Thermal Engineering-2
ME-202

MID-SEM PROJECT

Design And Optimization of Rocket Nozzle Using Method of Characteristics

The following works are performed and the project is prepared by ***Pranav Mittal (2K19/ME/167)*** *for the purpose of Mid-semester Project Evaluation for Thermal Engineering.*

and submitted it to Prof. Naushad Ahmad.

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ABSTRACT

The nozzle is the most crucial part of the rocket. It has to be designed for optimum efficiency to avoid wall wastage due to over-expansion and energy wastage due to under-expansion. In this project, we have a method based on the theory of characteristics presented for 2-D, supersonic nozzle design. The minimum length of the supersonic nozzle has been calculated for the optimum Mach number at the nozzle exit with the uniform flow at the diverging section of the nozzle by developing a MATLAB program.

Rocket Nozzle

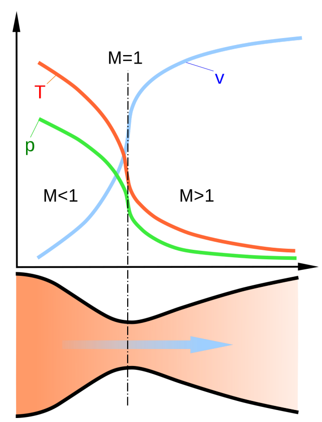
A rocket engine nozzle is a propelling nozzle (usually of the de Laval type) used in a rocket engine to expand and accelerate the combustion gases produced by burning propellants so that the exhaust gases exit the nozzle at hypersonic velocities.

Atmospheric Use:

The optimal size of a rocket engine nozzle to be used within the atmosphere is achieved when the exit pressure equals ambient (atmospheric) pressure, which decreases with increasing altitude. For rockets traveling from the Earth to orbit, a simple nozzle design is only optimal at one altitude, losing efficiency and wasting fuel at other altitudes.

Vacuum Use:

For nozzles that are used in a vacuum or at very high altitudes, it is impossible to match ambient pressure; rather, nozzles with a larger area ratio are usually more efficient. However, a very long nozzle has significant mass, a drawback in and of itself. A length that optimizes overall vehicle performance typically has to be found.

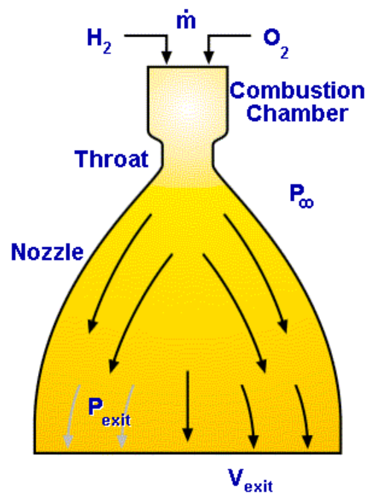


Convergent and divergent nozzle

Convergent-divergent nozzle. An arrangement in which the forward portion of the **nozzle** is **convergent**, which increases the pressure of the exhaust gases, while the aft section is **divergent** to increase gas velocity to the supersonic speed and avoid losses from under-expansion.

