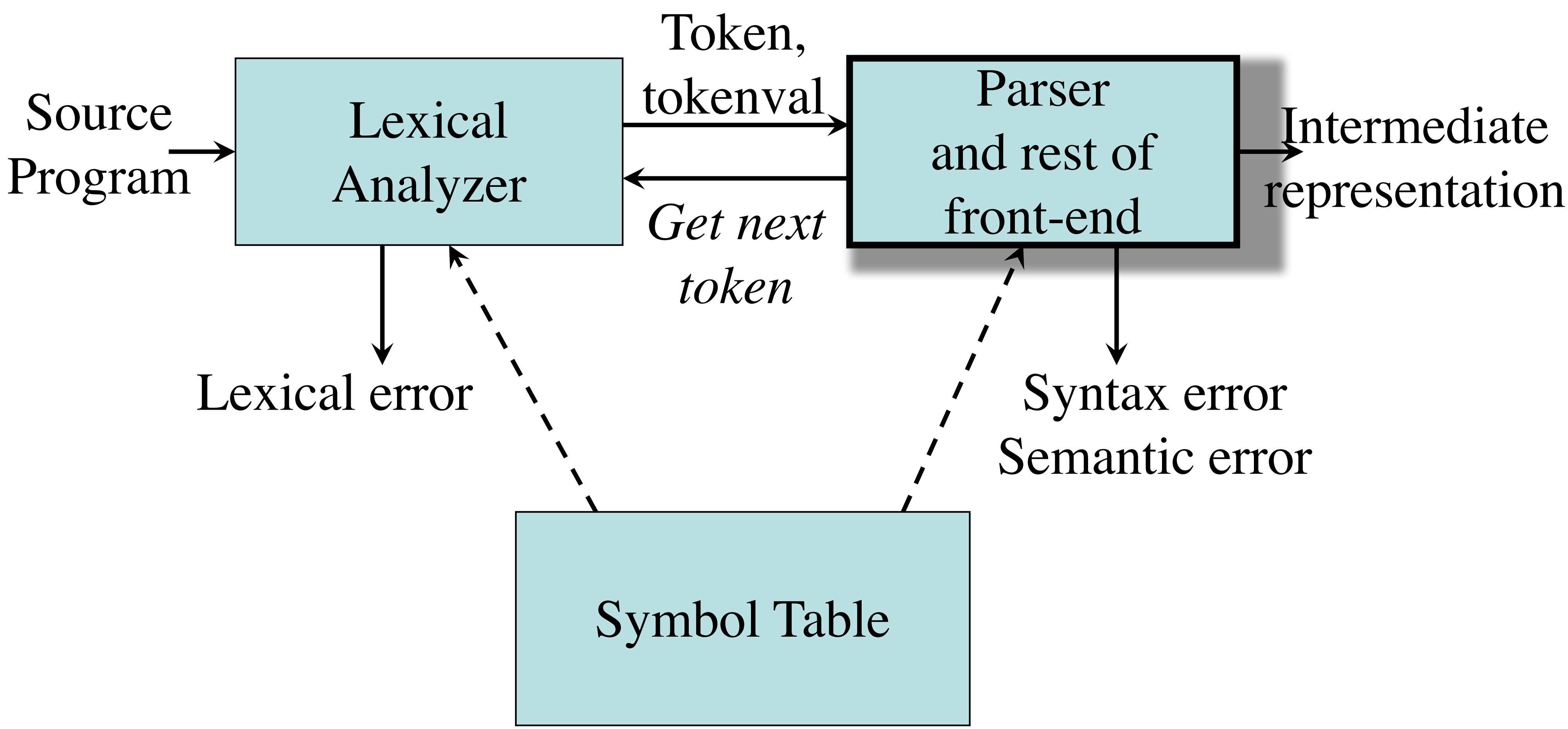
Syntax Analysis
Part I

Chapter 4

Position of a Parser in the Compiler Model



The Parser

- A parser implements a C-F grammar as a recognizer of strings
- The role of the parser in a compiler is twofold:
  1. To check syntax (= string recognizer)
     - And to report syntax errors accurately
  2. To invoke semantic actions
     - For static semantics checking, e.g. type checking of expressions, functions, etc.
     - For syntax-directed translation of the source code to an intermediate representation

Syntax-Directed Translation

- One of the major roles of the parser is to produce an *intermediate representation* (IR) of the source program using *syntax-directed translation* methods
- Possible IR output:
  - Abstract syntax trees (ASTs)
  - Control-flow graphs (CFGs) with triples, three-address code, or register transfer list notation
  - WHIRL (SGI Pro64 compiler) has 5 IR levels!

