# Assignment 08: Response Of Linear Systems To Arbitrary Inputs

## PH1050 Computational Physics

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## *Problem Statement*

This assignment concerns with the response of linear time invariant systems to arbitrary inputs. All systems we consider are Causal as well.

Part A

Choose a simple RC or LC circuit, the dynamic response (current or voltage response) of which can be cast as a first order initial value problem.

Analysis in time domain

1. Choose one RC or LC circuit and compute numerically its response to a step input. Compare the result with the analytical solution.

2. Generate an arbitrary signal which is a sum of several piecewise continuous signals. Compute the response of the circuit to this arbitrary input.

3. Choose a Causal square pulse such that its width is at least five orders smaller than the time response of the circuit you have chosen. Compute the response of the circuit to this narrow pulse input. This response is called the Impulse Response. Denote the Impulse Response as h(t) and plot the same as a function of time.

Analysis in frequency domain

1. Find the transfer function of the circuit and denote it as H($\omega$). Plot the absolute and relative phase angle of H($\omega$) as a function of $\omega$. You may also want to plot the Real and Imaginary parts of H($\omega$).

Part B

Choose a second order linear circuit such as a series LCR circuit with a suitable decay constant and natural frequency of oscillation. Set the constants so that the circuit is not over-damped.

1. Compute the impulse response of the circuit and plot the same.

2. Compute the response of the circuit to a causal square wave and plot the same.

3. Study the response to an arbitrary input. You may want to use the one that you have created earlier (section 2 of Part A).

# Aim

Part A:
To choose an RC circuit and using Kirchoffs Law:
1)Compute the current for a step Input and plot it.
2)To generate an arbitrary piece wise continuous input and compute the current and plot it.
3)Choose a Causal square pulse such that its width is at least five orders smaller than the time constant of the circuit chosen, find the current , and plot it.

Analysis in frequency domain
1)To find the Transfer function of the circuit as a function of $\omega$ and plot its phase angle, Real and Imaginary parts as functions of $\omega$.

Part B:
CHoose an LCR circuit and using Kirchoffs Law:
1)Compute and plot the current in the circuit to the impulse in Part 3 of A
2)Compute and plot the Current in the circuit to a causal square wave signal.
3)Compute the current in the circuit for the Piece Wise Continuous Input used in Part 2) of A , and to plot it as well.

# Introduction

This program requires the knowledge of basic circuit analysis such as potential difference acorss a capacitor, inductor, kirchoffs law etc. Apart from that, this program also uses mathematica functions such as NDSolve (to solve the differential equations), Plot (To plot the functions we obtain), Re [], Im[] and Arg[] to obtain the real part, imaginary and argument of a complex number .

# Code Organization

Part A
Part 1):
1) I declared a function stepSig[t] to represent the Step function which was the input voltage.
2) I plotted the input voltage.
3) I wrote Kirchofffs law eqn for the circuit and then solved it to get the charge on the capacitor as a function of time.
4) Then i plotted the charge on the capacitor as a function of time.
5) Then i differentiated the charge function to obtain the current as a function of time and then plotted it. (I also explained how the current agrees with our analytical solution)

Part 2):

1) I declared the function of Input voltage as a piece wise continuous function and then plotted it.
2) Then i wrote the equation of Kirchoffs law and solved it to obtain the charge on the capacitor as a function of time. Then i differentiated this to obtain current and then plotted it.

Part 3):
1)I declared the input signal as a UnitBox[] signal whose width is reduced by a factor of 10^5/(capacitance*R). Then i plotted it.
2)Then i wrote the equation of Kirchoffs law with this as the input voltage and then found the corresponding charge and current as functions of time and plotted them. (i also showed two different graphs for Current to show how the current is positive in the start and then it becomes negative)

Analysis In Frequency Domian:
1) I declared the transfer function as a function of $\omega$.
2)Then i plotted the phase angle, imaginary and real parts of the transfer function.

Part-B:
Part 1):
1) I chose the values of the variables for the LCR circuit.
2) Then i wrote Kirchoffs law equation and solved it to obtain the charge on the capacitor as a function of time. Then i found the current as a function of time and plotted them as functions of time.

For Parts 2) and 3):
i did the same thing, just the input voltages were different
in 2) , the input voltage is a SquareWave function whereas in 3) the input voltage is the same piece wise continuous function used in part A sub part 3).

