

... we can use the *matrix* function.

**Example 4.1.** Create a row vector  $u = [1, 2, 3]$

```
>>> from sympy import *
```

```
>>> Matrix([[1,2,3]])
```

```
Matrix([[1, 2, 3]])
```

**Example 4.2.** Create a column vector  $v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

```
>>> from sympy import *
```

```
>>> Matrix([[1],[2],[3]])
```

```
Matrix([
```

```
[1],
```

```
[2],
```

```
[3]])
```

**Example 4.3.** For vectors  $u = \begin{bmatrix} 2 \\ 5 \\ -3 \end{bmatrix}$  and  $v = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$  find,

a)  $u + v$

## CHAPTER 4. LINEAR ALGEBRA

b)  $u - v$

c)  $3.u$

d)  $2.u + 3.v$

```
>>> from sympy import *
```

```
>>> u=Matrix([[2],[5],[-3]])
```

```
>>> v=Matrix([[1],[0],[-2]])
```

```
>>> u+v
```

```
Matrix([
```

```
 [ 3],
```

```
 [ 5],
```

```
 [-5]])
```

```
>>> u-v
```

```
Matrix([
```

```
 [ 1],
```

```
 [ 5],
```

```
 [-1]])
```

```
>>> 3*u
```

```
Matrix([
```

```
 [ 6],
```

```
 [15],
```

```
 [-9]])
```

```
>>> 2*u+3*v
```

```
Matrix([
```

```
 [ 7],
```

```
 [10],
```

```
 [-12]])
```

more common are given below :

1. To create an identity matrix, use `eye`. `eye(n)` will create an  $n \times n$  identity matrix.

```
>>> from sympy import *
```

```
>>> eye(3)
```

```
Matrix([
```

```
[1, 0, 0],
```

```
[0, 1, 0],
```

```
[0, 0, 1]])
```

```
>>> eye(4)
```

```
Matrix([
```

```
[1, 0, 0, 0],
```

```
[0, 1, 0, 0],
```

```
[0, 0, 1, 0],
```

```
[0, 0, 0, 1]])
```

2. To create a matrix of all zeros, use `zeros`. `zeros(n, m)` creates an  $n \times m$  matrix of 0s.

```
>>> from sympy import *
```

```
>>> zeros(2,3)
```

```
Matrix([
```

```
[0, 0, 0],
```

#### CHAPTER 4. LINEAR ALGEBRA

```
>>> A*v
Matrix([
  [ 29],
  [ 65],
  [101]])
>>> A**3
Matrix([
  [ 468,  576,  684],
  [1062, 1305, 1548],
  [1656, 2034, 2412]])
```

**Note:** If we try  $v*A$  the it will gives error as Matrix size mismatch: (3,1) \* (3,3)

**Inverse:** Let  $A$  be any non singular matrix then there exists a matrix  $B$  such that

$$AB = BA = \text{Identity matrix.}$$

Matrix  $B$  is called inverse of matrix  $A$ . It is denoted by  $A^{-1}$ .

In python we can find inverse by **matrix.inv()** function.

For example

```
>>> from sympy import *
>>> # Consider matrices A and B,
>>> A=Matrix([[2,1,1],[1,2,1],[1,1,2]])
>>> B=Matrix([[1,1,1],[0,1,1],[0,0,1]])
>>> A.inv()
Matrix([
  [ 3/4, -1/4, -1/4],
  [-1/4,  3/4, -1/4],
  [-1/4, -1/4,  3/4]])
>>> B.inv()
Matrix([
  [1, -1,  0],
  [0,  1, -1],
  [0,  0,  1]])
```

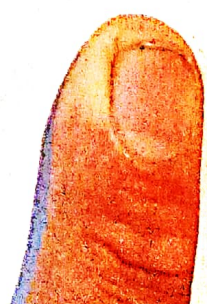
### 4.3 Accessing Rows and Columns, Deleting and Inserting Rows and Columns

To get an individual row or column of a matrix, use `row` or `col`. For example, `M.row(0)` will get the first row. `M.col(-1)` will get the last column.

```
>>> from sympy import *
>>> # Consider matrices A and B,
>>> A=Matrix([[2,1,1],[1,2,1],[1,1,2]])
>>> B=Matrix([[1,1,1],[0,1,1],[0,0,1]])
>>> A.row(0)
Matrix([[2, 1, 1]])
>>> B.row(2)
Matrix([[0, 0, 1]])
>>> A.col(-1)
Matrix([
[1],
[1],
[2]])
```

We can add or delete particular row in matrix. To delete a row or column, use `row_del` or `col_del`. These operations will modify the Matrix in place.

```
>>> from sympy import *
>>> # Consider matrix A
>>> A=Matrix([[2,1,1],[1,2,1],[1,1,2]])
>>> A.row_del(1)
>>> A
Matrix([
[2, 1, 1],
[1, 1, 2]])
>>> A.col_del(-1)
>>> A
Matrix([
[2, 1],
[1, 1]])
```



