Assignment 03: On Visualizing Vector Fields

PH1050 Computational Physics

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Problem Statement

Part1: Plot The Magnetic Field Lines of the Magnetic Field caused by a Current Carrying wire.

Aim

Part1: Given the magnetic field caused by a segment of the wire as a function of 'u','ρ' and 'z' , (where u represents the height of the segment from origin, ρ represents the radial distance of the point at which we want the magnetic field and z is the z coordinate of that point) we need to plot the Field Lines due to :
1)Infinite Wire
2)Finite Wire

Introduction

Part1:
We get the resultant Magnetic field as a function of 'L', 'z' and 'ρ' and use functions such as StramPlot[] and VectorPlot[] to plot the field lines.

Code Organization

Part1:
1) Integrate the Magnetic field function with respect to 'u' and put limits -L to L. This way we get Magnetic field function at a point due to a wire which runs from -L to L.
2)Put limit L->∞. This way we get the magnetic field due to an infinite current carrying wire, at a point.
3)The expression we get in 2) will be in cylindrical polar coordinates and hence, we convert these to Cartesian coordinates by using the /. "Replace All" function.
4)Using the expression for magnetic field in Cartesian coordinates , we plot the magnetic field lines using functions like "StreamPlot[]" and "VectorPlot[]".
5)Then to compute the magnetic field due to a finite wire , we use the expression in 1) and using /. (Replace all) , we put L=1 and z=0.01 to get the function which represents the field due to a finite wire.
6)We plot the function obtained in 5)using the same functions as in 4).

Code for computation

Part 1: Plotting Magnetic Field Lines

In[3]:=
```
(*Obtaining the expression for Magnetic Induction*)
```
$$f[u\_,\rho\_,z\_]=\frac{\rho}{\left(\rho^2+(z-u)^2\right)^{\frac{3}{2}}};$$
```
g[u_]=Integrate[f[u,ρ,z],{u,-L,L},Assumptions→{ρ>0,z>0,L>0}]
```

Out[4]=
$$\frac{z\left(-\frac{1}{\sqrt{(L-z)^2+\rho^2}}+\frac{1}{\sqrt{(L+z)^2+\rho^2}}\right)+L\left(\frac{1}{\sqrt{(L-z)^2+\rho^2}}+\frac{1}{\sqrt{(L+z)^2+\rho^2}}\right)}{\rho}$$

In[5]:=
```
(*Computing the Limit of the g[u] as L→ ∞*)
ISL=Limit[g[u],L→∞] (*ISL stands for Infinite straight line*)
```