Error Handling

- A good compiler should assist in identifying and locating errors
  - *Lexical errors*: important, compiler can easily recover and continue
  - *Syntax errors*: most important for compiler, can almost always recover
  - *Static semantic errors*: important, can sometimes recover
  - *Dynamic semantic errors*: hard or impossible to detect at compile time, runtime checks are required
  - *Logical errors*: hard or impossible to detect

Viable-Prefix Property

- The *viable-prefix property* of parsers allows early detection of syntax errors
  - Goal: detection of an error *as soon as possible* without further consuming unnecessary input
  - How: detect an error as soon as the prefix of the input does not match a prefix of any string in the language

Error is detected here

Prefix

```
...
for (;)
...
```

Error is detected here

Prefix

```
...
DO 10 I = 1;0
...
```

Error Recovery Strategies

- *Panic mode*
  - Discard input until a token in a set of designated synchronizing tokens is found
- *Phrase-level recovery*
  - Perform local correction on the input to repair the error
- *Error productions*
  - Augment grammar with productions for erroneous constructs
- *Global correction*
  - Choose a minimal sequence of changes to obtain a global least-cost correction

Grammars (Recap)

- Context-free grammar is a 4-tuple $G = (N, T, P, S)$ where
  - T is a finite set of tokens (*terminal* symbols)
  - N is a finite set of *nonterminals*
  - P is a finite set of *productions* of the form
    $\alpha \to \beta$
    where $\alpha \in (N \cup T)^* \, N \, (N \cup T)^*$ and $\beta \in (N \cup T)^*$
  - $S \in N$ is a designated *start symbol*

Notational Conventions Used

- Terminals
  $a,b,c,\ldots \in T$
  specific terminals: **0**, **1**, **id**, **+**
- Nonterminals
  $A,B,C,\ldots \in N$
  specific nonterminals: *expr*, *term*, *stmt*
- Grammar symbols
  $X,Y,Z \in (N \cup T)$
- Strings of terminals
  $u,v,w,x,y,z \in T^*$
- Strings of grammar symbols
  $\alpha,\beta,\gamma \in (N \cup T)^*$

Derivations (Recap)

- The *one-step derivation* is defined by
  $$\alpha\, A\, \beta \Rightarrow \alpha\, \gamma\, \beta$$
  where $A \rightarrow \gamma$ is a production in the grammar
- In addition, we define
  - $\Rightarrow$ is *leftmost* $\Rightarrow_{lm}$ if α does not contain a nonterminal
  - $\Rightarrow$ is *rightmost* $\Rightarrow_{rm}$ if β does not contain a nonterminal
  - Transitive closure $\Rightarrow^*$ (zero or more steps)
  - Positive closure $\Rightarrow^+$ (one or more steps)
- The *language generated by G* is defined by
  $$L(G) = \{w \in T^* \mid S \Rightarrow^+ w\}$$

Derivation (Example)

Grammar G = ({E}, {**+**,*****,**(**,**)**,**-**,**id**}, P, E) with productions P =

$$E \rightarrow E \mathbf{+} E$$
$$E \rightarrow E \mathbf{*} E$$
$$E \rightarrow \mathbf{(}\ E\ \mathbf{)}$$
$$E \rightarrow \mathbf{-}\ E$$
$$E \rightarrow \mathbf{id}$$

