Optimizing the Vehicle Routing Problem using genetic algorithm with local refinement

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1 Abstract

The Vehicle Routing Problem (VRP) is a widely studied problem in operational research. The problem involves delivering goods from one or more depots to customers with known requirements. The main objective is to find optimal or near-optimal routes that will satisfy various constraints while minimizing the overall cost. Here cost signifies the total distance travelled by the vehicles to fulfill all the customer requirements. The VRP has grabbed significant attention from researchers in recent years due to its crucial role in planning distribution systems for sectors like e-commerce, logistic industry, garbage collection, mail delivery and task sequencing. Different variations of the VRP exist, including VRP with Time Windows, VRP with Pick-Up and Delivery, and Capacitated VRP. Heuristics such as genetic algorithms, evolution strategies, and neural networks have gained considerable interest in solving VRPs efficiently. The proposed genetic algorithm approach with local refinement also give optimal or near optimal solutions for most the small or large instances of VRP.

Keywords: Vehicle Routing Problem (VRP) . genetic algorithm . local refinement

2 Introduction

The vehicle routing problem (VRP) is a combinatorial optimization problem that seeks to determine the best routes for a fleet of vehicles to deliver goods to a set of customers. It is a generalization of the traveling salesman problem (TSP) and was initially introduced by **George Dantzig and John Ramser** [1] in 1959 for petrol deliveries. The objective of the VRP is to minimize the overall cost of the routes.In 1964, **Clarke and Wright** [2] further improved the approach by introducing a greedy algorithm known as the savings algorithm. However, finding the optimal solution to the VRP is computationally challenging as it belongs to the class of **NP-hard** [3] problems. This means that the size of problems that can be optimally solved using mathematical programming or combinatorial optimization techniques may be limited. As a result, practical applications of the VRP often rely on heuristics rather than exact methods due to the large and frequent real-world instances that need to be solved. Heuristic approaches provide efficient and effective solutions for such scenarios, allowing for practical implementation and addressing the complexities associated with the problem.

Keywords: Travelling Salesperson Problem (TSP) . NP-Hard . Heuristics

3 Proposal

Since VRP is NP-Hard [3] problem in general, a better choice for solving VRP would be *genetic algorithms (GA)* [4]. Genetic algorithms (GAs) are widely used search and optimization strategies based on the principles of *Darwinian evolution*. In GAs, the parameters of the search space are represented as strings called *chromosomes*. The process begins by randomly initializing a population consisting of a set of chromosomes. The quality or fitness of a chromosome is assessed using a *fitness function*, i.e., total distance covered. Biologically inspired operators such as *selection, crossover, mutation* are then applied to generate the offspring population. These operators mimic the natural processes of selection, reproduction, and genetic variation. After creating the child population, *elitism* is often employed to evolve the next generation. Elitism involves selecting the most fit chromosomes from both the parent and child populations to ensure that the best solutions are preserved and carried forward. The termination of the genetic algorithm occurs when certain criteria are satisfied or when the maximum generation limit is reached. The solution is obtained from the best-fit chromosome in the final generation. Well-designed genetic algorithms are capable of generating near-global optimal solutions for NP-hard problems. However, these techniques are sometimes criticized due to their relatively longer computation times. By integrating **local refinement** [5] strategies into the genetic algorithm, its performance can be significantly enhanced. These strategies involve applying specific techniques to the solutions represented in the chromosomes. The objective is to refine the chromosomes locally, leading to faster convergence of the genetic algorithm. This approach enables the solutions to be fine-tuned quickly, ultimately improving the overall efficiency and effectiveness of the algorithm.

Keywords: Selection . Crossover . Mutation . Elitism . Local Refinement

4 Problem Definition

The Vehicle Routing Problem (VRP) is a combinatorial optimization problem that involves determining the optimal set of routes for a fleet of vehicles to deliver goods or services to a set of customers. Here in this paper we try to solve the one of the variants of the VRP that is the **Capacitated VRP (CVRP)** [6]. The problem is defined by the following components:

1. **Depots:** There are one or more depots where the vehicles start and end their routes. The depots serve as the central locations from which the goods or services are distributed.

2. **Customers:** There are a number of customers who have placed orders for goods or services. Each customer has a specific demand or requirement that needs to be fulfilled.

3. **Vehicles:** A fleet of vehicles is available to carry out the deliveries. Each vehicle has a limited capacity in terms of the maximum amount of goods or services it can carry.