## Code for computation

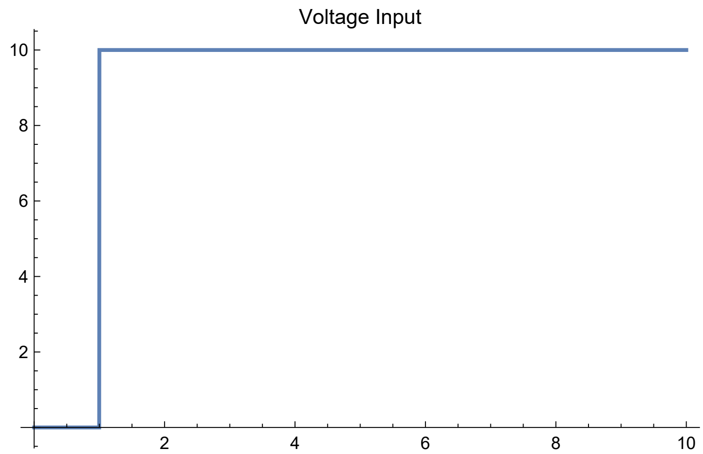
# Part A:

## Part 1):

In[415]:=

```
stepSig[t_] := 10 * HeavisideTheta[t - 1]; (*The step Input For our RC Circuit*)
Plot[stepSig[t], {t, 0, 10}, Exclusions → None, PlotLabel → "Voltage Input"]
```

Out[416]=




In[417]:=

```
R = 50;    (*Choosing values of resistance and Capacitence for our RC circuit*)
Cap = 1 / 100;
eqn = R q'[t] + q[t] / Cap == stepSig[t]; (*Kirchoffs Law for our RC Circuit*)
```

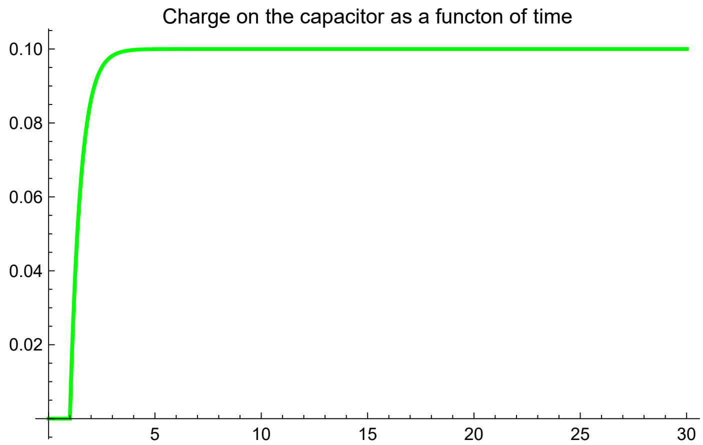
In[420]:=

```
sol = NDSolve[{eqn, q[0] == 0}, q, {t, 0, 30}];
(*Solving for charge as a function of time*)
```

In[421]:=

```
Plot[Evaluate[q[t] /. sol], {t, 0, 30}, PlotRange → All, PlotStyle → Green,
 PlotLabel → "Charge on the capacitor as a functon of time"]
(*Plotting the charge on the capacitor as a function of time*)
```

Out[421]=




In[423]:=

```
Q[t_] = q[t] /. sol
```

Out[423]=

{InterpolatingFunction[ Domain: {{0., 30.}} Output: scalar ][t]}

In[424]:=

```
Current[t_] = Q'[t]
```

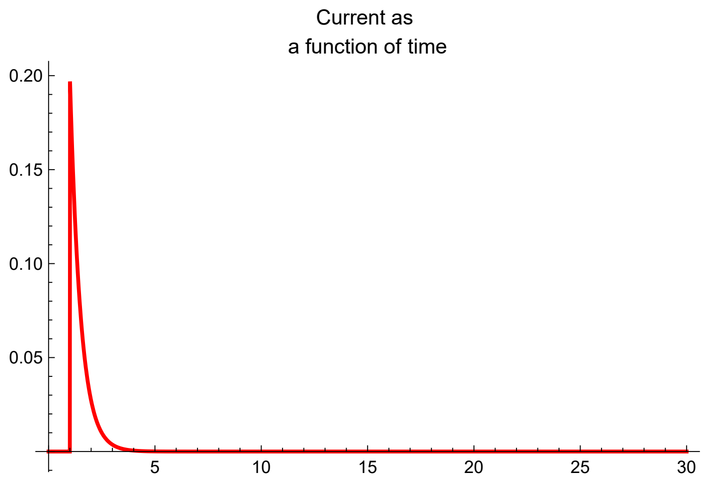
Out[424]=

{InterpolatingFunction[ Domain: {{0., 30.}} Output: scalar ][t]}

In[425]:=

```
Plot[Current[t], {t, 0, 30}, PlotStyle → Red, PlotRange → All, PlotLabel → "Current as
a function of time"]
```