Example derivations:

$$E \Rightarrow \mathbf{-}\ E \Rightarrow \mathbf{-\ id}$$

$$E \Rightarrow_{rm} E \mathbf{+} E \Rightarrow_{rm} E \mathbf{+\ id} \Rightarrow_{rm} \mathbf{id + id}$$

$$E \Rightarrow^{*} E$$

$$E \Rightarrow^{*} \mathbf{id + id}$$

$$E \Rightarrow^{+} \mathbf{id * id + id}$$

Chomsky Hierarchy: Language Classification

- A grammar G is said to be
  - *Regular* if it is *right linear* where each production is of the form
    $A \to w\, B$ or $A \to w$
    or *left linear* where each production is of the form
    $A \to B\, w$ or $A \to w$
  - *Context free* if each production is of the form
    $A \to \alpha$
    where $A \in N$ and $\alpha \in (N \cup T)^*$
  - *Context sensitive* if each production is of the form
    $\alpha\, A\, \beta \to \alpha\, \gamma\, \beta$
    where $A \in N$, $\alpha, \gamma, \beta \in (N \cup T)^*$, $|\gamma| > 0$
  - *Unrestricted*

Chomsky Hierarchy

$\mathcal{L}(\mathit{regular}) \subset \mathcal{L}(\mathit{context\ free}) \subset \mathcal{L}(\mathit{context\ sensitive}) \subset \mathcal{L}(\mathit{unrestricted})$

Where $\mathcal{L}(T) = \{ L(G) \mid G \text{ is of type } T \}$
That is: the set of all languages
generated by grammars G of type T

Examples:

Every *finite language* is regular! (construct a FSA for strings in $L(G)$)

$L_1 = \{ \mathbf{a}^n\mathbf{b}^n \mid n \geq 1 \}$ is context free

$L_2 = \{ \mathbf{a}^n\mathbf{b}^n\mathbf{c}^n \mid n \geq 1 \}$ is context sensitive

Parsing

- *Universal* (any C-F grammar)
  - Cocke-Younger-Kasimi
  - Earley
- *Top-down* (C-F grammar with restrictions)
  - Recursive descent (predictive parsing)
  - LL (Left-to-right, Leftmost derivation) methods
- *Bottom-up* (C-F grammar with restrictions)
  - Operator precedence parsing
  - LR (Left-to-right, Rightmost derivation) methods
    - SLR, canonical LR, LALR

Top-Down Parsing

- LL methods (Left-to-right, Leftmost derivation) and recursive-descent parsing

Grammar:

$E \to T \textbf{ + } T$

$T \to \textbf{(}\ E\ \textbf{)}$

$T \to \textbf{-}\ E$

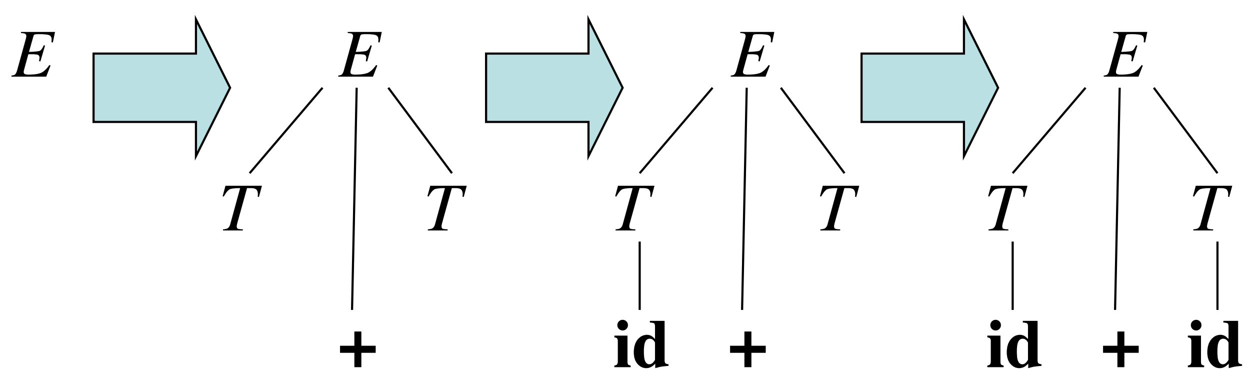
$T \to \textbf{id}$

Leftmost derivation:

$E \Rightarrow_{lm} T \textbf{ + } T$

$\Rightarrow_{lm} \textbf{id + } T$

$\Rightarrow_{lm} \textbf{id + id}$



Left Recursion (Recap)