Out[5]=
$$\frac{2}{\rho}$$

In[6]:=
```
(*Replacing ρ by (x^2+y^2)^(1/2)*)
ISL=ISL/.ρ→(x^2+y^2)^(1/2)
```

Out[6]=
$$\frac{2}{\sqrt{x^2+y^2}}$$

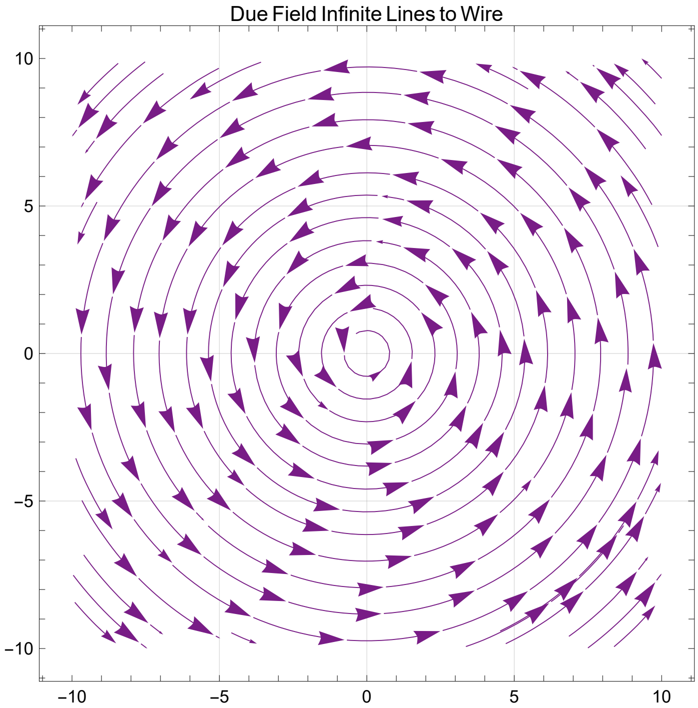
In[37]:=

(*This ISL = $\frac{\mu_0\ i}{4\ \pi}$*2/ρ along ephi cap
```
Converting this to its equivalent Cartesian Form:*)
ISLCart={ISL*(-y/ρ),ISL*(x/ρ)};
ISLCart=ISLCart/.ρ→(x^2+y^2)^(1/2)
(*Plotting the Vector Field*)
```

Out[38]=
$$\left\{-\frac{2\ y}{x^2+y^2},\frac{2\ x}{x^2+y^2}\right\}$$

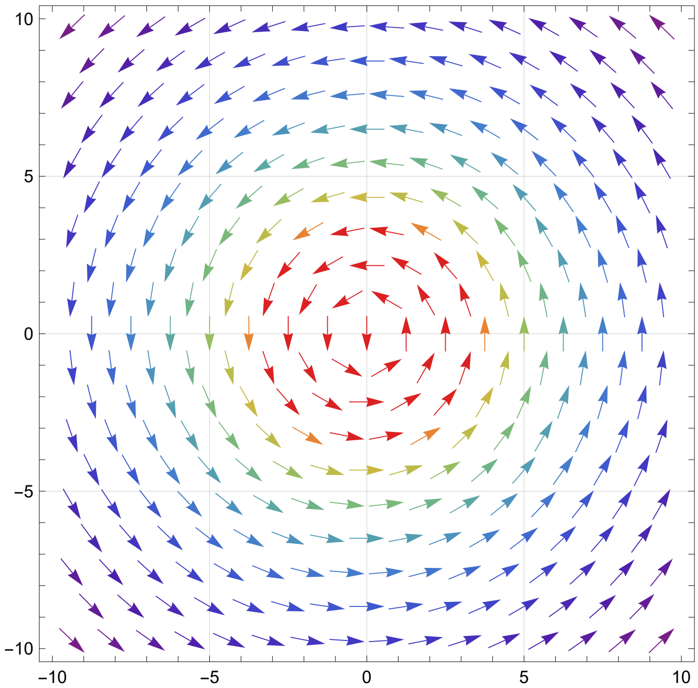
```
In[39]:= StreamPlot[ISLCart, {x, -10, 10}, {y, -10, 10}, StreamScale → Large,
          LabelStyle → {Black}, StreamColorFunction → "Rainbow", GridLines → Automatic,
          PlotLabel → Field Lines Due to Infinite Wire, LabelStyle → {FontSize → 16, Black}]
```

Out[39]=




```
In[40]:= VectorPlot[ISLCart, {x, -10, 10}, {y, -10, 10},
          VectorColorFunction → "Rainbow", GridLines → Automatic]
```

Out[40]=



In[10]:=
```
(*Getting the expression of magnetic induction by a short wire*)
g[u] (*This is the Magnetic Induction Due to a wire From -L to L, along ϕ cap*)
```

Out[10]=

$$\frac{z\left(-\frac{1}{\sqrt{(L-z)^2+\rho^2}}+\frac{1}{\sqrt{(L+z)^2+\rho^2}}\right)+L\left(\frac{1}{\sqrt{(L-z)^2+\rho^2}}+\frac{1}{\sqrt{(L+z)^2+\rho^2}}\right)}{\rho}$$

In[11]:=
```
MagFieldCart = {g[u] * (-y / ρ), g[u] * (x / ρ)}
```

Out[11]=

$$\left\{-\frac{y\left(z\left(-\frac{1}{\sqrt{(L-z)^2+\rho^2}}+\frac{1}{\sqrt{(L+z)^2+\rho^2}}\right)+L\left(\frac{1}{\sqrt{(L-z)^2+\rho^2}}+\frac{1}{\sqrt{(L+z)^2+\rho^2}}\right)\right)}{\rho^2},\right.$$
$$\left.\frac{x\left(z\left(-\frac{1}{\sqrt{(L-z)^2+\rho^2}}+\frac{1}{\sqrt{(L+z)^2+\rho^2}}\right)+L\left(\frac{1}{\sqrt{(L-z)^2+\rho^2}}+\frac{1}{\sqrt{(L+z)^2+\rho^2}}\right)\right)}{\rho^2}\right\}$$