Out[425]=



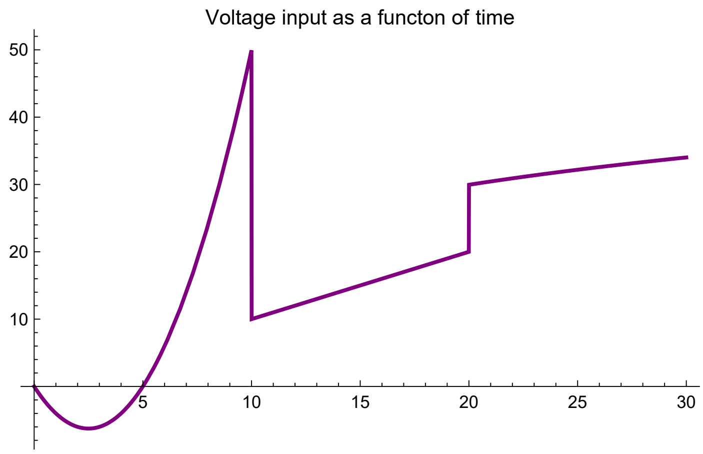
```
(*This plot of Current[t] that we have obtained is in agreement
  with the analytical/logical solution as initially the current is =
 0 as the voltage input is = 0 and then the voltage suddenly picks up and
   behaves as a DC source so the current increases and then dies out due to the
   potential difference between the capacitor being equal to that of the source*)
```

# Part 2):

In[426]:=

```
V[t_] := Piecewise[{{1 t^2 - 5 t, 0 ≤ t ≤ 10}, {t, 10 < t ≤ 20}, {10 Log[t], 20 < t}}]
Plot[V[t], {t, 0, 30}, Exclusions → None, PlotStyle → Purple,
 PlotLabel → "Voltage input as a functon of time"]
(*An arbitrary signal which is the sum of several piecewise functions*)
```

Out[427]=




In[428]:=

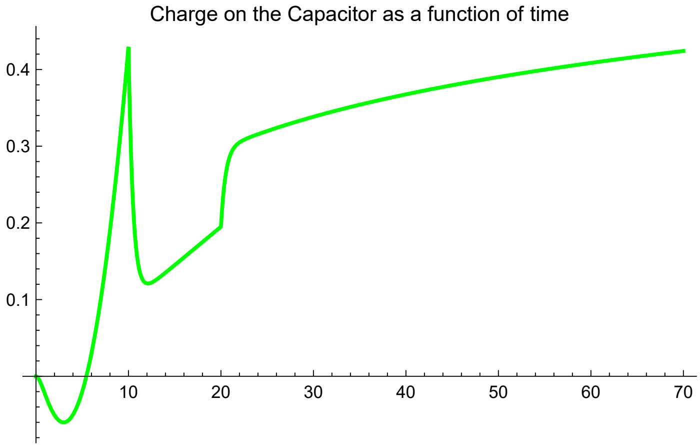
```
R = 50;
Cap = 1 / 100;
eqn = R q2'[t] + q2[t] / Cap == V[t];
sol2 = NDSolve[{eqn, q2[0] == 0}, q2, {t, 0, 100}];

(*Applying Kirchoffs Law*)
```

In[432]:=

```
Plot[Evaluate[q2[t] /. sol2], {t, 0, 70}, PlotRange → All, PlotStyle → Green,
 PlotLabel → "Charge on the Capacitor as a function of time"]
```

Out[432]=




In[433]:=

```
Q2[t_] = q2[t] /. sol2
```

Out[433]=

```
{InterpolatingFunction[ Domain: {{0., 100.}} Output: scalar ][t]}
```