Nozzle Overexpansion & Underexpansion

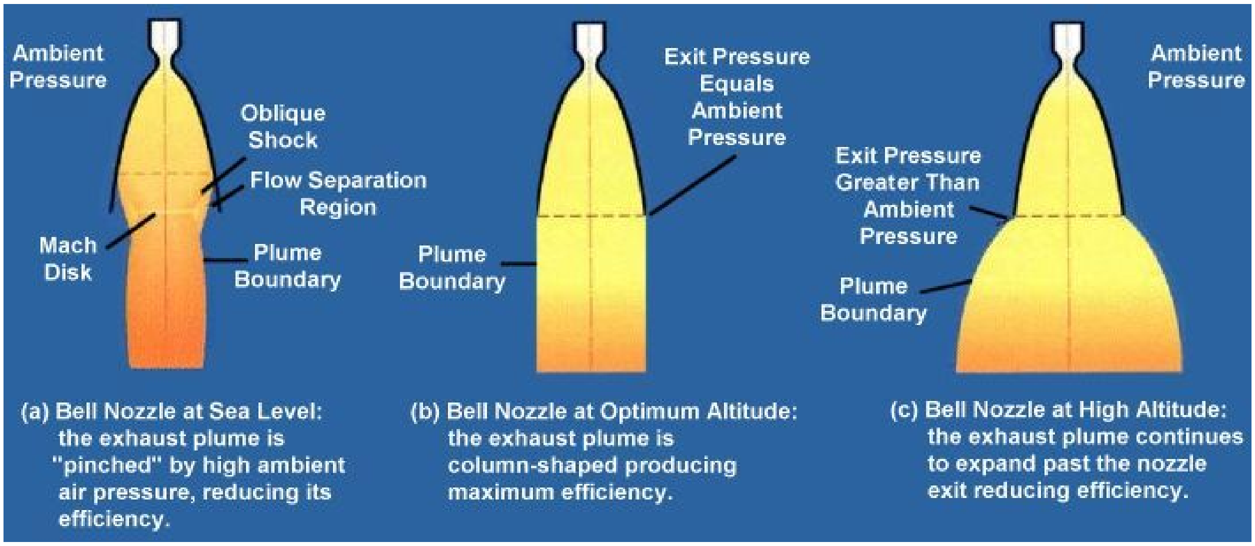
Expansion is the process that converts the thermal energy of combustion into kinetic energy to move an object forward. In other words, the hot gases created by burning fuel inside a jet or rocket engine are exhausted through a nozzle to produce thrust. It is the shape of this nozzle that is key to the expansion process. As that high-temperature flow is exhausted, it expands against the walls of the nozzle to create a force that pushes the vehicle forward.



Flow passing through a rocket nozzle

The behavior of this expansion process is largely dictated by pressure--both the pressure of the exhaust itself as well as the pressure of the external environment into which it exhausts. Of greatest concern is to design the shape and length of the nozzle so that it converts as much of that thermal energy into thrust as possible. **In an ideal nozzle that optimizes performance, the exit pressure (Pexit) will be equal to the ambient pressure of the external atmosphere (P¥).** The flow in this case is perfectly expanded inside the nozzle and maximizes thrust.

Unfortunately, this situation can only occur at one specific atmospheric pressure on a fixed-geometry nozzle. As we have seen previously, pressure decreases as altitude increases. Nozzle designers typically must select a shape that is optimum at only one altitude but minimizes the losses that occur at lower or higher altitudes. These losses result from the fact that the atmospheric pressure will either be higher than the exit pressure of the exhaust gases, i.e. at low altitudes, or lower than the exit pressure, i.e. at high altitudes.



The difference in flow behavior between (a) overexpansion, (b) ideal expansion, and (c) under expansion

This first case, where **the external pressure is higher than the exit pressure, is referred to as overexpanded**. When an overexpanded flow passes through a nozzle, the higher atmospheric pressure causes it to squeeze back inward and separate from the walls of the nozzle. **This "pinching" of the flow reduces efficiency because that extra nozzle wall is wasted** and does nothing to generate any additional thrust. Ideally, the nozzle should have been shorter to eliminate this unnecessary wall.

The opposite situation, in which **the atmospheric pressure is lower than the exit pressure, is called under expansion**. In this case, the flow continues to expand outward after it has exited the nozzle. **This behavior also reduces efficiency because external expansion does not exert any force on the nozzle wall.** This energy can therefore not be converted into thrust and is lost. Ideally, the nozzle should have been longer to capture this expansion and convert it into thrust.