In[12]:=
```
MagFieldCart = MagFieldCart /. ρ → (x^2 + y^2) ^ (1 / 2)
```

Out[12]=

$$\left\{-\frac{y\left(z\left(-\frac{1}{\sqrt{x^2+y^2+(L-z)^2}}+\frac{1}{\sqrt{x^2+y^2+(L+z)^2}}\right)+L\left(\frac{1}{\sqrt{x^2+y^2+(L-z)^2}}+\frac{1}{\sqrt{x^2+y^2+(L+z)^2}}\right)\right)}{x^2+y^2},\right.$$
$$\left.\frac{x\left(z\left(-\frac{1}{\sqrt{x^2+y^2+(L-z)^2}}+\frac{1}{\sqrt{x^2+y^2+(L+z)^2}}\right)+L\left(\frac{1}{\sqrt{x^2+y^2+(L-z)^2}}+\frac{1}{\sqrt{x^2+y^2+(L+z)^2}}\right)\right)}{x^2+y^2}\right\}$$

In[13]:=
```
MagFieldCart = MagFieldCart /. L → 1
MagFieldCart = MagFieldCart /. z → 0.01
```

Out[13]=

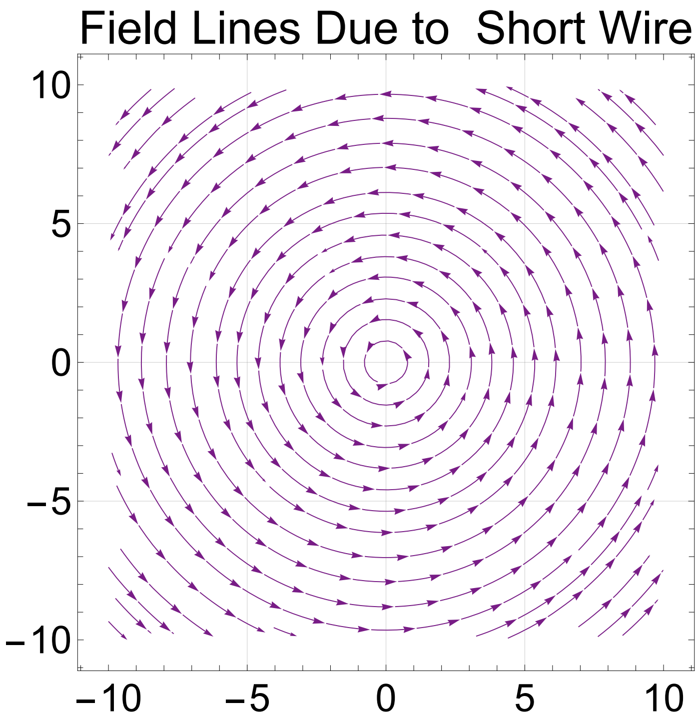
$$\left\{-\frac{y\left(\frac{1}{\sqrt{x^2+y^2+(1-z)^2}}+\frac{1}{\sqrt{x^2+y^2+(1+z)^2}}+z\left(-\frac{1}{\sqrt{x^2+y^2+(1-z)^2}}+\frac{1}{\sqrt{x^2+y^2+(1+z)^2}}\right)\right)}{x^2+y^2},\right.$$
$$\left.\frac{x\left(\frac{1}{\sqrt{x^2+y^2+(1-z)^2}}+\frac{1}{\sqrt{x^2+y^2+(1+z)^2}}+z\left(-\frac{1}{\sqrt{x^2+y^2+(1-z)^2}}+\frac{1}{\sqrt{x^2+y^2+(1+z)^2}}\right)\right)}{x^2+y^2}\right\}$$