4. **Routes:**The objective is to determine the optimal set of routes for the vehicles to visit all the customers while considering their demands, the vehicle capacities, and any additional constraints such as time windows (specific time intervals in which customers can be served) or vehicle operating costs.

5. **Objective Function:** The goal is to minimize the total cost associated with the delivery routes. This cost can be defined in various ways, including minimizing the total distance traveled, minimizing the number of vehicles used, or minimizing the overall delivery time.

The Vehicle Routing Problem can be *mathematically formulated* [7] as follows:

Given:

- A set of depots denoted by D, where $d = 1, 2, ..., n$.
- A set of customers denoted by C, where $c = 1, 2, ..., m$.
- Each customer c has a demand denoted by $q(c)$.
- A fleet of vehicles with a capacity denoted by Q.
- The distance between any two locations (depots and customers) denoted by $d(i, j)$, where i, $j \in D \cup C$ which we will be considering as V.

Decision Variables:

- $x(i, j)$ represents whether vehicle i travels directly from location i to location where $x(i, j) \in \{0, 1\}$.
- $y(i, c)$ represents whether vehicle i serves customer c where $y(i, c) \in \{0, 1\}$.

Objective Function:

- Minimize:

$$\sum_{i \in V} \sum_{j \in V} d(i, j) \ . \ x(i, j)$$

In this formulation $d(i, j)$ indicates the distance from node i to node j, where as $x(i, j)$ is a binary variable that has value 1 if the edge going from node i to node j is in the solution and 0 otherwise.

Constraints:

- Each customer c must be served by exactly one vehicle:

$$\sum y(i, c) = 1, \forall c \in C$$

- Each vehicle can only serve customers within its capacity limit:

$$\sum q(c) * y(i, c) \leq Q$$

.

- Each vehicle must start and end its route at a depot.

The goal is to find values for the decision variables $x(i, j)$ and $y(i, c)$ that satisfy all the constraints while minimizing the objective function, which represents the total distance traveled by the vehicles.

5 Proposed GA for VRP

Procedure GA: The following steps can be followed to solve a problem using the Genetic Algorithm [8]:

1. Begin by initializing the population, which is the **current population**.
2. Iterate through steps 3 to 5 until the **termination condition** is met.
3. Use the **selection operation** to obtain the mating pool from the current population.
4. Use the **crossover** and **mutation** operators on the mating pool to generate the new population.
5. Propagate the best chromosomes to the next generations.
6. Return the best solution found in the current population.

This section presents the GA-based solution for the VRP. The procedure of the proposed solution is illustrated in Figure 1. In the subsequent subsections, each step of the method is explained in detail.

