Monte Carlo Simulation of Radioactivity

1 Theory of the Monte Carlo Decay Simulation

In this simulation we model the radioactive decay of an ensemble of N unstable particles (or nucleons). Radioactive decay is inherently a stochastic process: each particle has a fixed probability per unit time of decaying, but the exact moment of decay for each individual particle cannot be predicted. Instead, the decay follows a probabilistic law.

1.1 Decay Probability

For a small time interval Δt, the probability that a given particle decays is taken to be a constant value p. This corresponds to the discretized form of an exponential decay law. In the continuous case, the number of particles $N(t)$ remaining at time t is

$$N(t) = N_0 e^{-\lambda t},$$

where λ is the decay constant. In the discrete Monte Carlo model, we approximate the relation between p and λ by

$$p \approx 1 - e^{-\lambda \Delta t}.$$

1.2 Monte Carlo Procedure

At every time step:

1. A set of $N(t)$ random numbers, uniformly distributed in $[0, 1]$, is generated. Each random number corresponds to one particle.

2. For each particle, if the random number r_i satisfies

   $$r_i \leq p,$$

   then that particle is considered to have decayed during the interval Δt.

3. The number of undecayed particles N and the number of decayed particles D are updated accordingly.

1.3 Time Evolution

The process continues while $N(t) > 0$ and $t < t_{\max}$. At each step, the population is updated according to

$$N(t + \Delta t) = N(t) - \Delta N, \qquad D(t + \Delta t) = D(t) + \Delta N,$$

where ΔN is the number of particles that decayed during the time step.

This algorithm produces a stochastic realization of the decay curve. For large numbers of particles and small Δt, the averaged results approach the analytical exponential decay law.

1.4 Interpretation

Because each decay event is determined by random sampling, individual runs of the simulation will fluctuate around the ideal exponential decay curve. The Monte Carlo method therefore provides an intuitive and computationally simple way to visualize and study the probabilistic nature of radioactive decay.