- Productions of the form

$$
\begin{aligned}
A \rightarrow\ & A\ \alpha \\
\mid\ & \beta \\
\mid\ & \gamma
\end{aligned}
$$

  are left recursive

- When one of the productions in a grammar is left recursive then a predictive parser loops forever on certain inputs

A General Systematic Left Recursion Elimination Method

Input: Grammar G with no cycles or ε-productions

Arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$

for $i = 1, \ldots, n$ **do**

  for $j = 1, \ldots, i\text{-}1$ **do**

    replace each

      $A_i \to A_j\,\gamma$

    with

      $A_i \to \delta_1\,\gamma \mid \delta_2\,\gamma \mid \ldots \mid \delta_k\,\gamma$

    where

      $A_j \to \delta_1 \mid \delta_2 \mid \ldots \mid \delta_k$

  enddo

  eliminate the *immediate left recursion* in A_i

enddo

Immediate Left-Recursion Elimination

Rewrite every left-recursive production

$$
\begin{aligned}
A &\rightarrow A\ \alpha \\
&\mid \beta \\
&\mid \gamma \\
&\mid A\ \delta
\end{aligned}
$$

into a right-recursive production:

$$
\begin{aligned}
A &\rightarrow \beta\ A_R \\
&\mid \gamma\ A_R \\
A_R &\rightarrow \alpha\ A_R \\
&\mid \delta\ A_R \\
&\mid \varepsilon
\end{aligned}
$$

Example Left Recursion Elim.

$$\left.\begin{array}{l} A \to B\ C \mid \mathbf{a} \\ B \to C\ A \mid A\ \mathbf{b} \\ C \to A\ B \mid C\ C \mid \mathbf{a} \end{array}\right\}\ \text{Choose arrangement: } A, B, C$$

$i = 1$: nothing to do

$i = 2, j = 1$: $B \to C\ A \mid \underline{A}\ \mathbf{b}$

$\Rightarrow$ $B \to C\ A \mid \underline{B\ C}\ \mathbf{b} \mid \underline{\mathbf{a}}\ \mathbf{b}$

$\Rightarrow_{\text{(imm)}}$ $B \to C\ A\ B_R \mid \mathbf{a}\ \mathbf{b}\ B_R$

$B_R \to C\ \mathbf{b}\ B_R \mid \varepsilon$

$i = 3, j = 1$: $C \to \underline{A}\ B \mid C\ C \mid \mathbf{a}$

$\Rightarrow$ $C \to \underline{B\ C}\ B \mid \underline{\mathbf{a}}\ B \mid C\ C \mid \mathbf{a}$

$i = 3, j = 2$: $C \to \underline{B}\ C\ B \mid \mathbf{a}\ B \mid C\ C \mid \mathbf{a}$

$\Rightarrow$ $C \to \underline{C\ A\ B_R}\ C\ B \mid \underline{\mathbf{a}\ \mathbf{b}\ B_R}\ C\ B \mid \mathbf{a}\ B \mid C\ C \mid \mathbf{a}$

$\Rightarrow_{\text{(imm)}}$ $C \to \mathbf{a}\ \mathbf{b}\ B_R\ C\ B\ C_R \mid \mathbf{a}\ B\ C_R \mid \mathbf{a}\ C_R$

$C_R \to A\ B_R\ C\ B\ C_R \mid C\ C_R \mid \varepsilon$

Left Factoring

- When a nonterminal has two or more productions whose right-hand sides start with the same grammar symbols, the grammar is not LL(1) and cannot be used for predictive parsing
- Replace productions

  $$A \to \alpha\ \beta_1 \mid \alpha\ \beta_2 \mid \ldots \mid \alpha\ \beta_n \mid \gamma$$

  with

  $$A \to \alpha\ A_R \mid \gamma$$
  $$A_R \to \beta_1 \mid \beta_2 \mid \ldots \mid \beta_n$$

Predictive Parsing

- Eliminate left recursion from grammar
- Left factor the grammar
- Compute FIRST and FOLLOW
- Two variants:
  - Recursive (recursive-descent parsing)
  - Non-recursive (table-driven parsing)