Out[14]=

$$\left\{-\frac{y\left(\frac{1}{\sqrt{0.9801+x^2+y^2}}+\frac{1}{\sqrt{1.0201+x^2+y^2}}+0.01\left(-\frac{1}{\sqrt{0.9801+x^2+y^2}}+\frac{1}{\sqrt{1.0201+x^2+y^2}}\right)\right)}{x^2+y^2},\right.$$
$$\left.\frac{x\left(\frac{1}{\sqrt{0.9801+x^2+y^2}}+\frac{1}{\sqrt{1.0201+x^2+y^2}}+0.01\left(-\frac{1}{\sqrt{0.9801+x^2+y^2}}+\frac{1}{\sqrt{1.0201+x^2+y^2}}\right)\right)}{x^2+y^2}\right\}$$

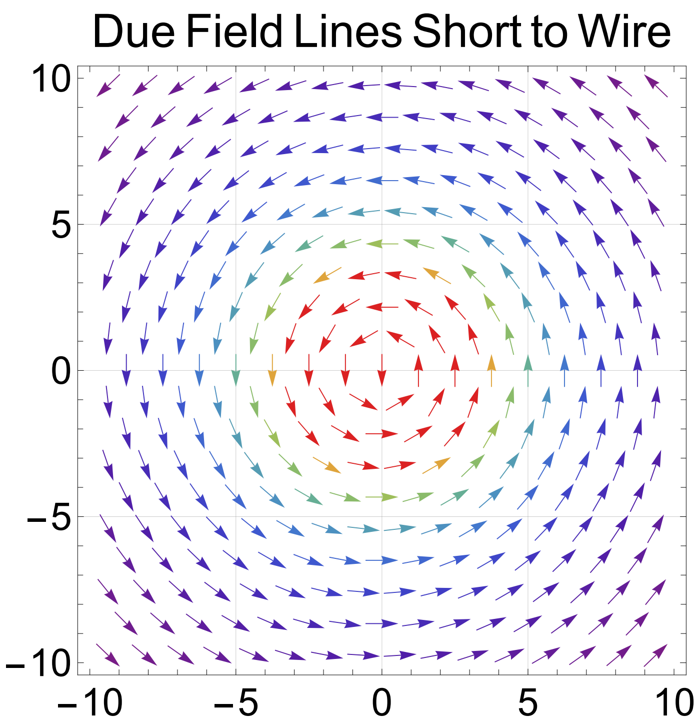
In[15]:=
```
StreamPlot[MagFieldCart, {x, -10, 10}, {y, -10, 10},
 PlotLabel → "Field Lines Due to  Short Wire", LabelStyle → {FontSize → 20, Black},
 StreamColorFunction → "Rainbow", GridLines → Automatic]
```

Out[15]=



In[16]:=
```
VectorPlot[MagFieldCart, {x, -10, 10}, {y, -10, 10},
 PlotLabel → Field Lines Due to Short Wire, LabelStyle → {FontSize → 20, Black},
 VectorColorFunction → "Rainbow", GridLines → Automatic]
```

Out[16]=



In[17]:=
```
(*To Confirm if the above reduces to the original result on putting L→∞*)

MagFieldCart2 = {g[u] * (-y / ρ), g[u] * (x / ρ)}
```

Out[17]=

$$\left\{-\frac{y\left(z\left(-\frac{1}{\sqrt{(L-z)^2+\rho^2}}+\frac{1}{\sqrt{(L+z)^2+\rho^2}}\right)+L\left(\frac{1}{\sqrt{(L-z)^2+\rho^2}}+\frac{1}{\sqrt{(L+z)^2+\rho^2}}\right)\right)}{\rho^2},\ \frac{x\left(z\left(-\frac{1}{\sqrt{(L-z)^2+\rho^2}}+\frac{1}{\sqrt{(L+z)^2+\rho^2}}\right)+L\left(\frac{1}{\sqrt{(L-z)^2+\rho^2}}+\frac{1}{\sqrt{(L+z)^2+\rho^2}}\right)\right)}{\rho^2}\right\}$$