In[434]:=

```
Current2[t_] = Q2'[t]
```

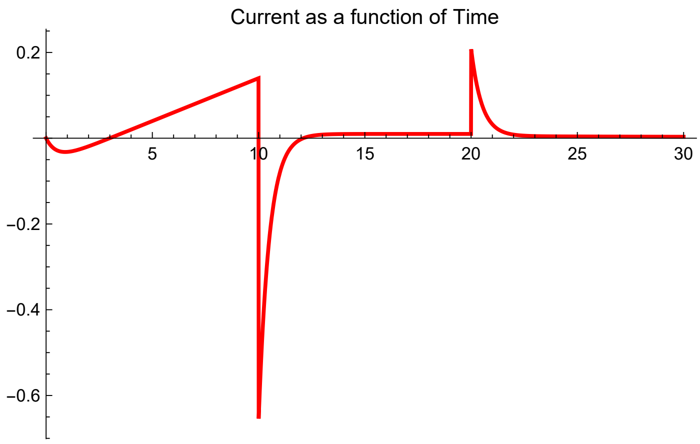
Out[434]=

```
{InterpolatingFunction[ Domain: {{0., 100.}} Output: scalar ][t]}
```

In[435]:=

```
Plot[Current2[t], {t, 0, 30}, PlotStyle → Red,
 PlotRange → All, PlotLabel → "Current as a function of Time"]
```

Out[435]=




# Part 3):

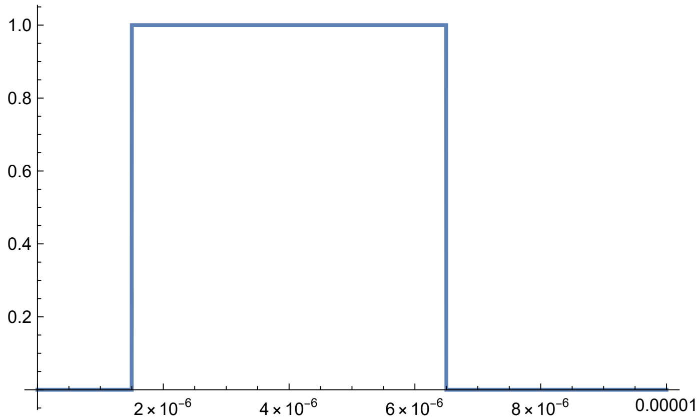
In[436]:=

```
w = 10^5 / (Cap R);
```

In[437]:=

```
PulseSig[t_] = UnitBox[w (t - 4 * 10^-6)];
Plot[PulseSig[t], {t, 0, 1 / 10^5}, Exclusions → None]
(*Causal square impulse with its width 10^5
 times smaller than the time constant of the RC circuit*)
```

Out[438]=



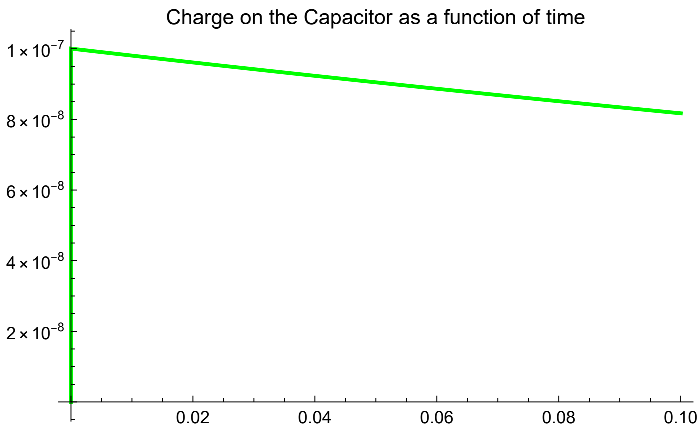

In[439]:=

```
eqn = R q3'[t] + q3[t] / Cap == PulseSig[t];
sol3 = NDSolve[{eqn, q3[0] == 0}, q3, {t, 0, 100}];
```

In[441]:=

```
Plot[Evaluate[q3[t] /. sol3], {t, 0, 1 / 10}, PlotRange → All,
 PlotStyle → Green, PlotLabel → "Charge on the Capacitor as a function of time"]
```

Out[441]=




In[442]:=

```
Q3[t_] = q3[t] /. sol3
```

Out[442]=

```
{InterpolatingFunction[ Domain: {{0., 100.}} Output: scalar ][t]}
```