FIRST (Revisited)

- FIRST(α) = { *the set of terminals that begin all strings derived from* α }

FIRST(a) = $\{a\}$ if $a \in T$
FIRST(ε) = $\{\varepsilon\}$
FIRST(A) = $\cup_{A\to\alpha}$ FIRST(α) for $A\to\alpha \in P$
FIRST($X_1X_2\ldots X_k$) =
if for all $j = 1, \ldots, i\text{-}1 : \varepsilon \in$ FIRST(X_j) **then**
add non-ε in FIRST(X_i) to FIRST($X_1X_2\ldots X_k$)
if for all $j = 1, \ldots, k : \varepsilon \in$ FIRST(X_j) **then**
add ε to FIRST($X_1X_2\ldots X_k$)

FOLLOW

- FOLLOW(A) = { *the set of terminals that can immediately follow nonterminal A* }

FOLLOW(A) =
for all ($B \to \alpha\ A\ \beta$) $\in P$ **do**
add FIRST(β)\{ε} to FOLLOW(A)
for all ($B \to \alpha\ A\ \beta$) $\in P$ and $\varepsilon \in$ FIRST(β) **do**
add FOLLOW(B) to FOLLOW(A)
for all ($B \to \alpha\ A$) $\in P$ **do**
add FOLLOW(B) to FOLLOW(A)
if A is the start symbol S **then**
add **$** to FOLLOW($A$)

LL(1) Grammar

- A grammar G is LL(1) if it is not left recursive and for each collection of productions
  $$A \to \alpha_1 \mid \alpha_2 \mid \ldots \mid \alpha_n$$
  for nonterminal A the following holds:

1. $\mathrm{FIRST}(\alpha_i) \cap \mathrm{FIRST}(\alpha_j) = \emptyset$ for all $i \neq j$
2. if $\alpha_i \Rightarrow^* \varepsilon$ then
   2.a. $\alpha_j \not\Rightarrow^* \varepsilon$ for all $i \neq j$
   2.b. $\mathrm{FIRST}(\alpha_j) \cap \mathrm{FOLLOW}(A) = \emptyset$ for all $i \neq j$

Non-LL(1) Examples

Grammar	*Not LL(1) because:*
$S \rightarrow S\ \mathbf{a} \mid \mathbf{a}$	Left recursive
$S \rightarrow \mathbf{a}\ S \mid \mathbf{a}$	FIRST($\mathbf{a}\ S$) $\cap$ FIRST($\mathbf{a}$) $\neq \varnothing$
$S \rightarrow \mathbf{a}\ R \mid \varepsilon$ $R \rightarrow S \mid \varepsilon$	For R: $S \Rightarrow^* \varepsilon$ and $\varepsilon \Rightarrow^* \varepsilon$
$S \rightarrow \mathbf{a}\ R\ \mathbf{a}$ $R \rightarrow S \mid \varepsilon$	For R: FIRST(S) $\cap$ FOLLOW(R) $\neq \varnothing$

Recursive-Descent Parsing (Recap)

- Grammar must be LL(1)
- Every nonterminal has one (recursive) procedure responsible for parsing the nonterminal's syntactic category of input tokens
- When a nonterminal has multiple productions, each production is implemented in a branch of a selection statement based on input look-ahead information