Method of Characteristics

Characteristics are ‘lines’ in a supersonic flow oriented in specific directions along which disturbances (pressure waves) are propagated. The Method of Characteristics (MOC) is a numerical procedure appropriate for solving, among other things, two-dimensional compressible flow problems. By using this technique, flow properties such as direction and velocity can be calculated at distinct points throughout a flow field. The three properties of characteristics are as follows:

Property 1: A characteristic in a two-dimensional supersonic flow is a curve or line along which physical disturbances are propagated at the local speed of sound relative to the gas.
Property 2: A characteristic is a curve across which flow properties are continuous, although they may have discontinuous first derivatives, and along which the derivatives are indeterminate.
Property 3: A characteristic is a curve along which the governing partial differential equations(s) may be manipulated into an ordinary differential equation(s).

It is in the region immediately after the sonic throat where the flow is turned away from itself that the air expands into supersonic velocity. This expansion happens rather gradually over the initial expansion region. **In the Prandtl-Meyer expansion scenario, it is assumed that the expansion takes place across a centered fan originating from an abrupt corner.** This phenomenon is typically modeled as a continuous series of expansion waves, each turning the airflow an infinitesimal amount along with the contour of the channel wall.

These expansion waves can be thought of as the opposite of shock compression waves, which slow airflow. This is governed by the Prandtl-Meyer function:

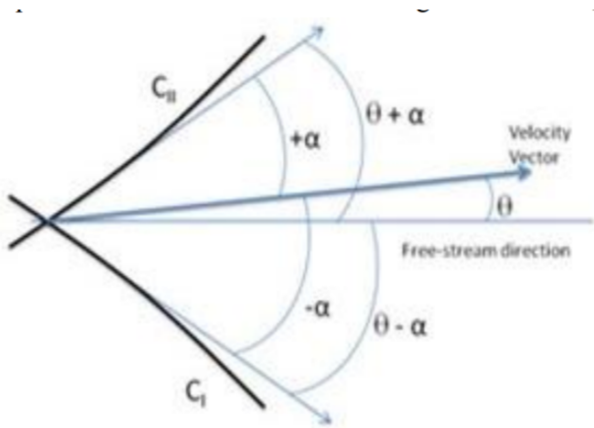
$$d\theta = \pm\sqrt{M^2 - 1}\,\frac{dV}{V}$$

Where the change in flow angle (relative to its original direction) is represented by $d\theta$. Integrating the above equation to give the following

$$v(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}\,tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M^2 - 1)}\ \text{-}\ tan^{-1}\sqrt{M^2 - 1}$$

The parameter ν is known as the Prandtl-Meyer angle.

Method of Characteristics analysis for this project used the following equations; In the Method of Characteristics equations the angle of the flow concerning the horizontal is given the symbol θ. The Mach angle α is defined as α = arcsin(1/M). The equations are with the reference are below,



Schematic Diagram of Characteristic Lines

$$\left(\frac{dy}{dx}\right)_I = \tan(\theta - \alpha)$$

$$\left(\frac{dy}{dx}\right)_{II} = \tan(\theta + \alpha)$$

$\theta + \upsilon(M) = \text{constant} = K_-$ (along C_- characteristic)
$\theta - \upsilon(M) = \text{constant} = K_+$ (along C_+ characteristic)

K_- and K_+ are constants along with their respective characteristics and are known as Riemann invariants.

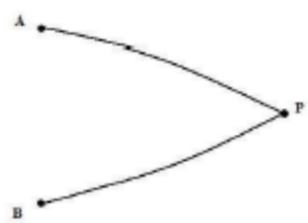
Where

θ **= $(K_- + K_+)/2$**

And

ν **= $(K_- - K_+)/2$**

Consider the intersection of two characteristic lines A and B at point P,



Intersection of characteristics lines

then we have,

$$m_I = \tan\left(\frac{(\theta-\alpha)_A + (\theta-\alpha)p}{2}\right)$$

$$m_{II} = \tan\left(\frac{(\theta-\alpha)_B + (\theta-\alpha)p}{2}\right)$$

And

$$y_P = y_A + m_1(x_P - x_A) \text{ and } y_P = y_B + m_{11}(x_P - x_B)$$

$$x_P = (y_A - y_B + m_{11}x_B - m_1x_A)/(m_{11} - m_1)$$

Thus, helping in developing the Supersonic nozzle.