In[443]:=

```
h[t_] = Q3'[t]
```

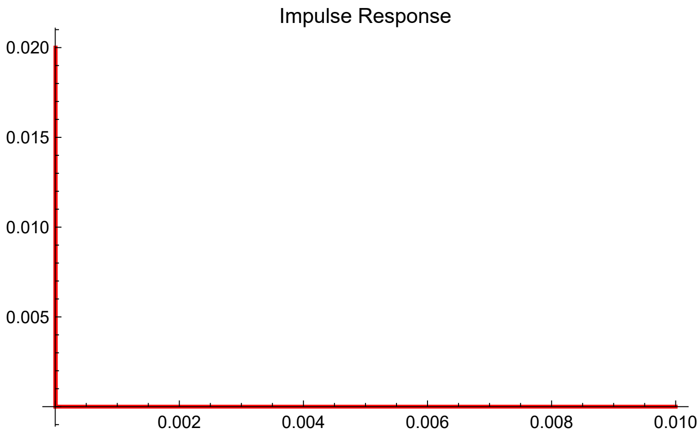
Out[443]=

```
{InterpolatingFunction[ Domain: {{0., 100.}} Output: scalar ][t]}
```

In[446]:=

```
Plot[h[t], {t, 0, 1 / 100}, PlotStyle → Red,
 PlotRange → All, PlotLabel → "Impulse Response"]
```

Out[446]=



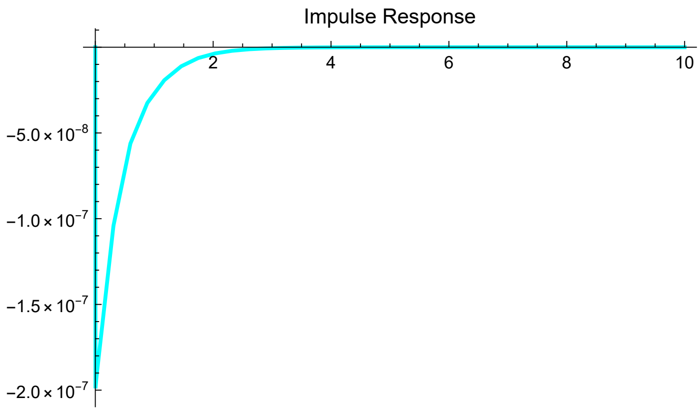

In[447]:=

```
(*Positive current when time is very small as the voltage hasnt died yet*)
```

In[448]:=

```
Plot[h[t], {t, 0, 10}, PlotRange → All, PlotLabel → "Impulse Response", PlotStyle → Cyan]
```

Out[448]=



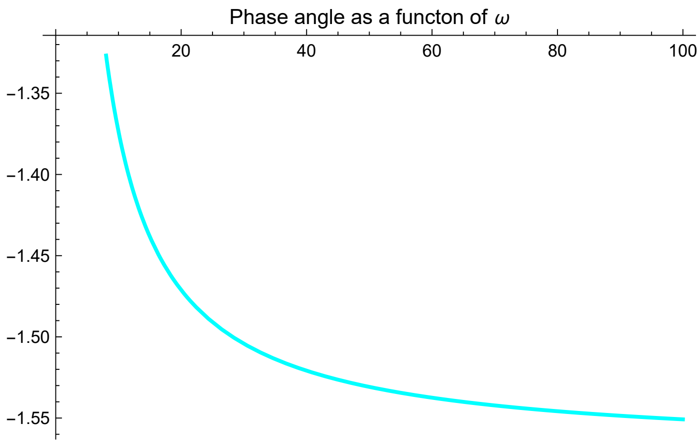

```
(*Negative current once the voltage has died and the capacitor is discharging*)
```

# Analysis In Frequency Domain:

In[449]:=

```
H[ω_] = 1 / (1 + I ω R Cap); (*Declaring the transfer function*)
PhaseAngle[ω_] = Arg[H[ω]];
Plot[PhaseAngle[ω], {ω, 0, 100}, PlotStyle → Cyan,
 PlotLabel → "Phase angle as a functon of ω"]
```

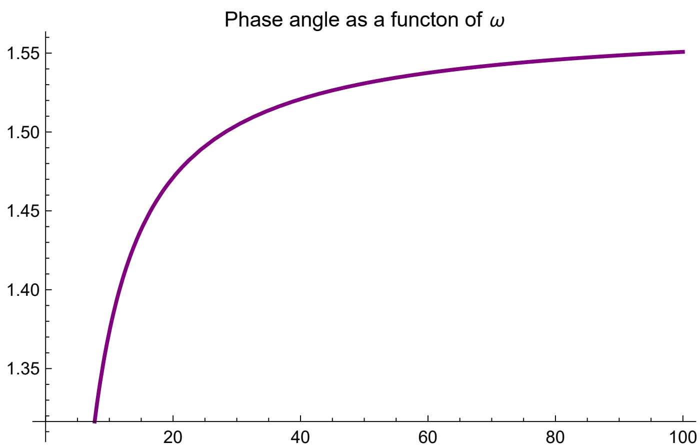
Out[451]=




In[452]:=

```
(*Absolute Phase Angle*)
Plot[Abs[PhaseAngle[ω]], {ω, 0, 100},
 PlotStyle → Purple, PlotLabel → "Phase angle as a functon of ω"]
```

