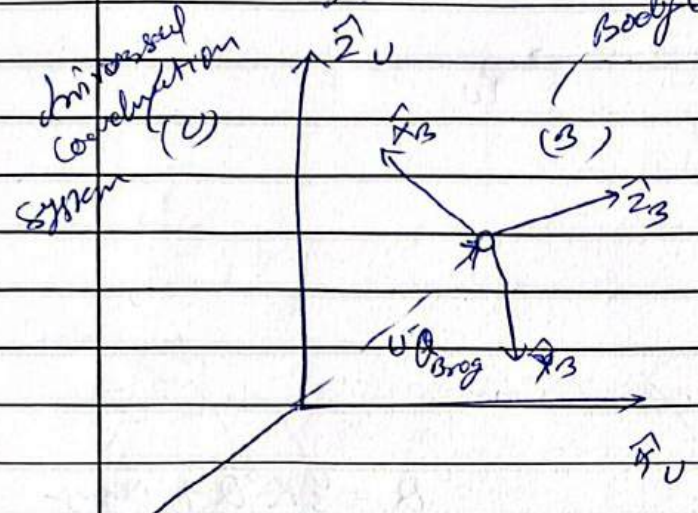
~~a set~~ a set of 3 vector

Rotation B with respect to U



$$\begin{bmatrix} n_x & s_x & a_x \\ n_y & s_y & a_y \\ n_z & s_z & a_z \end{bmatrix}$$

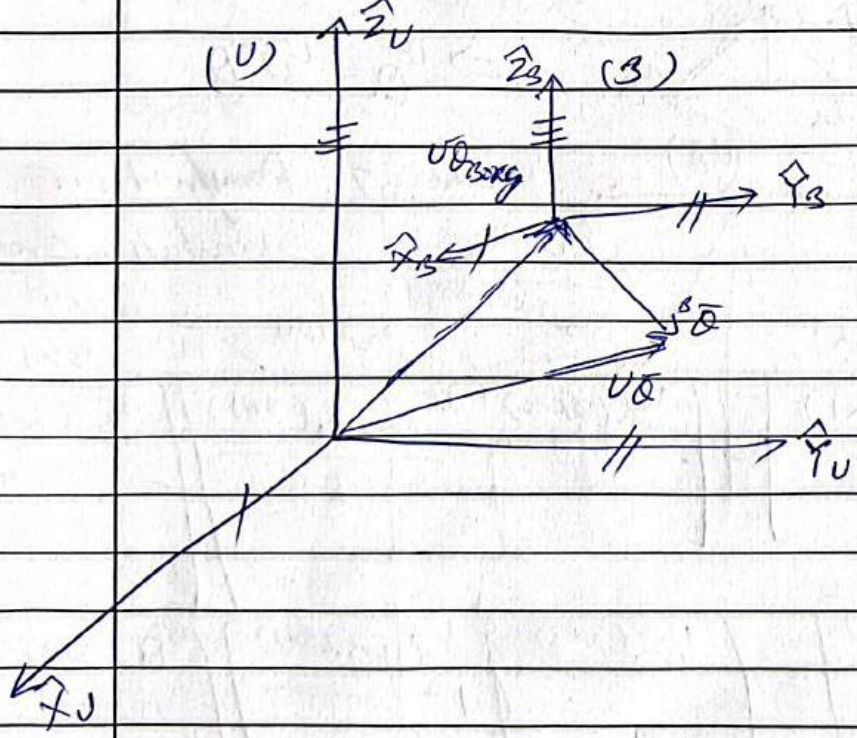
Frame Transformations:



Frame: A set of four vectors carrying position and orientation information

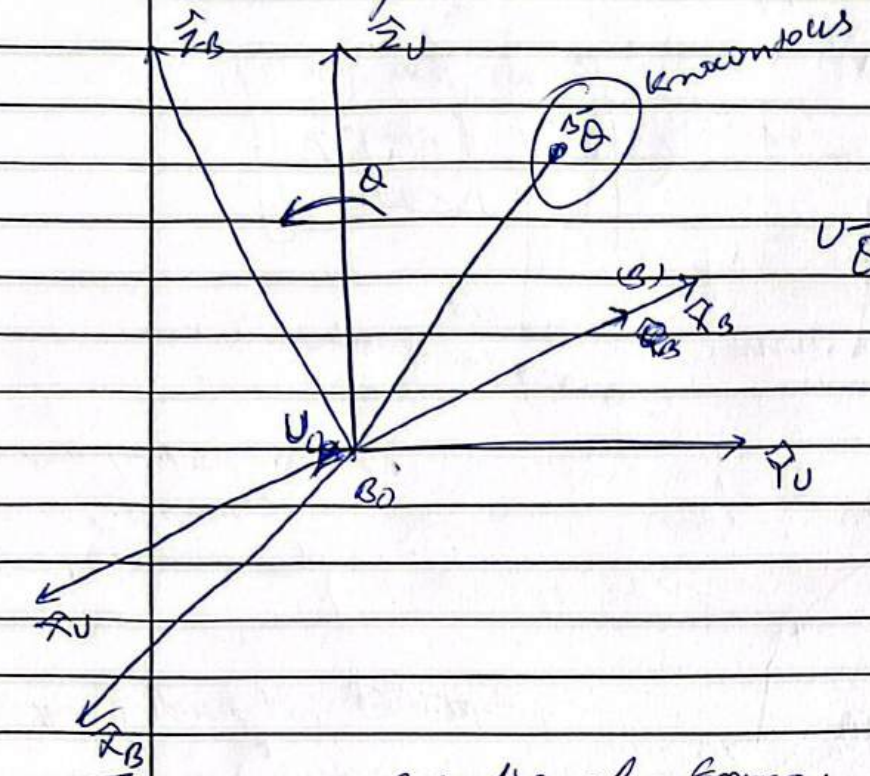
~~not parallel~~ not parallel anything

Frame Transformations.



$$^U\bar{Q} = {}^U\bar{Q}_{Borg} + {}^B\bar{Q}$$

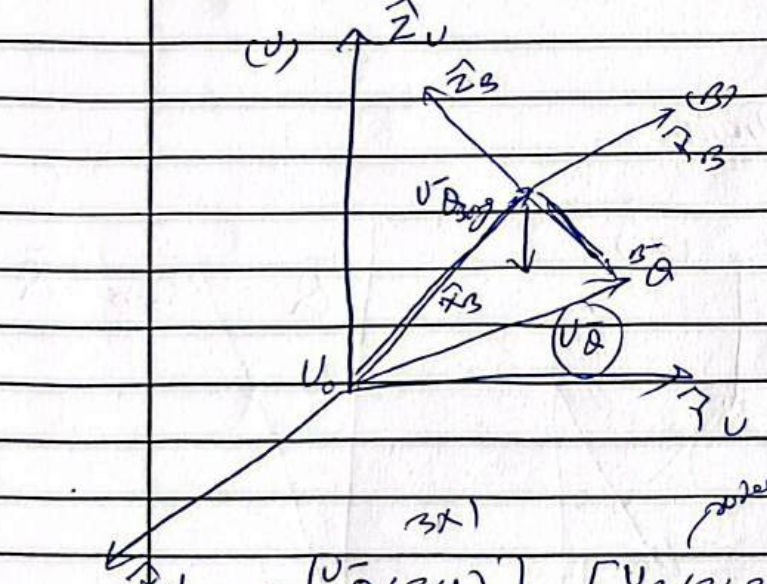
Rotation of a frame



$${}^U\bar{Q} = \underbrace{{}^U_BR}_{\text{known}}\,{}^B\bar{Q}$$

Translation & Rotation of a frame.

$${}^U\bar{Q} = \underbrace{{}^U_BR}_{\text{known}}\,{}^B\bar{Q} + \underbrace{{}^U\bar{Q}_{Borg}}_{\text{known}}$$



$${}^U\bar{Q} = {}^U_BR\,{}^B\bar{Q} + {}^U\bar{Q}_{Borg}$$

$$\Rightarrow {}^U\bar{Q} = {}^U_BT\,{}^B\bar{Q}$$

Where T: transformation = translation & rotation

$$\Rightarrow \underbrace{\begin{bmatrix} {}^U\bar{Q}(3\times1) \\ \hline \end{bmatrix}}_{3\times1} = \left[\begin{array}{c|c} \underbrace{{}^U_BR(3\times3)}_{\text{rotation}} & \underbrace{{}^U\bar{Q}_{Borg}(3\times1)}_{\text{position}} \\ \hline \end{array}\right] \underbrace{\begin{bmatrix} {}^B\bar{Q}(3\times1) \\ \hline \end{bmatrix}}_{3\times1}$$

vectors 3×4

$$\Rightarrow \underbrace{\begin{bmatrix} {}^U\bar{Q}(3\times1) \\ \hline 1 \end{bmatrix}}_{4\times1} = \underbrace{\left[\begin{array}{c|c} {}^U_BR(3\times3) & {}^U\bar{Q}_{Borg}(3\times1) \\ \hline 0\ 0\ 0 & 1 \end{array}\right]}_{4\times4} \underbrace{\begin{bmatrix} {}^B\bar{Q}(3\times1) \\ \hline 1 \end{bmatrix}}_{4\times1}$$

scaling factor

$[T] = 4\times4$

identity matrix so stop about

identity matrix

$[T]^{-1} = \frac{(\text{adj } T)}{|T|}$ Transposed Cofactor

$[T], [I]$ → this is identical $[T]$

so co-b column.

if $|T|$ becomes 2 then answer about something zero

identity matrix 4x4 is like $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

4x1 → perspective transformation.

Let $[T]$: Homogeneous transformation matrix

$$[T] = \left[\begin{array}{c|c} {}^{U}_{B}R_{(3\times3)} & {}^{U}O_{Borg}(3\times1) \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

Say find inverse also

Say

$$[T] = \left[\begin{array}{ccc|c} r_{11} & r_{12} & r_{13} & q_x \\ r_{21} & r_{22} & r_{23} & q_y \\ r_{31} & r_{32} & r_{33} & q_z \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Carry information of rotation

Carry information of the position

homogeneous transformation matrix

In matrix form:

$$\begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix}$$

$$Rot(\hat{z},\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Similar, we get

$$Rot(\hat{x},\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$Rot(\hat{y},\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

Scanned with CamScanner

$$\Rightarrow \begin{bmatrix} {}^{U}\theta_{(3\times1)} \\ \hline \end{bmatrix} = \left[\begin{array}{c|c} {}^{U}_{B}R_{(3\times3)} & {}^{U}\theta_{Borg\,(3\times1)} \\ \hline & \end{array}\right] \begin{bmatrix} {}^{B}\theta_{(3\times1)} \\ \hline \end{bmatrix} \begin{bmatrix} \\ \end{bmatrix} = 4\times4$$

have to be invertable or singular

$$\Rightarrow \begin{bmatrix} {}^{U}\theta_{(3\times1)} \\ \hline 1 \end{bmatrix} \left[\begin{array}{c|c} {}^{U}_{B}R_{(3\times3)} & {}^{U}\theta_{Borg\,(3\times1)} \\ \hline 0\ 0\ 0 & 1 \end{array}\right] \begin{bmatrix} {}^{B}\theta_{(3\times1)} \\ \hline 1 \end{bmatrix}$$

Why put 0 0 0 & 1 here

perspective transformation.

scaling factor

How $[T]$, $[I]$

row & column help operations

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

purposefully to make T invertable

Let $[T]$: Homogenous transformation matrix

$$[T] = \left[\begin{array}{c|c} {}^{U}_{B}R_{(3\times3)} & {}^{U}\theta_{(3\times1)} \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

Properties of Rotation matrix

(1) → Each row/column of a rotation matrix is a unit vector.

(2) → Inner (dot) product of each row of a rotation matrix with each other row becomes equal to 0. The same is true for each column also.

(3) → Rotation matrices are not commutative in nature.

$$ROT(\hat{X},\theta_1)\,ROT(\hat{Y},\theta_2) \neq ROT(\hat{Y},\theta_2)\,ROT(\hat{X},\theta_1)$$

(4) → Inverse of a rotation matrix is nothing but its transpose

$$ROT^{-1}(\hat{X},\theta) = ROT^{T}(\hat{X},\theta)$$

$${}^{A}_{B}T = {}^{B}_{A}T^{-1}$$

1st row → $\sqrt{\cos^2\theta + 0 + \sin^2\theta} = 1$

(1) 1st col

$$\sqrt{\cos^2\theta + 0 + \sin^2\theta} = 1$$

2nd part $\cos\theta \times 0 + 0 \times 1 + \sin\theta \times 0 = 0$ 1st row & 2nd row inner product

$$\cos\theta \times 0 + 0 \times 1 + \sin\theta \times 0 = 0$$

⇒ 1st column for inner product

$$\cos\theta \times 0 + 0 \times 1 + \sin\theta \times 0 = 0$$

3rd part

H A Numerical Example:

A frame {B} is rotated about $\hat{X}_U$ axis of the universal coordinate system by 45 degrees & translated along $\hat{X}_U$, $\hat{Y}_U$ and $\hat{Z}_U$ by 1, 2, & 3 units respectively. Let the position of a point Q in {B} is given by $[3.0\ 2.0\ 1.0]^T$. Determine ${}^{U}Q$

Solution:

$${}^{U}Q = {}^{U}_{B}R \times {}^{B}Q$$

$$= \underbrace{\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \cos 45 & -\sin 45 & 2 \\ 0 & \sin 45 & \cos 45 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{4\times 4} \underbrace{\begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix}}_{4\times 1}$$

$$U^{-}\theta = {}^{A}T_{B}\, {}^{B}\theta_{U} \qquad {}^{U}_{B}T$$

$$= \begin{bmatrix} 4 \\ 2\cos 45 - \sin 45 + 2 \\ 2\sin 45 + \cos 45 + 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2.707 \\ 5.121 \\ 1 \end{bmatrix}$$

following rule ⊗ robot in

Composite rotation matrix → sequence is very important

→ composite rotation matrix representing a rotation of α angle about $\hat{z}$ followed by a rotation of β angle about $\hat{Y}$ axis, followed by a rotation of γ angle about $\hat{X}$ axis.

$$ROT_{Composite} = \underbrace{ROT(\hat{X},\gamma)}_{3\times 3}\,\underbrace{ROT(\hat{Y},\beta)}_{3\times 3}\,\underbrace{ROT(\hat{Z},\alpha)}_{3\times 3}$$

3×3 = Composite matrix

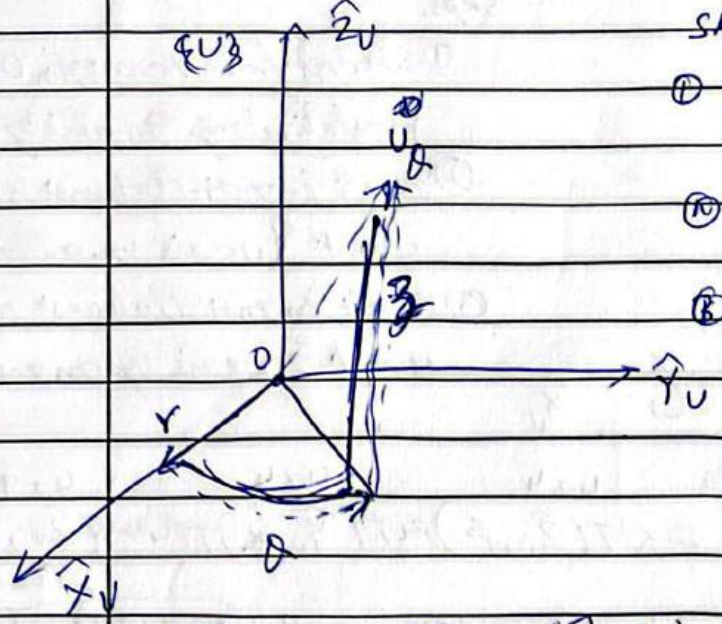
⇒ Representations of position in other than cartesian coordinate system.

position vector

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

3×3 in matrix 3×1

Cylindrical coordinate system



Steps:

① Starting from the origin O, translate by r units along $\hat{X}_U$ axis

② Rotate in anti-clockwise sense about $\hat{Z}_0$ axis by an angle θ

③ Translate along $\hat{Z}_U$ axis by z units

$$[T]_{composite} = TRANS(\hat{Z}_U, z)\, ROT(\hat{Z}_U, \theta)\, TRANS(\hat{X}_U, r)$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 & r\cos\theta \\ \sin\theta & \cos\theta & 0 & r\sin\theta \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We get $q_x = r\cos\theta$

$q_y = r\sin\theta$

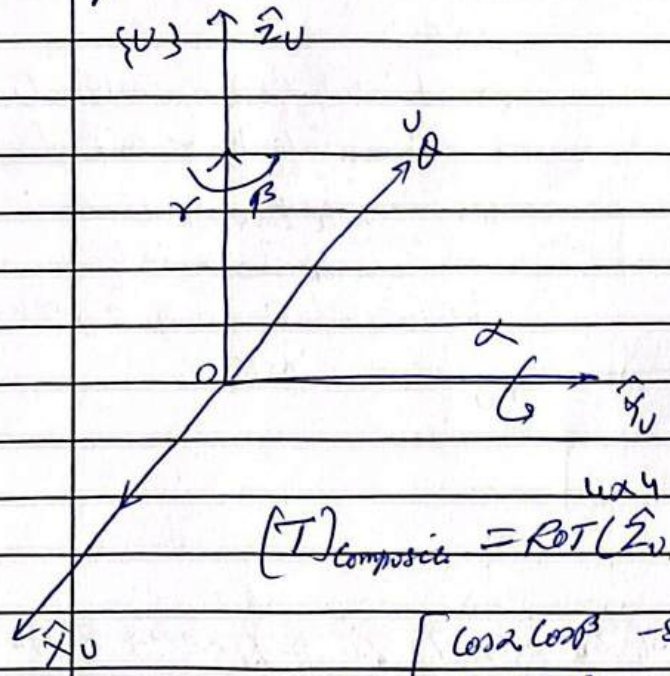
$q_z = z$

$$TRANS(\hat{X}_U, r) = \begin{bmatrix} 1 & 0 & 0 & r \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$ROT(\hat{Z}_U, \theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scanned with CamScanner

Spherical coordinate system.



Steps!

① Starting from the origin O, translate along $\hat{Z}_U$ axis by r units

② Rotate in anti-clockwise sense about $\hat{Y}_U$ axis by an angle α.