Algorithm

Input

Chamber pressure and temperature, Thurst or Mass Flow Rate, Altitude at which rocket nozzle is expected to operate and Coefficient of heat and R-value of the fuel used.

Calculations

From the input data,

Pressure ratio, Temperature ratio, Critical Temperature, Pressure and Velocity are calculated.

$$Pressure\ Ratio = \frac{Outside\ Pressure}{Chamber\ Pressure}$$

$$Temp\ Ratio = (Pressure\ Ratio)^{\frac{\gamma - 1}{\gamma}}$$

$$Critical\ (Throat)\ Temp = \frac{(2 \cdot \gamma \cdot R \cdot Chamber\_Temp)}{\gamma - 1}$$

$$Critical\ (Throat)\ Pressure = \left(\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}}\right) \cdot 2.068\ (air)$$

$$Critical\ (Throat)\ Velocity = \sqrt{\frac{(2 \cdot \gamma \cdot R \cdot Chamber\_Temp)}{\gamma + 1}}$$

Further,

Exit Velocity, temperature, Sound speed, Mach are calculated.

$$Exit\ Velocity = \sqrt{Throat\ Temp \cdot (1 - Temp\ Ratio)}$$

$$\text{Exit Temp} = (\text{Chamber } T) \cdot \left(\frac{\text{Pexit}}{\text{Pchamber}}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$\text{Exit Sound Speed} = \sqrt{\gamma \cdot R \cdot \text{Exit Temp}}$$

$$\text{Exit Mach} = \frac{\text{Exit Velocity}}{\text{Exit Sound Speed}}$$

Method of Characteristic Calculations

Using MOC as mentioned earlier and the following formulas, we can calculate x, y of the nozzle wall.

$$PM = \sqrt{\frac{\gamma+1}{\gamma-1}} \cdot \operatorname{atan}\sqrt{\frac{\gamma-1}{\gamma+1} \cdot (M^2 - 1)} - \operatorname{atan}\sqrt{M^2 - 1}$$

$$\text{Max Angle} = \frac{1}{2} \cdot \{PM(\text{Exit Mach})\}$$

$$\text{Angle Increment} = 2 \cdot ((90 - \text{Max Angle}) - \text{fix}(90 - \text{Max Angle}))$$

$$m_I = \tan\left(\frac{(\theta-\alpha)_A + (\theta-\alpha)p}{2}\right)$$

$$m_{II} = \tan\left(\frac{(\theta-\alpha)_B + (\theta-\alpha)p}{2}\right)$$

$y_P = y_A + m_1(x_P - x_A)$ **and** $y_P = y_B + m_{11}(x_P - x_B)$

$x_P = (y_A - y_B + m_{11}x_B - m_1x_A)/(m_{11} - m_1)$

Export X, Y Into Excel File

Then these x, y coordinates are exported to excel files. Further, these excel files can be imported to SolidWorks to generate the nozzle curve.