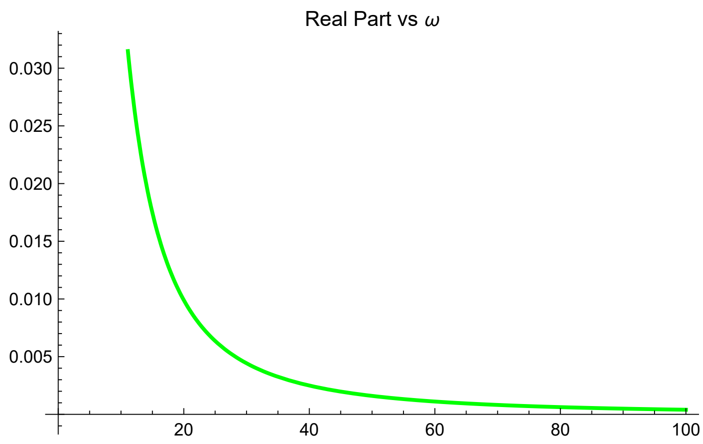
Out[452]=



In[453]:=

```
RealPart[ω_] = Re[H[ω]];
Plot[RealPart[ω], {ω, 0, 100}, PlotStyle → Green, PlotLabel → "Real Part vs ω"]
```

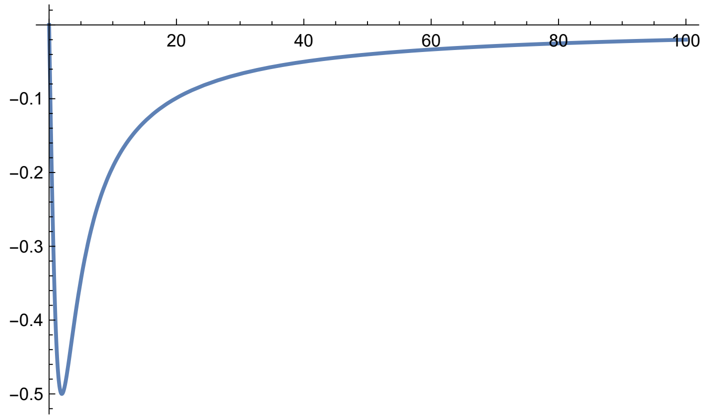
Out[454]=




In[455]:=

```
ImagPart[ω_] = Im[H[ω]];
Plot[ImagPart[ω], {ω, 0, 100}, PlotRange → Full]
```

Out[456]=




# Part-B

## Part 1):

In[457]:=

```
Res = 0.2;
L = 2;   (*Choosing values for the LCR circuit*)
Cap2 = 2;
PulseSig2[t_] = UnitBox[100 t];
eqn = L q4''[t] + Res q4'[t] + q4[t] / Cap2 == PulseSig2[t]; (*Writing Kirchoffs Law*)
```

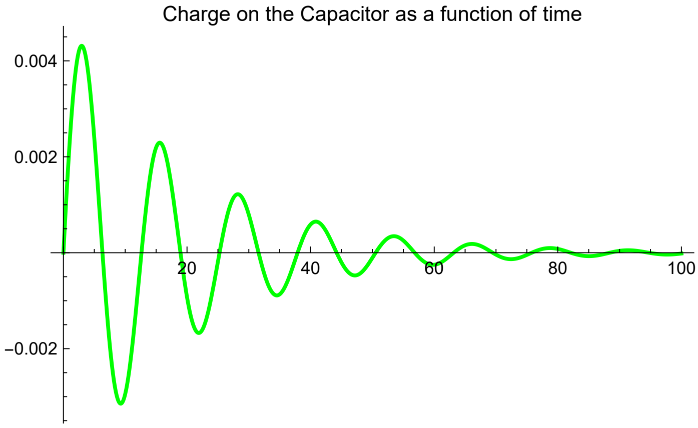
In[462]:=

```
sol4 = NDSolve[{eqn, q4[0] == 0, q4'[0] == 0}, q4, {t, 0, 100}];
```