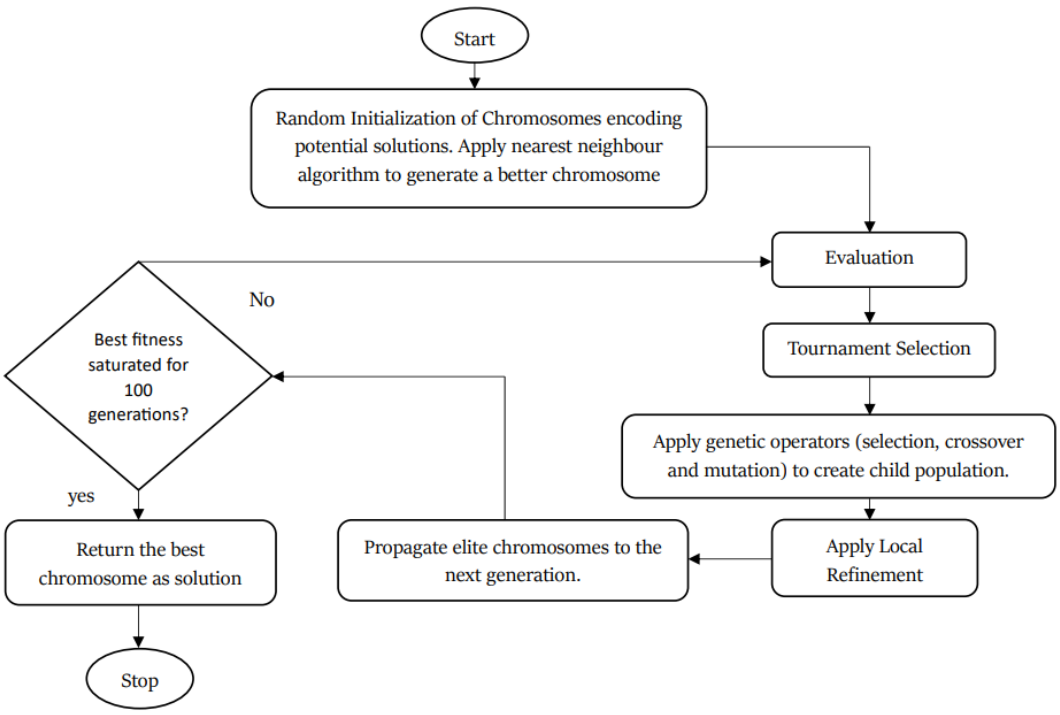

Figure 1: Flowchart of the proposed GA-based method for solving VRP

5.1 Chromosome encoding

In genetic algorithms (GAs), the feasible solutions for the optimization problem are represented using strings known as chromosomes. Here in the context of Vehicle Routing Problem the chromosomes are the random permutation of the cities and vehicles which cover all the cities exactly once. In our method we are taking the digit 1 as our vehicle and the other digits for the customers/cities. One example of valid chromosome is : 1 2 3 4 1 5 6 1 7 8. So the chromosome indicates that the first vehicle services the cities 2,3 and 4 while the second and third vehicle services the cities 5,6 and 7,8 respectively.

5.2 Population initialization

The initial population is consisting of a certain number of chromosomes that is a user defined parameter called population size. Each chromosome of the population is created randomly. A nearest neighbour algorithm called **Linear Sweep algorithm** [9] is implemented here to generate a better fitness chromosome so that a better chromosome is always present at the population from the starting. This will help for the algorithm to converge faster and reach to a near-optimal solution quickly.

5.3 Fitness Computation

The goodness of the solution is indicated by the fitness of that chromosome. Here the fitness of a particular chromosome is the total distance travelled by all the vehicles in that path provided that the path followed all the aforementioned constraints. Here our goal is to minimize this fitness value.

5.4 Genetic Operators

Selection, crossover and mutation are the three genetic operators used to create the next generation of population from the current generation.

5.4.1 Selection

To create the mating pool from the current population selection operator is used. The selected chromosomes will be further used for other operators. Here a popular *binary tournament selection* [4] is used. , which involves selecting two chromosomes randomly from the current population and opting the fittest one to move on to the mating pool. This process repeats until the mating pool is full.

5.4.2 Crossover

Crossover means exchanging the genetic elements between the chromosomes. In each crossover operation, two randomly chosen chromosome from the mating pool exchange their city sequences and produce a new offspring. *Crossover probability* μ_c is used to control the crossover operation and is repeated ρ times, where ρ is the population size. The crossover probability μ_c is kept in the range of 0.8 - 0.9 throughout all the generations.

5.4.3 Mutation

Mutation is a crucial process in generating a new and improved generation. It involves making random changes to the genetic sequence of a chromosome. In our study, we have implemented mutation by randomly selecting two points in the chromosome and swapping those selecting cities to create a new path. *Mutation probability* μ_p is used to control the mutation operation and is repeated ρ times, where ρ is the population size. The mutation probability μ_p is kept in the range of 0.1 - 0.3 throughout all the generations.

5.4.4 Local Refinement

Each chromosome is locally updated after the mutation operation. Local refinement is done as follows. First, we take one of the vehicle path and then apply TSP on the path to get the optimal path for that vehicle only and subsequently do the same for the next vehicle paths. For example, take a chromosome 1 2 3 4 5 6 1 7 8 9 so here we first take the first vehicle path that is 1 2 3 4 5 6 1 and apply to TSP to get the minimum distance path. This refinement strategy helps the solution to get a near optimal solution quickly and faster convergence.

5.4.5 Elitism

Elitism is utilized to prevent the loss of high-quality solutions caused by the unpredictable nature of genetic operators. To achieve this, the parent and child populations from a specific generation are combined, and the top ρ solutions based on their fitness values are selected to carry forward to the next generation. This guarantees the retention of the best chromosome obtained thus far, safeguarding its inclusion in subsequent iterations.

5.4.6 Termination Condition

The process of calculating fitness, performing selection, crossover, mutation, local refinement, and elitism is repeated over multiple generations. This loop continues until the best fitness value remains unchanged for the last 100 generations, indicating saturation. The output of the genetic algorithm is determined by the best solution, which is the one that achieves the lowest coverage value during the optimization process.