In[18]:=
```
MagFieldCart2 = MagFieldCart2 /. ρ → (x^2 + y^2)^(1 / 2)
(*Here I just did the same things as in MagFieldCart,
The only thing different here is that instead of L=1 I have used the Limit L→∞*)
Limit[MagFieldCart2, L → ∞]
```

Out[18]=

$$\left\{-\frac{y\left(z\left(-\frac{1}{\sqrt{x^2+y^2+(L-z)^2}}+\frac{1}{\sqrt{x^2+y^2+(L+z)^2}}\right)+L\left(\frac{1}{\sqrt{x^2+y^2+(L-z)^2}}+\frac{1}{\sqrt{x^2+y^2+(L+z)^2}}\right)\right)}{x^2+y^2},\ \frac{x\left(z\left(-\frac{1}{\sqrt{x^2+y^2+(L-z)^2}}+\frac{1}{\sqrt{x^2+y^2+(L+z)^2}}\right)+L\left(\frac{1}{\sqrt{x^2+y^2+(L-z)^2}}+\frac{1}{\sqrt{x^2+y^2+(L+z)^2}}\right)\right)}{x^2+y^2}\right\}$$

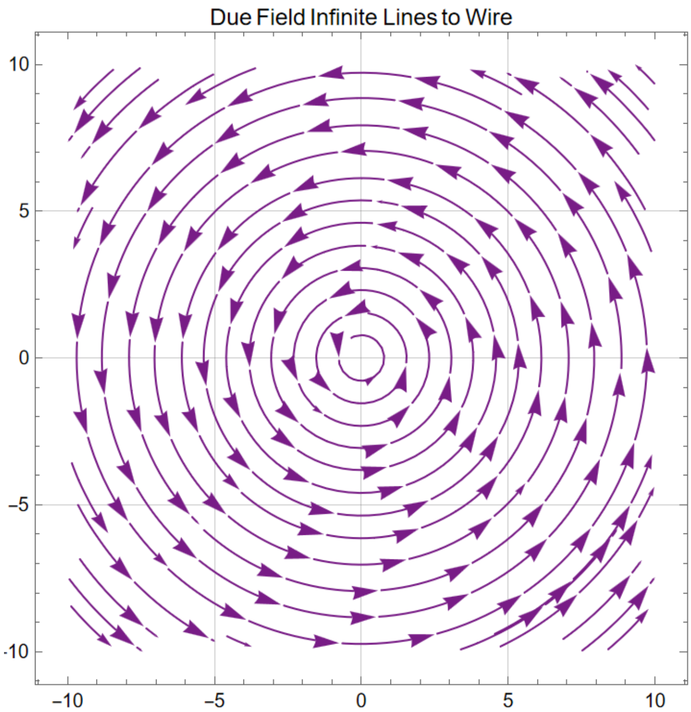