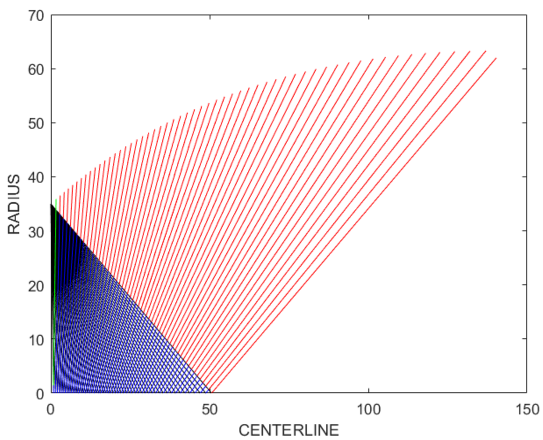
Results

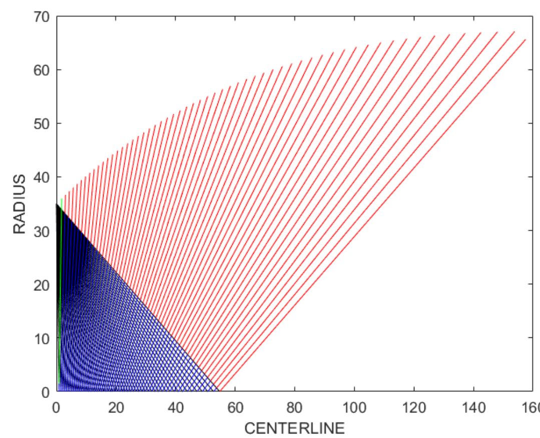
Generated optimum rocket nozzle for 4000 N and 5000 N Thrust at Altitude of 7500 m and 8500 m respectively.

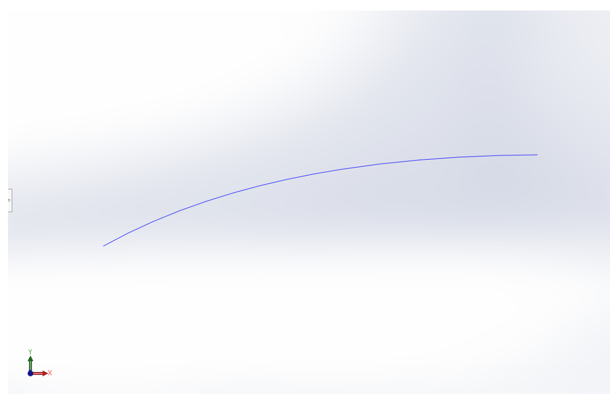
PARAMETER	VALUE
Chamber Pressure (Pa)	2.27E+06
Chamber Temperature (K)	1200
Thrust (N)	4000
Mass Flow Rate	0
Altitude (m)	7500
Coefficient of Heats (γ)	1.4
R	355

PARAMETER	VALUE
Chamber Pressure (Pa)	2.27E+06
Chamber Temperature (K)	1200
Thrust (N)	5000
Mass Flow Rate	0
Altitude (m)	8500
Coefficient of Heats (γ)	1.4
R	355