Using FIRST and FOLLOW in a Recursive-Descent Parser

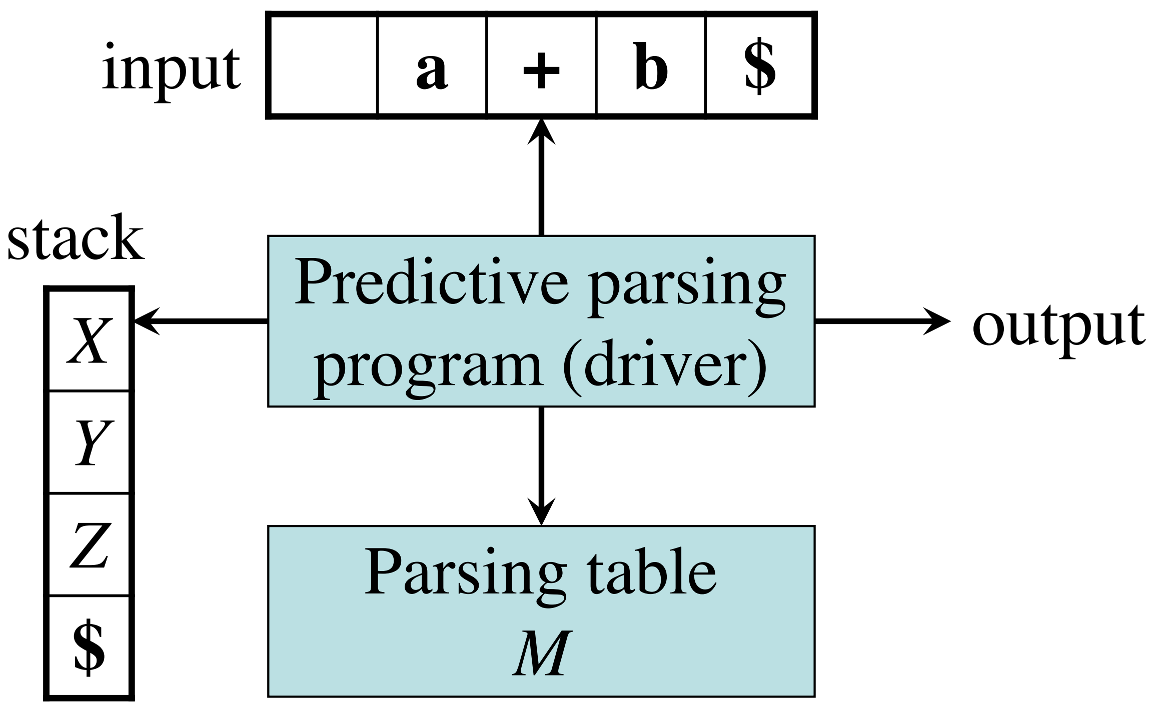
$$
\begin{aligned}
expr &\rightarrow term\ rest \\
rest &\rightarrow \mathbf{+}\ term\ rest \\
&\quad \mid \mathbf{-}\ term\ rest \\
&\quad \mid \varepsilon \\
term &\rightarrow \mathbf{id}
\end{aligned}
$$

procedure *rest*();
begin
  if *lookahead* in FIRST(**+** *term rest*) **then**
    match(**'+'**); *term*(); *rest*()
  else if *lookahead* in FIRST(**-** *term rest*) **then**
    match(**'-'**); *term*(); *rest*()
  else if *lookahead* in FOLLOW(*rest*) **then**
    return
  else error()
end;

where FIRST(**+** *term rest*) = { **+** }
FIRST(**-** *term rest*) = { **-** }
FOLLOW(*rest*) = { **$** }

Non-Recursive Predictive Parsing: Table-Driven Parsing

- Given an LL(1) grammar $G = (N, T, P, S)$ construct a table $M[A,a]$ for $A \in N$, $a \in T$ and use a *driver program* with a *stack*



Constructing an LL(1) Predictive Parsing Table

```
for each production A → α do
        for each a ∈ FIRST(α) do
                add A → α to M[A,a]
        enddo
        if ε ∈ FIRST(α) then
                for each b ∈ FOLLOW(A) do
                        add A → α to M[A,b]
                enddo
        endif
enddo
Mark each undefined entry in M error
```

Example Table

$E \rightarrow T\ E_R$
$E_R \rightarrow \mathbf{+}\ T\ E_R \mid \varepsilon$
$T \rightarrow F\ T_R$
$T_R \rightarrow *\ F\ T_R \mid \varepsilon$
$F \rightarrow (\ E\ ) \mid \mathbf{id}$