③ Rotate in anti-clockwise sense about $\hat{Z}_U$ axis by an angle β.

$$[T]_{composite} = \underset{4\times4}{ROT(\hat{Z}_U,\beta)}\,\underset{4\times4}{ROT(\hat{Y}_U,\alpha)}\,\underset{4\times4}{TRANS(\hat{Z}_U,r)}$$

$$= \begin{bmatrix} \cos\alpha\cos\beta & -\sin\beta & \sin\alpha\cos\beta & r\sin\alpha\cos\beta \\ \cos\alpha\sin\beta & \cos\beta & \sin\alpha\sin\beta & r\sin\alpha\sin\beta \\ -\sin\alpha & 0 & \cos\alpha & r\cos\alpha \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Position

we get

$$q_x = r\sin\alpha\cos\beta$$
$$q_y = r\sin\alpha\sin\beta$$
$$q_z = r\cos\alpha$$

Cartesian coordinate system

~~Representation~~

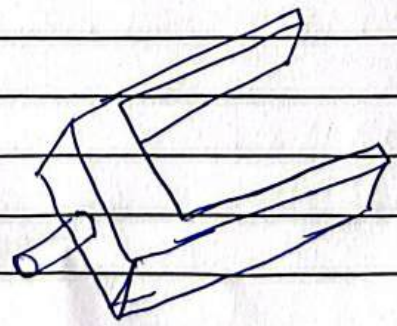
⇒ Representation of orientation in other than Cartesian coordinate system ⇐

$$\begin{bmatrix} n_x & s_x & a_x \\ n_y & s_y & a_y \\ n_z & s_z & a_z \end{bmatrix}$$

3×3 matrix in Cartesian

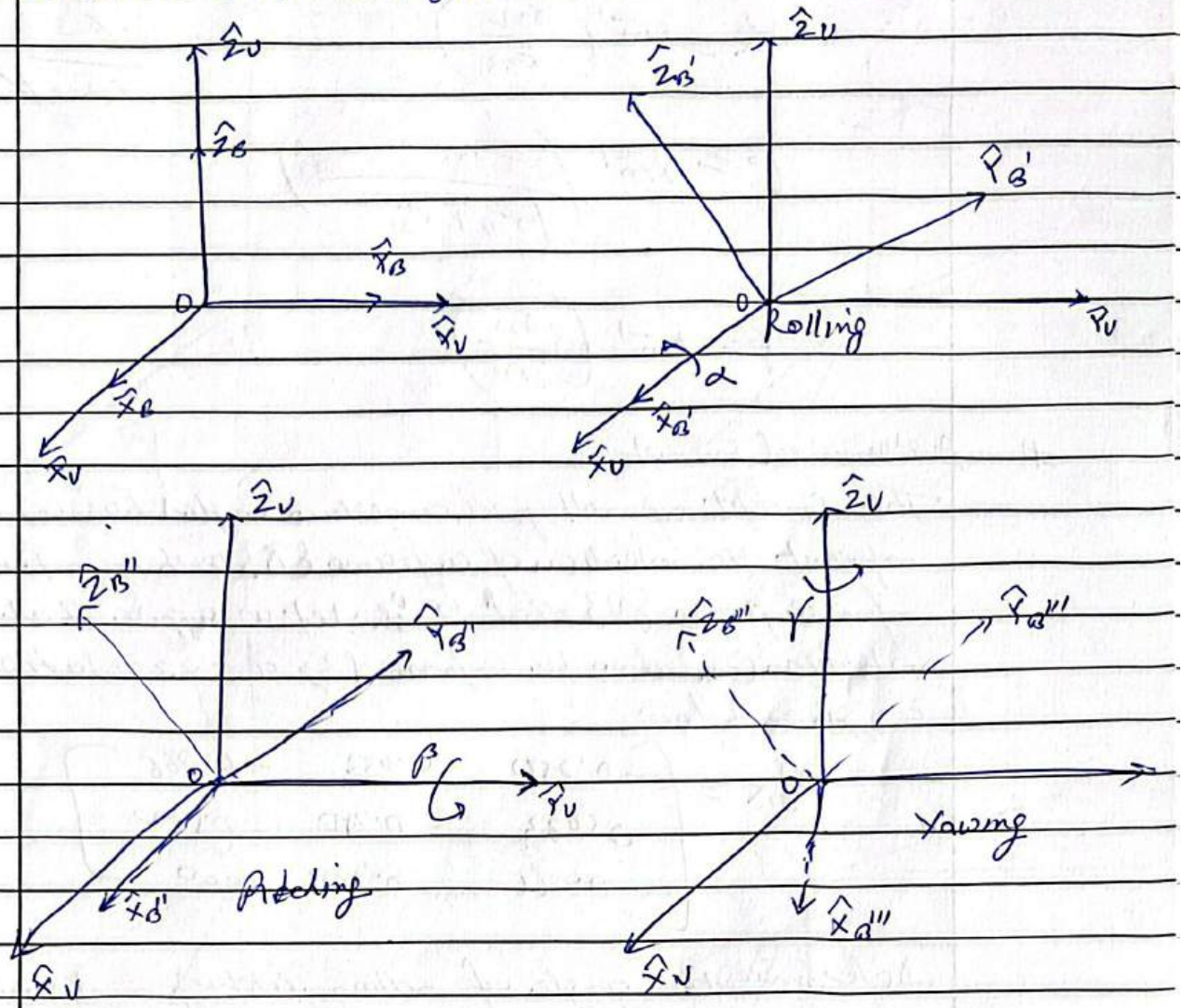
approach vector

sliding vector



B B prime

Roll, Pitch and Yaw Angles:



Rolling: about X
Pitching: " Y
Yawing: " Z

$\boxed{R - P - Y}$ Sequence

$$^{V}_{B}R_{composite} : r_{RPY} = \underset{3\times3}{ROT(\hat{Z}_V,\gamma)}\,\underset{3\times3}{ROT(\hat{Y}_V,\beta)}\,\underset{3\times3}{ROT(\hat{X}_V,\alpha)}$$

$$= \begin{bmatrix} c\beta c\gamma & -\cos\gamma + s\alpha s\beta c\gamma & s\alpha s\gamma + c\alpha s\beta c\gamma \\ c\beta s\gamma & c\alpha c\gamma + s\alpha s\beta s\gamma & -s\alpha c\gamma + c\alpha s\beta s\gamma \\ -s\beta & c\beta c\alpha & c\alpha c\beta \end{bmatrix}_{3\times3}$$

with compact with

$$^{V}_{B}R = \begin{array}{c} \text{normal vector} \quad \text{slide vector} \quad \text{approach vector} \\ \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}_{3\times3} \end{array}$$

Known

$C\beta \to \cos\beta$
$S\alpha \to \sin\alpha$

we get

$$\alpha = \tan^{-1}\left(\frac{r_{32}}{r_{33}}\right) \qquad \frac{C\beta\, S\alpha}{C\alpha\, C\beta} = \tan\alpha$$

$$\beta = \tan^{-1}\left(\frac{-r_{31}}{\sqrt{r_{11}^2 + r_{21}^2}}\right)$$

$$\gamma = \tan^{-1}\left(\frac{r_{21}}{r_{11}}\right)$$

A numerical Example:

The concept of roll, pitch and yaw angles has been used to represent the rotation of a frame {B} with respect to the reference frame {U} that is ${}^{U}_{B}R$. Let us suppose that the above rotation can also be expressed by a 3×3 rotation matrix as given below

$$ {}^{U}_{B}R = \begin{bmatrix} -0.250 & 0.433 & -0.866 \\ 0.433 & -0.750 & -0.500 \\ -0.866 & -0.500 & 0.00 \end{bmatrix} $$

Determine the angle of rolling, pitching and yawing

Solution:

Angle of rolling $\alpha = \tan^{-1}\frac{r_{32}}{r_{33}} = \tan^{-1}\frac{-0.500}{0.00} = 90^\circ$

angle of pitching $\beta = \tan^{-1}\frac{-r_{31}}{\sqrt{r_{11}^2 + r_{21}^2}} = \tan^{-1}\frac{0.866}{\sqrt{-0.250^2 + 0.433^2}}$

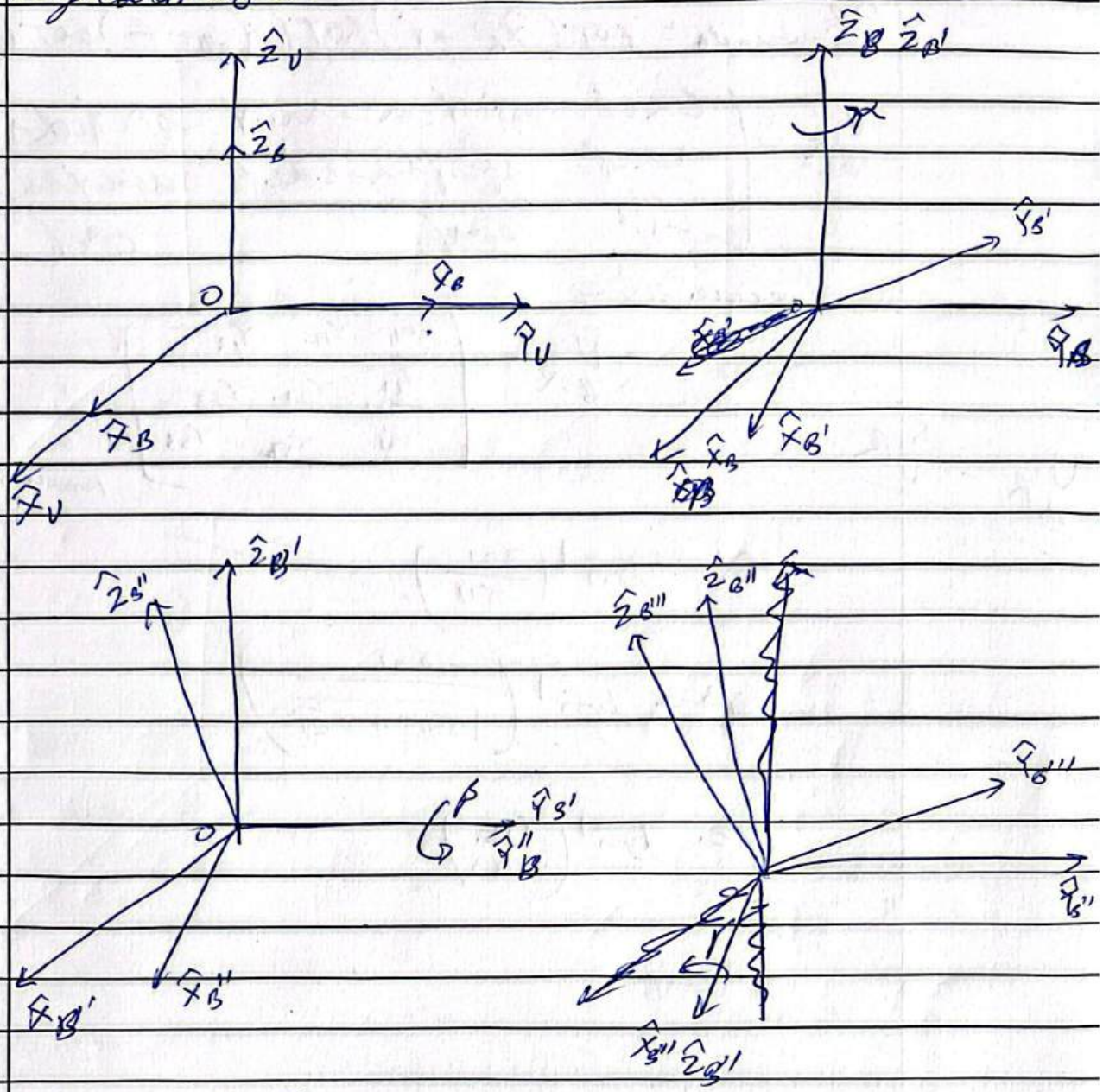
$= 40.89^\circ$

angle of yawing $\gamma = \tan^{-1}\frac{-r_{21}}{r_{11}} = \tan^{-1}\frac{0.433}{-0.250}$

$= -59.99 \approx -60^\circ$

Scanned with CamScanner

Using Euler Angles:



$$ {}^{V}_{B}R = {}^{B}_{V}R^{-1} $$

Steps:

(i) Rotate {B} about $\hat{z}_B$ by angle α in anti-clockwise sense.

(ii) Rotate {B} about $\hat{y}_{B'}$ by an angle β in anti-clockwise sense.

(iii) Rotate {B} about $\hat{x}_{B''}$ by an angle γ in anti-clockwise sense.

Scanned with CamScanner

$$ {}^{B}_{U}R_{euler\,angles} = \overset{3\times3}{ROT(\hat{X}_{B''}, -\gamma)}\;\overset{3\times3}{ROT(\hat{Y}_{B''}, -\beta)}\;\overset{3\times3}{ROT(\hat{Z}_{B}, -\alpha)} = 3\times3 $$

$$ {}^{U}_{B}R = \begin{bmatrix} c\alpha c\beta & s\beta s\gamma c\alpha - s\alpha c\gamma & s\beta c\gamma c\alpha + s\alpha s\gamma \\ s\alpha c\beta & s\beta s\gamma s\alpha + c\alpha c\gamma & s\beta c\gamma s\alpha - s\gamma c\alpha \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix} $$

we compare with

$$ {}^{U}_{B}R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}_{3\times3} \text{ known} $$

$$ {}^{U}_{B}R = {}^{B}_{U}R $$

$$ \alpha = \tan^{-1}\left(\frac{r_{21}}{r_{11}}\right) $$

$$ \beta = \tan^{-1}\left(\frac{-r_{31}}{\sqrt{r_{11}^2 + r_{21}^2}}\right) $$

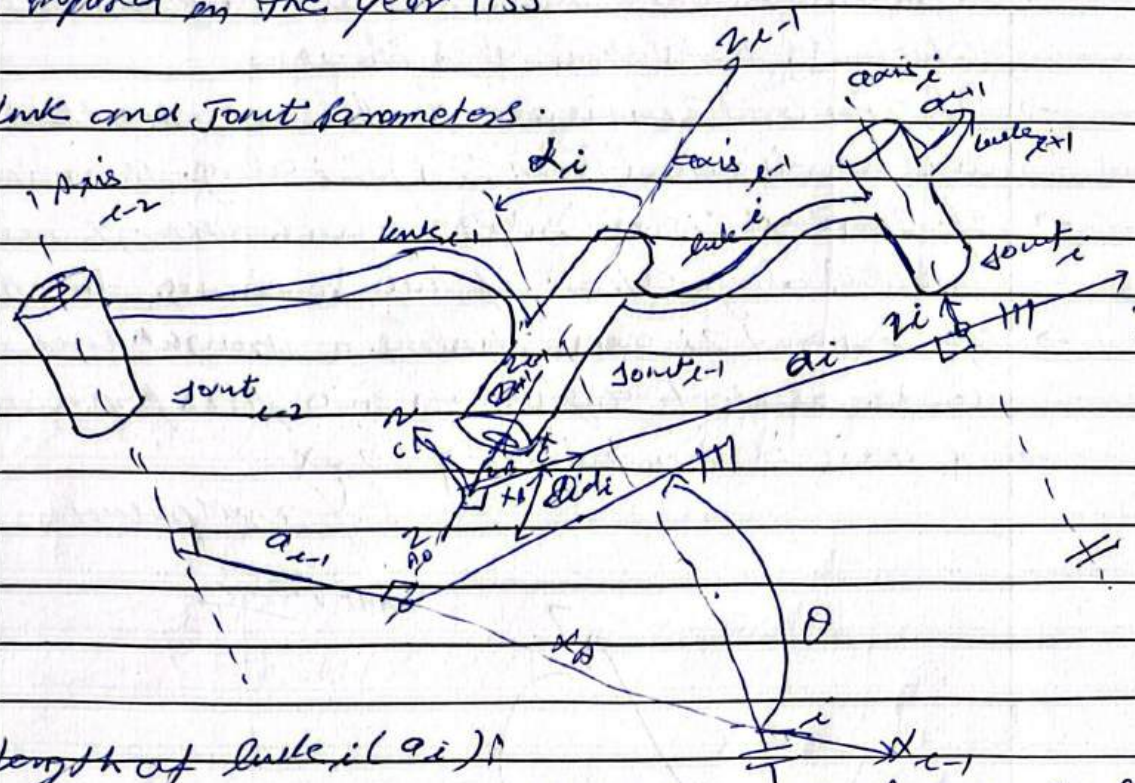
$$ \gamma = \tan^{-1}\left(\frac{r_{32}}{r_{33}}\right) $$

Scanned with CamScanner

Denavit - Hartenberg notations:

→ Proposed in the year 1955

Link and Joint parameters:



⇒ length of link i (a_i): It is the mutual perpendicular distance between Axis i-1 and Axis i

⇒ Angle of twist of link i (α_i): It is defined as the angle between axis i-1 and Axis i.

Structure → link parameter

relative position of neighbouring links → Joint parameter

⇒ Offset of link i (d_i): It is the distance measured from a point where a_{i-1} intersect the axis i-1 to the point where a_i intersects the Axis i-1 measured along the said axis.

⇒ Joint Angle (θ_i): It is defined as the angle between the extension of a_{i-1} and a_i measured about the axis i-1.

Notes:

→ Revolute Joint: θ_i is variable

→ Prismatic Joint: d_i is variable.

Scanned with CamScanner

from last image

Note: Y axis is perpendicular to the plane of the paper and moving inside it.

Rules for Coordinates Assignment:

→ Z_i is an axis about which the rotation is considered or along which the translation takes place.

→ If Z_{i-1} and Z_i axis are parallel to each other, X axis will be directed from Z_{i-1} to Z_i along their common normal.

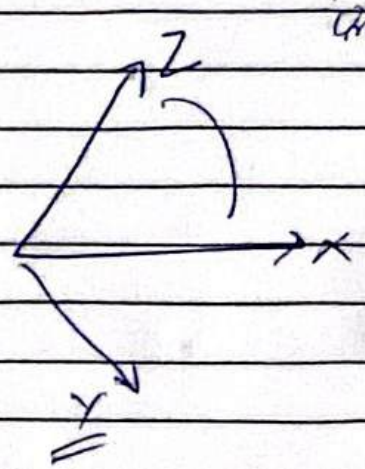
→ If Z_{i-1} and Z_i axis intersect each other, X axis can be selected along either of two remaining directions.

→ If Z_{i-1} and Z_i axes act along a straight line, X axis can be selected anywhere in a plane perpendicular to them.

→ Y axis is decided as $Y = Z \times X$

cross product

unit vector

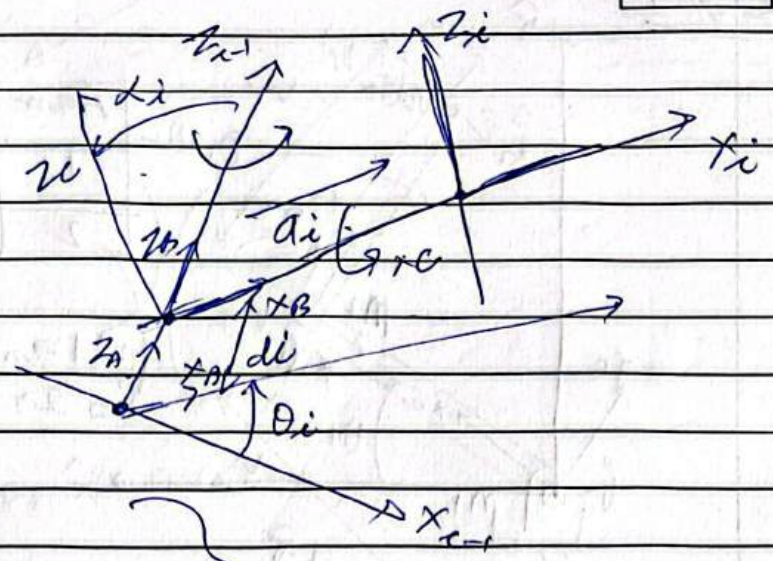


Therefore

$$^{i-1}_{i}T = {}^{i-1}_{A}T\,{}^{A}_{B}T\,{}^{B}_{C}T\,{}^{C}_{i}T$$

$$= ROT(Z, \theta_i)\,TRANS(Z, d_i)\,ROT(X, \alpha_i)\,TRANS(X, a_i)$$

$$= Screw_Z\,Screw_X$$



$^{i-1}_{A}T = ROT(z, \theta_i)$

$^{A}_{B}T \rightarrow TRANS(z, d_i)$ — Servo z

$^{B}_{C}T \rightarrow ROT(x, \alpha_i)$

$^{C}_{i}T \rightarrow TRANS(x, a_i)$ — Servo x

$$^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Usando Composição de matriz inversa

Novo, $^{i}_{i-1}T = \left[^{i-1}_{i}T \right]^{-1}$

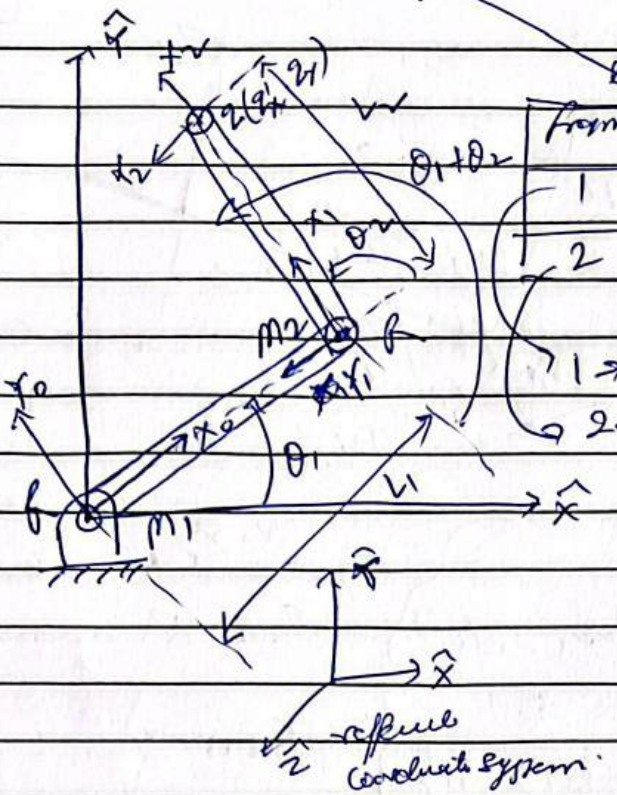
$$= \begin{bmatrix} c\theta_i & s\theta_i & 0 & -a_i \\ -s\theta_i c\alpha_i & c\theta_i c\alpha_i & s\alpha_i & -d_i s\alpha_i \\ s\theta_i s\alpha_i & -c\theta_i s\alpha_i & c\alpha_i & -d_i c\alpha_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Vi

Sequence is very important from last explanations, it is from screw rules

2 dof Serial manipulator

Example)



D-H parameters table

Frame	θ_i	d_i	α_i	a_i
1	θ_1	0	0	L_1
2	θ_2	0	0	L_2

1 → means 1 with reference to zero

2 → " 2 " " 1

$\boxed{D-H}$

Forward kinematics

$$^{Base}_{2}T = {}^{Base}_{1}T\,{}^{1}_{2}T$$

$$^{Base}_{1}T = \underset{4\times4}{ROT(\hat{Z},\theta_1)}\,\underset{4\times4}{TRANS(\hat{X},L_1)} \Rightarrow 4\times4 \text{ matrix}$$

$$= \begin{bmatrix} c_1 & -s_1 & 0 & L_1c_1 \\ s_1 & c_1 & 0 & L_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times4}$$

from table

$$^{Base}_{1}T = ROT(Z,\theta_1)\,Trans(X,L_1)$$

$$^{1}_{2}T = ROT(Z,\theta_2)\,Trans(X,L_2)$$

$$^{1}_{2}T = ROT(\hat{Z},\theta_2)\,TRANS(\hat{X},L_2)$$

$$= \begin{bmatrix} c_2 & -s_2 & 0 & L_2c_2 \\ s_2 & c_2 & 0 & L_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$ {}^{Base}_{2}T = {}^{Base}_{1}T\,{}^{1}_{2}T = \begin{bmatrix} C_{12} & -S_{12} & 0 & L_1C_1 + L_2C_{12} \\ S_{12} & C_{12} & 0 & L_1S_1 + L_2S_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$\cos\theta_1$ position $\cos(\theta_1+\theta_2)$ $= q_x$ $= q_y$

$S_{12} \Rightarrow \sin(\theta_1+\theta_2)$

$q_x = L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2)$

$q_y = L_1\sin\theta_1 + L_2\sin(\theta_1+\theta_2)$

forward kinematic: To determine the position & orientation of endeffector of Robot provided the length of links are known and joints angles are known.

