COEP Technological University Pune
Department of Mathematics
(MA- 20004) - VECTOR CALCULUS AND PARTIAL DIFFERENTIAL EQUATIONS
S.Y. B.Tech. Semester III (All Branches)
Tutorial 3 (AY: 2023-24)

Questions on CO1

1. Define a vector function and a scalar function. Give an example of each.
2. Define a vector field and a scalar field. Give an example of each.
3. Define the derivative of a vector function. What is its significance in mechanics and in geometry?
4. Define gradient of a scalar function, divergence and curl of a vector function and explain their physical significance.

Questions on CO2 and CO3

1. Let $f(x,y) = e^{xy}sin(x+y)$. Find the direction, starting at $(0, \frac{\pi}{2})$, in which f is changing fastest.
   Ans: $\pm i$
2. Consider the pressure field given by $f(x,y) = 9x^2 + 4y^2$. Sketch the isobar for pressure 36. Also find the region in which the pressure varies between 36 and 144.
   Ans: ellipse and region between two ellipses.
3. Determine and sketch the isotherms of the temperature fields
   (a) $T = x^2 - y^2$ (b) $T = x/(x^2+y^2)$.
   Ans: (a) $y = \pm x$ for $T = 0$ and hyperbolas for other constant temperatures.
   (b) T not defined at origin. Zero temp on y-axis and circles with centers on x-axis for other constant temperatures.
4. Determine the level surfaces of the scalar fields
   (a) $f = x^2 + y^2 - z$ (b) $f = y^2 - z$.
   Ans: (a) paraboloids (b) parabolic cylinders
5. Sketch the vector fields given by the vector functions
   (a) $\overline{v} = y\mathbf{i} - x\mathbf{j}$ (b) $\overline{v} = \mathbf{i} + \mathbf{j}$
6. Sketch the following curves and identify them:
   (a) $\overline{r(t)} = [2 + 4cost, 2sint, 0]$
   Ans: circle in xy plane
   (b) $\overline{r(t)} = [-2, 2 + 5cost, -1 + 5sint]$
   Ans: circle in the plane $x = -2$
7. Find the parametric representation of the circle in the $yz-$ plane with center $(4,0)$ and passing through $(0,3)$. Sketch it.
   Ans: $[0, 4 + 5cost, 5sint]$
8. Find the parametric representation of the helix $x^2 + y^2 = 25, z = arctan(y/x)$. Sketch it.
   Ans: $[5cost, 5sint, t]$
9. Find the tangent and the unit tangent vector to the given curve at the given point:
   (a) $\overline{r(t)} = [cost, sint, 9t]$ Point $P(1, 0, 18\pi)$
   Ans: $[0, 1, 9], 1/\sqrt{82}[0, 1, 9]$
   (b) $\overline{r(t)} = [t, 4/t, 0]$ Point $P(4, 1, 0)$
   Ans: $[1, -1/4, 0]$, $4/\sqrt{17}[1, -1/4, 0]$
10. Find the length for the curve in (a) part of above question number 9, from $t = 0$ to $t = 2\pi$.
    Ans: $2\pi\sqrt{82}$.