In[463]:=

```
Plot[Evaluate[q4[t] /. sol4], {t, 0, 100}, PlotRange → All, PlotStyle → Green,
 PlotLabel → "Charge on the Capacitor as a function of time"]
```

Out[463]=




In[465]:=

```
Q4[t_] = q4[t] /. sol4
```

Out[465]=

{InterpolatingFunction[ Domain: {{0., 100.}} Output: scalar ][t]}

In[466]:=

```
Current4[t_] = Q4'[t]
```

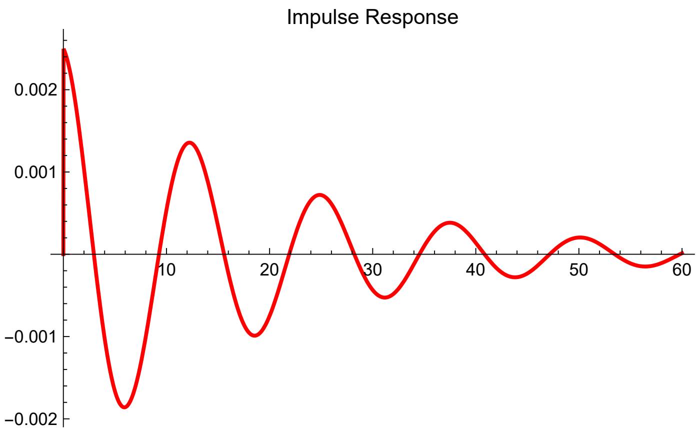
Out[466]=

{InterpolatingFunction[ Domain: {{0., 100.}} Output: scalar ][t]}

In[467]:=

```
Plot[Current4[t], {t, 0, 60}, PlotStyle → Red,
 PlotRange → All, PlotLabel → "Impulse Response"]
```

Out[467]=



# Part 2):

In[468]:=

```
sqrSig[t_] = SquareWave[t];
eqn = L q5''[t] + Res q5'[t] + q5[t] / Cap2 == sqrSig[t];
```

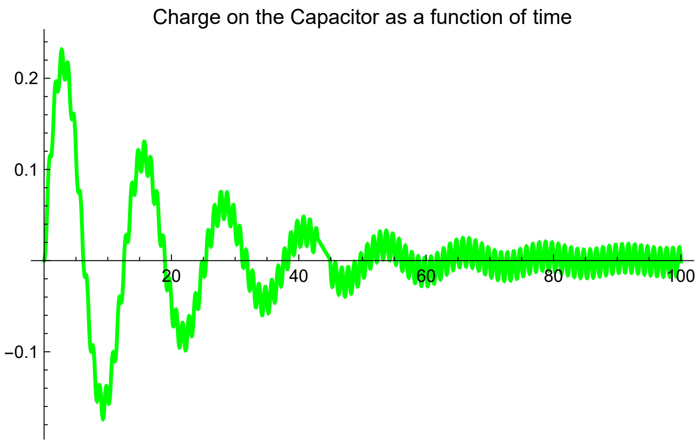
In[470]:=

```
sol5 = NDSolve[{eqn, q5[0] == 0, q5'[0] == 0}, q5, {t, 0, 100}];
```

In[471]:=

```
Plot[Evaluate[q5[t] /. sol5], {t, 0, 100}, PlotRange → All, PlotStyle → Green,
 PlotLabel → "Charge on the Capacitor as a function of time"]
```

Out[471]=




In[472]:=

```
Q5[t_] = q5[t] /. sol5
```

Out[472]=

{InterpolatingFunction[  Domain: {{0., 100.}} Output: scalar ][t]}

In[473]:=

```
Current5[t_] = Q5'[t]
```

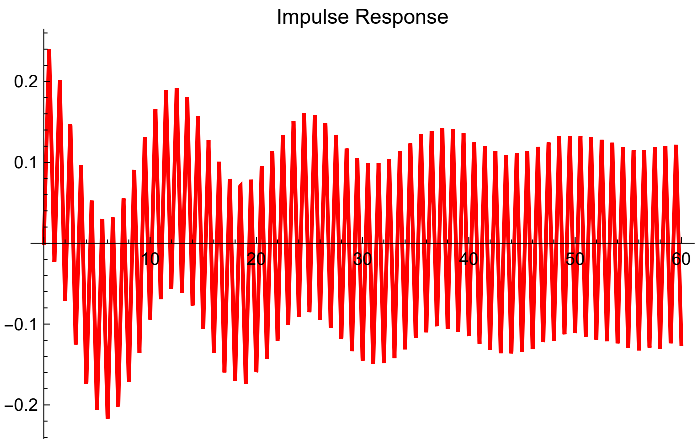
Out[473]=

{InterpolatingFunction[  Domain: {{0., 100.}} Output: scalar ][t]}

In[474]:=

```
Plot[Current5[t], {t, 0, 60}, PlotStyle → Red,
 PlotRange → All, PlotLabel → "Impulse Response"]
```