$\left.\begin{matrix} L_1 & L_2 \\ \theta_1 & \theta_2 \end{matrix}\right]$ known → If these are known then can not I find the position & orientation of endeffector with respect to base coordinate system of the robot.

~~Hence~~ determine θ_1 & θ_2

Inverse kinematics:

$$ {}^{Base}_{2}T = \begin{bmatrix} c\phi & -s\phi & 0 & q_x \\ s\phi & c\phi & 0 & q_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

q_x = known, q_y = known

~~The purpose is determine the joint angle θ_1 & θ_2 provided the position~~ and orientation of end effector with respect to base coordinate frame and length of links are known.

$$q_x = L_1c_1 + L_2c_{12} \quad \text{--- (I)}$$

$$q_y = L_1s_1 + L_2s_{12} \quad \text{--- (II)}$$

Above adding (I) & (II)

$$q_x^2 + q_y^2 = L_1^2 + L_2^2 + 2L_1L_2c_{12}c_1 + 2L_1L_2s_{12}s_1,$$

$$c_2 = \frac{q_x^2 + q_y^2 - L_1^2 - L_2^2}{2L_1L_2}$$

Unknown → θ_1 & θ_2

$$q_x^2 + q_y^2 - L_1^2 - L_2^2 = 2L_1L_2c_2$$

$$c_2 = \frac{q_x^2 + q_y^2 - L_1^2 - L_2^2}{2L_1L_2}$$

$$\theta_2 = \cancel{\arccos} \arccos\left(\frac{q_x^2 + q_y^2 - L_1^2 - L_2^2}{2L_1L_2}\right) \rightarrow 2 \text{ values}$$

$$q_x = L_1c_1 + L_2c_1c_2 - L_2s_1s_2$$

$$= c_1\underbrace{(L_1 + L_2c_2)}_{\text{known}} - s_1\underbrace{(L_2s_2)}_{\text{known}}$$

$$L_1 + L_2c_2 = \rho\sin\psi$$

$$L_2s_2 = \rho\cos\psi$$

Introduction of Robotics

Hard Automation:- Conventional machinery package, sewing, manufacturing small parts.
Cannot handle product design variations. mass production.
Adjustability possible.
Programmable Automation:- Flexible because of Computer Control.
Can handle variations. batch production.
Autonomous:- Endowed with decision making capability through use of sensors.
Animals:- Autonomous to some extent.
Humans: Highly autonomous. Endowed with greater ~~intell~~ intelligence.

The Quest.

Is it possible to develop machine which can be autonomous like animals and humans.
Yes! Developments in digital electronics. microprocessor and sensor and actuator technologies have made this possibility.

Utility of such Systems.

Hazardous and repetitive tasks.
painting, handling toxic material. welding. loading and unloading jobs from machine. assembly operations. Safety.
Handling fragile and ~~to~~ tiny parts.
Remote Areas:- Inspections and maintenance tasks in ~~dan~~ dangerous areas, handling explosive.
Exploring the deep sea, space and other planets.

Status Today.

Industrial robots, medical robots, mobile robots, (wheeled/legged) Hopping and Running machines, Robotic Aircraft and Boats, Robots Toys, Robots for Entertainments, Robots for Cleaning of Home and Industry.

Scanned with CamScanner

Extrapolating from Nature

Arms and fingers to manipulate objects. legs for locomotion
muscles as actuators!
Eyes provide vision, nose for smelling, ears for hearing, tongue
for taste, skin for feeling.
nerves for communication & brain the controller.

Scanned with CamScanner

$q_x = c_1 \rho \sin\psi$

$\quad = -s_1 \rho \cos\psi$

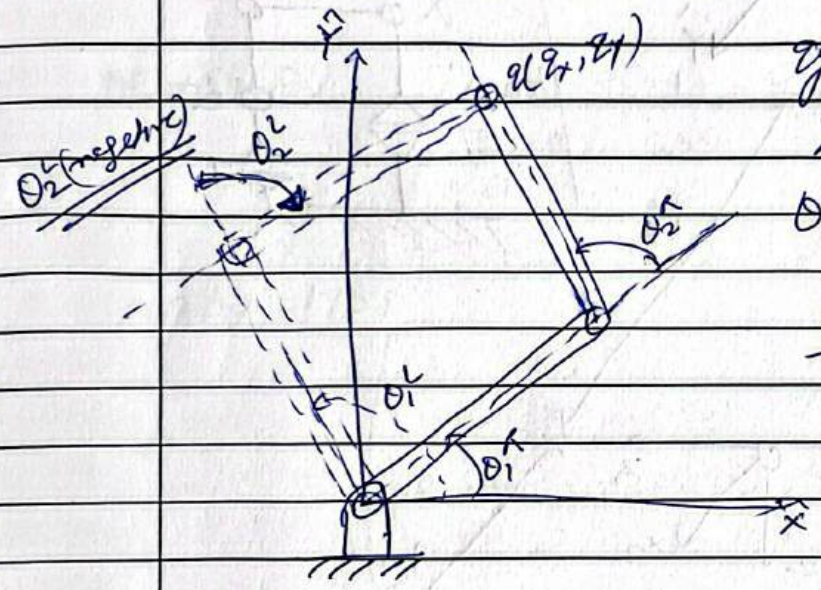
$q_x = \rho \sin(\psi - \theta_1)$

Know Know

$\rho = \sqrt{(L_1 + L_2 c_2)^2 + (L_2 s_2)^2}$

$\psi = \tan^{-1} \dfrac{L_1 + L_2 c_2}{L_2 s_2}$

then can find $\underline{\theta_1}$



$q_y = \rho \cos(\psi - \theta_1)$ — ①

$q_x = \rho \sin(\psi - \theta_1)$

$\theta_1 = \psi - \arctan\left(\dfrac{q_x}{q_y}\right)$ — ② 2 values

$\dfrac{q_x}{q_y} = \tan(\psi - \theta_1)$

$\rho \neq 0$

$\psi - \theta_1 = \tan^{-1}\left(\dfrac{q_x}{q_y}\right)$

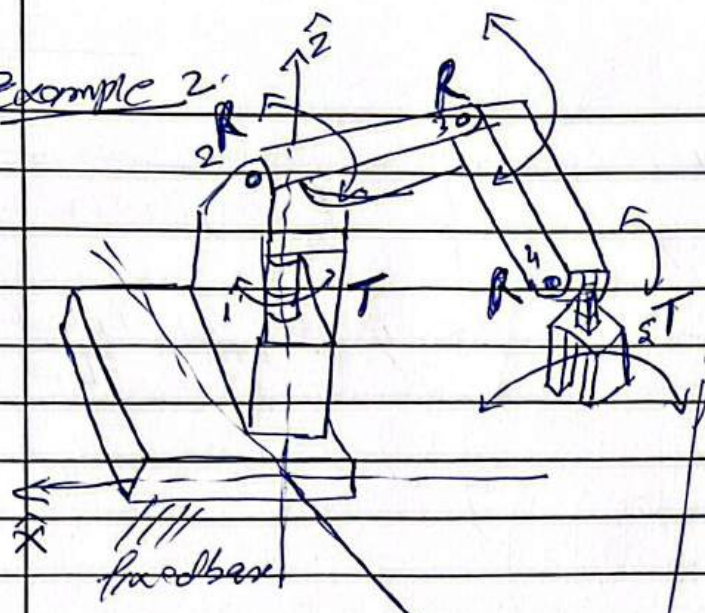
(θ_1, θ_2) RH

(θ_1, θ_2) LH

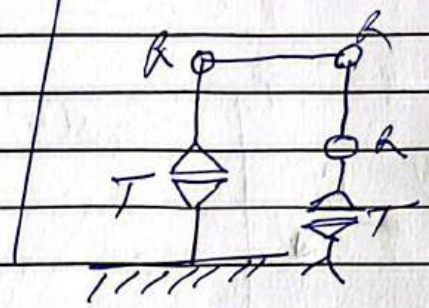
Example 2:

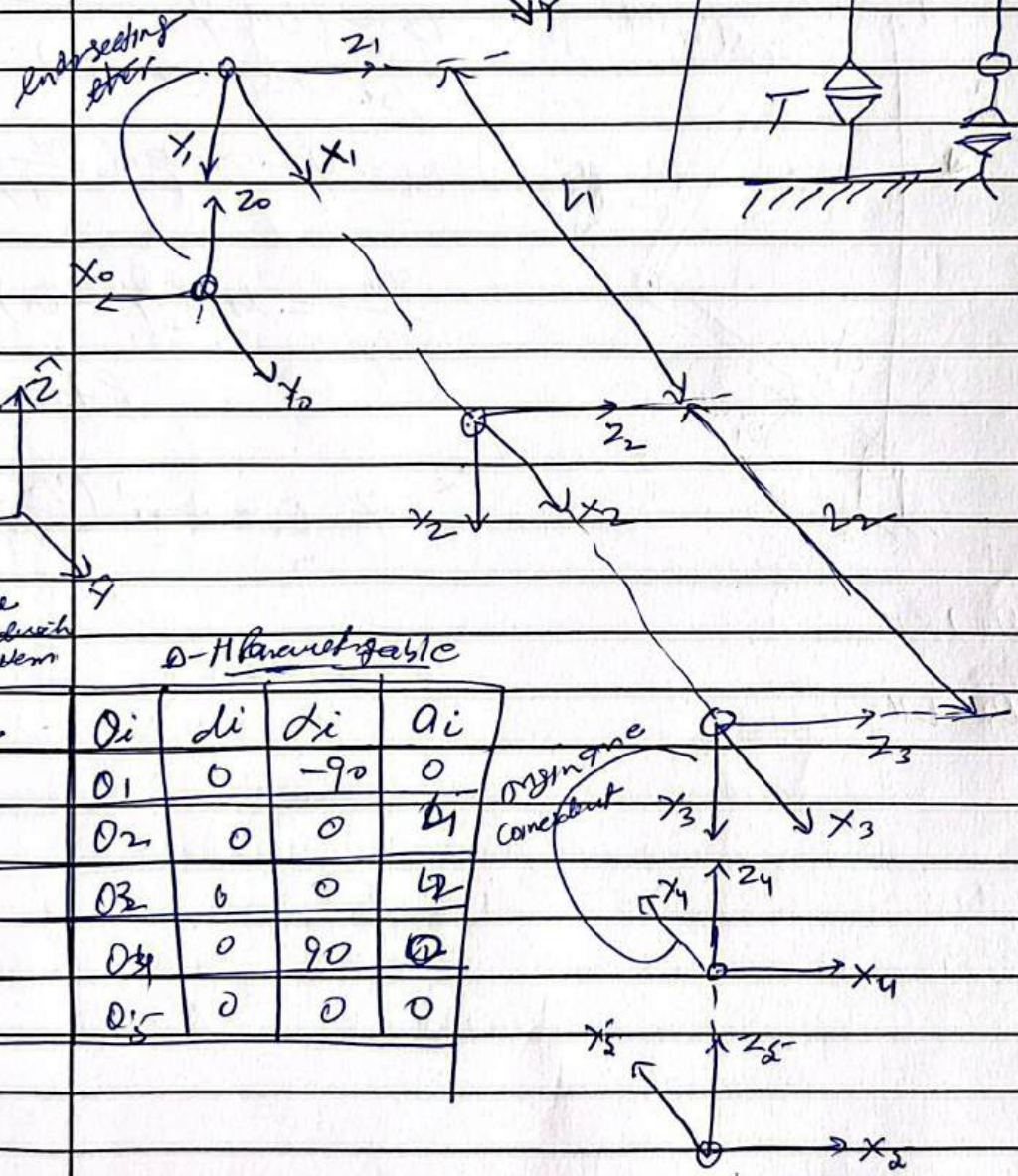
MINIMOVER (5DOF)

T-R-R-R-T ⇒ manipulator



Kinematics diagram:





D-H Parameter table

Frame	θ_i	d_i	α_i	a_i
1	θ_1	0	-90	0
2	θ_2	0	0	L_1
3	θ_3	0	0	L_2
4	θ_4	0	90	0
5	θ_5	0	0	0

forward kinematics

$$ {}^0_5T = {}^0_1T\,{}^1_2T\,{}^2_3T\,{}^3_4T\,{}^4_5T $$

$$ {}^0_1T = Rot(\hat{Z}, \theta_1)\,Rot(\hat{X}, -90) $$

$$ = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Here, c_1 & s_1 denotes $c\theta_1$ (or $\cos\theta_1$) & $s\theta_1$ (or $\sin\theta_1$), respectively.

$$ Rot(z, \theta) = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ Rot(x, -90) $$

$$ = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-90) & -\sin(-90) & 0 \\ 0 & \sin(-90) & \cos(-90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^0_1T^{-1} = \begin{bmatrix} c_1 & s_1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^1_2T = Rot(\hat{Z}, \theta_2)\,Trans(\hat{X}, L_1) = \begin{bmatrix} c_2 & -s_2 & 0 & L_1c_2 \\ s_2 & c_2 & 0 & L_1s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^2_3T = Rot(\hat{Z}, \theta_3)\,Trans(\hat{X}, L_2) $$

$$ = \begin{bmatrix} c_3 & -s_3 & 0 & L_2c_3 \\ s_3 & c_3 & 0 & L_2s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Scanned with CamScanner

Scanned with CamScanner

$${}^{3}_{4}T = Rot(\hat{z}, \theta_4)\, Rot(\hat{x}, 90)$$

$$= \begin{bmatrix} C_4 & 0 & S_4 & 0 \\ S_4 & 0 & -C_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{4}_{5}T = Rot(\hat{z}, \theta_5) = \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}_{5}T = {}^{0}_{1}T\, {}^{1}_{2}T\, {}^{2}_{3}T\, {}^{3}_{4}T\, {}^{4}_{5}T = \begin{bmatrix} U_{11} & U_{12} & U_{13} & P_x \\ U_{21} & U_{22} & U_{23} & P_y \\ U_{31} & U_{32} & U_{33} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times 4}$$

$U_{11} = C_1 C_{234} C_5 - S_1 S_5$

$U_{12} = -C_1 C_{234} S_5 - S_1 C_5$

$U_{13} = C_1 S_{234}$

$U_{21} = S_1 C_{234} C_5 + C_1 S_5$

$U_{22} = -S_1 C_{234} S_5 + C_1 C_5$

$U_{23} = S_1 S_{234}$

$U_{31} = -S_{234} C_5$

$U_{32} = S_{234} S_5$ —— $\sin(\theta_2+\theta_3+\theta_4)$

$U_{33} = C_{234}$

$P_x = C_1(L_1 C_2 + L_2 C_{23})$

$P_y = S_1(L_1 C_2 + L_2 C_{23}) \rightarrow C_{23} = \cos(\theta_2+\theta_3)$

$P_z = -L_1 S_1 - L_2 S_{23}$

Inverse kinematics

$$ {}^{0}_{5}T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \text{known} $$

$$ {}^{0}_{5}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T\,{}^{3}_{4}T\,{}^{4}_{5}T $$

$$ \Rightarrow {}^{0}_{1}T^{-1}\left({}^{0}_{5}T\right) = {}^{1}_{2}T\,{}^{2}_{3}T\,{}^{3}_{4}T\,{}^{4}_{5}T $$

$$ {}^{0}_{1}T^{-1}\,{}^{0}_{1}T = [I] $$

$$ {}^{0}_{1}T^{-1} \Rightarrow \begin{bmatrix} c_1 & s_1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = $$

$$ \begin{bmatrix} c_{234}c_5 & -c_{234}s_5 & -s_{234} & l_4c_1 + l_2c_{23} \\ s_{234}c_5 & -s_{234}s_5 & -c_{234} & l_4s_1 + l_2s_{23} \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \checkmark\ {}^{1}_{5}T $$

$$ \Rightarrow \begin{bmatrix} r_{11}c_1 + r_{21}s_1 & r_{12}c_1 + r_{22}s_1 & r_{13}c_1 + r_{23}s_1 & p_xc_1 + p_ys_1 \\ -r_{31} & -r_{32} & -r_{33} & -p_z \\ -r_{11}s_1 + r_{21}c_1 & -r_{12}s_1 + r_{22}c_1 & -r_{13}s_1 + r_{23}c_1 & -p_xs_1 + p_yc_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ = \begin{bmatrix} c_{234}c_5 & -c_{234}s_5 & s_{234} & l_4c_2 + l_2c_{23} \\ s_{234}c_5 & -s_{234}s_5 & -c_{234} & l_4s_2 + l_2s_{23} \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Scanned with CamScanner

$$q_x C_1 + q_y S_1 = L_1 C_2 + L_2 C_{23} \quad \text{—— (1)}$$

$$-q_x S_1 + q_y C_1 = 0 \quad \text{—— (2)}$$

$$q_z = -L_1 S_2 - L_2 S_{23} \quad \text{—— (3)}$$

$$S_{234} = r_{13} C_1 + r_{23} S_1 \quad \text{—— (4)}$$

$$C_{234} = r_{33} \quad \text{—— (5)}$$

$$-r_{11} S_1 + r_{21} C_1 = S_5 \quad \text{—— (6)}$$

$$-r_{12} S_1 + r_{22} C_1 = C_5 \quad \text{—— (7)}$$

from (2) $-q_x S_1 + q_y C_1 = 0$

$$\Rightarrow \theta_1 = \arctan\left(\frac{q_y}{q_x}\right)$$

By squaring & adding equation (1) & (3)

$$q_x^2 + q_y^2 + q_z^2 = L_1^2 + L_2^2 + 2L_1L_2C_3$$

$$\Rightarrow \theta_3 = \arccos\left(\frac{q_x^2 + q_y^2 + q_z^2 - L_1^2 - L_2^2}{2L_1L_2}\right)$$

$$L_1 C_2 + L_2 C_{23} = q_x C_1 + q_y S_1$$

$$\Rightarrow (L_1 + L_2 C_3) C_2 - (L_2 S_3) S_2 = q_x C_1 + q_y S_1$$

let us assume $L_1 + L_2 C_3 = \rho \sin\alpha$ and $L_2 S_3 = \rho \cos\alpha$,

where $\rho \neq 0$ and $\rho = \sqrt{(L_1 + L_2 C_3)^2}$; $\alpha = \arctan\left(\frac{L_1 + L_2 C_3}{L_2 S_3}\right)$

Thus, the above expression can be written as follows:

$$\rho \sin\alpha\, C_2 - \rho \cos\alpha\, S_2 = q_x C_1 + q_y S_1 \quad \text{—— (8)}$$

$$\rho \sin(\alpha - \theta_2) = q_x C_1 + q_y S_1 \quad \text{—— (8)}$$

$$\rho \cos(\alpha - \theta_2) = -q_z \quad \text{—— (9)}$$

$$\tan(\alpha - \theta_2) = \frac{x_c c_1 + y_c s_1}{-z_3}$$

$$\Rightarrow \theta_2 = \alpha - \arctan\left(\frac{x_c c_1 + y_c s_1}{-z_3}\right)$$

$$\theta_2 + \theta_3 + \theta_4 = \arctan\left(\frac{r_{13} c_1 + r_{23} s_1}{r_{33}}\right)$$

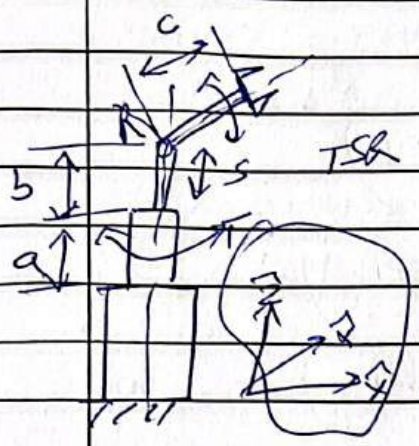
$$\Rightarrow \theta_4 = \arctan\left(\frac{r_{13} c_1 + r_{23} s_1}{r_{33}}\right) - \theta_2 - \theta_3$$

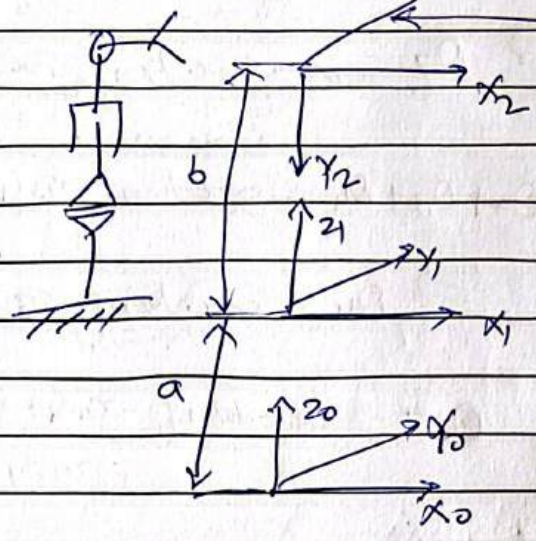
$$\theta_5 = \arctan\left(\frac{-r_{11} s_1 + r_{21} c_1}{-r_{12} s_1 + r_{22} c_1}\right)$$

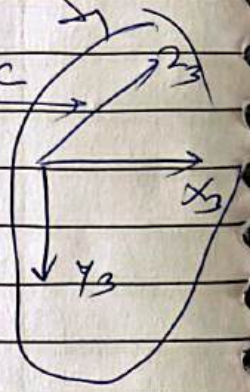
Scanned with CamScanner

Copied from 2

An Example





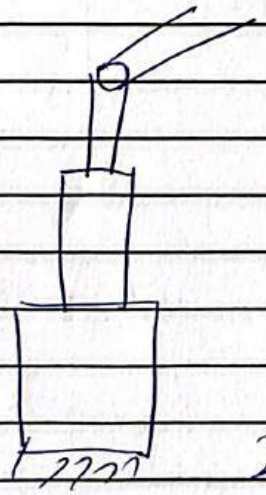


D-H Parameters' table

frame	θ_i	d_i	α_i	a_i
1	θ_1	a^*	0	0
2	[illegible]	b	-90	0
3	θ_3	0	0	c

$a^* \rightarrow$ fixed offset

3 DOF



Inverse kinematics

frame	θ_i	d_i	α_i	a_i
1	0	a	0	0
2	θ_2	b^*	-90	0
3	θ_3	0	0	c

Scanned with CamScanner

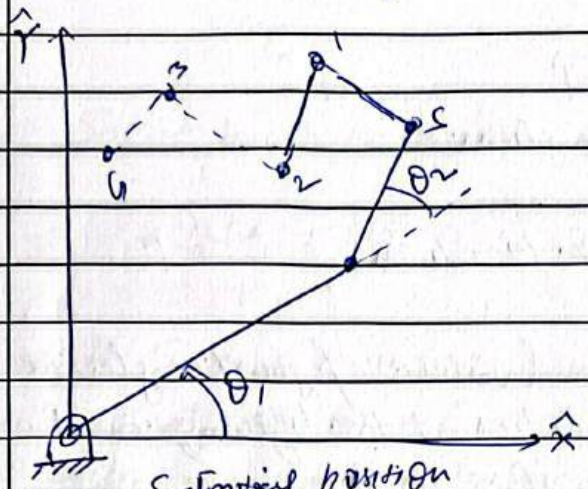
Trajectory Planning & path same motion planning

(3)

21. Trajectory Planning

Aim:

To determine time history of position, velocity and acceleration of end-effector of a manipulator, while moving from an initial position to a final position through some intermediate / via points



S - Initial position
G - Final position
1, 2, 3 - Intermediate points

Points	Cartesian scheme	Joint-space scheme
S	(x_s, x_s)	(θ_1^s, θ_2^s)
1	(x_1, y_1)	(θ_1^1, θ_2^1)
2	(y_2, x_2)	(θ_1^2, θ_2^2)
3	(x_3, y_3)	(θ_1^3, θ_2^3)
G	(x_G, y_3)	(θ_1^G, θ_2^G)

Trajectory planning

Trajectory Planning

Cartesian scheme:
→ Computationally expensive as inverse kinematics problem has to be solved, on-line

Joint-space scheme

Joint-space scheme:

→ To fit a smooth (continuous) curve through $(\theta_f, \theta_i, \theta^2, \theta_i^3, \theta_i^4)$

→ first and second order derivative must be continuous.