6 Result and Discussion

The above-mentioned solution of the VRP is implemented using MATLAB programming language in MATLAB 2021a with the system configuration: Intel(R) Pentium(R) CPU G4400 @ 3.30GHz, 3300 MHz, 2 Core(s), 2 Logical Processor(s), Windows 10 Operating System and 4 GB RAM. The input instances are taken from the standard VRP libraries: (Augerat, 1995) [10] (Christofides and Eilon, 1969) (Christofides and Eilon, 1969). The optimal solutions are also given in the above aforementioned libraries.

6.1 Result

instances	mean solution	optimal solution	Error(%)
E-n13-k4	247	247	0
P-n16-k8	450	450	0
P-n20-k2	216	216	0
E-n23-k3	534	534	0
E-n30-k3	562.87	534	**0.18**
A-n32-k5	833.34	784	**0.19**
A-n38-k5	794.49	730	**0.23**
A-n45-k6	1056.67	944	**0.26**
E-n51-k5	652.72	521	**0.49**
A-n61-k9	1423.47	1034	**0.61**
A-n80-k10	2753.34	1763	**0.70**

6.2 Discussion

The problem instance is given like this, instance E-n13-k4 that there is 13 customers/cities to serve and 4 vehicles are there to supply them. Vehicle capacity is

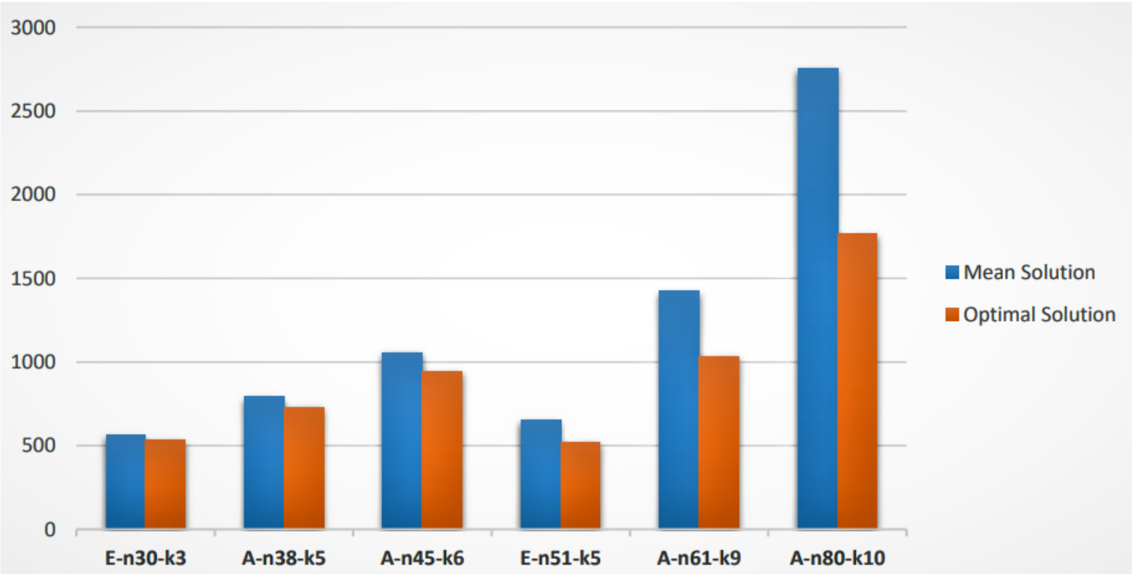

Figure 2: Bar chart for showing performance Comparison for the obtained solutions.
X-axis: Problem instance Y-axis: Total distance

