

## Assignment 3

let  $s$  be the source vertex &  $t$  be the sink vertex. let ~~the~~ graph  $G = (V, E)$  be given.

~~Primal~~ Primal

Objective: maximize  $\sum_{v=(s,v) \in E} x_{sv}$

Decision variable :  $x_{uv} \quad \forall (u,v) \in E$   
 $\& \quad u,v \in V$

Constraints : 1)  $x_{uv} \leq w_{uv}$  where  $w_{uv}$  is the weight of edge  $(u, v)$

$$2) \sum_u x_{uv} = \sum_w x_{vw}$$

inflow = outflow

3)  $x_{uv} \geq 0 \quad \forall (u,v) \in E$

For standard form, inequalities are replaced by equalities by introducing slack variables.



Dual

~~The~~ Decision Variable :  $x_{uv} \forall (u,v) \in E$   
 $z_v \forall v \in V$

Objective : minimize  $\sum_{(u,v) \in E} w_{uv} \cdot d_{uv}$

where  $w_{uv}$  is weight of edge  $(u,v)$

Constraints :  $x_{uv} - z_u + z_v \geq 0$   
 $\forall (u,v) \in E, u \neq s, v \neq t$

$d_{sv} + z_v \geq 1 \quad \forall (s,v) \in E$

$d_{ut} - z_u \geq 0 \quad \forall (u,t) \in E$

$d_{uv} \geq 0 \quad \forall (u,v) \in E$

$z_v \in \mathbb{R} \quad \forall v \in V$

The problem is converted to standard form by introducing slack variables to remove inequalities & writing

$$z_v = z_v^+ - z_v^- \quad \forall v \in V$$

$$\text{s.t. } z_v^+, z_v^- \geq 0$$