Various Trajectory functions:

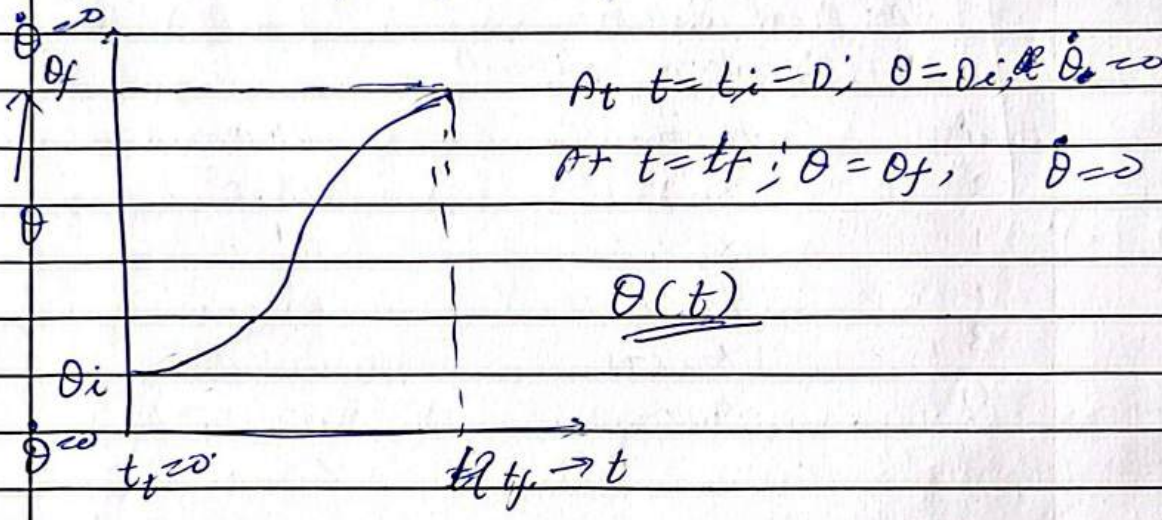
→ Cubic polynomial

→ fifth-order polynomial

→ Linear trajectory function.

* Polynomial Trajectory function:

Case 1: Initial and final values of joint angle are known, and angular velocities at the beginning and end of the cycle are kept equal to zero.



Let us consider cubic polynomial

$\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$

where c_0, c_1, c_2, c_3 are the coefficients.

Differentiate $\theta(t)$ with respect to time to get angular velocity.

$\dot{\theta} = c_1 + 2c_2 t + 3c_3 t^2$ also function of $\dot{\theta}(t)$

Apply the initial conditions to angular displacement and velocity equations we get

$$C_0 = \theta_i \quad \text{--- (1)}$$

$$C_1 = 0 \quad \text{--- (10)}$$

$$C_0 + C_1 t_f + C_2 t_f^2 + C_3 t_f^3 = \theta_f \quad \text{--- (3)}$$

$$C_1 + 2C_2 t_f + 3C_3 t_f^2 = 0 \quad \text{--- (4)}$$

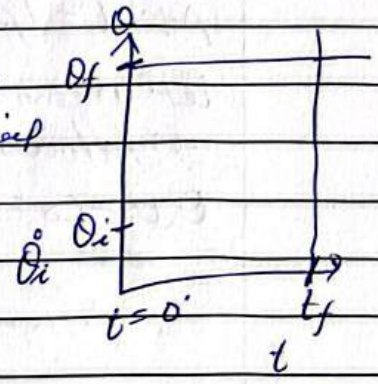
Solving above equations, we get

$$\theta(t) = \theta_i + \frac{3(\theta_f - \theta_i)}{t_f^2} t^2 - \frac{2(\theta_f - \theta_i)}{t_f^3} t^3$$

Case 2: Initial and final values of joint angle are known and angular velocities at the beginning and end of the cycle are ~~assuming~~ assumed to have non zero values.

At $t = t_i = 0$, $\theta = \theta_i$, $\dot{\theta} = \dot{\theta}_i$

At $t = t_f$; $\theta = \theta_f$, $\dot{\theta} = \dot{\theta}_f$.



Let us consider a third order polynomial of the form.

$$\theta(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3$$

Where C_0, C_1, C_2, C_3 are the coefficients.

Differentiate $\theta(t)$ w.r.t respect to time to get angular velocity.

$$\dot{\theta}(t) = C_1 + 2C_2 t + 3C_3 t^2$$

Apply the initial condition to get angular displacement and velocity equations we get

$$C_0 = \theta_i \quad \text{--- (1)}$$

$$C_1 = \dot{\theta}_i \quad \text{--- (10)}$$

$$C_0 + C_1 t_f + C_2 t_f^2 + C_3 t_f^3 = \theta_f \quad \text{--- (3)}$$

$$C_1 + 2C_2 t_f + 3C_3 t_f^2 = \dot{\theta}_f \quad \text{--- (4)}$$

Scanned with CamScanner

Solving above equation, we get

$$C_0 = \theta_i$$

$$C_1 = \dot{\theta}_i$$

$$C_2 = \frac{3(\theta_f - \theta_i)}{t_f^2} - \frac{2}{t_f}\dot{\theta}_i - \frac{1}{t_f}\dot{\theta}_f$$

$$C_3 = -\frac{2(\theta_f - \theta_i)}{t_f^3} + \frac{1}{t_f^2}(\dot{\theta}_f + \dot{\theta}_i)$$

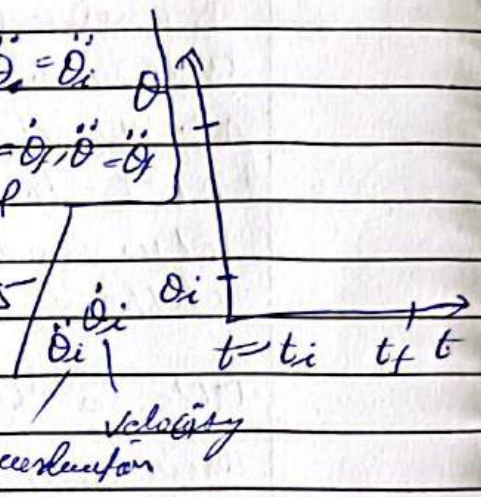
Case 3: Initial and final values of joint angle, angular velocities and angular accelerations at the beginning and end of the cycle are assumed to have non zero values.

At $t = t_i = 0$; $\theta = \theta_i$, $\dot{\theta} = \dot{\theta}_i$ & $\ddot{\theta} = \ddot{\theta}_i$

At $t = t_f$; $\theta = \theta_f$, $\dot{\theta} = \dot{\theta}_f$, $\dot{\theta} = \dot{\theta}_f$, $\ddot{\theta} = \ddot{\theta}_f$

Let us consider a fifth-order polynomial as follows:

$$\theta(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3 + C_4 t^4 + C_5 t^5$$



Differentiate $\theta(t)$ with respect to time once to get angular velocity and twice to get angular acceleration.

$$\dot{\theta}(t) = C_1 + 2C_2 t + 3C_3 t^2 + 4C_4 t^3 + 5C_5 t^4$$

$$\ddot{\theta}(t) = 2C_2 + 6C_3 t + 12C_4 t^2 + 20C_5 t^3$$

Apply the initial conditions to angular displacement, velocity and acceleration equation, we get

$$C_0 = \theta_i \quad \text{—(i)}$$

$$C_1 = \dot{\theta}_i \quad \text{—(ii)}$$

$$C_2 = \frac{1}{2}\ddot{\theta}_i \quad \text{—(iii)}$$

$$2C_2 + 6C_3 t_f + 12C_4 t_f^2 + 20C_5 t_f^3 = \ddot{\theta}_f \quad \text{—(6)}$$

$$C_0 + C_1 t_f + C_2 t_f^2 + C_3 t_f^3 + C_4 t_f^4 + C_5 t_f^5 = \theta_f \quad \text{—(4)}$$

$$C_1 + 2C_2 t_f + 3C_3 t_f^2 + 4C_4 t_f^3 + 5C_5 t_f^4 = \dot{\theta}_f \quad \text{—(5)}$$

Scanned with CamScanner

Solving above equations, we get.

$c_0 = \theta_i$

$c_1 = \dot{\theta}_i$

$c_2 = \frac{1}{2}\ddot{\theta}_i$

$c_3 = 20(\theta_f - \theta_i) - (8\dot{\theta}_f + 12\dot{\theta}_i)t_f - (3\ddot{\theta}_i - \ddot{\theta}_f)t_f^2$

$c_4 = 30(\theta_i - \theta_f) + (14\dot{\theta}_f + 16\dot{\theta}_i)t_f + (3\ddot{\theta}_i - 2\ddot{\theta}_f)t_f^2$

$$c_5 = \frac{12(\theta_f - \theta_i) - 6(\dot{\theta}_f + \dot{\theta}_i)t_f - (\ddot{\theta}_i - \ddot{\theta}_f)t_f^2}{2t_f^5}$$

A Numerical Example:

→ A single-link robot with a revolute joint is motionless at $\theta = 20°$. It is desired to move the joint in a smooth manner to $\theta = 80°$ in 4.0 seconds. Find a suitable cubic polynomial to generate this motion and bring the manipulator to rest at the goal.

Solution

Cubic Polynomial

$$\theta(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3 \quad \text{--- (1)}$$

Conditions:

At time $t = t_i = 0$

$\theta = \theta_i = 20°$

$\dot{\theta} = 0$;

At time $t = t_f = 4.0s$

$\theta = \theta_f = 80°$

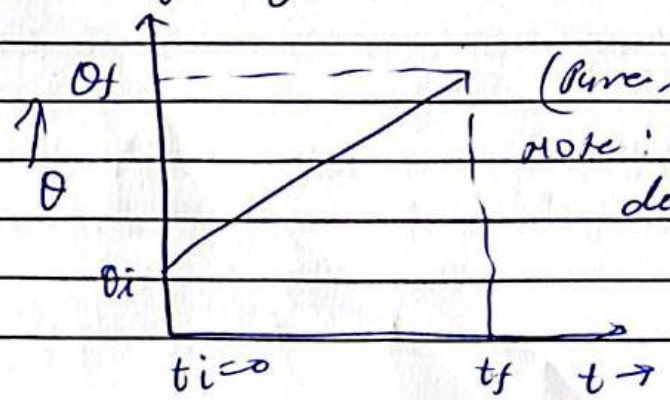
$\dot{\theta} = 0$;

$$\theta(t) = \theta_i + \frac{3(\theta_f - \theta_i)}{t_f^2} t^2 - \frac{2(\theta_f - \theta_i)}{t_f^3} t^3$$

$$= 20 + \frac{3(80-20)}{(4.0)^2} t^2 - \frac{2(80-20)}{(4.0)^3} t^3$$

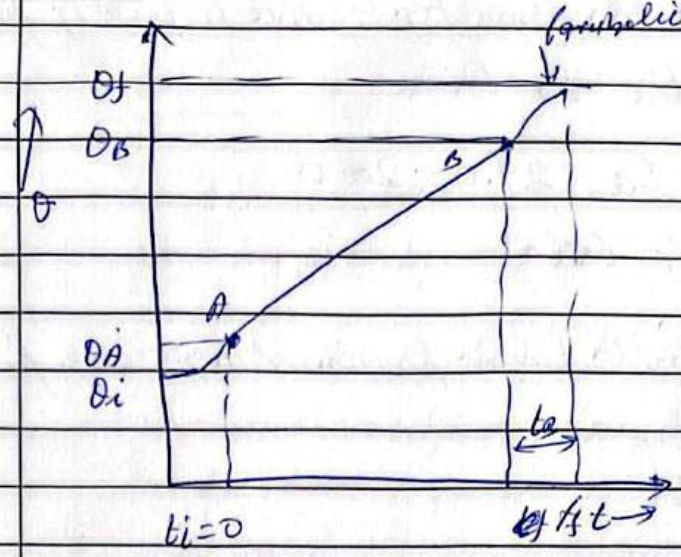
$$= 20 + 11.25 t^2 - 1.875 t^3$$

⇒ Linear Trajectory Function



(Pure linear Trajectory function)

Note: Infinite acceleration and declaration.



modified linear trajectory function with parabolic blends.

<u>A Numerical Example</u>: A linear trajectory function with parabolic blends at its two ends is to be obtained to satisfy the following conditions given below.

At time $t = t_i = 0$, $\theta = \theta_i = 20.0°$, $\dot{\theta} = 0.0$,

At time $t = t_f = 12.0s$, $\theta = \theta_f = 74.0°$, $\dot{\theta} = 0.0$;

total cycle time $t_c = t_f - t_i = 12.0s$

time duration at each of the blend portion $t_b = 3.0s$

magnitude of acceleration/deceleration

$\ddot{\theta} = 2.0$ degree/s^2

Determine angular displacement and velocity at two functions of parabolic blends with the straight portion of trajectory function.

<u>Solution</u>. At point A angular displacement.

$$\theta_A = \theta_i + \frac{1}{2} \times \ddot{\theta} \times t_b^2 = 20.0 + \frac{1}{2} \times (2.0) \times (3.0)^2$$
$$= 29.0°$$

$s = ut + \frac{1}{2}at^2$

Angular velocity

$$\dot{\theta}_A = \dot{\theta}_i + \ddot{\theta} \times t_b = 0.0 + 2.0 \times 3.0$$
$$= 6.0 \text{ degree/s}$$

$v = u + at$

At point B: from the symmetry of the trajectory function:

$$\theta_f - \theta_B = \theta_A - \theta_i$$

$$74.0 - \theta_B = 29.0 - 20.0$$

$$\theta_B = 65.0^\circ$$

Angular velocity in the linear portion of trajectory function

$$= \frac{\theta_B - \theta_A}{t_f - t_B}$$

$$= \frac{65.0 - 29.0}{12.0 - 2 \times 3.0} = \frac{36.0}{6.0} = 6.0 \text{ degrees/s}$$

To maintain continuity of the trajectory function at point B, $\dot{\theta}_B$ should be equal to the velocity of the linear portion, that is, 6.0 degree per second.

Singularity Checking Through Jacobian

23 Singularity checking

Jacobian → multi-dimensional form of derivatives.

$$f = f_1(x_1 - - x_n)$$

$$\frac{\partial f_1}{\partial x_1}, \frac{\partial f_1}{\partial x_2}, - - - -$$

Let us consider six functions and each of which is a function of six independent variables.

$y_1 = f_1(x_1, x_2, x_3, x_4, x_5, x_6)$

$y_2 = f_2(x_1, x_2, x_3, x_4, x_5, - x_6)$

$\vdots$

$y_6 = f_6(x_1, x_2, x_3, x_4, x_5, x_6)$

$$Y = \begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_6 \end{Bmatrix} \quad X = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_6 \end{Bmatrix}$$

in vector notation: $Y = f(X)$

now,

$$\delta y_1 = \frac{\partial f_1}{\partial x_1}\delta x_1 + \frac{\partial f_1}{\partial x_2}\delta x_2 + - + \frac{\partial f_1}{\partial x_6}\delta x_6$$

$$\delta y_2 = \frac{\partial f_2}{\partial x_1}\delta x_1 + \frac{\partial f_2}{\partial x_2}\delta x_2 + - + \frac{\partial f_2}{\partial x_6}\delta x_6$$

$\vdots$

$$\delta y_6 = \frac{\partial f_6}{\partial x_1}\delta x_1 + \frac{\partial f_6}{\partial x_2} + - - + \frac{\partial f_6}{\partial x_6}\delta x_6$$

In vector notation: $\delta Y = J(X)\delta X$

where $J(X)$ is Jacobian

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & - - - & \frac{\partial f_1}{\partial x_6} \\ \vdots & & & \\ \frac{\partial f_6}{\partial x_1} & \frac{\partial f_6}{\partial x_2} & - - & \frac{\partial f_6}{\partial x_6} \end{bmatrix}$$

Scanned with CamScanner

Also,

$$\lim_{\delta t \to 0} \frac{\delta y}{\delta t} = \lim_{\delta t \to 0} J(x) \frac{\delta x}{\delta t}$$

$$\Rightarrow \dot{y} = J(x)\dot{x}$$

$$\lim_{\delta t \to 0} \frac{\delta y}{\delta t} = J(x) \lim_{\delta t \to 0} \frac{\delta x}{\delta t}$$

$$\frac{dy}{dt} = J(x)\frac{dx}{dt}$$

$$\dot{y} = J(x)\dot{x}$$

in Robotics

$$V = J(\theta)\dot{\theta}$$

Cartesian velocity — Jacobian — Joint velocity

$$\dot{y} = J(x)\dot{x}$$
$$V = J(\theta)\dot{\theta}$$

$J(\theta)$

$$\Rightarrow J^{-1}(\theta)V = J^{-1}(\theta)J(\theta)\dot{\theta}$$

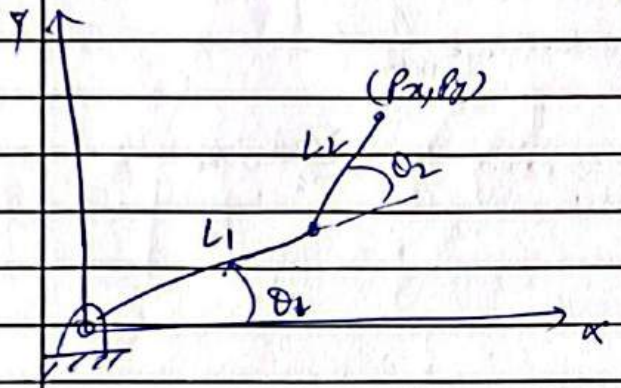
$$\Rightarrow J^{-1}(\theta)V = \dot{\theta}$$

$$J^{-1}(\theta) = \frac{\operatorname{adj} J(\theta)}{|J(\theta)|}$$

$|J(\theta)| = 0$

→ check singularity of a manipulator

Two DOF Serial manipulator



$$P_x = L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2)$$

$$\frac{\partial P_x}{\partial \theta_1} = -L_1\sin\theta_1 - L_2\sin(\theta_1+\theta_2)$$

$$\frac{\partial P_x}{\partial \theta_2} = -L_2\sin(\theta_1+\theta_2)$$

$$P_y = L_1\sin\theta_1 + L_2\sin(\theta_1+\theta_2)$$

$$\frac{\partial P_y}{\partial \theta_1} = L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2)$$

$$\frac{\partial P_y}{\partial \theta_2} = L_2\cos(\theta_1+\theta_2)$$

$$J(\theta_1, \theta_2) = \begin{bmatrix} \frac{\partial P_x}{\partial \theta_1} & \frac{\partial P_x}{\partial \theta_2} \\ \frac{\partial P_y}{\partial \theta_1} & \frac{\partial P_y}{\partial \theta_2} \end{bmatrix}$$

Scanned with CamScanner

$\boxed{-\sin(\theta_1+\theta_2)}$

Jacobian $J(\theta) = \begin{pmatrix} -L_1S_1 - L_2S_{12} & -L_2S_{12} \\ L_1C_1 + L_2C_{12} & L_2C_{12} \end{pmatrix}$

now

$$\dot{\theta} = J^{-1}(\theta)\, V$$

$J^{-1}(\theta)$ should exist, that is $|J(\theta)| \neq 0$

for singularity $J(\theta) = 0$

$$\Rightarrow L_1L_2S_2 = 0$$

Now

$L_1 \neq 0, L_2 \neq 0$

$S_2 = 0$

$\sin\theta_2 = 0$

So $\theta_2 = 0^\circ$ or 180°

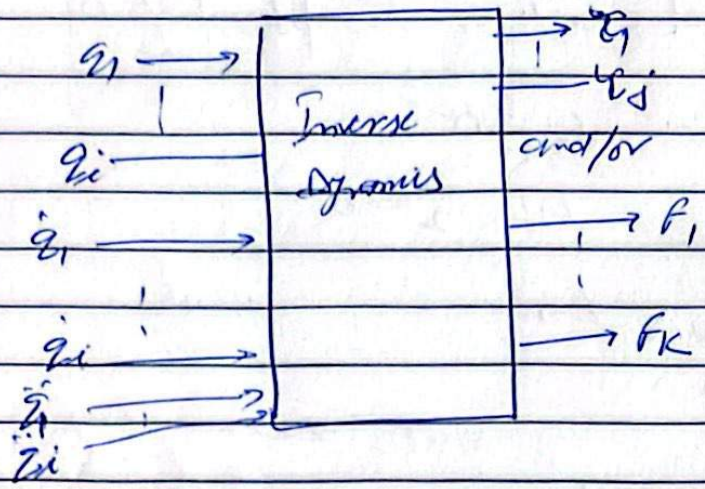
when $\theta_2 = 0^\circ$, $\rightarrow$ fully stretched

$\theta_2 = 180^\circ$; $\Rightarrow$ folded back situation.

