Applied Stochastic Processes (MATH F424)
Stock Price Prediction Using Optimal Stopping Theory

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1 Introduction

The prediction of stock prices has been a fundamental challenge in financial markets, with numerous models developed to forecast returns and guide investment decisions. This project focuses on the application of Stochastic Processes, Martingale-based Optimal Stopping Theory, and Geometric Brownian Motion (GBM) to predict stock price returns.

Our code can be found at https://github.com/Pranav-Saxena/ASP-Project-2024

2 Theory

2.1 Martingales

A martingale models a fair game in stochastic processes. A process $\{M_t \mid t \in T\}$ is a martingale with respect to a filtration $\{\mathcal{F}_t\}$ and probability measure P if:

- $E[|M_t|] < \infty$ for all $t \in T$,
- M_t is $\mathcal{F}_t$-measurable for all $t \in T$, and
- $E[M_{t+1} \mid \mathcal{F}_t] = M_t$ for all $t \in T$.

2.2 Geometric Brownian Motion (GBM)

Geometric Brownian Motion (GBM) models stock prices using:

$$S_t = S_0 \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right) t + \sigma W_t\right),$$

where S_t is the stock price at time t, S_0 the initial stock price, μ the drift, σ the volatility, and W_t the Wiener process.

2.3 Optimal Stopping Theory

Let $\{X_t\}_{t\geq 0}$ be a martingale. If T is an optimal stopping time, then:

$$E[X_T] = E[X_0],$$

Process $\{X_t\}_{t\geq 0}$ follows Optimal Stopping Theory if either:

- $T < C$ for some constant C, or
- $P[T < \infty] = 1$ and $E[|X_T|] < \infty$.

The probability of reaching target price a in time t is given by:

$$P[T_a < t] = 2P\left[z > \frac{|a|}{\sqrt{t}}\right]$$

3 Implementation

3.1 Prediction Using Optimal Stopping Theory

We run simulations using **Geometric Brownian Motion (GBM)** on the historical stock data of the NSE stocks **BAJAJ Finserv** and **HAL**. The next price is computed using the following formula:

$$\texttt{next\_price} = \texttt{path}[-1] \cdot \exp\left((\mu - 0.5 \cdot \sigma^2) \cdot \Delta t + \sigma \cdot dW\right)$$

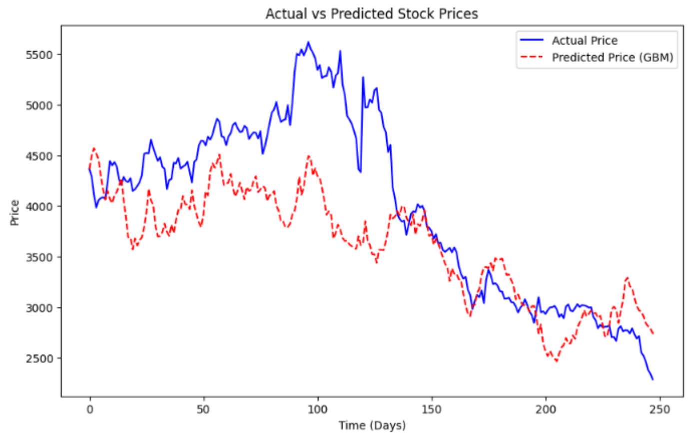

Figure 1: Stock Price Simulation Using Geometric Brownian Motion

The results we found seem to be accurate, with our simulated Geometric Brownian Motion (GBM) giving a good match to the actual historical data.

Currently, we use log returns to compute the values of μ and σ for each stock. The log returns are calculated as the natural logarithm of the ratio of consecutive prices:

$$\text{log\_returns} = \log\left(\frac{\text{price}_t}{\text{price}_{t-1}}\right)$$

The mean of these log returns gives the expected daily return, which is then annualized by multiplying by 246 (assuming 246 trading days per year) to obtain μ. Similarly, the standard deviation of the log returns represents the daily volatility, which is annualized by multiplying by $\sqrt{246}$ to obtain σ. These annualized values are then used in our Geometric Brownian Motion model for stock price simulations.

Backtesting Results:

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Initial Balance : 10000
Mean Final Balance: 9312.50
Mean Return on Investment (ROI): -6.87%
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The predicted value of the mean ROI appears to be accurate, as the recent trend of the HAL stock has been negative, which would ultimately result in a loss, leading to a negative ROI.

We also tested the estimation of the probability of reaching a target price within a specific time using the formula:

$$P[T_a < t] = 2P\left(z > \frac{|a|}{\sqrt{t}}\right)$$

and obtained the results:

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Current Price : 2287.0
Probability of reaching price 2500 in 36.00 days: 0.2133
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The probability appears to be accurate, as the current stock trend is negative. This results in a having a lower probability of 0.2133 to get a better return.

3.2 Estimation Using LSTM

We then extend our method to using LSTM for stock price prediction for comparison.
We can observe that the predicted values using LSTM are more accurate compared to simulations using Geometric Brownian Motion. Therefore, we can use our trained LSTM model to predict future stock values and make trading decisions accordingly.

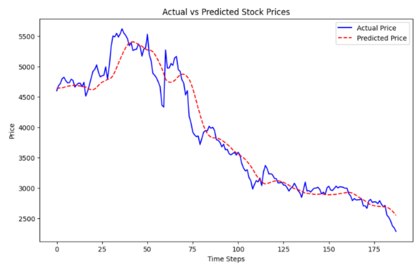

Figure 2: Actual vs Predicted Stock Prices

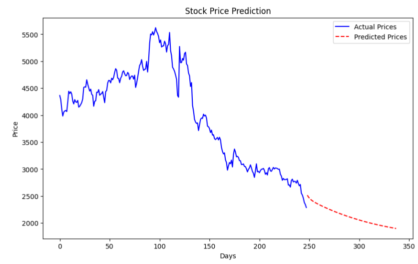

Figure 3: Prediction of Future Stock Prices

4. Conclusion

Optimal Stopping Theory (OST) can be effectively applied to stock price prediction to determine the best time to enter or exit the market, optimizing profits by deciding when to 'stop' based on predicted price movements.

For enhanced accuracy in predicting stock prices, Long Short-Term Memory (LSTM) models can be trained on historical stock data to capture long-term dependencies in price trends. Combining the predictions from LSTMs with OST can help in making informed trading decisions by identifying the optimal points to trade.

Currently, we calculate μ and σ using log returns of stock data. However, we could further refine these estimates by employing statistical techniques like Maximum Likelihood Estimation (MLE) or GARCH models. These methods could provide more accurate estimates of μ and σ, improving the application of OST for better trading accuracy.