$A \rightarrow \alpha$	FIRST(α)	FOLLOW(A)
$E \rightarrow T\ E_R$	**(id**	**$)**
$E_R \rightarrow \mathbf{+}\ T\ E_R$	**+**	**$)**
$E_R \rightarrow \varepsilon$	ε	
$T \rightarrow F\ T_R$	**(id**	**+ $)**
$T_R \rightarrow *\ F\ T_R$	*	**+ $)**
$T_R \rightarrow \varepsilon$	ε	
$F \rightarrow (\ E\ )$	(	*** + $)**
$F \rightarrow \mathbf{id}$	**id**	*** + $)**

	id	**+**	*	(	)	**$**
E	$E \rightarrow T\ E_R$			$E \rightarrow T\ E_R$		
E_R		$E_R \rightarrow \mathbf{+}\ T\ E_R$			$E_R \rightarrow \varepsilon$	$E_R \rightarrow \varepsilon$
T	$T \rightarrow F\ T_R$			$T \rightarrow F\ T_R$		
T_R		$T_R \rightarrow \varepsilon$	$T_R \rightarrow *\ F\ T_R$		$T_R \rightarrow \varepsilon$	$T_R \rightarrow \varepsilon$
F	$F \rightarrow \mathbf{id}$			$F \rightarrow (\ E\ )$		

LL(1) Grammars are Unambiguous

Ambiguous grammar

$S \to \mathbf{i}\ E\ \mathbf{t}\ S\ S_R \mid \mathbf{a}$

$S_R \to \mathbf{e}\ S \mid \varepsilon$

$E \to \mathbf{b}$

$A \to \alpha$	FIRST(α)	FOLLOW(A)
$S \to \mathbf{i}\ E\ \mathbf{t}\ S\ S_R$	**i**	**e $**
$S \to \mathbf{a}$	**a**	
$S_R \to \mathbf{e}\ S$	**e**	**e $**
$S_R \to \varepsilon$	ε	
$E \to \mathbf{b}$	**b**	**t**

Error: duplicate table entry

	a	**b**	**e**	**i**	**t**	**$**
S	$S \to \mathbf{a}$			$S \to \mathbf{i}\ E\ \mathbf{t}\ S\ S_R$		
S_R			$S_R \to \varepsilon$ $S_R \to \mathbf{e}\ S$			$S_R \to \varepsilon$
E		$E \to \mathbf{b}$				

Predictive Parsing Program (Driver)

```
push($)
push(S)
a := lookahead
repeat
        X := pop()
        if X is a terminal or X = $ then
                match(X) // moves to next token and a := lookahead
        else if M[X,a] = X → Y1Y2…Yk then
                push(Yk, Yk-1, …, Y2, Y1) // such that Y1 is on top
                … invoke actions and/or produce IR output …
        else    error()
        endif
until X = $
```

Example Table-Driven Parsing

Stack	Input	Production applied
$\$\underline{E}$	$\underline{\mathbf{id}}\mathbf{+id*id\$}$	$E \to T\ E_R$
$\$E_R\underline{T}$	$\underline{\mathbf{id}}\mathbf{+id*id\$}$	$T \to F\ T_R$
$\$E_RT_R\underline{F}$	$\underline{\mathbf{id}}\mathbf{+id*id\$}$	$F \to \mathbf{id}$
$\$E_RT_R\underline{\mathbf{id}}$	$\underline{\mathbf{id}}\mathbf{+id*id\$}$	
$\$E_R\underline{T}_R$	$\underline{\mathbf{+}}\mathbf{id*id\$}$	$T_R \to \varepsilon$
$\$\underline{E}_R$	$\underline{\mathbf{+}}\mathbf{id*id\$}$	$E_R \to \mathbf{+}\ T\ E_R$
$\$E_RT\underline{\mathbf{+}}$	$\underline{\mathbf{+}}\mathbf{id*id\$}$	
$\$E_R\underline{T}$	$\underline{\mathbf{id}}\mathbf{*id\$}$	$T \to F\ T_R$
$\$E_RT_R\underline{F}$	$\underline{\mathbf{id}}\mathbf{*id\$}$	$F \to \mathbf{id}$
$\$E_RT_R\underline{\mathbf{id}}$	$\underline{\mathbf{id}}\mathbf{*id\$}$	
$\$E_R\underline{T}_R$	$\underline{\mathbf{*}}\mathbf{id\$}$	$T_R \to *\ F\ T_R$
$\$E_RT_RF\underline{*}$	$\underline{\mathbf{*}}\mathbf{id\$}$	
$\$E_RT_R\underline{F}$	$\underline{\mathbf{id}}\mathbf{\$}$	$F \to \mathbf{id}$
$\$E_RT_R\underline{\mathbf{id}}$	$\underline{\mathbf{id}}\mathbf{\$}$	
$\$E_R\underline{T}_R$	$\underline{\mathbf{\$}}$	$T_R \to \varepsilon$
$\$\underline{E}_R$	$\underline{\mathbf{\$}}$	$E_R \to \varepsilon$
$\underline{\mathbf{\$}}$	$\underline{\mathbf{\$}}$	