Scanned with CamScanner

24 Robot Dynamics

The purpose of dynamics is to determine the amount of force if it is a linear joint or the amount of torque if it is a rotary joint. Which is the reason behind the movement of robotics least robotics joints.



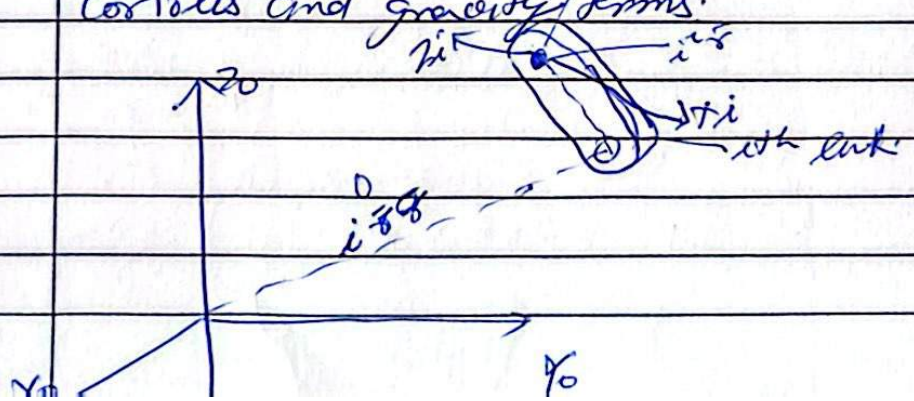
Inverse Dynamics

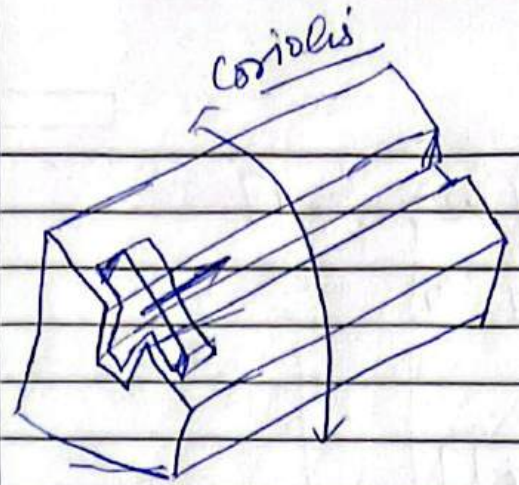
$j = i$, when all are rotary joints
$K = i$, when all are linear joints
$j+K = i$, when there are j rotary joints and K linear joints

q = generalized coordinates
$q = \theta$ for rotary joints
$= d$ for linear joints

Robot Dynamics

→ To determine joint torques/forces.
→ Robotic joint torque consists of inertia, centrifugal and coriolis and gravity terms.





Reaction force / Reaction torque: that why Coriolis not of motor.

Inertia Term: Depends on mass distribution of the links and it is expressed in terms of moment of inertia tensor.

Let ${}^{i}_{i}\bar{r}$ = position of a fixed point lying on i-th rigid link expressed in its own coordinate system.

$$= \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix}_{4\times 1}$$

The same point can be expressed in base coordinate system as follows:

$${}^{0}_{i}\bar{r} = {}^{0}_{i}T \, {}^{i}_{i}\bar{r}$$

where ${}^{0}_{i}T = {}^{0}_{1}T \, {}^{1}_{2}T \, {}^{2}_{3}T \cdots {}^{i-1}_{i}T$

Inertia Tensor of i-th link:

$$J_i = \int {}^{i}_{i}\bar{r} \, {}^{i}_{i}\bar{r}^{T} \, dm$$

$$= \begin{bmatrix} \int x_i^2 dm & \int x_i y_i dm & \int x_i z_i dm & \int x_i dm \\ \int x_i y_i dm & \int y_i^2 dm & \int y_i z_i dm & \int y_i dm \\ \int x_i z_i dm & \int y_i z_i dm & \int z_i^2 dm & \int z_i dm \\ \int x_i dm & \int y_i dm & \int z_i dm & \int dm \end{bmatrix}$$

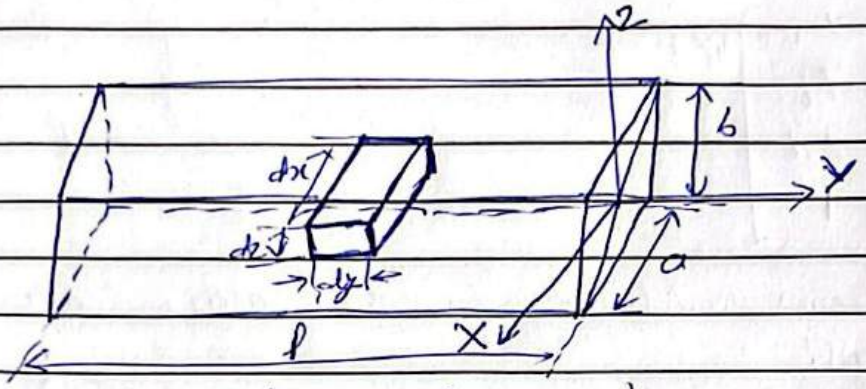
$$\underset{i}{\underset{\sim}{r}} = \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix}_{4\times 1}, \quad \underset{i}{\underset{\sim}{r}}^T = [x_i, y_i, z_i, 1]_{1\times 4}$$

$$\begin{pmatrix} x_i \\ y_i \\ z_i \\ 1 \end{pmatrix} \times [x_i \; y_i \; z_i \; 1]$$

$$= \begin{pmatrix} x_i^2 & x_i y_i & z_i x_i & x_i \\ x_i y_i & y_i^2 & y_i z_i & y_i \\ z_i x_i & y_i z_i & z_i^2 & z_i \\ x_i & y_i & z_i & 1 \end{pmatrix}$$

Various Cases

Case 1 :- Link with Rectangular Cross-section



Moment of Inertia (Positive Value)

Differential mass $dm = \rho\, dx\, dy\, dz$

$$I_{xx} = \int_{-b/2}^{b/2}\int_{-l}^{0}\int_{-a/2}^{a/2} (y^2+z^2)\, \rho\, dx\, dy\, dz$$

$$= m\left(\frac{l^2}{3} + \frac{b^2}{12}\right)$$

$$m = \underbrace{\rho a b l}_{\text{volume}}$$

mass

$$I_{yy} = \int_{-b/2}^{b/2}\int_{-l}^{0}\int_{-a/2}^{a/2} (x^2+z^2)\, \rho\, dx\, dy\, dz$$

$$= m\left(\frac{a^2}{12} + \frac{b^2}{12}\right)$$

Scanned with CamScanner

$$I_{zz} = \int_{-b/2}^{b/2}\int_{-l}^{0}\int_{-a/2}^{a/2} (x^2+y^2)\,\rho\,dx\,dy\,dz$$

$$= m\left(\frac{l^2}{3} + \frac{a^2}{12}\right)$$

product of Inertia (positive/negative/zero)

$$I_{xy} = \int_{-b/2}^{b/2}\int_{-l}^{0}\int_{-a/2}^{a/2} xy\,\rho\,dx\,dy\,dz = 0$$

$$I_{yz} = \int_{-b/2}^{b/2}\int_{-l}^{0}\int_{-a/2}^{a/2} yz\,\rho\,dx\,dy\,dz = 0$$

$$I_{zx} = \int_{-b/2}^{b/2}\int_{-l}^{0}\int_{-a/2}^{a/2} zx\,\rho\,dx\,dy\,dz = 0$$

$$\int x\,dm = \int_{-b/2}^{b/2}\int_{-l}^{0}\int_{-a/2}^{a/2} x\rho\,dx\,dy\,dz = 0$$

$$\int y\,dm = -m\frac{l}{2} = m\bar{y}_i$$

$$\int z\,dm = 0$$

Mass Center $= (\bar{x}_i, \bar{y}_i, \bar{z}_i)$

$$= (0, -l/2, 0)$$

$$\int dm = m$$

Inertia tensor, J_i can be written as

$$J_i = \begin{bmatrix} \frac{-I_{xx}+I_{yy}+I_{zz}}{2} & I_{xy} & I_{zx} & m_i\bar{x}_i \\ I_{xy} & \frac{I_{xx}-I_{yy}+I_{zz}}{2} & I_{yz} & m_i\bar{y}_i \\ I_{zx} & I_{yz} & \frac{I_{xx}+I_{yy}-I_{zz}}{2} & m_i\bar{z}_i \\ m_i\bar{x}_i & m_i\bar{y}_i & m_i\bar{z}_i & m_i \end{bmatrix}$$

Scanned with CamScanner

$$= \begin{bmatrix} \frac{ma^2}{12} & 0 & 0 & 0 \\ 0 & \frac{ml^2}{3} & 0 & -\frac{ml}{2} \\ 0 & 0 & \frac{mb^2}{12} & 0 \\ 0 & -\frac{ml}{2} & 0 & m \end{bmatrix}$$

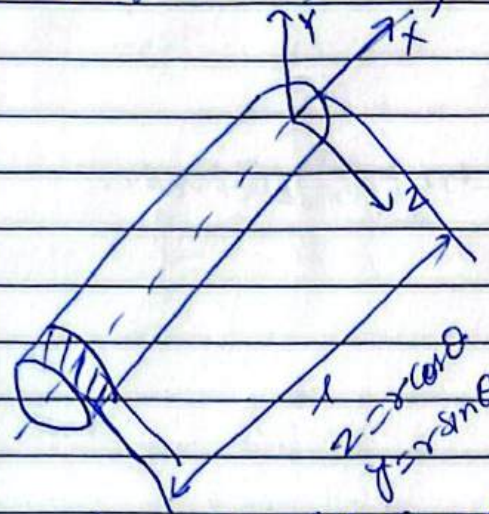
4x4

for a slender bar:
($l >> a$ and $b >> b$)

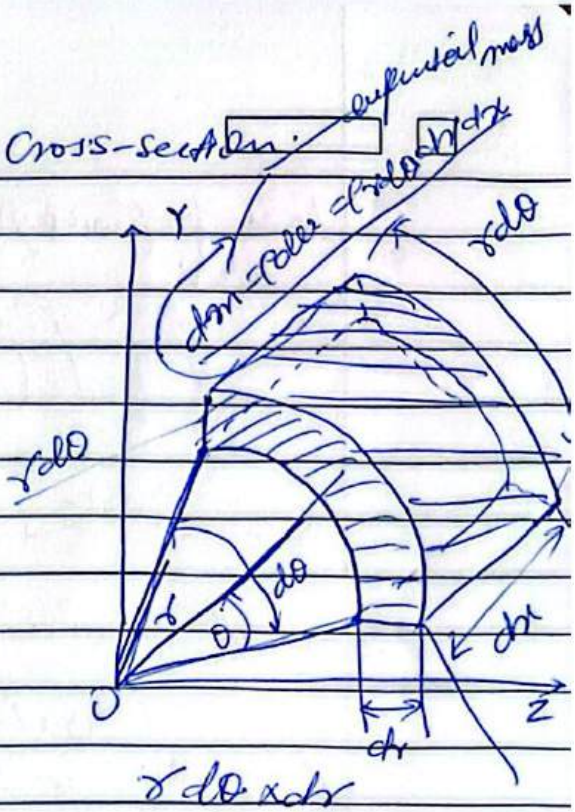
$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{ml^2}{3} & 0 & -\frac{ml}{2} \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{ml}{2} & 0 & m \end{bmatrix}$$

Scanned with CamScanner

Case 2: Robotic link of circular cross-section



R: Radius of the circular link



$r\,d\theta \times dr$

$dV = r\,d\theta\,dr \times dx$

Let us consider a link of length l having circular cross-section of radius r

$$y = r\sin\theta$$
$$z = r\cos\theta$$

Volume of small element $dv = r\,d\theta\,dr\,dx$

mass of small element $dm = \rho dv$, where ρ = density

Moment of inertia

$$I_{xx} = \int_v (y^2+z^2)\,dm = \int_{-l}^{0}\int_0^r\int_0^{2\pi} r^2 \rho\, r\,d\theta\,dr\,dx = \frac{1}{2} m r^2$$

$$\boxed{\text{where } m = \pi r^2 \times l \times \rho}$$

$$I_{yy} = \int_v (x^2+z^2)\,dm$$
$$= \int_{-l}^{0}\int_0^r\int_0^{2\pi} (x^2 + r^2\cos^2\theta)\,\rho\, r\,d\theta\,dr\,dx$$
$$= \frac{ml^2}{3} + \frac{mr^2}{4}$$

Scanned with CamScanner

$$I_{zz} = \int_V (x^2+y^2)\,dm$$

$$= \int_{-l}^{0}\int_0^{r}\int_0^{2\pi} (x^2+r^2\sin^2\theta)\,\rho\, r\,dr\,d\theta\,dx$$

$$= \frac{ml^2}{3} + \frac{mr^2}{4}$$

product of Inertia

$$I_{xy} = \int_V xy\,dm$$

$$= \int_{-l}^{0}\int_0^{r}\int_0^{2\pi} xr\sin\theta\,\rho\, r\,dr\,d\theta\,dx$$

$$= 0$$

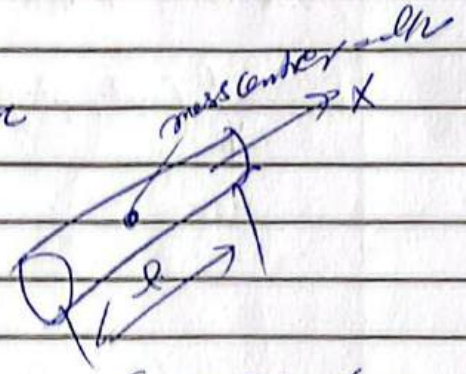
Similar $I_{yz} = 0$; $I_{zx} = 0$

$$I_{yz} = \int_V yz\,dm$$

$$I_{zx} = \int_V zx\,dm$$

$$\int_V x\,dm = \int_{-l}^{0}\int_0^{r}\int_0^{2\pi} x\,\rho\, r\,dr\,d\theta\,dx$$

$$= -\frac{1}{2}ml$$



$$\int_V y\,dm = 0$$

$$\int_V z\,dm = 0$$

$$\int dm = m$$

mass center $= (\bar{x}, \bar{y}, \bar{z}) = (-l/2, 0, 0)$

Inertia Tensor

$$J_i = \begin{bmatrix} \frac{ml^2}{3} & 0 & 0 & -\frac{ml}{2} \\ 0 & \cancel{\frac{mr^2}{4}} & 0 & 0 \\ 0 & 0 & \cancel{\frac{mr^2}{4}} & 0 \\ -\frac{ml}{2} & 0 & 0 & m \end{bmatrix}$$

for a slender link ($l \gg r$)

$$J_i = \begin{bmatrix} \frac{ml^2}{3} & 0 & 0 & -\frac{ml}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{ml}{2} & 0 & 0 & m \end{bmatrix}$$

Determination of Robotic Joint Torque

Lagrange - Euler formulation:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \tau_i ,$$

where $i = 1, 2 \ldots n$

n = No. of Joint

L : Lagrangian function

$L = \underbrace{K(K.E)}_{\text{Kinetic Energy}} - \underbrace{P(P.E)}_{\text{Potential Energy}}$

q_i = Generalized Coordinates.

$q_i = \theta_i$ for a rotary joint

$= d_i$ for a prismatic joint

$\dot{q}_i$ = first time (t) derivative of q_i

τ : Generalized torque for a rotary joint.

: Generalized force for a linear joint.

Let us consider i-th link of a serial manipulator positional a fixed point lying on this link.

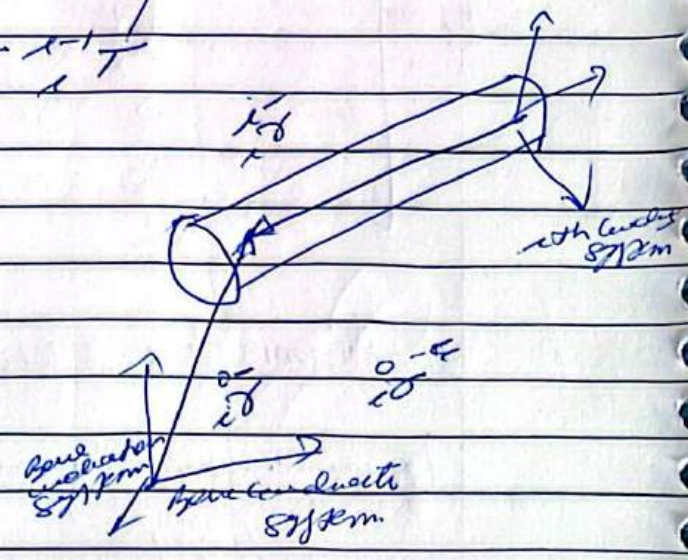
$$\underset{\sim}{r}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix}$$

Scanned with CamScanner

$$ {}^{0}_{i}\bar{r} = {}^{0}_{i}T \, {}^{i}_{i}\bar{r} $$

transformation matrix

where ${}^{0}_{i}T = {}^{0}_{1}T \, {}^{1}_{2}T \cdots {}^{i-1}_{i}T$



Determination of kinetic Energy (K) of the manipulator

velocity of a particle of link i w.r.t to base coordinate system

$$ {}^{0}_{i}\bar{V} = \frac{d}{dt}\left({}^{0}_{i}\bar{r}\right) $$

$$ {}^{0}_{i}\bar{V} = \frac{d}{dt}\left({}^{0}_{i}T \, {}^{i}_{i}\bar{r}\right) = {}^{0}_{1}\dot{T} \, {}^{1}_{2}T \cdots {}^{i-1}_{i}T \, {}^{i}_{i}\bar{r} + \cdots + {}^{0}_{1}T \, {}^{1}_{2}T \cdots $$

$$ \cdots {}^{i-1}_{i}\dot{T} \, {}^{i}_{i}\bar{r} + {}^{0}_{i}T \, {}^{i}_{i}\dot{\bar{r}} $$

$$ = \sum_{j=1}^{i} \frac{\partial \, {}^{0}_{i}T}{\partial q_j} \dot{q}_j \Big) \, {}^{i}_{i}\bar{r}, \text{ as } {}^{i}_{i}\dot{\bar{r}} = 0 $$

Let $\frac{\partial \, {}^{0}_{i}T}{\partial q_j} = U_{ij}$ Therefore, ${}^{0}_{i}\bar{V} = \sum_{j=1}^{i} U_{ij} \dot{q}_j \Big) \, {}^{i}_{i}\bar{r}$

Note: $U_{ijk} = \frac{\partial U_{ij}}{\partial q_k}$

Kinetic energy of the particle having differential mass dm

$$dK_i = \frac{1}{2}(\dot{x}_i^2 + \dot{y}_i^2 + \dot{z}_i^2)\,dm = \frac{1}{2} Tr\left({}^{0}_{i}\dot{\bar{V}}\,{}^{0}_{i}\dot{\bar{V}}^{T'}\right) dm$$

where Tr: Trace of a matrix

$$dK_i = \frac{1}{2} Tr\left[\sum_{a=1}^{i} U_{ia}\,{}^{i}\dot{q}_a\,{}^{i}_{i}\bar{r}\left[\sum_{b=1}^{i} U_{ib}\,{}^{i}\dot{q}_b\,{}^{i}_{i}\bar{r}\right]^{T'}\right] dm$$

$$= \frac{1}{2} Tr\left[\sum_{a=1}^{i}\sum_{b=1}^{i} U_{ia}\,{}^{i}_{i}\bar{r}\,{}^{i}_{i}\bar{r}^{T'}\,U^{T'}_{ib}\,\dot{q}_a\,\dot{q}_b\right] dm$$

$$= \frac{1}{2} Tr\left[\sum_{a=1}^{i}\sum_{b=1}^{i} U_{ia}\left({}^{i}_{i}r\,dm\,{}^{i}_{i}r^{T'}\right) U^{T'}_{ib}\,\dot{q}_a\,\dot{q}_b\right]$$