Out[19]=

$$\left\{-\frac{2y}{x^2+y^2},\ \frac{2x}{x^2+y^2}\right\}$$

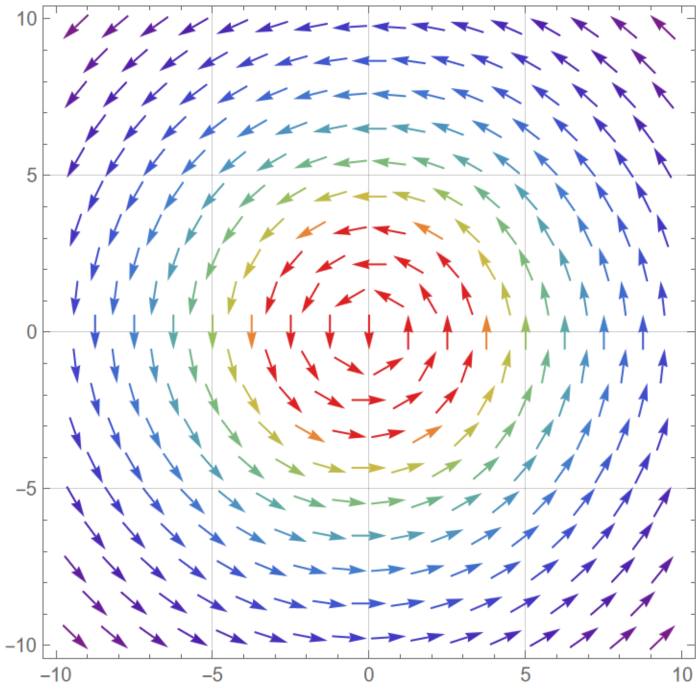
```
(*We got the expected result*)
```

Results

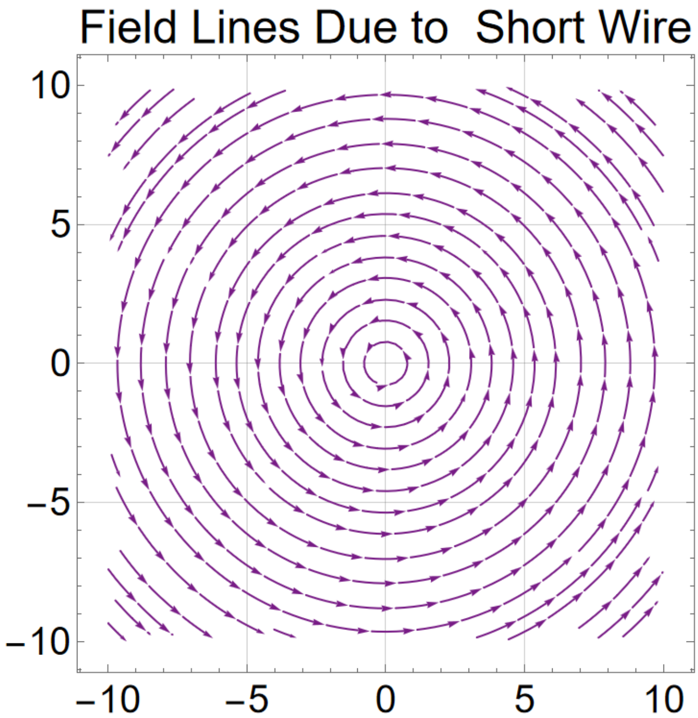
1)Field lines due to infinite current carrying
wire:



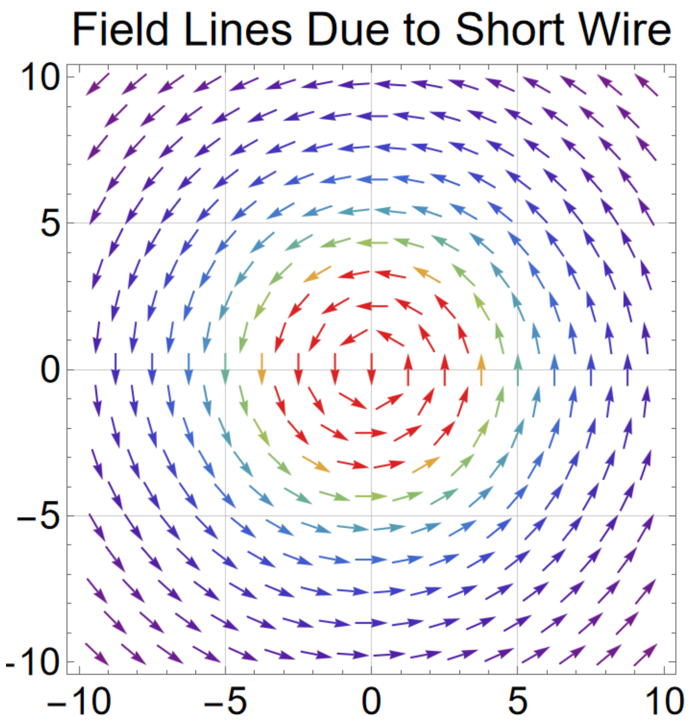
2)Vector plot of the field lines:



3)Field Lines due to a short Wire



4)Vector Plot of the Field Lines:



Discussions

Part 1:
This assignment shows how one can plot field lines due to various current carrying configurations by using the functions Integrate[] ,/. Replace All and Plotting functions like StreamPlot[] and VectorePlot[].

Comments

References

1. Wolfram Documentation.