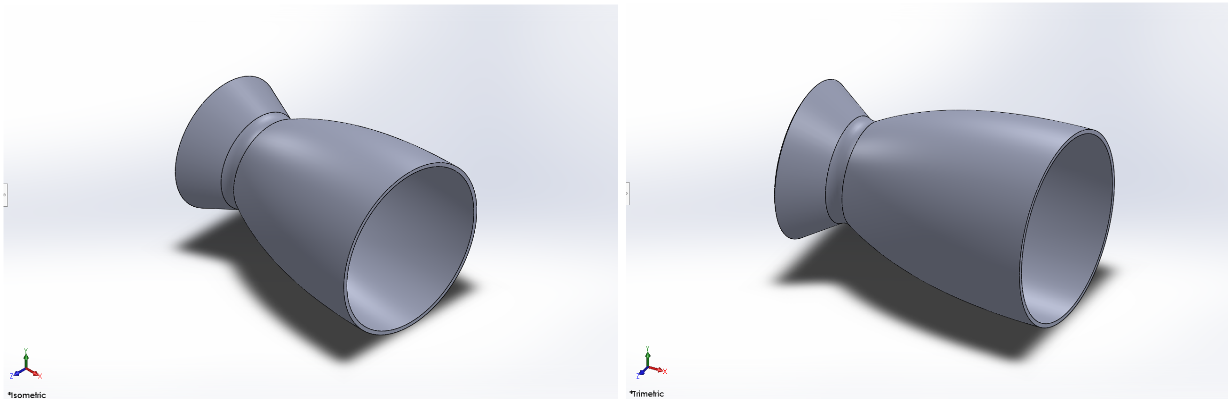
Input Values



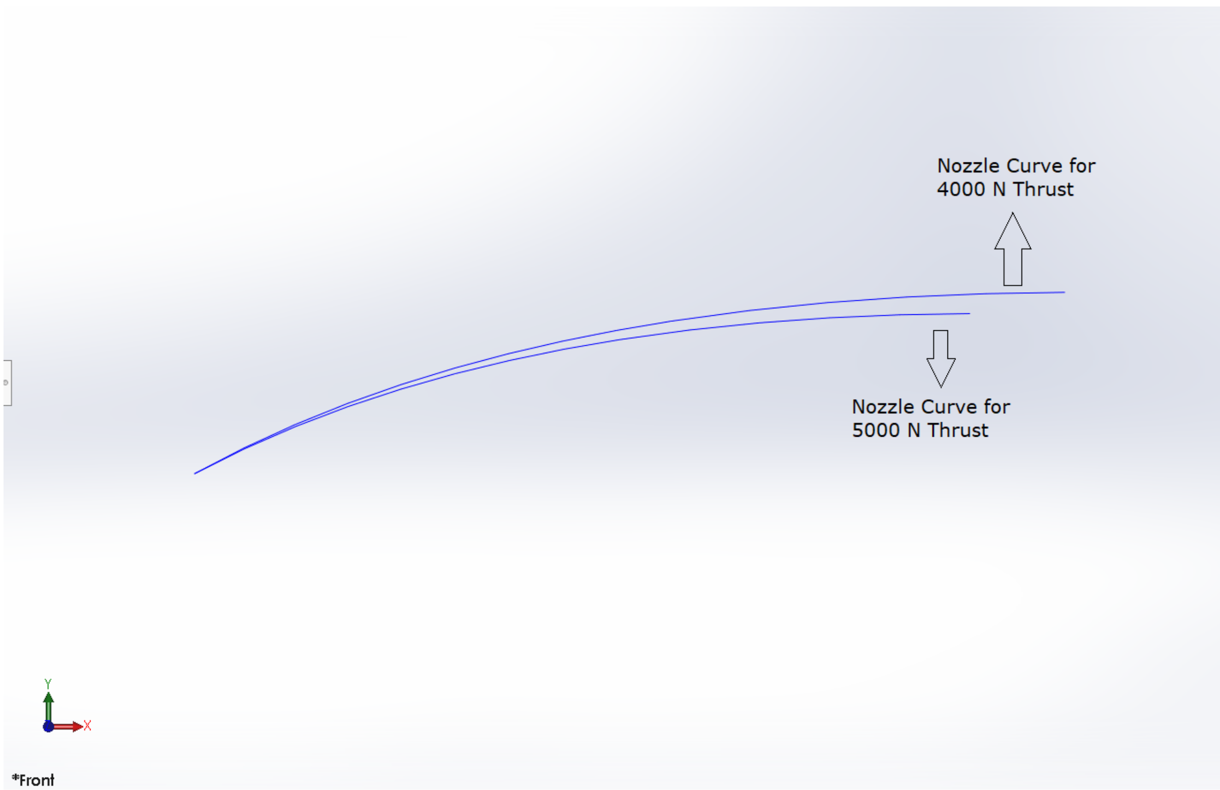




Generated Curves



3-D Model of Rocket Nozzle



Comparison Between The Generated Curve

Conclusion

Supersonic nozzles have many applications. They are usually subjected to complex flow patterns. The computer is a must to achieve high accuracy and large calculations needed with modern high-speed applications. Hence, a computerized approximation approach might be a better method to tackle such a problem. Characteristic method is the most appropriate method to be used with the supersonic nozzle design. To address the actual conditions, the consideration should be made on account of the phenomena due to the viscous effect, pressure difference concerning the backpressure, heat conduction, etc. The design presented here can be utilized to compare with the other nozzle designs regarding the specific design conditions. The simulation program of the practical supersonic nozzle can be developed depending on this design with the considerations of the losses taking place in real-time.

What this project has done is, it has examined the essential feature required to achieve steady, sustained supersonic flow. That feature is the nozzle contour. A profound appreciation for the process by which the desired exit Mach number is achieved.

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