T' → trace
↳ Calculate $[\dot{x}_i, \dot{y}_i, \dot{z}_i]$

$$\begin{aligned} {}^{0}_{i}\dot{\bar{V}} &= \begin{bmatrix}\dot{x}_i \\ \dot{y}_i \\ \dot{z}_i\end{bmatrix}\begin{bmatrix}\dot{x}_i & \dot{y}_i & \dot{z}_i\end{bmatrix} \\ &= \begin{bmatrix}\dot{x}_i^2 & \dot{x}_i\dot{y}_i & \dot{z}_i\dot{x}_i \\ \dot{x}_i\dot{y}_i & \dot{y}_i^2 & \dot{y}_i\dot{z}_i \\ \dot{z}_i\dot{x}_i & \dot{y}_i\dot{z}_i & \dot{z}_i^2\end{bmatrix} \quad Tr \end{aligned}$$

Kinetic energy of i-th link

$$K_i = \int dK_i = \frac{1}{2} Tr\left[\sum_{a=1}^{i}\sum_{b=1}^{i} U_{ia}\left(\int {}^{i}_{i}r\, {}^{i}_{i}r^{T}\, dm\right) U^{T}_{ib}\dot{q}_a \dot{q}_b\right]$$

$$= \frac{1}{2} Tr\left[\sum_{a=1}^{i}\sum_{b=1}^{i} U_{ia} J_i U^{T}_{ib}\dot{q}_a \dot{q}_b\right]$$

Where inertia tensor

$$J_i = \int {}^{i}_{i}r\, {}^{i}_{i}r^{T}\, dm$$

Total K.E. of the serial manipulator having n links

$$K = \sum_{i=1}^{n} K_i = \sum_{i=1}^{n} \frac{1}{2} Tr\left[\sum_{a=1}^{i}\sum_{b=1}^{i} U_{ia} J_i U^{T}_{ib}\dot{q}_a \dot{q}_b\right]$$

~~Kinetic~~ ~~Total K.E~~

Determination of potential Energy of the manipulator

Potential energy of i-th link

$$P_i = -m_i \bar{g}\, {}^{0}_{i}r = -m_i \bar{g}\left(\underbrace{{}^{0}_{i}T}_{4\times4}\ \underbrace{{}^{i}_{i}r}_{4\times1}\right) \quad \frac{4\times1}{1\times4} \qquad \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix}$$

Where $\bar{g} = (g_x, g_y, g_z, 0)$

Total potential energy of the manipulator

$$P = \sum_{i=1}^{n} P_i = \sum_{i=1}^{n} -m_i \bar{g}\, {}^{0}_{i}T\, {}^{i}_{i}r$$

Now,

$$L = K - P$$

$$= \frac{1}{2}\sum_{i=1}^{n}\sum_{a=1}^{i}\sum_{b=1}^{i}\left[Tr\left(U_{ia} J_i U^{T}_{ib}\right)\dot{q}_a \dot{q}_b\right] + \sum_{i=1}^{n} m_i \bar{g}\left({}^{0}_{i}T\, {}^{i}_{i}r\right)$$

Scanned with CamScanner

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \tau_i$

using lagrange-Euler equation, we get

$$\tau_i = \sum_{c=1}^{n} D_{ic}\ddot{q}_c + \sum_{c=1}^{n}\sum_{d=1}^{n} h_{icd}\dot{q}_c\dot{q}_d + C_i ,$$

where $i = 1, 2, \ldots, n$

Inertia term

$$D_{ic} = \sum_{j=\max(i,c)}^{n} Tr\left(U_{jc} J_j U_{ji}^{T}\right) \quad i, c = 1, 2, \ldots, n$$

coriolis and centrifugal term

$$h_{icd} = \sum_{j=\max(i,c,d)}^{n} Tr\left(U_{jcd} J_j U_{ji}^{T}\right) \quad i, c, d = 1, 2 \ldots n$$

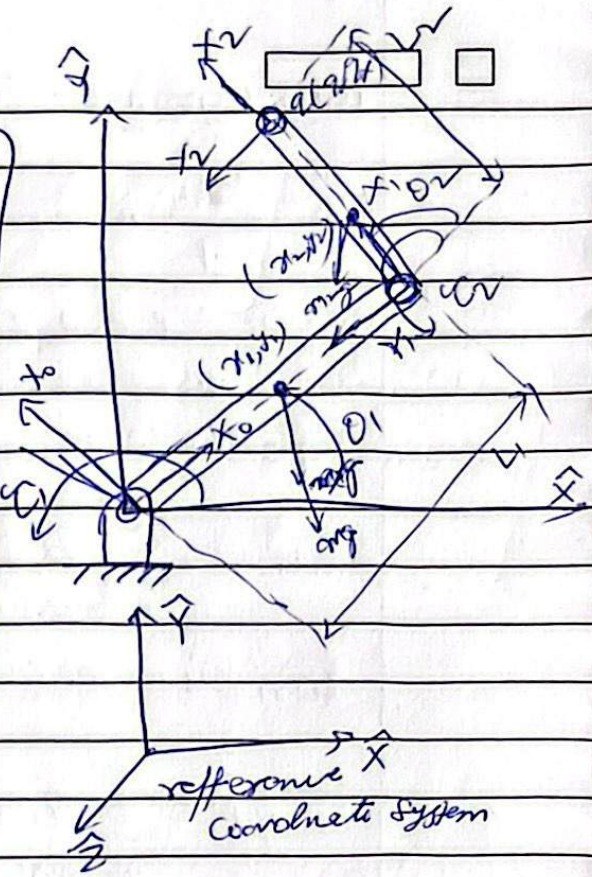
Gravity term

$$C_i = \sum_{j=i}^{n}\left(-m_j \bar{g} U_{ji} {}^{j}\bar{r}\right) \quad i = 1, 2, \ldots n$$

Scanned with CamScanner

An Example

$$ {}^{0}T = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & L_1C\theta_1 \\ S\theta_1 & C\theta_1 & 0 & L_1S\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$



$$ {}^{0}_{2}T = \begin{bmatrix} C\theta_{12} & -S\theta_{12} & 0 & L_1C\theta_1 + L_2C\theta_{12} \\ S\theta_{12} & C\theta_{12} & 0 & L_1S\theta_1 + L_2S\theta_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ \boxed{\tau_1 = (D_{11}\ddot{\theta}_1 + D_{12} + D_{12}\ddot{\theta}_2) + h \ldots \quad \text{from previous equation}} $$

$$ \tau_1 = (D_{11}\ddot{\theta}_1 + D_{12}\ddot{\theta}_2) + h_{111}\dot{\theta}_1^2 + h_{112}\dot{\theta}_1\dot{\theta}_2 + h_{121}\dot{\theta}_1\dot{\theta}_2 + h_{122}\dot{\theta}_2^2 + c_1 $$

$$ \tau_2 = (D_{21}\ddot{\theta}_1 + D_{22}\ddot{\theta}_2) + h_{211}\dot{\theta}_1^2 + h_{212}\dot{\theta}_1\dot{\theta}_2 + h_{221}\dot{\theta}_1\dot{\theta}_2 + h_{222}\dot{\theta}_2^2 + c_2 $$

$$ U_{11} = \frac{\partial\, {}^{0}_{1}T}{\partial\theta_1} = \begin{bmatrix} -S\theta_1 & -C\theta_1 & 0 & -L_1S\theta_1 \\ C\theta_1 & -S\theta_1 & 0 & L_1C\theta_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} $$

$$ U_{21} = \frac{\partial\, {}^{0}_{2}T}{\partial\theta_1} = \begin{bmatrix} -S\theta_{12} & -C\theta_{12} & 0 & -L_1S\theta_1 - L_2S\theta_{12} \\ C\theta_{12} & -S\theta_{12} & 0 & L_1C\theta_1 + L_2C\theta_{12} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} $$

Scanned with CamScanner

$$U_{22} = \frac{\partial_2^0 T}{\partial \theta_2} = \begin{bmatrix} -s\theta_{12} & -c\theta_{12} & 0 & -L_2 s\theta_{12} \\ -c\theta_{12} & -s\theta_{12} & 0 & L_2 c\theta_{12} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Inertia tensor for link 1 & Link 2

$$J_1 = \begin{bmatrix} \frac{m_1 L_1^2}{3} & 0 & 0 & -\frac{1}{2} m_1 L_1 \\ 0 & \frac{m_1 r^2}{4} & 0 & 0 \\ 0 & 0 & \frac{m_1 r^2}{4} & 0 \\ -\frac{1}{2} m_1 L_1 & 0 & 0 & m_1 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} \frac{m_2 L_2^2}{3} & 0 & 0 & -\frac{1}{2} m_2 L_2 \\ 0 & \frac{m_2 L_2^2}{4} & 0 & 0 \\ 0 & 0 & \frac{m_2 r^2}{4} & 0 \\ -\frac{1}{2} m_2 L_2 & 0 & 0 & m_2 \end{bmatrix}$$

$$D_{11} = Tr(U_{11} J_1 U_{11}^T) + Tr(U_{21} J_2 U_{21}^T)$$

$$= \left(\frac{1}{3} m_1 + m_2\right) L_1^2 + \frac{1}{3} m_2 L_2^2 + m_2 L_1 L_2 c\theta_2 + \frac{1}{4} r^2 (m_1 + m_2)$$

$$D_{12} = Tr(U_{22} J_2 U_{21}^T)$$

$$= \frac{1}{3} m_2 L_2^2 + \frac{1}{4} m_2 r^2 + \frac{1}{2} m_2 L_1 L_2 c\theta_2$$

$$D_{21} = Tr(U_{21} J_2 U_{22}^T)$$

$$= \frac{1}{3} m_2 L_2^2 + \frac{1}{4} m_2 r^2 + \frac{1}{2} m_2 L_1 L_2 c\theta_2$$

$$D_{22} = Tr(U_{22} J_2 U_{22}^T)$$

$$= \frac{1}{3} m_2 L_2^2 + \frac{1}{4} m_2 r^2$$

$$h_{111} = Tr(U_{111} J_1 U_{11}^T) + Tr(U_{211} J_2 U_{21}^T),$$

Scanned with CamScanner

$$U_{111} = \frac{\partial U_{11}}{\partial \theta_1} = \begin{bmatrix} -c\theta_1 & s\theta_1 & 0 & -L_1 c\theta_1 \\ -s\theta_1 & -c\theta_1 & 0 & -L_1 s\theta_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$U_{211} = \frac{\partial U_{21}}{\partial \theta_1} = \begin{bmatrix} -c\theta_{12} & s\theta_{12} & 0 & -L_1 c\theta_1 - L_2 c\theta_{12} \\ -s\theta_{12} & -c\theta_{12} & 0 & -L_1 s\theta_1 - L_2 s\theta_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underline{h_{111} = 0}$$

$$h_{112} = Tr\left(U_{212} J_2 U_{21}^T\right),$$

$$U_{212} = \frac{\partial U_{21}}{\partial \theta_2} = \begin{bmatrix} -c\theta_{12} & s\theta_{12} & 0 & -L_2 c\theta_{12} \\ -s\theta_{12} & -c\theta_{12} & 0 & -L_2 s\theta_{12} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$h_{121} = -L_2 m_2 L_1 L_2 s\theta_2$$

$$G = -m_1 \bar{g} U_{11}{}^1\bar{r} - m_2 \bar{g} U_{21}{}^2\bar{r}$$

$$G = \sum_{j=1}^{2} \left(-m_j \bar{g} U_{j1}{}^j\bar{r}\right)$$

$$= -m_1 \bar{g} U_{11}{}^1\bar{r} - m_2 \bar{g} U_{21}{}^2\bar{r}$$

Substituting the values of $\bar{g} = (0 \; -g \; 0 \; 0)$, $U_{11}, U_{21}, {}^1\bar{r} = \left(-\frac{L_1}{2} \; 0 \; 0 \; 1\right)^T$ and ${}^2\bar{r} = \left(-\frac{L_2}{2} \; 0 \; 0 \; 1\right)^T$ in the above expression, we get

$$G = \tfrac{1}{2} m_1 g L_1 c\theta_1 + m_2 g L_1 c\theta_1 + \tfrac{1}{2} m_2 g L_2 c\theta_{12}$$

$\boxed{\text{g is in the negative opposite to y direction then putting at place of y we used } -g}$

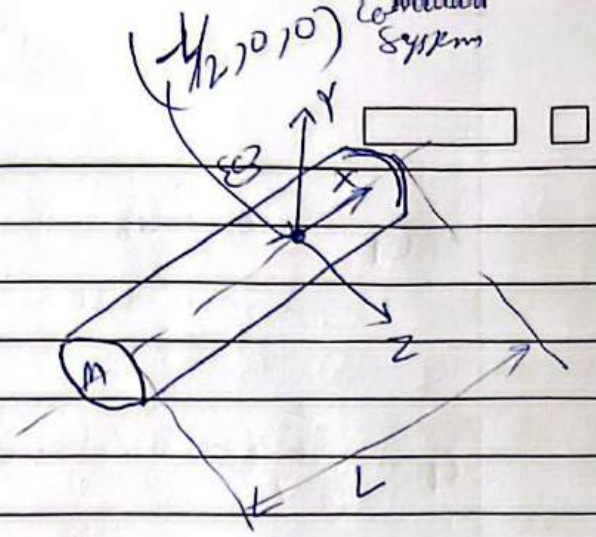
$$G_2 = -m_2 \bar{g} U_{22}{}^2\bar{r}$$

Substituting the value of $\bar{g} = (0 \; -g \; 0 \; 0)$, $U_{22}, {}^2\bar{r} = \left(-\frac{L_2}{2} \; 0 \; 0 \; 1\right)^T$ in the above expression, we get

$$G_2 = \tfrac{1}{2} m_2 g L_2 c\theta_{12}$$

$$\tau_1 = \left(\left(\tfrac{1}{3}m_1 + m_2\right)L_1^2 + \tfrac{1}{3}m_2 L_2^2 + m_2 L_1 L_2 c\theta_2 + \cancelto{0}{\tfrac{1}{4}r_2^2(m_1+m_2)}\right)\ddot{\theta}_1 + \left(\tfrac{1}{3}m_2 L_2^2 + \cancelto{0}{\tfrac{1}{4}m_2 r_2^2} + \tfrac{1}{2}m_2 L_1 L_2 c\theta_2\right)\ddot{\theta}_2 - m_2 L_1 L_2 s\theta_2 \dot{\theta}\dot{\theta}_2 - \tfrac{1}{2}m_2 L_1 L_2 s\theta_2 \dot{\theta}_2^2 + \tfrac{1}{2}m_1 g L_1 c\theta_1 + m_2 g L_1 c\theta_1 + \tfrac{1}{2}m_2 g L_2 c\theta_1$$

$$\tau_2 = \left(\left(\tfrac{1}{3}m_2 L_2^2 + \cancelto{0}{\tfrac{1}{4}m_2 r_2^2} + \tfrac{1}{2}m_2 L_1 L_2 c\theta_2\right)\ddot{\theta}_1 + \left(\tfrac{1}{3}m_2 L_2^2 + \cancelto{0}{\tfrac{1}{4}m_2 r_2^2}\right)\ddot{\theta}_2 + \tfrac{1}{2}m_2 L_1 L_2 s\theta_2 \dot{\theta}_1^2 + \tfrac{1}{2}m_2 g L_2 c\theta_{12}$$



Another approach:

$I_{xx} = \frac{1}{2} m r^2$

$I_{yy} = \frac{1}{3} m L^2 + \frac{1}{4} m r^2$

$I_{zz} = \frac{1}{3} m L^2 + \frac{1}{4} m r^2$

Parallel axis theorem:

Coordinate system at the centre of mass

$$I_{zz} = I_{zz} - m(\bar{x}^2 + \bar{y}^2)$$

$$= \frac{1}{3} m L^2 + \frac{1}{4} m r^2 - m\left(\frac{L^2}{4} + 0\right)$$

$$= \frac{1}{12} m L^2 + \frac{1}{4} m r^2$$

Kinetic energy (first link)

$$K_1 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} I_1 \omega_1^2,$$

$$v_1 = \frac{d}{dt}\left(\frac{L_1}{2}\theta_1\right) = \frac{1}{2} L_1 \dot{\theta}_1$$

$$I_1 = \frac{1}{12} m_1 L_1^2 + \frac{1}{4} m_1 r^2$$

$$K_1 = \frac{1}{8} m_1 L_1^2 \dot{\theta}_1^2 + \frac{1}{8} m_1 r^2 \dot{\theta}_1^2$$

Potential energy (first link)

$$P_1 = -m_1(-g)\frac{L_1}{2} s\theta_1 = \frac{1}{2} m_1 g L_1 s\theta_1$$

Second link

$$\bar{x}_2 = L_1 c\theta_1 + \frac{L_2}{2} c\theta_{12}; \quad \bar{y}_2 = L_1 s\theta_1 + \frac{L_2}{2} s\theta_{12}.$$

$$\dot{\bar{x}}_2 = -L_1 s\theta_1 \dot{\theta}_1 - \frac{L_2}{2} s\theta_{12}(\dot{\theta}_1 + \dot{\theta}_2),$$

$$\dot{\bar{y}}_2 = L_1 c\theta_1 \dot{\theta}_1 + \frac{L_2}{2} c\theta_{12}(\dot{\theta}_1 + \dot{\theta}_2)$$

Scanned with CamScanner

$$v_2^2 = \dot{x}_2^2 + \dot{y}_2^2$$

$$= L_1^2\dot{\theta}_1^2 + \frac{L_2^2}{4}(\dot{\theta}_1^2 + 2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2) + L_1L_2c\theta_2(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2)$$

Kinetic energy $\quad K_2 = \frac{1}{2}m_2v_2^2 + \frac{1}{2}I_2\omega_2^2$

$$K_2 = \frac{1}{2}m_2L_1^2\dot{\theta}_1^2 + \frac{m_2L_2^2}{8}(\dot{\theta}_1^2 + 2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2) + \frac{1}{2}m_2L_1L_2c\theta_2(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2) + \frac{1}{2}\left(\frac{1}{12}m_2L_2^2 + \frac{1}{4}m_2r^2\right)(\dot{\theta}_1 + \dot{\theta}_2)^2$$

$$= \frac{1}{2}m_2L_1^2\dot{\theta}_1^2 + \left(\frac{1}{6}m_2L_2^2 + \frac{1}{8}m_2r^2\right)(\dot{\theta}_1 + \dot{\theta}_2)^2 + \frac{1}{2}m_2L_1L_2c\theta_2\dot{\theta}_1^2 + \frac{1}{2}m_2L_1L_2c\theta_2\dot{\theta}_1\dot{\theta}_2$$

Potential energy

$$P_2 = -m_2(-g)L_1s\theta_1 - m_2(-g)\frac{L_2}{2}s\theta_{12}$$

$$= m_2gL_1s\theta_1 + \frac{1}{2}m_2gL_2s\theta_{12}$$

Lagrangian

$$L = K_1 + K_2 - P_1 - P_2$$

$$= \frac{1}{6}m_1L_1^2\dot{\theta}_1^2 + \frac{1}{8}m_1r^2\dot{\theta}_1^2 + \frac{1}{2}m_2L_1^2\dot{\theta}_1^2 + \left(\frac{1}{6}m_2L_2^2 + \frac{1}{8}m_2r^2\right)(\dot{\theta}_1 + \dot{\theta}_2)^2 + \frac{1}{2}m_2L_1L_2c\theta_2\dot{\theta}_1^2 + \frac{1}{2}m_2L_1L_2c\theta_2\dot{\theta}_1\dot{\theta}_2 - \frac{1}{2}m_1gL_1s\theta_1 - m_2gL_1s\theta_1 - \frac{1}{2}m_2gL_2s\theta_{12}$$

~~Lagrange-Euler equation (f~~

Lagrange-Euler equation (first joint)

$$\tau_1 = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1}$$

Scanned with CamScanner

$$\frac{\partial L}{\partial \theta_1} = -\left(\tfrac{1}{2}m_1 + m_2\right) g L_1 C\theta_1 - \tfrac{1}{2} m_2 g L_2 C\theta_{12};$$

$$\frac{\partial L}{\partial \dot\theta_1} = \left(\tfrac{1}{3}m_1 + m_2\right) L_1^2 \dot\theta_1 + \tfrac{1}{3} m_2 L_2^2 (\dot\theta_1 + \dot\theta_2) + \tfrac{1}{2} m_2 L_1 L_2 C\theta_2 (2\dot\theta_1 + \dot\theta_2) + \tfrac{1}{4} m_1 r^2 \dot\theta_1 + \tfrac{1}{4} m_2 r^2 (\dot\theta_1 + \dot\theta_2)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta_1}\right) = \left(\tfrac{1}{3}m_1 + m_2\right) L_1^2 \ddot\theta_1 + \tfrac{1}{3} m_2 L_2^2 (\ddot\theta_1 + \ddot\theta_2) + \tfrac{1}{2} m_2 L_1 L_2 C\theta_2 (2\ddot\theta_1 + \ddot\theta_2) + \tfrac{1}{2} m_2 L_1 L_2 (2\dot\theta_1 + \dot\theta_2)(-S\theta_2)\dot\theta_2 + \tfrac{1}{4} m_1 r^2 \ddot\theta_1 + \tfrac{1}{4} m_2 r^2 (\ddot\theta_1 + \ddot\theta_2)$$

$$Q_1 = \left(\left(\tfrac{1}{3}m_1 + m_2\right) L_1^2 + \tfrac{1}{3} m_2 L_2^2 + m_2 L_1 L_2 C\theta_2 + \tfrac{1}{4} r^2 (m_1 + m_2)\right)\ddot\theta_1 + \left(\tfrac{1}{3} m_2 L_2^2 + \tfrac{1}{4} m_2 r^2 + \tfrac{1}{2} m_2 L_1 L_2 C\theta_2\right)\ddot\theta_2 - m_2 L_1 L_2 S\theta_2 \dot\theta_1 \dot\theta_2 - \tfrac{1}{2} m_2 L_1 L_2 S\theta_2 \dot\theta_2^2 + \tfrac{1}{2} m_1 g L_1 C\theta_1 + m_2 g L_1 C\theta_1 + \tfrac{1}{2} m_2 g L_2 C\theta_{12}$$