Panic Mode Recovery

Add synchronizing actions to undefined entries based on FOLLOW

Pro:	Can be automated
Cons:	Error messages are needed

FOLLOW(E) = {) $ }
FOLLOW(E_R) = {) $ }
FOLLOW(T) = { **+**) $ }
FOLLOW(T_R) = { **+**) $ }
FOLLOW(F) = { **+** *) $ }

	id	**+**	*	(	)	$
E	$E \to T\ E_R$			$E \to T\ E_R$	***synch***	***synch***
E_R		$E_R \to \mathbf{+}\ T\ E_R$			$E_R \to \varepsilon$	$E_R \to \varepsilon$
T	$T \to F\ T_R$	***synch***		$T \to F\ T_R$	***synch***	***synch***
T_R		$T_R \to \varepsilon$	$T_R \to *\ F\ T_R$		$T_R \to \varepsilon$	$T_R \to \varepsilon$
F	$F \to \mathbf{id}$	***synch***	***synch***	$F \to (\ E\ )$	***synch***	***synch***

synch: the driver pops current nonterminal A and skips input till synch token or skips input until one of FIRST(A) is found

Phrase-Level Recovery

Change input stream by inserting missing tokens
For example: **id id** is changed into **id * id**

Pro:	Can be fully automated
Cons:	Recovery not always intuitive

Can then continue here

	id	**+**	*****	**(**	**)**	**$**
E	$E \rightarrow T\ E_R$			$E \rightarrow T\ E_R$	***synch***	***synch***
E_R		$E_R \rightarrow \mathbf{+}\ T\ E_R$			$E_R \rightarrow \varepsilon$	$E_R \rightarrow \varepsilon$
T	$T \rightarrow F\ T_R$	***synch***		$T \rightarrow F\ T_R$	***synch***	***synch***
T_R	***insert ****	$T_R \rightarrow \varepsilon$	$T_R \rightarrow *\ F\ T_R$		$T_R \rightarrow \varepsilon$	$T_R \rightarrow \varepsilon$
F	$F \rightarrow \mathbf{id}$	***synch***	***synch***	$F \rightarrow (\ E\ )$	***synch***	***synch***

insert *: driver inserts missing * and retries the production

Error Productions

$E \rightarrow T\, E_R$
$E_R \rightarrow \mathbf{+}\, T\, E_R \mid \varepsilon$
$T \rightarrow F\, T_R$
$T_R \rightarrow *\, F\, T_R \mid \varepsilon$
$F \rightarrow (\, E\,) \mid \mathbf{id}$

Add "*error production*":

$$T_R \rightarrow F\, T_R$$

to ignore missing *, e.g.: **id id**

Pro:	Powerful recovery method
Cons:	Manual addition of productions

	id	**+**	*****	**(**	**)**	**$**
E	$E \rightarrow T\, E_R$			$E \rightarrow T\, E_R$	***synch***	***synch***
E_R		$E_R \rightarrow \mathbf{+}\, T\, E_R$			$E_R \rightarrow \varepsilon$	$E_R \rightarrow \varepsilon$
T	$T \rightarrow F\, T_R$	***synch***		$T \rightarrow F\, T_R$	***synch***	***synch***
T_R	$T_R \rightarrow F\, T_R$	$T_R \rightarrow \varepsilon$	$T_R \rightarrow *\, F\, T_R$		$T_R \rightarrow \varepsilon$	$T_R \rightarrow \varepsilon$
F	$F \rightarrow \mathbf{id}$	***synch***	***synch***	$F \rightarrow (\, E\,)$	***synch***	***synch***