Out[474]=




# Part 3):

In[475]:=

```
V[t_] := Piecewise[{{1 t^2 - 5 t, 0 ≤ t ≤ 10}, {t, 10 < t ≤ 20}, {10 Log[t], 20 < t}}]
```

In[476]:=

```
eqn = L q6''[t] + Res q6'[t] + q6[t] / Cap2 == V[t];
```

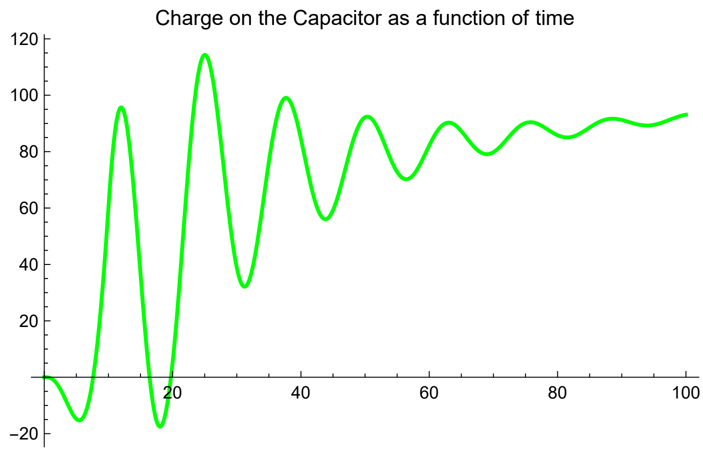
In[477]:=

```
sol6 = NDSolve[{eqn, q6[0] == 0, q6'[0] == 0}, q6, {t, 0, 100}];
```

In[478]:=

```
Plot[Evaluate[q6[t] /. sol6], {t, 0, 100}, PlotRange → All, PlotStyle → Green,
 PlotLabel → "Charge on the Capacitor as a function of time"]
```

Out[478]=




In[479]:=

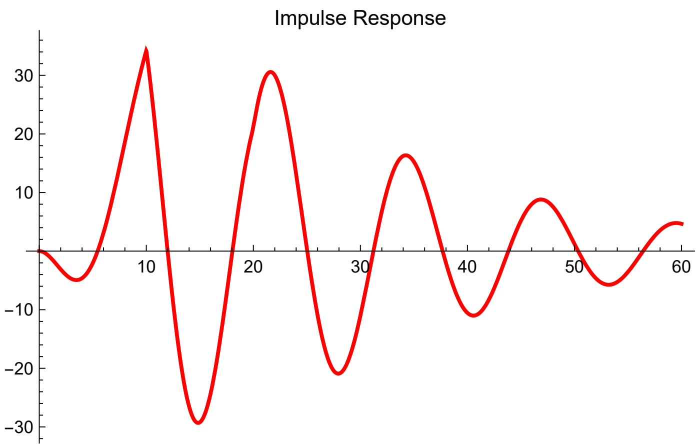
```
Q6[t_] = q6[t] /. sol6
```

Out[479]=

```
{InterpolatingFunction[ Domain: {{0., 100.}} Output: scalar ][t]}
```

In[480]:=
```
Current6[t_] = Q6'[t]
```
Out[480]=
```
{InterpolatingFunction[ Domain: {{0., 100.}} Output: scalar ][t]}
```

In[481]:=
```
Plot[Current6[t], {t, 0, 60}, PlotStyle → Red,
 PlotRange → All, PlotLabel → "Impulse Response"]
```
Out[481]=




## Results

Please refer to output no- 416, 421, 425, 427, 432, 435, 438, 441, 446, 448, 451, 452, 454, 456, 463, 467, 471, 474, 478, 481.

## Comments

This assignment required a bit of basic knowledge of circuit analysis. It was a fun but lengthy assignment to do.

## References

https://reference.wolfram.com/language/
https://chat.openai.com/