Second joint

$$Q_2 = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta_2}\right) - \frac{\partial L}{\partial \theta_2}$$

$$\frac{\partial L}{\partial \theta_2} = \tfrac{1}{2} m_2 L_1 L_2 (-S\theta_2)\dot\theta_1^2 + \tfrac{1}{2} m_2 L_1 L_2 (-S\theta_2)\dot\theta_1 \dot\theta_2 - \tfrac{1}{2} m_2 g L_2 C\theta_{12};$$

$$\frac{\partial L}{\partial \dot\theta_2} = \left(\tfrac{1}{3} m_2 L_2^2 + \tfrac{1}{4} m_2 r^2\right)(\dot\theta_1 + \dot\theta_2) + \tfrac{1}{2} m_2 L_1 L_2 C\theta_2 \dot\theta_1;$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta_2}\right) = \left(\tfrac{1}{3} m_2 L_2^2 + \tfrac{1}{4} m_2 r^2 + \tfrac{1}{2} m_2 L_1 L_2 C\theta_2\right)\ddot\theta_1 + \left(\tfrac{1}{3} m_2 L_2^2 + \tfrac{1}{4} m_2 r^2\right)\ddot\theta_2 - \tfrac{1}{2} m_2 L_1 L_2 S\theta_2 \dot\theta_1 \dot\theta_2$$

Scanned with CamScanner

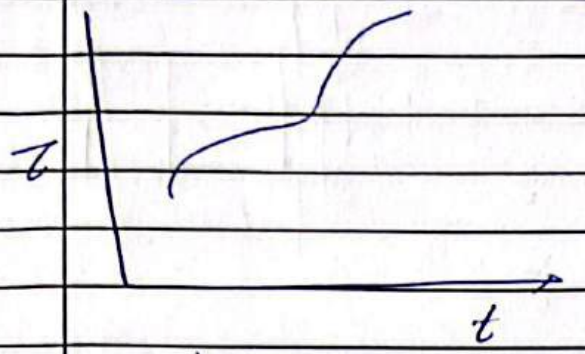
$$\tau_2 = \left(\left(\tfrac{1}{3} m_2 L_2^2 + \tfrac{1}{4} m_2 r^2 + \tfrac{1}{2} m_2 L_1 L_2 \cos\theta_2 \right) \ddot{\theta}_1 + \left(\tfrac{1}{3} m_2 L_2^2 + \tfrac{1}{4} m_2 r^2 \right) \ddot{\theta}_2 \right) + \tfrac{1}{2} m_2 L_1 L_2 \sin\theta_2 \dot{\theta}_1^2 + \tfrac{1}{2} m_2 g L_2 \cos\theta_{12}$$

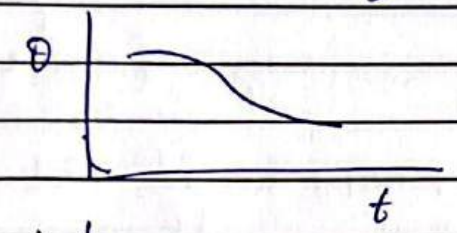
Control of motor

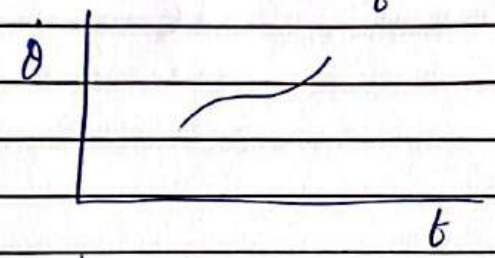
→ A DC motor is connected at each joint of a robot, where torque is proportional of the armature current.

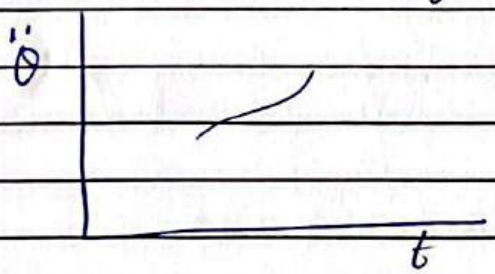
$\tau_m \propto I_A$ — armature current.

$\tau = k_m \cdot I_A$ — Constant of proportionality or motor constant.









→ Joint torque can be represented as follows

$$\tau = \Delta(\theta)\ddot{\theta} + h(\theta, \dot{\theta}) + c(\theta)$$

where

$\Delta(\theta)$: inertia terms

$h(\theta, \dot{\theta})$: Coriolis and ~~Cor~~ Centrifugal terms

$c(\theta)$: Gravity terms.

Scanned with CamScanner

Let us consider Partioned control Scheme:

$$\tau = \alpha\tau' + \beta$$

Where $\alpha = D(\theta)$

$$\beta = h(\theta,\dot\theta) + c(\theta) + F(\theta,\dot\theta)$$

Now, τ' can be written as follows:

$$\tau' = \ddot\theta_d + k_p E + k_D \dot E \quad \text{(for PD control law)}$$

proportional gain value

derivative gain

proportional derivative

E = error

$$E = \theta_d - \theta$$

rate of change of error $\quad \dot E = \dot\theta_d - \dot\theta$

$$\tau' = \ddot\theta_d + k_p E + k_I \int E\,dt + k_D \dot E \quad \text{(for PID control law)}$$

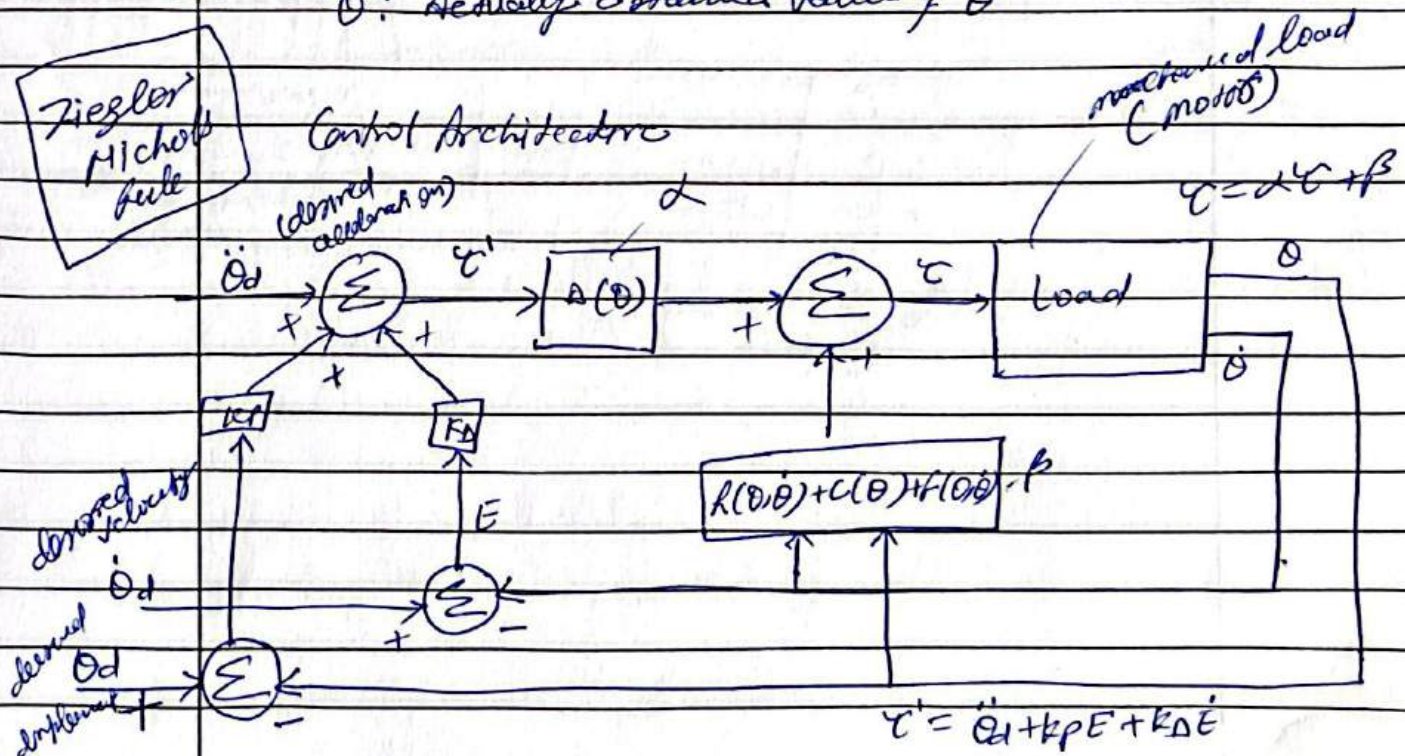
Where E = error $= \theta_d - \theta$

Where θ_d: Desired value of θ

θ: Actually obtained value of θ

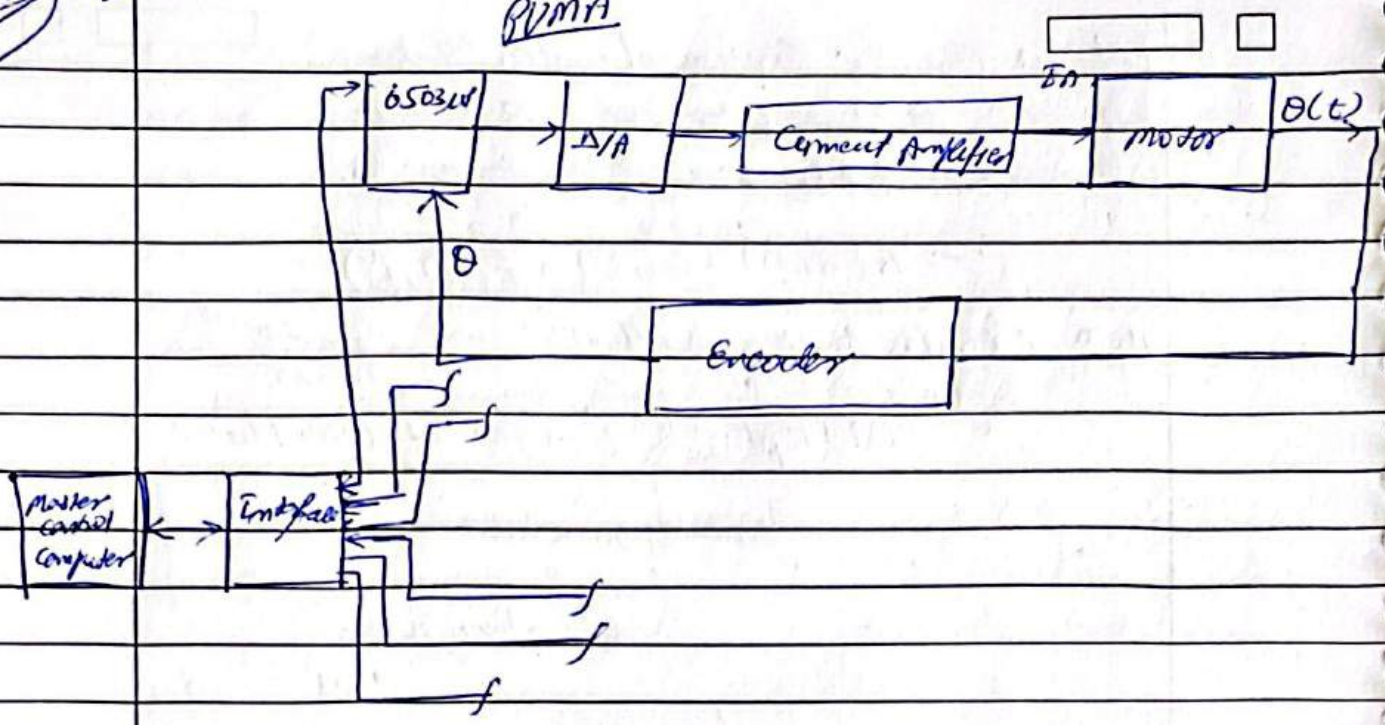
Ziegler Nichols Rule

Control Architecture



PID Controller

PUMA



31 Sensors

Sensors

→ Human-beings collect information of the surroundings using their sensors, namely eyes, ears, nose, palm etc., in order to perform various tasks.

→ A sensor is used to take measurements of physical variables.

→ A sensor requires calibration.

→ Sensors are used to build intelligent robots.

Classification of Sensors

- Internal Sensors
  (Used to operate the drivers)
  ex: position sensors, velocity sensors, acceleration sensors, force/moment sensors.
- External Sensors
  (used to collect information of the environment)
  ex: Temperature sensors, visual sensor, proximity sensor, Acoustic Sensor

2. Sensors

- Contact Sensor
  Physical contact b/w sensor mounted on robot and object
  - Touch Sensor/Tactile Sensor
    Binary Sensor
    (indicates presence or absence of an object)
    ex: micro switch, limit switch
  - Force Sensor/Analog Sensor (not only the contact is made but also the force is measured)
    ex: Sensor using strain gauge
- Non-Contact Sensors
  (No physical contact)
  Proximity Sensor
  Visual Sensor
  Acoustic Sensor
  Range sensor

Characteristics of Sensors

- Range : Difference between the maximum and minimum values of the input that can be measured.
- Response : Should be capable of responding to the changes in minimum time.
- Accuracy :- deviation from exact quality
- Sensitivity = change in output / change in input
- Linearity : Constant sensitivity.
- Repeatability : Deviation from reading to reading, when these are taken for a number of times under identical conditions.
- Resolution.

Touch Sensor

- and to indicate whether contact has been made b/w two objects.
- does not determine the magnitude of contact force.
- ex : micro-switch, limit switch.

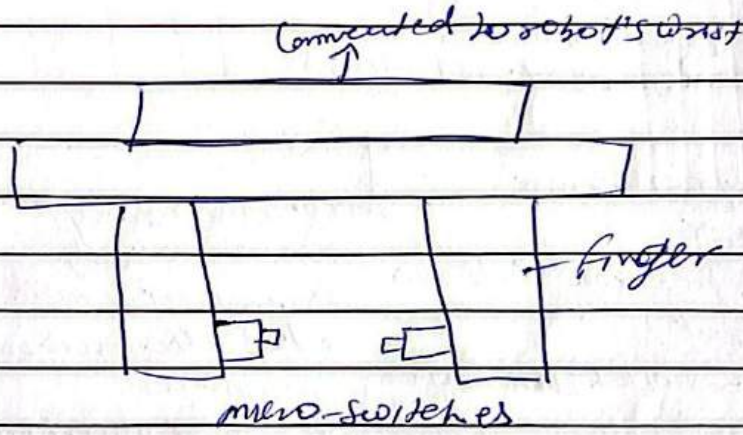


figure: micro-switches placed on two fingers of a robotic hand.

Position Sensor.

① potentiometer
- linear potentiometer
- Angular potentiometer

- Optical Encoder
  - Absolute Optical Encoder
  - Incremental Optical Encoder

Linear Variable Differential Transformer (LVDT)

Force/Moment Sensor

Range Sensor

Proximity Sensors
- Inductive
- Hall-effect
- Capacitive

Scanned with CamScanner

34 Robot Vision (also known as Computer Vision)

Digital Image Processing.

→ To extract, characterize and interpret objects present in an image / photograph captured using a camera.

① Step 1: Capturing image of the environment using CCD Camera.

② Step 2: Light intensity is measured along a particular direction say Y using Electron Beam Scanner (in which the charge accumulated in photo-site is proportional to light intensity).

Analog plot of light intensity is digitized and it is known as A/D Conversion or digitizing.

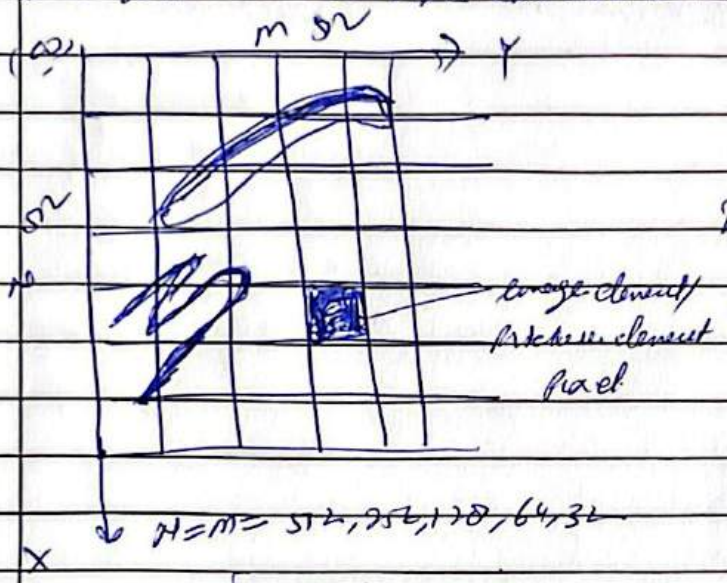
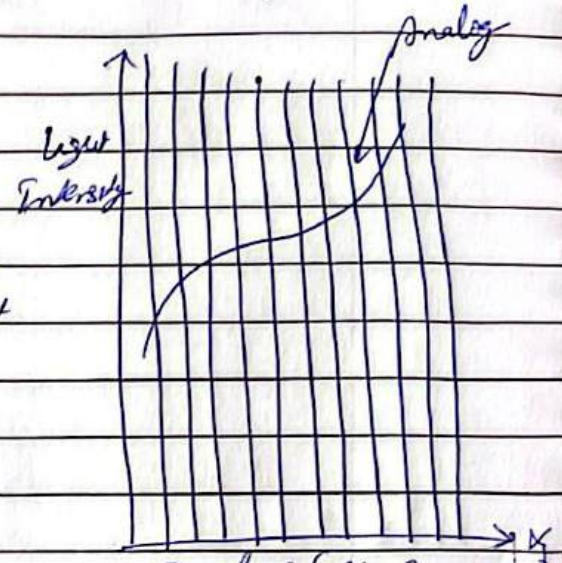


Image Capturing



Sampling (A/D Conversion)

③ Image is stored as an array of pixels (each pixel may have different light intensity values). It is known as frame grabbing

④ Preprocessing of the data collected in step 3 is done for noise reduction, restoration of lost information etc.

Preprocessing using filter (3×3

-1	-1	-1
-1	8	-1
-1	-1	-1

mask

5 Neighborhood Averaging:

$$P(x,y) = \frac{1}{R} \sum_{(n,m) \in S} f(n,m)$$

(c) median filtering

Step 5 Thresholding.

(6) Edge detection

→ Laplacian operator $L[P(x,y)] = \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}$

⇒ Boundary Descriptors.

Signature:

Image Capturing
↓
Sampling
↓
A/D → digitizing
↓
frame grabing
↓
Preprocessing
↓
Thresholding
↓
Edge detection.
↓
Object Identification.

Scanned with CamScanner

Scanned with CamScanner

Topic 8

37 Robot motion planning

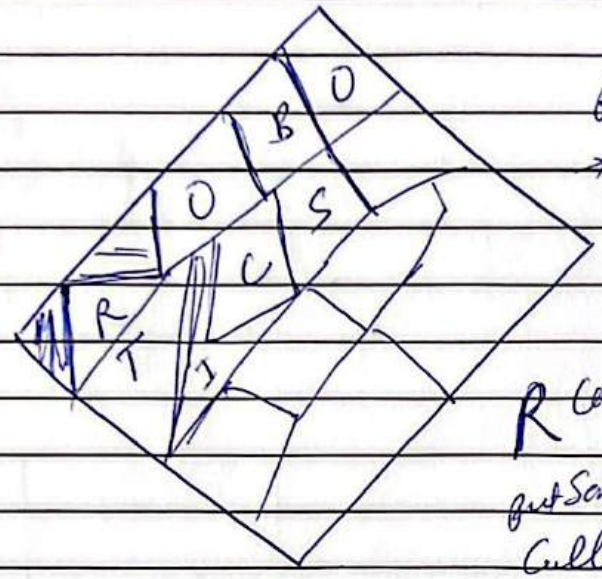
Aim

To determine the course of action/path while moving from an initial position to a final position.