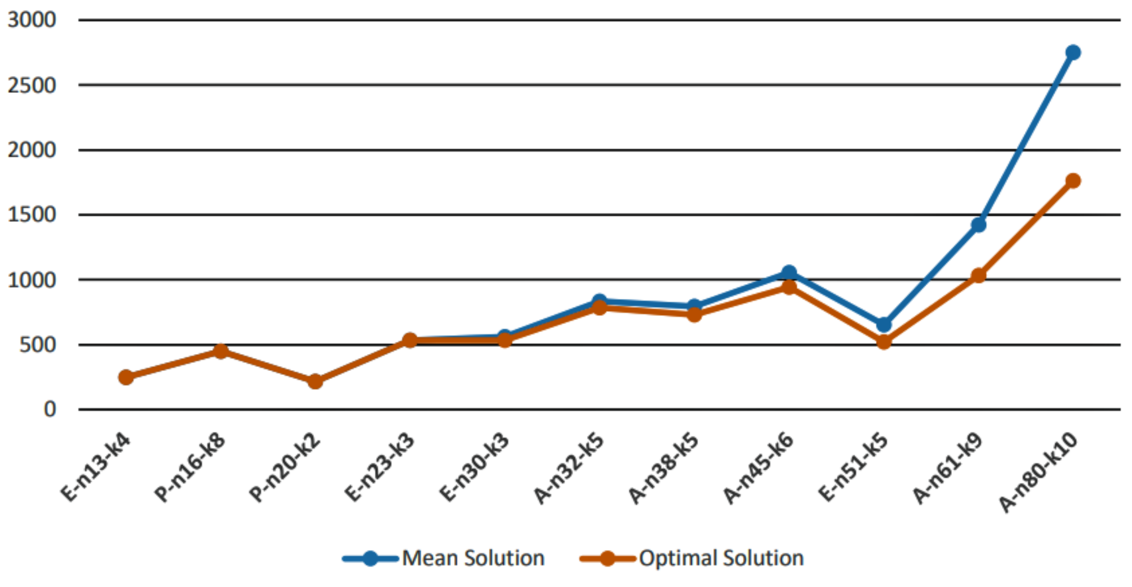

Figure 3: Line chart for showing difference in the result with increasing the size of the problem.
X-axis: Problem instance Y-axis: Total distance

also given in all the instances.

Mean solution means the same instance is executed five times and the mean of all that 5 given solution is taken for the result. As a lot of randomness is there in GA so the mean solution will give the indication that how good is the solution actually is. This ensures the quality of the overall solution.

Error Rate is defined as:

$$Error\ rate = \frac{meansolution - optimalsolution}{optimalsolution * cities}$$

Here the error rate is defined as the difference between the optimal and mean result with respect to that of optimal result and the no. of cities. Cities are taken here for evaluation because suppose for a 10 city instance the difference is 50 but again for a 50 city instance the difference is 50 also then the later solution is more desirable than the initial one.

7 Conclusion

In this article, we have introduced a genetic algorithm (GA) approach for addressing the vehicle routing problem (VRP). The GA-based approach we propose incorporates a local refinement strategy at the end of the genetic operators. By incorporating this local refinement strategy, the overall convergence rate of the approach is improved. Our future aim with this study is to develop a more efficient method for solving VRP using GA. Additionally, we seek to minimize the error rate for larger city instances.

References

[1] Dantzig, George B., and John H. Ramser. "The truck dispatching problem." Management science 6.1 (1959): 80-91.

[2] ClarkeG, WrightJW. "Schedulingofvehiclesfroma centraldepottoanumberofdeliverypoints." OperationsResearch 12.4 (1964): 568-581.

[3] Toth, Paolo, and Daniele Vigo, eds. The vehicle routing problem. Society for Industrial and Applied Mathematics, 2002.

[4] Goldberg, D.E., Holland, J.H. Genetic Algorithms and Machine Learning. Machine Learning 3, 95–99 (1988). https://doi.org/10.1023/A:1022602019183

[5] Atta, Soumen, Priya Ranjan Sinha Mahapatra, and Anirban Mukhopadhyay. "Solving maximal covering location problem using genetic algorithm with local refinement." Soft Computing 22 (2018): 3891-3906.

[6] Ralphs, Ted K., et al. "On the capacitated vehicle routing problem." Mathematical programming 94 (2003): 343-359.

[7] Zirour, Mourad. "Vehicle routing problem: models and solutions." Journal of Quality Measurement and Analysis JQMA 4.1 (2008): 205-218.

[8] Roy, Samir, and Udit Chakraborty. Soft computing. Pearson Education India, 2013.

[9] Kumar, VV Senthil, and R. Jayachitra. "Linear sweep algorithm for vehicle routing problem with simultaneous pickup and delivery between two depots with several nodes." Global Journal of Pure and Applied Mathematics 12.1 (2016): 897-908.

[10] Augerat, Philippe, et al. "Computational results with a branch and cut code for the capacitated vehicle routing problem." (1995).