To insert rows or columns, use `row_insert` or `col_insert`. These operations do not operate in place.

```
>>> from sympy import *
>>> # Consider matrix A
>>> A=Matrix([[2,1,1],[1,2,1],[1,1,2]])
>>> A=A.row_insert(1, Matrix([[0,5, 4]]))
>>> A
Matrix([
[2, 1, 1],
[0, 5, 4],
[1, 2, 1],
[1, 1, 2]])
>>> A=A.col_insert(0, Matrix([[0],[5],[6],[4]]))
>>> A
Matrix([
[0, 2, 1, 1],
[5, 0, 5, 4],
[6, 1, 2, 1],
[4, 1, 1, 2]])
```



following examples:

**Example 4.4.** *Solve the following system of equations:*

$$3x + 2y - z = 3 \quad 2x - 2y + 4z = 6 \quad 2x - y + 2z = 9$$

```
>>> from sympy import *  
>>> x, y, z = symbols("x, y, z")  
>>> A = Matrix([[3, 2, -1], [2, -2, 4], [2, -1, 2]])  
>>> b = Matrix([3, 6, 9])  
>>> linsolve((A, b), [x, y, z])  
FiniteSet((6, -11, -7))
```

**Example 4.5.** *Solve the following system of equations:*

$$7x + 6y - 8z = 3 \quad 7x - 2y + 2z = 0 \quad 6x - y - 2z = 9$$

```
>>> from sympy import *  
>>> x, y, z = symbols("x, y, z")  
>>> A = Matrix([[7, 6, -8], [7, -2, 2], [6, -1, -2]])
```

```
>>> b = Matrix([3, 0, 9])
>>> linsolve((A, b), [x, y, z])
FiniteSet((-9/79, -276/79, -489/158))
```

**Example 4.6.** Solve the following system of equations:

$$x + 2y + 3z = 3 \quad 4x + 5y + 6z = 6 \quad 7x + 8y + 9z = 9$$

```
>>> from sympy import *
>>> x, y, z = symbols("x, y, z")
>>> A = Matrix([[1, 2, 3], [4, 5, 6], [7, 8, 9]])
>>> b = Matrix([3, 6, 9])
>>> linsolve((A, b), [x, y, z])
FiniteSet((z - 1, 2 - 2*z, z))
```



## CHAPTER 4. LINEAR ALGEBRA

Here are some examples:

**Example 4.7.** *Solve the following system:*

$$2x + y = 5; \quad x + 2y = 7$$

```
>>> from sympy import *
>>> A = Matrix([[2, 1], [1, 2]])
>>> B = Matrix([5, 7])
>>> A.gauss_jordan_solve(B)
(Matrix([
[1],
[3]]), Matrix(0, 1, []))
```

**Example 4.8.** *Solve the following system:*

$$x + 2y + 3z = 3; \quad 4x + 5y + 6z = 6 \quad 7x + 8z = 1$$

```
>>> from sympy import *
>>> A = Matrix([[1, 2, 3], [4, 5, 6], [7, 8, 10]])
>>> B = Matrix([3, 6, 9])
>>> sol, params = A.gauss_jordan_solve(B)
>>> sol
Matrix([
[-1],
[ 2],
[ 0]])
>>> params
Matrix(0, 1, [])
```

### 4.5.3 LU-decomposition Method

to solve system  $AX = B$  using L

given in of syms. Consider the following examples:

**Example 4.9.** Solve the following system using LU Decomposition method.

$$\begin{pmatrix} 6 & 18 & 3 \\ 2 & 12 & 1 \\ 4 & 15 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 19 \\ 0 \end{pmatrix}$$

```
>>> # First we import sympy.
>>> from sympy import *
>>> # We define variables x, y, z.
>>> from sympy.abc import x, y, z
>>> # We need to give augmented matrix AB .
>>> AB = Matrix([[6,18, 3,3], [2, 12, 1,19], [4, 15, 1,0]])
>>> solve_linear_system_LU(AB,[x,y,z])
{x: -14, y: 3, z: 11}
```

**Example 4.10.** Solve the following system using LU Decomposition method.

$$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 8 \\ 6 \end{pmatrix}$$

```
>>> # First we import sympy.
>>> from sympy import *
>>> # We define variables x, y, z.
>>> from sympy.abc import x, y, z
>>> # We need to give augmented matrix AB .
>>> AB = Matrix([[1,2, 2,7], [2, 1, 2,8], [2, 2, 1,6]])
>>> solve_linear_system_LU(AB,[x,y,z])
{x: 7/5, y: 2/5, z: 12/5}
```