Robot motion planning

- Gross motion planning / Free space motion planning
  - manipulation
  - navigation
- Fine motion planning / Compliant motion planning

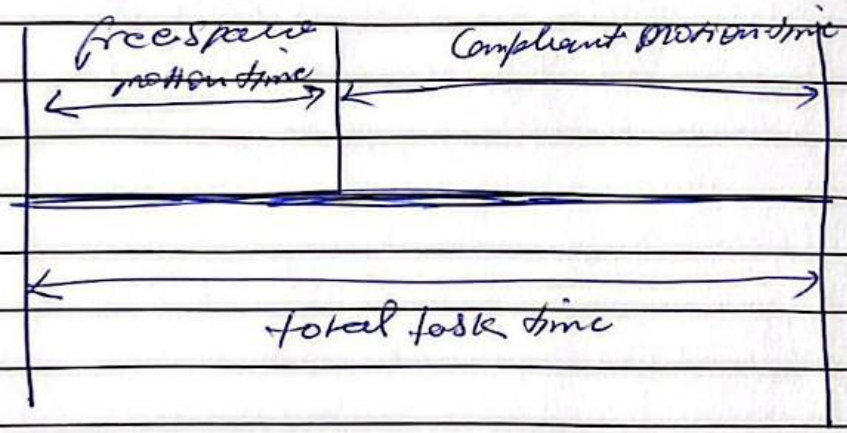
Strong / Strong / Weak



ROBOTICS
→ free space

R Compliant motion

Put some amount of force towards R called Compliant motion.



Sequence of Robotics Action:

Task identification ⇒ motion planning ⇒ Kinematics

⇓

Trajectory planning

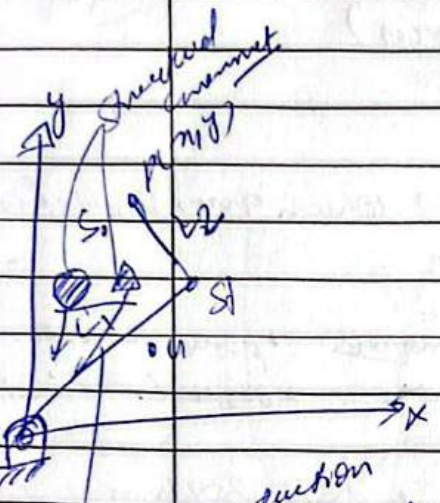
⇓

motion action ⇐ Control scheme ⇐ Dynamics

① motion planning

Environment — Structured environment — find path problem

Environment — Unstructured environment — dynamic motion ~~problem~~ planning



moving in one direction then problem becomes very difficult

motion planning Approaches

Global approach / Act-after-Thinking process / off-line planning (motion planning with complete information)

If structured environment then go for global approach

Local approach / Act-while-Thinking process / online planning (motion planning with incomplete information)

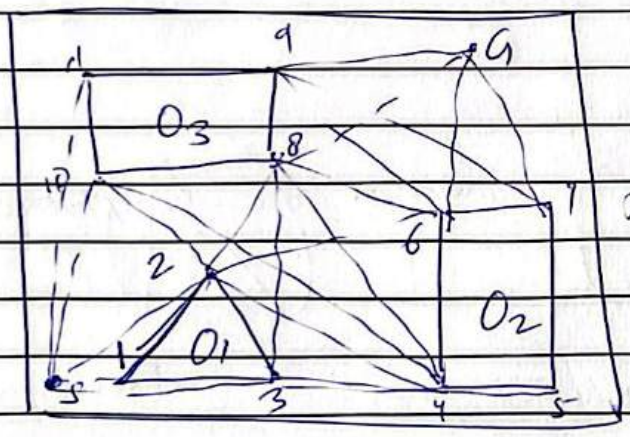
If moving obstacle or environment is dynamic or environment is unstructured then go for local approach

Motion Planning Schemes

- Traditional scheme / Algorithmic approaches
  - Graph-based method
    - I) Visibility graph - 1967
    - II) Voronoi diagram
    - III) Cell decomposition
    - IV) Tangent graph
    - V) Accessibility graph
  - Analytical approaches
    - I) Potential field Approach
    - II) Path velocity Decomposition
    - III) Incremental Planning
    - IV) Probabilistic Approach
    - V) Relative velocity Approach
    - VI) Reactive Control strategies (Behavior Based Robotics)
- Non-Traditional schemes (using Soft Computing)
  - I) Fuzzy logic-based
  - II) Neural network-based approaches.

Visibility Graph (Nilsson 1969)

→ It connects those vertices of obstacles, which are visible from one another.



Consider
- point robot
- obstacles fixed

find path Bottom.

$S \to 2 \to 8 \to G$

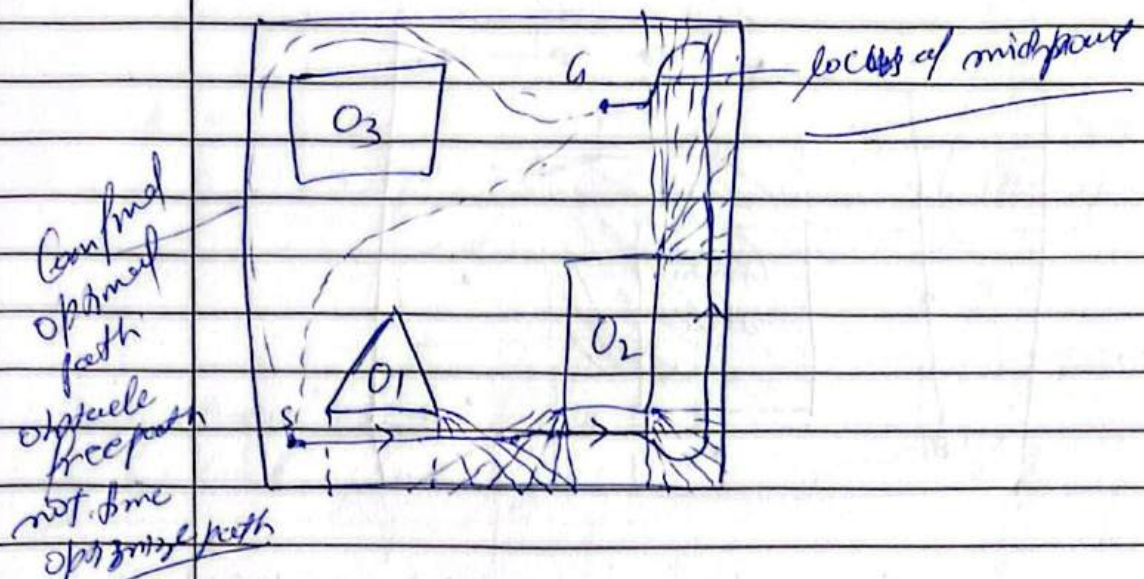
$S \to 3 \to 6 \to G$

⋮

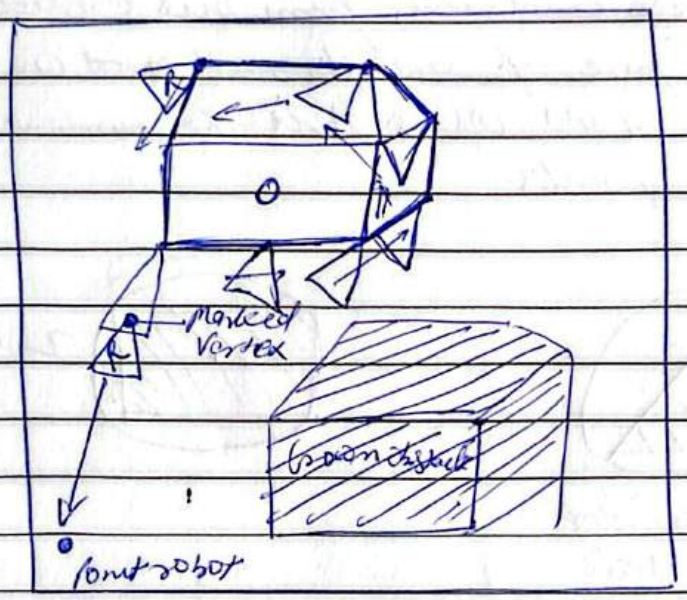
Time optimal path

Voronoi Diagram (Dumlaing et al. 1986)

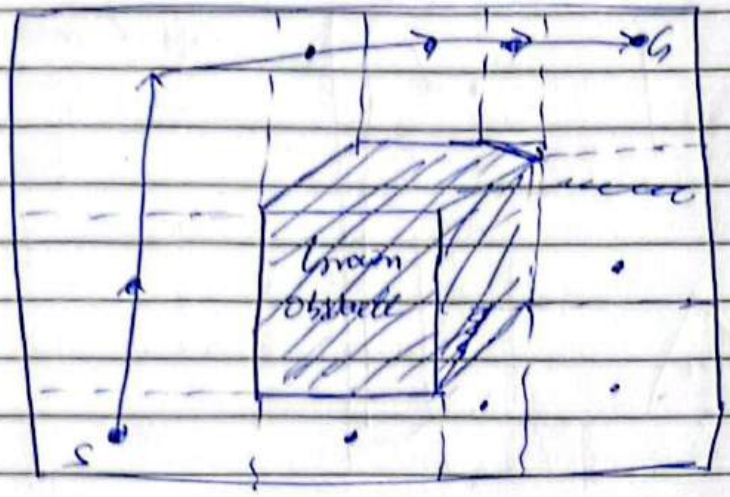
→ Represents the locus of points those are equidistant from at least two of the boundaries (obstacles).



Cell Decomposition (Lozano Perez, 1983)
Configuration space (C-Space)

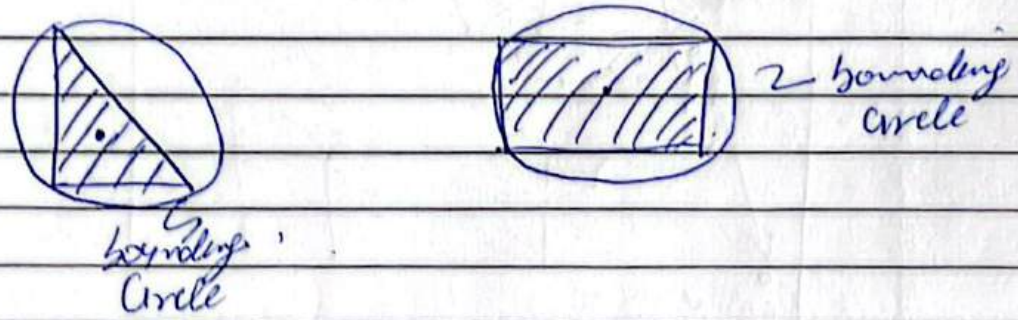


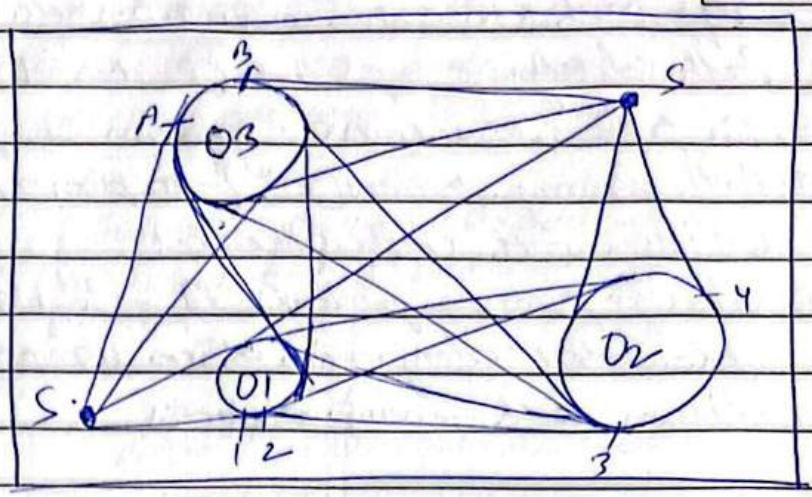
→ Robot's free area is divided into a number of small regions called cells. A connectivity graph is then constructed and searched.



Tangent graph (Liu & Arimoto 1991)

→ Tangents are drawn from the starting point to the visible obstacle and then from one obstacle to another.
→ A path comprise of tangents and ~~arcs~~ circular arcs.
→ Complexity: $O(n^2)$, where n is the number of control points.





S→1→2→3→4→G

S→A→B→G

Computational Complexity Problem
of Kare

Approaches to solve moving obstacle problems

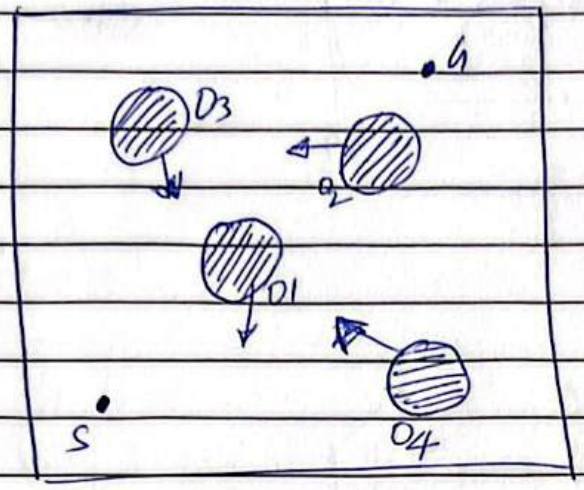
Path Velocity Decomposition (Kant and Zucker, 1984)

This problem is decomposed into two sub-problems as follows:

(i) Path planning problem (PPP) - to plan a path to avoid collision with static obstacles

(ii) Velocity planning problem (VPP) - to plan the velocity of the robot along the above planned path to avoid collision with moving obstacles.

known as Dynamic motion planning problem.



Drawbacks:

→ The path may not be always good, particularly when there are many obstacles.

→ As there is a sudden change of velocity of the robot, it will have jerky motion, which is not desirable.

Accessibility Graph (Fujimura and Samet, 1988)

→ Generalization of the visibility graph. $t = t_1$ $t = t_2$

→ At a particular instant of time, motion planning problem in dynamic environment is converted into find-path problem which is solved using visibility graph.

→ In dynamic environment, visibility graph will go on changing.

Scanned with CamScanner

Incremental Planning Scheme (~~Stocker~~ Stiller, 1987)

Plan a collision-free path based on predicted positions of obstacles

⇓

Follow the path until it becomes invalid (Validity is checked based on the data collected using sensor(s)/cameras)

⇓

Adjust the predictions & plan a new collision-free path

⇓

Follow the path until it becomes invalid

⇓

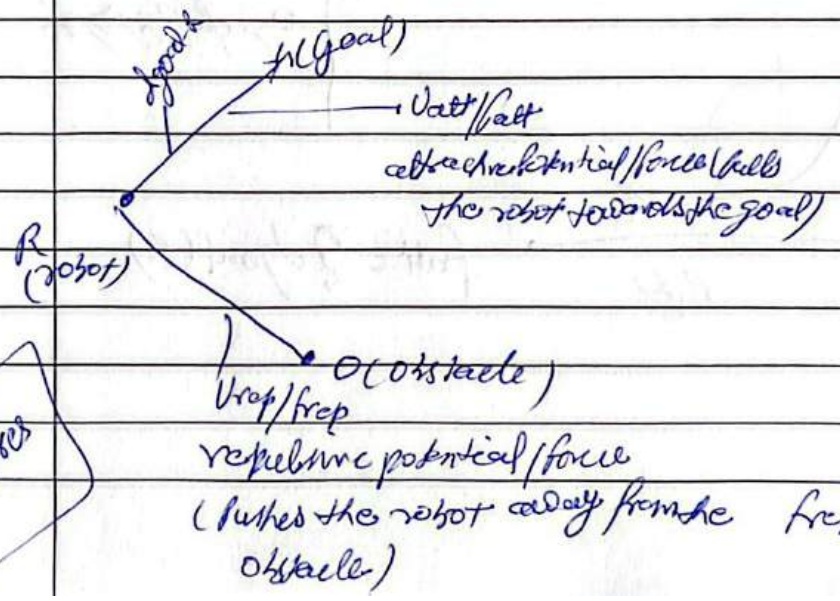
Continue until the robot reaches its destination

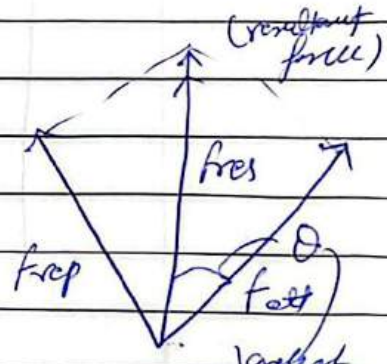
Relative Velocity Scheme

→ Consider relative velocity of the robot with respect to obstacles.

→ Dynamic motion planning problem is converted into several static problems.

Potential Field Approach (Khatib, 1986)





$$F_{att} = \frac{d\,U_{att}}{d\,d_{goal-R}}$$ (angle of deviation)

$$F_{rep} = \frac{d\,U_{rep}}{d\,d_{RO}}$$

speed, Fres

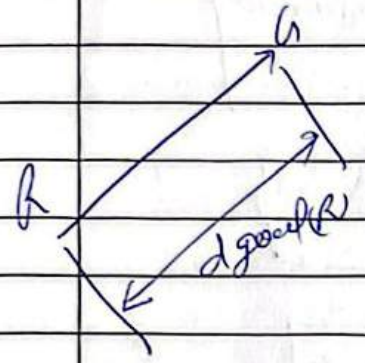
→ Speed of the robot ∝ fres.

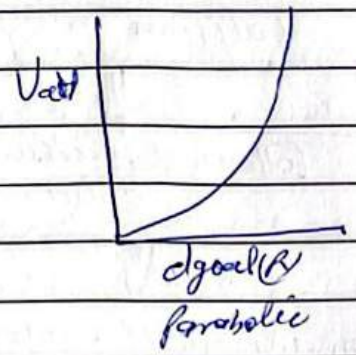
→ Direction of movement of the robot is along the direction of resultant forces.

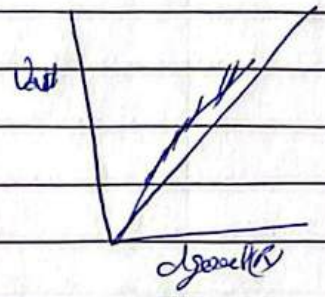
Attractive potential

$$U_{att}(R) = \frac{1}{2}\xi d^2_{goal}(R), \text{ if parabolic}$$

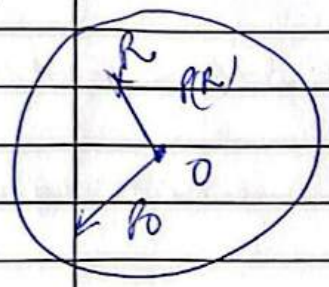
$$= \xi d_{goal}(R), \text{ if conic well}$$



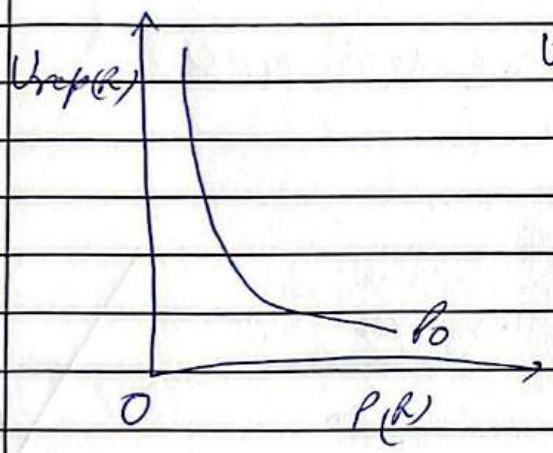




Repulsive potential



$U_{rep}(R) = 0$



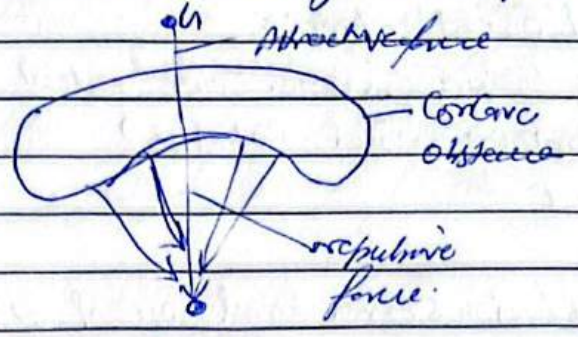
$$U_{rep}(R) = \begin{cases} \frac{1}{2}\eta\left(\frac{1}{\rho(R)} - \frac{1}{\rho_0}\right)^2, & \text{if } \rho(R) \le \rho_0 \\ 0, & \text{if } \rho(R) \ge \rho_0 \end{cases}$$

$$f_{att} = \xi d_{goal}(R)$$

Drawbacks

→ Solution depends on the chosen potential function

→ ~~Change~~ Chance of local minima problem - when the attractive force is balanced by the repulsive force.



→ When the robot travels in a narrow corridor, it experiences repulsive forces simultaneously from the opposite side, and consequently the motion becomes unstable.

→ Unable to find a path among closely spaced obstacles

Reactive Control Strategy (Brooks, 1986)

→ Robotic action is decomposed into some independent primitive behaviours like move-to-goal, avoid-obstacle, etc.

→ Basic behaviours are controlled at different layers of control architecture.

→ Basic behaviours are coordinated by a central mechanism (Behaviour-Based-Robotics)

Drawbacks:

→ The behaviours are hard-wired thus it is unable to handle behaviours which the programmer did not foresee beforehand.

→ The number of layer increases with the complexity of the problem. It requires a large amount of computer memory.

Computational Complexity

→ Canny & Reif (1987)
motion planning for a point robot among moving obstacles in 2D plane (with bounded velocity is NP-hard.

a) no

a) without 3rd velocity — NP-hard

b) velocity bounded 3rd — PSPACE-hard

41 Intelligent Robot

→ Intelligent Robot - Adaptive motion planner & Controller (AI to be merge with Robotics)!

ex! Robot Soccer

→ Ultimate Goal of the RoboCup: "By the mid-21st Century, a team of autonomous humanoid robots shall beat the human world cup champion team under the official regulations of FIFA.

1997 Deeper Blue Gary Kasparov - chess champion

42 Biped Walking